In-medium dressed quark evolution in a light-front Hamiltonian approach

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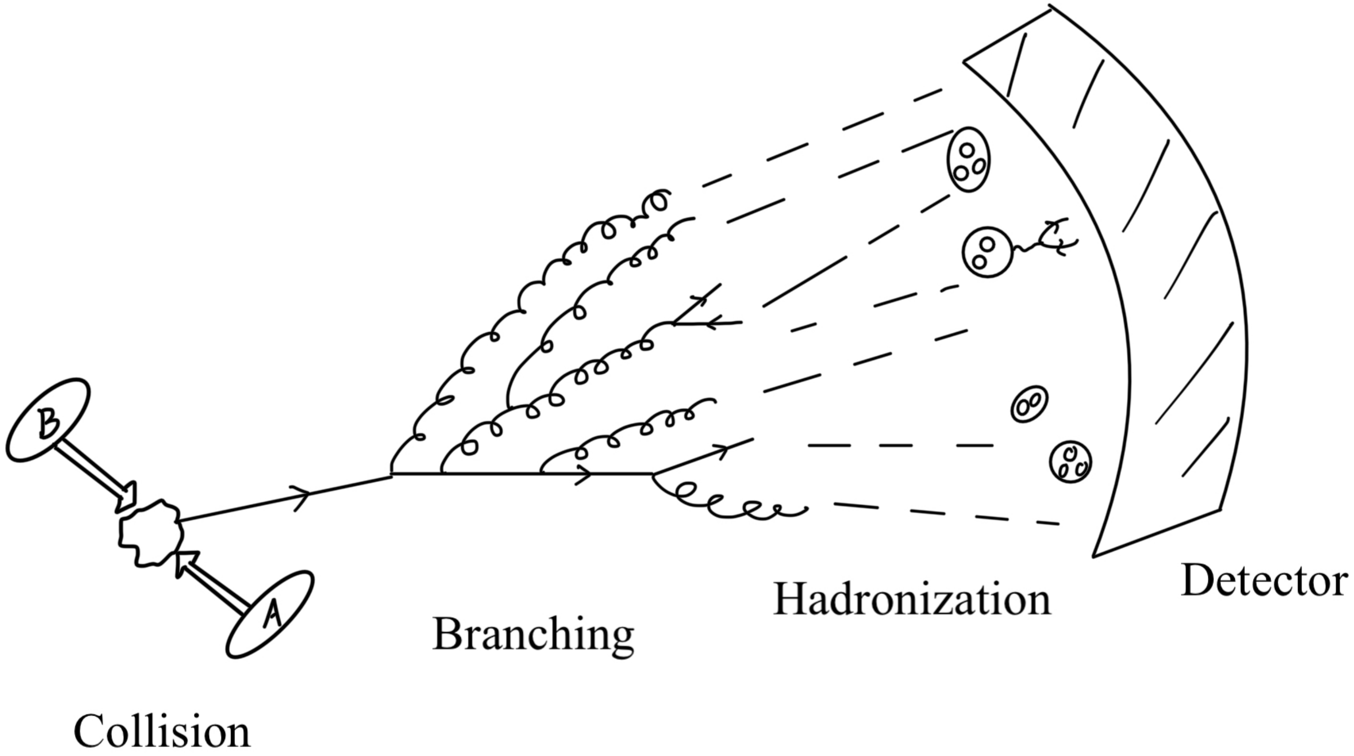
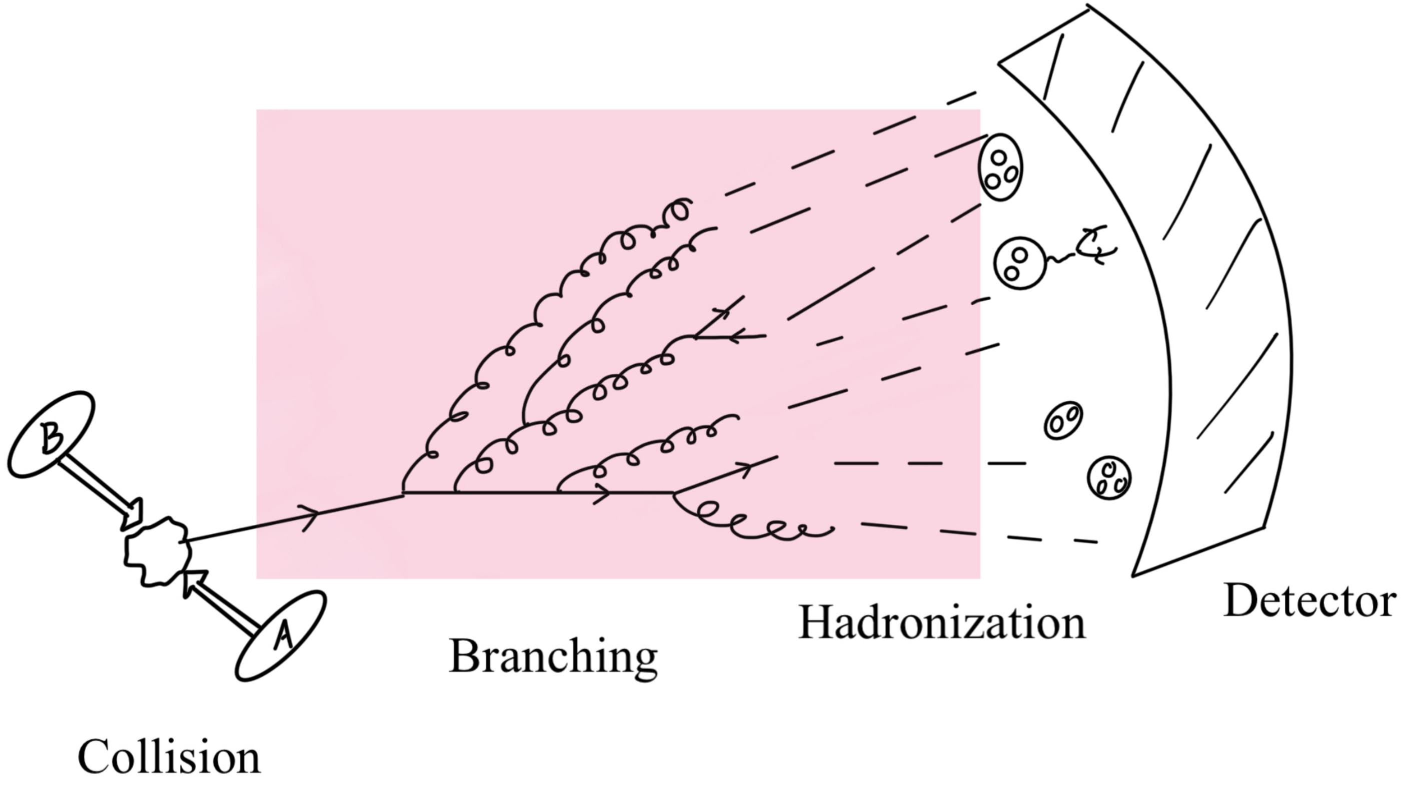
What is a jet?

In high-energy collisions, a jet is a collimated beam of particles produced by the splitting of a common ancestor (quark or gluon).



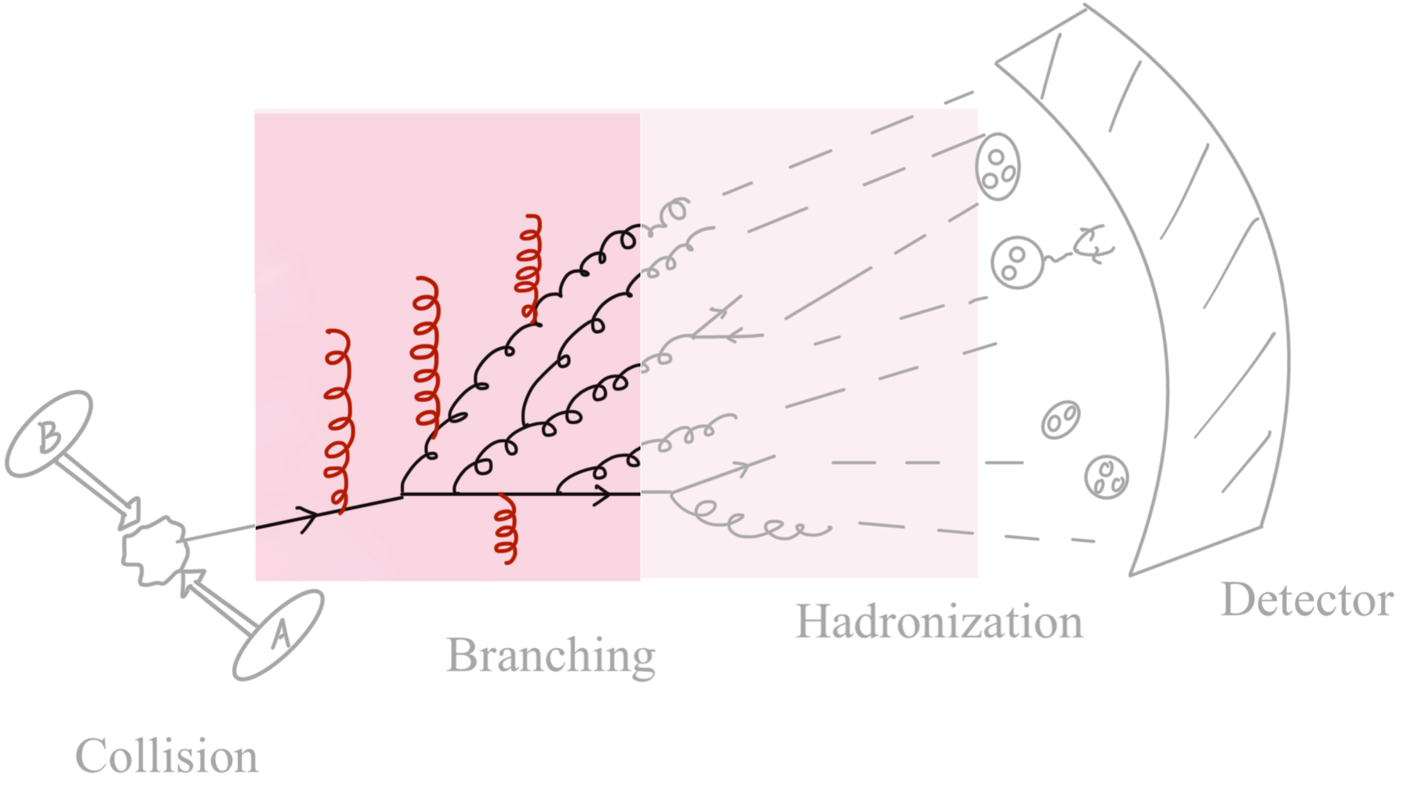
What is a jet?

A probe of matter, a tool to understand interaction.



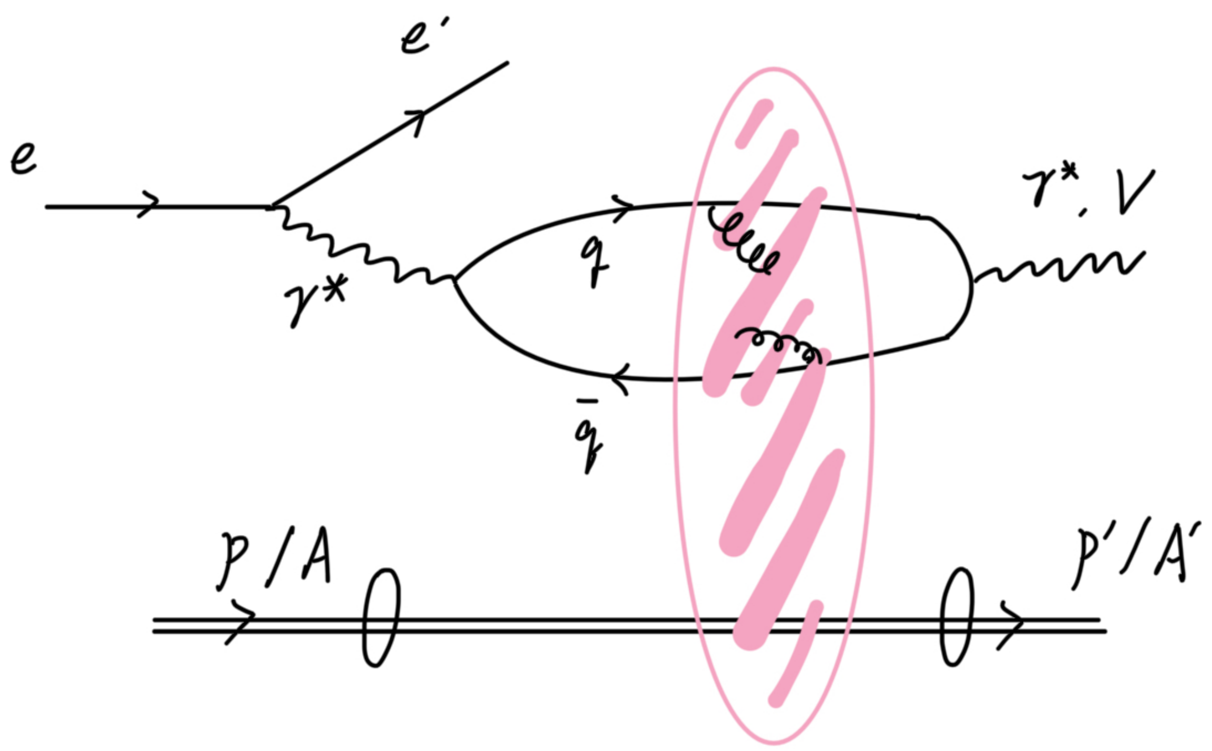
What is a jet?

An energetic QCD state that evolves and interacts.



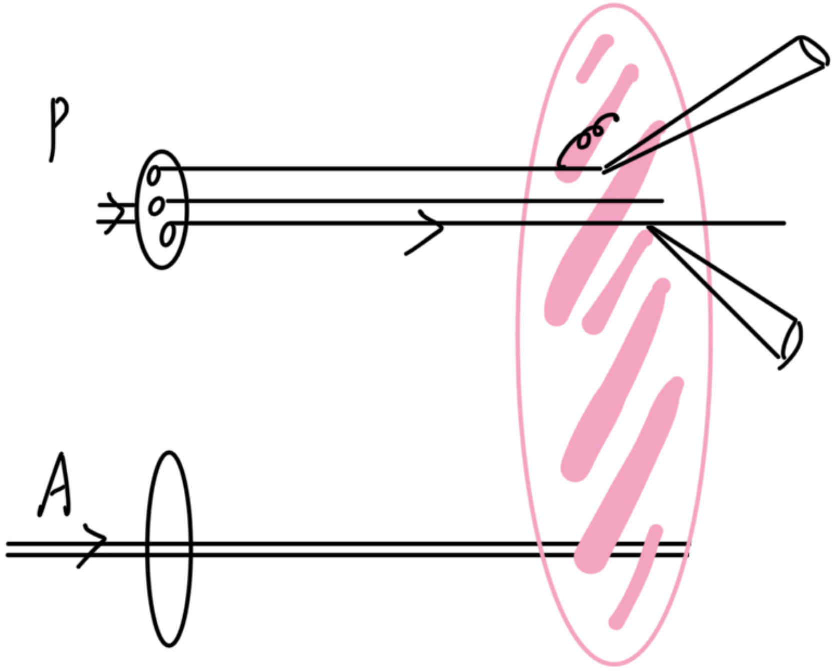
Quark jet scattering off a color field in:

Deep inelastic scattering e+p/A



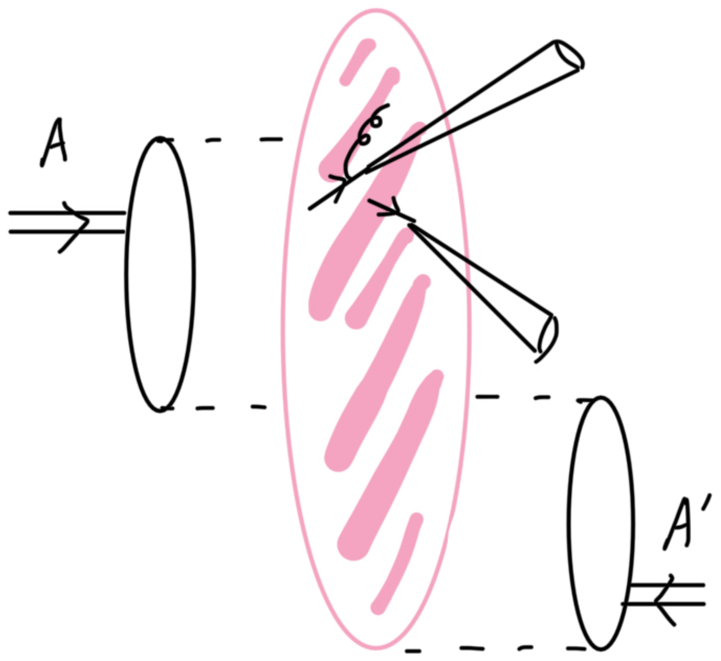
Quark jet scattering off a color field in:

Proton nucleus scattering p+A



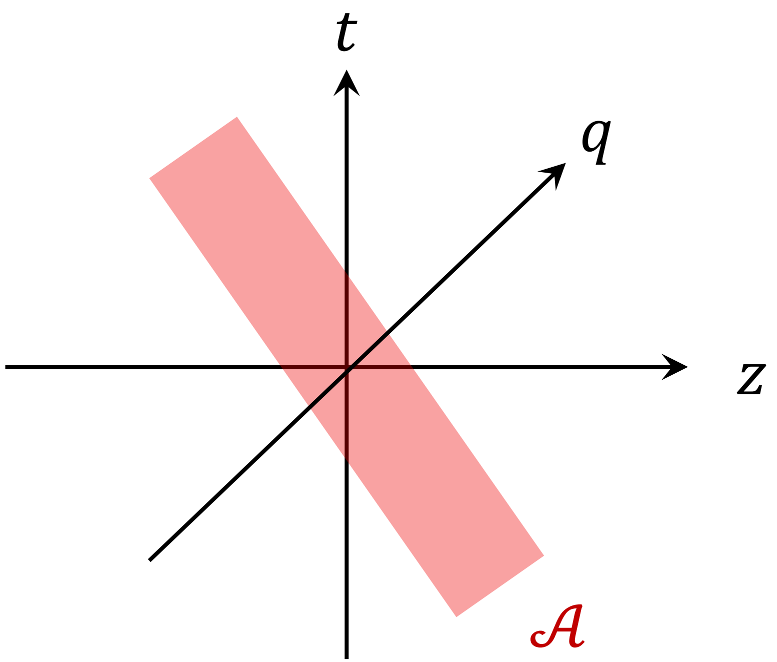
Quark jet scattering off a color field in:

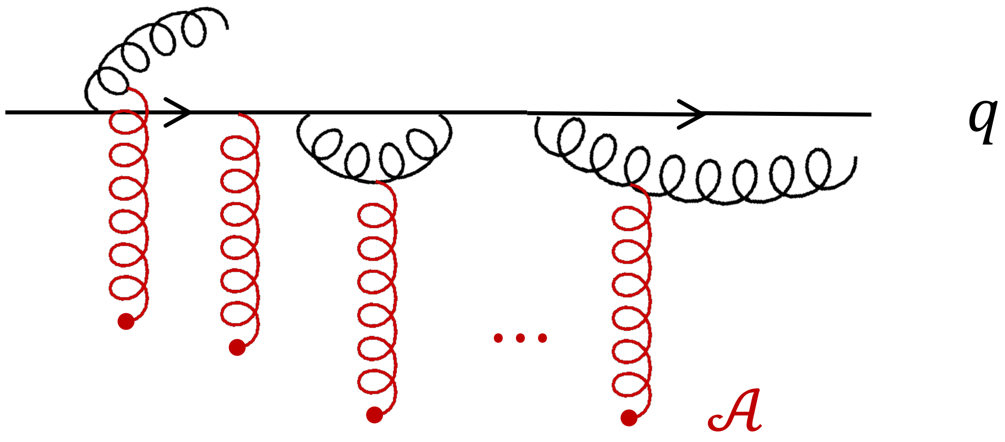
Heavy ion collisions A+A



Quark jet scattering off a color field

- The fundamental process



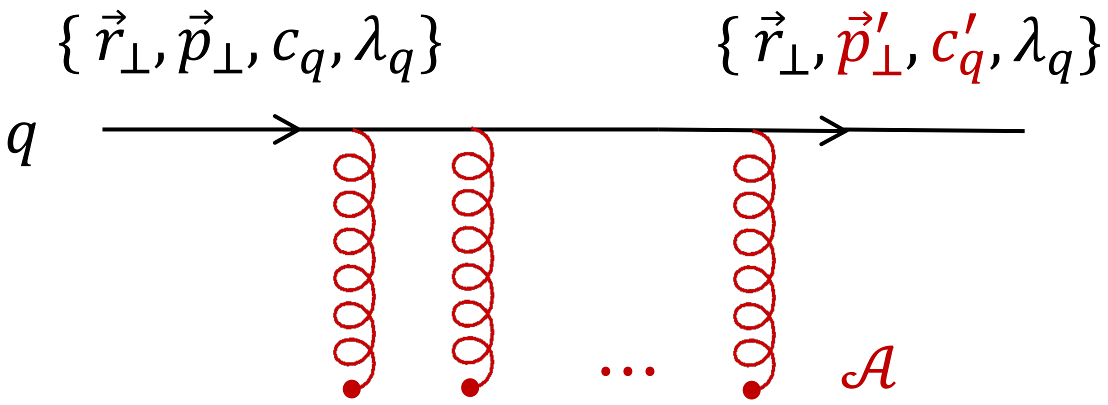


At high energy, the target has many gluons:
$\mathcal{A}_\mu \gg 1/g$
$\Rightarrow$ described as a classical gluon field (Color Glass Condensate, MV model)

What has been established and approximated?

○ Eikonal limit

Quark is infinitely energetic: $p^+ \equiv p^0 + p^z = \infty$



⇒ Wilson line: eikonal scattering amplitude, resummation of $\mathcal{A}_\mu$ in the path-ordered exponential

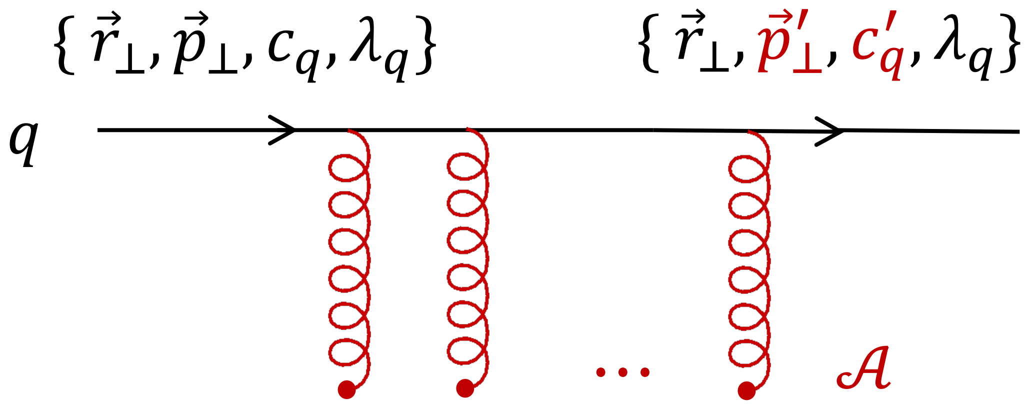
What has been established and approximated?

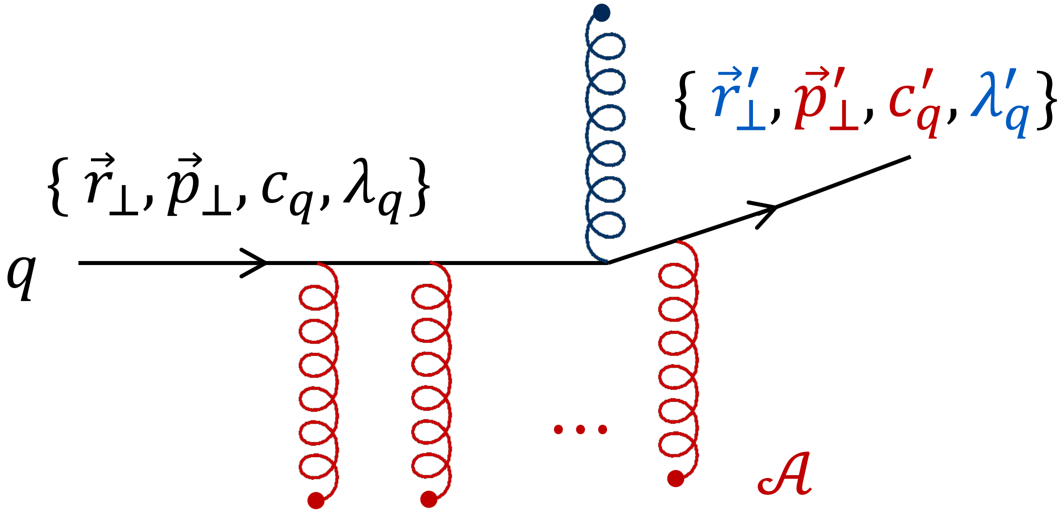
○ Eikonal limit

Quark is infinitely energetic:

$$p^+ \equiv p^0 + p^z = \infty$$

⇒ Sub-eikonal effects

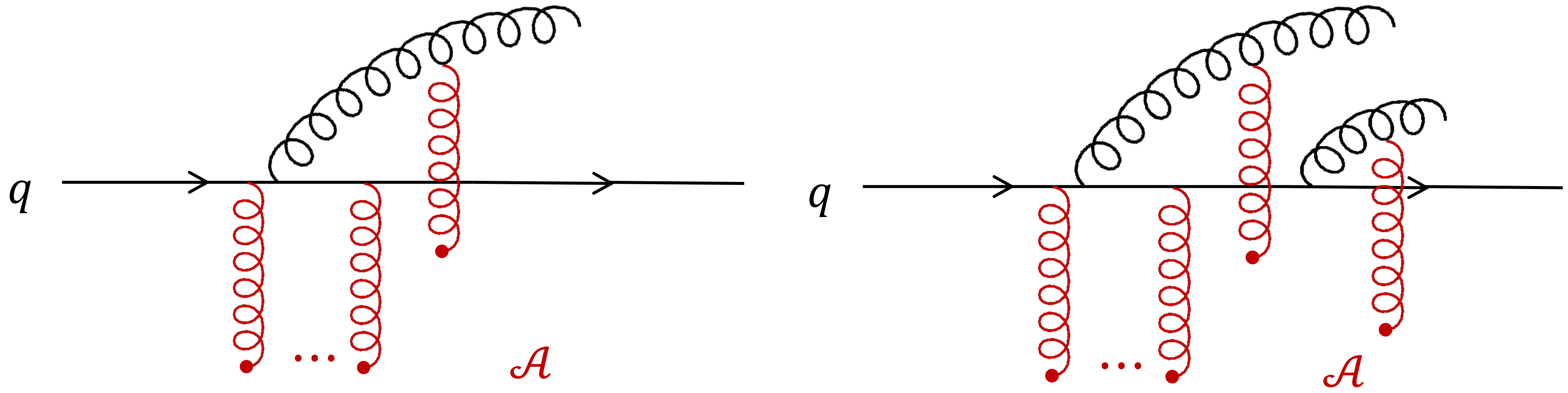




What has been established and approximated?

- **Perturbative-based approaches**

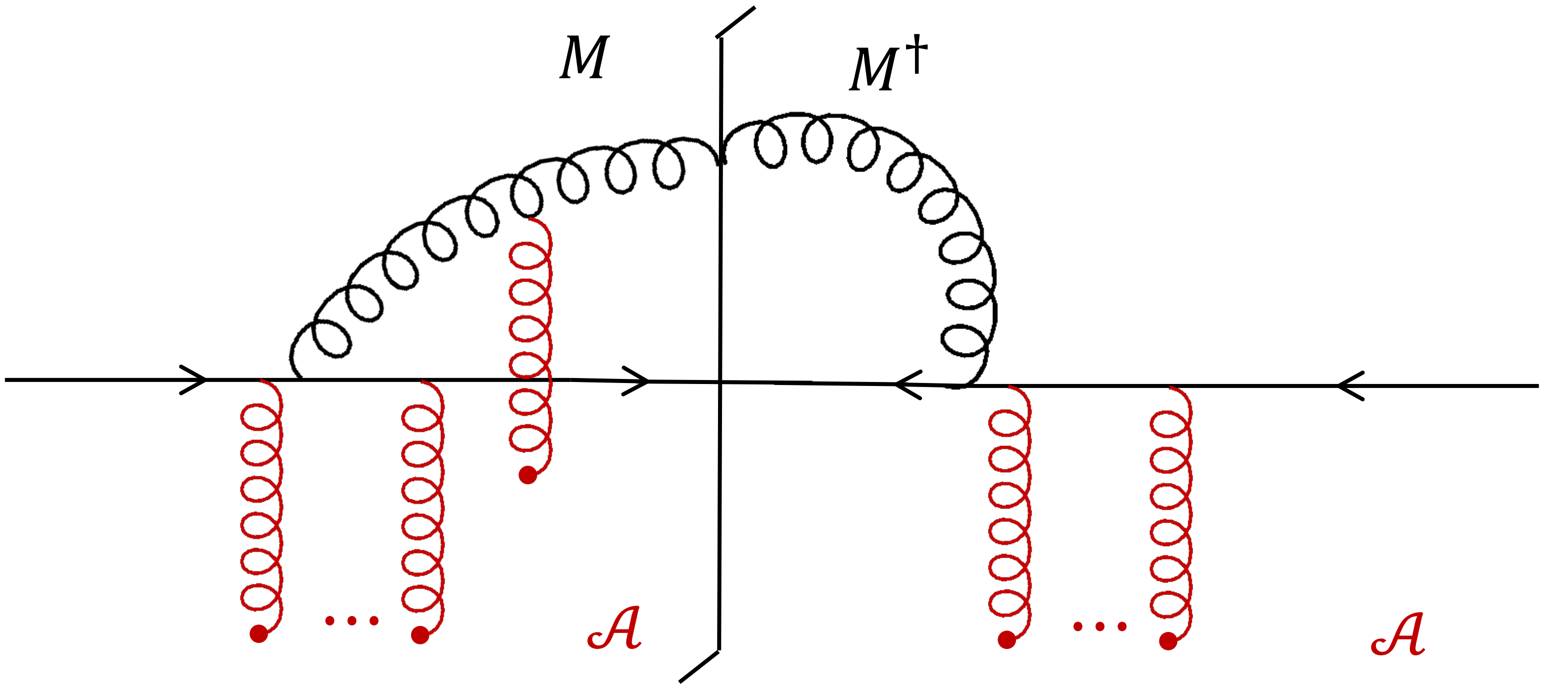
  Expansion in powers of the coupling: one gluon emission at NLO, and two gluons at NNLO



What has been established and approximated?

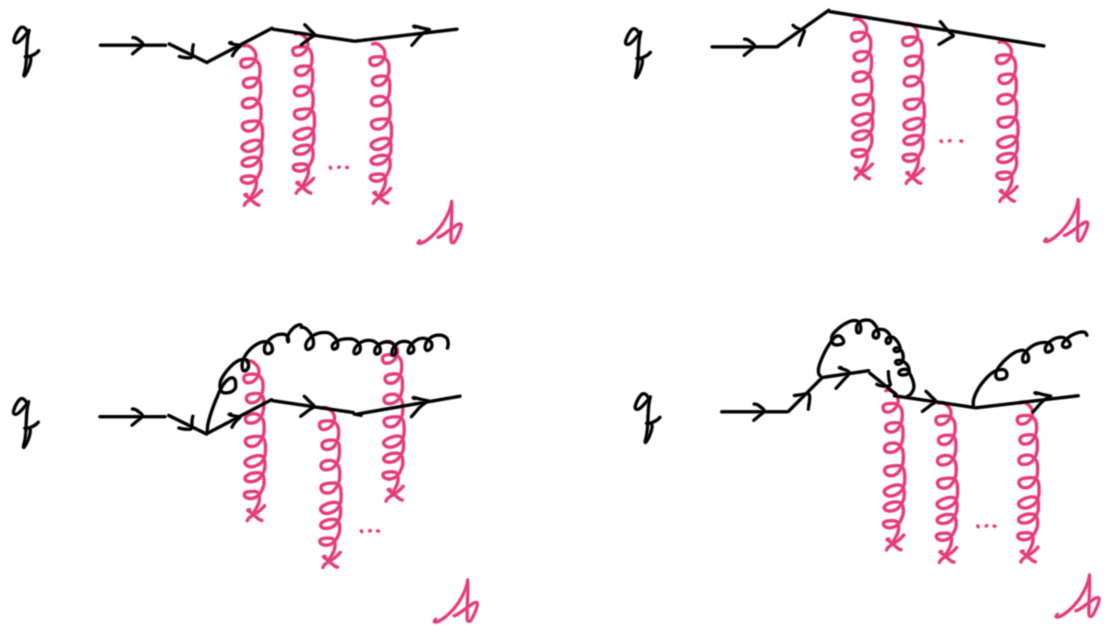
- **Perturbative-based approaches**

Calculation is on the probability level



What are the differences here?

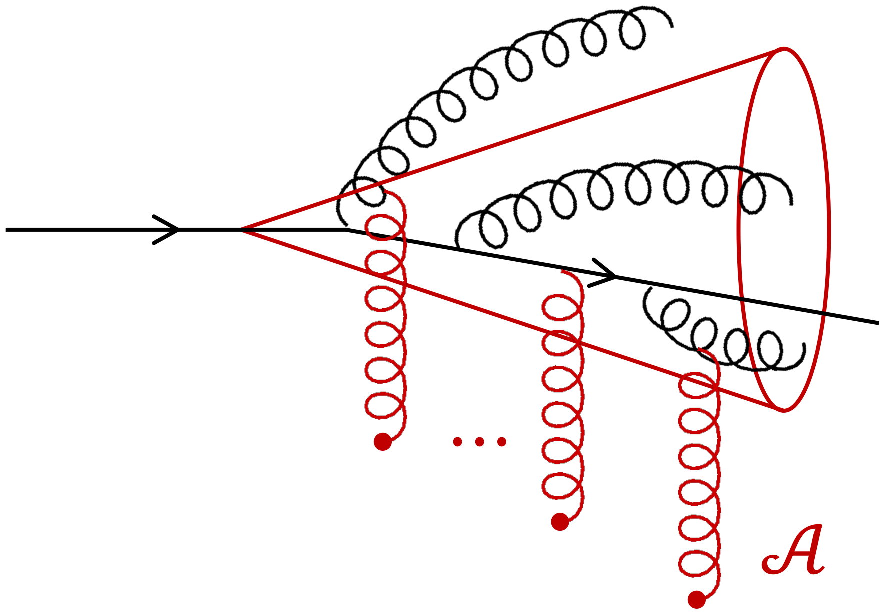
- **Non-perturbative approach**
  - ⇒ beyond eikonal
- **Amplitude level computation**
  - ⇒ jet is tracked as an evolving quantum state
- **Real-time simulation**
  - ⇒ accessibility to intermediate state



What are the differences here?

- **Dressed quark states**

    ⇒ distinguish jet intrinsic and external gluons

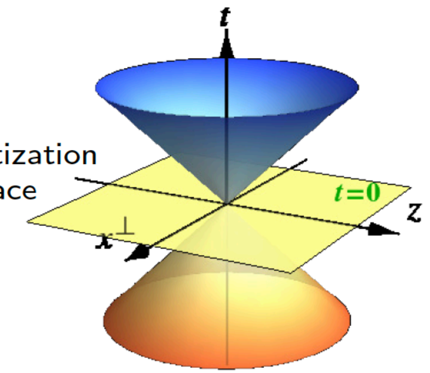
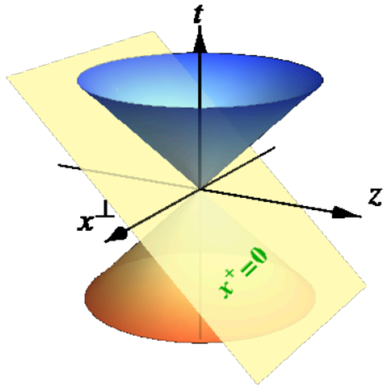
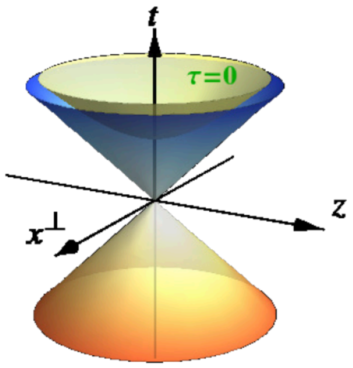


Outline

- ❑ Methodology
    - The light-front Hamiltonian approach: BLFQ & tBLFQ
- ❑ Application to jet physics
    1. Dressed quark
    2. In-medium dressed quark evolution
- ❑ Summary and outlooks

Light-front Hamiltonian approach: BLFQ & tBLFQ[1]

➢ Light-front dynamics

	instant form	front form	point form
time variable	$t = x^0$	$x^+ \triangleq x^0 + x^3$	$\tau \triangleq \sqrt{t^2 - \vec{x}^2 - a^2}$
quantization surface			
Hamiltonian	$H = P^0$	$P^- \triangleq P^0 - P^3$	P^μ
kinematical	$\vec{P}, \vec{J}$	$\vec{P}^\perp, P^+, \vec{E}^\perp, E^+, J^-$	$\vec{J}, \vec{K}$
dynamical	$\vec{K}, P^0$	$\vec{F}^\perp, P^-$	$\vec{P}, P^0$
dispersion relation	$p^0 = \sqrt{\vec{p}^2 + m^2}$	$p^- = (\vec{p}_\perp^2 + m^2)/p^+$	$p^\mu = m v^\mu \;\; (v^2 = 1)$

1. J. P. Vary, H. Honkanen, Jun Li, P. Maris, S. J. Brodsky, A. Harindranath, G. F. de Teramond, P. Sternberg, E. G. Ng, C. Yang., Phys. Rev. C81, 035205 (2010); X. Zhao, A. Ilderton, P. Maris, and J. P. Vary, Phys. Rev. D88, 065014 (2013).

Light-front Hamiltonian approach: BLFQ & tBLFQ[1]

➢ Hamiltonian formalism

- Bound states: eigenstates of the light-front Hamiltonian

$$P^-|\phi\rangle = P^-_\phi|\phi\rangle$$

$$\Updownarrow$$

$$\underbrace{(P^-P^+ - \vec{P}_\perp^2)}_{H_{\mathrm{LC}}}|\phi\rangle = M^2|\phi\rangle$$

- Time-dependent process: the state obeys the time-evolution equation

$$\frac{1}{2}P^-(x^+)|\psi(x^+)\rangle = i\frac{\partial}{\partial x^+}|\psi(x^+)\rangle$$

Light-front Hamiltonian approach: BLFQ & tBLFQ[1]

Basis representation

- Optimal basis encodes certain symmetries of the system, and it is the key to computational efficiency

$$|\psi; x^+\rangle = \sum_\beta c_\beta(x^+)|\beta\rangle$$

Operators

$$\begin{pmatrix} \langle 1|U|1\rangle & \langle 1|U|2\rangle & \dots & \langle 1|U|n\rangle \\ \langle 2|U|1\rangle & \langle 2|U|2\rangle & \dots & \langle 2|U|n\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \langle n|U|1\rangle & \langle n|U|2\rangle & \dots & \langle n|U|n\rangle \end{pmatrix}$$

State

$$\begin{pmatrix} c_1 \\ c_2 \\ \vdots \\ c_n \end{pmatrix}$$

Light-front Hamiltonian approach: BLFQ & tBLFQ[1]

➢ Computational method

- **Basis Light-Front Quantization (BLFQ)**: the bound state is solved by diagonalizing the Hamiltonain matrix

$$H_{\mathrm{LC}} \rightarrow \begin{pmatrix} M_1^2 & & & \\ & M_2^2 & & \\ & & \ddots & \\ & & & M_2^2 \end{pmatrix}$$

Eignestates → LF wavefunctions
Eigenvalues → M^2

Light-front Hamiltonian approach: BLFQ & tBLFQ[1]

➢ Computational method

- **Basis Light-Front Quantization (BLFQ)**
- **time-dependent BLFQ (tBLFQ)**: the evolving state is solved by sequential matrix multiplications of the evolution operators

$$\begin{pmatrix} c_1(x^+) \\ c_2(x^+) \\ \vdots \\ c_n(x^+) \end{pmatrix} = \left[\quad U_n \quad \right] \cdots \left[\quad U_2 \quad \right] \left[\quad U_1 \quad \right] \begin{pmatrix} c_1(0) \\ c_2(0) \\ \vdots \\ c_n(0) \end{pmatrix}$$

$$U_k = \mathcal{T}_+ \exp\left[-\frac{\mathrm{i}}{2} \int_{x_{k-1}^+}^{x_k^+} dz^+ P^-(z^+) \right], \qquad x_n^+ = x^+$$

Light-front Hamiltonian approach: BLFQ & tBLFQ[1]

➢ Computational method

- **Basis Light-Front Quantization (BLFQ)**
- **time-dependent BLFQ (tBLFQ)**

✓ *First-principles*

✓ *Non-perturbative*

✓ *Quantum amplitude*

Outline

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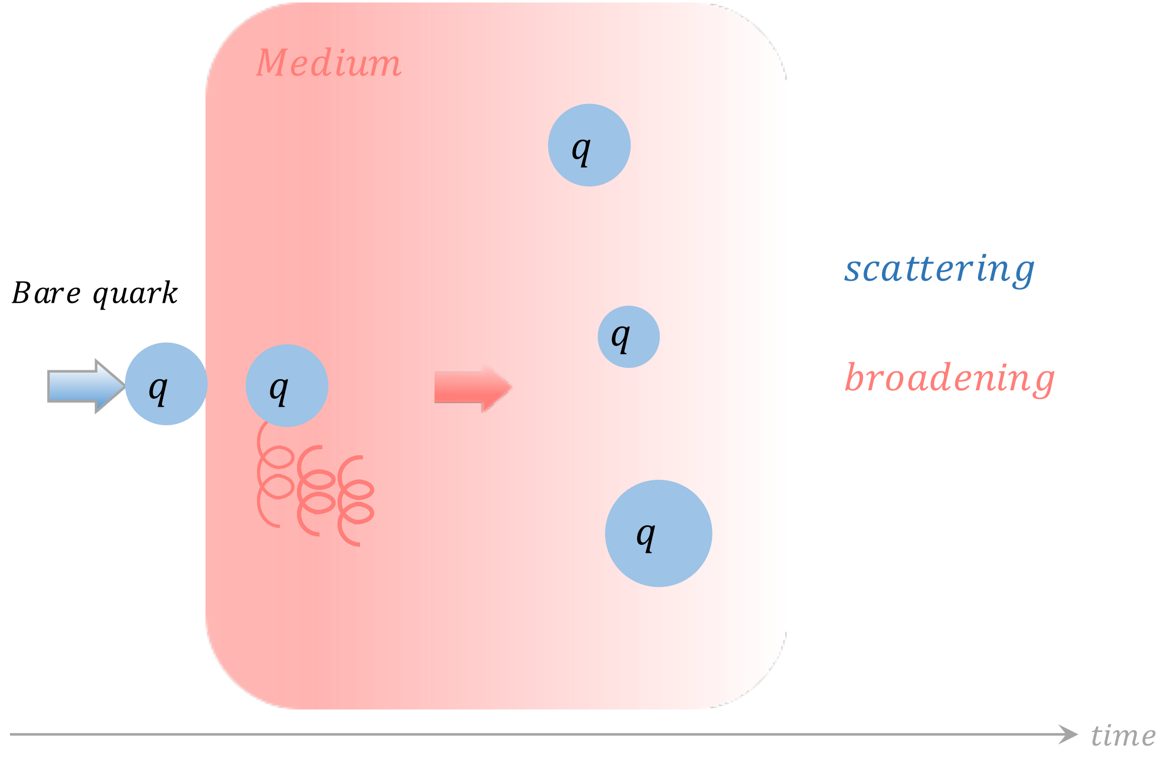
- ❑ Application to jet physics
  1. Dressed quark
  2. In-medium dressed quark evolution
- ❑ Summary and outlooks

Applications of tBLFQ to Jet evolution

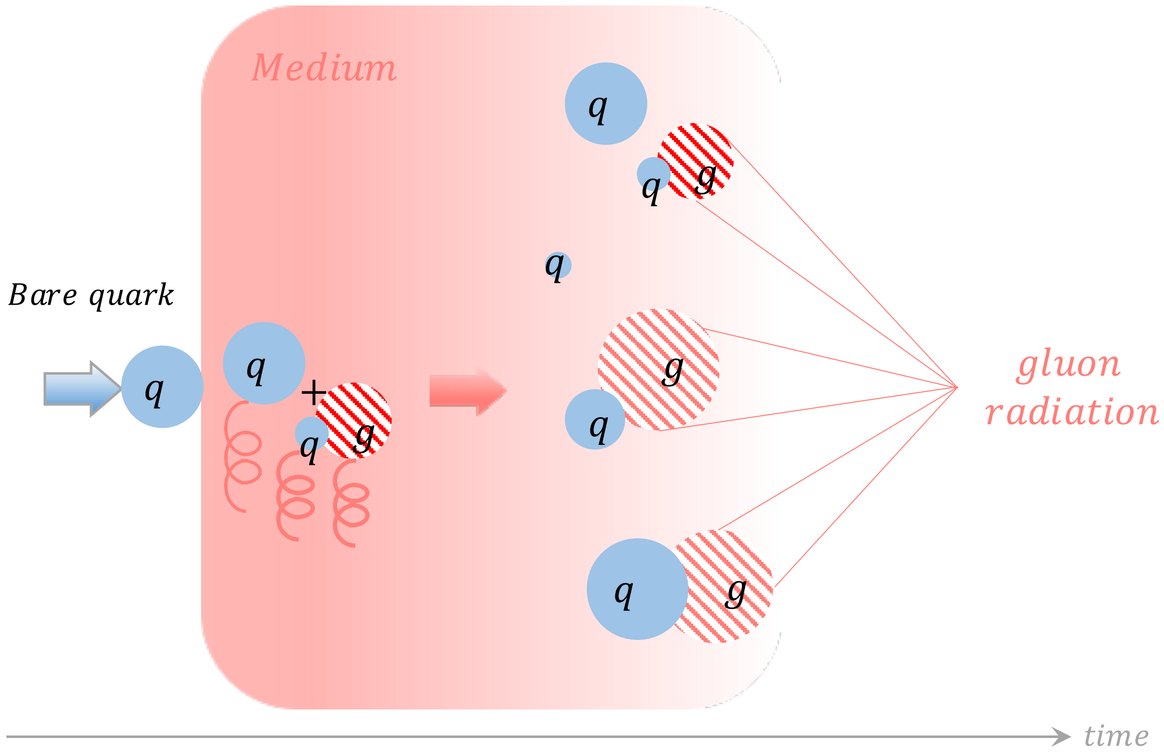
I. $|q\rangle$: quark jet scattering off a color field[1]



1. Phys.Rev.D 101(2020)7, 076016, ML, X. Zhao, P. Maris, G. Chen, Y. Li, K. Tuchin and J. P. Vary

Applications of tBLFQ to Jet evolution

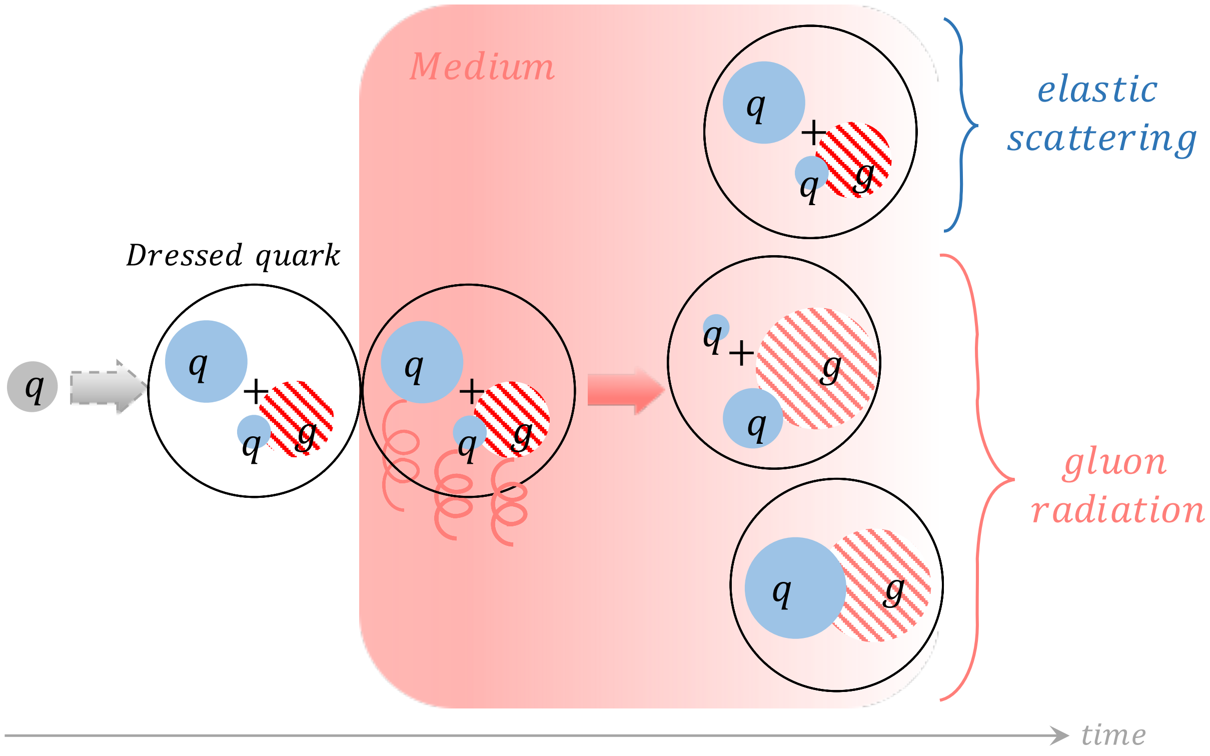
II. $|q\rangle + |qg\rangle$: quark jet scattering and gluon emission[1]



1. Phys.Rev.D 104 (2021) 5, 056014, ML, T. Lappi and X. Zhao; Phys.Rev.D 108 (2023) 3, 3, ML, T. Lappi, X. Zhao and C. A. Salgado

Applications of tBLFQ to Jet evolution

III. $|q\rangle + |qg\rangle$: dressed quark scattering and gluon emission



Applications of tBLFQ to Jet evolution

- **Basis representation:** discrete momentum states

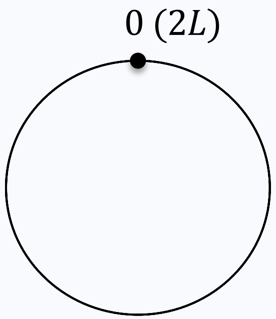
$$P^-_{\mathrm{KE}}|\beta\rangle = P^-_\beta|\beta\rangle,\ \beta_l = \{k_l^x, k_l^y, k_l^+, \lambda_l, c_l\}, (l = q, g)$$

$$|q\rangle : |\beta_q\rangle; \quad |qg\rangle : |\beta_{qg}\rangle = |\beta_q\rangle \otimes |\beta_g\rangle$$

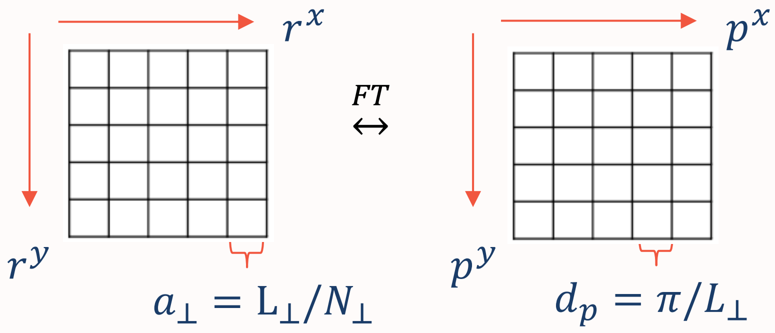
- The longitudinal space
  - $x^- = [0, 2L]$
  - $p_l^+ = \frac{2\pi}{L} k_l^+$

    $k_q^+ = \frac{1}{2}, \frac{3}{2}, \dots, K + \frac{1}{2}$

    $k_g^+ = 1, 2, \dots, K$



- The transverse coordinate space
  - $r_l^\perp = [-N_\perp, \dots, N_\perp - 1]\, L_\perp / N_\perp$
  - $p_l^\perp = \frac{2\pi}{2L_\perp} k_l^\perp, k_l^\perp = -N_\perp, \dots, N_\perp - 1$



Basis size:

$$N_{tot} = (2N_\perp)^2 \times 2 \times 3 + K \times (2N_\perp)^4 \times 4 \times 24$$

Dressed quark

- **QCD eigenstates in** $|q\rangle + |qg\rangle$
  - The dressed quark state is described as the eigenstate of the light-front QCD Hamiltonian with the quark quantum numbers:

$$\mathcal{L}_{QCD} = -\frac{1}{4}\mathrm{F}_{\mathrm{a}}^{\mu\nu}F_{\mu\nu}^{a} + \bar{\psi}(i\gamma^{\mu}D_{\mu} - m)\psi \quad \rightarrow \quad \mathrm{P}_{QCD}^{-} = \mathrm{P}_{KE}^{-} + V_{qg}$$

$$P_{QCD}^{-}|\phi\rangle = P_{\phi}^{-}|\phi\rangle$$

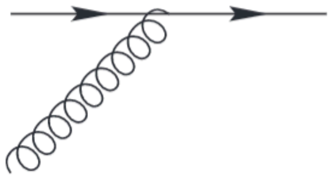
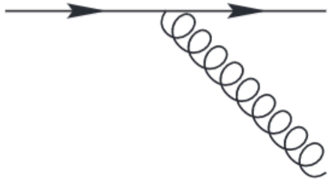
$$\Updownarrow$$

$$\underbrace{(P_{QCD}^{-}P^{+} - \vec{P}_{\perp}^{2})}_{H_{\mathrm{LC}}}|\phi\rangle = M^{2}|\phi\rangle$$

Dressed quark

- **QCD eigenstates in $|q\rangle + |qg\rangle$**
  - Sector-dependent mass renormalization

$$H_{\mathrm{LC}}(\delta m)|\phi\rangle = m_q^2|\phi\rangle$$

Fock sector	$\|q\rangle$	$\|qg\rangle$
$\langle q\|$	$P^-_{KE}(m_Q = m_q + \delta m)$	
$\langle qg\|$		$P^-_{KE}(m_q)$

Dressed quark

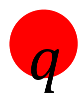
- **Boost Invariance**
  - The internal structure of the QCD eigenstate is boost invariant

$$|\phi\rangle = |\phi\rangle_{CM} \otimes |\phi\rangle_{rel}$$

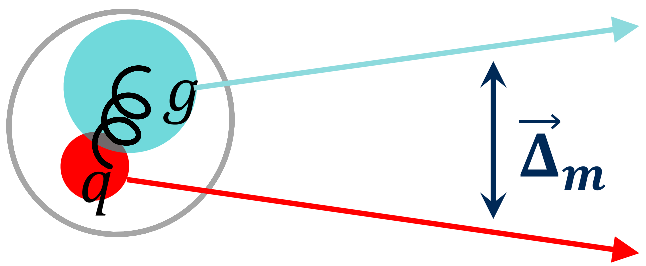
Dressed quark

- **Boost Invariance**
  - The internal structure of the eigenstates is boost invariant

$$|\phi\rangle = \left|\phi(\{P^+, \vec{P}_\perp)\right\rangle_{CM} \otimes \left|\phi_q, \phi_{qg}\big(\boldsymbol{z}, \vec{\boldsymbol{\Delta}}_{\boldsymbol{m}}\big)\right\rangle_{rel}$$



$$\{p_Q^+ = P^+, \vec{p}_{\perp,Q} = \vec{P}_\perp\}$$



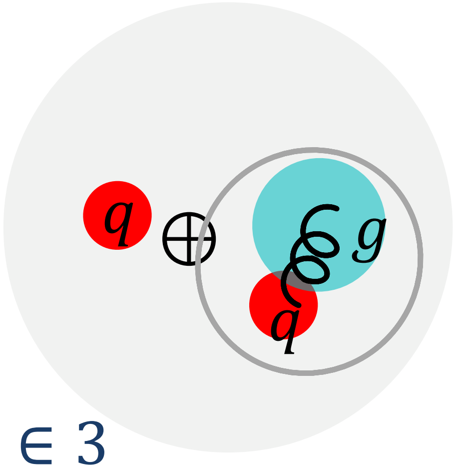
$$\{p_g^+ = \boldsymbol{z}P^+, \vec{p}_{\perp,g} = \vec{\Delta}_m + z\vec{P}_\perp\}$$

$$\{p_q^+ = (1-z)P^+, \vec{p}_{\perp,q} = -\vec{\Delta}_m + (1-z)\vec{P}_\perp\}$$

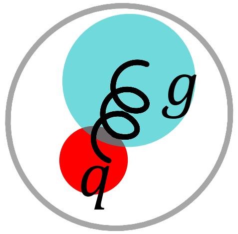
Dressed quark

- **Color rotation invariance**
  - The wavefunction is invariant under the color rotation in each irreducible representation of SU(N_c)

$$|\phi\rangle = |\phi\rangle_{color-triplet} \otimes |\phi\rangle_{spin\,\&\,spatial}$$
$$|\phi'\rangle = |\phi'\rangle_{color-excited} \otimes |\phi'\rangle_{spin\,\&\,spatial}$$



$c_Q \in 3$

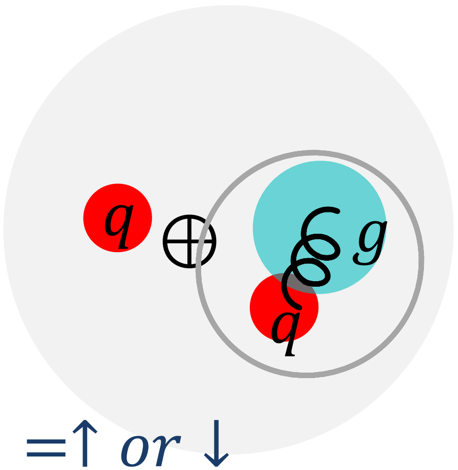


$c_{qg} \in \bar{6} \oplus 15$

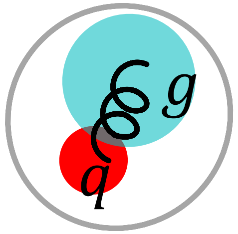
Dressed quark

- **Spin rotation symmetry**
  - The wavefunction is invariant under the spin rotation in each helicity subspace

$$|\phi\rangle = |\phi\rangle_{quark-helicity} \otimes |\phi\rangle_{helicity\,\&\,spatial}$$
$$|\phi'\rangle = |\phi'\rangle_{helicity-uncoupled} \otimes |\phi'\rangle_{helicity\,\&\,spatial}$$



$\lambda_Q =\uparrow or \downarrow$



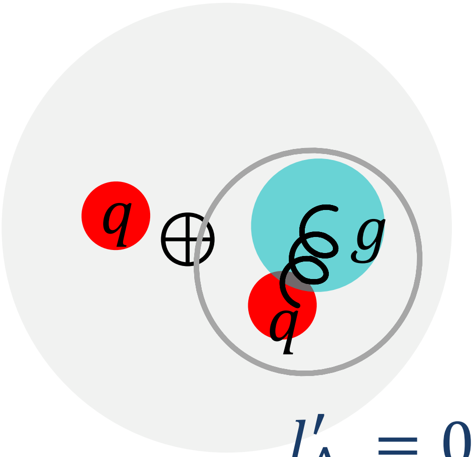
helicity-uncoupled qgs
(e.g., $|qg(\lambda_q, \lambda_g =\downarrow,\downarrow)\rangle$ cannot couple to $|q(\lambda_q =\uparrow)\rangle$)

Dressed quark

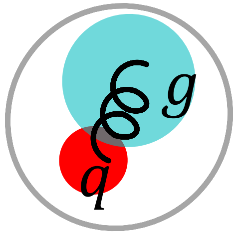
- **Discrete rotational symmetry**
  - The eigenstates can be labeled by a qg-relative orbital angular momentum number l'_Δ

$$\vec{\Delta}_m = \{\Delta_m = |\vec{\Delta}_m|, \theta = \arg\vec{\Delta}_m\}$$

$$\phi_{qg}(\Delta_m, \theta) \sim \exp(i\theta l'_\Delta)$$



$l'_\Delta = 0$

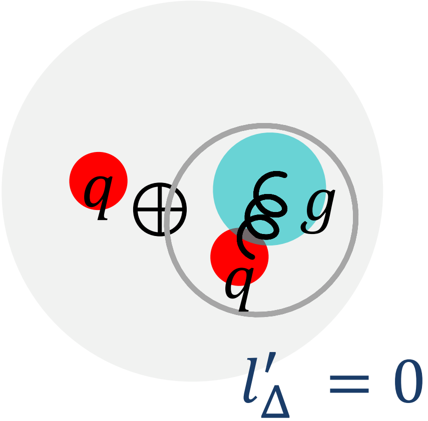


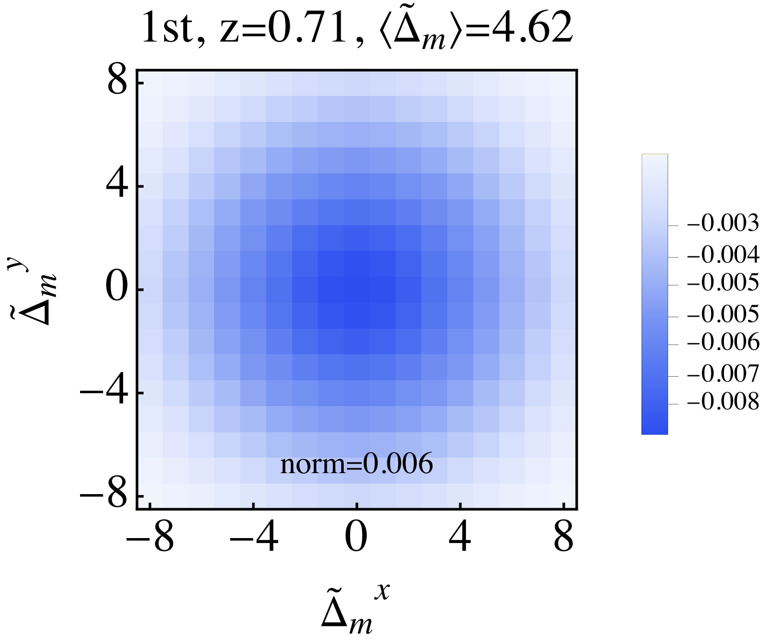
$l'_\Delta \neq 0$

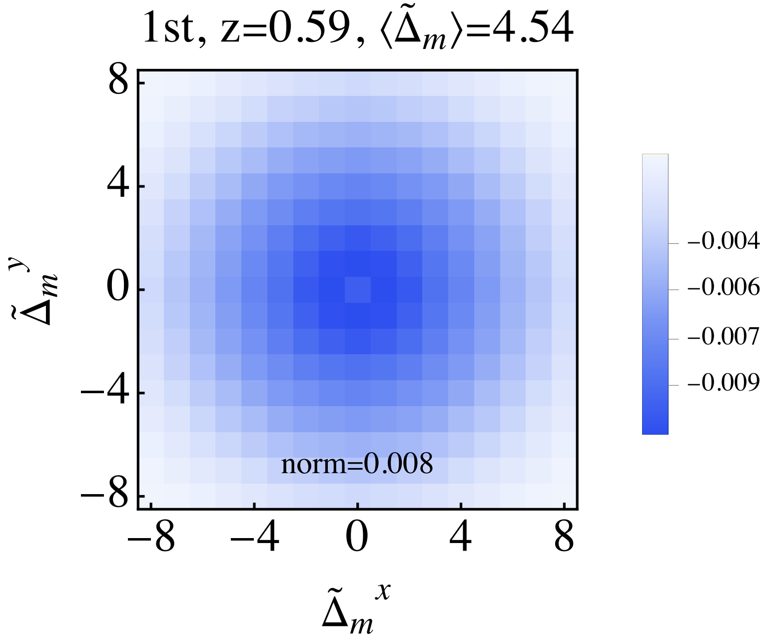
Orbital-angular excited qg

Dressed quark

- **Discrete rotational symmetry**
  - Dressed state wavefunction example

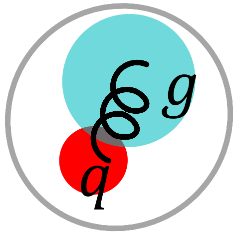




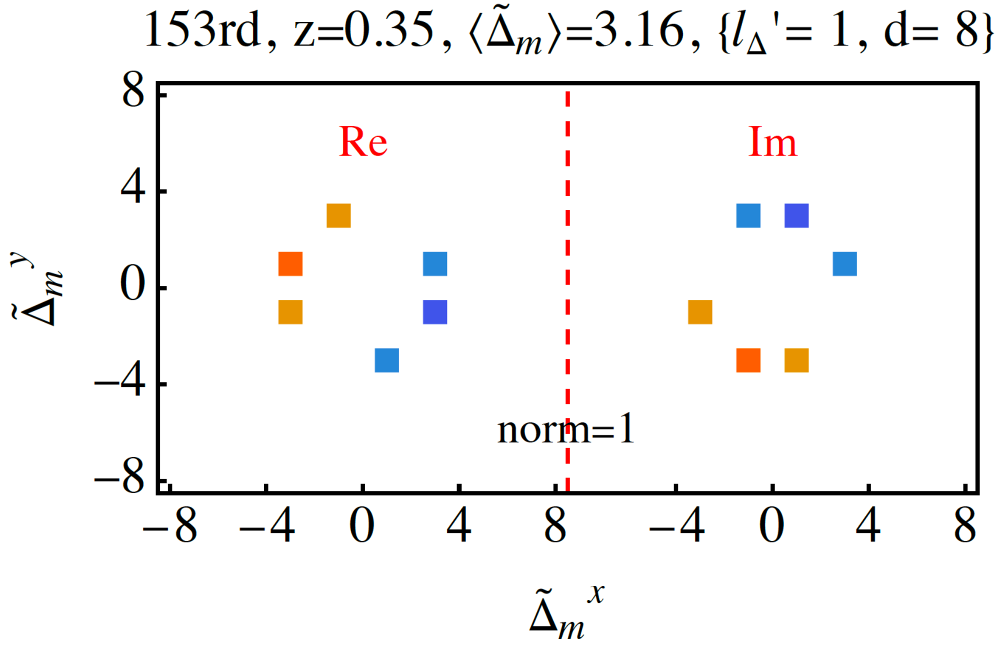


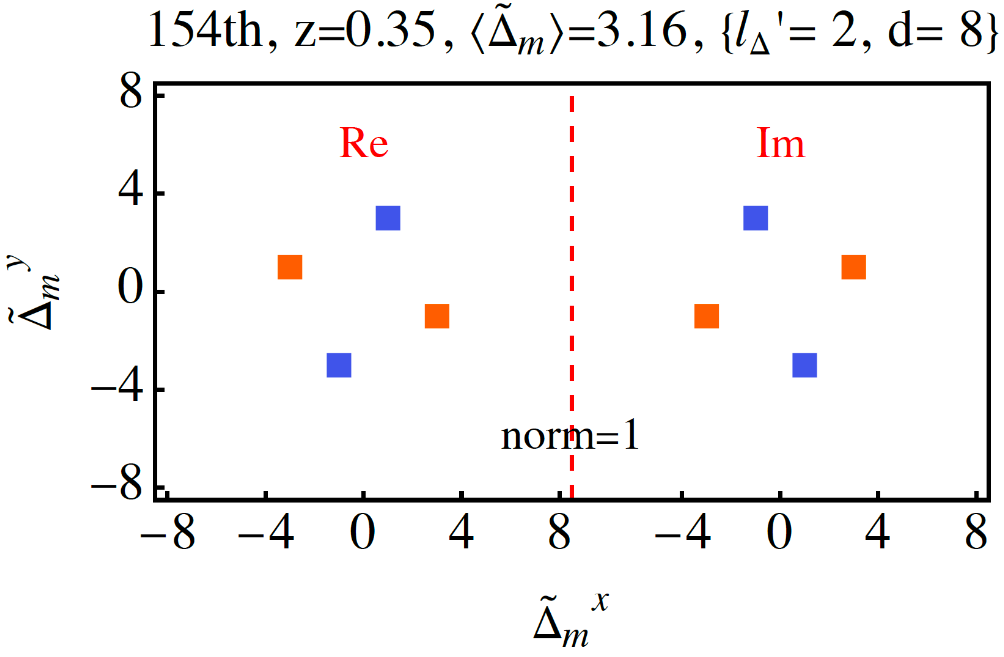
Dressed quark

- **Discrete rotational symmetry**
  - Orbital-angular excited qg wavefunction examples



$l'_\Delta \neq 0$





Dressed quark

- **Invariance and symmetries**

Total basis size: $N_{tot} = (2N_{\perp})^2 \times 2 \times 3 + K \times (2N_{\perp})^4 \times 4 \times 24$

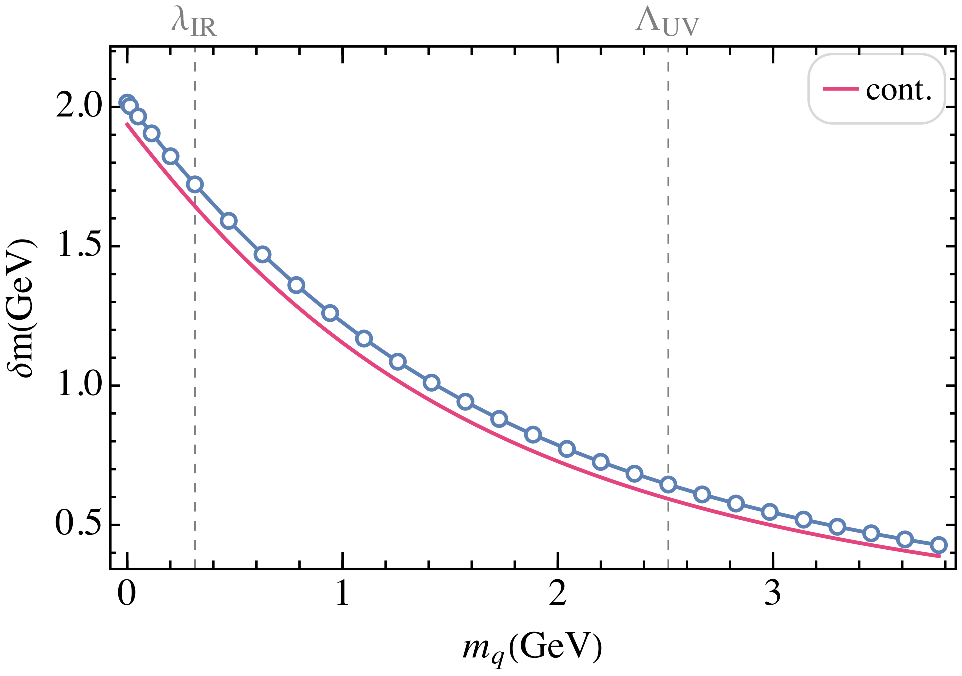
- Boost invariance
- Color rotation invariance
- Spin rotation symmetry
- Discrete rotational symmetry

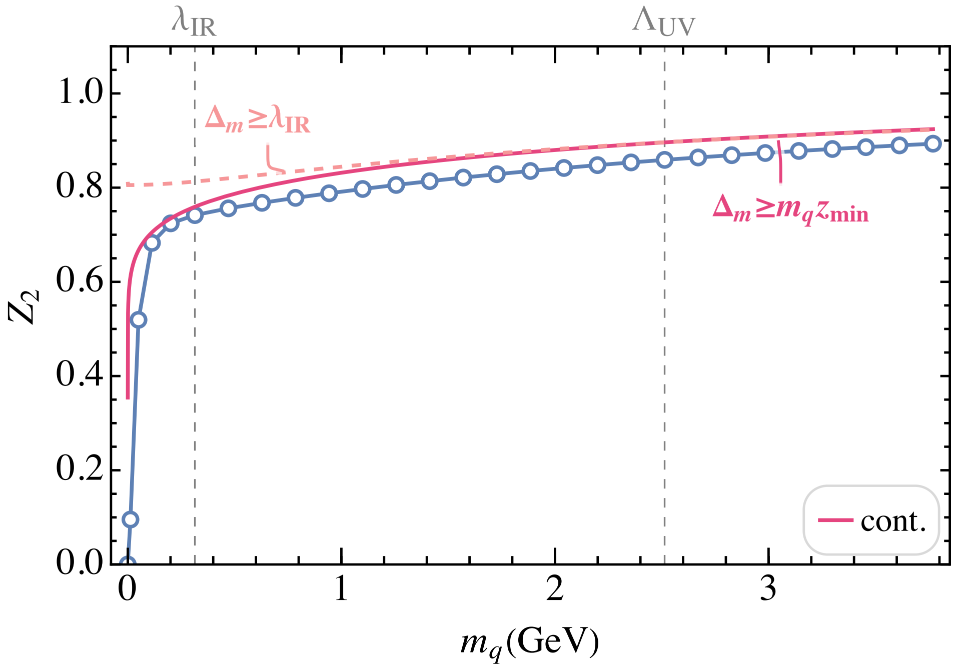
Basis size of matrix diagonalization: $1 + K \times (2N_{\perp})^2$

Dressed quark

- **QCD eigenstates in $|q\rangle + |qg\rangle$**
  - Sector-dependent mass renormalization

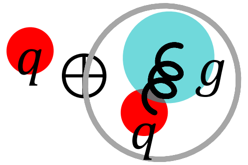
$$H_{\rm LC}(\delta m)|\phi\rangle = m_q^2|\phi\rangle$$

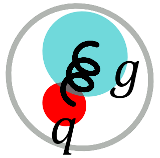


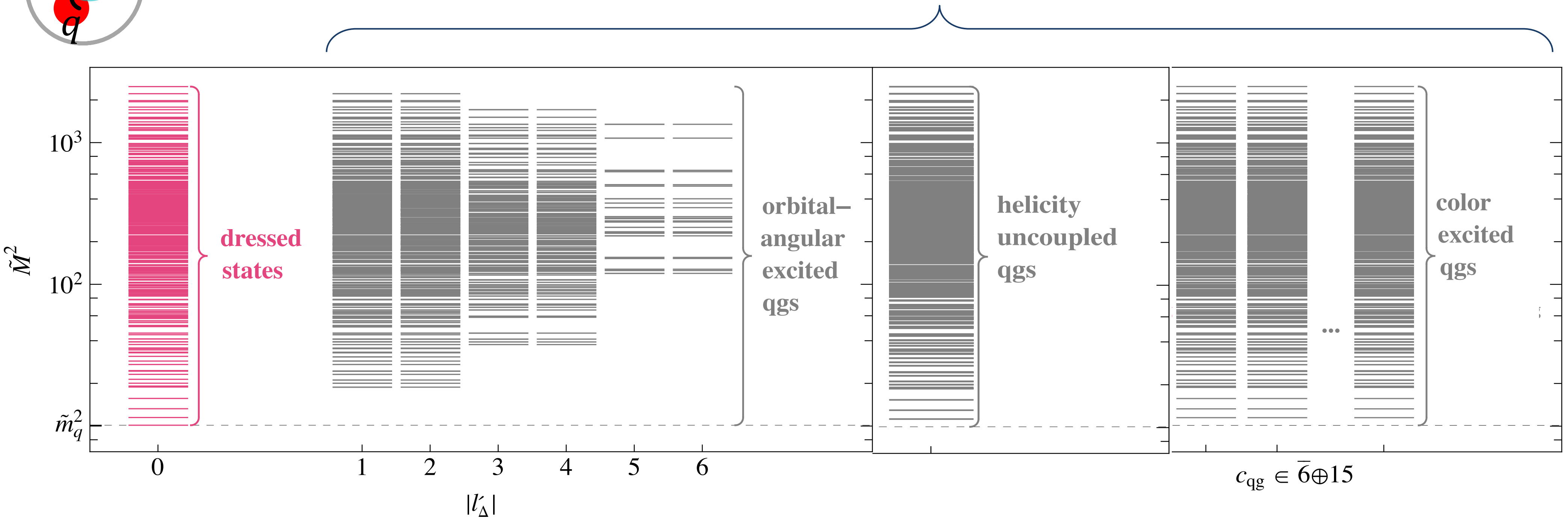


Dressed quark

- **QCD eigenstates in $|q\rangle + |qg\rangle$**
  - the dressed states
  - quark-gluon states

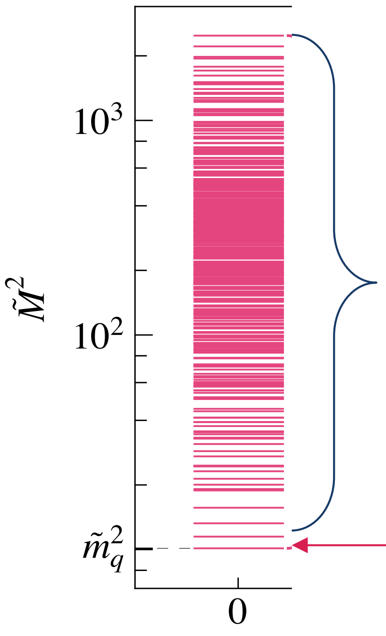






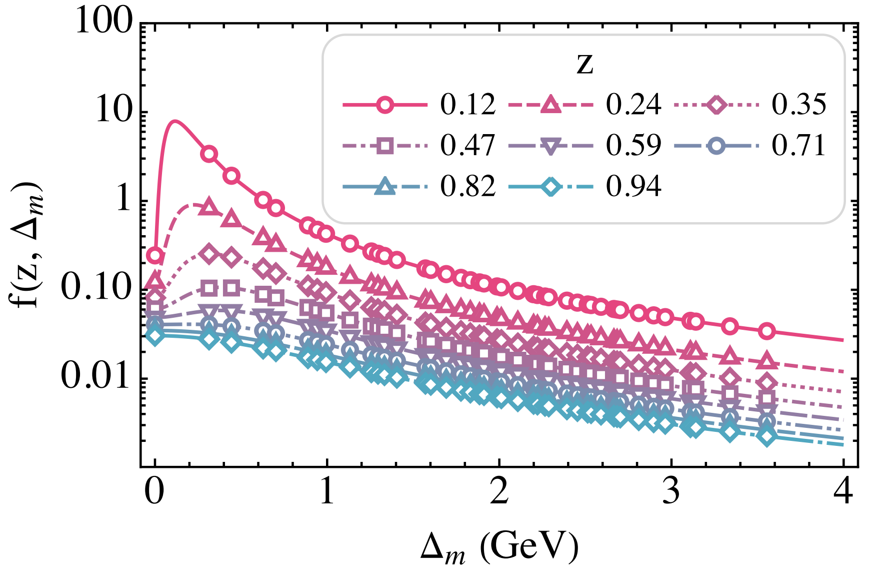
Dressed quark

- **QCD eigenstates in $|q\rangle + |qg\rangle$**
  - the dressed states



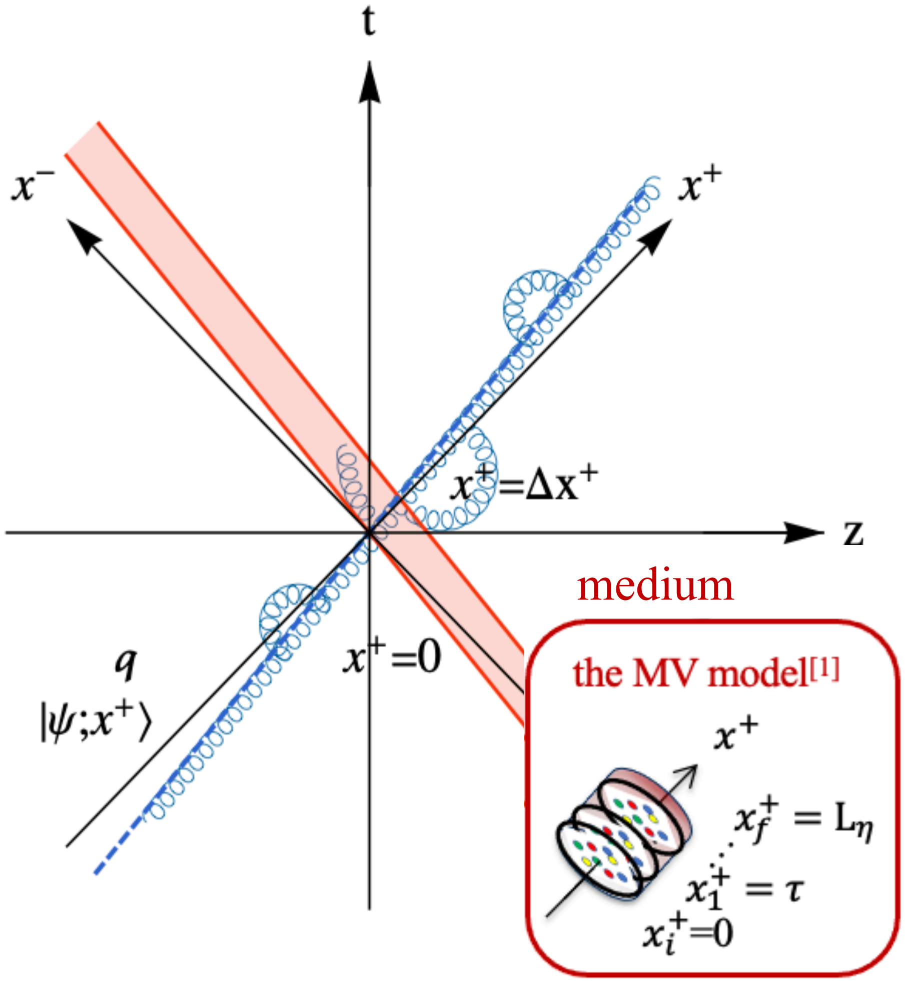
dressed quark-gluon states

dressed quark



In-medium quark jet evolution

- **The physics process**



- The medium, $\mathcal{A}(x^+,\vec{x}_\perp)$, is a classical gluon field[1]
  - Color charges

$$\langle \rho_a(x)\rho_b(y)\rangle = g^2\tilde{\mu}^2\delta_{ab}\delta^{(3)}(x-y)$$

  - The color field

$$(m_g^2 - \nabla_\perp^2)\mathcal{A}_a^-(x^+,\vec{x}_\perp) = \rho_a(x^+,\vec{x}_\perp)$$

where m_g is a chosen infrared regulator.

  - Saturation scale: $Q_s^2 = C_F\,(g^2\tilde{\mu})^2 L_\eta/(2\pi)$

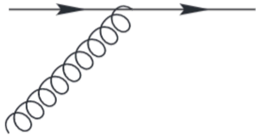
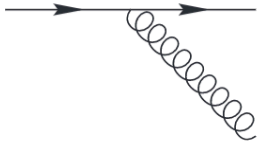
[1]L. D. McLerran and R. Venugopalan, Phys. Rev. D49, 2233 (1994); L. D. McLerran and R. Venugopalan, Phys. Rev. D49, 3352 (1994); L. D. McLerran and R. Venugopalan, Phys. Rev. D50, 2225 (1994).

In-medium quark jet evolution

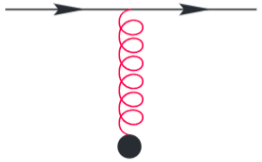
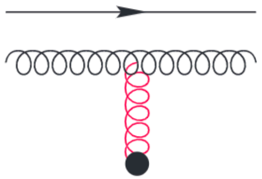
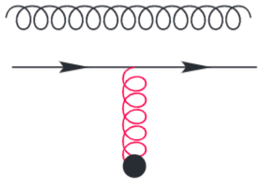
- **The evolution Hamiltonian**

$$\partial_\mu + ig(A_\mu + \mathcal{A}_\mu)$$

$$\mathcal{L}_{QCD} = -\frac{1}{4}\mathrm{F}_{\mathrm{a}}^{\mu\nu} F_{\mu\nu}^{a} + \bar{\psi}\left(i\gamma^{\mu} D_{\mu} - m\right)\psi \quad \rightarrow \; \mathrm{P}^{-}(x^{+}) = \mathrm{P}_{QCD}^{-} + V_{\mathcal{A}}(x^{+})$$

Fock sector	$\vert q\rangle$	$\vert qg\rangle$
$\langle q\vert$	$\mathrm{P}_{KE}^{-}(m_Q = m_q + \delta m)$	
$\langle qg\vert$		$\mathrm{P}_{KE}^{-}(m_q)$

$+$

Fock sector	$\vert q\rangle$	$\vert qg\rangle$
$\langle q\vert$		
$\langle qg\vert$		 

In-medium quark jet evolution

- **Solve the time-evolution equation**

$$\frac{1}{2}V_I(x^+)|\psi; x^+\rangle_\mathrm{I} = i\frac{\partial}{\partial x^+}|\psi; x^+\rangle_\mathrm{I}$$

  - P^-_{KE} as a phase factor:

$$|\psi; x^+\rangle_\mathrm{I} = \mathrm{e}^{\frac{\mathrm{i}}{2}\mathrm{P}^-_{KE}\,x^+}|\psi; x^+\rangle, \qquad \mathrm{V_I(x^+)} = \mathrm{e}^{\frac{\mathrm{i}}{2}\mathrm{P}^-_{KE}\,x^+}V(x^+)e^{-\frac{\mathrm{i}}{2}\mathrm{P}^-_{KE}\,x^+}$$

  - Time evolution as a product of many small timesteps

$$|\psi; x^+\rangle_\mathrm{I} = \lim_{n\to\infty}\prod_{k=1}^{n}\underbrace{\mathcal{T}_+\exp\{-\frac{\mathrm{i}}{2}\int_{x^+_{k-1}}^{x^+_k}dz^+V_I(z^+)\}}\,|\psi; 0\rangle_\mathrm{I}$$

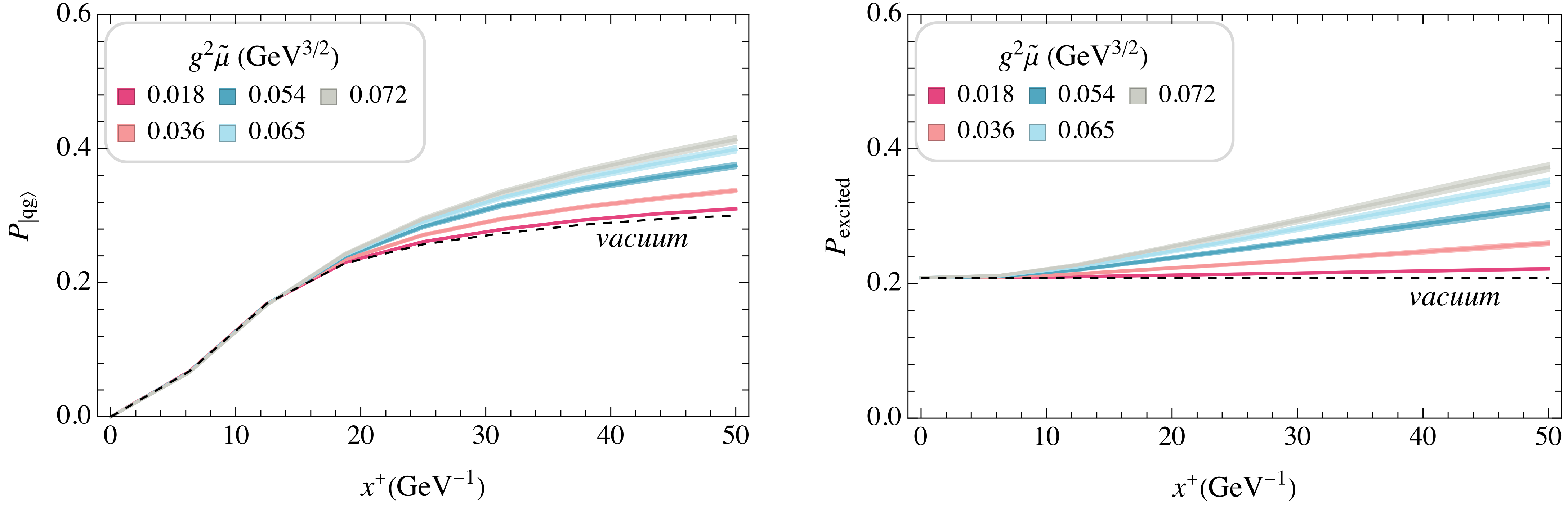
$$\mathcal{T}_+\exp\{-\frac{\mathrm{i}}{2}\int_{x^+_{k-1}}^{x^+_k}dz^+V_{\mathcal{A},I}(z^+)\}\times\,\mathcal{T}_+\exp\{-\frac{\mathrm{i}}{2}\int_{x^+_{k-1}}^{x^+_k}dz^+V_{qg,I}(z^+)\}$$

matrix exponential in coordinate space + Fast Fourier Transform, $\sim O(N_{tot}\log N_{tot})$

4th-order Runge-Kutta method, $\sim O(N_{tot})$

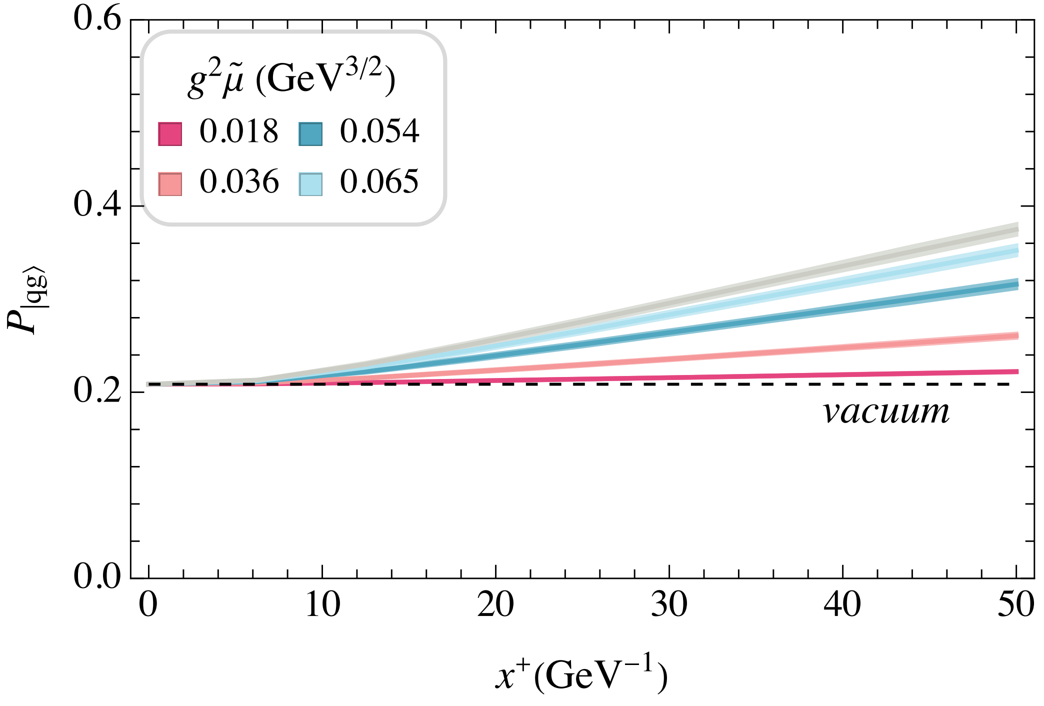
Evolution results: gluon emission

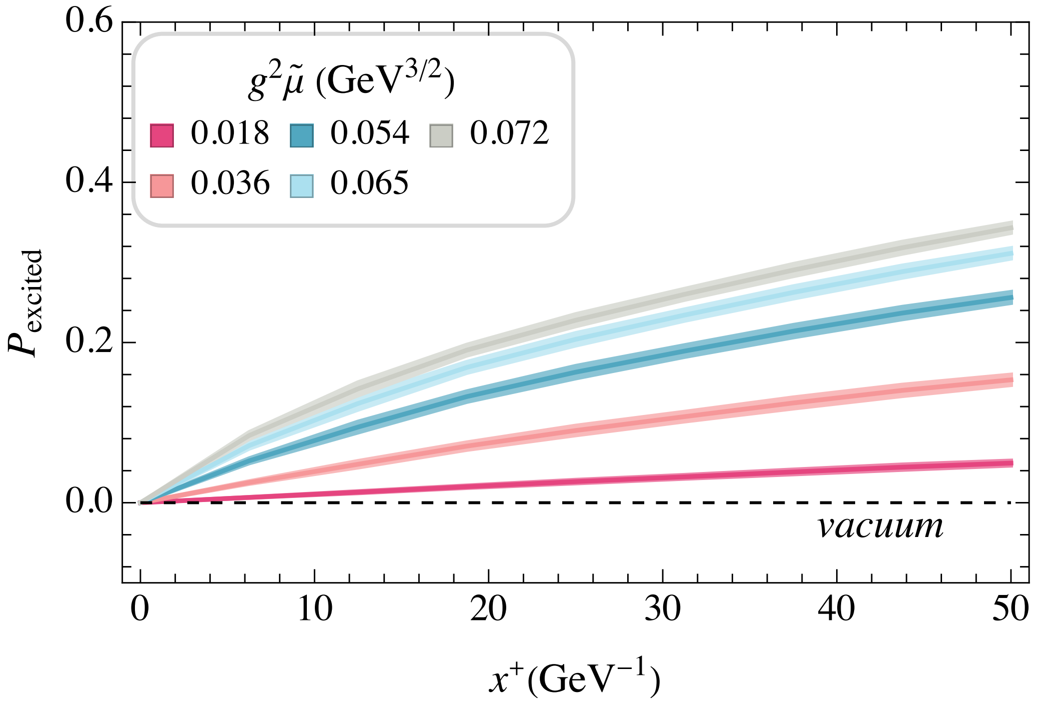
- **Bare quark initial state**
  - Total gluon: $P_{|qg\rangle}$
  - External gluon: $P_{excited}$



Evolution results: gluon emission

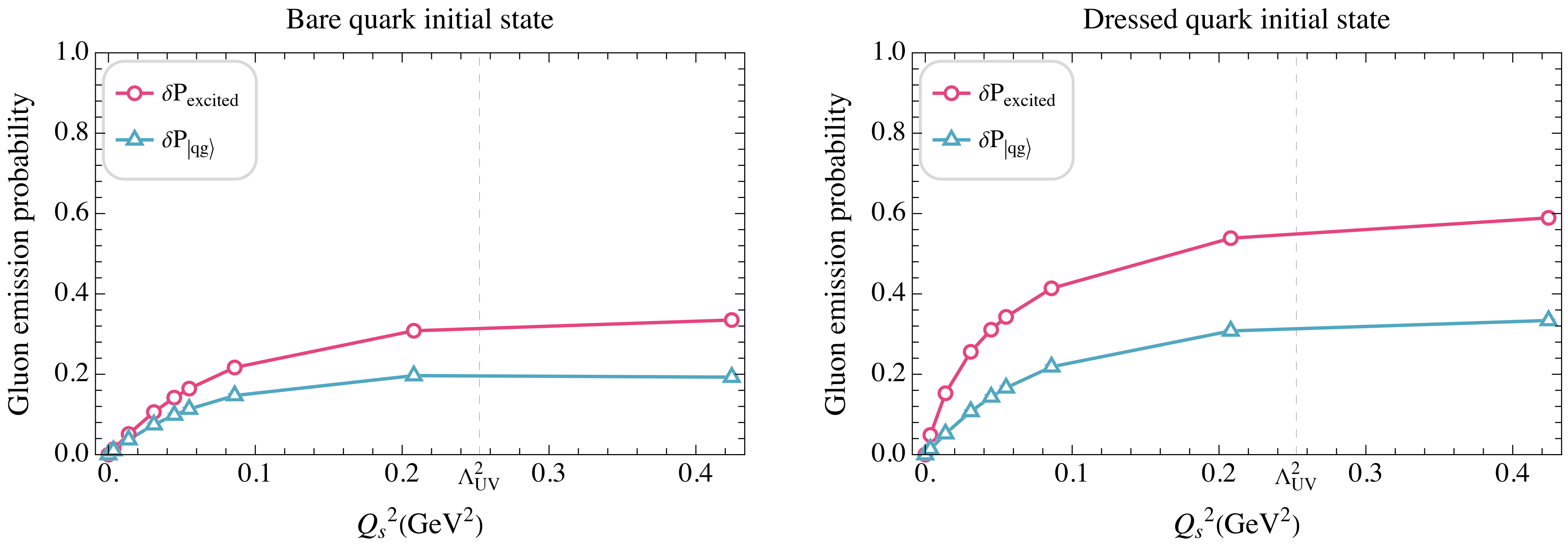
- **Dressed quark initial state**
  - Total gluon: $P_{|qg\rangle}$
  - External gluon: $P_{excited}$





Evolution results: gluon emission

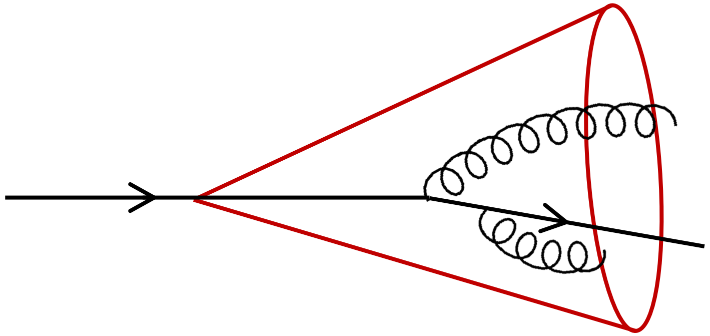
- **Medium-induced gluon emission**
  - Total gluon: $\delta P_{|qg\rangle}(Q_s) = P_{|qg\rangle}(Q_s) - P_{|qg\rangle}(Q_s = 0)$
  - External gluon: $\delta P_{excited}(Q_s) = P_{excited}(Q_s) - P_{excited}(Q_s = 0)$



Evolution results: momentum broadening

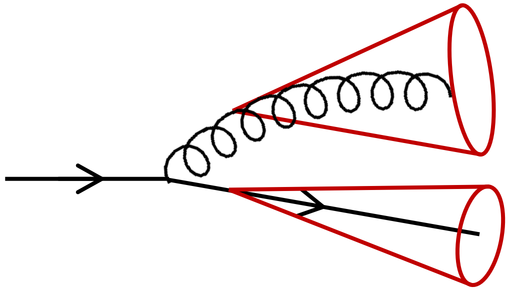
- **Which momentum?**
  - Center of mass (CM) momentum, taking the full state as a jet

$$\langle P_{\perp,CM}^2 \rangle = P_{|q\rangle} \langle P_{\perp}^2 \rangle_{|q\rangle} + P_{|qg\rangle} \langle P_{\perp}^2 \rangle_{|qg\rangle}$$



  - Quark momentum, in-jet or dijet structure

$$\langle P_{\perp,q}^2 \rangle = P_{|q\rangle} \langle P_{\perp}^2 \rangle_{|q\rangle} + P_{|qg\rangle} \langle p_{\perp,q}^2 \rangle_{|qg\rangle}$$

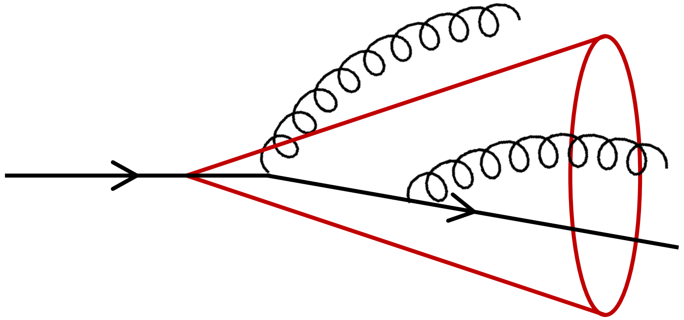


  - Gluon momentum, in-jet or dijet structure

$$\langle P_{\perp,g}^2 \rangle = P_{|qg\rangle} \langle p_{\perp,g}^2 \rangle_{|qg\rangle}$$

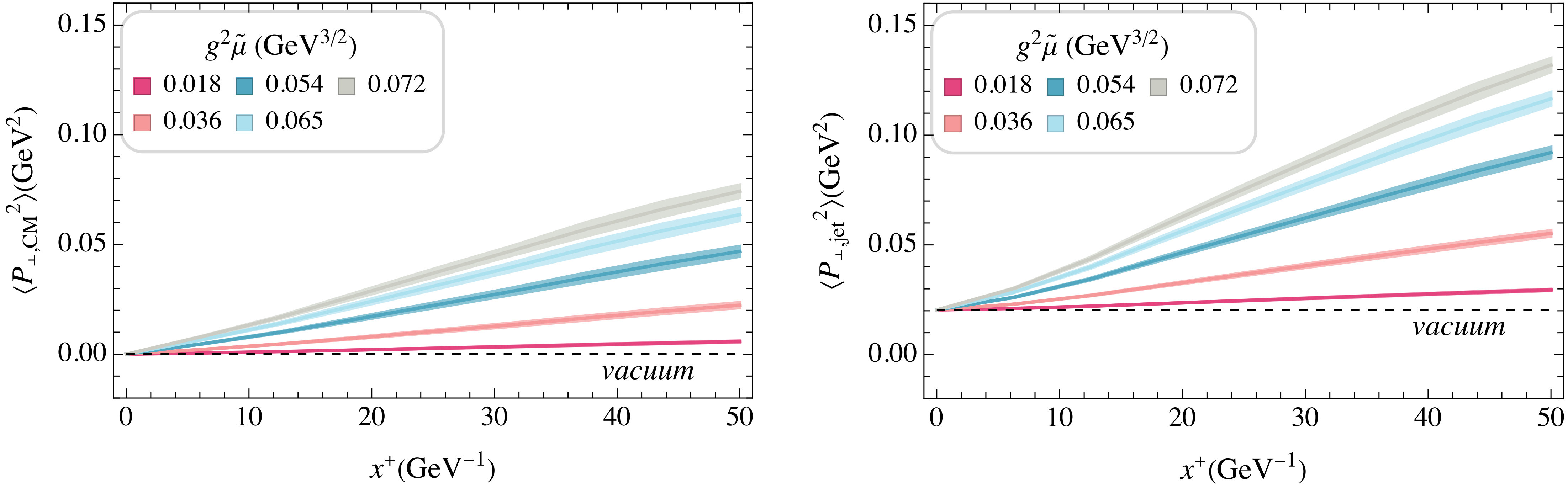
  - Jet momentum, taking away external gluons

$$\langle P_{\perp,jet}^2 \rangle = \Sigma_d^{dressed\ quark} |\psi_d^2| \langle P_{\perp}^2 \rangle_d + \Sigma_e^{excited\ states} |\psi_e^2| \langle P_{\perp,q}^2 \rangle_e$$



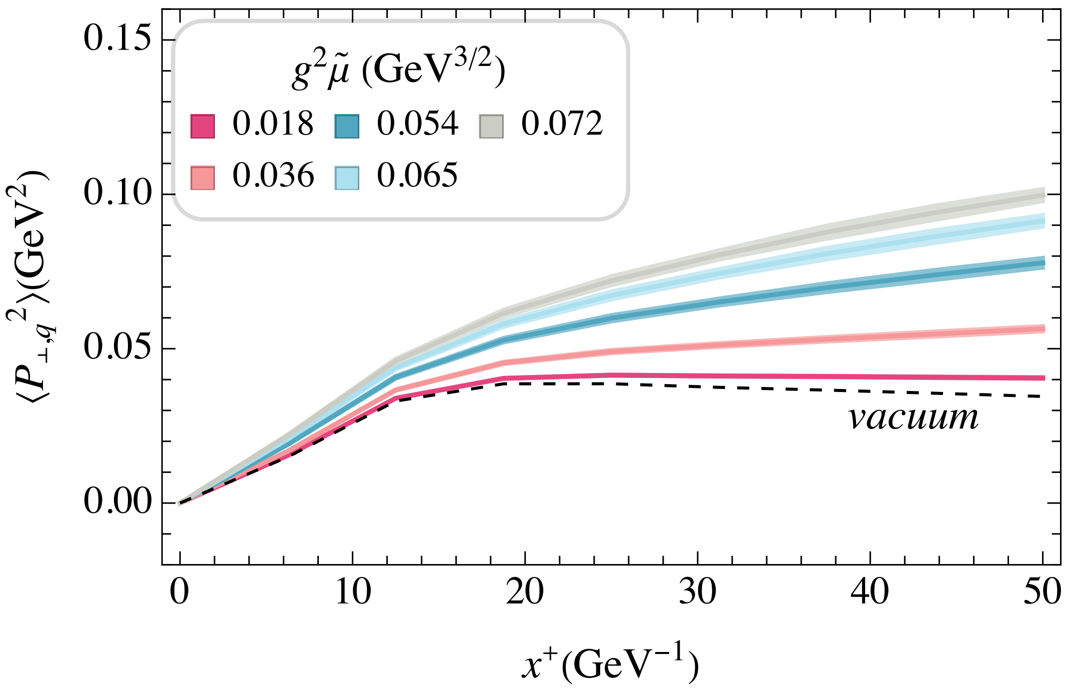
Evolution results: momentum broadening

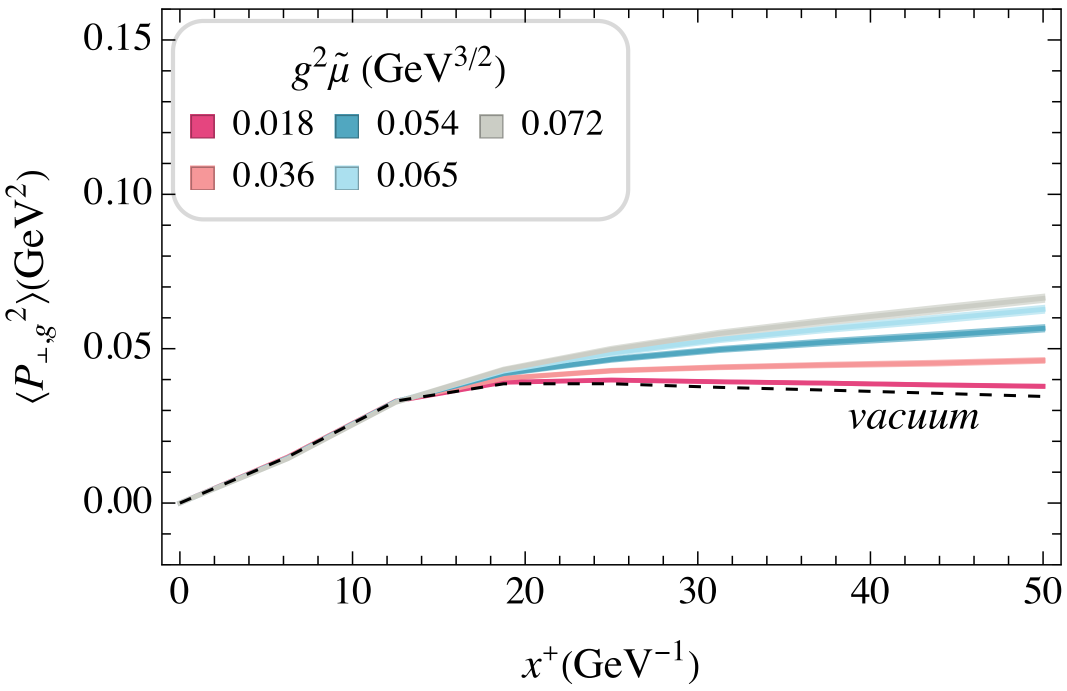
- **Bare quark initial state**
  - Momentum broadening is more profound at a stronger medium
  - Jet momentum is larger than CM momentum



Evolution results: momentum broadening

- **Bare quark initial state**
  - Quark and gluon momenta change in vacuum
  - Quark momentum increases faster than gluon in medium





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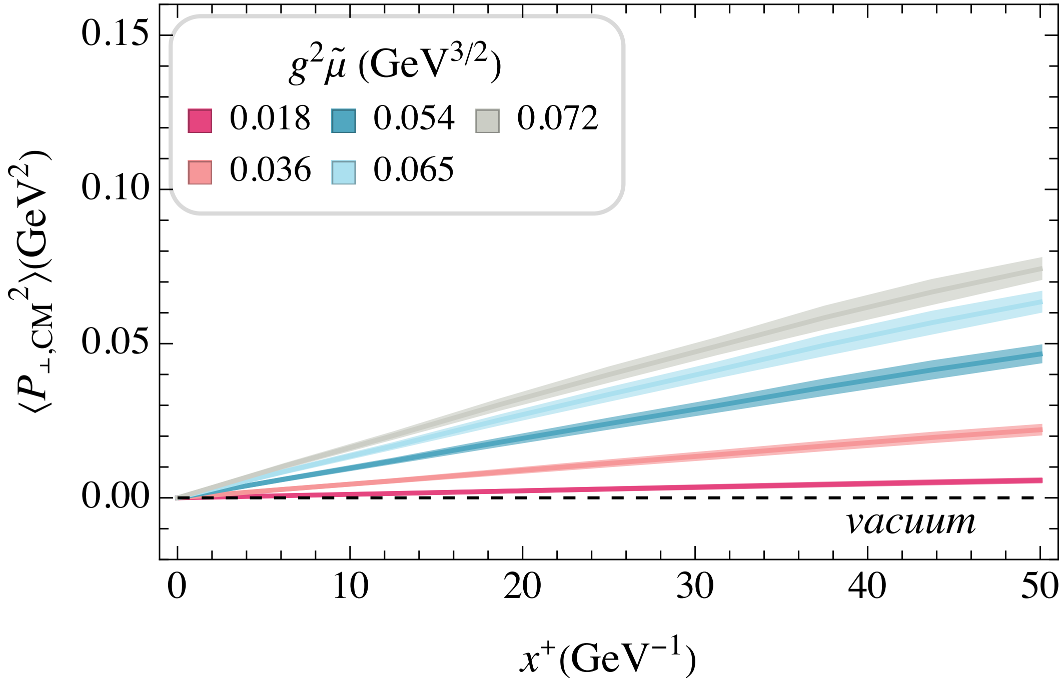

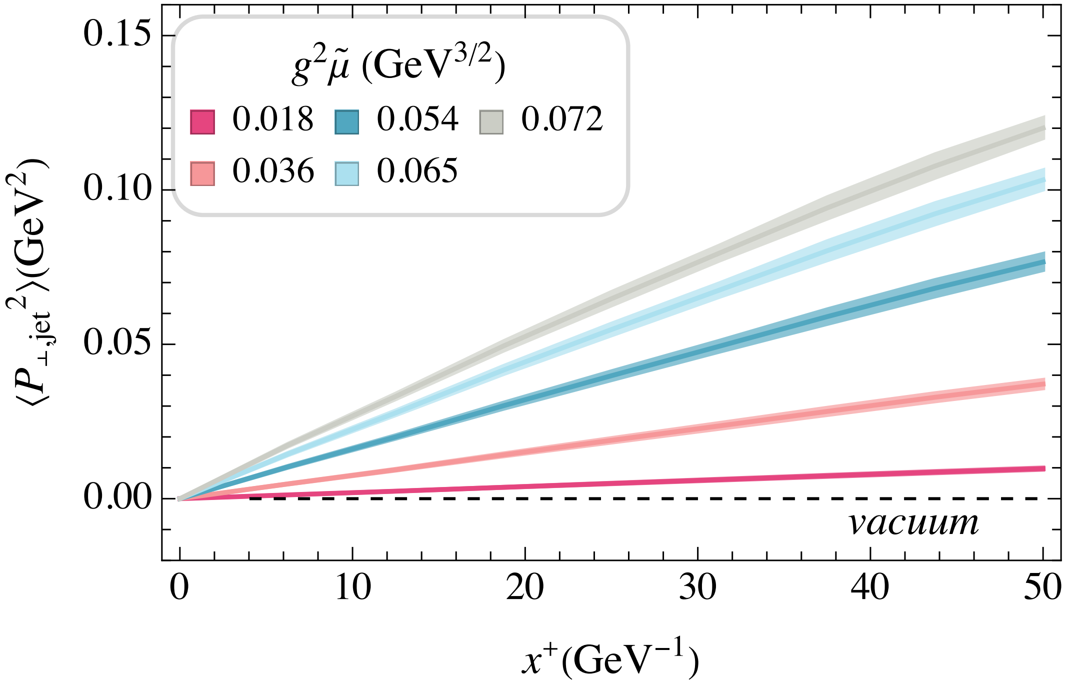

Evolution results: momentum broadening

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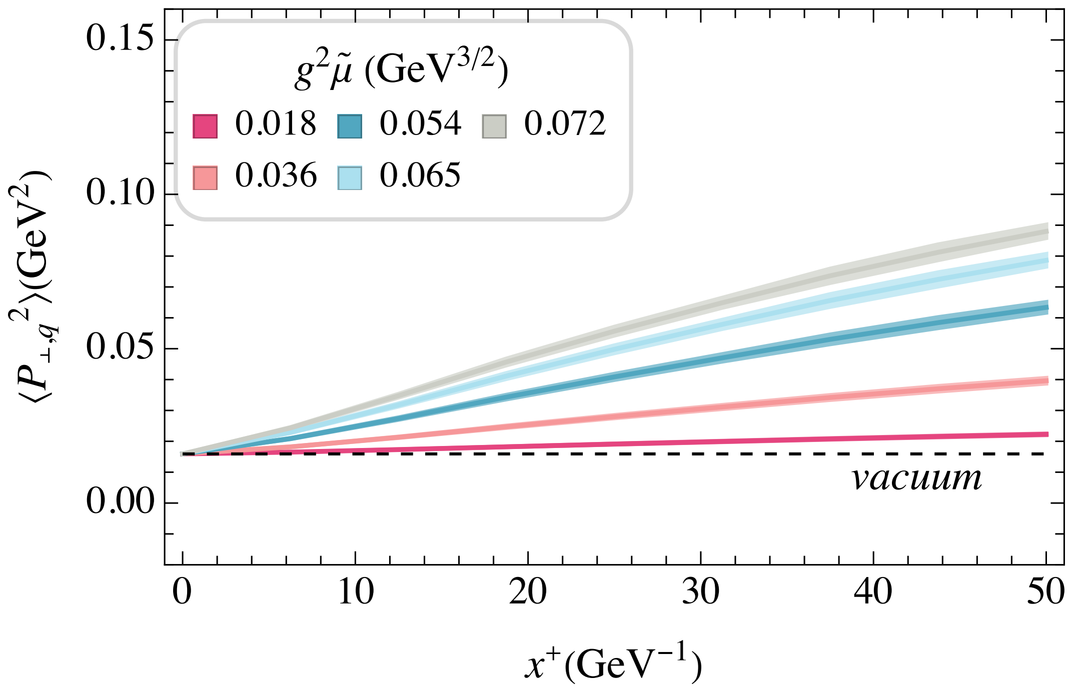
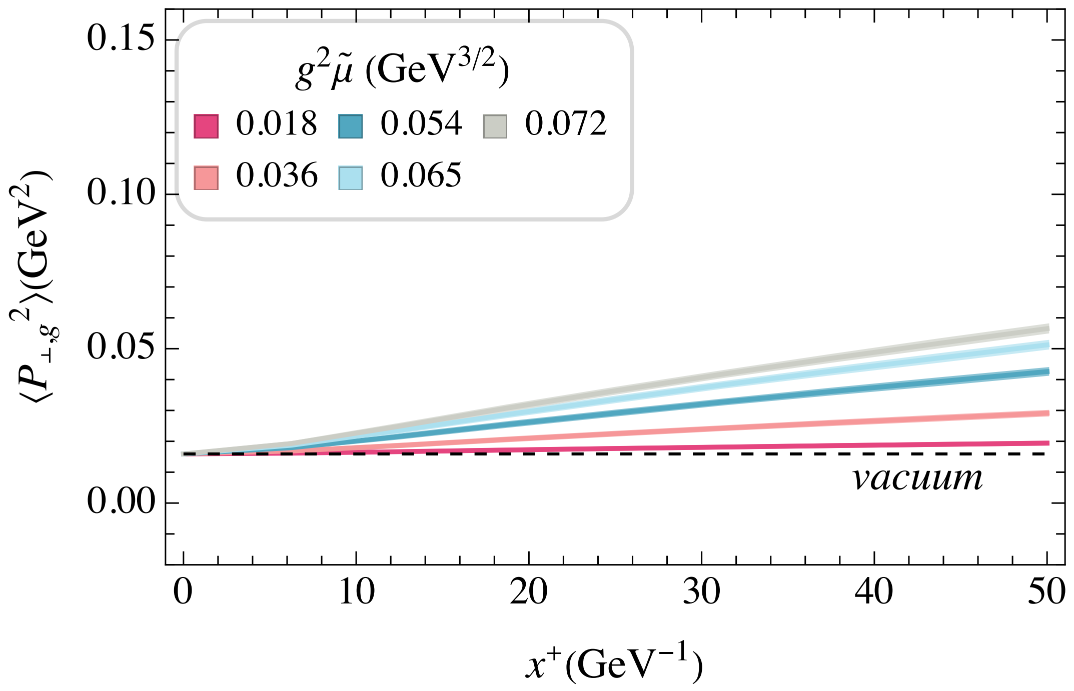
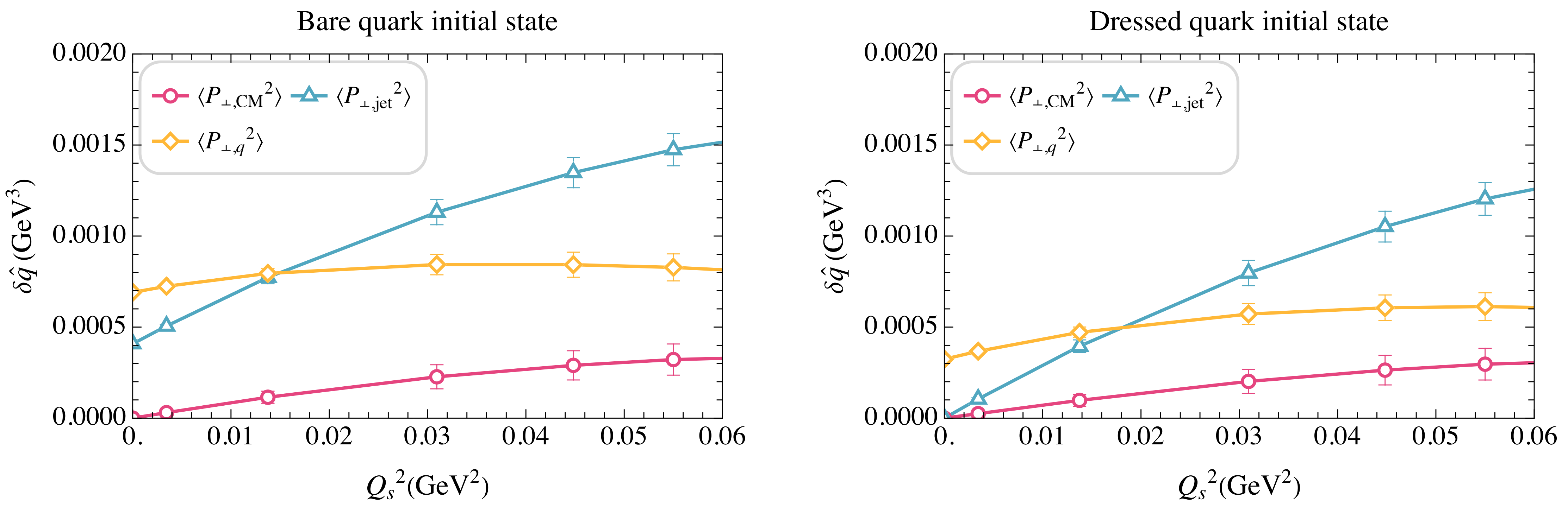




Evolution results: momentum broadening

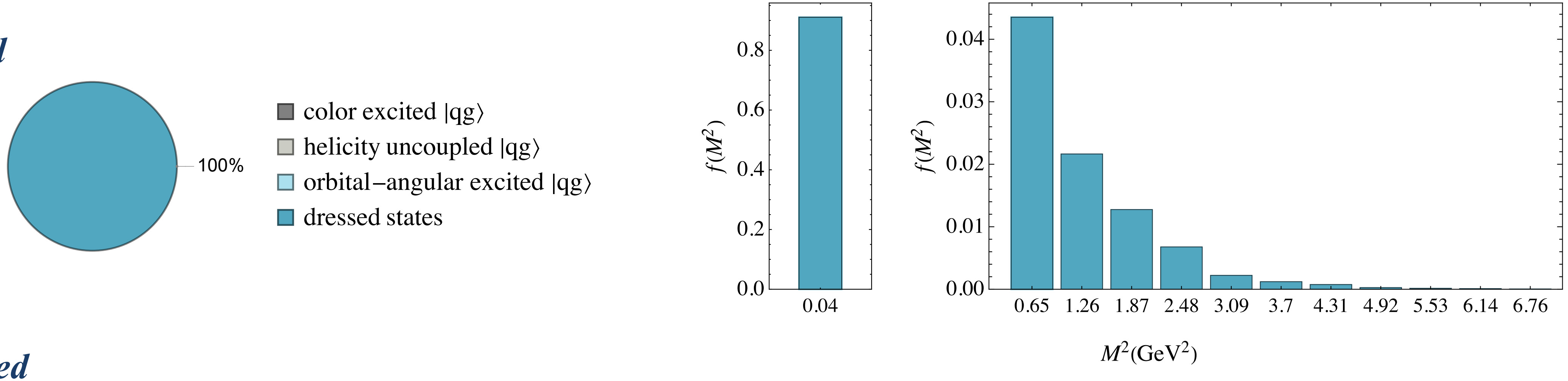
- **“NLO” Fock sector contribution**
  - Quenching parameter $\hat{q} \equiv \Delta\langle P_\perp^2\rangle/\Delta x^+$
  - Non-eikonal effect: $\delta\hat{q} = \hat{q} - \hat{q}_{single\ q}$



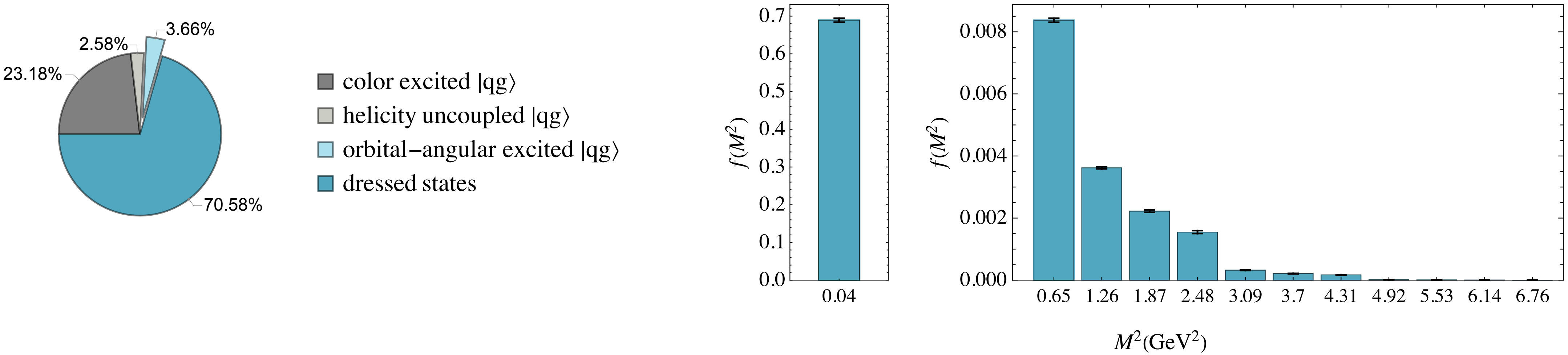
Evolution results: spectral distribution

- **Bare quark initial & evolved states**

initial



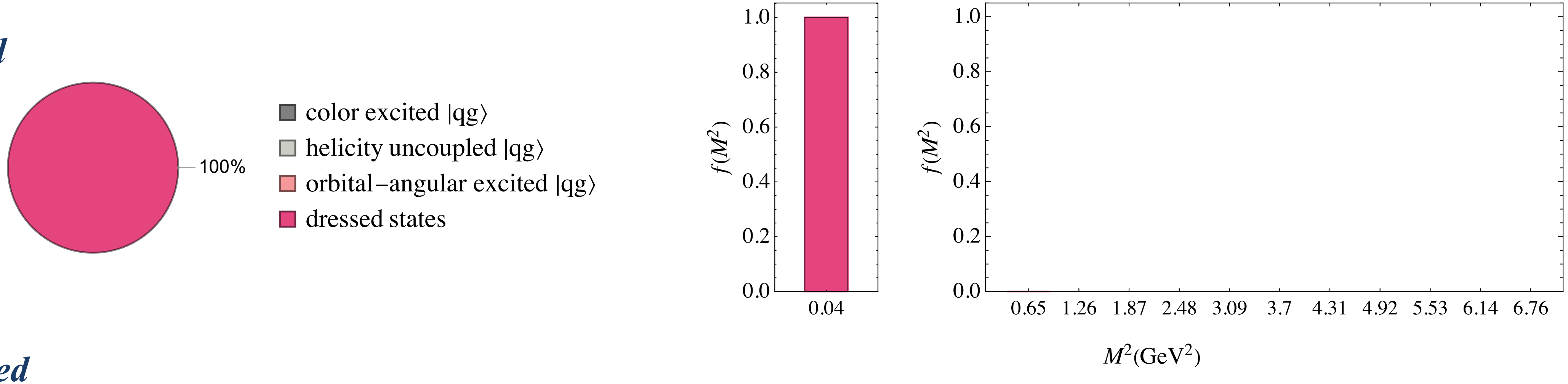
evolved



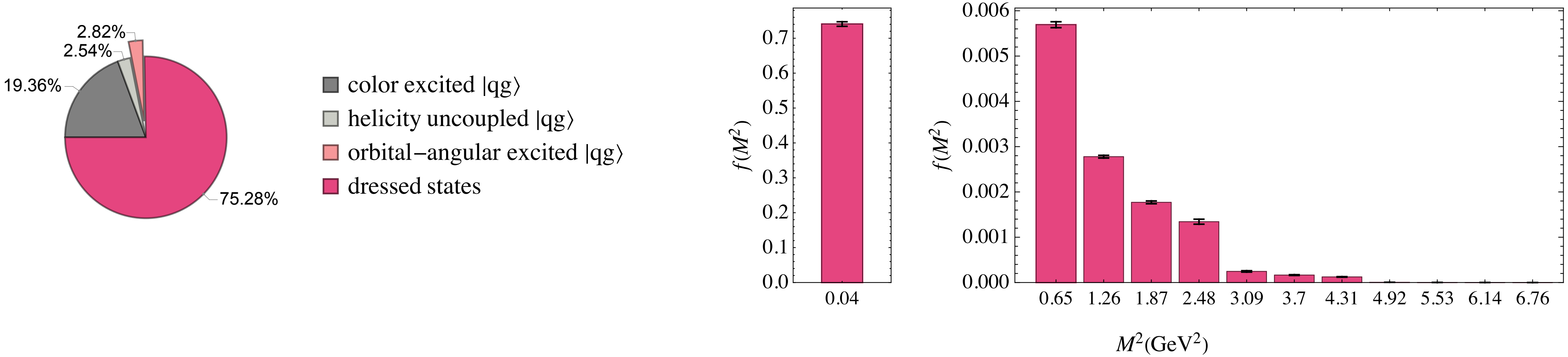
Evolution results: spectral distribution

- **Dressed quark initial & evolved states**

initial



evolved



Summary and outlooks

- We applied a light-front Hamiltonian approach, BLFQ and tBLFQ, to study in-medium quark jet evolution:
  1. we obtained the dressed quark states and the excited states
  2. we analyzed gluon emission from non-perturbative perspectives
  3. we extracted non-eikonal effect of momentum broadening

- Further applications
  1. Jet phenomenon, in-jet and dijet structures
  2. Quantum simulation of QCD jets → [W. Qian's seminar on Thursday]
  3. Jet evolution in Glasma

Thank you!

Evolution results: cross section

- **Bare/dressed quark scattering off the medium**

$$\frac{d\sigma}{d^2b} = \sum_{\phi_{out}} |M(\phi_{out};\psi_{in})|^2 = \sum_{\phi_{out}} |\langle\phi_{out}|\psi_{out}\rangle - \langle\phi_{out}|\psi_{in}\rangle|^2$$