High Energy Jets
High Energy Behaviour for the LHC

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with HEJ collaborators

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Durham University

IP3

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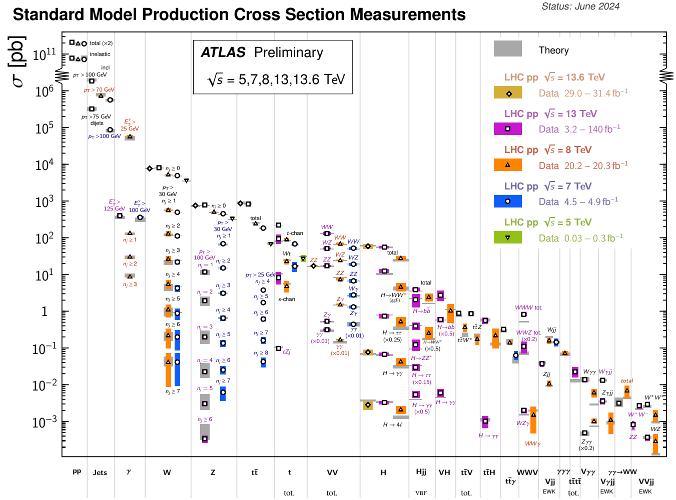
Lessons on Perturbative QCD at High Energies



Incredible agreement between fixed order calculations and total cross sections.

Will discuss situations where perturbative corrections **beyond fixed order** are required to obtain good agreement for **distributions**.

This would be old news in the case of p_t-hierarchies (requiring p_t-resummation, parton showers etc.)

New situation: considerations of logarithmic corrections of **High Energy-origin** are necessary for relevant distributions

Introduction

Outline of talk:

1. Amplitudes in the High Energy Limit
2. From amplitudes to cross sections
3. Progress towards full next-to-leading logarithmic accuracy

High Energy Jets:

- **Factorisation of matrix elements** using **currents** retains analytic properties such as **crossing symmetries**
- systematic **power expansion of QCD amplitudes** for real emissions
- all-order **leading and sub-leading logarithmic corrections**
- Recent experimental results showcasing the importance of the high-energy corrections

Regge theory

Regge theory describes scattering from a **central potential** in terms of the projections on Legendre polynomial and states of **definite orbital angular momentum** (partial wave analysis)

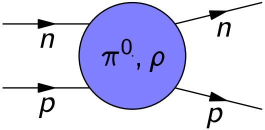
The analysis of **analytic scattering amplitudes** in terms of Regge Theory: Regge (1959)

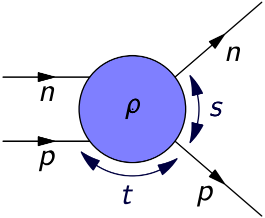
$$\mathcal{M} = \sum_i \Gamma_i(t)\,(s)^{j_i}$$

At **large energies** s, the contribution from particle of **highest spin** j **dominates**

$$\mathcal{M} \to \Gamma(t)\,(s)^{j}$$

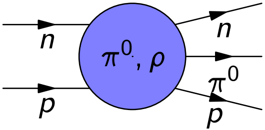
Regge limit: $s \gg -t$ or $s \gg p_t^2$





Multi-Regge theory

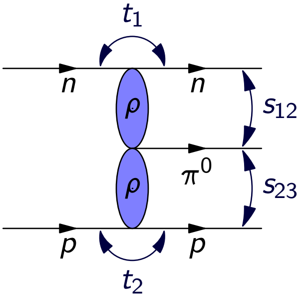
Large $\boldsymbol{s}$ of course leads to the possibility of **multi-particle production**



Multi-Regge **limit**: Brower, DeTAR, Weis (1974)

$$s_{12}, s_{23} \gg p_{t_i}^2, |t_i|, \quad |t_i| \sim |t_j|, \quad |p_{t_i}| \sim |p_{t_j}|$$

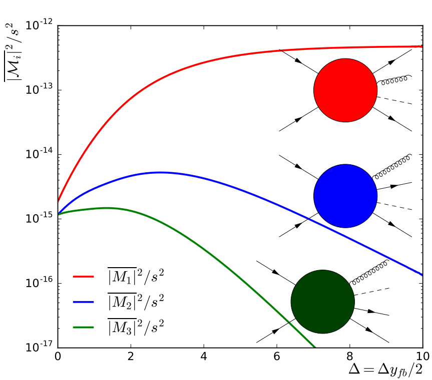
$$\mathcal{M} = s_{12}^j s_{23}^j\, \Gamma\left(t_1, t_2, s/(s_{12}s_{23})\right)$$

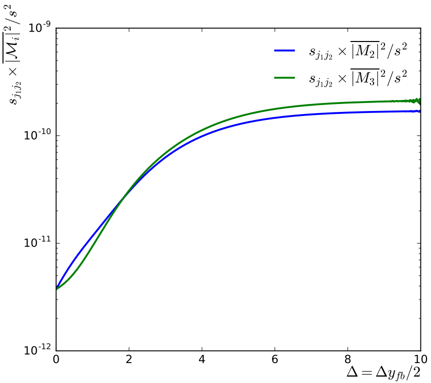


No underlying theory for strong interactions; derives constraints on the high energy behaviour based on the constraints from an **analytic scattering amplitude**.

Scaling of QCD Amplitudes

All QCD tree-level processes involving also Higgs bosons, W, Z and photon production respect the behaviour deduced by the partial wave analysis





The **scaling** for different kinematic evaluations of the same amplitude is exactly as predicted by Regge theory applied to the **planar graph** connecting the rapidity-ordered configuration.

M. Heil, A. Maier, J.M. Smillie, JRA, arXiv:1706.01002

Perurbative Corrections in the High Energy Limit

Building approximations used for all-order evaluations. Standard approach:

$qQ \to qQ : \overline{|\mathcal{M}|}^2 \propto \frac{s^2 + u^2}{t^2}.$

In the high energy limit $s \sim u \gg t \to -k_\perp^2$ so $\hat{\sigma} \propto \frac{1}{k_{1,\perp}^2 k_{2,\perp}^2}$.

In the limit where all $s_{ij} \gg p_t^2$ $2 \to n$ (in LL configurations) factorise: $\hat{\sigma} \propto \prod_i^n \frac{1}{k_{i,\perp}^2}$.

Since the $2 \to 3$ **real emision** perturbative corrections have $|M|^2/s^2 \to \mathrm{constant}$ for large $\Delta y_{fb} \sim \log(s/p_t^2)$, integration over the rapidity of the middle parton will contribute a correction $\alpha_s \Delta y_{fb} \sim \alpha_s \log(s/p_t^2)$.

The other orderings of momenta (and other processes) contribute sub-leading corrections which can be included at next-to-leading order.

Perurbative Corrections in the High Energy Limit

The **virtual** corrections also exhibit universal logarithmic terms in the **colour octet** channel

$$\mathcal{A}_4^{1-loop}(\bar{q},\bar{Q};Q,q) = g^4\left[\left(\delta_{i_1 i_3}\delta_{i_2 i_4} - \frac{1}{N_c}\delta_{i_1 i_4}\delta_{i_2 i_3}\right) a_{4;1}(1,2;3,4)\right] + \delta_{i_1 i_3}\delta_{i_2 i_4} a_{4;2}(1,2;3,4)$$

$$a_{4;1}(-,-;+,+) = c_\Gamma\ a_{4;0}(-,-;+,+)\ F_{a,1}^{--}(\varepsilon, s_{!2}, s_{13}, s_{14})$$

$$F_{a;1}^{--}(\varepsilon, s_{12}, s_{13}, s_{14}) = \left(-\frac{\mu^2}{s_{14}}\right)^{\varepsilon}\left\{N_c\left[-\frac{2}{\varepsilon^2} - \frac{3}{\varepsilon} + \frac{11}{3} - \frac{2}{\varepsilon}\log\frac{s_{14}}{s_{12}} + \frac{13}{9} + \pi^2\right] + \cdots\right\} - \frac{1}{\varepsilon}\beta_0$$

Logarithmic structure predicted to all orders (BFKL, Regge, VDD,...)**.**

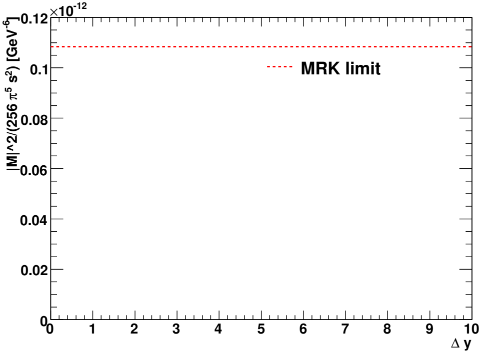
Control perturbative corrections of $\alpha_s^n \log^n(s/p_t^2)$ (leading logarithm) and $\alpha_s^{n+1}\log^n(s/p_t^2)$ (NLL)

Comparison of 3-jet scattering amplitudes

Study the Born level $qQ \to qgQ$ in just a slice in phase space:

40GeV jets in transverse Mercedes.

Rapidities at $-\Delta y, 0, \Delta y$.



JRA, J.M. Smillie, arXiv:0908.2786

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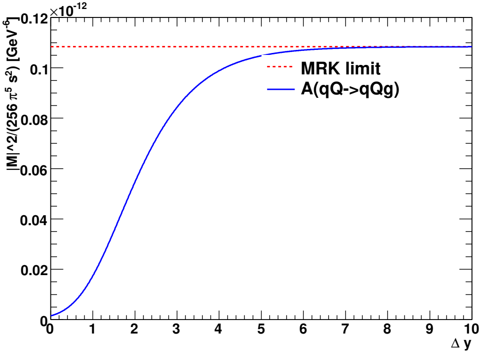

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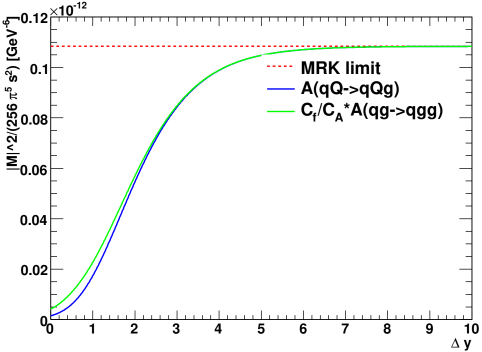

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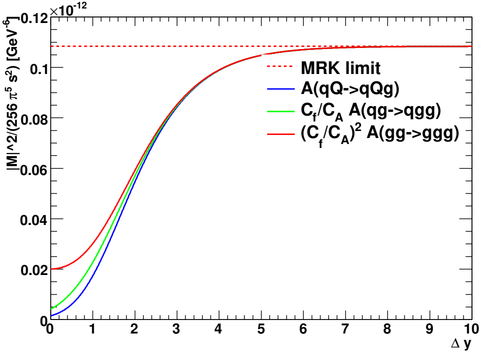

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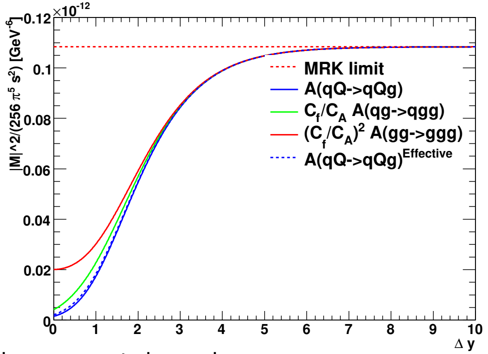

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High Energy Jets (HEJ):

1) Inspiration from Fadin&Lipatov: Factorisation across t-channel
2) Maintain analyticity and crossing symmetry
3) Respect Lorentz invariance
4) Exact Gauge invariance. Not just asymptotically.

Analytic Constraints

How did the high energy limit get it so wrong for such a simple process?
Need a method to "analytically reconstruct" the amplitude from the understanding of the high energy behaviour.

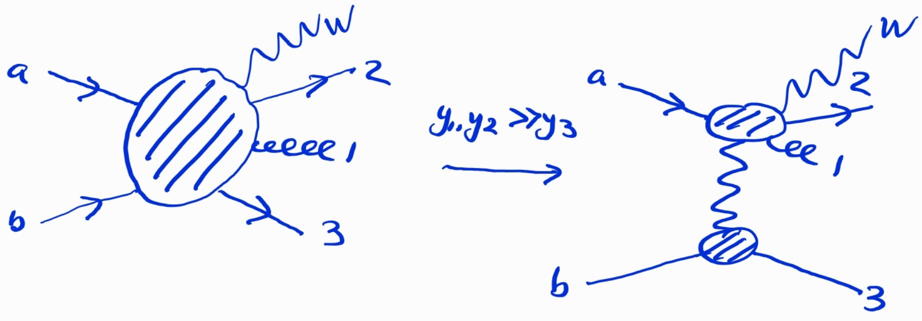
$$\mathcal{M} \propto j_\mu(p_a, p_1)\, j^\mu(p_b, p_2)/t$$

- Simple description; each current depends on momenta of relevant quarks only.
- Same helicity: contributes s, opposite contributes u.
- Ensures **crossing symmetry**, analyticity, Lorentz invariance.

Will require these constraints on the "analytic reconstruction" by insisting on contractions of currents.
Even $gq \to qg$ and $gg \to gg$ factorises into "currents" depending only on "local" momenta.
Higher logarithmic corrections will require two-particle production currents...
All these are gauge invariant.

NLL Real components for Reggeisation

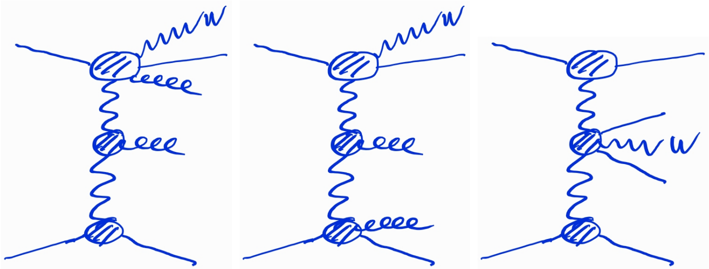
Consider $pp \to W3j$. Next-to-leading logarithmic corrections arise e.g. from the amplitudes in the quasi-multi-Regge-kinematic limit, where the invariant mass between one pair of partons is not large.



Amplitude expressed as $\mathcal{M} = I^\mu(a, w, 1, 2)\ J_\mu(b, 3)/t$. Full crossing symmetry, gauge invariance etc. in each component.

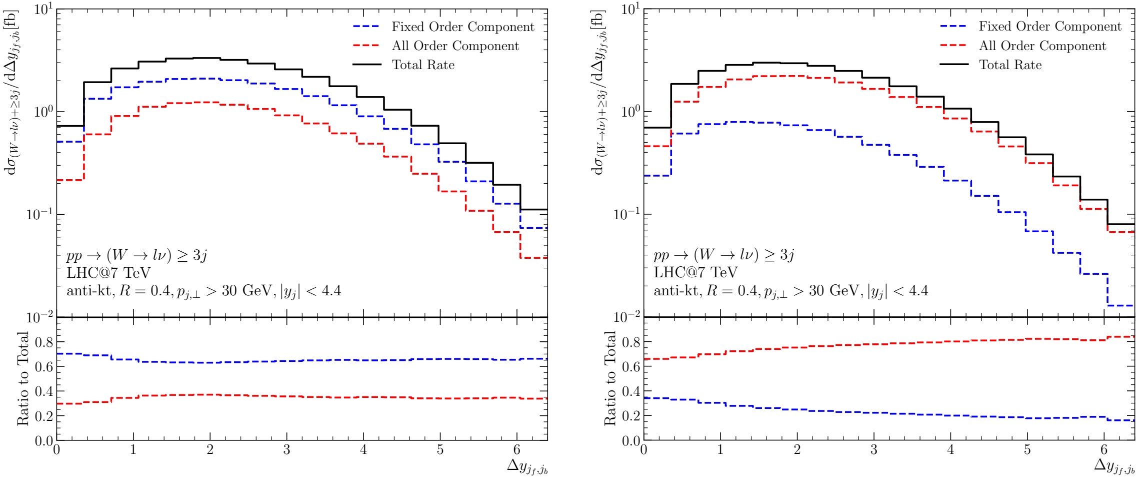
NLL Real components for Reggeisation

Can calculate higher order corrections with NLL components by explicit MC integration over the regulated amplitudes, represented by a Reggeised graph



Virtual corrections encoded in the t-channel propagators.
Matching: Sub-processes and phase space points not reached with LL or NLL Reggeised description are treated at fixed order (additive). Resummation points are matched to n-jet matrix elements (multiplicative).

Impact of NLL corrections for W3J

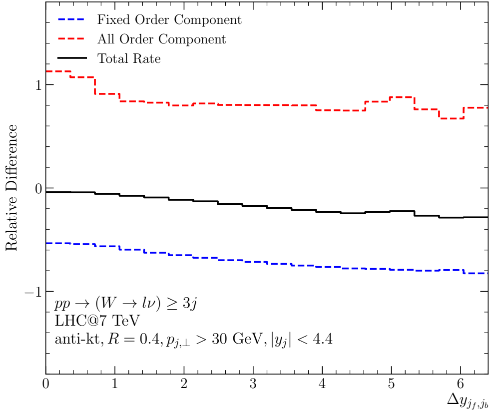


LL channels resummation only

sub-leading channels included

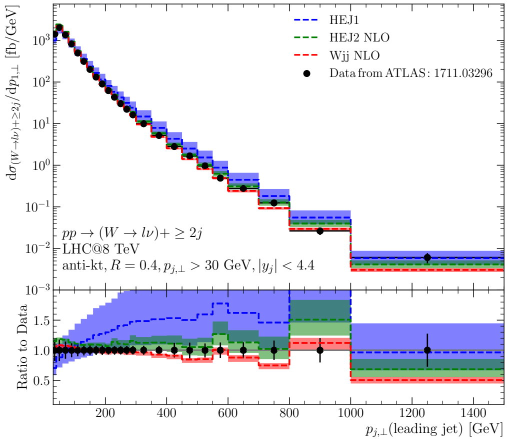
arxiv:2012.10310

Impact of NLL corrections for W3J



Much less fixed order matching, much bigger resummation component. Final result of the inclusive distribution changes by no more than 25%. arxiv:2012.10310

Comparison to Data (WJJ)



The NLL terms included and improvement in matching are sufficient to ensure the predictions agree well with data even in the most difficult regions of phase space. arxiv:2012.10310

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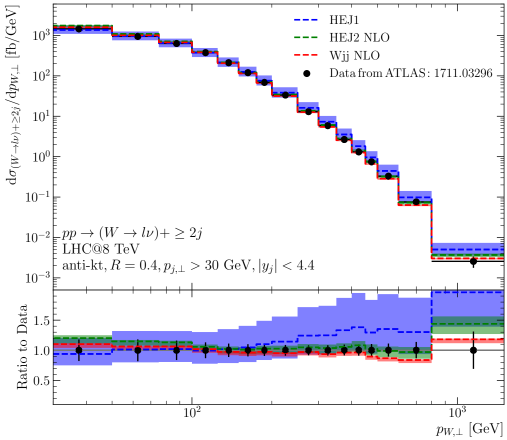
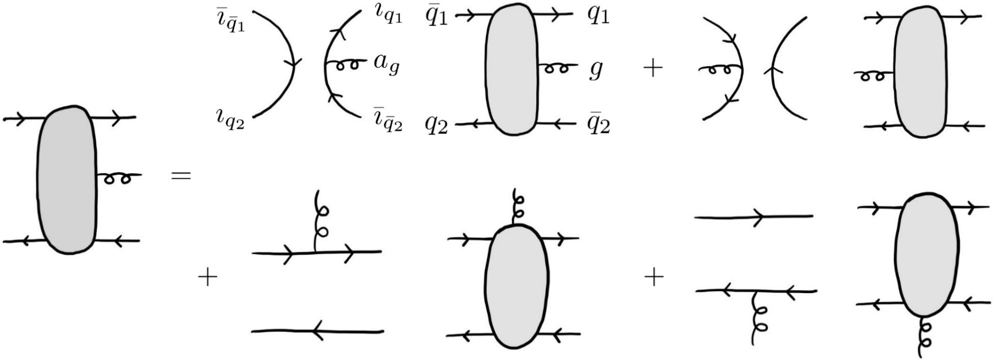


The NLL terms included and improvement in matching are sufficient to ensure the predictions agree well with data even in the most difficult regions of phase space.

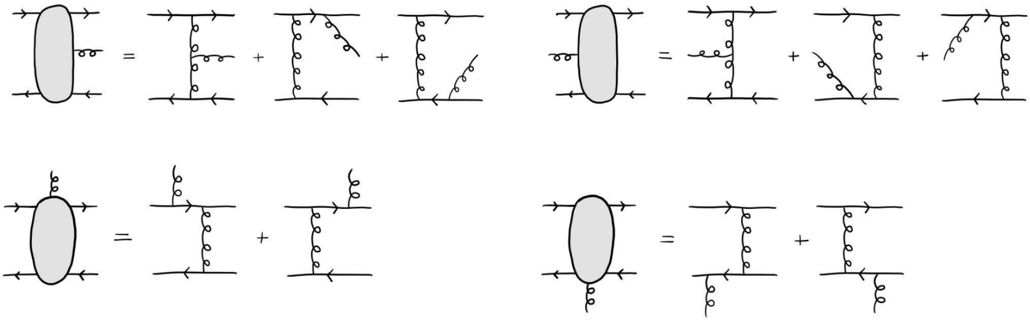
Calculation of two-particle current

Extract two gluon current from 2 $\to$ 3 amplitude (see thesis of E. Byrne):



The use of a colour basis helps ensuring gauge invariance of the few approximations which are necessary

Calculation of two-particle current, III



Most contributions already kinematically factorised. For some helicity configurations the factorisation is exact.

Calculation of two-particle current, IV

$$M(\bar{q}_1^{\ominus}, q_1^{\oplus}, \bar{q}_2^{\ominus}, q_2^{\oplus}, g^{\lambda_g}) = J_{\bar{q}qg^*\,\mu_{t_1}}(p_{\bar{q}_1}^{\ominus}, p_{q_1}^{\oplus}, p_{t_1}) \left(\frac{-i}{t_1}\right) J^{\mu_{t_1}}_{\bar{q}qgg^*}(p_{\bar{q}_2}^{\ominus}, p_{q_2}^{\oplus}, p_g^{\lambda_g}, -p_{t_1}),$$

$$J^{\mu_{t_1}}_{\bar{q}qgg^*}(p_{\bar{q}_2}^{\ominus}, p_{q_2}^{\oplus}, p_g^{\lambda_g}, -p_{t_1}) = -ig_s^2 \epsilon_{\mu_g}(p_g^{\lambda_g}, p_r) \Bigg\{ \frac{1}{s_{q_2 g}} [q_2|\mu_g(q_2+g)\mu_{t_1}|\bar{q}_2\rangle$$
$$+ \frac{1}{t_2}\left(\eta^{\mu_{t_1}\mu_{t_2}}(p_{t_1}+p_{t_2})^{\mu_g} - 2p_g^{\mu_{t_2}}\eta^{\mu_{t_1}\mu_g} + 2p_g^{\mu_{t_1}}\eta^{\mu_{t_2}\mu_g} - t_1 \frac{2p_{\bar{q}_1}^{\mu_g}}{s_{\bar{q}_1 g}}\right)[q_2|\mu_{t_2}|\bar{q}_2\rangle \Bigg\}.$$

Gauge invariance: J evaluated with $\epsilon_{\mu_g} \to p_g^{\mu}$ is 0.
Lorentz invariance: Contraction of four vectors
Crossing symmetry: Observed by each component
In this form all the $1/\varepsilon$ poles are maintained. Same collinear and soft structure as full QCD. Requires no approximation to the virtual corrections (the NLO expansion of the virtual corrections agrees with the virtual corrections in full QCD). Work in progress.

Resummation

Consider the **perturbative expansion** of an observable

$$R = r_0\alpha^2 + r_1\alpha_s^3 + r_2\alpha^4 + r_3\alpha^5 + r_4\alpha^6 + \cdots$$

Fixed order pert. QCD will calculate a fixed number of terms in this expansion. For multi-scale processes r_n may contain **large logarithms** so that $\alpha_s \ln(\cdots)$ is large.

$$R = \left(r_0^{LL} + r_0^{SL}\right)\alpha_s^2 + \left(r_1^{LL}\ln(\cdots) + r_1^{NLL} + r_1^{SL}\right)\alpha_s^3 + \cdot$$
$$= \alpha_s^2 \sum_{n=0} r_n^{LL}(\alpha_s \ln(\cdots))^n + \alpha_s^3 \sum_{n=1} r_n^{NLL}(\alpha_s \ln(\cdots))^{n-1} + \text{sub-leading terms}$$

Replace the perturbative parameter α_s with $\boldsymbol{\alpha_s \ln(\cdots)}$. Useful if the terms can be **summed to all orders** in the perturbative expansion (LLA, NLLA).

The High Energy Logarithm

The High Energy Logarithm has as argument the hierarchy between the partonic centre of mass and the transverse jet momentum $\ln(\hat{s}/p_t^2) \sim \Delta y_{fb}$. This will dominate the behaviour of the perturbative corrections at large $\hat{s}/pt^2$.

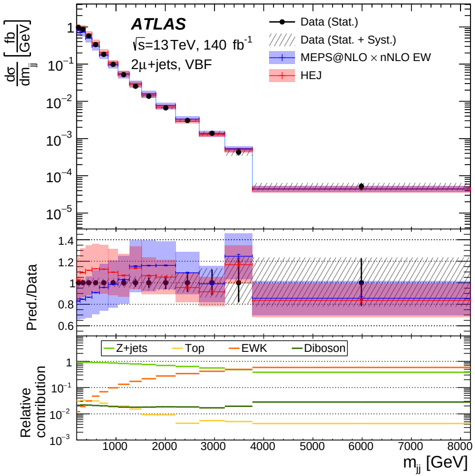
Relevant for regions of large m_{jj} (VBF, VBS,...). Obviously even more important for higher energy colliders, but will here discuss recent results from LHC. Already clear effects in the relevant regions.

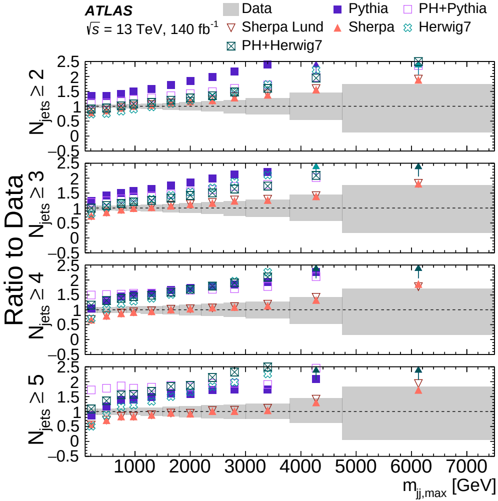
Two ATLAS studies:

- arxiv:2405.20206 $R_{32} = \sigma_{3j}/\sigma_{2j}$ studied vs. m_{jj}.
- arxiv:2403.02793 Missing transverse momentum and jets.

Systematic deviations in $\log \hat{s}/p_t^2 \sim \Delta y$ for perturbative predictions not controlling the high energy logarithmic corrections.

$d\sigma/d\max(m_{jj})$



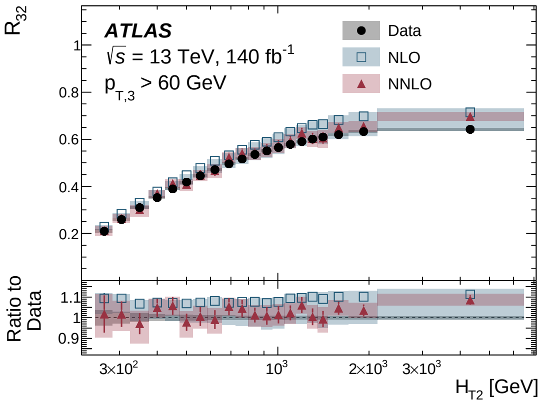


Systematic deviations in $\ln(m_{jj})$ for perturbative predictions not controlling the high energy logarithmic corrections.

R_{32} measurement

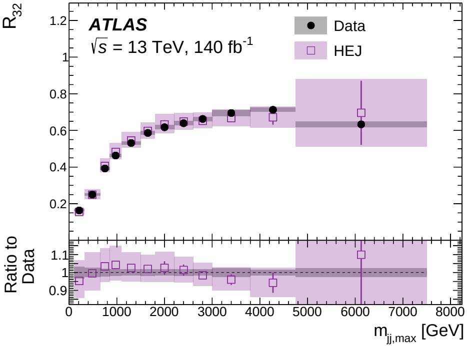
$R_{32} = \frac{\sigma_{3j}}{\sigma_{2j}}$ both measured vs. H_{T2}, m_{jj} and rapidity differences.
Simple cuts: Jets defined with anti-k_t, $R = 0.4$, $p_t > 60$GeV/c, $|y| < 4.4$.
arxiv:2405.20206

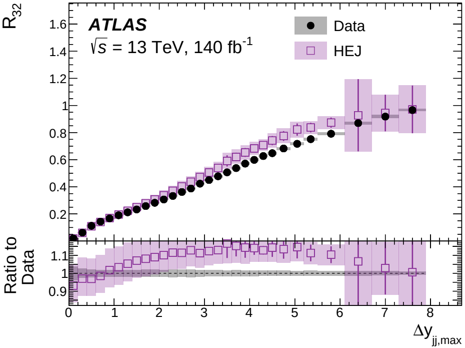
R_{32}



Plot from arxiv:2405.20206 (ATLAS).
Fixed order obtains reliable prediction vs. transverse momentum

R_{32}





Plots from arxiv:2405.20206 (ATLAS).

Data clearly has $R_{32} \sim \alpha_s \Delta y$. A leading logarithmic control should get the slope right (up to α_s corrections from NLL).

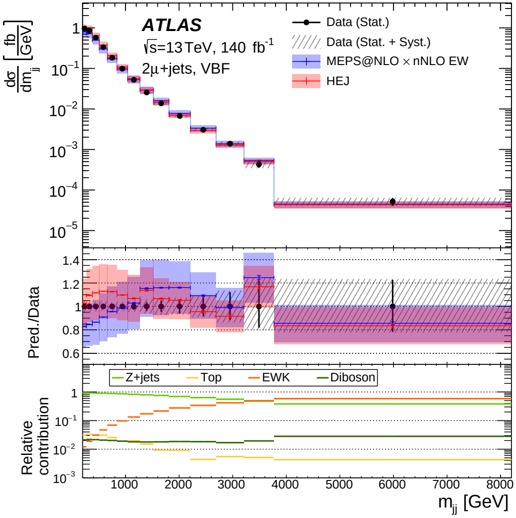
HEJ does indeed get the shape and slope right to within $\sim \alpha_s$ corrections.

Z plus jets

arxiv:2403.02793 ATLAS: Missing transverse momentum and jets.

Attribute	≥ 1 jet	VBF
$\Delta\phi\,(\text{jet}, p_{\mathrm{T}}^{\mathrm{miss}})$	> 0.4 for four leading p_{T} jets	
Hadronic τ-lepton	None with $p_{\mathrm{T}} > 20$ GeV, $\lvert\eta\rvert < 1.37$ or $1.52 < \lvert\eta\rvert < 2.47$	
Leading jet p_{T} [GeV]	> 120	> 80
Sub-leading jet p_{T} [GeV]	–	> 50
Leading jet $\lvert y\rvert$	< 2.4	< 4.4
Sub-leading jet $\lvert y\rvert$	–	< 4.4
Dijet invariant mass m_{jj} [GeV]	–	> 200
$\lvert\Delta y_{jj}\rvert$	–	> 1
In-gap jets	–	None with $p_{\mathrm{T}} > 30$ GeV

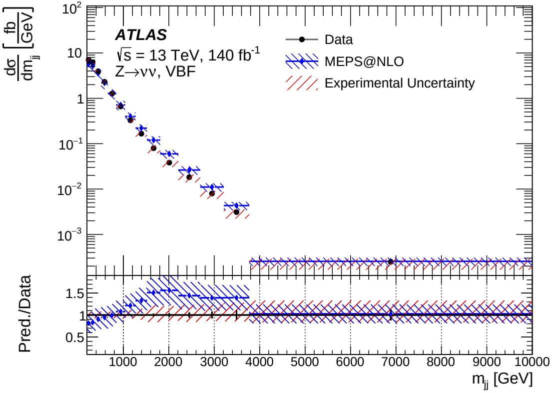
Z plus jets



Blue: State of the art prediction, but without high energy logarithms.
Red: High Energy Jets

Similar for W+jets arxiv:2403.02793 (ATLAS)

Z plus jets neutrino channel

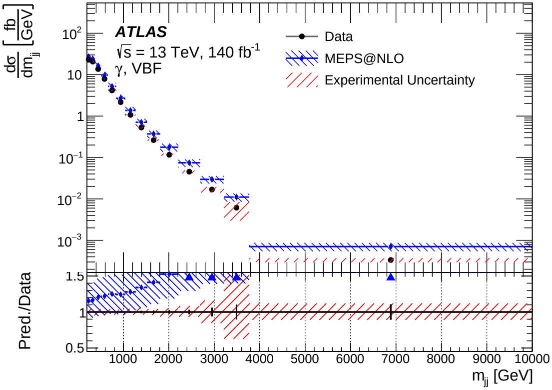


Blue: State of the art prediction, but without high energy logarithms.

High Energy Jets: In progress

arxiv:2403.02793 (ATLAS)

Photon + jets



Blue: State of the art prediction, but without high energy logarithms.

High Energy Jets: In progress

arxiv:2403.02793 (ATLAS)

Where can the understanding of HE logs help the experimental programme

A better understanding/description of the radiation pattern (i.e. where the additional jets are radiated) will help in suppressing the “QCD” background in VBF/VBS studies.

Starts impacting other measurements already at the LHC

The high energy logarithms will be even more important at any future collider - need to start preparing the predictions now, but we need input form experiments to understand which directions to push first.

Conclusions

Unsurprisingly, the inclusion of sub-leading logarithms leads to

- small changes in the leading regions of phase space
- a better description in sub-leading regions of phase space

Hall-marks of a well-behaved perturbative expansion.
Further improvements ongoing, calculation of full NLL currents with both virtual and real corrections using FKS regularisation.