Undecay

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Based on *JHEP 05 (2024) 158*, with
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Continuous BSM

Unparticle physics = SM coupled to CFT Georgi, '07

$$\mathcal{L}_{\text{SM}} + \langle \mathcal{O}\mathcal{O} \rangle \varphi^2 + \langle \mathcal{O}\mathcal{O}\mathcal{O} \rangle \varphi^3 + \ldots$$

$$\langle \mathcal{O}(x)\mathcal{O}(y) \rangle \propto (x-y)^{-2\Delta}$$

$$\langle \mathcal{O}(p)\mathcal{O}(-p) \rangle \propto (p^2)^{\Delta-2} + P_{[\Delta-2]}(p^2)$$

Spectral density: $(1 < \Delta < 2)$

$$\sigma(\mu^2) \propto (\mu^2)^{\Delta-2}$$

gapless continuum

Conformal breaking → Discretum

Conformal breaking → Gapped continuum

For instance,

$$\langle \mathcal{O}(p)\mathcal{O}(-p)\rangle \propto (p^2 - \mu_0^2)^{\Delta-2}$$

Pheno of charged continuum:

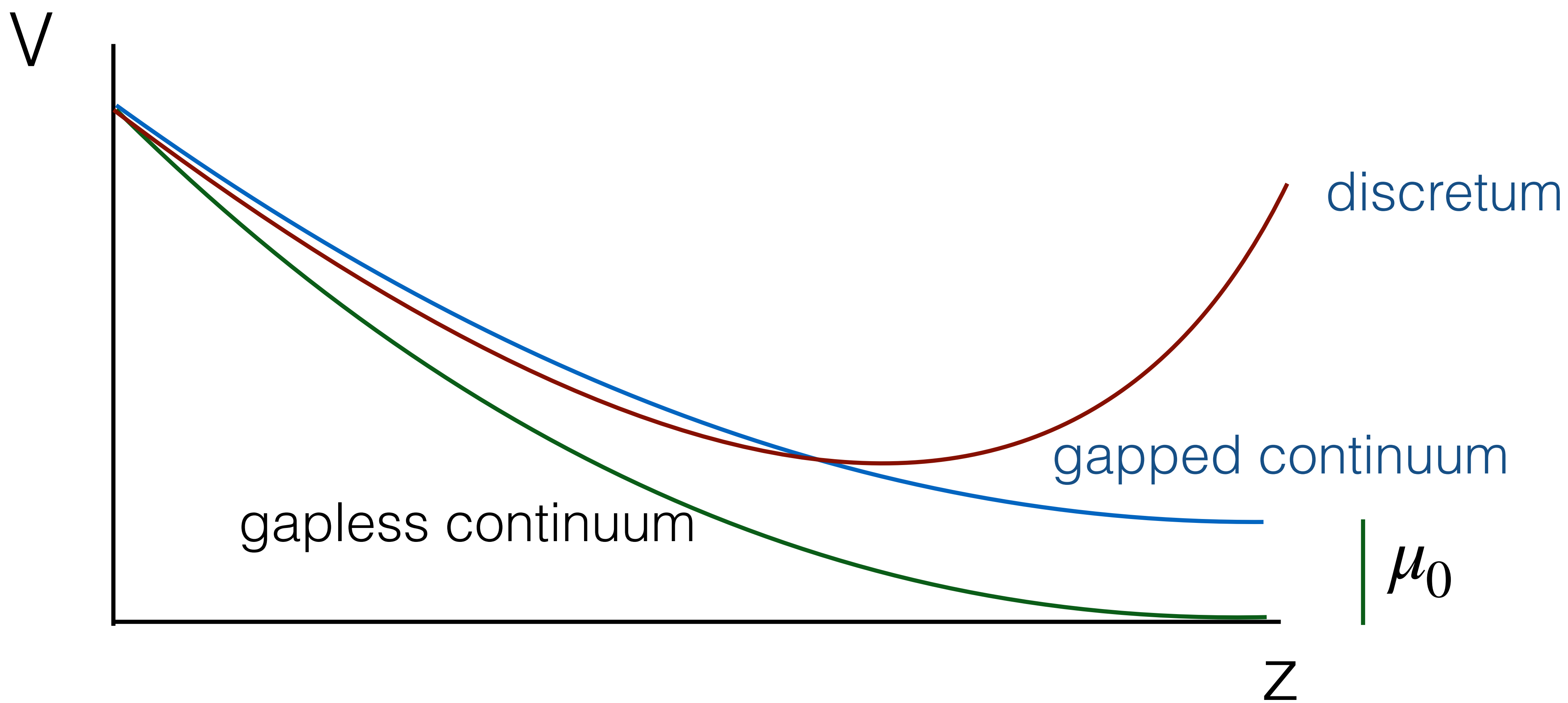
- UnHiggs scenario   Stancato, Terning '08; Falkowski, MPV '08, '09
- SM + continuum   Csaki et al '18

AdS/CFT

Cacciapaglia, Marandella, Terning '08

Falkowski, MPV '08

Schrödinger potential



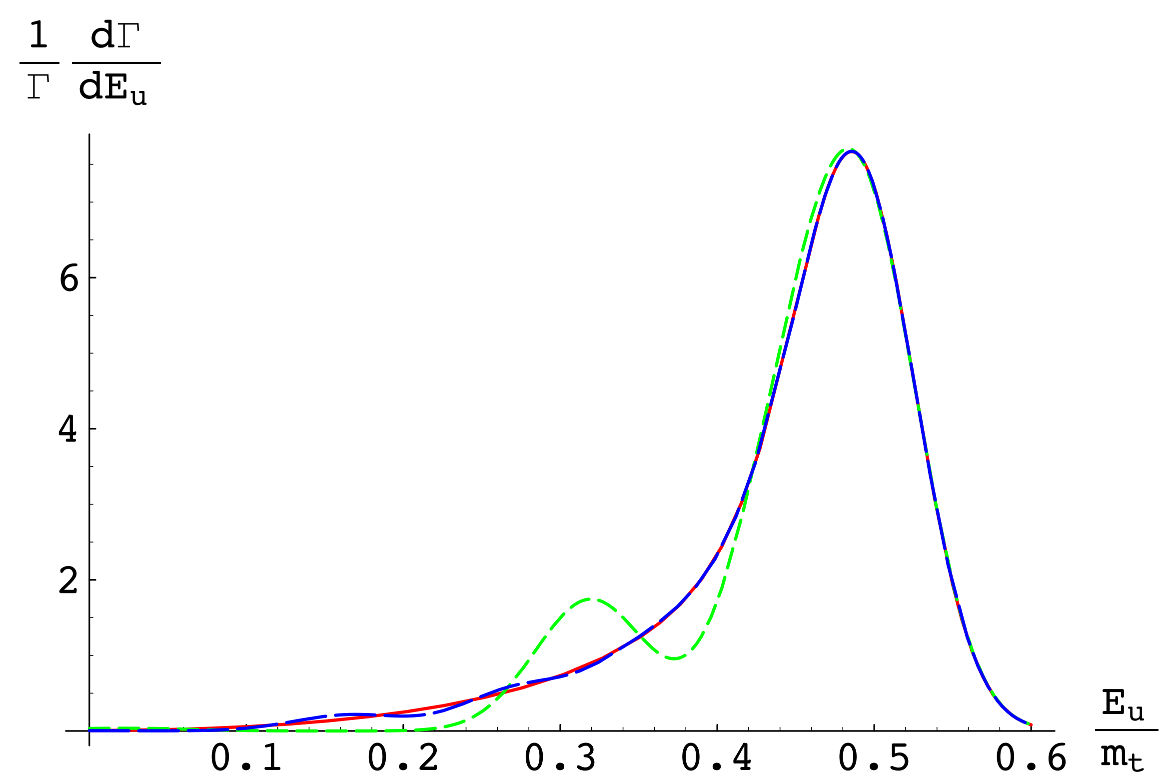
Compressed BSM

Examples of BSM with compressed discrete spectrum:

- Large extra dimensions — Arkani-Hamed, Dimopoulos, Dvali ’98; Giudice, Rattazzi, Wells ’98; Dudas, Dienes, Gherghetta ´00

- Clockwork models — Kaplan, Rattazzi ’15; Giudice, McCullough ‘16

Discretization ⟷ Continuum limit



MPV ‘08

$$\lambda \bar{u} \gamma^{\mu} (1-\gamma_5) t \partial_{\mu} \mathcal{O}_d + \mathrm{h.c.}$$

To decay or not to decay

Does unparticle stuff with linear couplings to the SM fields decay?

- Yes Delgado, Espinosa, No, Quirós '08 ⟵ Pole on 2nd Riemann sheet
- No Stephanov '07

$$\Delta_{\mathcal{U}} = \sum_n \frac{F_n^2}{q^2 - M_n^2 + i\varepsilon} \quad , \qquad F_n^2 \sim \Delta^2 \to 0.$$

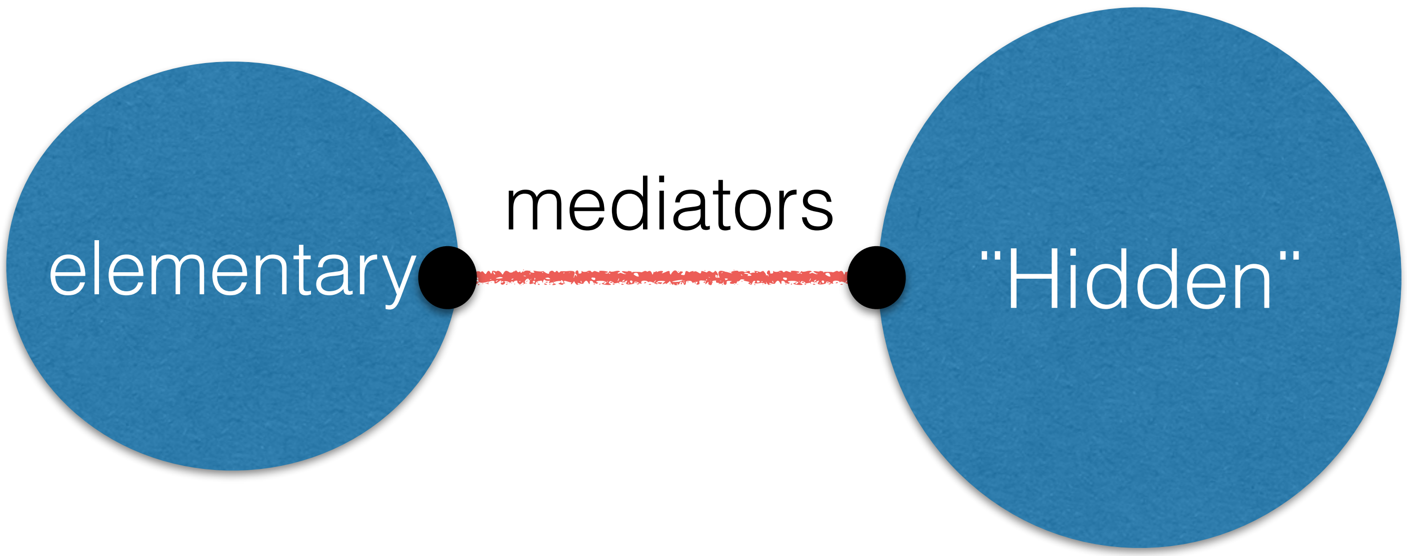
- Yes and no! This talk

  (to be clarified)

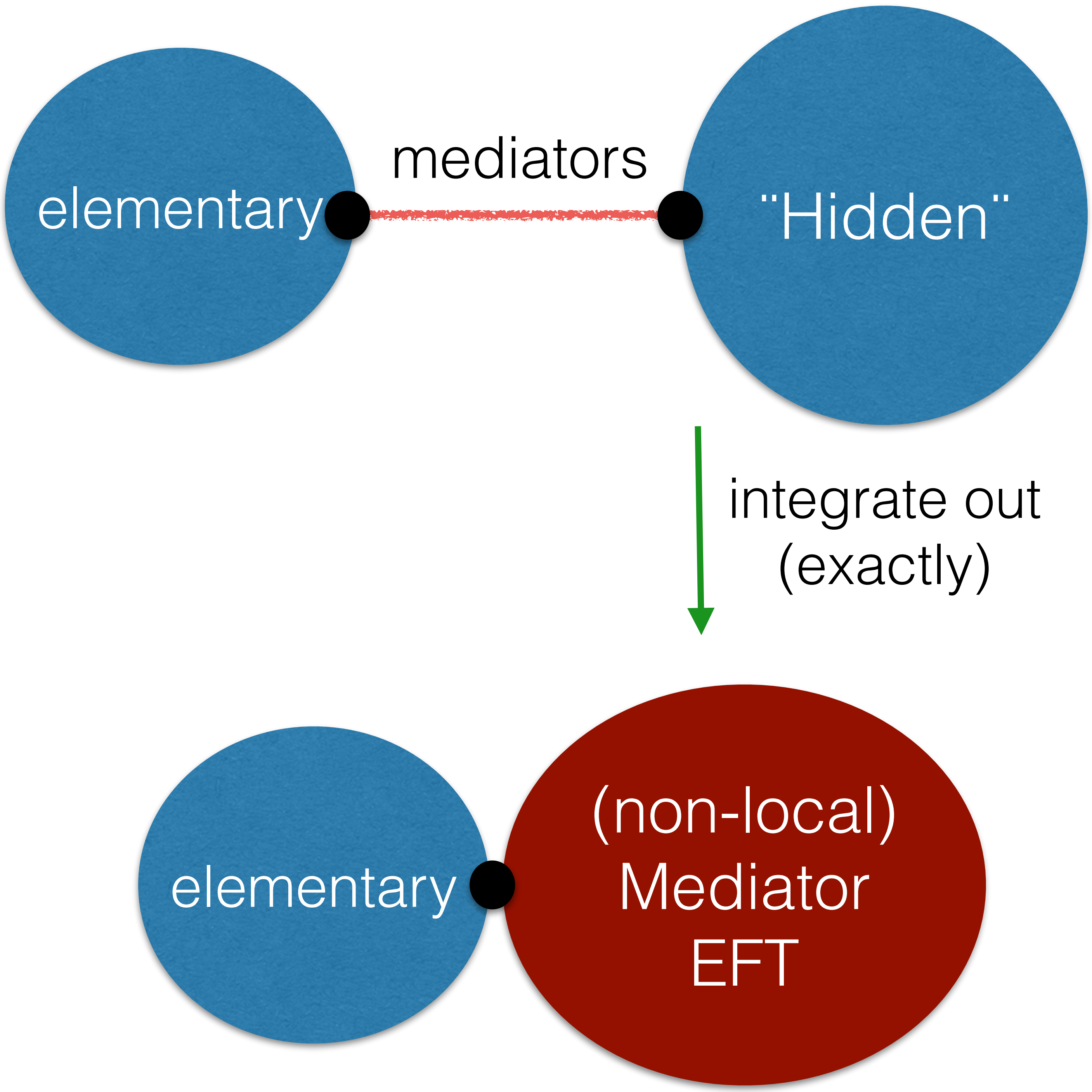
Outline

- Setup
- Spectrum
- Unitarity
- Real processes
- Time evolution
- Holographic picture
- Conclusions

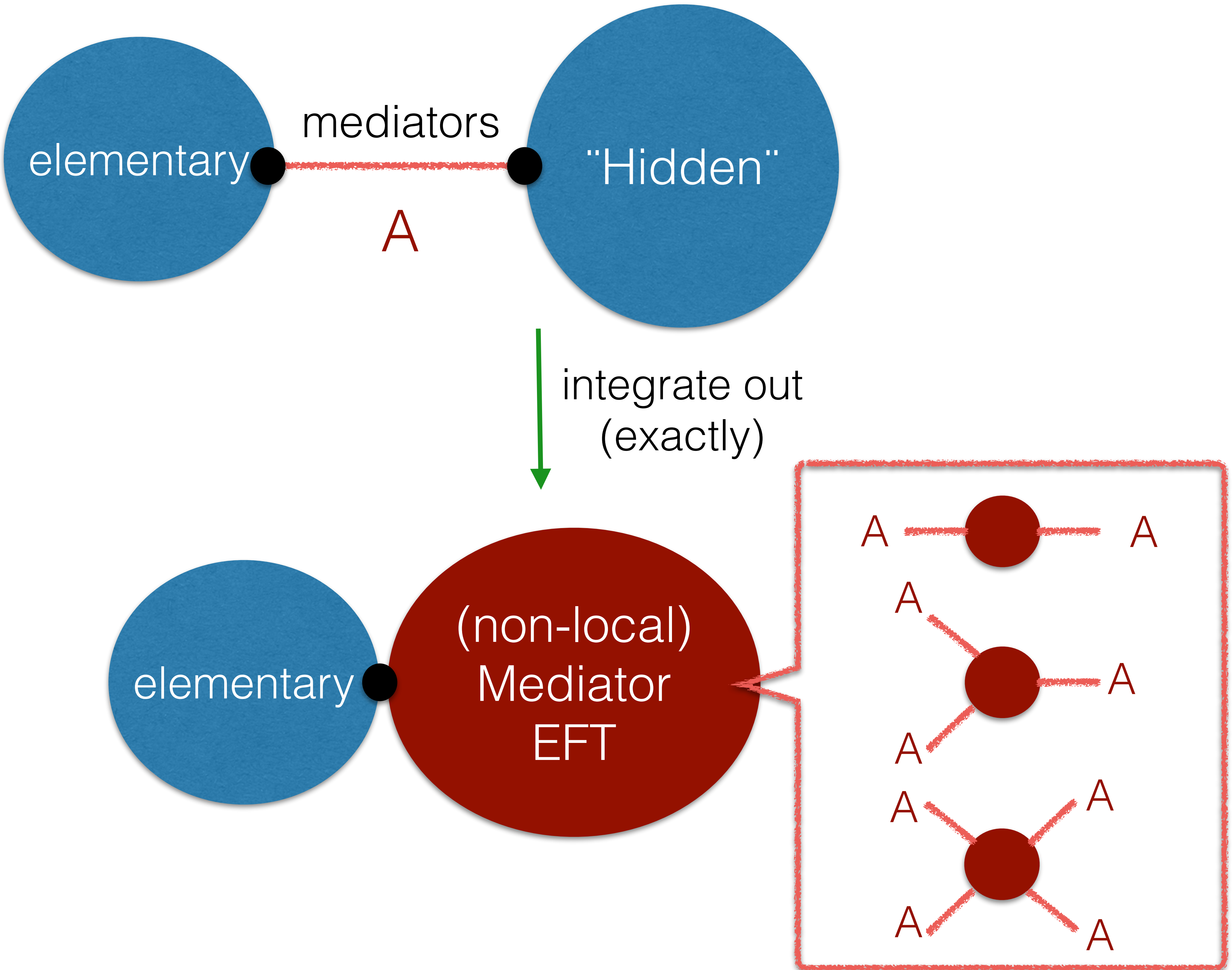
Typical scenario



Typical scenario



Typical scenario



Toy model

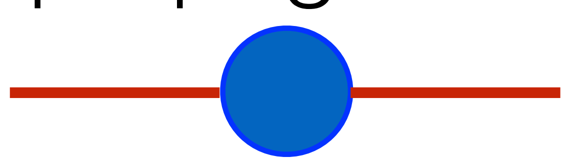
elementary field ⟶ massless complex scalar φ

composite field (mediator) ⟶ real scalar A

$$\mathcal{L} = -\frac{1}{2} A \Pi(-\partial^2) A + \partial_\mu \varphi^\dagger \partial^\mu \varphi + g A \varphi^\dagger \varphi$$

only two-point function Π for simplicity (large N)

dressed A propagator:



$$iG(p^2) = \frac{i}{\Pi(p^2) + \Sigma(p^2)}$$

with selfenergy $\Sigma(p^2) = -\frac{g^2}{16\pi^2} \log \frac{-p^2}{M^2}$

Spectrum

Spectral density: $$\sigma(\mu^2) = -\frac{1}{\pi}\,\mathrm{Im}\,G(\mu^2 + i0^+)$$

in free theory

(g=0)

$$\sigma^{(0)}(\mu^2) = -\frac{1}{\pi}\,\mathrm{Im}\,G^{(0)}(\mu^2 + i0^+)$$
$$= \frac{1}{\pi}\frac{\mathrm{Im}\,\Pi(\mu^2 + i0^+)}{|\Pi(\mu^2 + i0^+)|^2}.$$

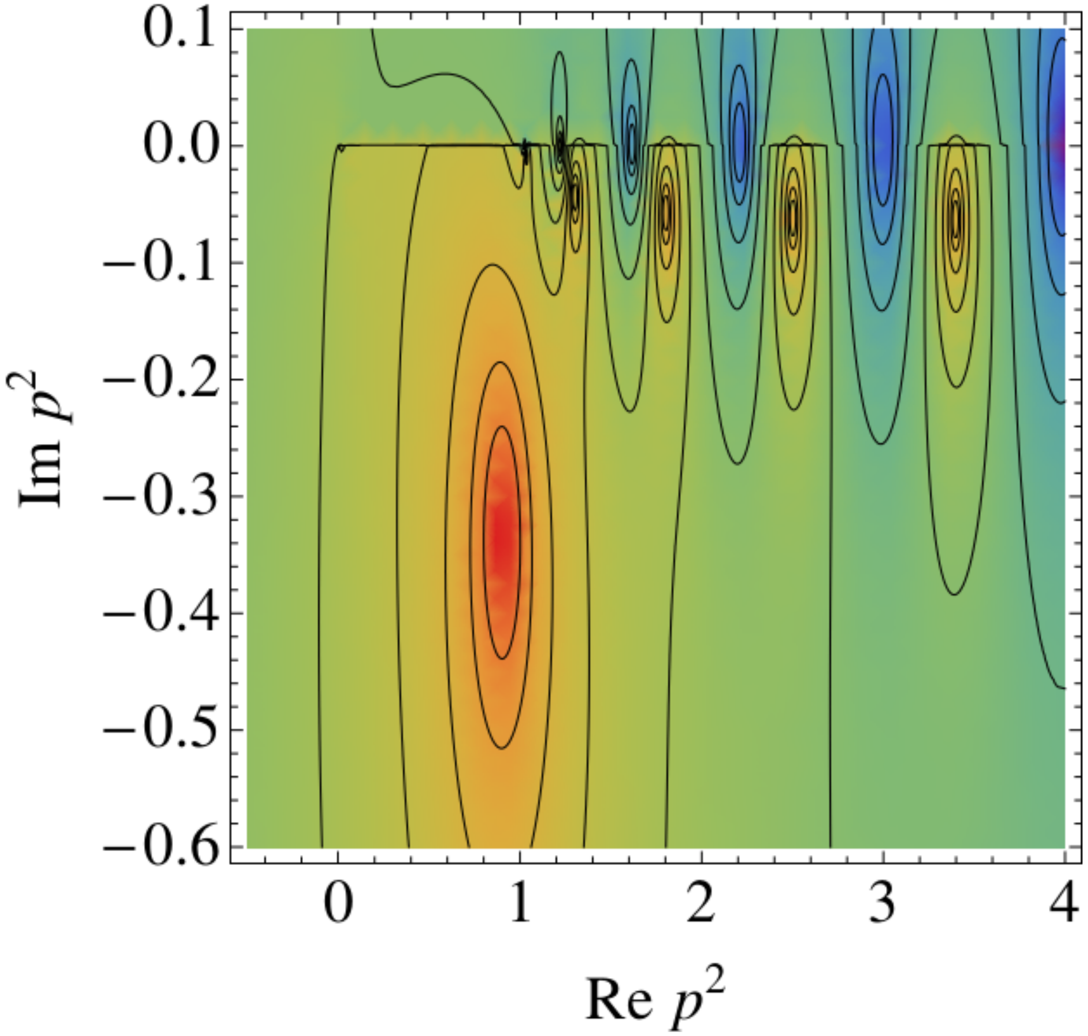
Discretum $$\sigma^{(0)}(\mu^2) = \sum_n F_n \delta(\mu^2 - m_n^2)$$

Continuum smooth $\sigma^{(0)}$

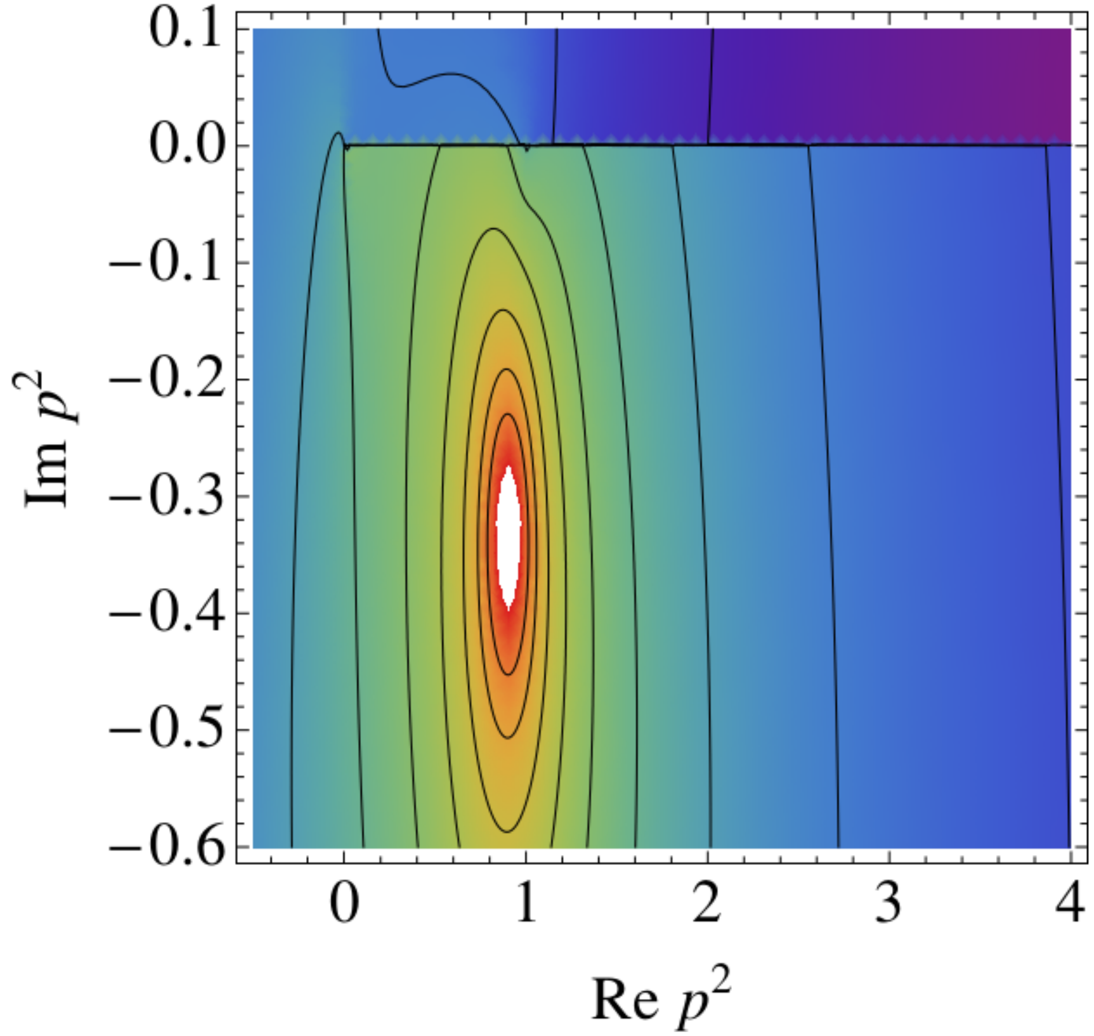
I assume a mass gap

In all cases, σ is smooth (due to Σ in denominator)

Analytic structure of propagator



Discretum



Continuum

Equivalent "KK" form

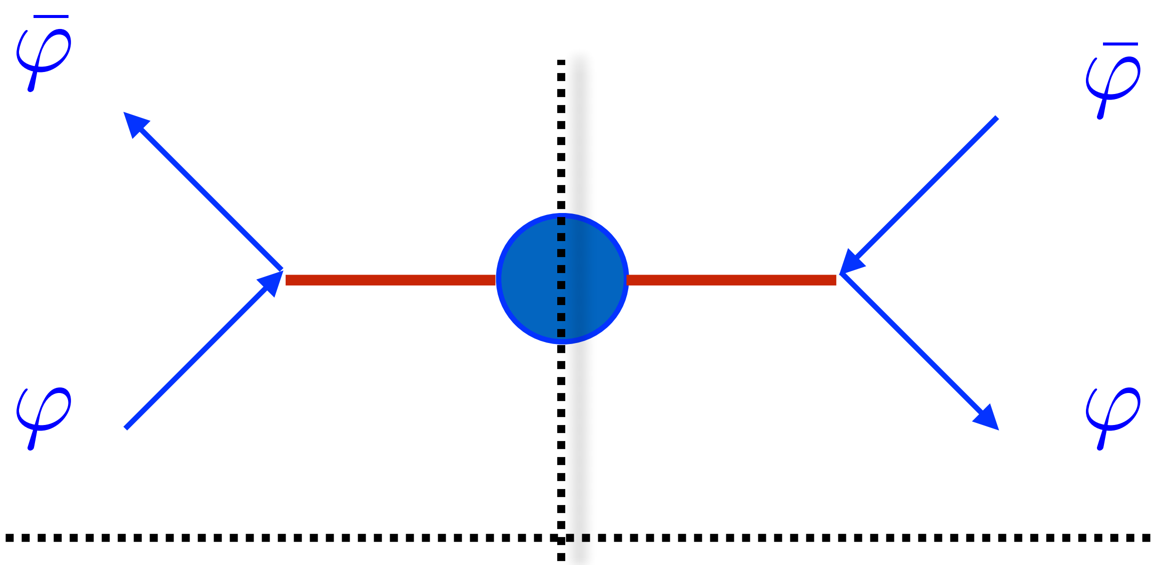
$$\mathcal{L}' = \partial_\nu \varphi^\dagger \partial^\nu \varphi + \int_0^\infty d\mu^2 \sigma^{(0)}(\mu^2) \left(\frac{1}{2} \partial_\nu B_\mu \partial^\nu B_\mu - \frac{\mu^2}{2} B_\mu^2 + g B_\mu \varphi^\dagger \varphi \right)$$

with constraint $\quad A = \int_0^\infty d\mu^2 \sigma^{(0)}(\mu^2) B_\mu$

- *A* creates many (generalized) energy eigenstates with different masses
- Each B_μ creates a (generalized) energy eigenstate with well-defined mass

(free fields)

Unitarity



(s channel)

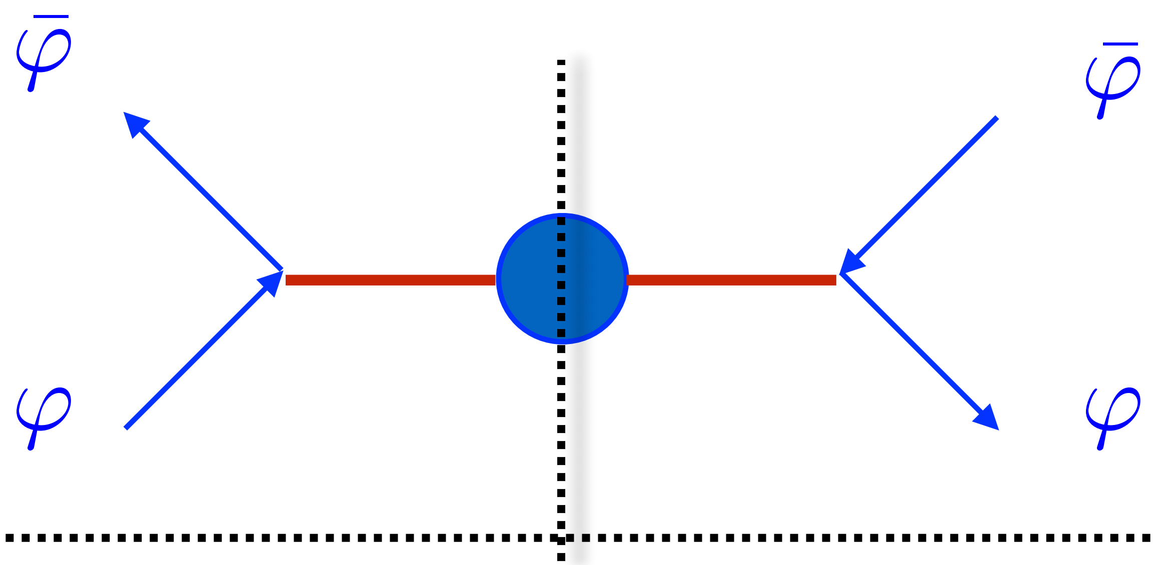
Optical theorem

$$\mathrm{Im}\,\mathcal{M}_{\varphi}(s,0) = \frac{1}{2}\tilde{\sigma}_{\varphi}(p_1, p_2).$$

Veltman '63:

Unstable particles do not enter as final states in unitarity relations

Unitarity



(s channel)

Optical theorem

$$\mathrm{Im}\,\mathcal{M}_{\varphi}(s,0) \stackrel{?}{=} \frac{1}{2}\tilde{\sigma}_{\varphi}(p_1,p_2).$$

$$\tilde{\sigma}_{\varphi}^{(s)\bar{\varphi}\varphi}(p_1,p_2) = \frac{g^4}{8\pi}\frac{1}{|\Pi(s+i0^+)+\Sigma(s+i0^+)|^2}$$

$$\mathrm{Im}\,\mathcal{M}_{\varphi}^{(s)}(s,0) = g^2\left(\mathrm{Im}\,\Pi(s+i0^+)+\frac{g^2}{16\pi}\right)\frac{1}{|\Pi(s+i0^+)+\Sigma(s+i0^+)|^2}$$

Discretum ✔

Continuum Excess in Im *M* 🤔

$$\Delta\,\mathrm{Im}\,\mathcal{M}_{\varphi}^{(s)}(s,0) = \frac{g^2\,\mathrm{Im}\,\Pi(s+i0^+)}{|\Pi(s+i0^+)+\Sigma(s+i0^+)|^2} > 0.$$

Unitary theory ⇒ missing final states in continuum

$$\Delta\,\mathrm{Im}\,\mathcal{M}_{\varphi}^{(s)}(s,0) = \frac{g^2\,\mathrm{Im}\,\Pi(s+i0^+)}{|\Pi(s+i0^+)+\Sigma(s+i0^+)|^2} > 0.$$

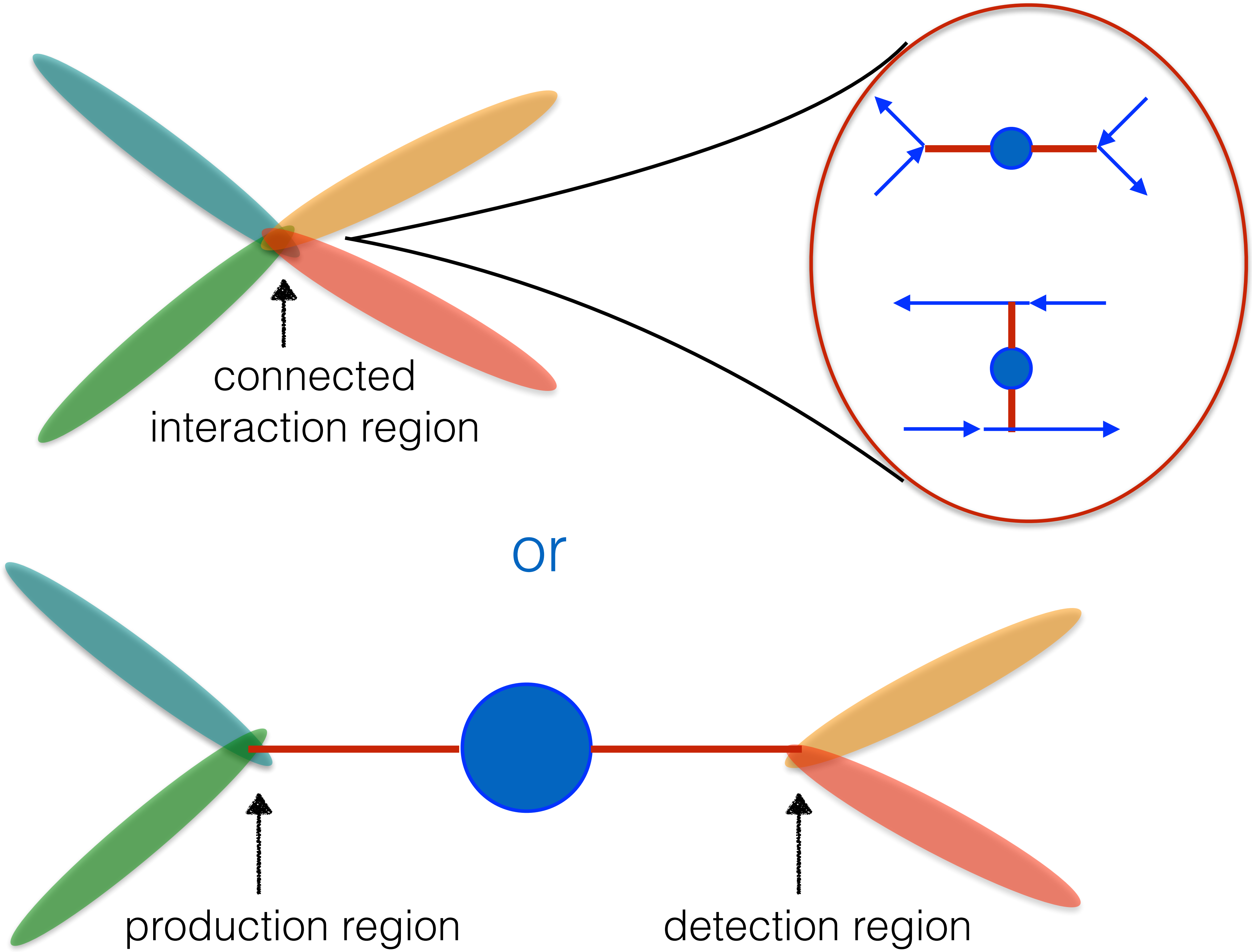
Unitary theory ⇒ missing final states in continuum

- Not multi-φ particle states
- Must be states in the hidden sector
- Missed in the EFT? ➞ Not really! *A* interpolating field

Mysteries (from EFT point of view):

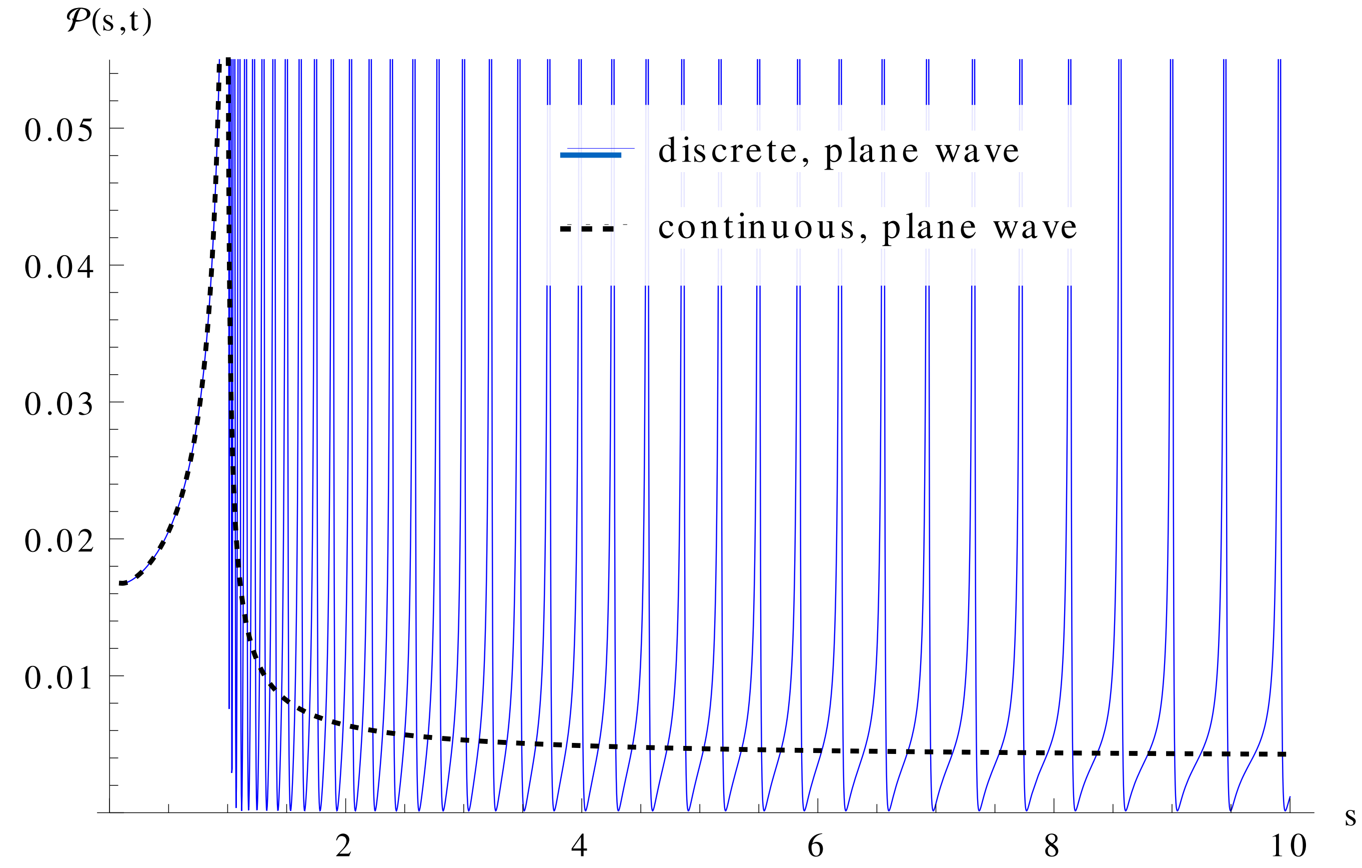
- Nature of hidden asymptotic states
- ***A* does decay… but not completely in continuum!**
- Decay law?
- Discretum to continuum transition: not continuous

Scattering in real life



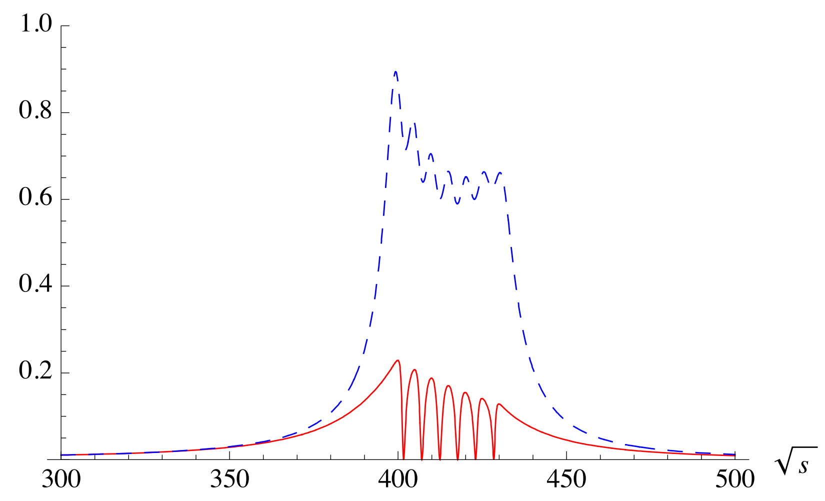
Wave packets ➞ Field smearing $A_\tau(t,\vec{x}) = \int_{-\infty}^{\infty} dt' f_\tau(t-t') A(t',\vec{x})$

Plane waves

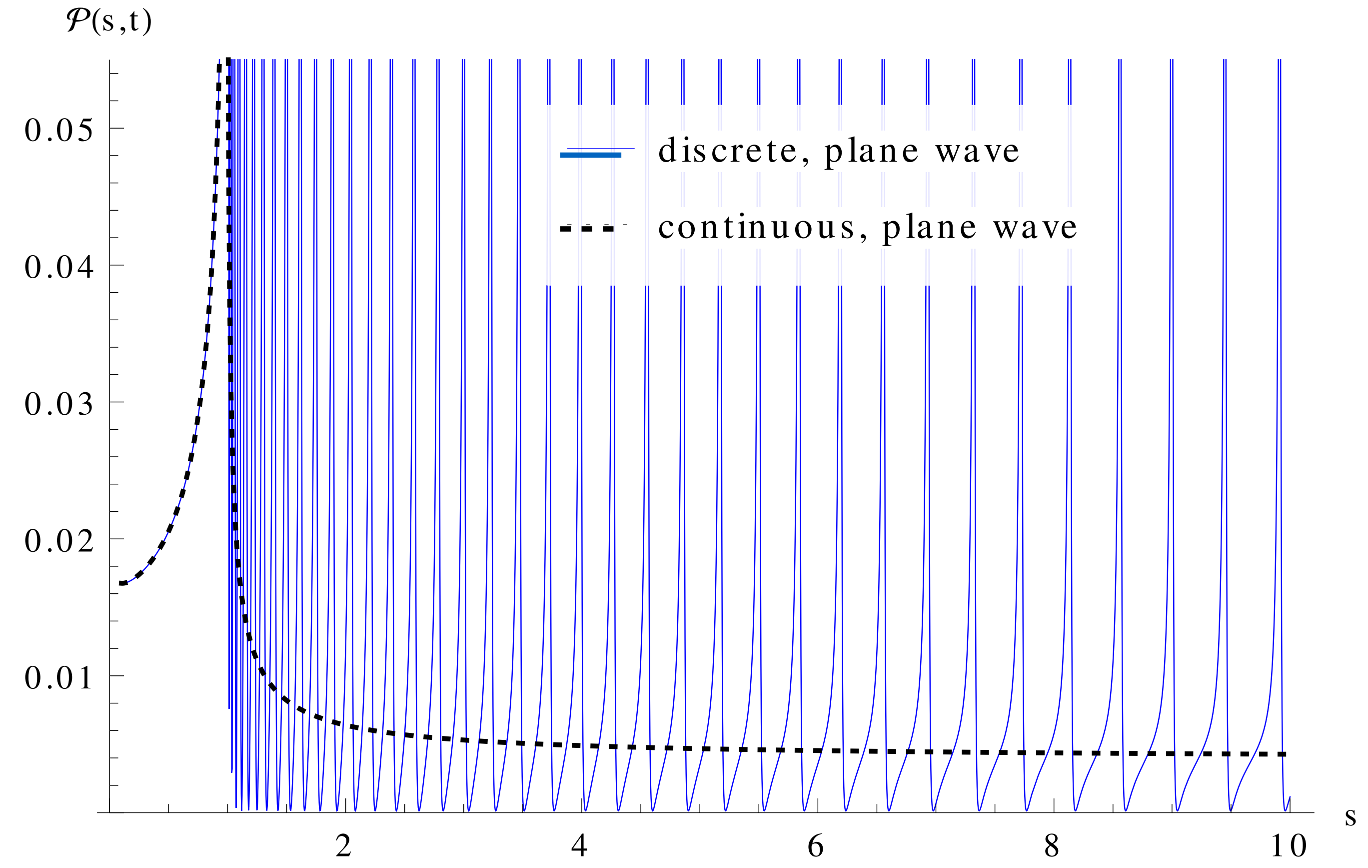


(width mixing taking into account by resummed propagator)

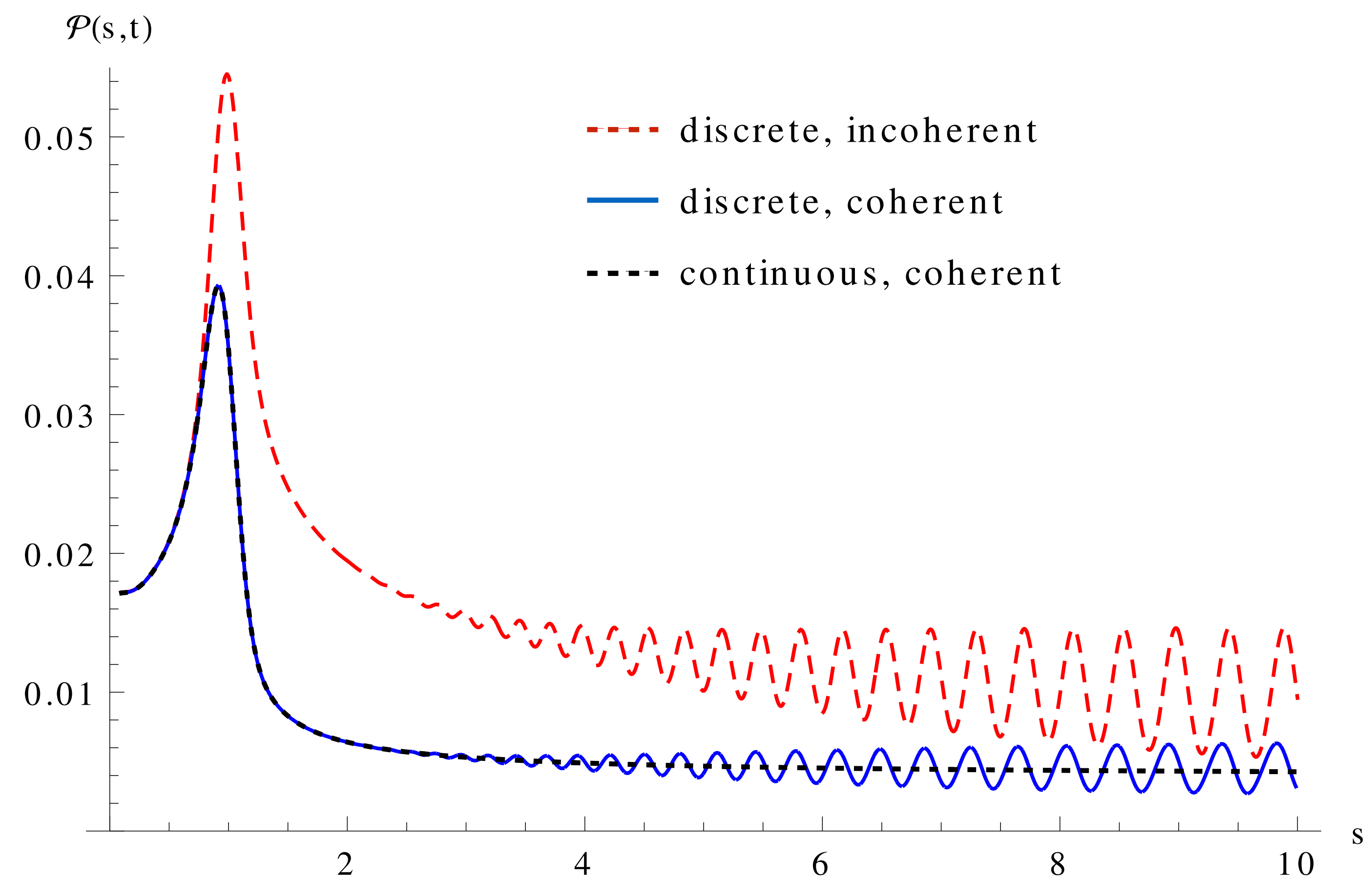
Cacciapaglia, Deandrea, De Curtis '09



Plane waves



Wave packets



Time evolution: free theory

At time t=0, A creates from the vacuum a particular one-particle state

$$A(0,\vec{x})|0\rangle = \int \frac{d^3p}{(2\pi)^3} e^{-i\vec{x}\cdot\vec{p}} \underbrace{\int_0^\infty d\mu^2 \rho(\mu^2) Z_{0\mu}^{\frac{1}{2}} \frac{1}{2\omega_{\mu,p}} |\mu,\vec{p}\rangle_0}_{|\mathcal{A}^0_{\vec{p}}\rangle}$$

Non-trivial evolution $\quad |\mathcal{A}^0_{\vec{p}}, t\rangle = e^{-itH_0}|\mathcal{A}^0_{\vec{p}}\rangle$

Overlap with initial state:

$$\langle \mathcal{A}^0_{\vec{p}}|\mathcal{A}^0_{\vec{q}}, t\rangle = (2\pi)^3\delta^3(\vec{p}-\vec{q}) \int_0^\infty d\mu^2 e^{-it\omega_{\mu,p}} \frac{\sigma^{(0)}(\mu^2)}{2\omega_{\mu,p}}$$

$$\overset{(t\gtrsim 0)}{=} (2\pi)^3\delta^3(\vec{p}-\vec{q}) i\tilde{G}^{(0)}(t,\vec{p})$$

time smearing

Survival probability: $\mathcal{P}_{\text{sur}}(t) = |\langle \mathcal{A}_h^{0\tau}|\mathcal{A}_h^{0\tau}, t\rangle|^2$

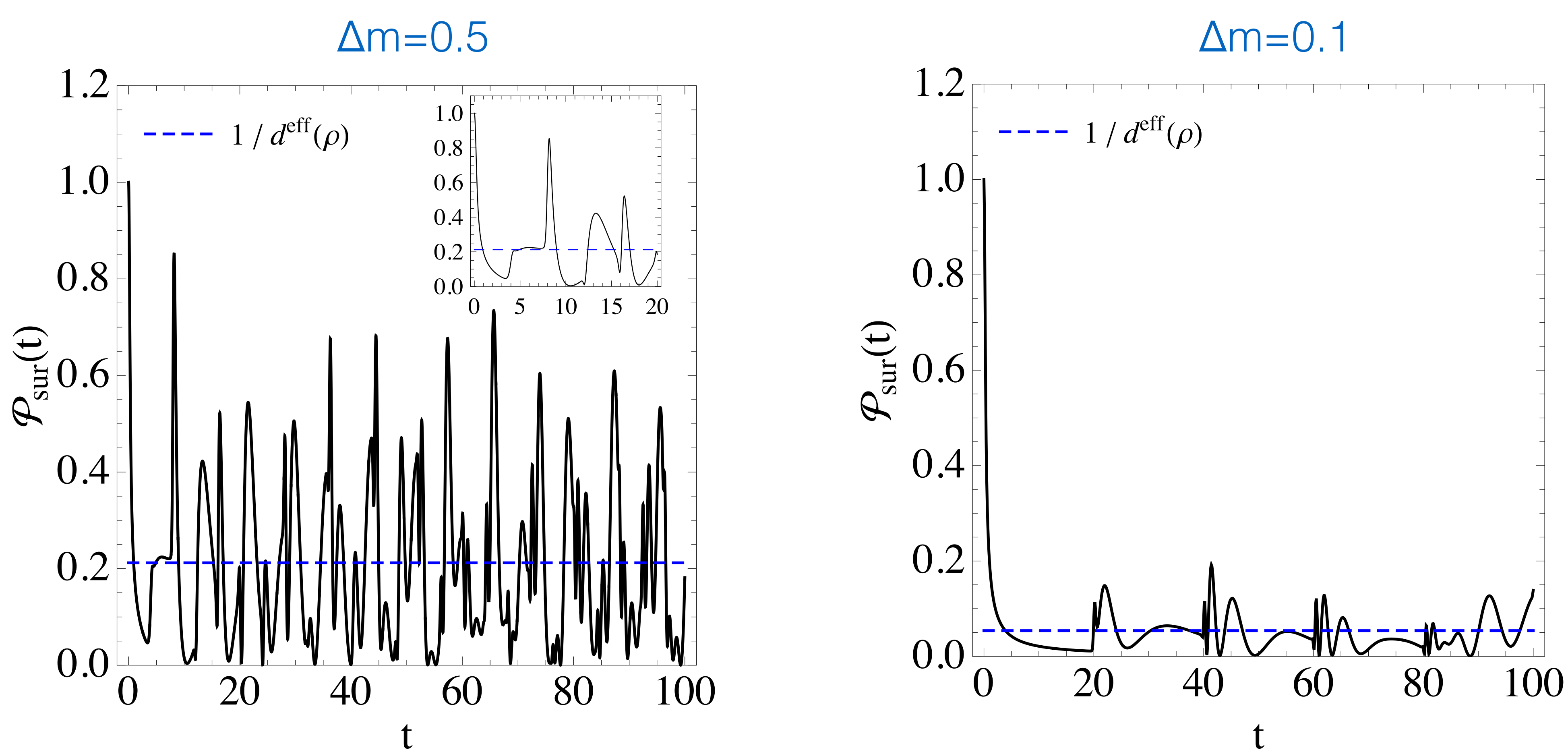
Discretum

$$i\tilde{G}^{(0)}_{\tau}(t,\vec{p}_0) = \sum_{\nu} \alpha_{\nu,p} e^{-it\omega_{\nu,p}}$$

Survival probability is almost periodic… but fast equilibration:

$$\rho := \langle |\mathcal{A}^{0\tau}_h, t\rangle\langle\mathcal{A}^{0\tau}_h, t|\rangle_t \qquad d^{\rm eff}(\rho) = \frac{1}{\mathrm{Tr}(\rho^2)}$$

$$\langle \mathcal{P}_{\rm sur}(t) - \frac{1}{d^{\rm eff}(\rho)}\rangle_t \quad \leq \frac{1}{d^{\rm eff}(\rho)}$$



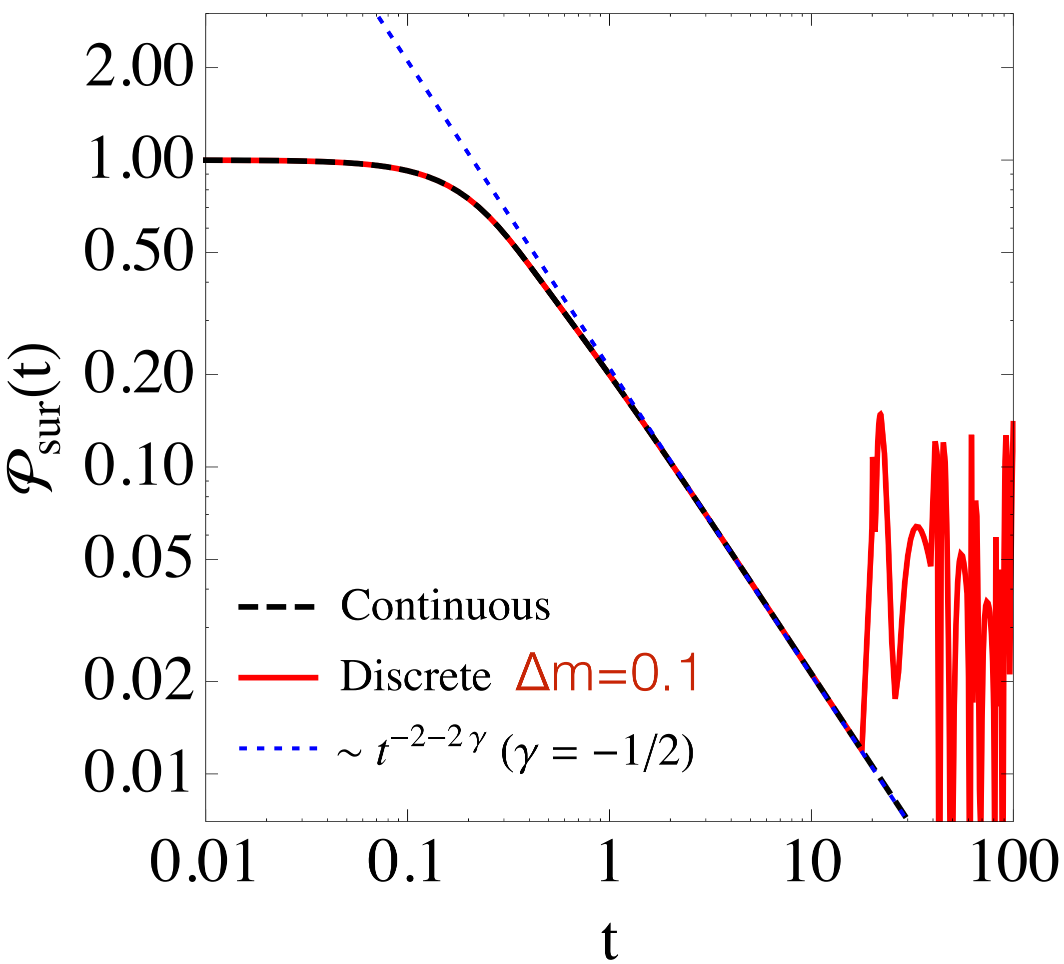
Continuum

Riemann-Lebesgue lemma $\Rightarrow$ $\lim_{t\to\infty} \mathcal{P}_{\rm sur}(t) = 0$

In the free theory, the initial state **decays** into orthogonal linear combinations of continuous modes created by A

"invisible decay"

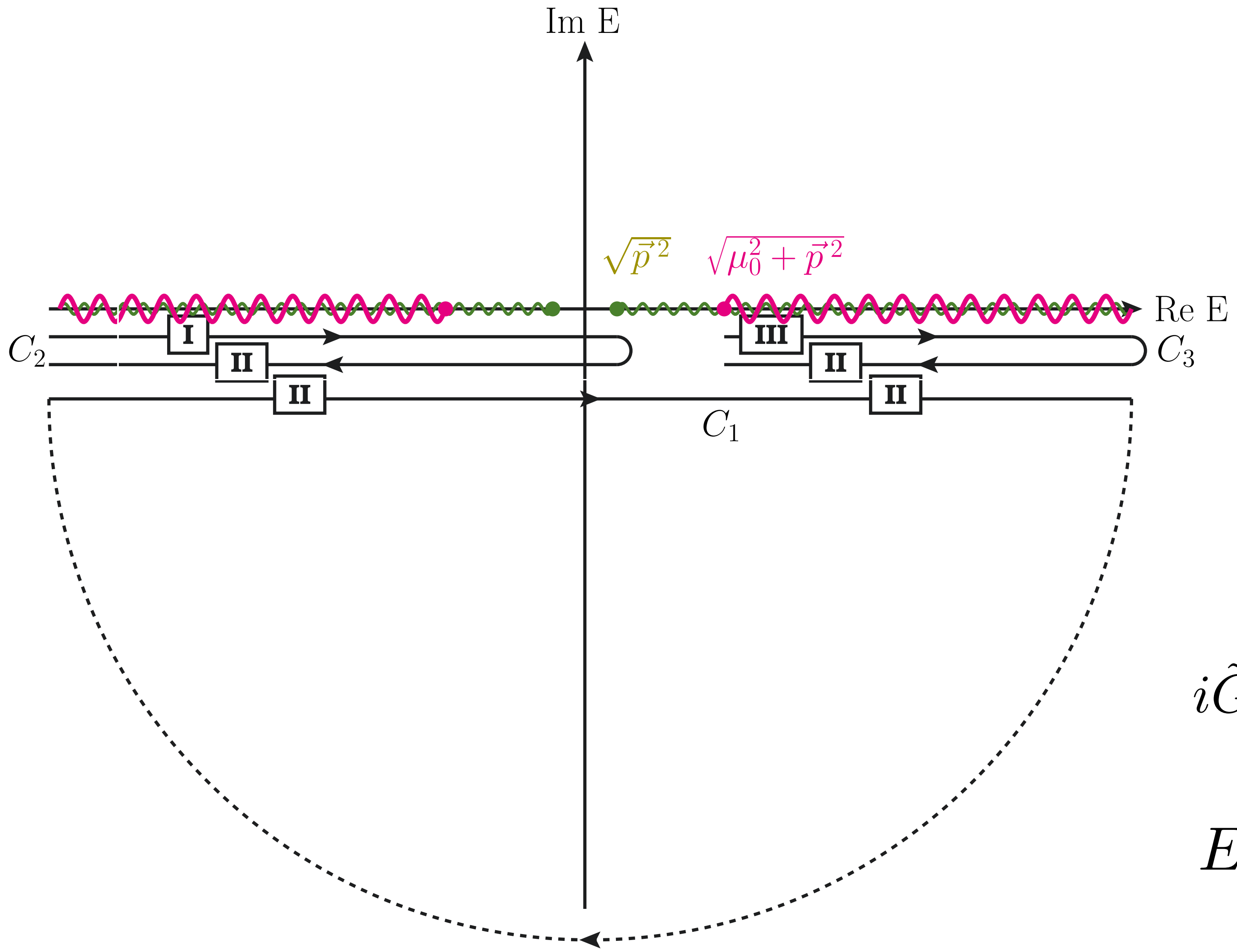
$\frac{\sigma^{(0)}(\mu^2)}{\omega_{\mu,p}} \sim (\mu^2 - \mu_0^2)^\gamma$ $\longrightarrow$ $\mathcal{P}_{\rm sur}(t) \propto t^{-2(1+\gamma)}$ at large times



Time evolution with interactions

$$i\tilde{G}_\tau(t,\vec{p}) = \int_0^\infty d\mu^2 e^{-it\omega_{\mu,p}} \frac{\sigma_\tau(\mu^2,\vec{p}^2)}{2\omega_{\mu,p}}$$

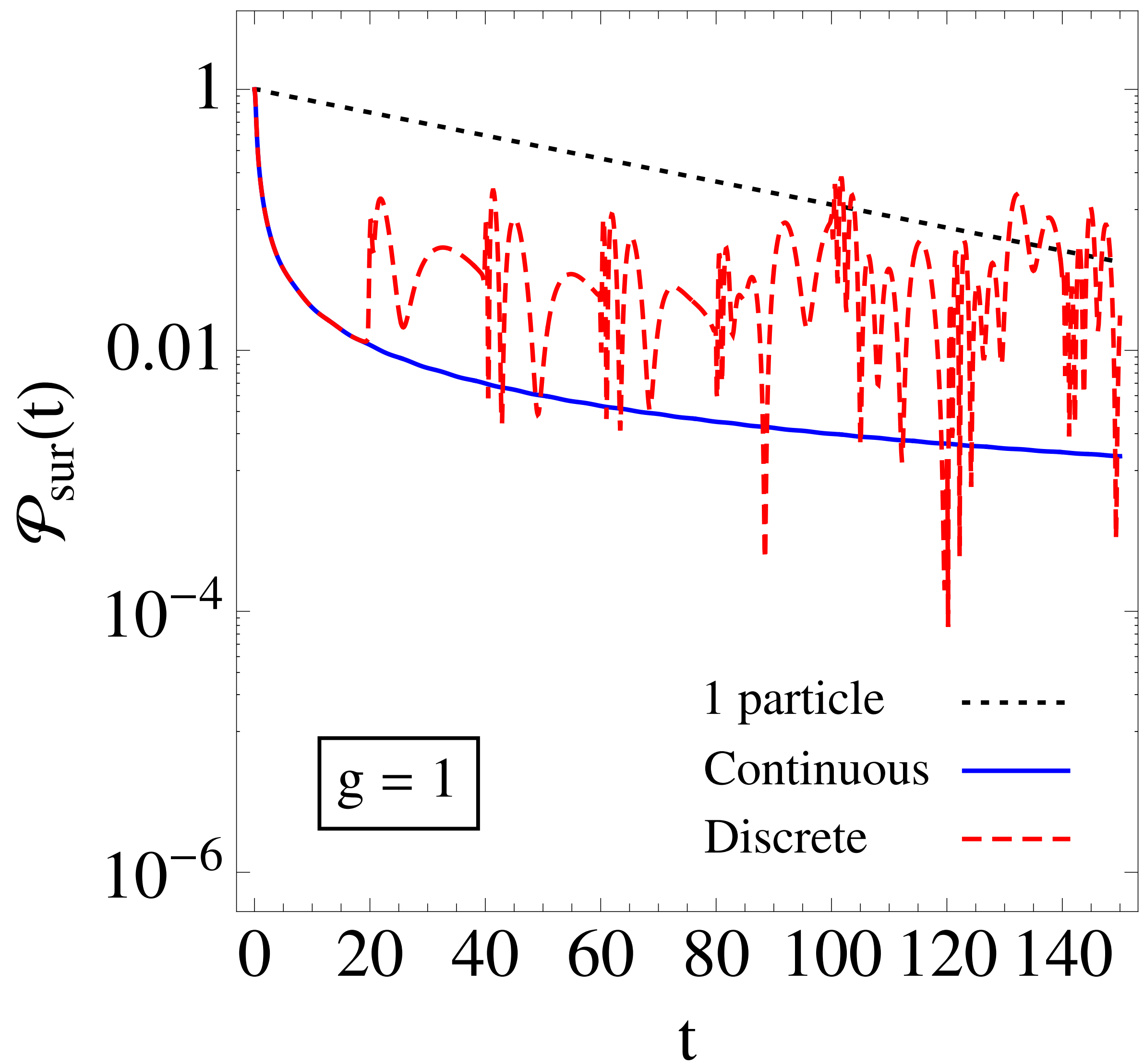
RL $\Rightarrow$ $\lim_{t\to\infty} \mathcal{P}_{\rm sur}(t) = 0$ in both continuum and discrete cases

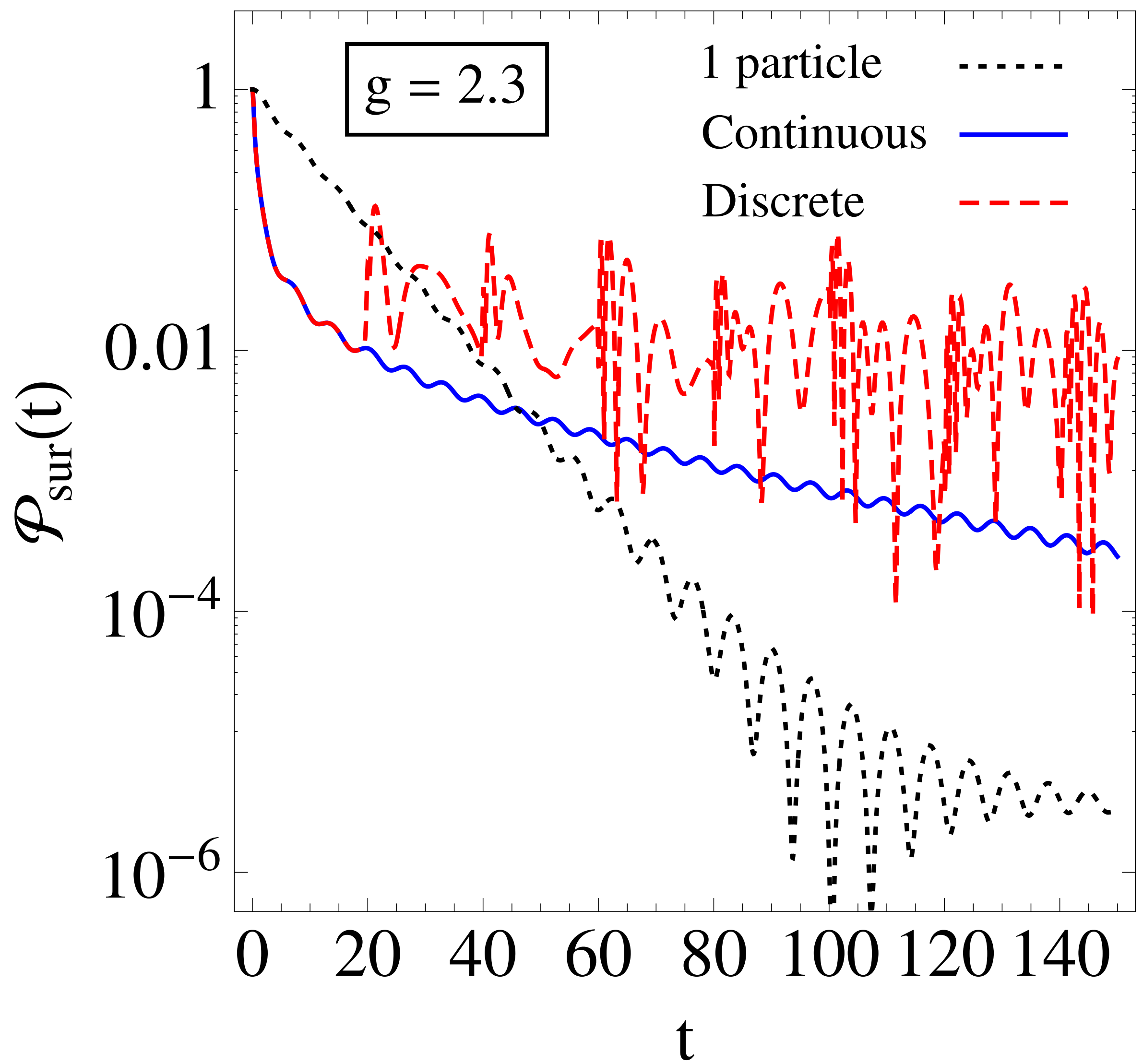


$$i\tilde{G}_\tau^{C_1,\rm poles}(t,\vec{p}) = \sum_n \frac{\mathcal{Z}_n}{2E_n}\frac{1}{1+E_n^2\tau^2}e^{-itE_n}$$

$$E_n = \omega_n - i\frac{\Lambda_n}{2}$$

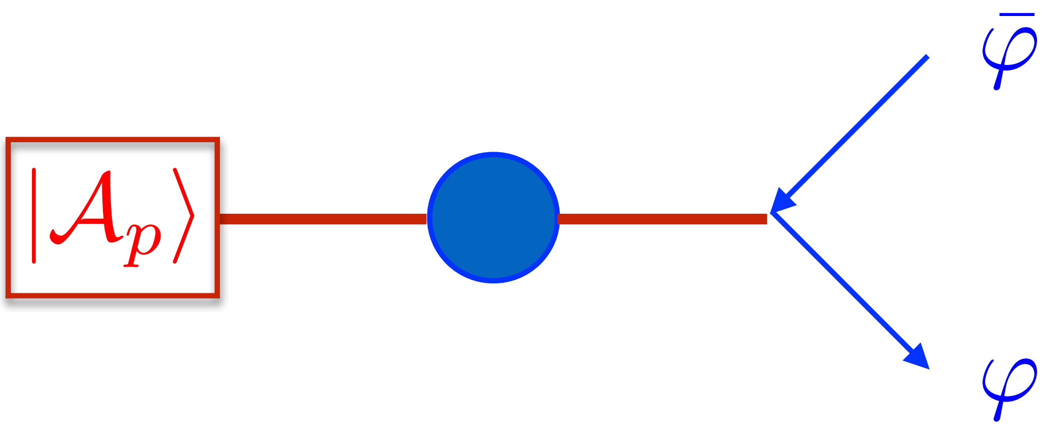
Survival of initial state





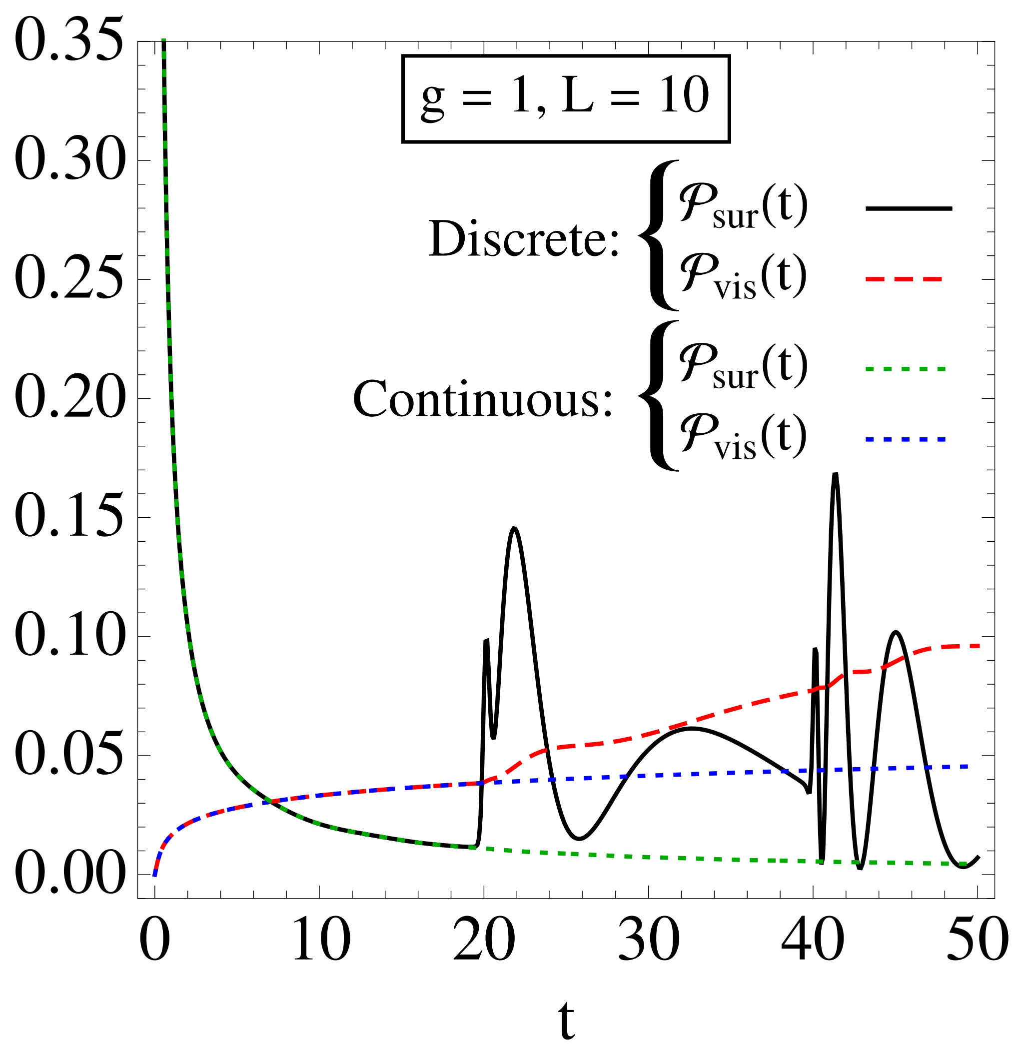
(discrete case agrees with axion oscillations in Dudas, Dienes, Gherghetta ´00)

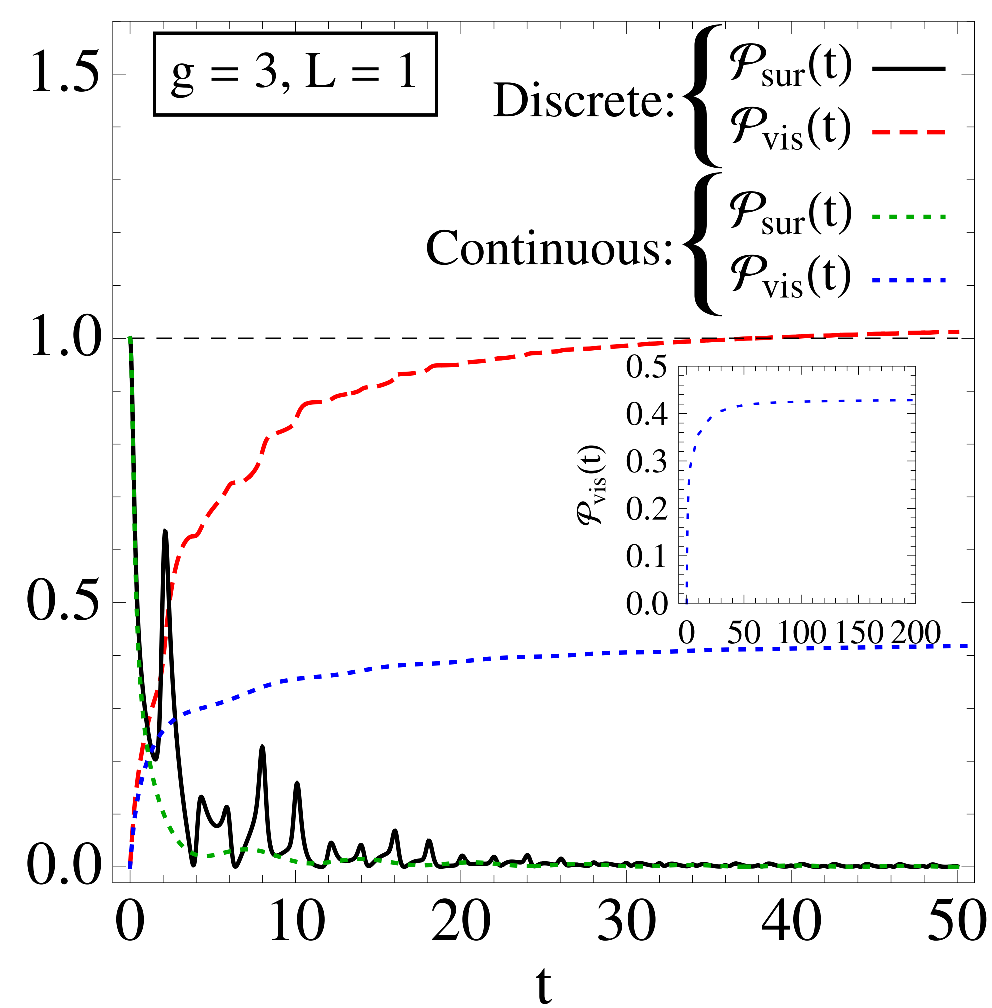
Visible decay



$$\frac{d\mathcal{P}_{\bar{\varphi}\varphi}}{dt}(t_1) \simeq \frac{g^2}{8\pi}\frac{1}{|\tilde{G}_\tau(0,\vec{0})|}\left|\tilde{G}_\tau(t_1,\vec{p}_0)\right|^2$$

$$\mathcal{P}_{\rm vis}(t) \simeq \int_0^t dt_1 \frac{d\mathcal{P}_{\bar{\varphi}\varphi}}{dt}(t_1)$$

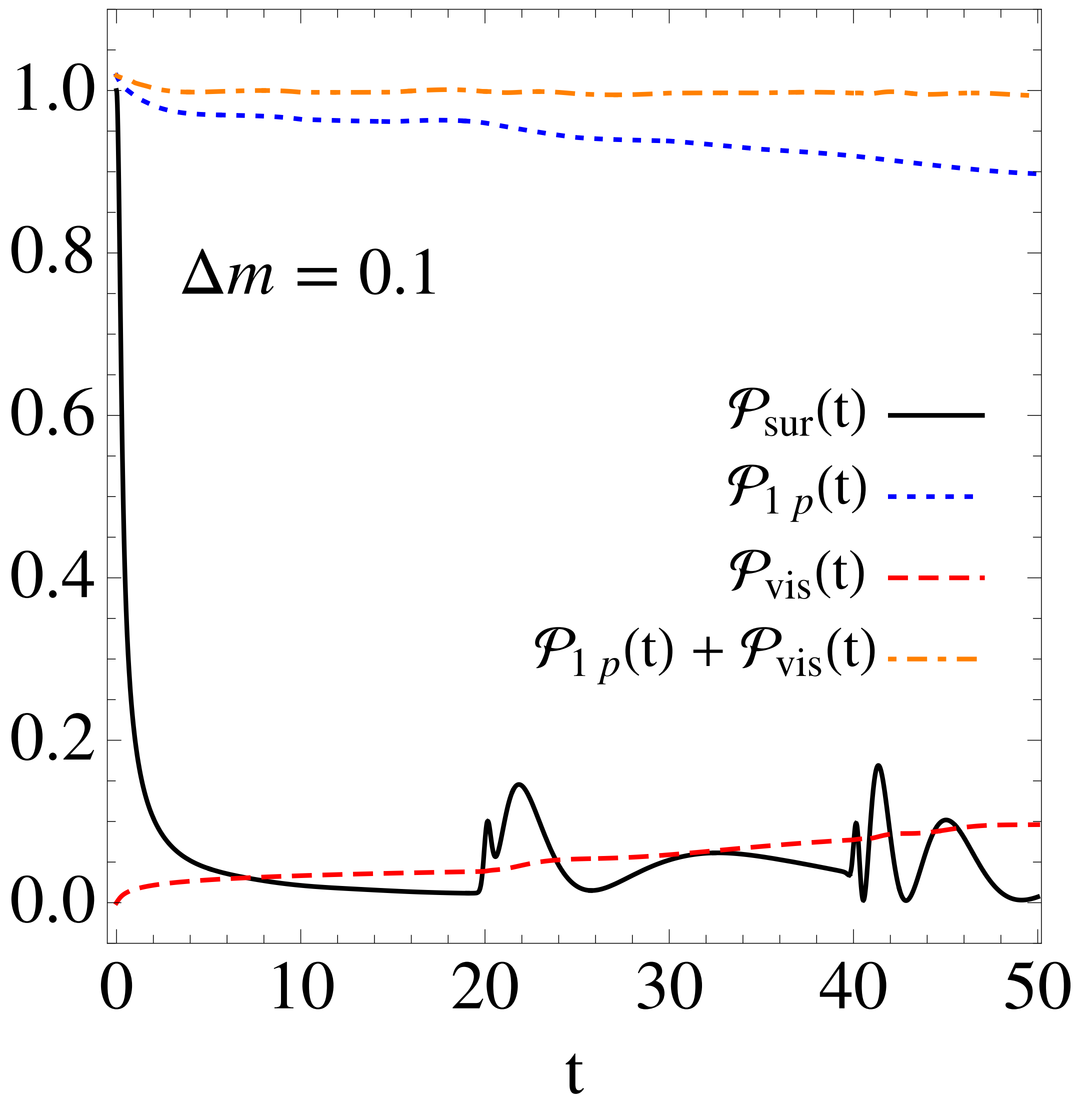




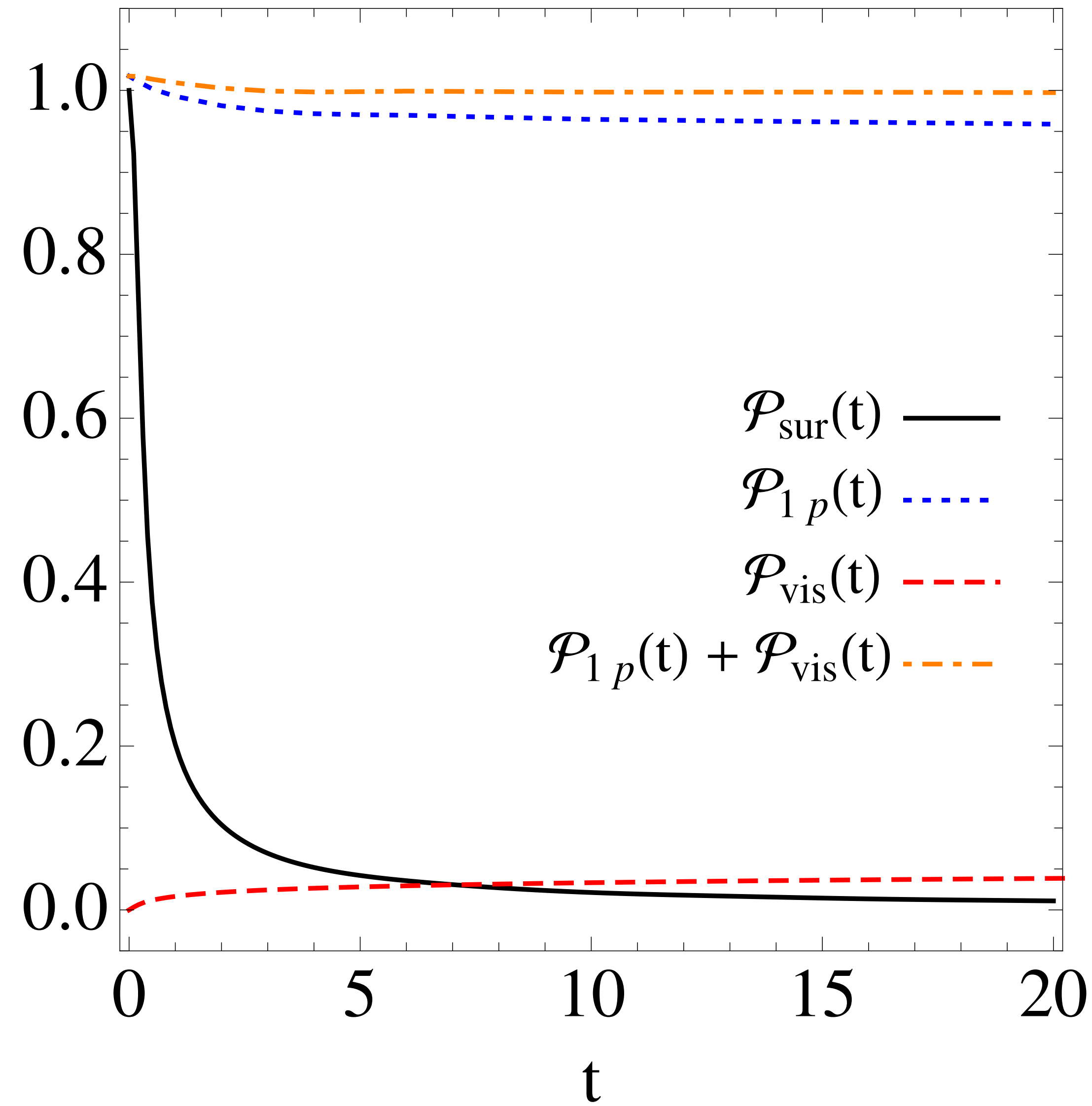
$\lim_{t\to\infty} \mathcal{P}_{\rm vis} < 1$ in continuous case!

Summary of time evolution

Discretum



Continuum



Unitarity: $\mathscr{P}_{1P}(t) + \mathscr{P}_{vis}(t) = 1$ $\quad (\mathscr{P}_{1P} = \mathscr{P}_{sur} + \mathscr{P}_{osc})$

(possible decoherence not included in the plots)

$P_{sur} \to 0$ in all cases

Decay is slower than exponential

Unitarity: $P_{sur}(t) + P_{vis}(t) + P_{osc}(t) = 1$

Discretum:

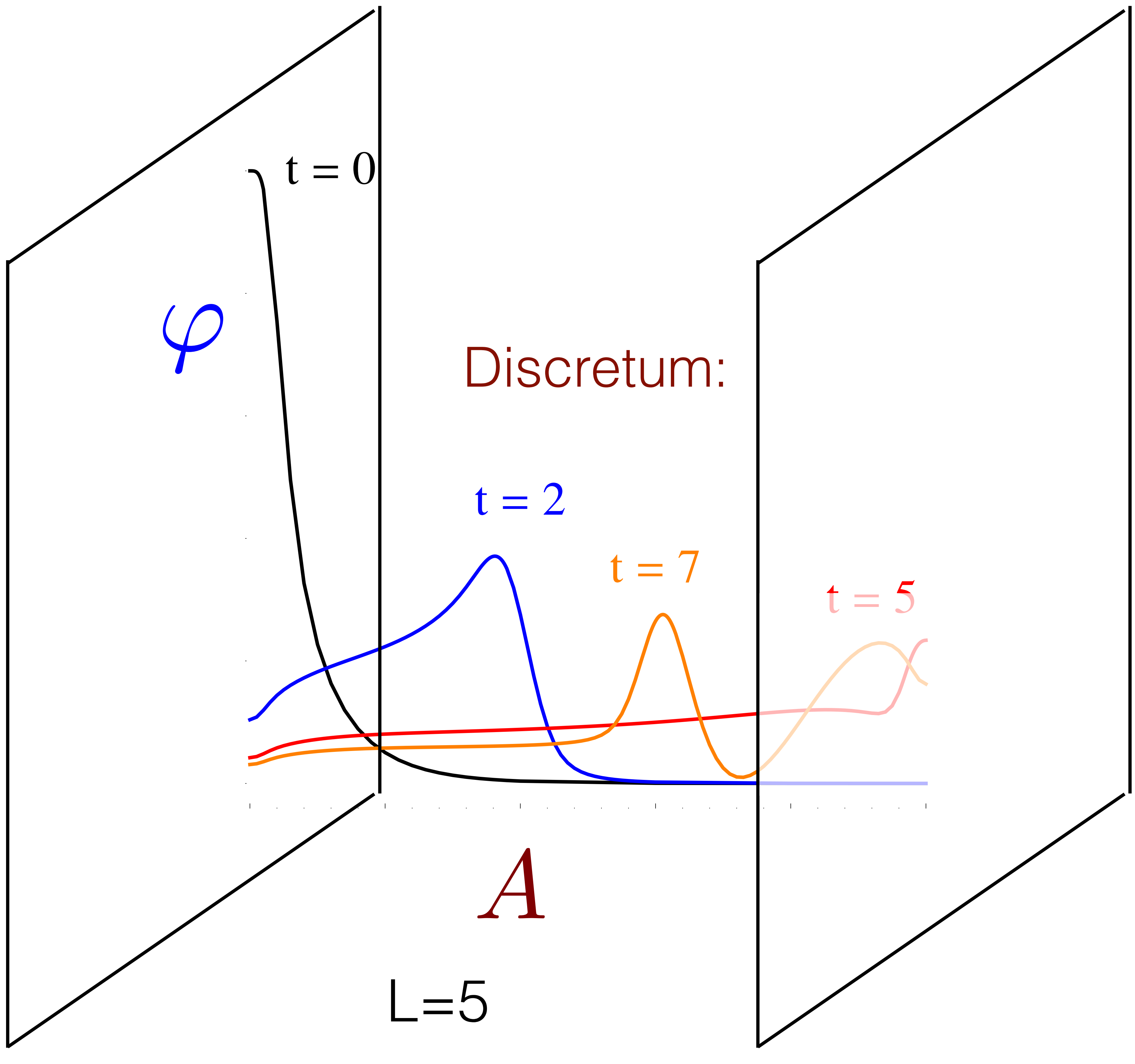
- Correlated oscillations in P_{sur} and P_{vis}
- $P_{vis} \to 1$

Continuum:

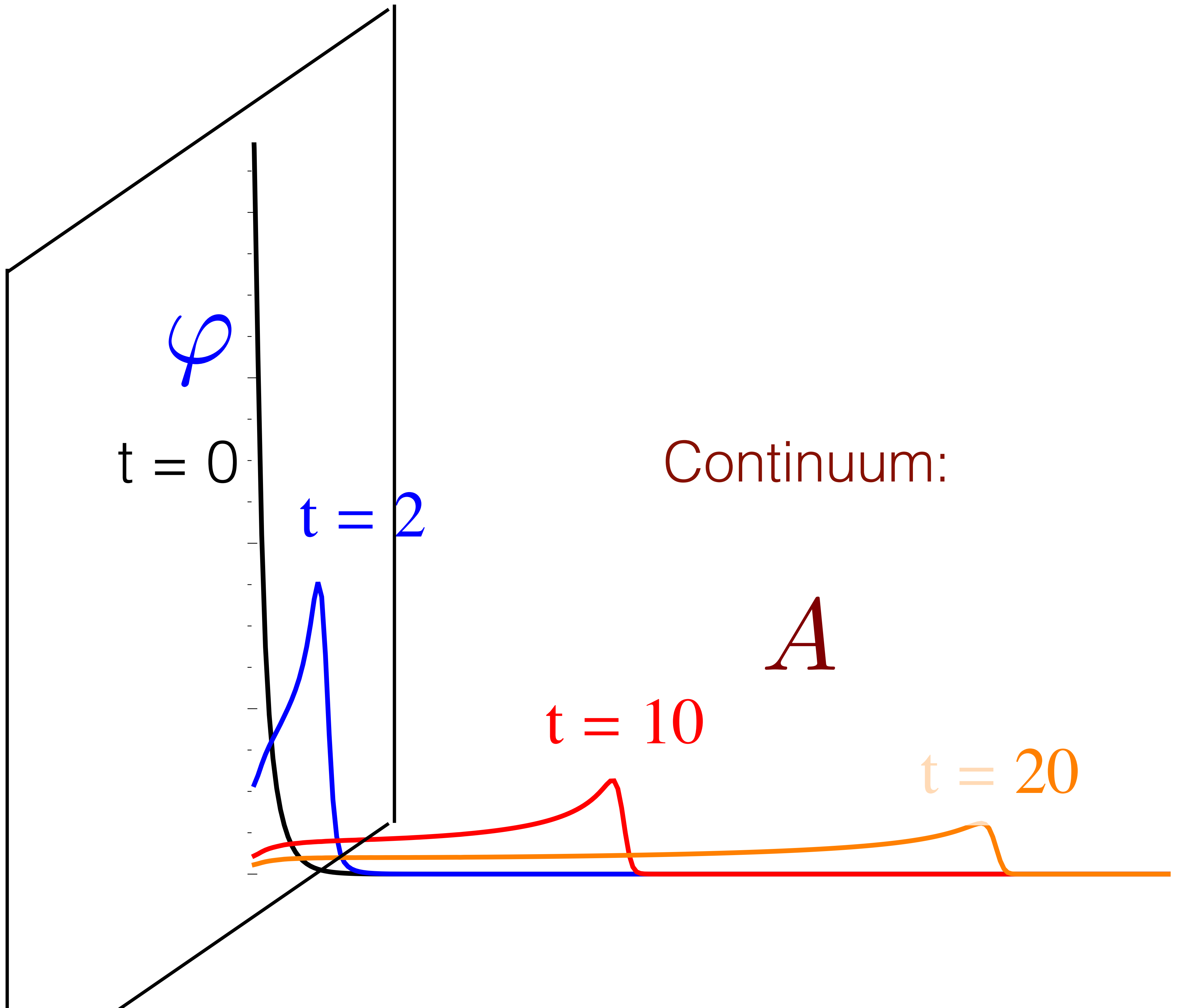
- $P_{vis} \not\to 1$
- $P_{osc} \not\to 0$ invisible decay

For $t \lesssim \frac{1}{\Delta m}$, discretum behaves like continuum

Holographic picture



Holographic picture



Conclusions: pheno / model building

(for both continuum and compressed discretum)

- Continuous spectra are ubiquitious in QFT
- Cannot treat each mode separately
- No resonant peaks ➞ more elusive, different searches
- Suppressed visible BR
- Non-standard decay law (far detectors)
- Stable or nearly-stable stuff
- Dark matter without Z_2 symmetry?

(continuous dark matter with Z_2 : Csaki et al '21)