

# Heavy Flavor Kinetics in Hot QCD

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# Outline

- ☞ Task: thermal production of  $c\bar{c}$  in the QGP
- ☞ Quasiparticle model
- ☞ QGP hydrodynamics: 1D perfect vs (2+1)D viscous
- ☞ Rate equation &  $c\bar{c}$  production rate
- ☞  $N_{c\bar{c}}(\tau)$  in hot deconfined matter

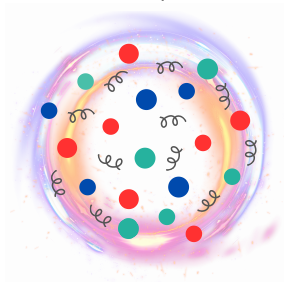
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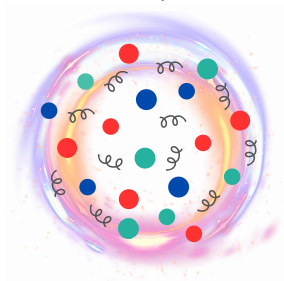


strongly-interacting particles,  
constant (bare) masses  $m_i$

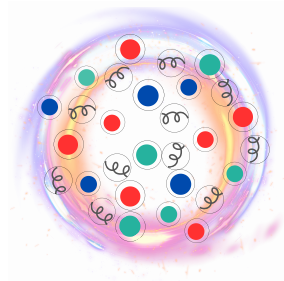
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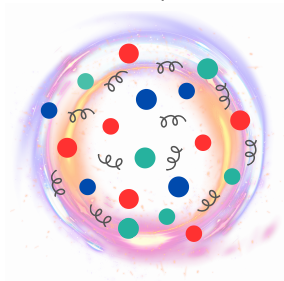
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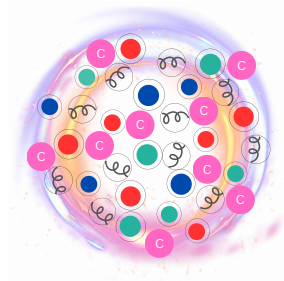
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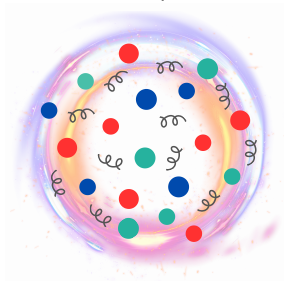
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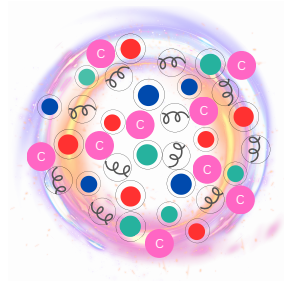
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# Quasiparticle Model - Effective Approach to QCD

Quasiparticles are „dressed” with effective masses  $m_i[G(T), T]$ :

$$m_i[G(T), T] = \sqrt{(m_i^0)^2 + \Pi_i[G(T), T]} \quad (1)$$

Self-energies  $\Pi_i$  from pQCD - HTL (Hard Thermal Loops):

[M. Bluhm et al. EPJ C 49 (2007), R. D. Pisarski, Nucl. Phys. A 498 (1989)]

$$\text{gluons: } \Pi_g[G(T), T] = \left(3 + \frac{N_f}{2}\right) \frac{G^2(T)}{6} T^2 \quad (2)$$

$$\text{quarks: } \Pi_{l,s}[G(T), T] = 2 \left[ m_{l,s}^0 \sqrt{\frac{G^2(T) T^2}{6}} + \frac{G^2(T) T^2}{6} \right] \quad (3)$$



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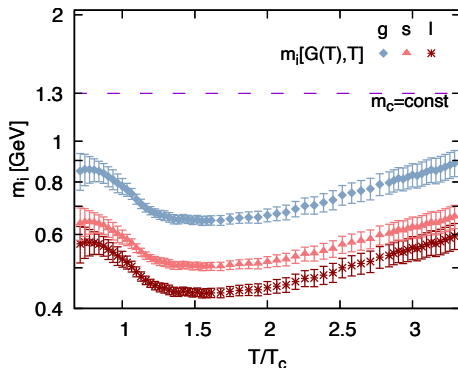
➡ Effective coupling  $G(T)$  – reliable thermodynamics – lattice QCD

[V.M. M. Bluhm, C. Sasaki, K. Redlich, PRD 100 (2019)]

# Quasiparticle Model

$$s(T) \simeq \sum_{i=g,l,s,\dots} \int d^3p \left( [1 \pm f_i^0] \ln[1 \pm f_i^0] \mp f_i^0 \ln f_i^0 \right) = \text{lattice data} \rightarrow G(T)$$

$$f_i^0(E_i) : E_i[G(T), T] = \sqrt{p^2 + m_i^2[G(T), T]} \quad (4)$$



[V.M, Acta Phys.Polon.Supp. 17 (2024), V.M, M. Bluhm, C. Sasaki, K. Redlich, PRD 100 (2019);

lattice: S. Borsanyi, et al., Phys. Lett. B 730 (2014) (Wuppertal-Budapest)]

# Rate Equation

describes time/temperature evolution of the number density function:

[Biro et al., PRC 48 '(1993); Zhang et al., PRC 77 (2008)]

$$\partial_\mu (n_c u^\mu) = R_{l\bar{l} \rightarrow c\bar{c}} + R_{s\bar{s} \rightarrow c\bar{c}} + R_{gg \rightarrow c\bar{c}} - R_{c\bar{c} \rightarrow l\bar{l}} - R_{c\bar{c} \rightarrow s\bar{s}} - R_{c\bar{c} \rightarrow gg} \quad (5)$$

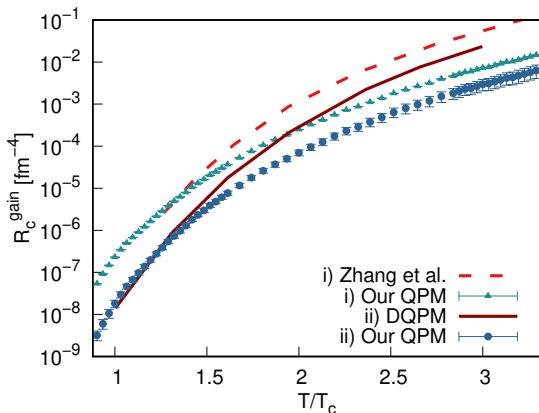
Applying the detailed balance:

$$\partial_\mu (n_c u^\mu) = \left[ \bar{\sigma}_{l\bar{l} \rightarrow c\bar{c}} (n_l^0)^2 + \bar{\sigma}_{s\bar{s} \rightarrow c\bar{c}} (n_s^0)^2 + \frac{1}{2} \bar{\sigma}_{gg \rightarrow c\bar{c}} (n_g^0)^2 \right] \times \left( 1 - \frac{n_c^2}{(n_c^0)^2} \right) \quad (6)$$

$$n_i^0 = d_i \int d^3 p f_i^0 [E_i(T)] \quad (7)$$

$$R_c^{gain} = \left[ \bar{\sigma}_{l\bar{l} \rightarrow c\bar{c}} (n_l^0)^2 + \bar{\sigma}_{s\bar{s} \rightarrow c\bar{c}} (n_s^0)^2 + \frac{1}{2} \bar{\sigma}_{gg \rightarrow c\bar{c}} (n_g^0)^2 \right] \quad (8)$$

# Charm Quark Production Rate



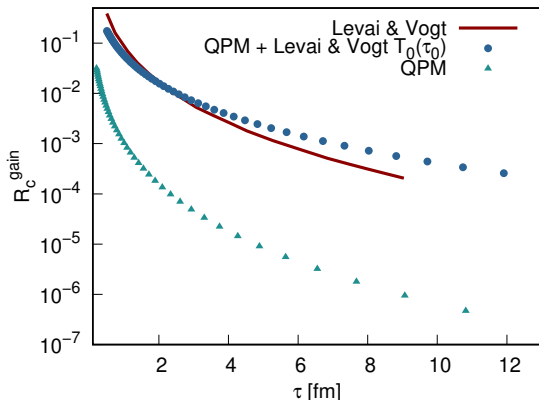
i)  $M_c = 1.3$  GeV, Total rate

ii)  $M_c = 1.5$  GeV, Production rate from quarks only

[DQPM: T. Song, I. Grishmanovskii, O. Soloveva, E. Bratkovskaya, arXiv:2404.00425 (2024);  
Zhang et al., Phys. Rev. C 77 (2008)]

# Charm Quark Production Rate

$$\text{Bjorken scaling: } T = T_0 \left( \frac{\tau}{\tau_0} \right)^{-1/3}$$



Levai & Vogt: Total rate for  $M_c = 1.2$  GeV,  $T_0 = 0.82$  GeV,  $\tau_0 = 0.5$  fm;

QPM: Total rate for  $M_c = 1.3$ ,  $T_0 = 0.624$  GeV,  $\tau_0 = 0.2$  fm.

[V.M., C. Sasaki, K. Redlich: to appear on ArXiv; P. Levai and R. Vogt, PRC 56 (1997)]

# Rate Equation

$$\partial_\mu(n_c u^\mu) = \left[ \bar{\sigma}_{ll\bar{l}\rightarrow c\bar{c}} (n_l^0)^2 + \bar{\sigma}_{s\bar{s}\rightarrow c\bar{c}} (n_s^0)^2 + \frac{1}{2} \bar{\sigma}_{gg\rightarrow c\bar{c}} (n_g^0)^2 \right] \times \left( 1 - \frac{n_c^2}{(n_c^0)^2} \right) \quad (9)$$

$$n_c[T(\tau)] = ? \quad (10)$$

\* LHS depends on the QGP evolution:

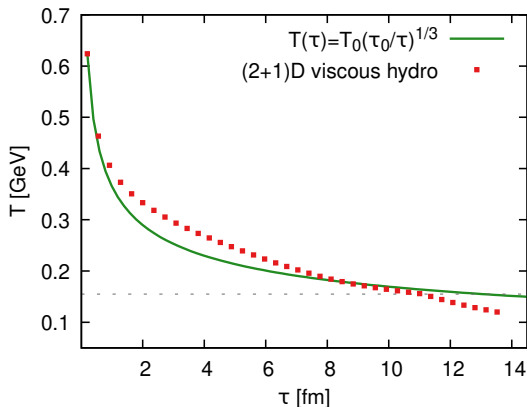
i) 1D Bjorken flow, ideal fluid: no dissipation,

$$T = T_0 \left( \frac{\tau}{\tau_0} \right)^{-1/3} \quad (11)$$

ii) (2+1)D expansion, viscous fluid +  $(\eta/s)(T)$

[V.M., M. Bluhm, K. Redlich, C. Sasaki, PRD100 '19; Auvinen, Eskola, Huovinen, Niemi, Paatelainen, Petreczky, PRC 102 '20]

# QGP Evolution: 1D vs (2+1)D

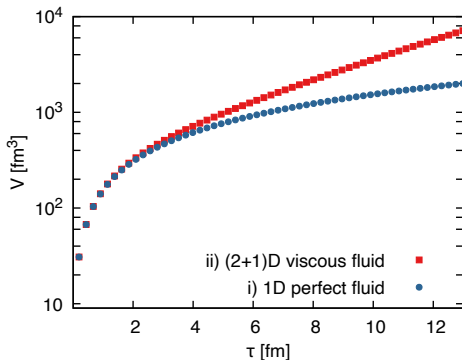


Common initial conditions:  $T_0 = 0.624$  GeV,  $\tau_0 = 0.2$  fm.

[Auvinen, Eskola, Huovinen, Niemi, Paatelainen, Petreczky, PRC 102 '20]

# Volume of the QGP

$$V(\tau) = \pi R^2 \tau \quad (12)$$



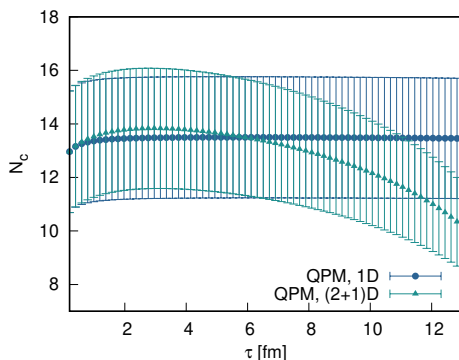
i) 1D ideal Bjorken dynamics:  $R = R_0 = 7$  fm

ii) (2+1)D viscous expansion:  $R(\tau) = R_0 + (\tau - \tau_0)^2 a/2$ ,  $a = 0.07 \text{ fm}^{-1}$

[P. Braun-Munzinger, K. Redlich, EPJ C 16 (2000), Zhang et al., Phys. Rev. C 77 (2008)]



# Charm Quark Evolution



1D:  $\tau_{\text{fin}} \simeq 13$  fm vs (2+1)D:  $\tau_{\text{fin}} \simeq 11$  fm

$$\text{Initial charm quark number: } \frac{dN_c}{dy} = 12.95 \pm 2.27 \quad (13)$$

(rapidity density for most central Pb-Pb collisions at  $\sqrt{s} = 5.02$  TeV)

# Summary

- ☞ **Quasiparticle model** – effective well-established tool connecting non-perturbative and perturbative QCD regimes.
- ☞ **Charm quarks** – minor thermal production in both ideal 1D- and viscous (2+1)D-expanding plasma, consistent with SHM predictions.
- ☞ **Possibilities** – quasihadrons out of chemical equilibrium, finite  $\mu$ , ...

# Extreme QCD 2025

- **Dates:** July 2 – 4, 2025
- **Venue:** University of Wroclaw, Wroclaw, Poland
- **Student support:** fee, travel, accommodation
- **Web:** INDICO  
INSPIRE



XQCD 2025

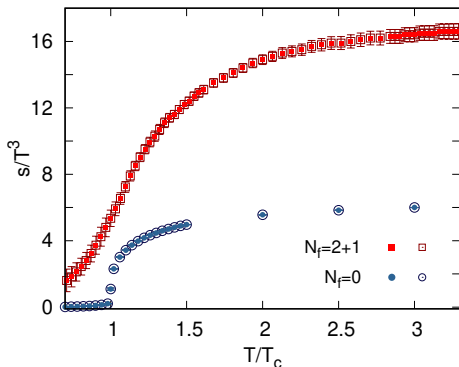


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# Quasiparticle Model

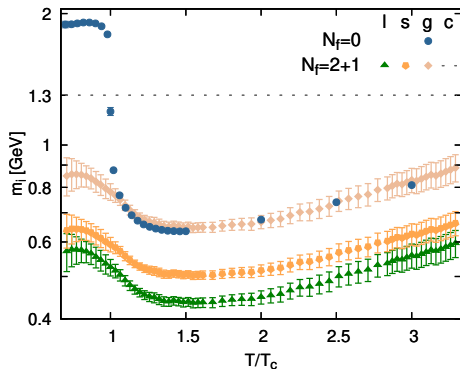
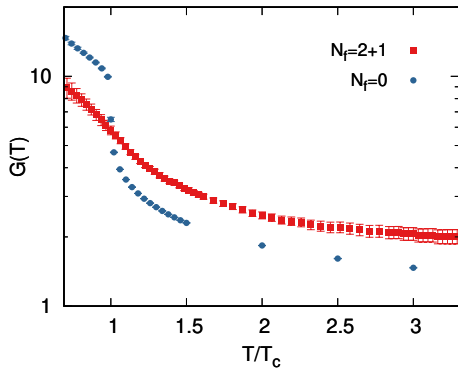
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$$f_i^0(E_i) : E_i[G(T), T] = \sqrt{p^2 + m_i^2[G(T), T]} \quad (14)$$



[V.M, M. Bluhm, C. Sasaki, K. Redlich, PRD 100 (2019); lattice: S. Borsanyi, et al., Phys. Lett. B 730 (2014) (Wuppertal-Budapest)]

# Effective Coupling and Masses



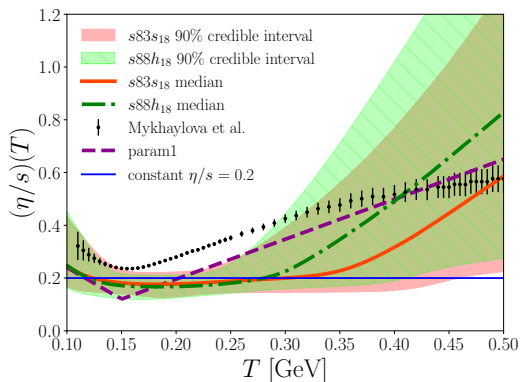
$$m_i[G(T), T] \gg m_l^0 = 5 \text{ MeV}, m_s^0 = 95 \text{ MeV}$$

[V.M., M. Bluhm, K. Redlich, C. Sasaki, PRD100 (2019), V.M. C. Sasaki, PRD 103 (2021)]

# Shear Viscosity

(reaction to flow) [Hosoya, Kajantie, NPB250 '85]

$$\eta = \frac{1}{15T} \sum_{i=g,l,s,\dots} d_i \int \frac{d^3p}{(2\pi)^3} \frac{p^4}{E_i^2} f_i^0 (1 \pm f_i^0) \tau_i \quad (15)$$



[V.M., M. Bluhm, K. Redlich, C. Sasaki, PRD100 '19; Auvinen, Eskola, Huovinen, Niemi, Paatelainen, Petreczky, PRC 102 '20]