Anatomy of Critical Fluctuations in Hadronic Matter

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References:

[1] M. Marczenko, K Redlich, C. Sasaki PRD 107, (2023) 5, 054046

[2] V. Koch, M. Marczenko, K Redlich, C. Sasaki, PRD 109 (2024) 1, 014033

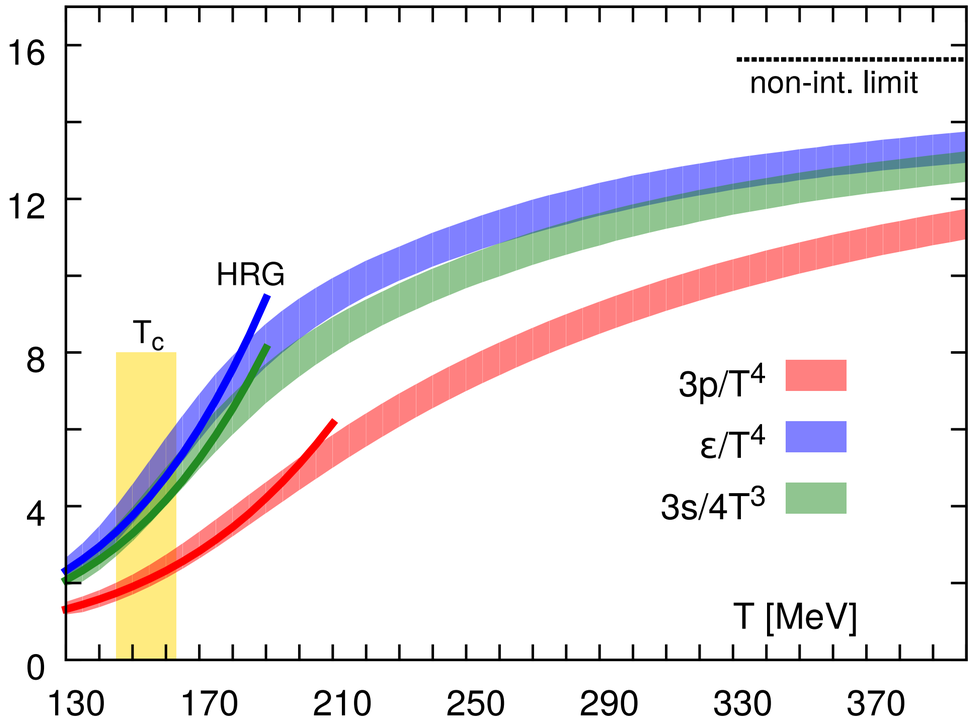
[3] M. Marczenko, PRD 110 (2024) 1, 014018

[4] M. Marczenko, K Redlich, C. Sasaki arXiv:2410.21746 (2024)

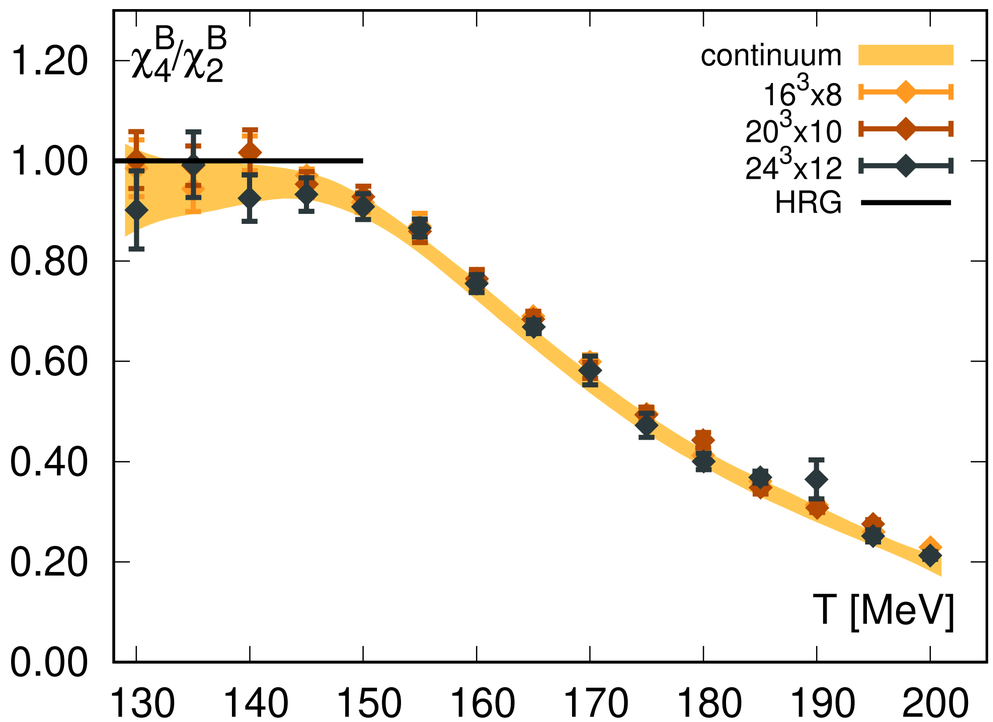
December 02, 2024 - 24th ZIMÁNYI SCHOOL WINTER WORKSHOP ON HEAVY ION PHYSICS

Lattice QCD vs Hadron Resonance Gas

Bazavov et al, 2014



Borsányi et al, 2023



Pressure in the HRG model

$$P^{\mathrm{HRG}} = \sum_{i \in \mathrm{had}} P^{\mathrm{id}}\left(T, \mu_i; m_i\right)$$

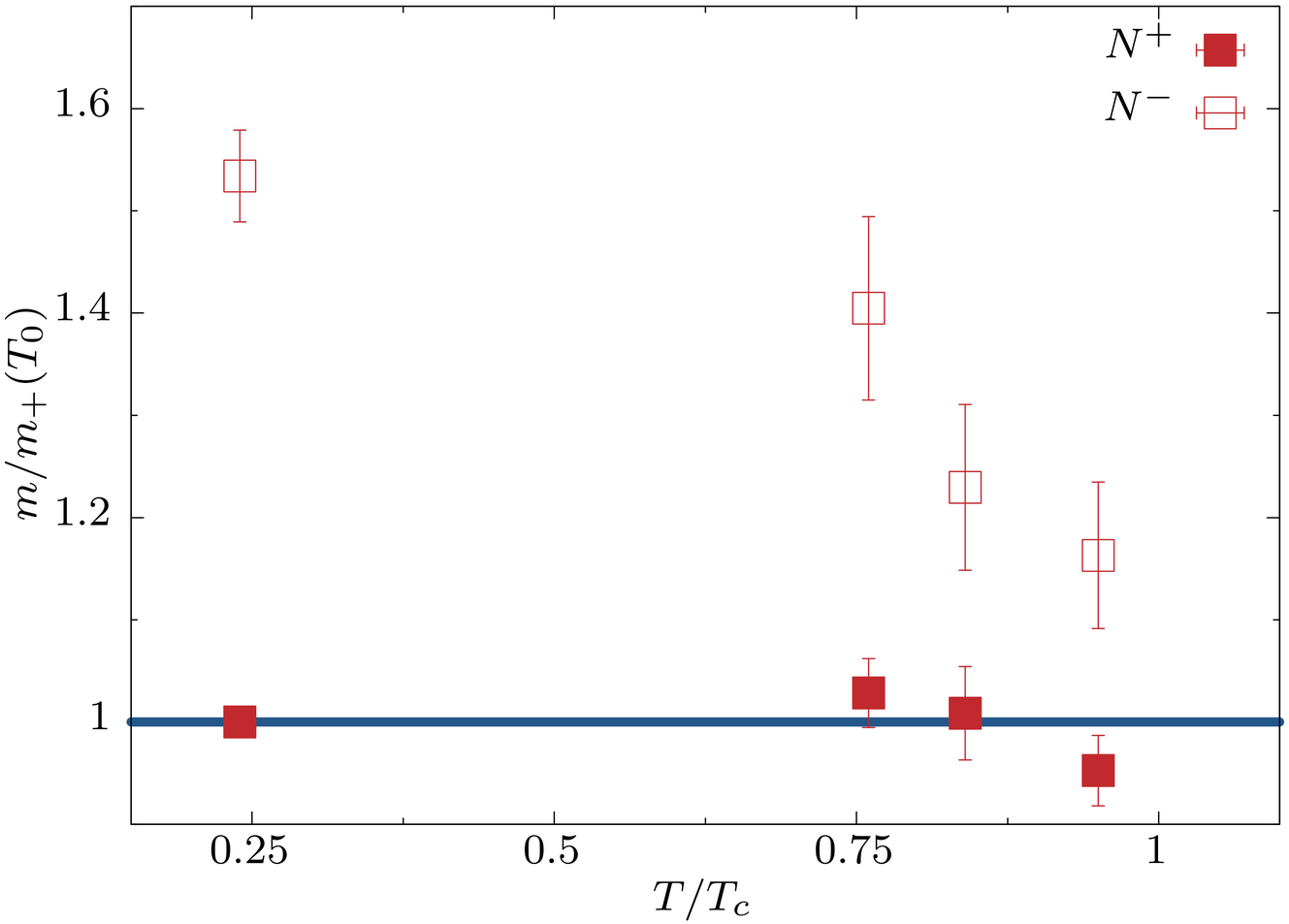
Agreement with LQCD EoS up to $\simeq T_c$

Taylor expansion of LQCD EoS

$$\frac{P}{T^4} = \sum_{k=0}^{\infty} \left(\frac{\mu_B}{T}\right)^k \frac{\chi_k^B}{k!}, \text{ where } \chi_k^B = \frac{\partial^k P/T^4}{\partial\left(\mu_B/T\right)^k}$$

Kurtosis: $\dfrac{\chi_4^B}{\chi_2^B} \sim B^2$: breakdown $\sim T_c$: changeover to QGP

Parity Doubling in Lattice QCD Aarts et al, 2017, 2019



- N^+ nucleon stays nearly unchanged
- N^- chiral partner drops mass towards T_c
- Chiral partners $N^\pm$ degenerate at T_c
- Chiral parents stay massive
- Seen for octet and decouplet of baryons

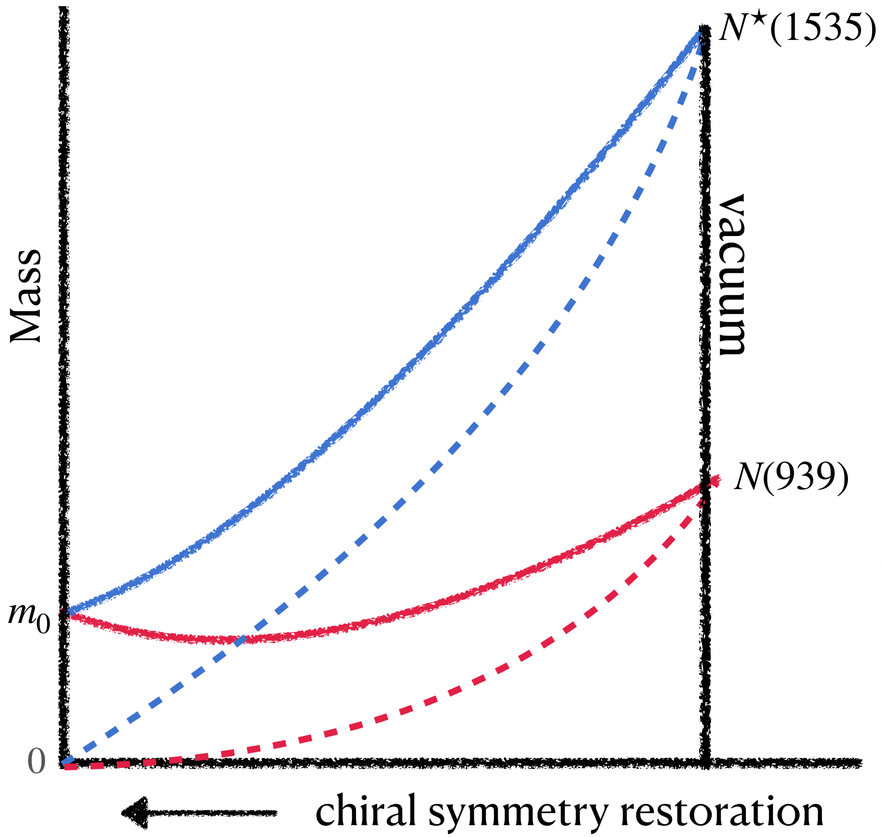
Imprint of chiral symmetry restoration in the baryonic sector

LQCD results still obtained with heavy m_π far from continuum limit

Parity Doublet Model a'la DeTar, Kunihiro 1989

- SU(2) chiral transformation of 2 nucleons → how to assign 2 independent rotation to them?

$$\mathscr{L}_{\text{mass}} \sim m_0 \left(\bar{\psi}_1 \gamma_5 \psi_2 + \bar{\psi}_2 \gamma_5 \psi_1\right) \Longrightarrow M_\pm = \frac{1}{2}\left(\sqrt{4m_0^2 + a^2\sigma^2} \mp b\sigma\right) \xrightarrow{\sigma \to 0} m_0$$



N_- $\boxed{N(1535)\ 1/2^-}$

$I(J^P) = \frac{1}{2}(\frac{1}{2}^-)$ Status: ****

N_+ $\boxed{p}$ $\boxed{n}$

$I(J^P) = \frac{1}{2}(\frac{1}{2}^+)$ Status: ****

Caution

- N(1535): the lower-lying resonance
  - πN & ηN interactions
  - similar to $f_0(500)$ vs σ in LSMa

For multiplicity $N_B = N_+ + N\_$ Net-baryon number: $\langle N_B \rangle = \langle N_+ \rangle + \langle N\_ \rangle$

Second-order fluctuations of the net-baryon number:

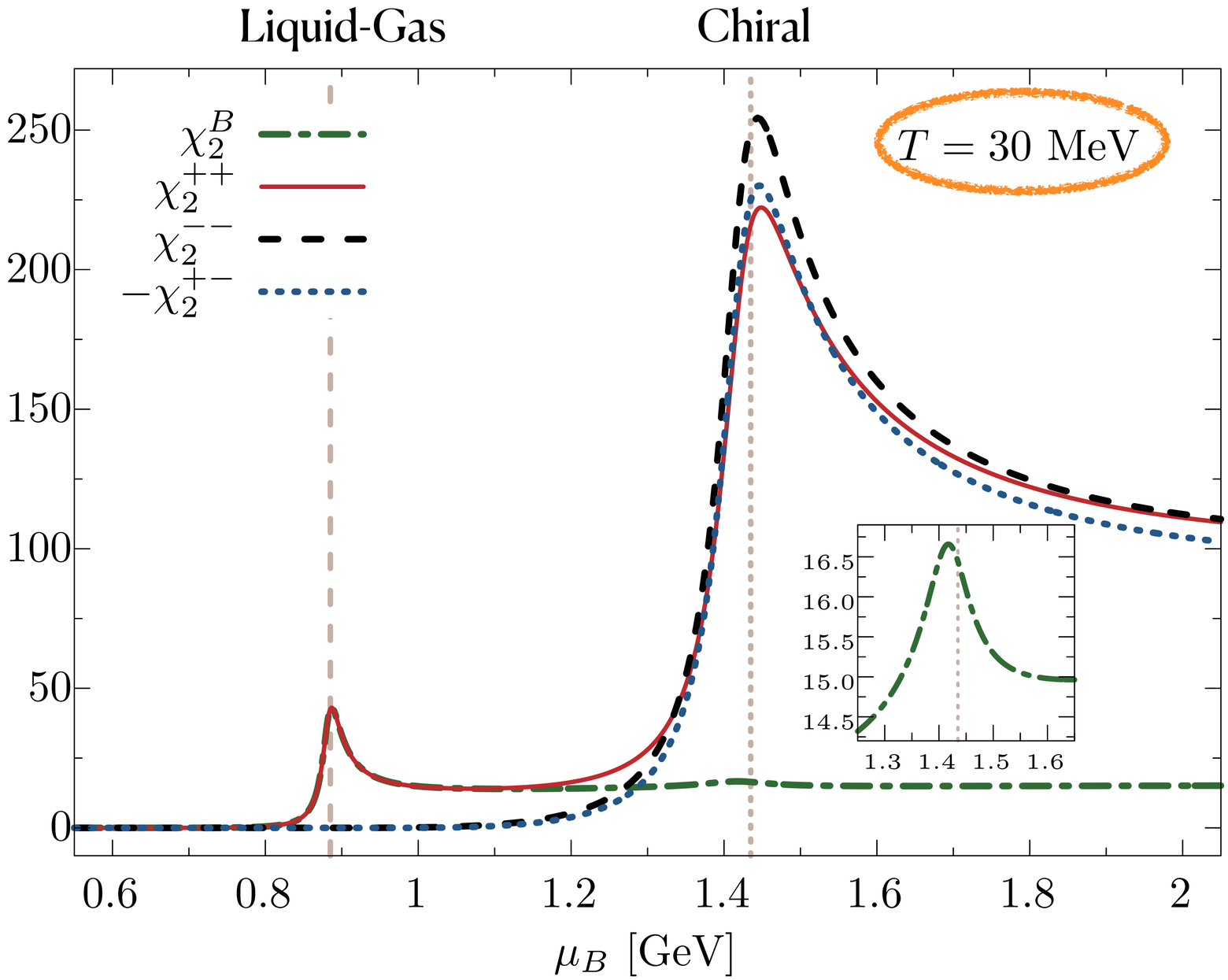
$$\langle \delta N_B \delta N_B \rangle = \langle (\delta N_+)^2 \rangle + \langle (\delta N\_)^2 \rangle + 2\langle \delta N_+ \delta N\_ \rangle$$

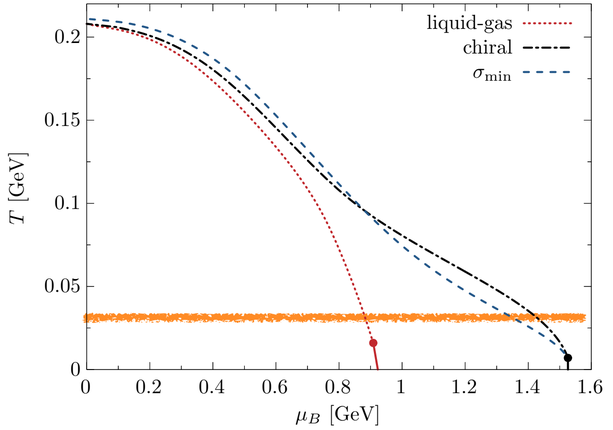
$$\langle \delta N_\alpha \delta N_\beta \rangle = VT^3 \chi_n^{\alpha\beta} \quad \longleftrightarrow \quad \chi_2^{\alpha\beta} = \frac{d^2 P/T^4}{d(\mu_\alpha/T) d(\mu_\beta/T)}$$

$$\chi_2^B = \chi_2^{++} + \chi_2^{--} + 2\chi_2^{+-}$$

- What are the individual contributions of parity partners N_+ and $N\_$?
- What is the strength and sign of the correlation χ_2^{+-}?
- Is net-proton a good proxy for net-baryon fluctuations? $\chi_2^B = \chi_2^{++} + \chi_2^{--} + 2\chi_2^{+-}$

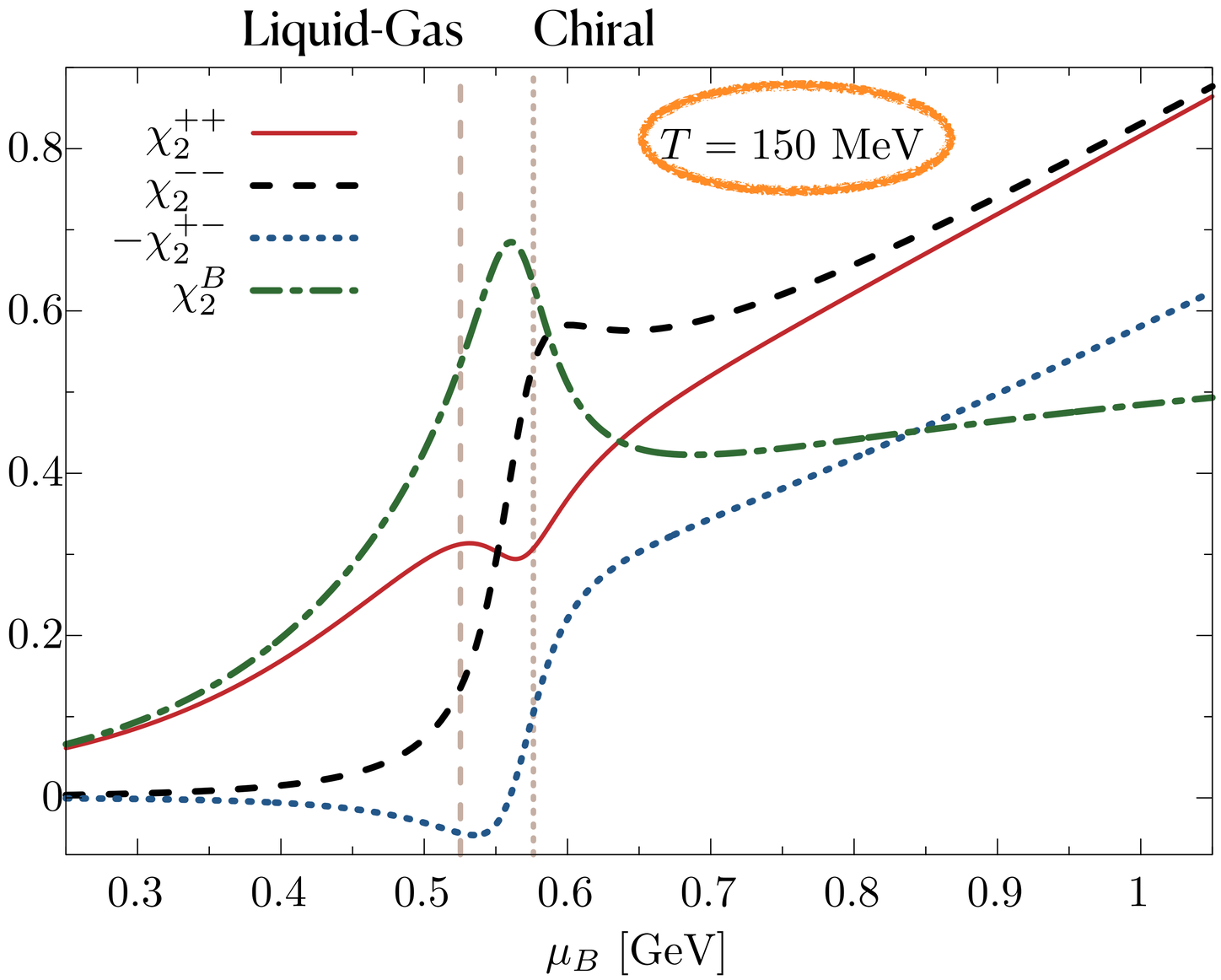
Fluctuations at liquid-gas and chiral transitions

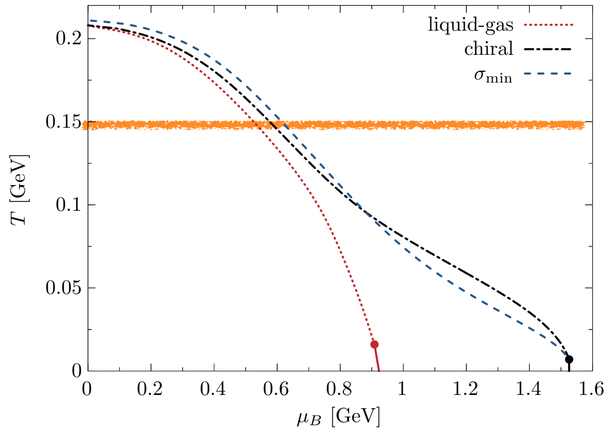




$$\chi_2^B = \chi_2^{++} + \chi_2^{--} + 2\chi_2^{+-}$$

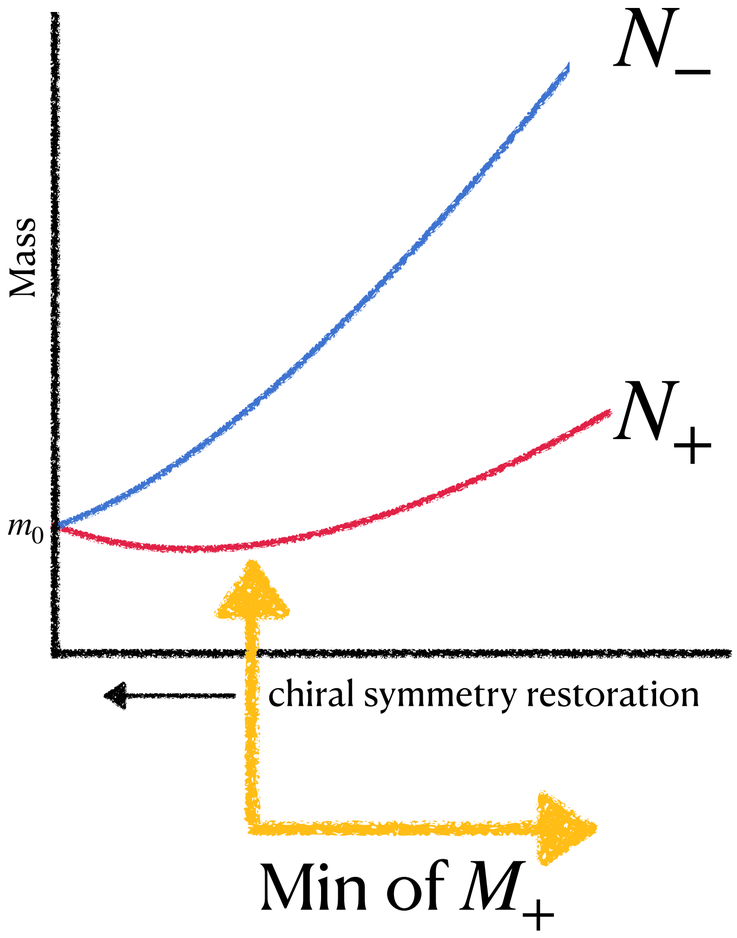
Increasing T ⟶ peaks get closer

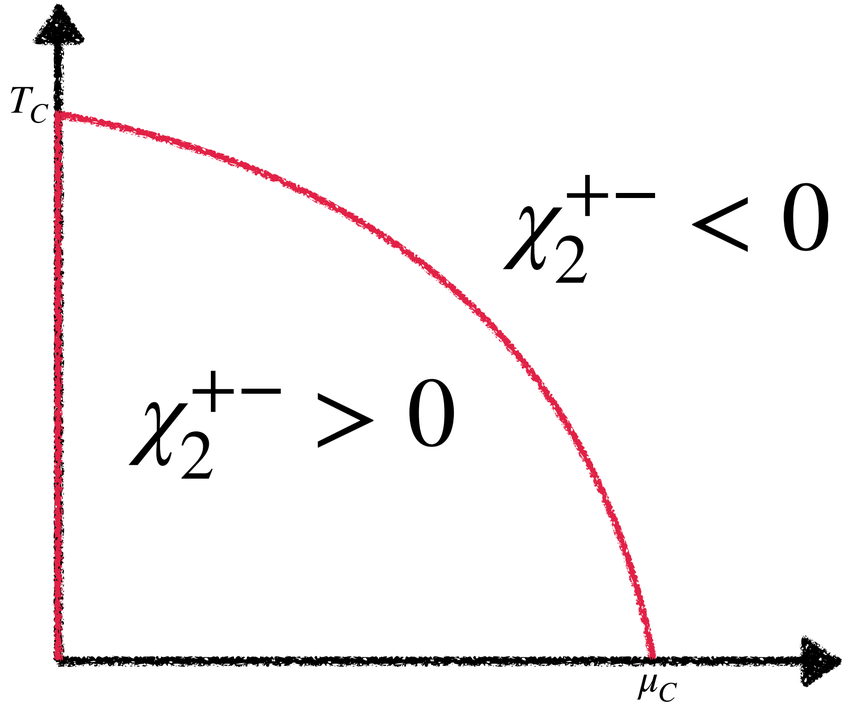




- Qualitative difference of χ_2^{++} and χ_2^{--}
- Stronger signal left in χ_2^B

Idealized behavior of the χ_2^{+-} correlator $\longrightarrow$ no repulsive forces





$$\chi_2^{+-} \sim \frac{\partial m_+}{\partial \sigma}\frac{\partial m_-}{\partial \sigma}$$

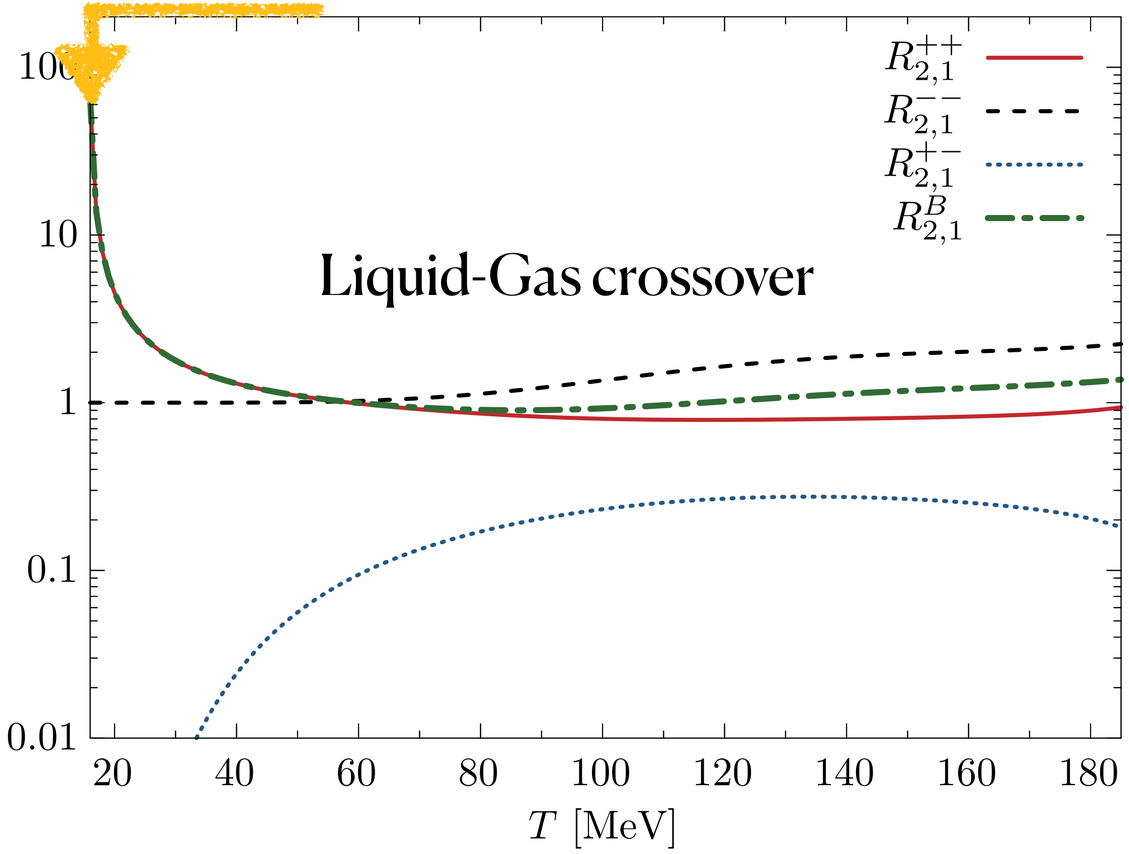
but also repulsion

Correlations of between different baryon species e.g., $N^{\pm}\Delta^{\mp}$, behave similarly

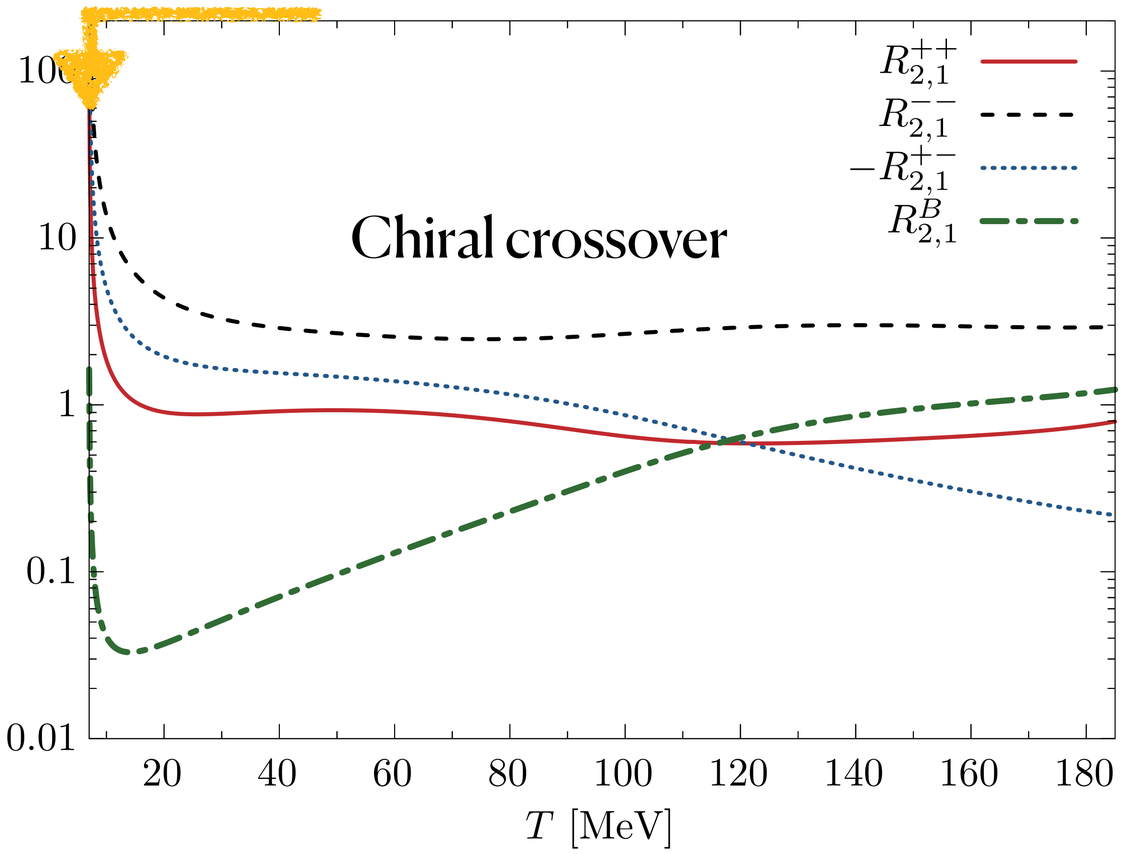
Change of the sign of χ_2^{+-} linked to the chiral phase boundary → interesting quantity to calculate in LQCD

$R_{2,1} = \chi_2/\chi_1$ along phase boundary

Liquid-Gas CP



Chiral CP



Fluctuations dominated by positive parity

↓

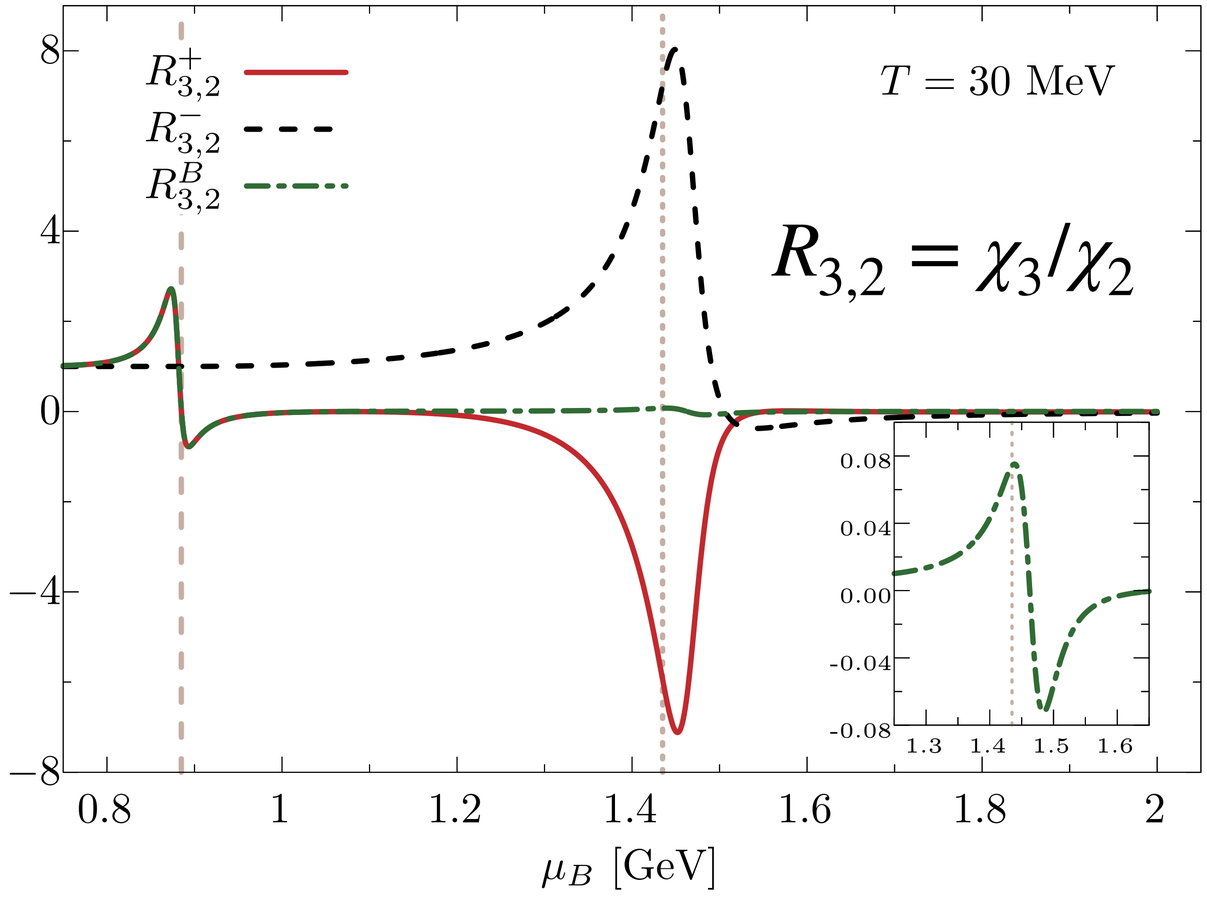
Net-baryon ~ Net-nucleon

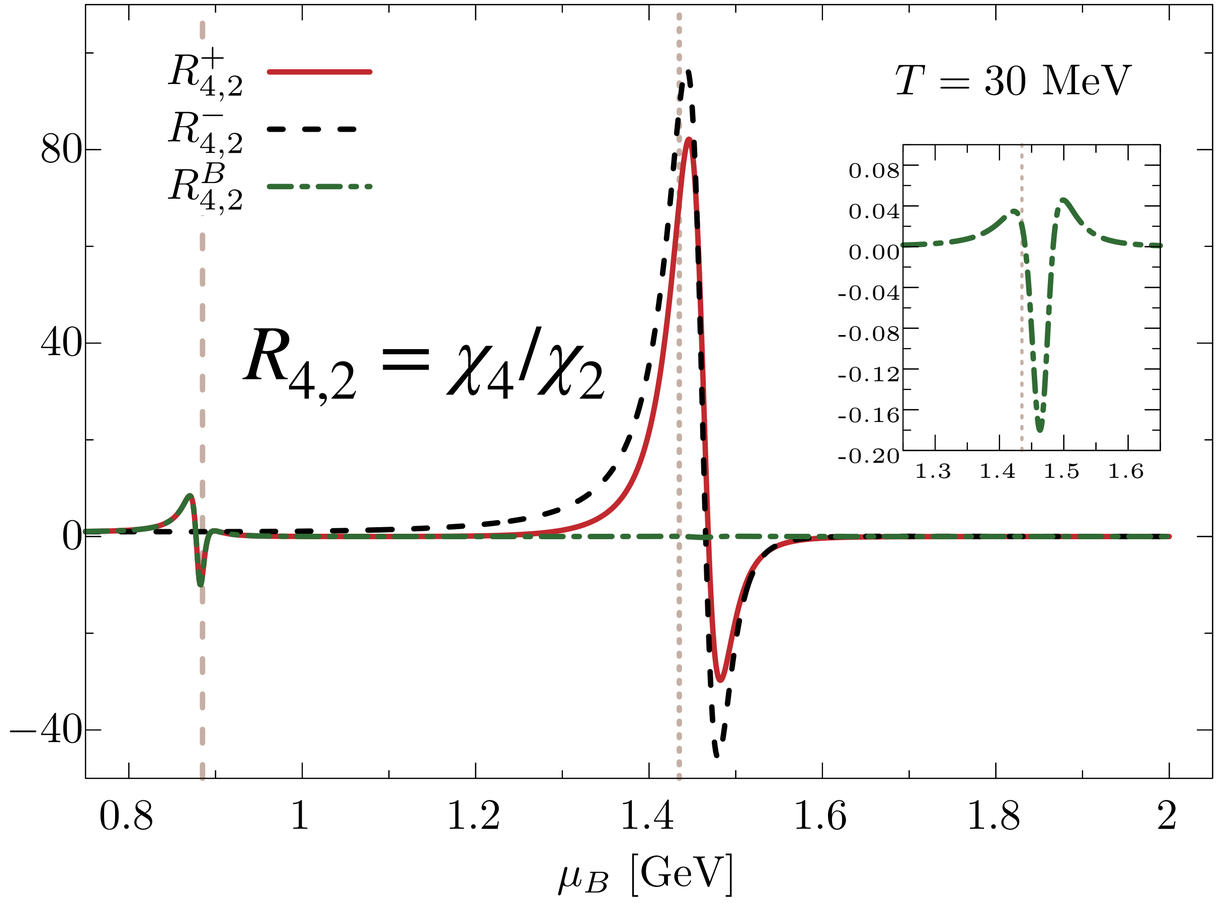
Presence of chiral partners + correlations

↘ ↙

Net-baryon ≪ Net-nucleon

Higher-Order Fluctuations of Parity Partners

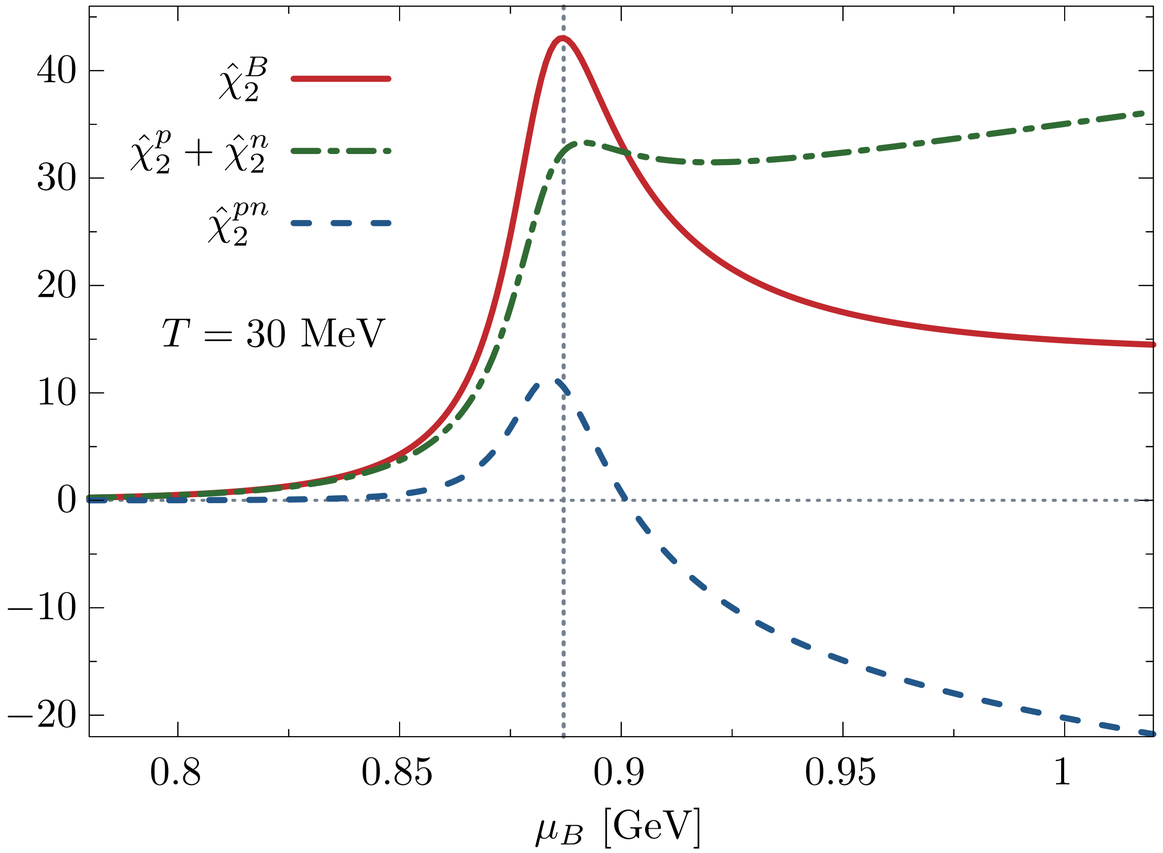


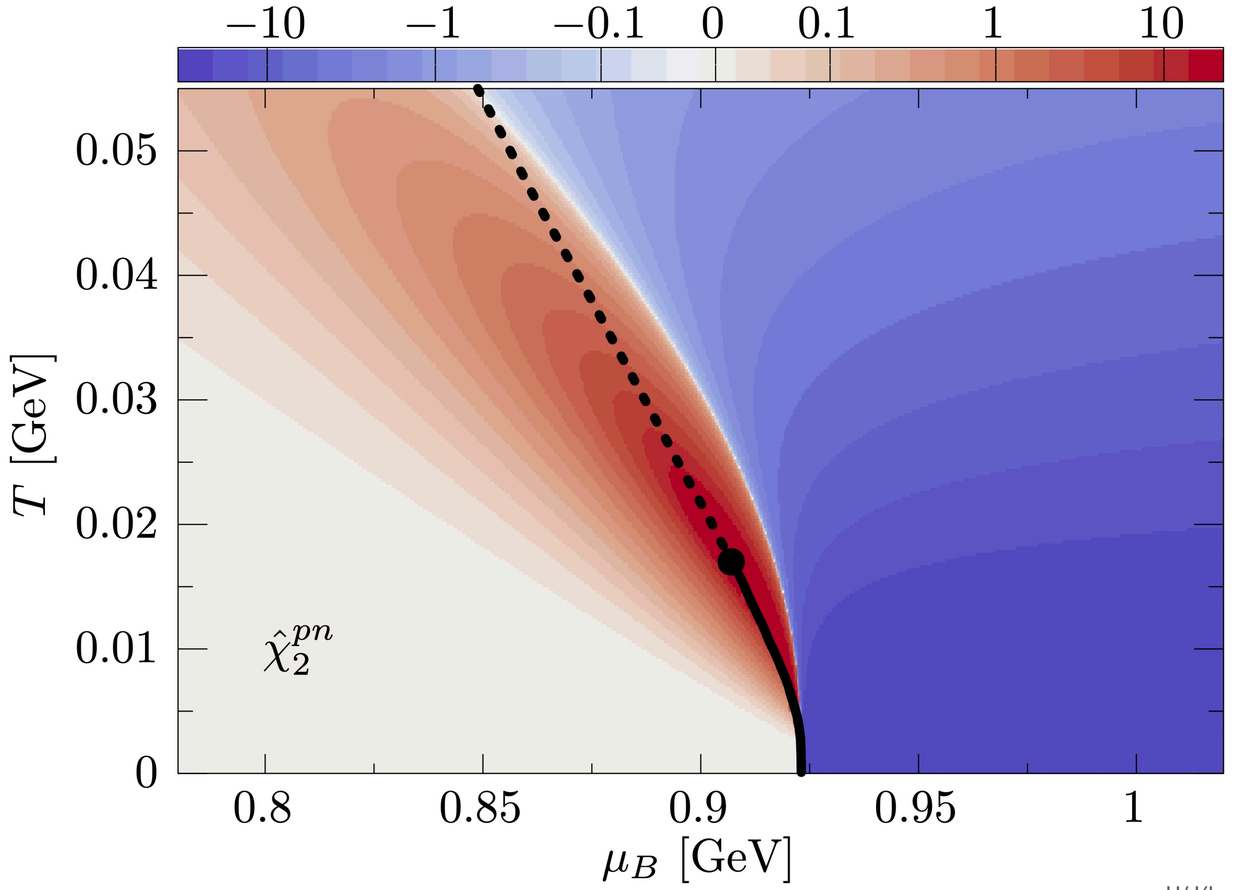


The net-proton fluctuations do not necessarily reflect the net-baryon fluctuations at the chiral phase boundary

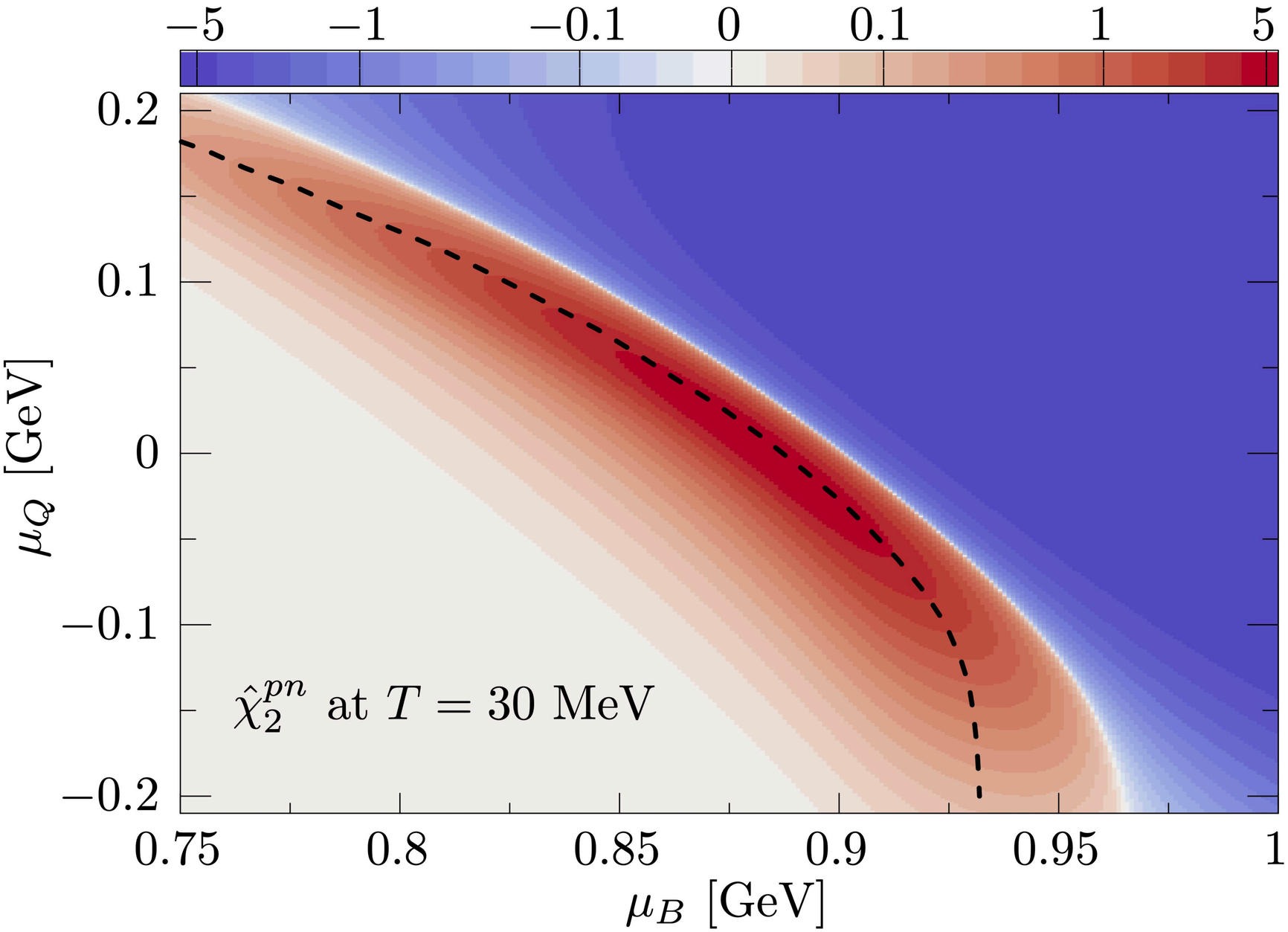
Isospin Correlations Near the Liquid-Gas Transition

$$\chi_2^B = \chi_2^{++} + \ldots \simeq \chi_2^p + \chi_2^n + \chi_2^{pn} \neq 2\chi_2^p$$





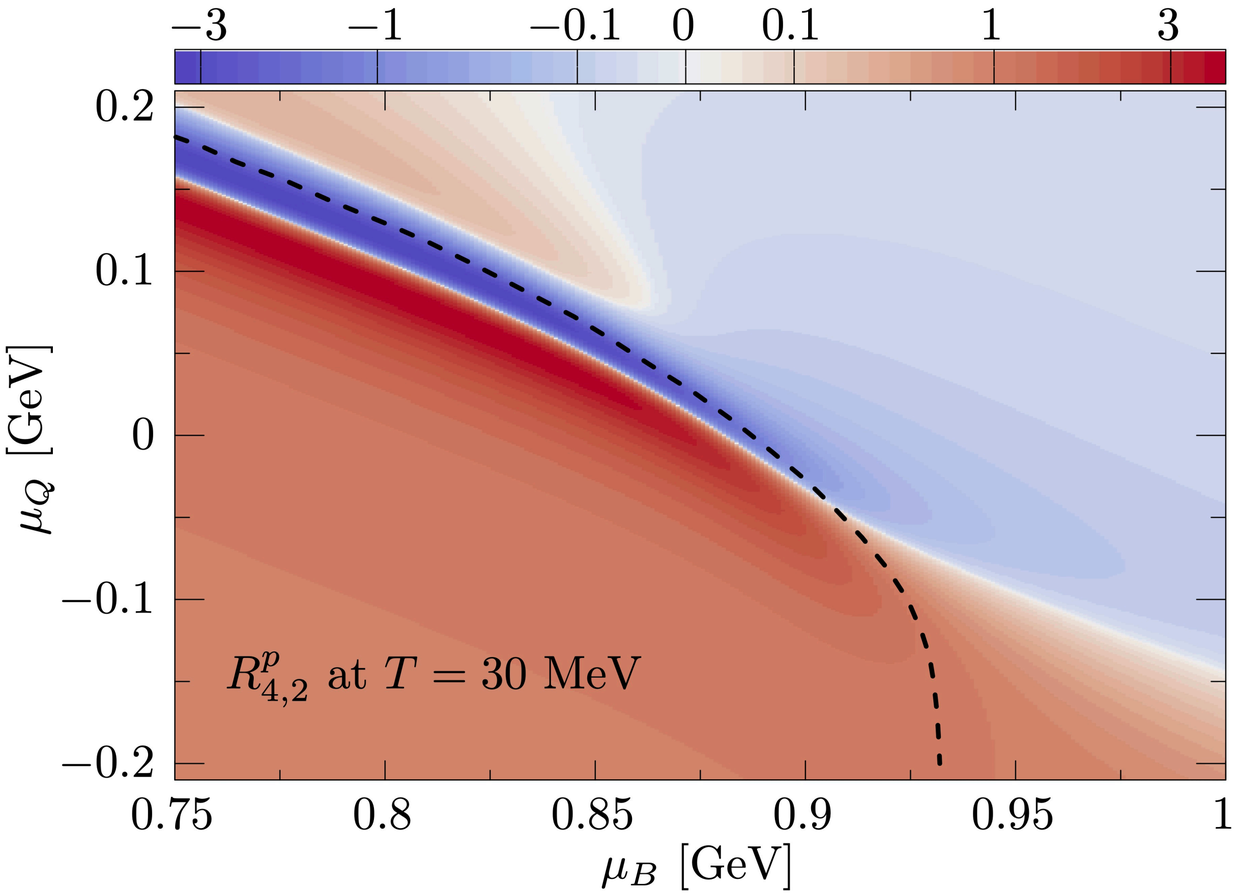
Isospin Correlations Near the Liquid-Gas Transition

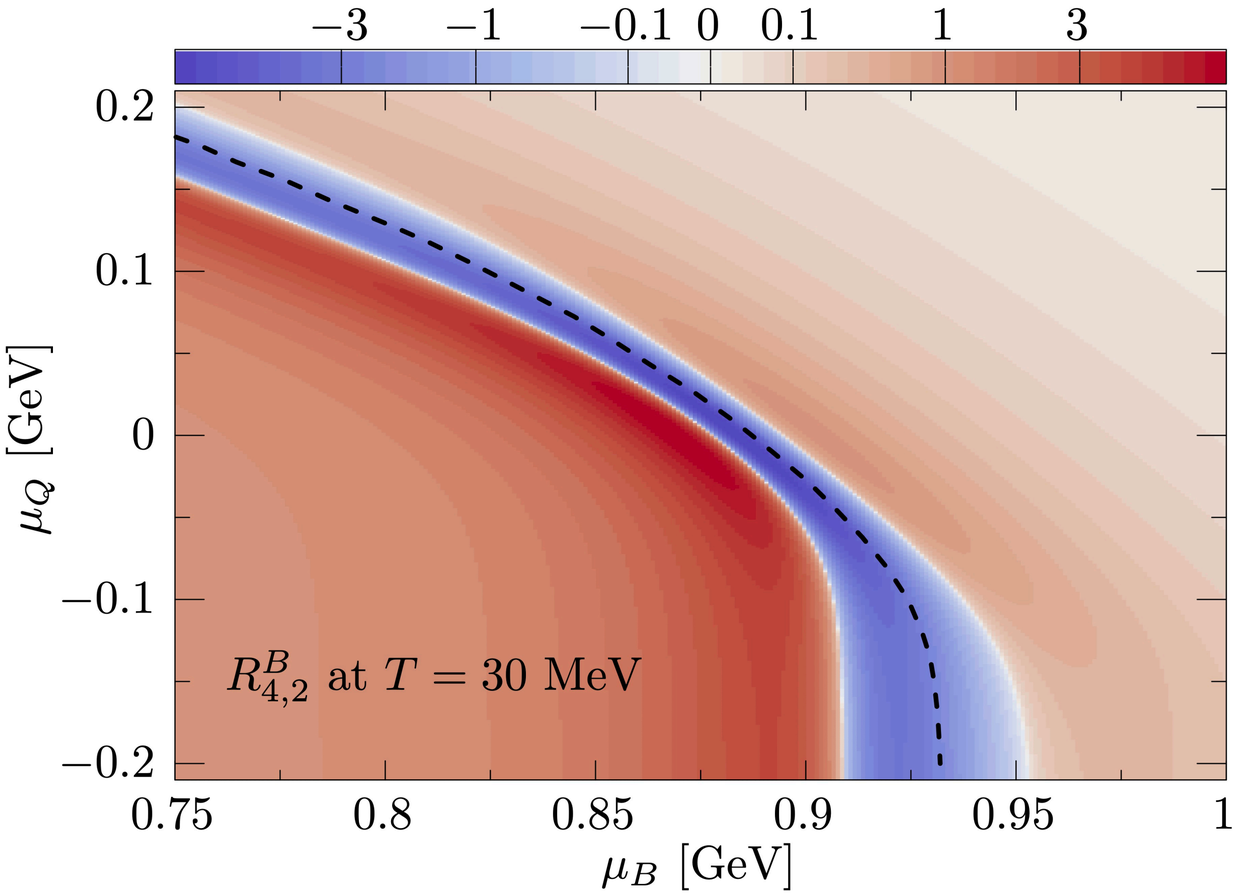


Isospin Correlations Near the Liquid-Gas Transition

$$R^p_{4,2} = \chi^p_4/\chi^p_2$$

$$R^B_{4,2} = \chi^B_4/\chi^B_2$$





Summary

Non-trivial correlations between baryonic chiral partners

χ_2^{proton} may not reflect χ_2^B at the chiral or LG phase boundary

Interesting to calculate χ_2^{+-} in other non-perturbative approaches

Thank You

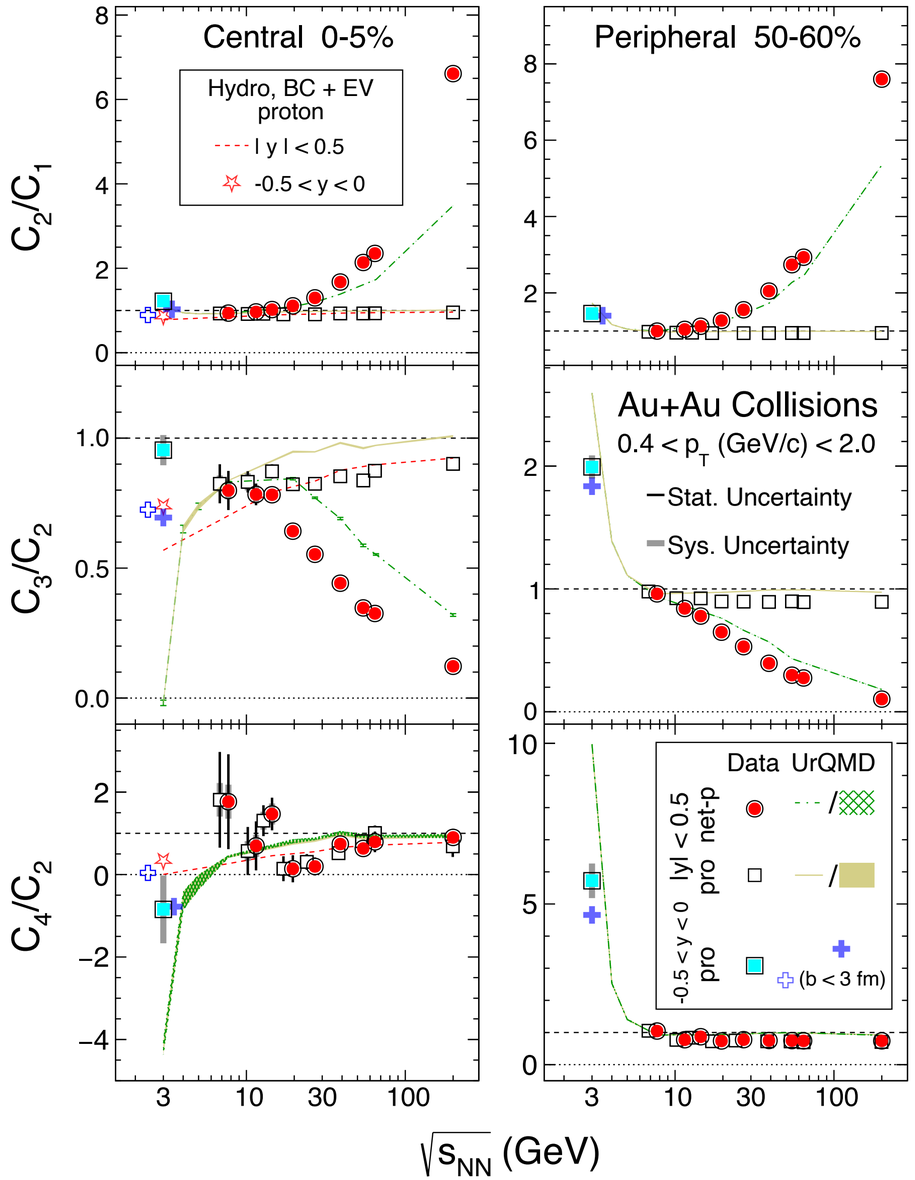
Cumulants vs Susceptibilities

Mean: M	$\langle N_B \rangle$	C_1
Variance: σ^2	$\langle (\delta N_B)^2 \rangle$	C_2
Skewness: S	$\langle (\delta N_B)^3 \rangle / \sigma^3$	$C_3/C_2^{3/2}$
Kurtosis: K	$\langle (\delta N_B)^4 \rangle / \sigma^3 - 3$	C_4/C_2^2

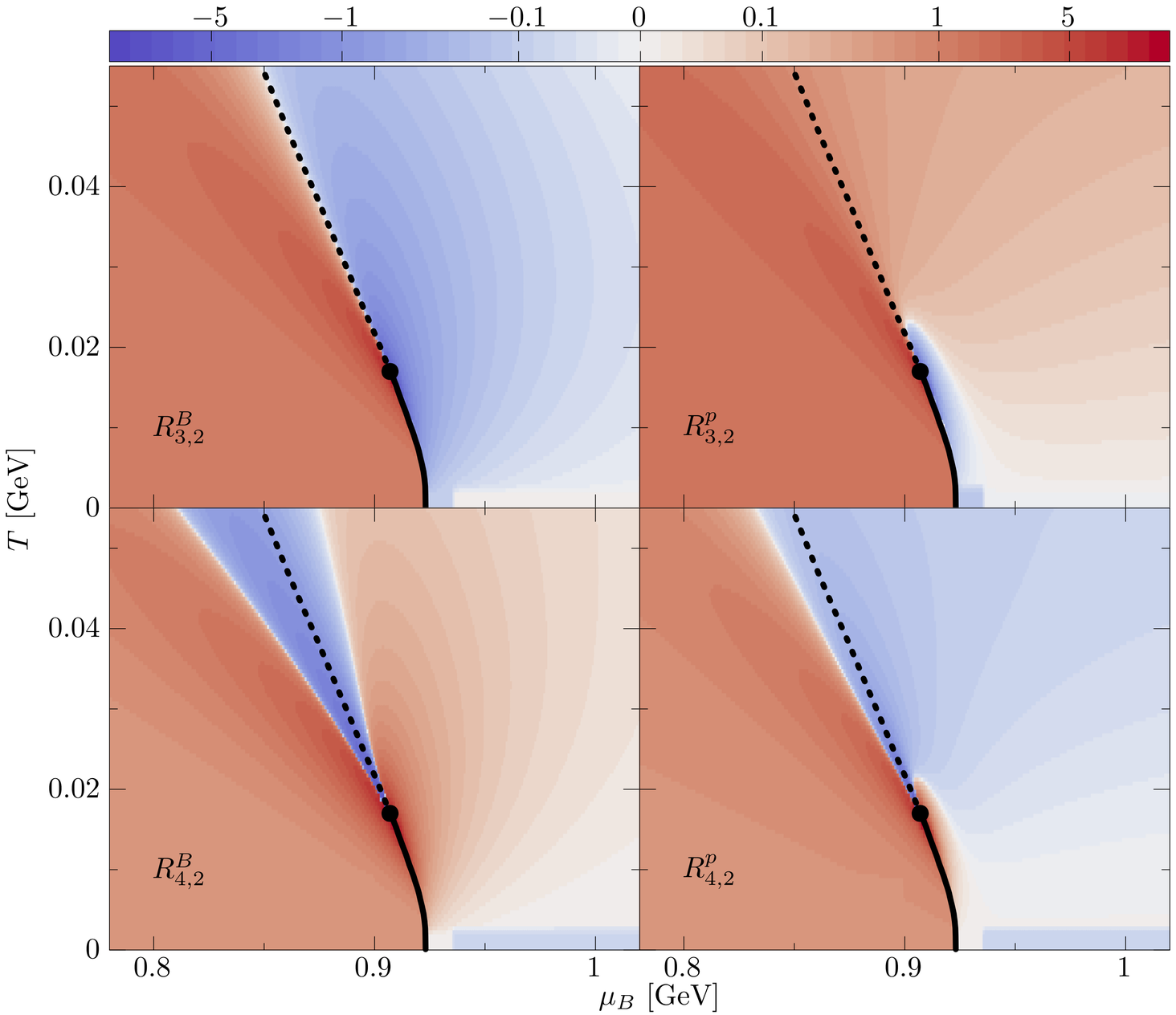
$$C_n \equiv VT^3 \frac{d^n P/T^4}{d\left(\mu_B/T\right)^n}\bigg|_T \longleftrightarrow \chi_n^B \equiv \frac{d^n P/T^4}{d(\mu_B/T)^n}\bigg|_T$$

$$C_n = VT^3 \chi_n^B$$

STAR, 2023

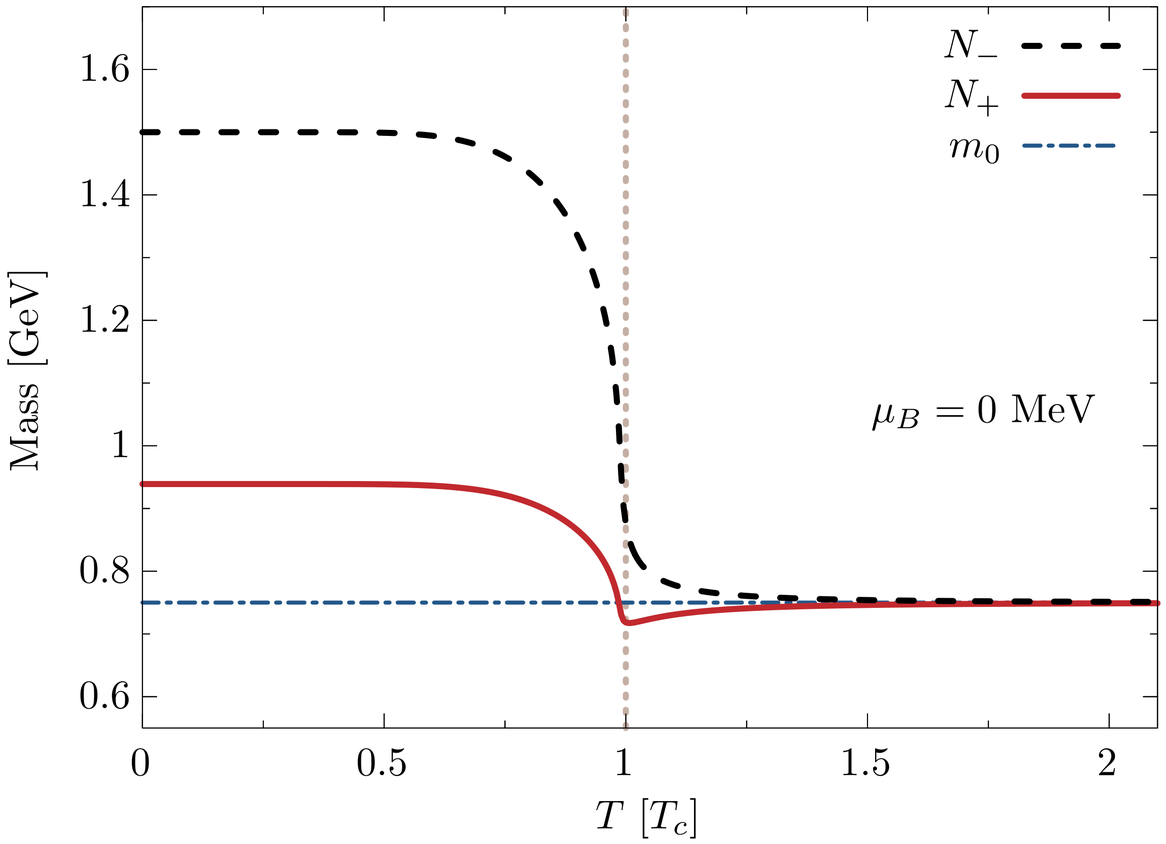


Isospin Correlations Near the Liquid-Gas Transition



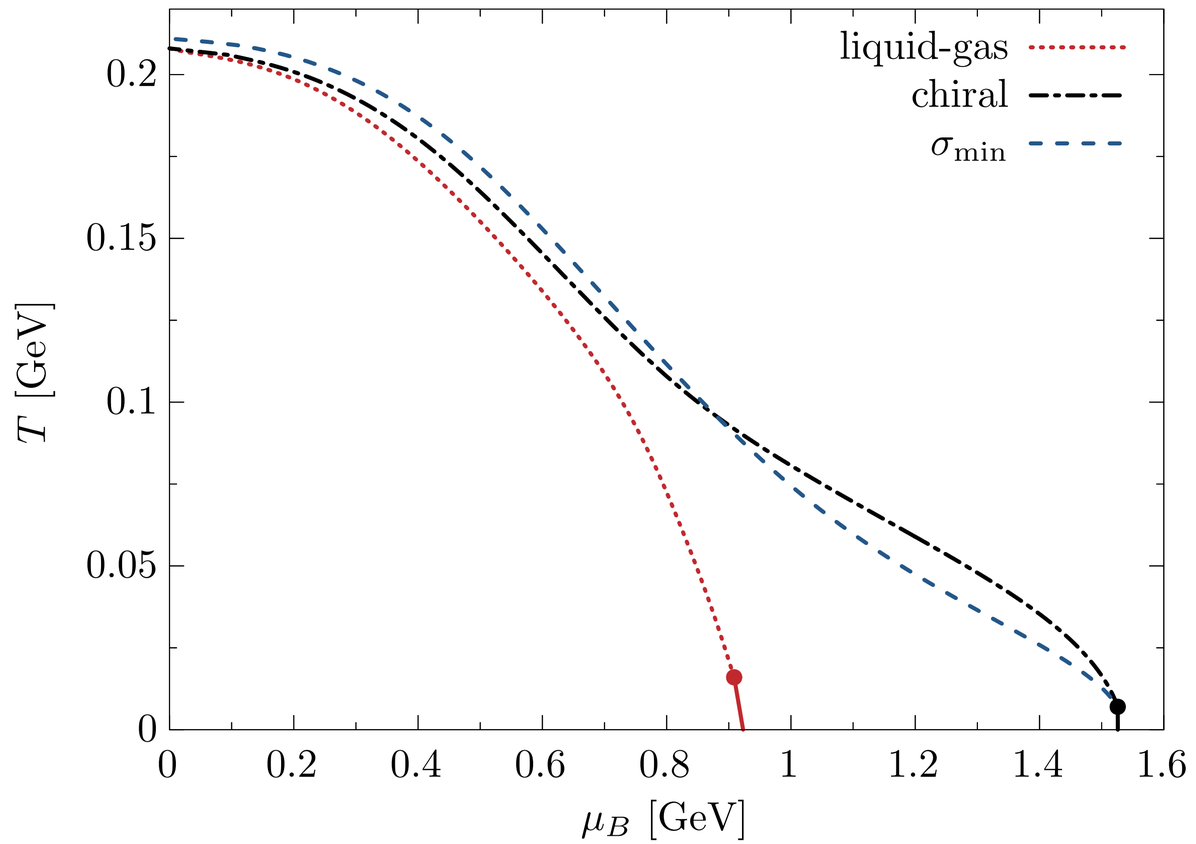
Chiral Criticality in Parity Doubling Model

In-medium masses

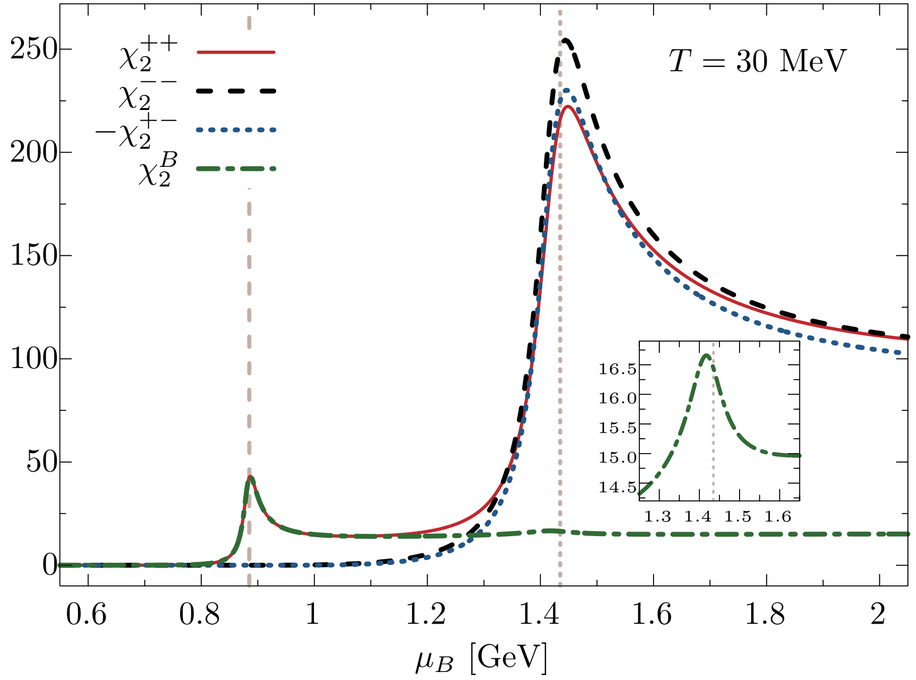


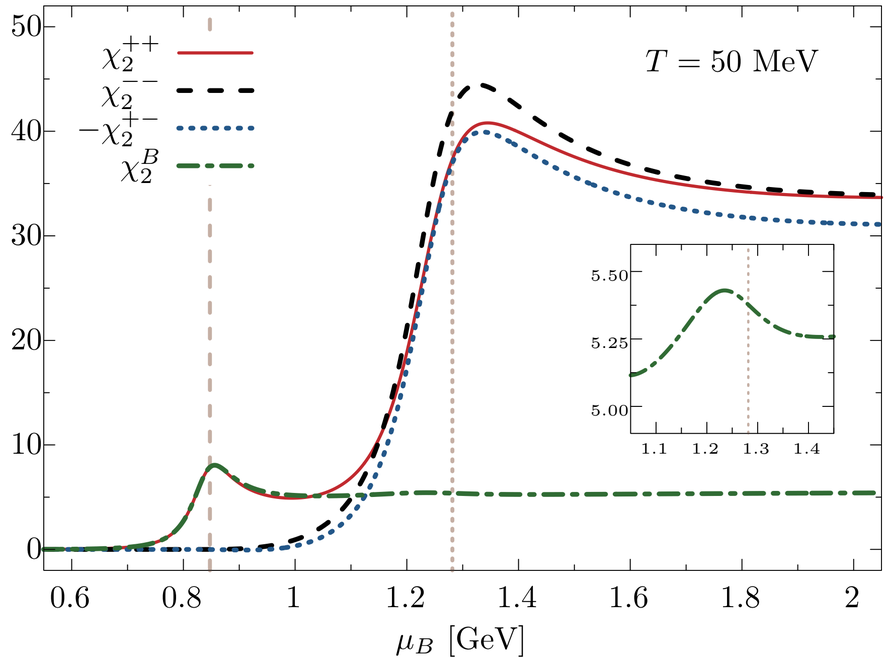
- M_- decreases monotonically
- M_+ has a minimum at $\sigma_{\min} = 2\dfrac{b}{a}\dfrac{m_0}{\sqrt{a^2 - b^2}}$

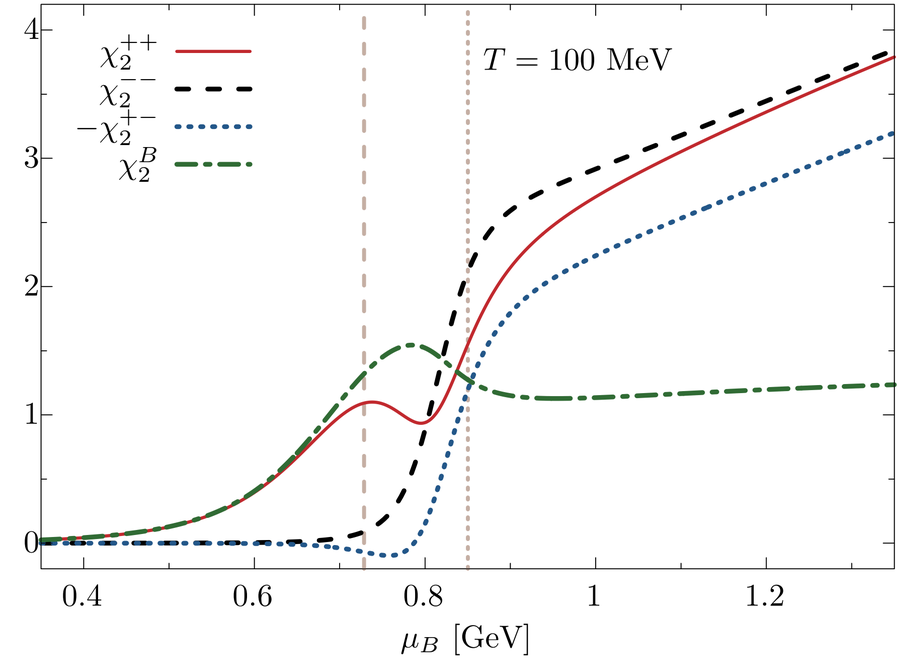
Phase diagram with liquid gas and chiral PTs

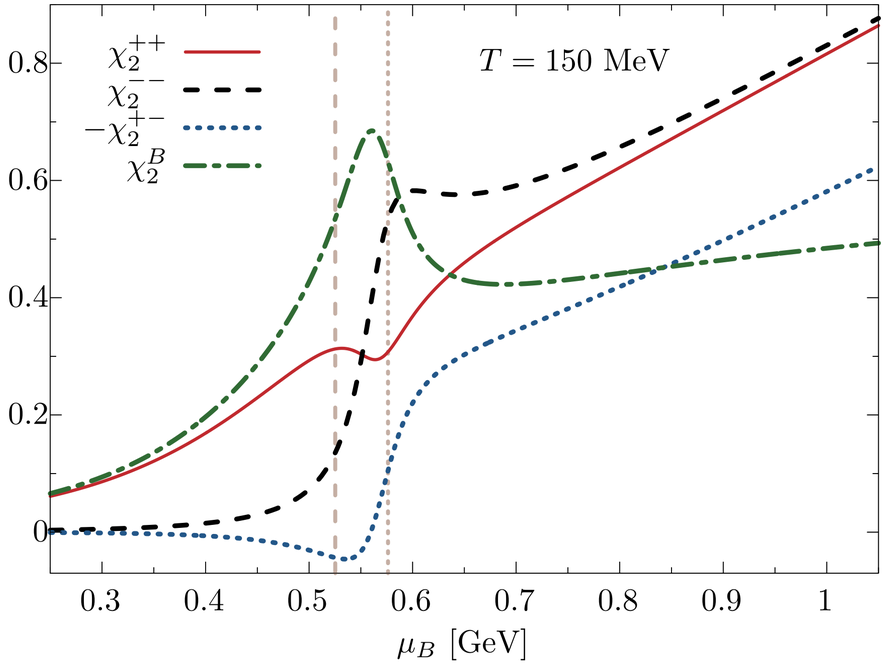


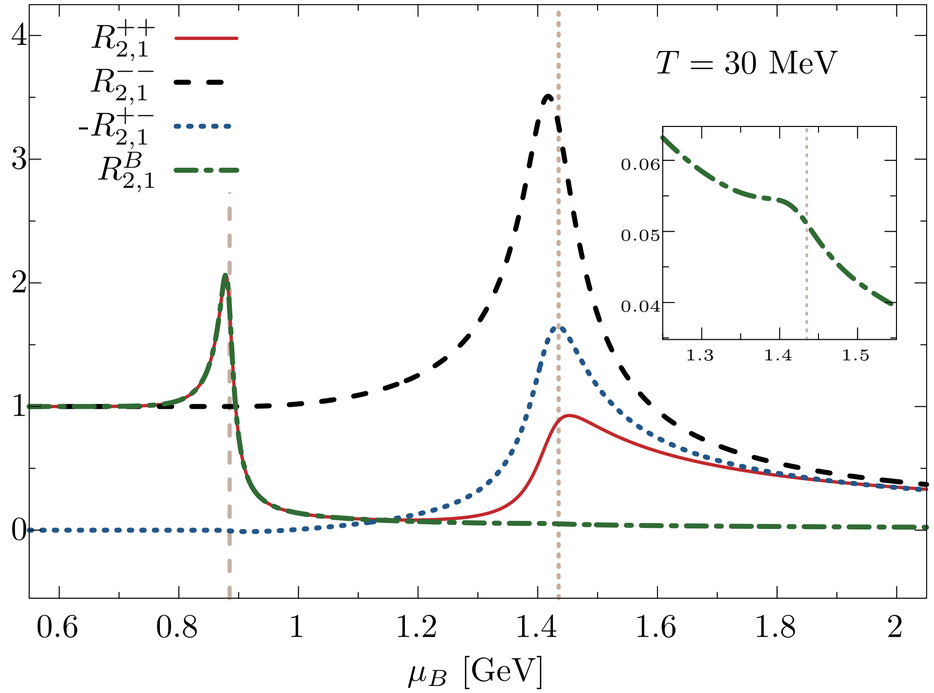
- Position of $\sigma_{\min}$ closely related to the chiral phase transition

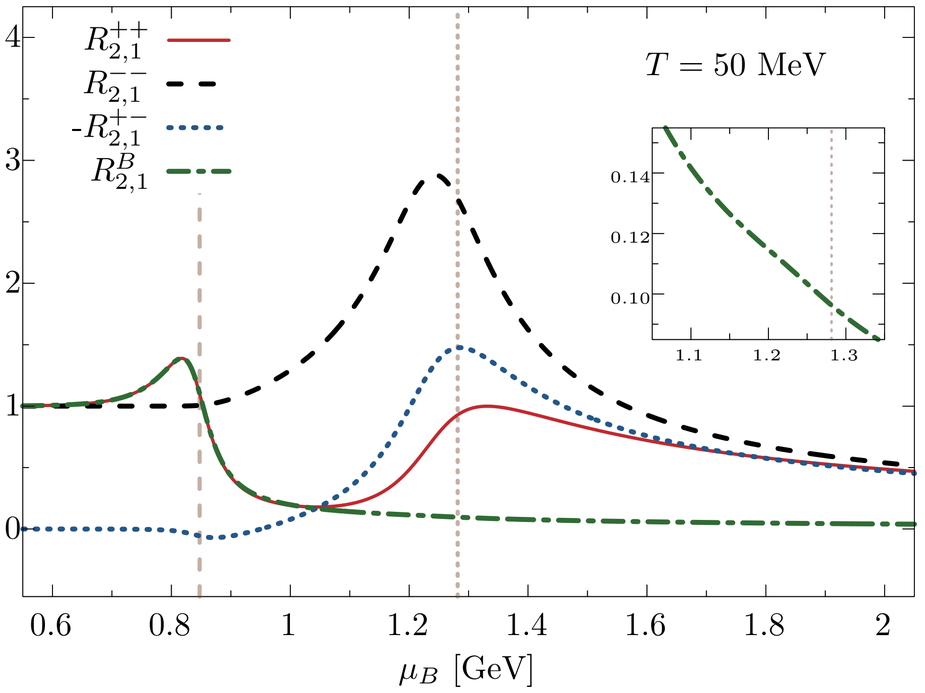

χ_2^{++}
χ_2^{--}
$-\chi_2^{+-}$
χ_2^B
$T = 30$ MeV
μ_B [GeV]

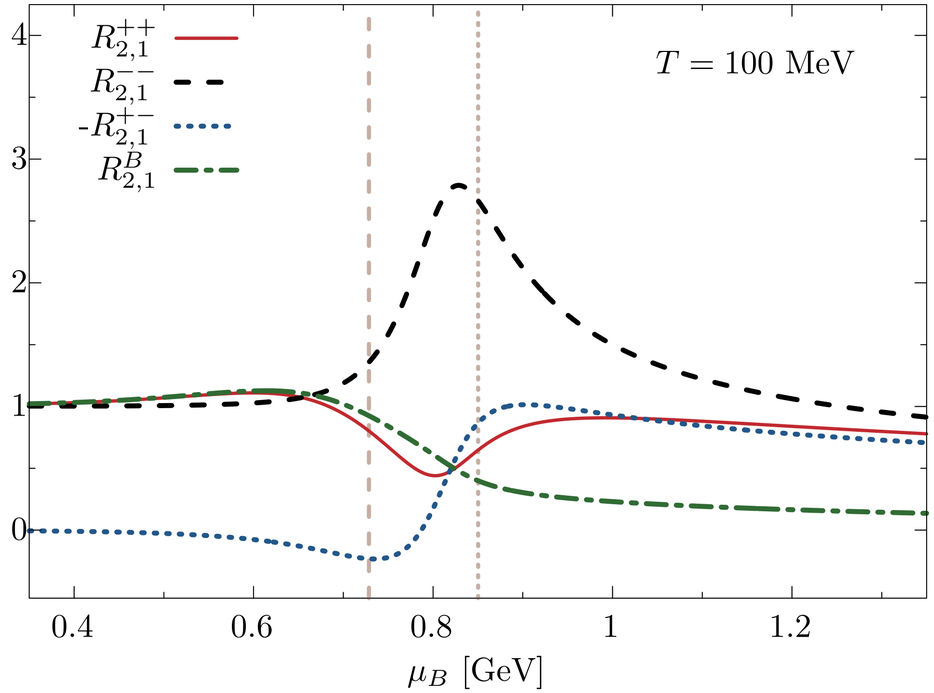

χ_2^{++}
χ_2^{--}
$-\chi_2^{+-}$
χ_2^B
$T = 50$ MeV
μ_B [GeV]

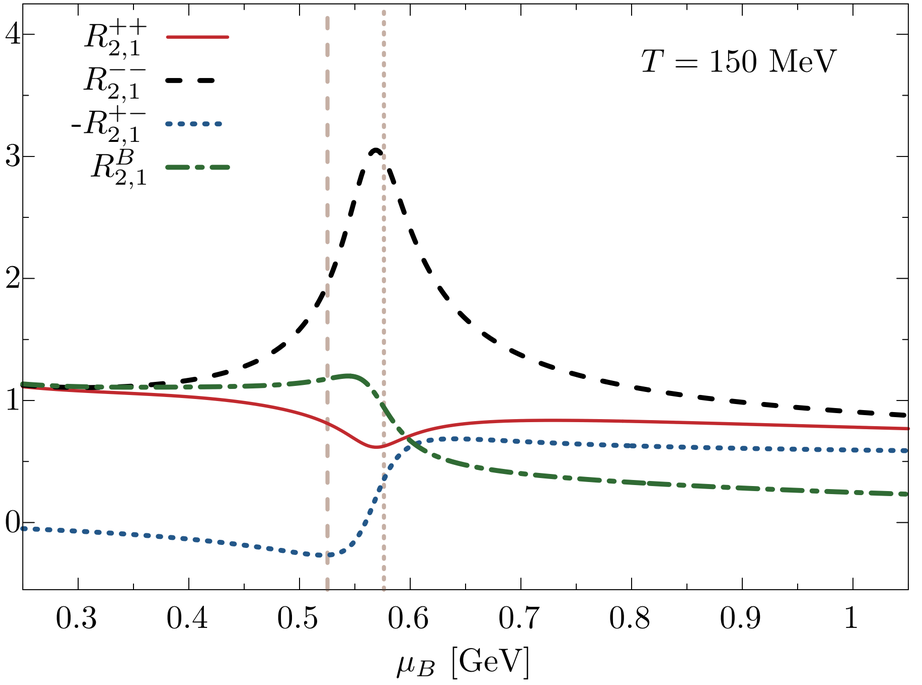

χ_2^{++}
χ_2^{--}
$-\chi_2^{+-}$
χ_2^B
$T = 100$ MeV
μ_B [GeV]

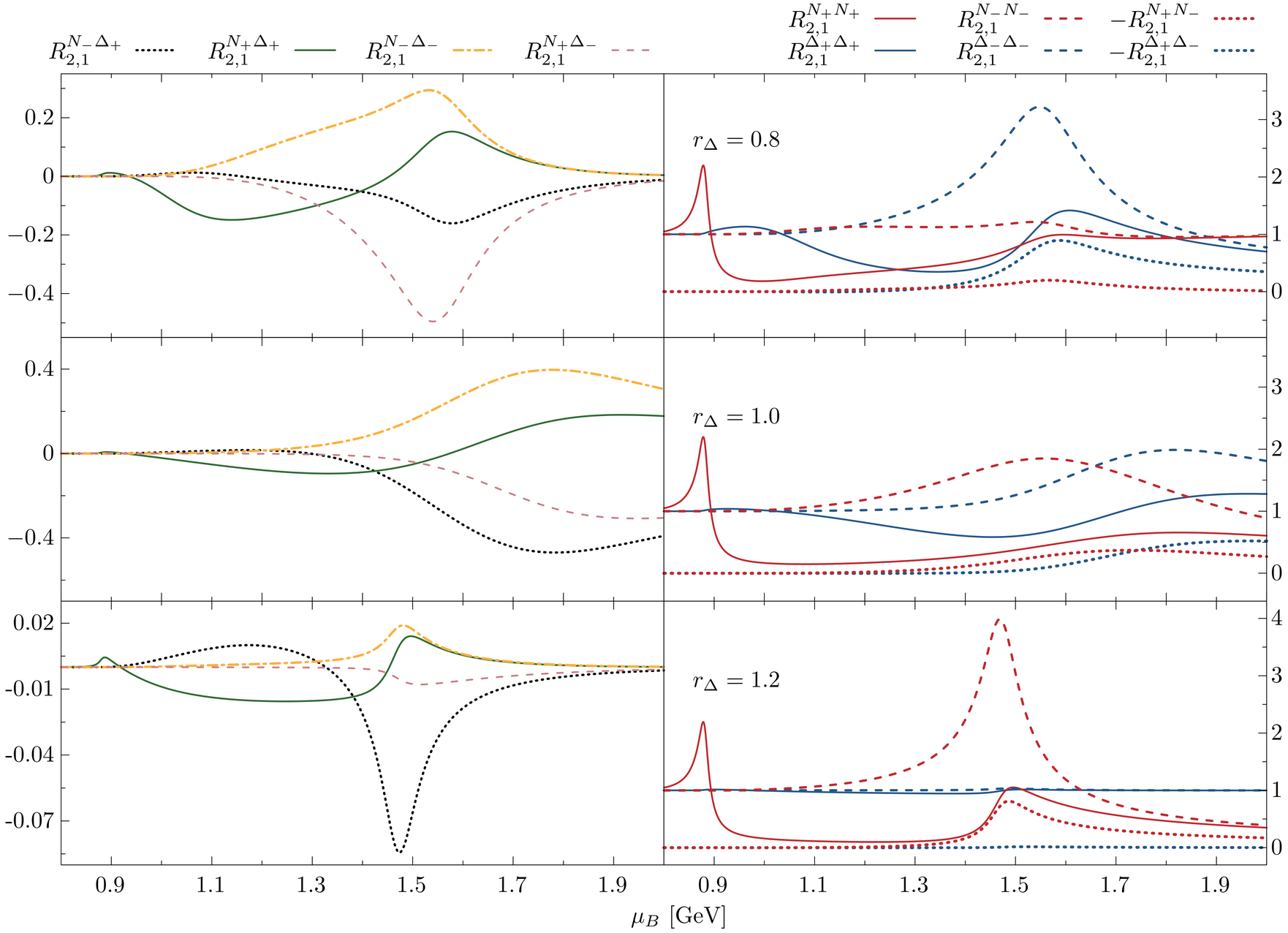

χ_2^{++}
χ_2^{--}
$-\chi_2^{+-}$
χ_2^B
$T = 150$ MeV
μ_B [GeV]


$R^{++}_{2,1}$
$R^{--}_{2,1}$
$-R^{+-}_{2,1}$
$R^{B}_{2,1}$
$T = 30$ MeV
μ_B [GeV]


$R^{++}_{2,1}$
$R^{--}_{2,1}$
$-R^{+-}_{2,1}$
$R^{B}_{2,1}$
$T = 50$ MeV
μ_B [GeV]

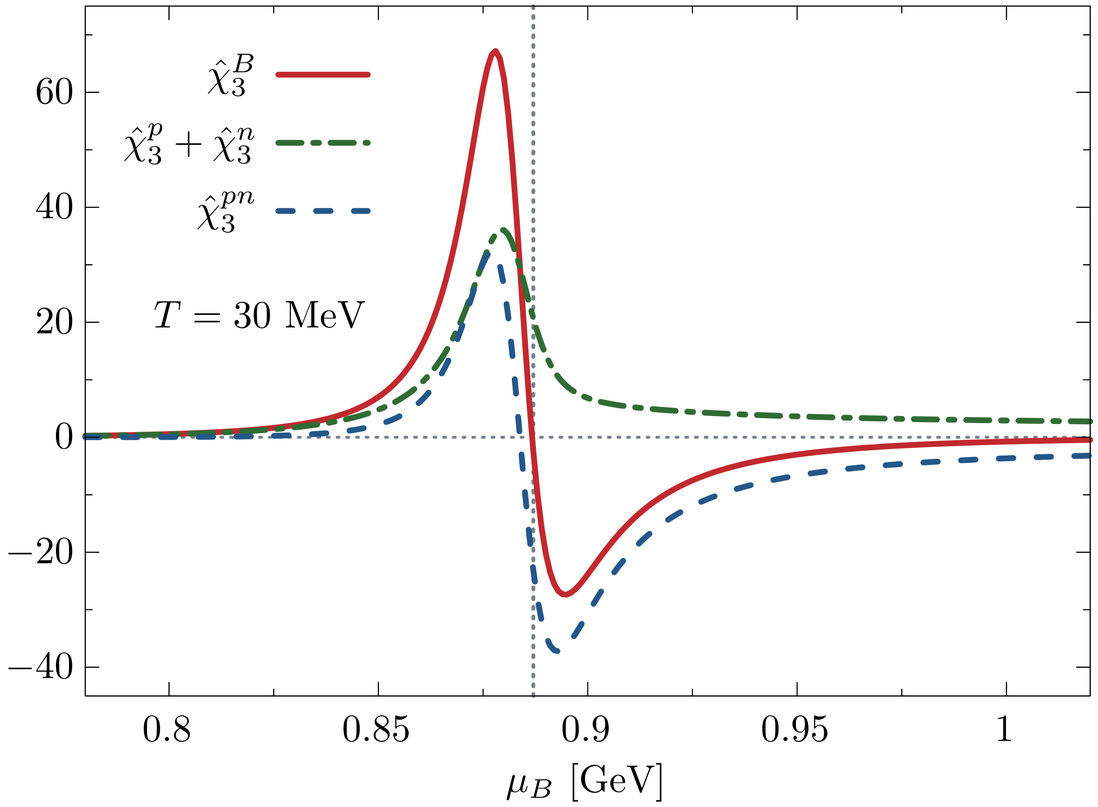

$R^{++}_{2,1}$
$R^{--}_{2,1}$
$-R^{+-}_{2,1}$
$R^{B}_{2,1}$
$T = 100$ MeV
μ_B [GeV]

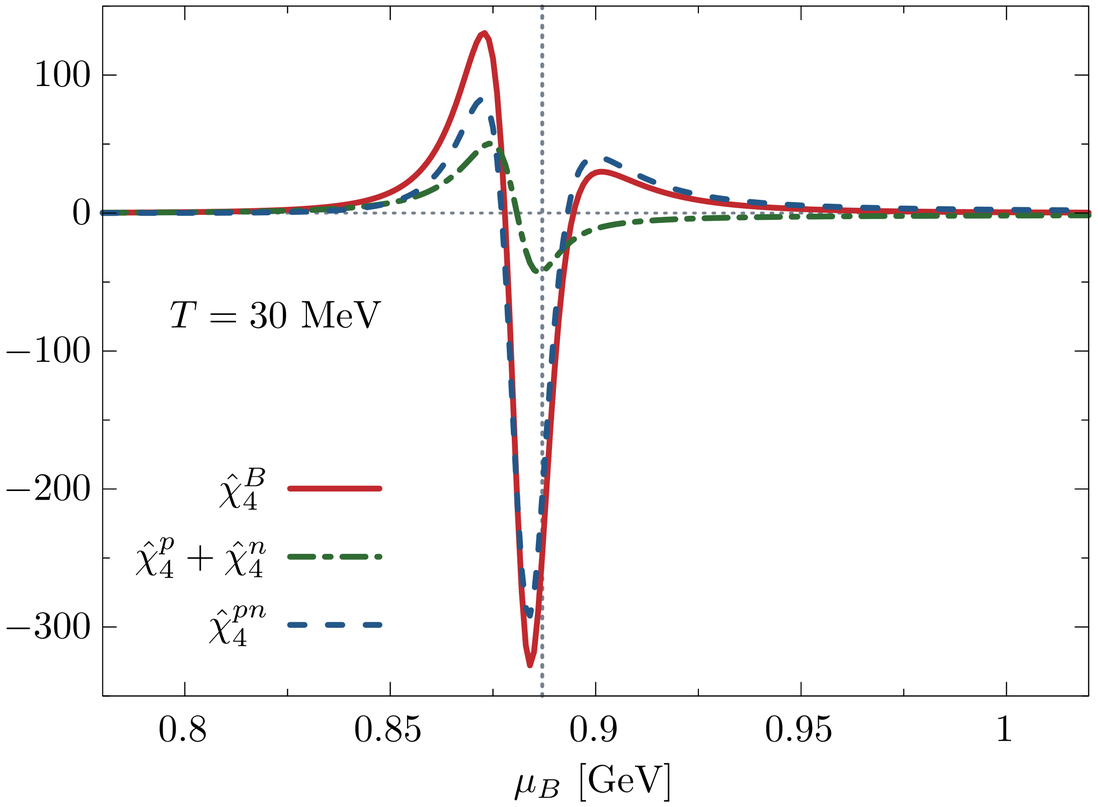

$R^{++}_{2,1}$
$R^{--}_{2,1}$
$-R^{+-}_{2,1}$
$R^{B}_{2,1}$
$T = 150$ MeV
μ_B [GeV]


$R^{N-\Delta+}_{2,1}$
$R^{N+\Delta+}_{2,1}$
$R^{N-\Delta-}_{2,1}$
$R^{N+\Delta-}_{2,1}$
$R^{N+N+}_{2,1}$
$R^{\Delta+\Delta+}_{2,1}$
$R^{N-N-}_{2,1}$
$R^{\Delta-\Delta-}_{2,1}$
$-R^{N+N-}_{2,1}$
$-R^{\Delta+\Delta-}_{2,1}$
$r_\Delta = 0.8$
$r_\Delta = 1.0$
$r_\Delta = 1.2$
0.2
0
−0.2
−0.4
0.4
0
−0.4
0.02
-0.01
-0.04
-0.07
3
2
1
0
4
3
2
1
0
0.9
1.1
1.3
1.5
1.7
1.9
μ_B [GeV]

Isospin Correlations Near the Liquid-Gas Transition

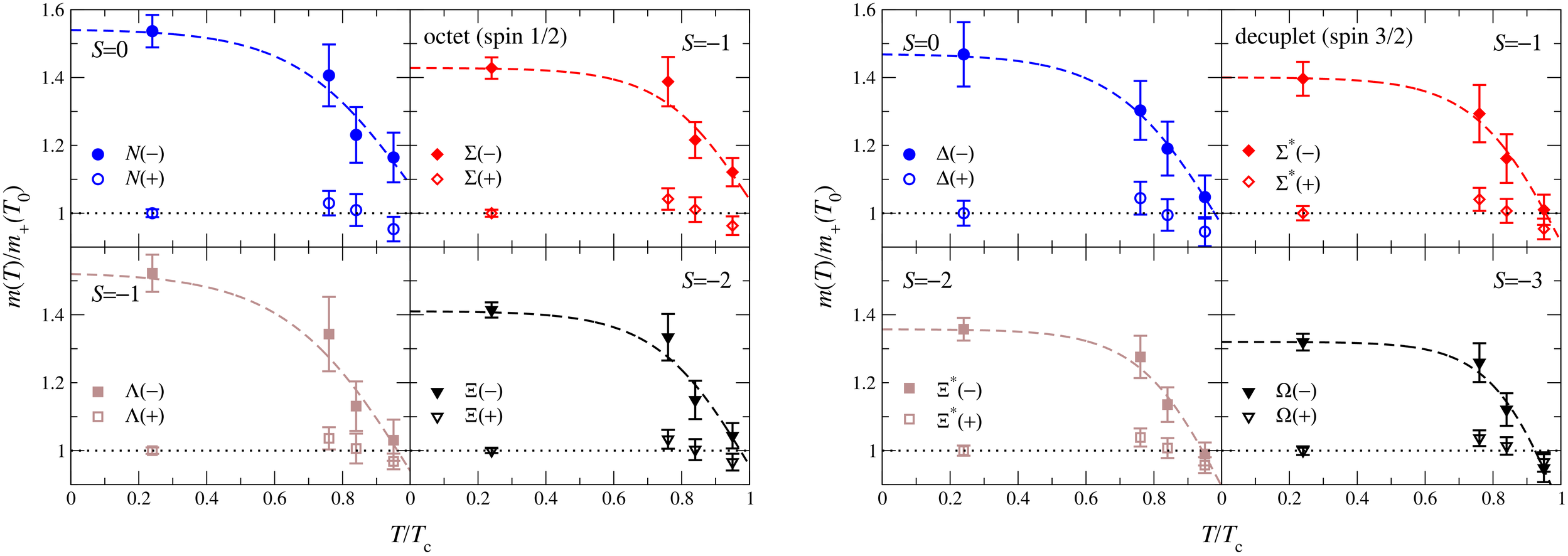
$$\chi_2^B = \chi_2^{++} + \ldots \simeq \chi_2^p + \chi_2^n + \chi_2^{pn} \neq 2\chi_2^p$$





Imprint of chiral symmetry restoration in the baryonic sector

Aarts et al, 2019



Clear evidence for partial restoration of chiral symmetry in the strange baryon sector

Influence of the strength of the repulsive interactions

- Clear suppression of fluctuations with increasing repulsive vector interactions
- Increase of fluctuations due to in-medium chiral masses is reduced via negative correlations
- With particular repulsion strength, fluctuations are pushed down to HRG results with vacuum masses