QCD phase diagram and the chiral transition

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[PRD 109 (2024) 11, 114516, arXiv: 2403.09390]

[PRD 105 (2022) 7, 074511, arXiv: 2202.09184]

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[arXiv: 2405.10196]

[PRD 105 (2022) 3, 034513, arXiv: 2110.15933]

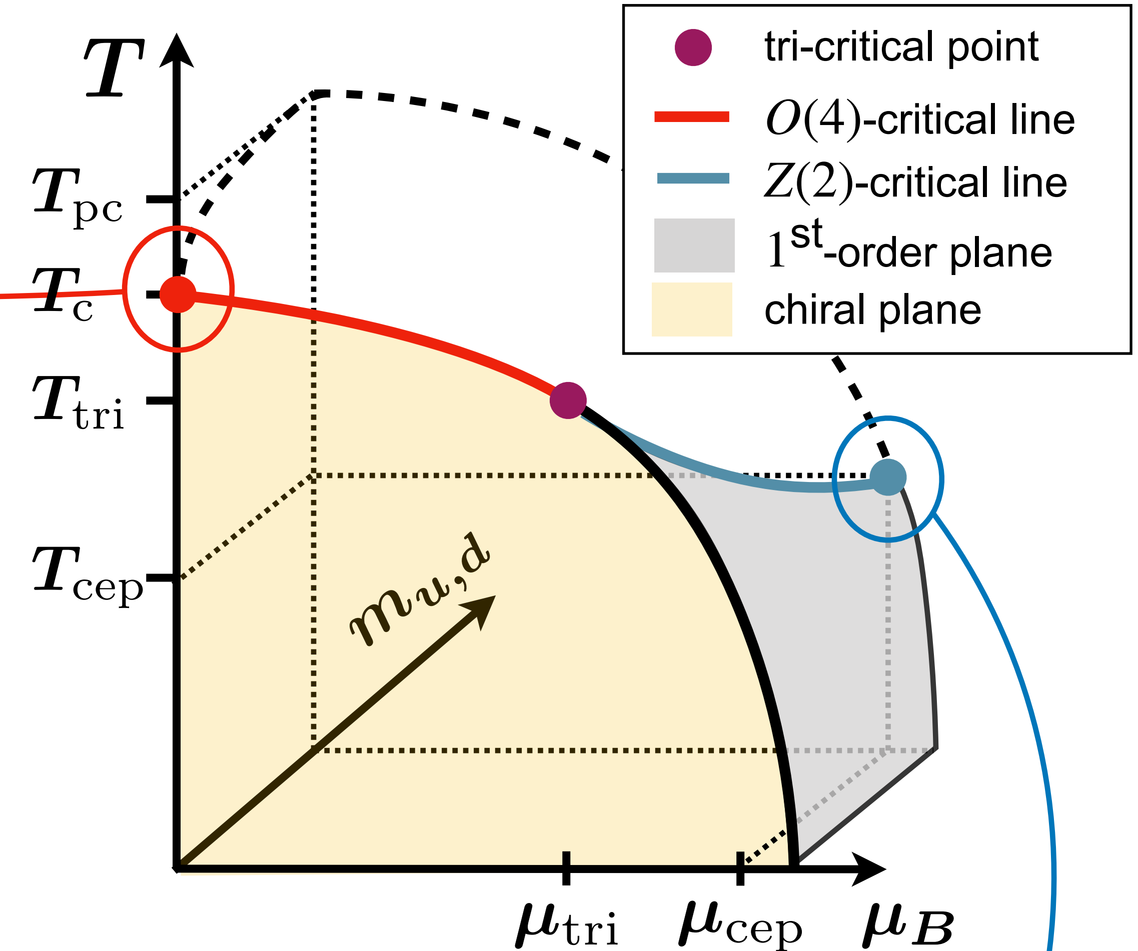
Budapest, December 2-6, 2024

Spontaneous chiral symmetry breaking

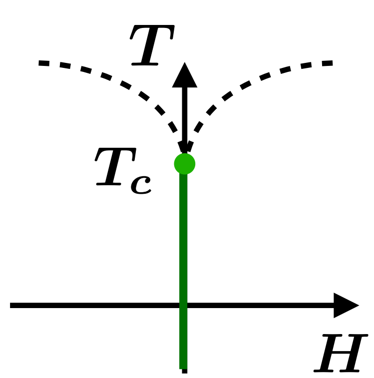
$$\mathrm{U}(1)_V \times \cancel{\mathrm{U}(1)}_A \times \mathrm{SU}(N_f)_L \times \mathrm{SU}(N_f)_R$$

$$\to \mathrm{U}(1)_V \times \cancel{\mathrm{U}(1)}_A \times \mathrm{SU}(N_f)_V \times \cancel{\mathrm{SU}(N_f)}_A$$

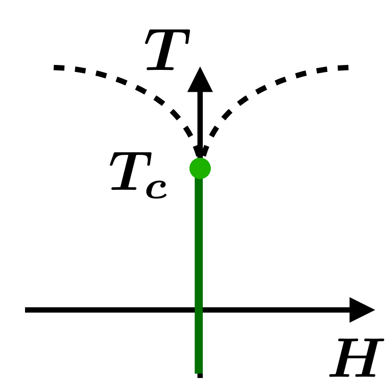
The QCD phase diagram



O(4)-symmetric model

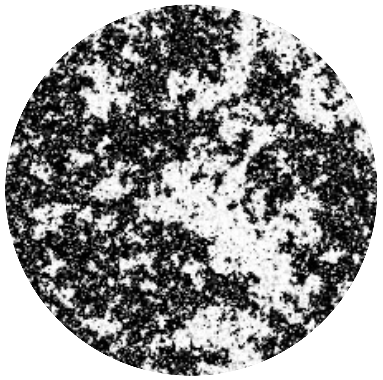


Z(2)-symmetric model



- Map is defined by the scaling directions and few non-universal constants T_c, t_0, h_0, … → need to be determined
- Critical exponents, critical amplitudes and scaling functions are universal and well known and can be used
- Verifying universality classes is more difficult

Chiral scaling fields and operators



Scaling hypotheses:

Free energy: $f_s(t,h,L) = b^{-d} f_s(b^{y_t} t, b^{y_h} h, L^{-1} b)$

Effective model O(4)/O(2)/Z(2):

Scaling fields

t reduced temperature

h reduced symmetry breaking field

L^{-1} inverse system size

map QCD to the effective model ⟷

controlled by non-universal parameters:

t_0, h_0, l_0
T_c, H_c

(2+1)-flavor QCD:

Scaling fields

$$t = \frac{1}{t_0}\left(\Delta T + \kappa_2^l \hat{\mu}_l^2 + \kappa_2^s \hat{\mu}_s^2 + 2\kappa_{11}^{ls} \hat{\mu}_l \hat{\mu}_s\right)$$

$$\Delta T = \frac{T - T_c}{T_c}$$

$$h = \frac{1}{h_0}(H - H_c), \quad H = \frac{m_l}{m_s}$$

$$l = l_0 L^{-1}$$

- We aim on the determination of the parameters, including $\kappa_2^l, \kappa_2^s, \kappa_{11}^{ls}$
- So far no evidence for $H_c > 0$ in (2+1)-flavor QCD
  - ➡ We assume $H_c = 0$ and determine $T_c \equiv T_c^0$

Chiral scaling fields and operators

Order parameter

Remove multiplicative UV divergences

$$M_l = \frac{m_s}{f_K^4} \frac{T}{V} \frac{\partial \ln Z}{\partial m_l}$$

$$\frac{\partial}{\partial m_l} = \frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d}$$

→

Equation of state

$$M_l = h^{1/\delta} f_G(z)$$
+ sub-leading

With scaling variable

$$z = t/h^{1/\beta\delta}$$

Magnetic susceptibility

$$\chi_l = m_s \frac{\partial}{\partial m_l} M_l$$

Renormalized order parameter

$$M = M_l - H\chi_l$$

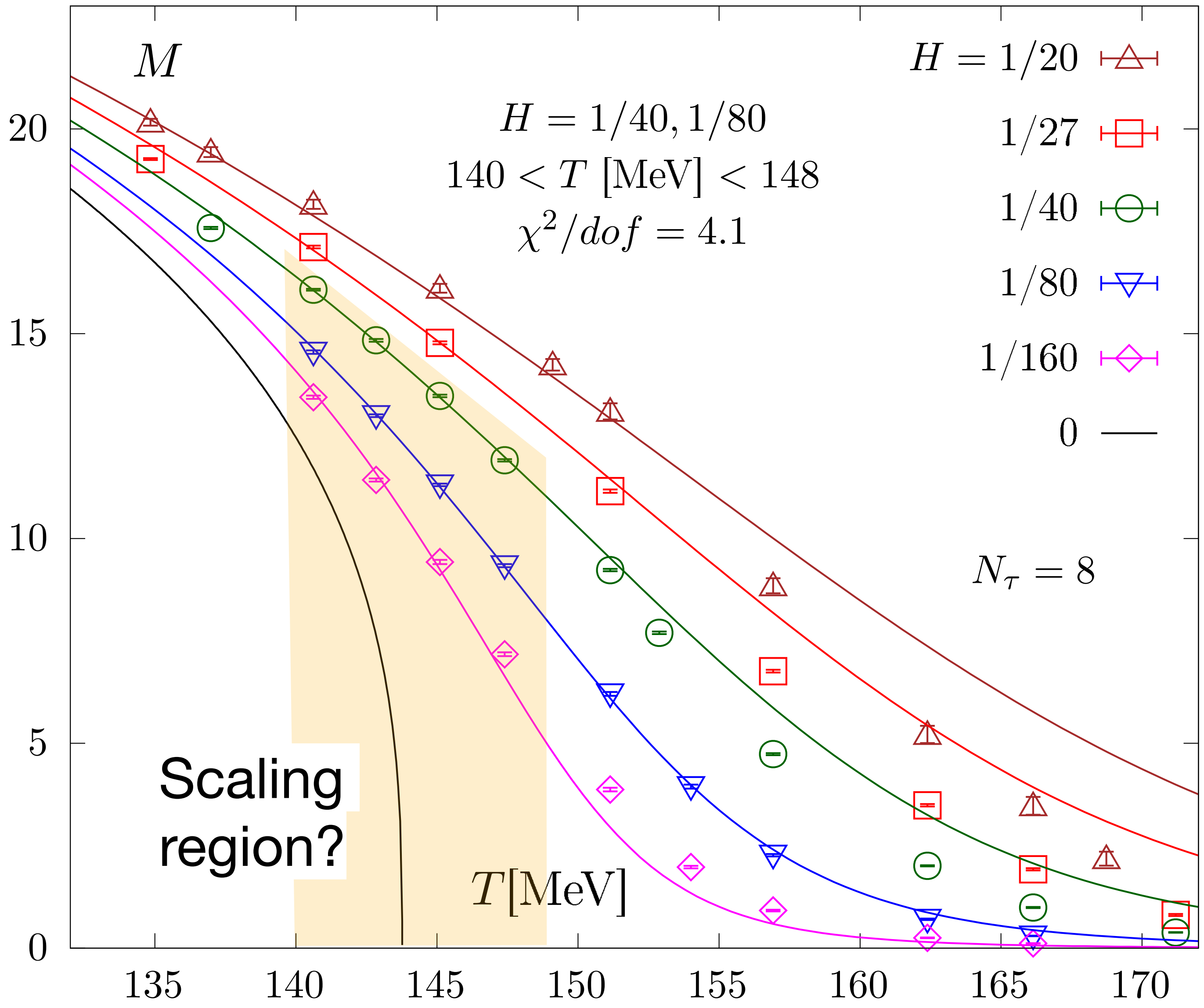
→

Equation of state

$$M = h^{1/\delta}\left(f_G(z) - f_\chi(z)\right)$$
+ sub-leading

- $f_G(z)$ and $f_\chi(z)$ are the well known universal scaling functions of a single scaling variable z [Karsch, Neumann, Sarkar '23]
- This version of the renormalised order parameter has advantages:
  - ➡ No explicit contribution from the strange condensate
  - ➡ Direct relation to the scaling function of the free energy [Kotov et. al, '21]

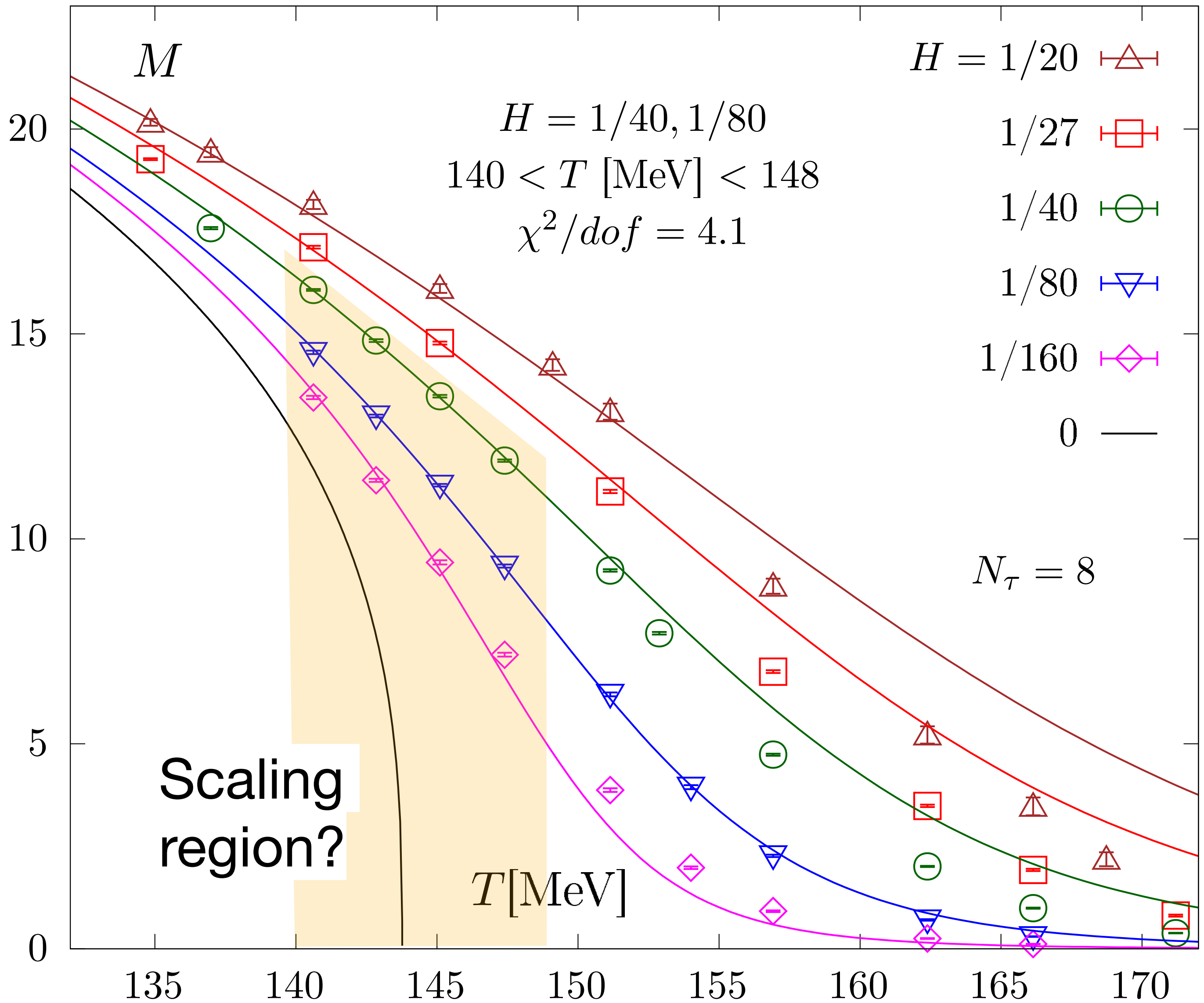
$$M = h_0^{-1/\delta} H^{1/\delta} \left(f_G(z) - f_\chi(z) \right)$$



[Ding et al. (HotQCD) '24]

- $N_\tau = 8$ (with updated statistics)
- Corresponding pion masses: $m_\pi \simeq$ 180 MeV, 140 MeV, 110 MeV, 80 MeV, 55 MeV.
- Use O(2) scaling functions and exponents due to staggered fermions
- Fit results for $N_\tau = 8$
  $T_c^0 = 143.7(2)$ MeV
  $z_0 = 1.42(6)$
  $h_0^{-1/\delta} = 39.2(4)$
  [Ding et al. (HotQCD) '24]
- Continuum estimate: $T_c^0 = 132^{+2}_{-6}$ MeV [Ding et al. (HotQCD) '19]

EoS fit: universality class is assumed

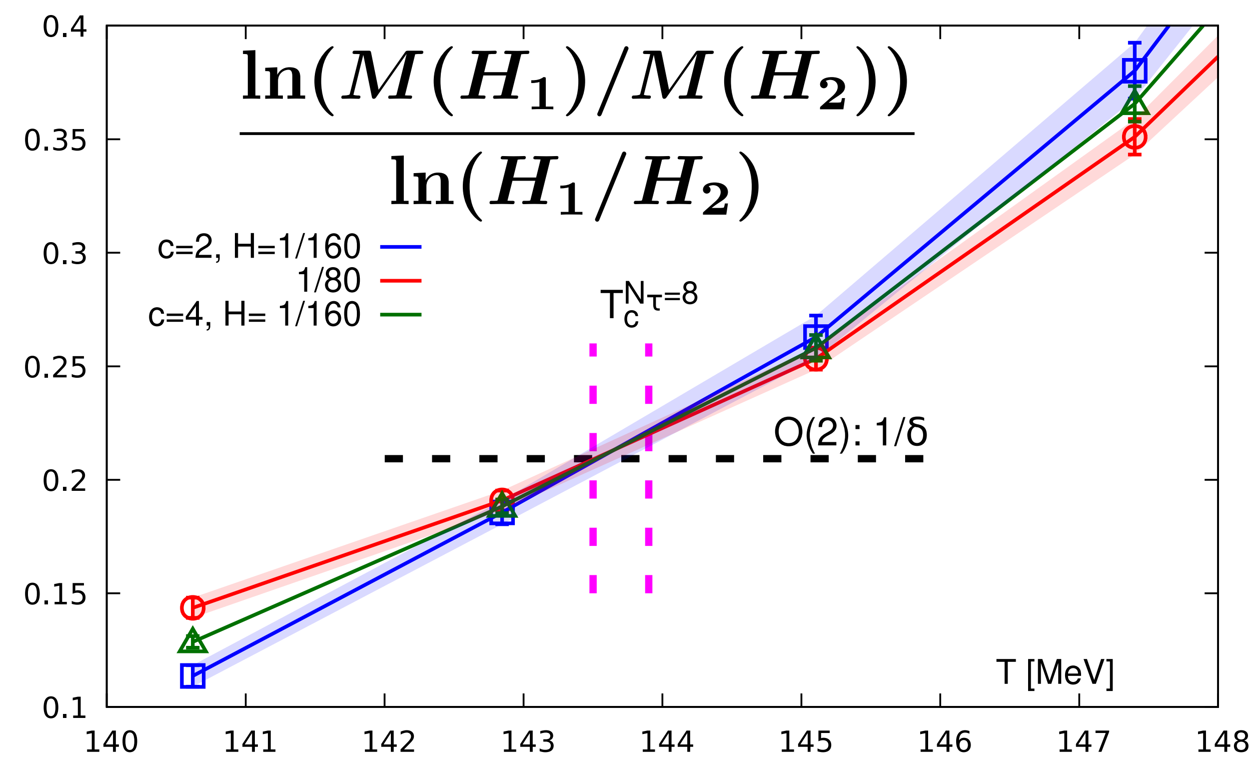


[Ding et al. (HotQCD) '24]

New parameter free method

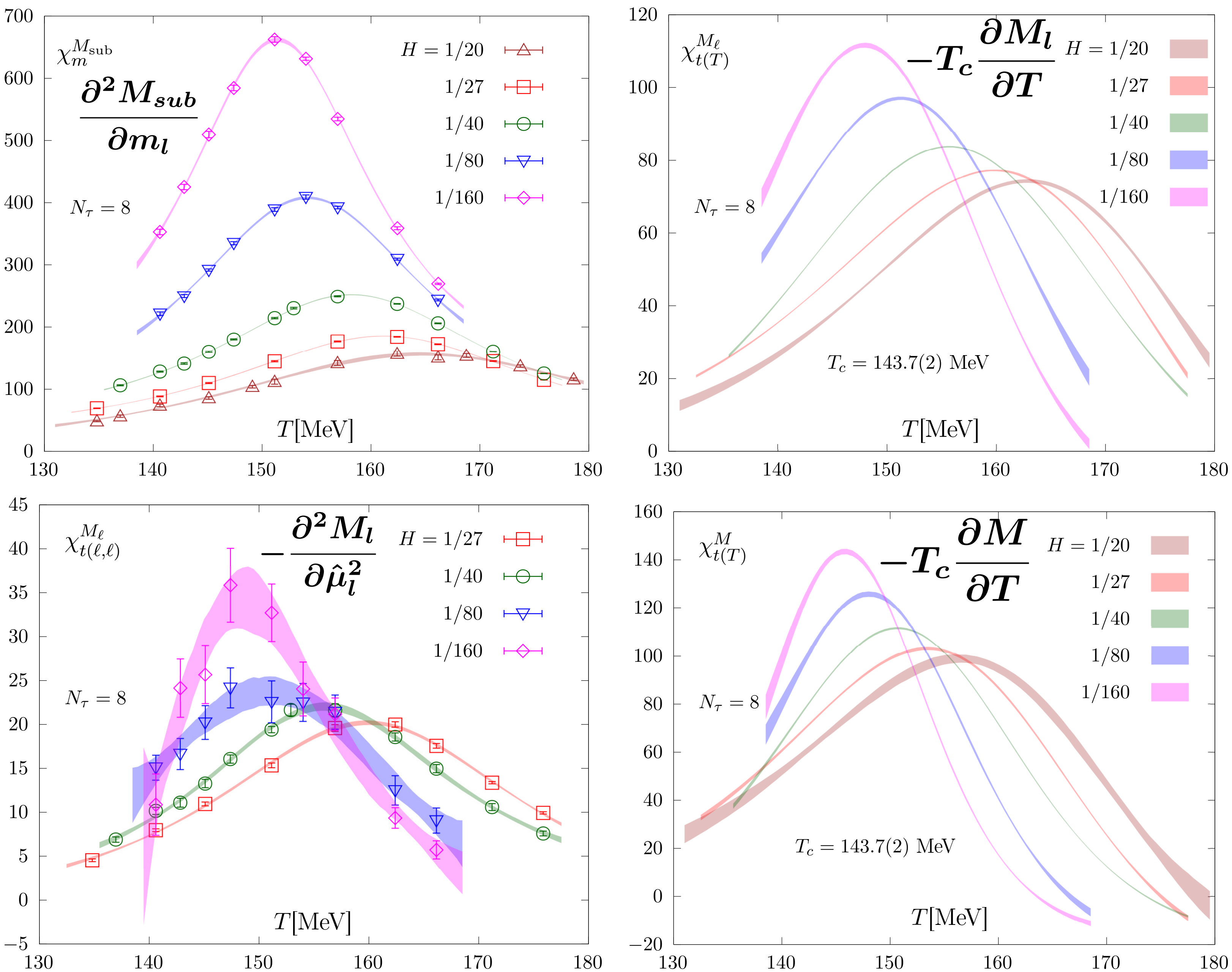
➡ **ratios of the renormalized order parameter can be used to define define T_c and δ**

[Mitra et al., Lattice '24]

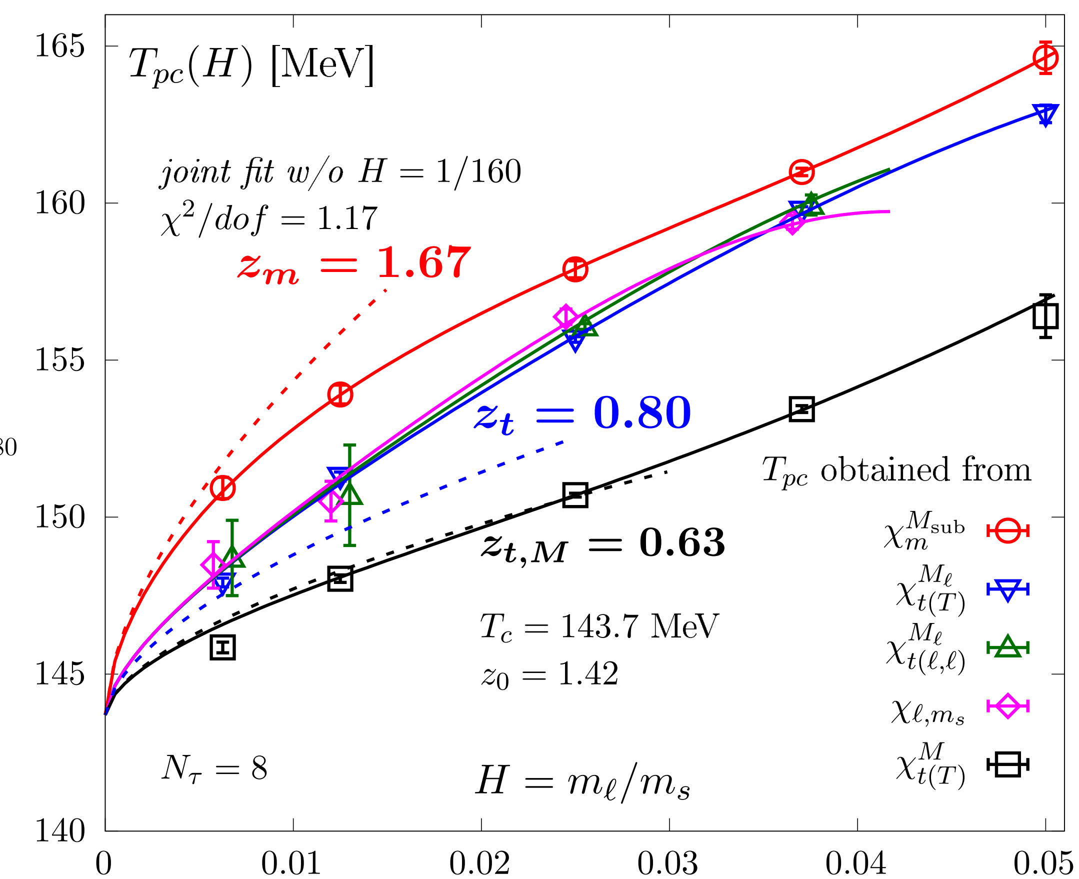


- Intersection at $(T_c, 1/\delta)$
- Obtained T_C and δ consistent with EoS fit and O(2) universality class

- Determine peak positions of various susceptibilities
  - ➡ Definition of pseudo critical line (constant z value)



$$T_{pc,x} = T_c \left(1 + \frac{z_x}{z_0} H^{1/\beta\delta} + \text{corrections to scaling}\right)$$



- Fit to peak positions give T_c in good agreement with EoS fits

[Ding et al. (HotQCD) '24]

- Temperature like scaling field

$$t = \frac{1}{t_0}\left(\Delta T + \kappa_2^l \hat{\mu}_l^2 + \kappa_2^s \hat{\mu}_s^2 + 2\kappa_{11}^{ls}\hat{\mu}_l\hat{\mu}_s\right)$$

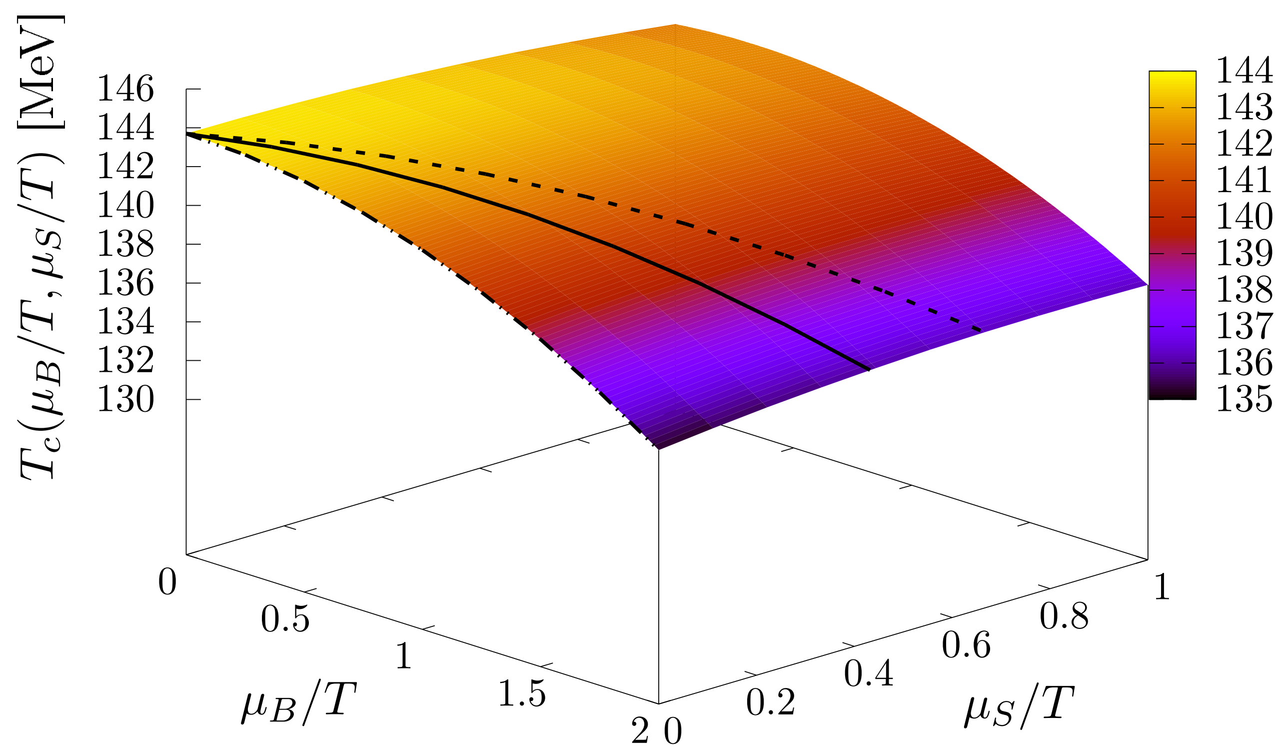
- Ratio of mixed susceptibilities are related to the curvature coefficients

$$\kappa_2^l = \frac{1}{2T_c}\left(\frac{\partial^2 M_l/\partial\hat{\mu}_l^2}{\partial M_l/\partial T}\right)\Bigg|_{(T=T_c,\vec{\mu}=0)}$$

$$\kappa_{11}^{ls} = \frac{1}{2T_c}\left(\frac{\partial^2 M_l/\partial\hat{\mu}_l\hat{\mu}_s}{\partial M_l/\partial T}\right)\Bigg|_{(T=T_c,\vec{\mu}=0)}$$

$$\kappa_2^s = \frac{1}{2T_c}\left(\frac{\partial^2 M_l/\partial\hat{\mu}_s^2}{\partial M_l/\partial T}\right)\Bigg|_{(T=T_c,\vec{\mu}=0)}$$

- results may be transformed to the hadronic basis



[Ding et al. (HotQCD) '24]

$m_l = 0 \quad (N_\tau = 8)$

$$\kappa_2^{B,\hat{\mu}_S=0} \equiv \kappa_2^B = 0.015(1)$$
$$\kappa_2^{B,n_S=0} = 0.893(35)\kappa_2^B$$
$$\kappa_2^{B,\hat{\mu}_s=0} = 0.968(23)\kappa_2^{n_S=0}$$

$m_l = m_s/27 \quad$ (cont.)

$$\kappa_2^{B,n_S=0} = \begin{cases} 0.012(4) & \text{[HotQCD '19]} \\ 0.0153(8) & \text{[BW '20]} \end{cases}$$

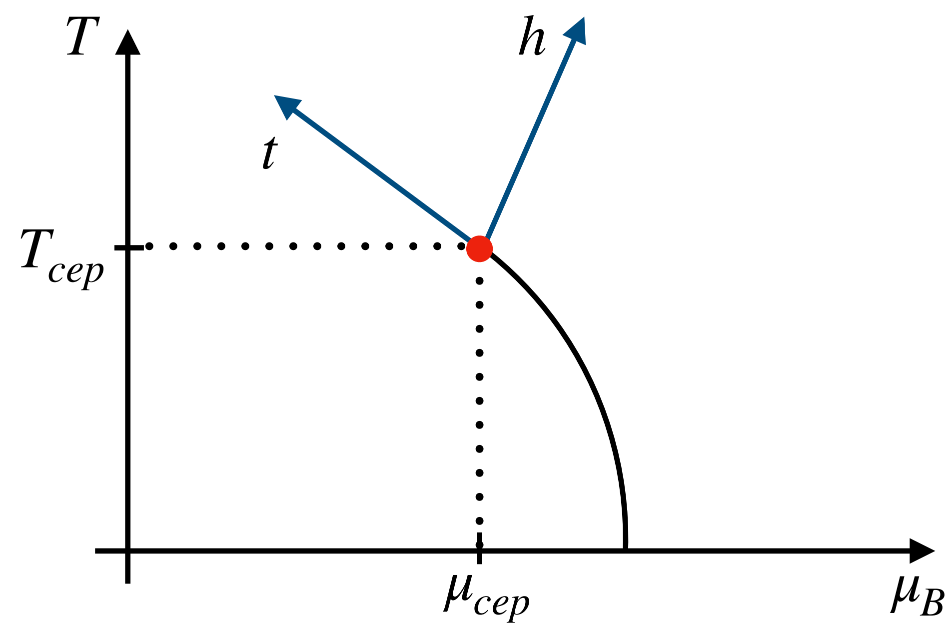
[Ding et al. (HotQCD) '24]

Mixing of scaling fields:

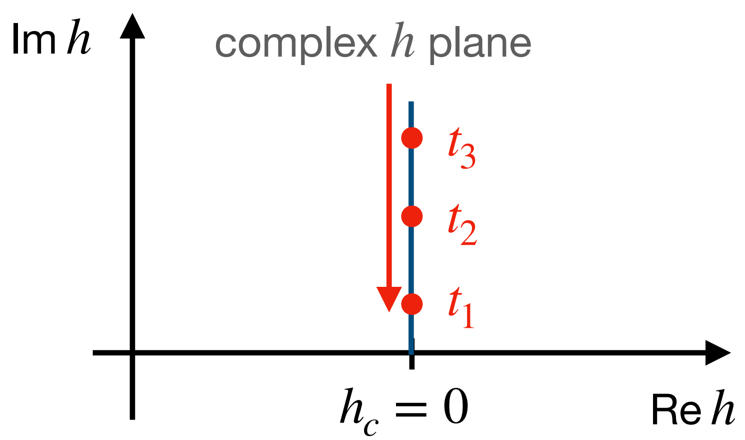
- Scaling fields are unknown, a frequently used ansatz is given by a linear mixing of T, μ_B

$$t = A_t \Delta T + B_t \Delta \mu_B,$$
$$h = A_h \Delta T + B_h \Delta \mu_B,$$

with $\Delta T = T - T^{\mathrm{CEP}}$ and $\Delta \mu_B = \mu_B - \mu_B^{\mathrm{CEP}}$



Lee-Yang edge:



- Poles approach critical point along imaginary h-axis **[Yang,Lee'59]**
- $t/h^{1/\beta\delta} = z_c$ is const. and universal

Fit Ansatz:

- For a constant $z = z_c$ we obtain

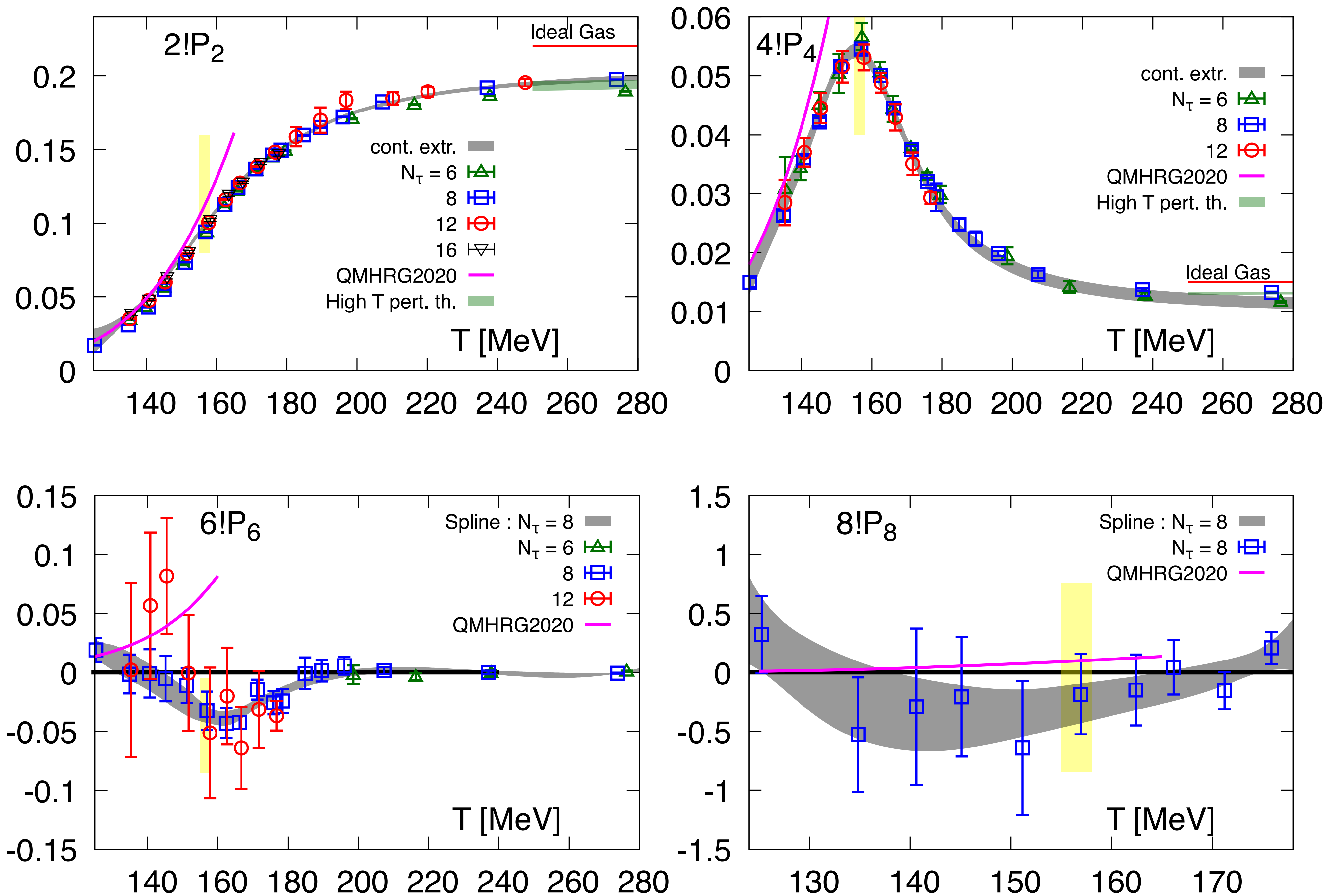
$$\mathrm{Re}[\mu_{\mathrm{LYE}}] = \mu_B^{\mathrm{CEP}} + c_1 \Delta T + c_2 \Delta T^2 + O(\Delta T^3)$$
$$\mathrm{Im}[\mu_{\mathrm{LYE}}] = c_3 \Delta T^{\beta\delta},$$

[Stephanov, Phys. Rev. D, 73.9, 094508 (2006)]

- The fit parameter c_1 gives the (inverse) slope of the 1st order line at the critical point: $c_1 = -A_h/B_h$

- Detecting phase transitions via Padé and post-Padé approximants has a long history in statistical and high energy physics
- They are often used in combination with perturbation theory
- QCD is non-perturbative in the vicinity of the phase
- The numerical calculation of the pressure series in μ_B is difficult

$$\Delta\hat{p} \equiv \frac{p(T,\mu_B)}{T^4} - \frac{p(T,0)}{T^4} = \sum_{k=1}^{\infty} P_{2k}(T)\hat{\mu}_B^{2k}$$

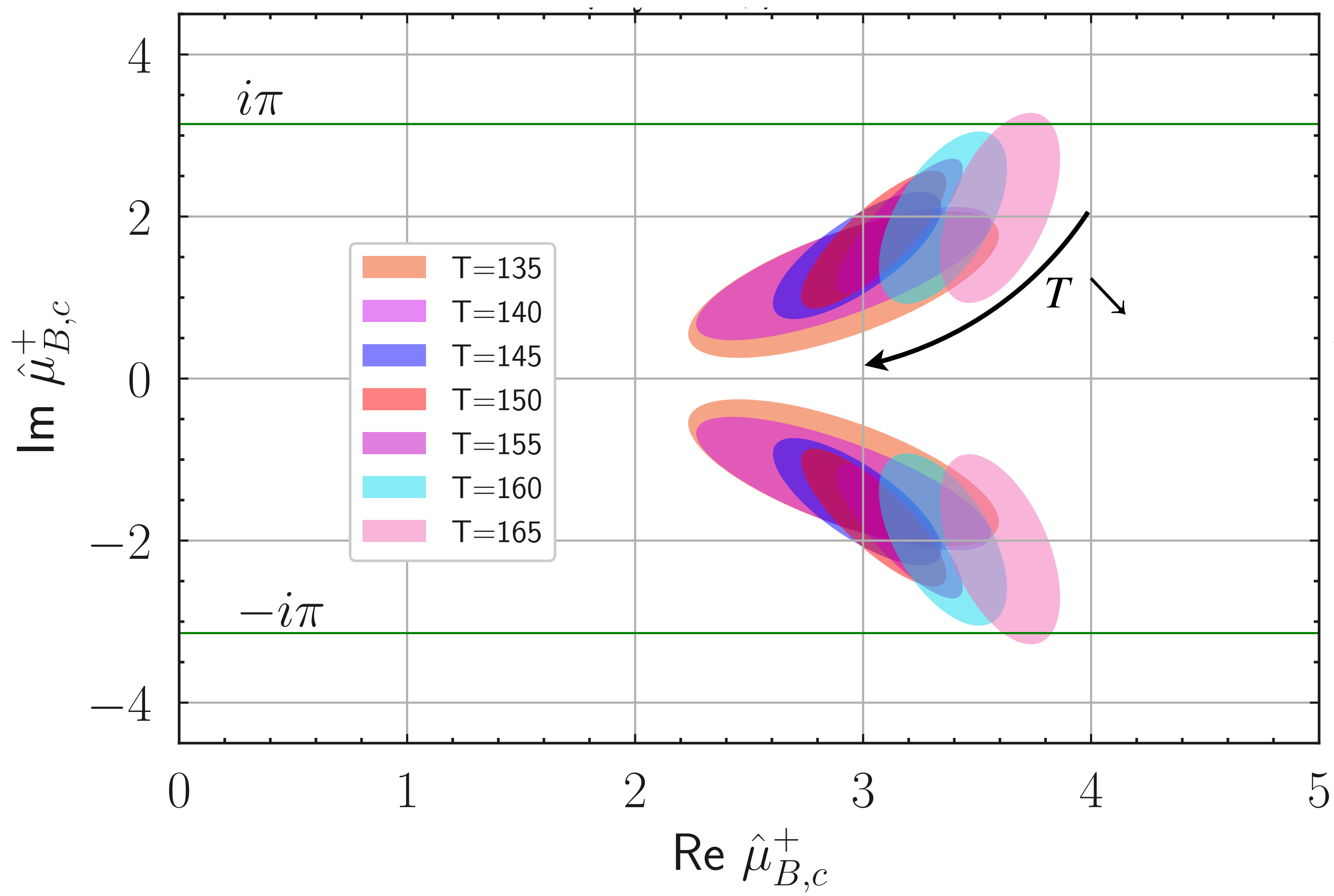


HotQCD, PRD 108 (2023) 1, 014510, arXiv: 2212.09043]

Padé resummation of the Taylor series about $\mu_B = 0$

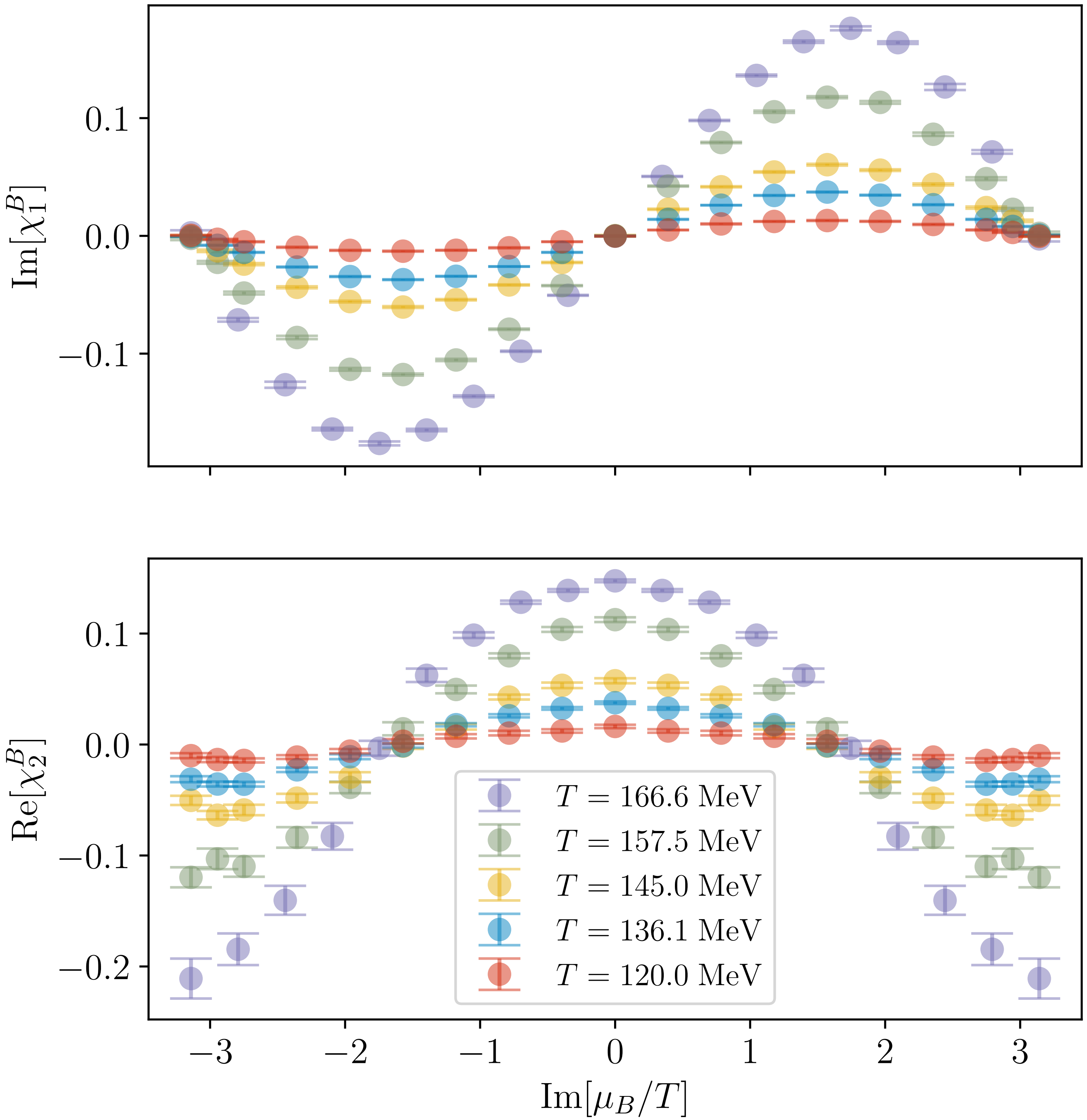
$$P[4,4] = \frac{P_2\hat{\mu}_B^2 + (P_4 + (P_2^2 P_8)/P_4^2)\hat{\mu}_B^4}{1 + ((P_2 P_8)/P_4^2)\hat{\mu}_B^2 - (P_8/P_4)\hat{\mu}_B^4}$$

- Construct [4,4]-Padé from 8th order Taylor Expansion
- Calculate complex roots of the denominator
- Find apparent approach to the real axis with decreasing temperature
- Can also be combined with conformal maps **[Basar, 2312.06952]**



[PRD 105 (2022) 7, 074511, arXiv: 2202.09184]

Lattice size: $36^3 \times 6$



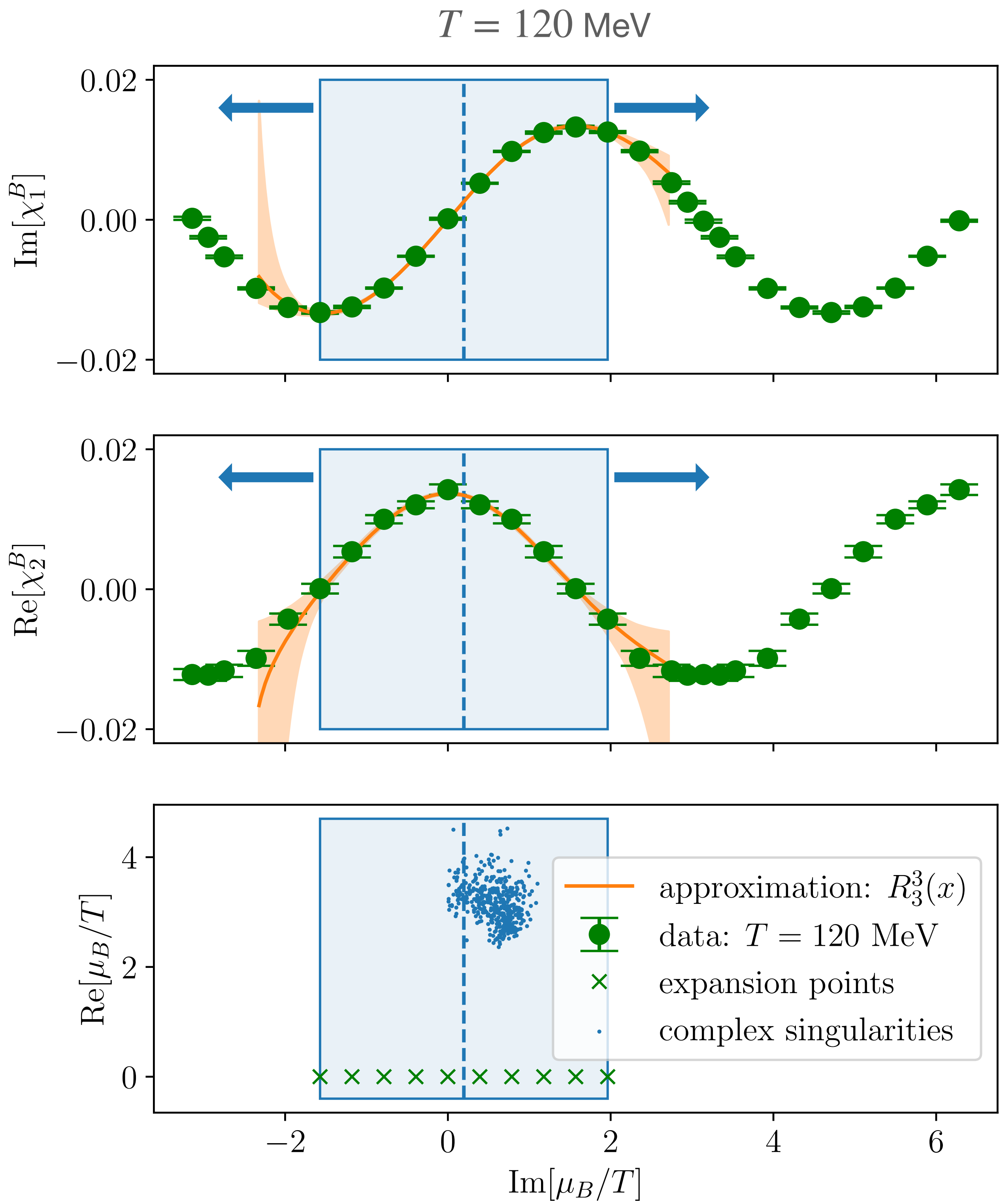
[arXiv: 2403.09390]

Observables:

* Derivatives of $\ln Z$, w.r.t $\hat{\mu}_B = \mu_B/T$

$$\chi_n^B(T) = \frac{V}{T^3}\left(\frac{\partial}{\partial \hat{\mu}_B}\right)^n \ln Z(T, \hat{\mu}_B)$$

* $\ln Z$ is even in $\hat{\mu}_B = i\theta$ and periodic, with periodicity 2π
* Choose 10 equidistant $\hat{\mu}_B$-points in $[0, i\pi]$, all further points are obtained by periodicity and parity
* Odd (even) derivatives are imaginary (real) at $\hat{\mu}_B = i\theta$

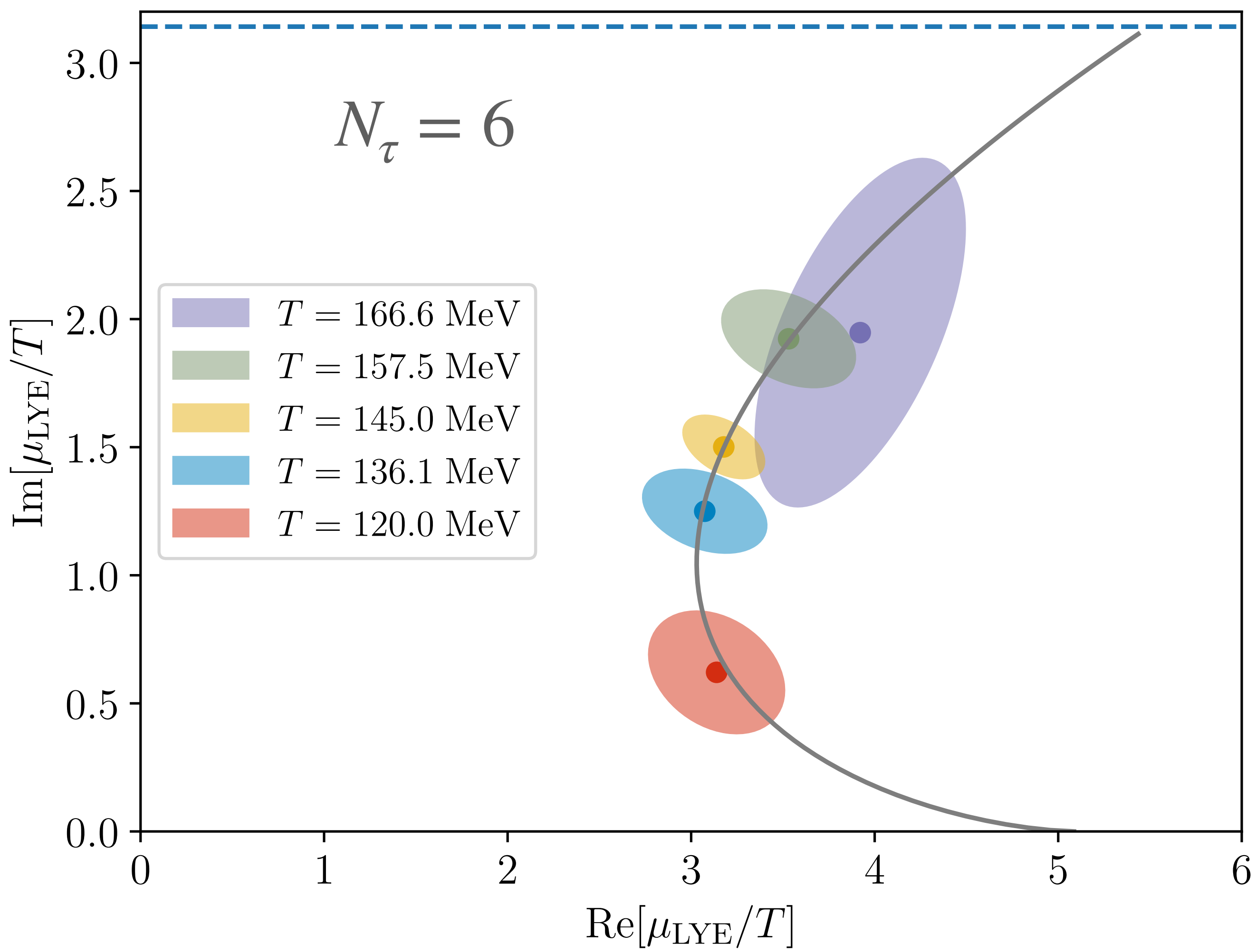


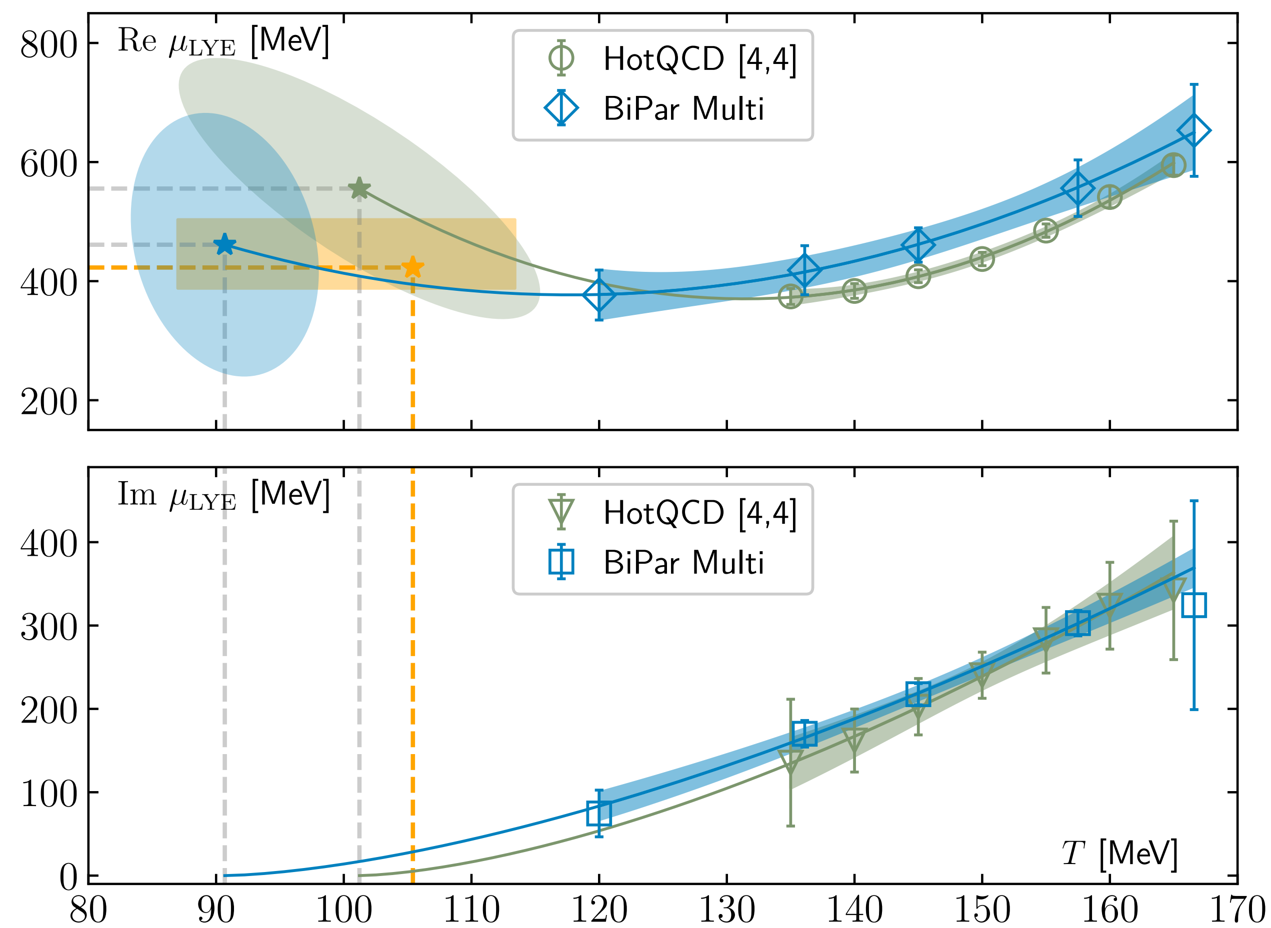
[arXiv: 2405.10196]

Procedure:

- Perform simultaneous fits to χ_1^B and χ_2^B for each temperature
- Use [3,3]-Padé
- Varry length of the fit interval in $[\pi, 2\pi]$ and the center of the interval in $[-\pi/2, +\pi/2]$
- bootstrap over the data by assuming independent and normal distributed errors
- Calculate roots of the denominator and keep only roots in the first quadrant
- Collect all the results for Lee-Yang scaling fits. We have 55 different intervals per temperature.

- Perform one fit for $N_\tau = 8$ and $\mathcal{O}(10^5)$ fits for $N_\tau = 6$



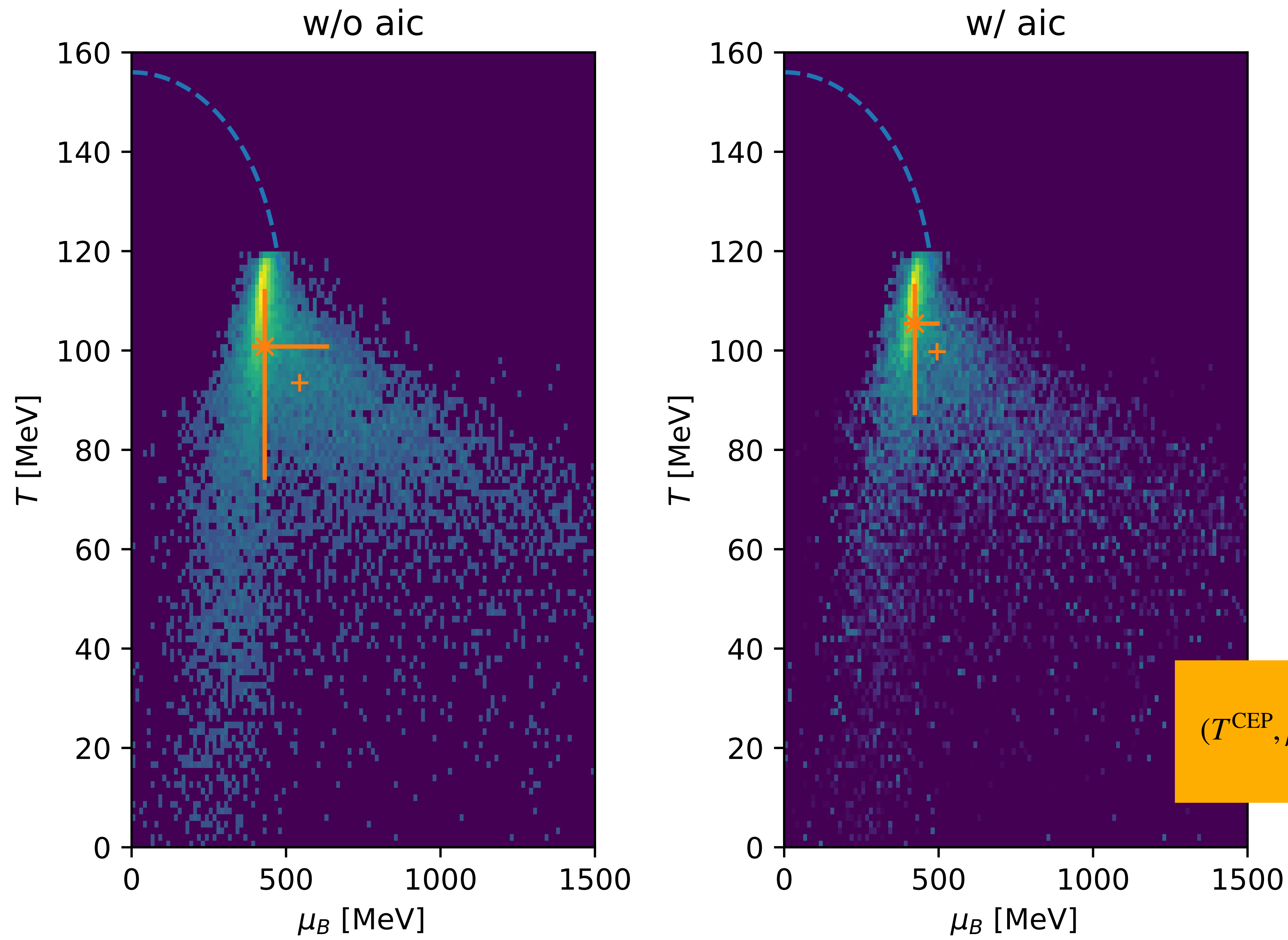


- Ellipses show 1σ confidence region, using the Pearson correlation coefficient
- $N_\tau = 6$ singularities show here are chosen on the basis of the χ^2 of the scaling fit ("best fit")

$$\mathrm{Re}[\mu_{\mathrm{LYE}}] = \mu_B^{\mathrm{CEP}} + c_1 \Delta T + c_2 \Delta T^2$$
$$\mathrm{Im}[\mu_{\mathrm{LYE}}] = c_3 \Delta T^{\beta\delta},$$

- Orange box shows the AIC weighted result for $N_\tau = 6$, based on $\mathcal{O}(10^5)$ scaling fits

Statistical analysis of fits



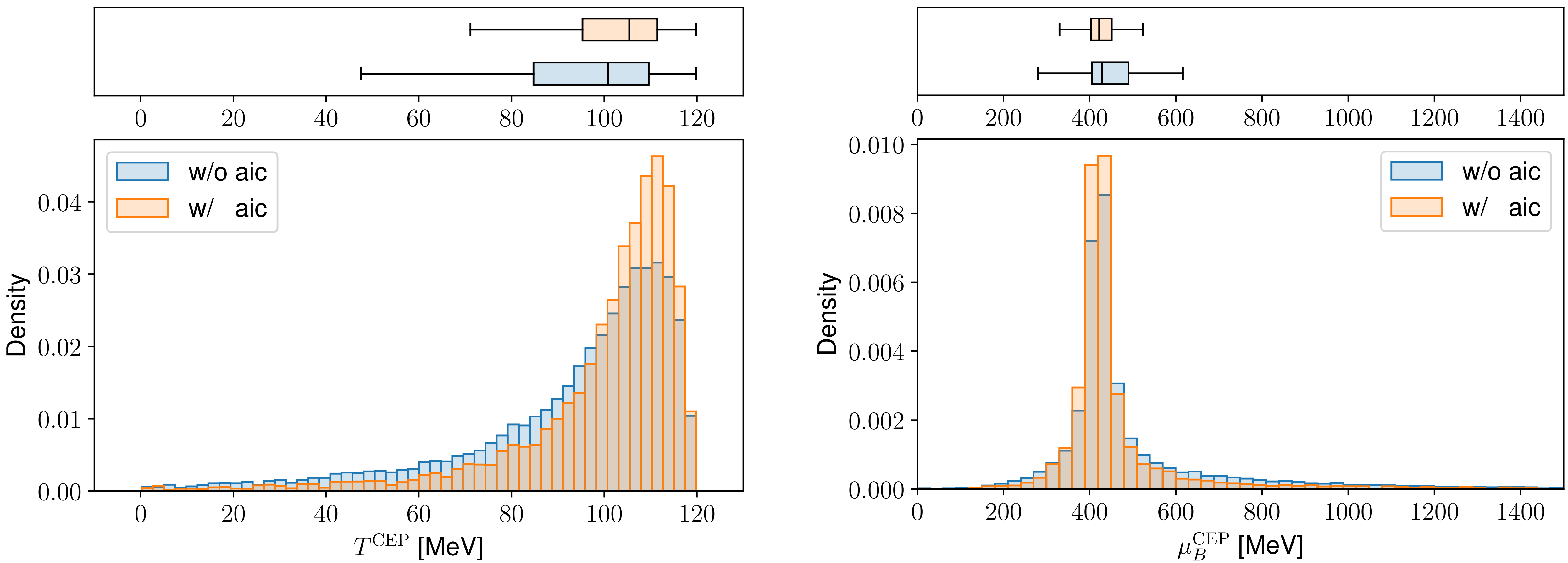
- Dashed line indicates the continuum extrapolated crossover line

$(T^{\mathrm{CEP}}, \mu_B^{\mathrm{CEP}}) = (105^{+8}_{-18}, 422^{+80}_{-35})$ MeV

- Histogram over the T^{CEP} and μ_B^{CEP} from the $\mathcal{O}(10^5)$ fits
- Error bars are based on the inner 68-percentile
- Observe interesting structure

- Universal scaling is a very powerful tool if the scaling fields and the universality class are known.
- Transition temperate in the chiral limit, pseudo-critical line and curvature coefficients are obtained from scaling fits.
- Pseudo-critical lines correspond (asymptotically) to a constant real $z = t/h^{1/\beta\delta}$, the Lee-Yang edge to a universal complex z_c
- New Strategy: Determine the QCD critical point by the temperature scaling of the Lee-Yang edge singularity
- Technically this requires Pade or multi-point Pade analysis of $\ln Z$ derivatives. The later eliminates the need for the calculation of high order expansion coefficients but introduces some interval dependence.
- Find encouraging results for $N_\tau = 6$: $(T^{\mathrm{CEP}}, \mu_B^{\mathrm{CEP}}) = (105^{+8}_{-18}, 422^{+80}_{-35})$ MeV .
- No continuum result yet
- Current estimates of the cutoff effects increase μ_B^{CEP} towards $\mu_B^{\mathrm{CEP}} \approx 650$ MeV, which is consistent with FRG and DSE results

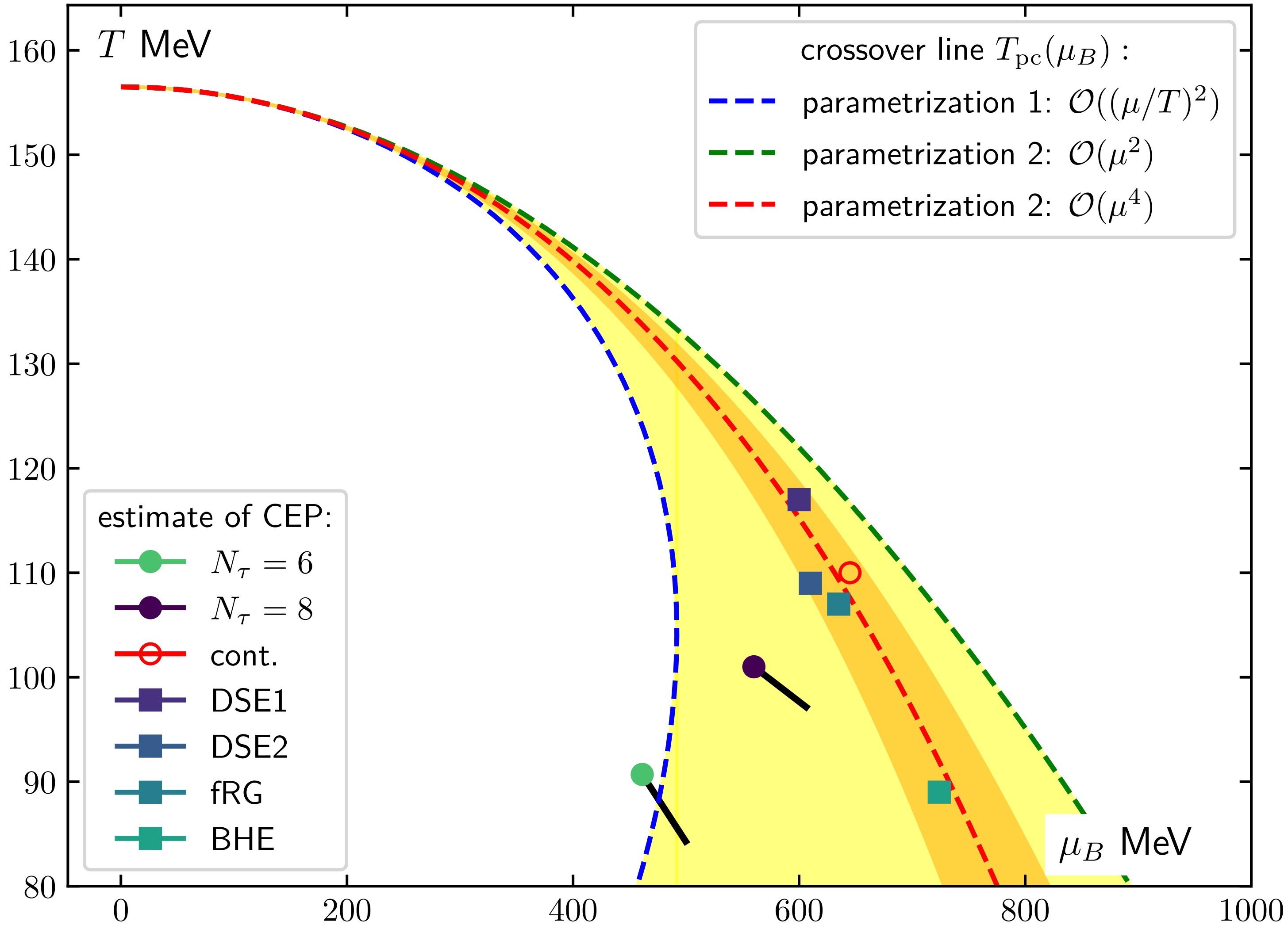
Statistical analysis of fits



	$N_\tau = 6$ multi-point Padé			$N_\tau = 8$ [4,4]-Padé		
	T^{CEP} [MeV]	μ_B^{CEP} [MeV]	μ_B/T	T^{CEP} [MeV]	μ_B^{CEP} [MeV]	μ_B/T
best fit	90.7 ± 7.7	461.2 ± 220	5.09 ± 0.68	101 ± 15	560 ± 140	5.5 ± 1.7
weight-1	$105.4 + 8.0 - 18.4$	$422.9 + 80.5 - 34.9$	$3.92 + 1.52 - 0.24$			
weight-2	$100.8 + 11.6 - 26.8$	$430.9 + 208.2 - 42.2$	$4.20 + 4.13 - 0.47$			
	c_1	c_2	c_3	c_1	c_2	c_3
best fit	-6.2 ± 9.2	0.115 ± 0.090	0.424 ± 0.086	-12.3 ± 8.1	0.203 ± 0.059	0.55 ± 0.25

* For $N_\tau = 8$: similar results by **[Basar, arXiv: 2312.06952]**

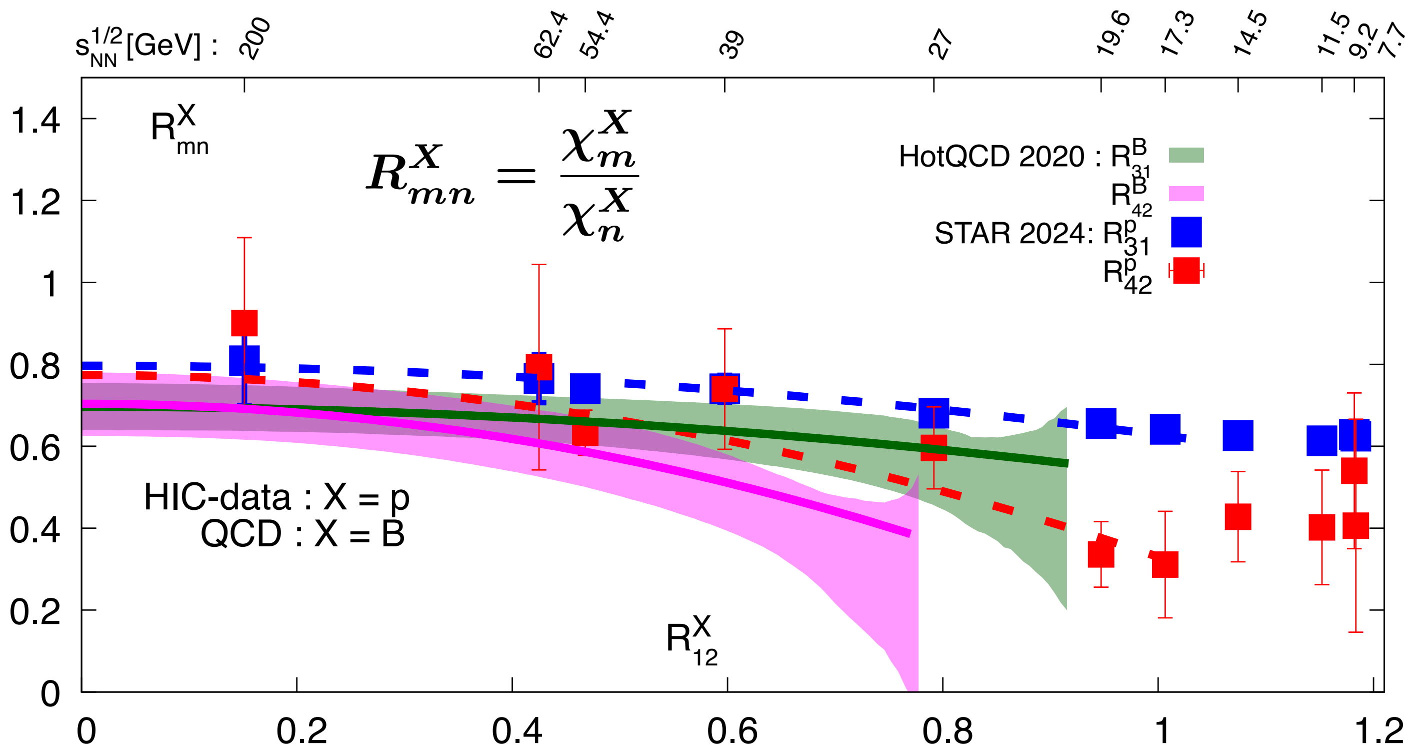
Crossover line and cutoff effects



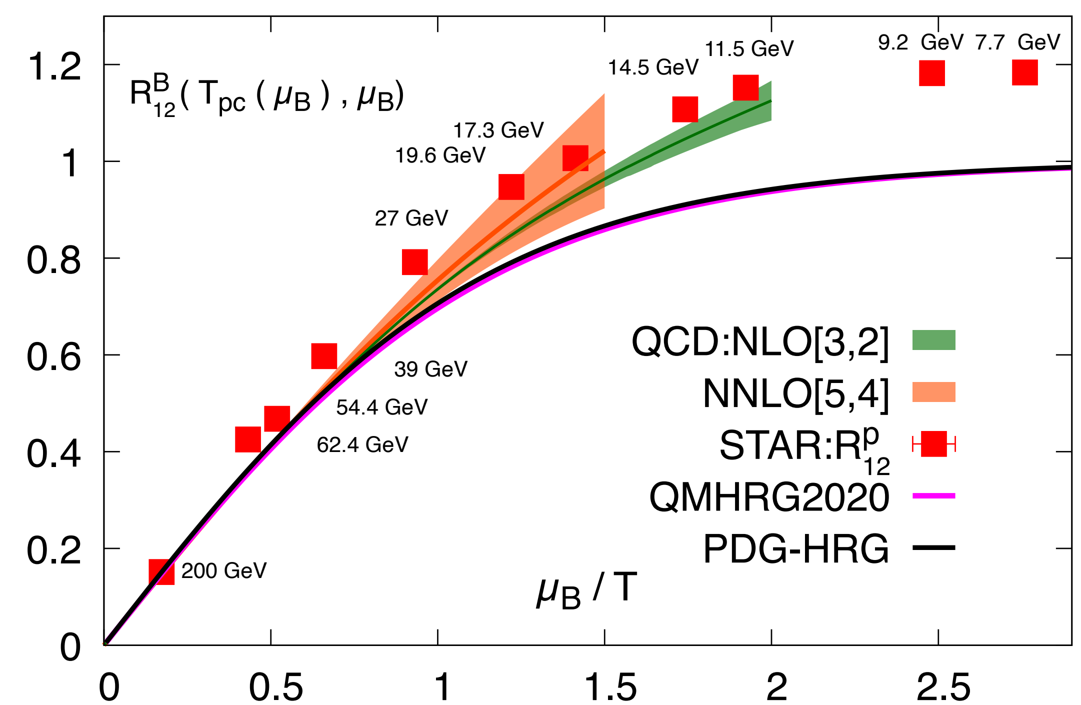
- Continuum estimate might suffer from large systematic effects (Padé vs multi-point Padé)
- $\kappa_2 = \bar{\kappa}_2 = -0.015(1)$
  [HotQCD, 2403.09390]
- Many results seem to favour a small $\bar{\kappa}_4 \approx -0.0002(1)$

Parametrizations of the crossover line:

- 1.) $$T_{pc}(\mu_B) = T_{pc}(0)\left[1 + \kappa_2^B\left(\frac{\mu_B}{T}\right)^2 + \kappa_4^B\left(\frac{\mu_B}{T}\right)^4\right]$$
- 2.) $$T_{pc}(\mu_B) = T_{pc}(0)\left[1 + \bar{\kappa}_2^B\left(\frac{\mu_B}{T_{pc}(0)}\right)^2 + \bar{\kappa}_4^B\left(\frac{\mu_B}{T_{pc}(0)}\right)^4\right]$$



[Karsch, Goswami, XQCD '24]



[Bollweg et al. '24]

- QCD and STAR results are in good agreement for $\sqrt{s_{NN}} < 19.6\ GeV$
- Slight vertical shift might suggest that freeze-out temperature is slightly below T_{pc}

- HRG model calculations based on noninteracting, point-like hadrons will always lead to $R^B_{12} < 1$
  ➡ HRG can not reproduce STAR data