Anomalous transport from lattice QCD

Gergely Endrődi

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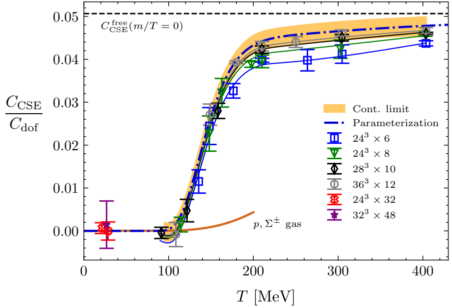
in collaboration with:

Bastian Brandt, Eduardo Garnacho, Javier Hernández, Gergely Markó,
Laurin Pannullo, Leon Sandbote, Dean Valois

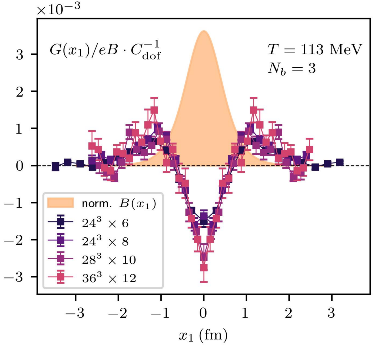
Appetizer

first fully non-perturbative determination
of in-equilibrium anomalous transport coefficients

chiral separation effect



(local) chiral magnetic effect



Brandt, Endrődi, Garnacho, Markó, JHEP 02 142

Brandt, Endrődi, Garnacho, Markó, Valois, 2409.17616

Brandt, Endrődi, Garnacho, Markó, JHEP 09 092

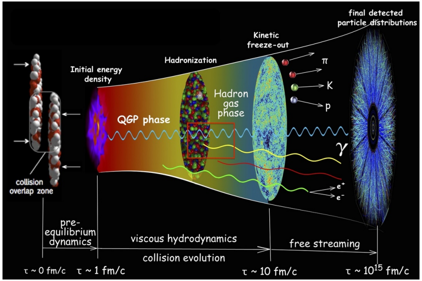
Outline

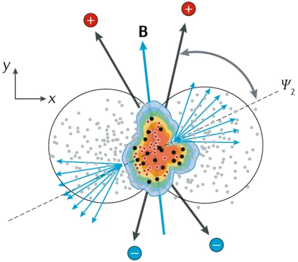
- introduction: anomalous transport phenomena
- in-equilibrium chiral magnetic effect
- in-equilibrium chiral separation effect
- local in-equilibrium chiral magnetic effect
- out-of-equilibrium chiral magnetic effect
- summary

Introduction

Quarks and gluons in extreme conditions

- heavy ion collisions $T \lesssim 10^{12}\,{}^\circ C = 200$ MeV, $n \lesssim 0.12$ fm^{-3}
  $B \lesssim 10^{19}$ G $= 0.3$ GeV$^2/e$





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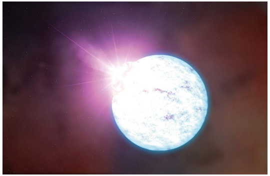
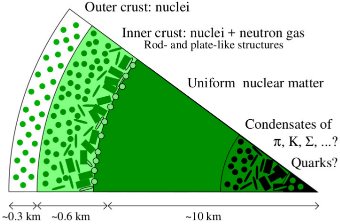
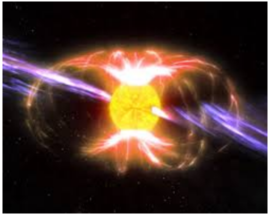
- neutron stars $T \lesssim 1$ MeV, $n \lesssim 2$ fm^{-3}
  magnetars $B \lesssim 10^{15}$ G







Lattimer, Nature Astronomy 2019

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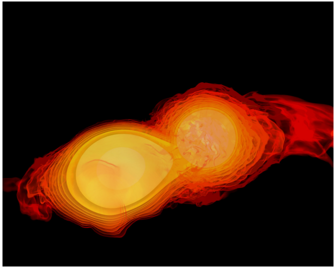
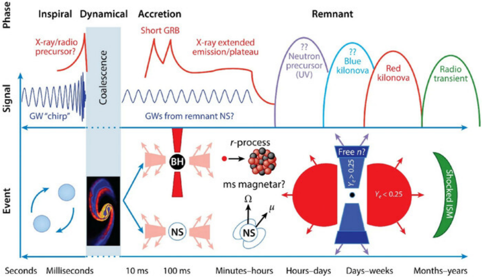
- neutron star mergers $T \lesssim 50$ MeV





astro.uni-frankfurt.de/rezzolla

Quarks and gluons in extreme conditions

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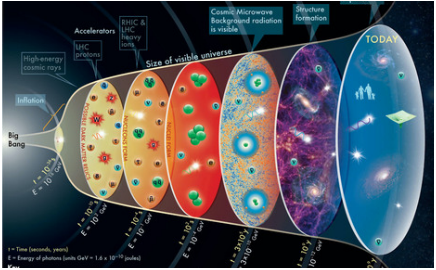
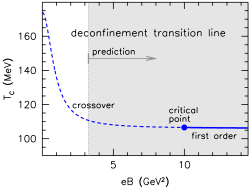
- eary universe, QCD epoch $T \lesssim 200$ MeV, standard scenario: $n \approx 0$
  B from electroweak epoch Vachaspati '91 Enqvist, Olesen '93

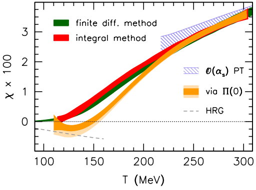


Magnetic fields impact on

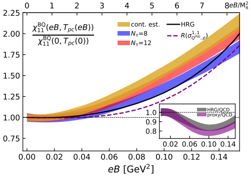
- phase diagram
  - 🔗 Endrődi, JHEP 07 (2015)



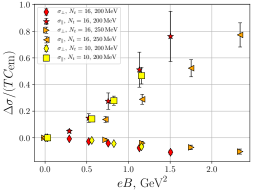
- equation of state
  - 🔗 Bali et al. JHEP 07 (2020)



- fluctuations
  - 🔗 Ding et al. PRL 132 (2024)



- transport phenomena
  - 🔗 Astrakhantsev et al. PRD 102 (2020)



- anomalous transport phenomena

Anomalous transport

- usual transport:
  vector current due to electric field

$$\langle \boldsymbol{J} \rangle = \sigma \cdot \boldsymbol{E}$$

- chiral magnetic effect (CME)
  Fukushima, Kharzeev, Warringa, PRD 78 (2008)
  vector current due to chirality and magnetic field

$$\langle \boldsymbol{J} \rangle = \sigma_{\mathrm{CME}} \cdot \boldsymbol{B}$$

- chiral separation effect (CSE)
  Son, Zhitnitsky, PRD 70 (2004) Metlitski, Zhitnitsky, PRD 72 (2005)
  axial current due to baryon number and magnetic field

$$\langle \boldsymbol{J}_5 \rangle = \sigma_{\mathrm{CSE}} \cdot \boldsymbol{B}$$

Phenomenological and theoretical relevance

- experimental observation of CME in condensed matter systems
  Li, Kharzeev, Zhan et al., Nature Phys. 12 (2016)
- experimental searches for CME and related observables in heavy-ion collisions
  STAR collaboration, PRC 105 (2022)
- serves as indirect way to probe topological fluctuations and CP-odd domains in heavy-ion collisions
- recent reviews: Kharzeev, Liao, Voloshin, Wang, PPNP 88 (2016)
  Kharzeev, Liao, Tribedy, 2405.05427

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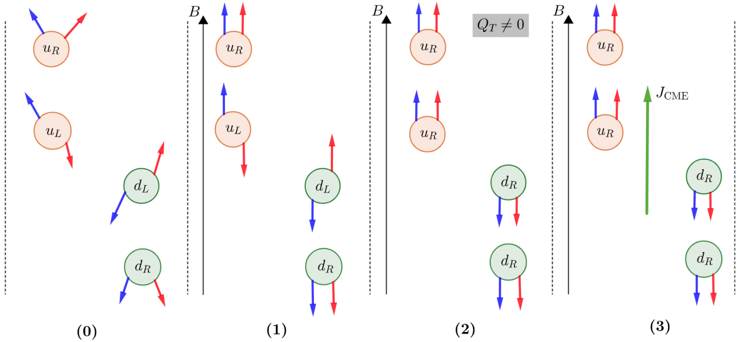
- disclaimer: this talk is not about feasibility of experimental detection, but about the theory of anomalous transport

General (handwaving) argument

- spin, momentum chiral magnetic effect



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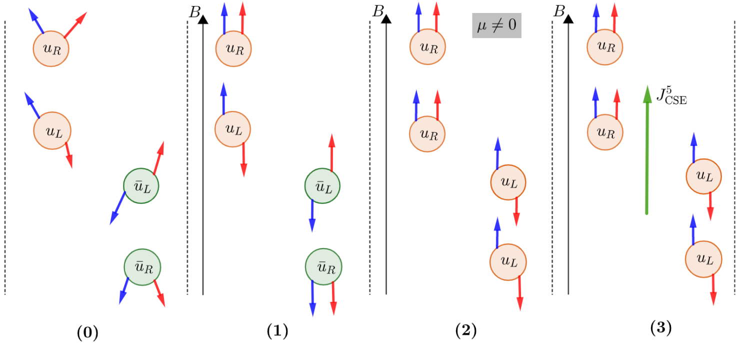

General (handwaving) argument – issues

- quantum theory requires ultraviolet regularization
- massless vs. massive fermions
- strong interactions between fermions
- in-equilibrium vs. out-of-equilibrium nature

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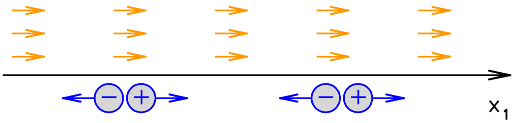
In-equilibrium vs. out-of-equilibrium

- example: charge transport due to electric field $\boldsymbol{E} \parallel \boldsymbol{e}_1$



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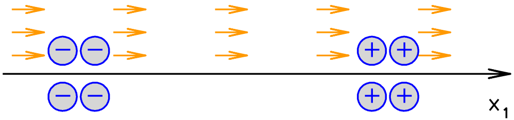

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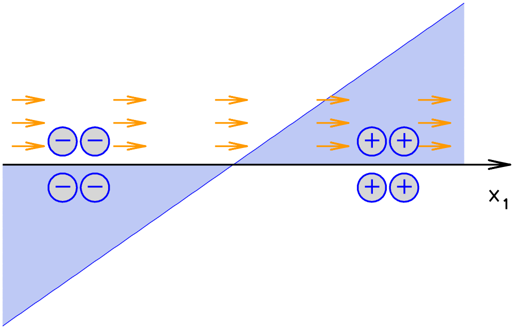

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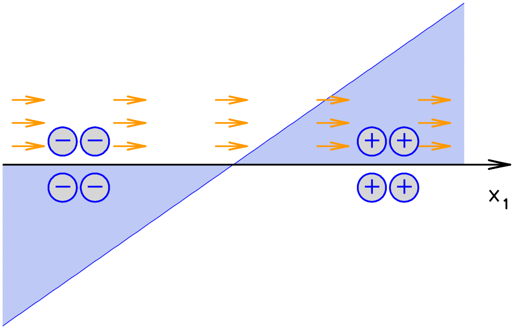

- out-of equilibrium linear response:
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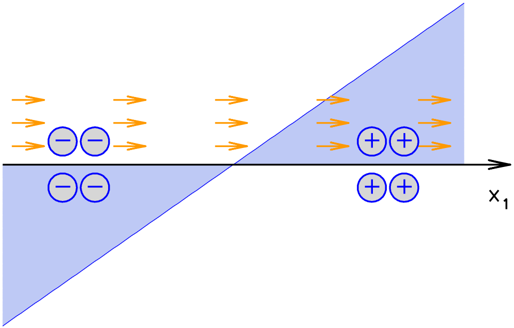

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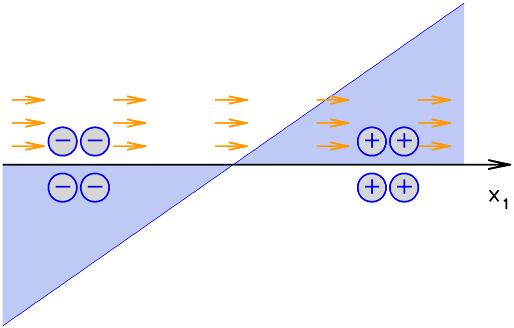

- out-of equilibrium linear response:
  time-dependent response to time-dependent perturbation (electric conductivity)
- leading to an equilibrium distribution (electric polarization/susceptibility)
- same story can be told for CME

No currents in equilibrium

- **Bloch's theorem**: Bohm Phys. Rev. 75 (1949) N. Yamamoto, PRD 92 (2015)
  persistent electric currents do not exist in ground state of quantum systems
- applies to conserved currents
- applies to global (spatially averaged) currents
- applies in the thermodynamic limit ($V \to \infty$)

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- in-equilibrium CSE is possible
- in-equilibrium local CME currents are possible

Chiral magnetic effect in equilibrium

CME and inconsistencies

- parameterize chiral imbalance n_5 by a chiral chemical potential μ_5
  🔗 Fukushima, Kharzeev, Warringa, PRD 78 (2008)
- CME for weak chiral imbalance ($\boldsymbol{B} = B\boldsymbol{e}_3$)

$$\langle J_3 \rangle = \sigma_{\mathrm{CME}}\, B = C_{\mathrm{CME}}\, \mu_5 B + \mathcal{O}(\mu_5^3)$$

- from Bloch's theorem it follows that in equilibrium

$$C_{\mathrm{CME}} = 0 \checkmark$$

- several results in the literature give incorrectly

$$C_{\mathrm{CME}} = \frac{1}{2\pi^2} \lightning$$

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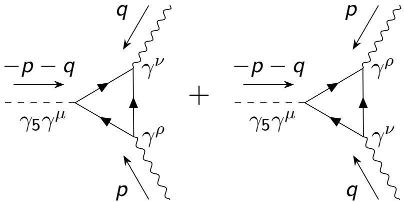
  careful regularization is required

Perturbation theory

- triangle diagram

$$\Gamma^{\mu\nu\rho}_{AVV}(p+q,p,q) =$$ 

- gives in-equilibrium CME coefficient

$$C_{\mathrm{CME}} = \lim_{p,q\to 0} \frac{1}{q_1} \Gamma^{023}_{AVV}(p+q,p,q)$$

- also gives the axial anomaly Peskin-Schroeder 19.2

$$\langle \partial_\mu J_5^\mu \rangle \sim (p+q)_\mu \Gamma^{\mu\nu\rho}_{AVV}(p+q,p,q) A_\nu A_\rho$$

Regularization sensitivity – anomaly

- naive regularization

$$(p+q)_\mu \Gamma^{\mu\nu\rho}_{AVV}(p+q,p,q)A_\nu A_\rho = mP_5(p,q) \ ↯$$

- Pauli-Villars regularization
  (regulator particles $s = 1,2,3$ with $c_s = \pm 1$ and $m_s \to \infty$)

$$(p+q)_\mu \Gamma^{\mu\nu\rho}_{AVV}(p+q,p,q)A_\nu A_\rho = mP_5(p,q) + \sum_{s=1}^{3} c_s m_s P^{\nu\rho}_{5,s}(p,q)A_\nu A_\rho$$

$$\xrightarrow{m_s\to\infty} mP_5(p,q) + \frac{\epsilon^{\alpha\beta\nu\rho}F_{\alpha\nu}F_{\beta\rho}}{16\pi^2} \ \checkmark$$

Regulator sensitivity – CME

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- Pauli-Villars regularization

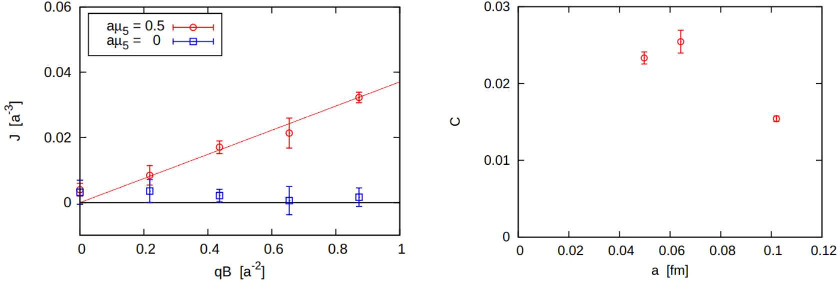
$$C_{\mathrm{CME}} = \lim_{p,q,p+q\to 0} \frac{1}{q_1} \Gamma^{023}_{AVV}(p+q,p,q) = \frac{1}{2\pi^2} + \sum_{s=1}^{3} \frac{c_s}{2\pi^2} = 0 \ \checkmark$$

- in equilibrium, C_{CME} vanishes due to anomalous contribution

CME in equilibrium – lattice simulations

Regularization sensitivity on the lattice

- seminal lattice determination of $\langle J_3 \rangle$ at $B \neq 0$, $\mu_5 \neq 0$ A. Yamamoto, PRL 107 (2011)



- coefficient $C_{\mathrm{CME}} \approx 0.025 \sim 1/(4\pi^2)$ ⚡

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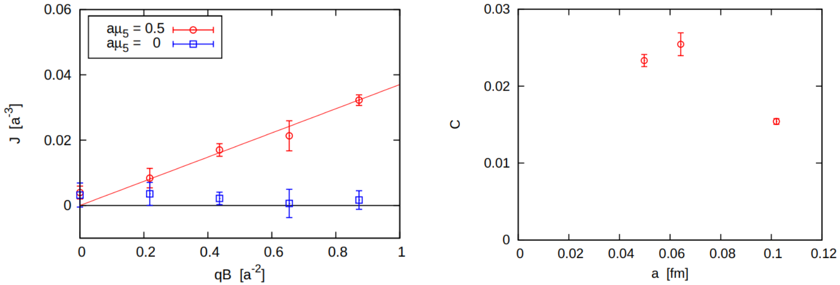

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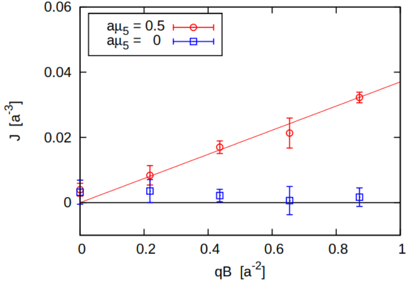

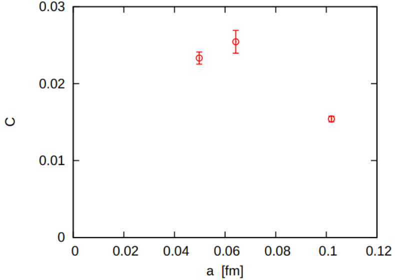

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$$J_\nu^{\rm non-cons} = \bar\psi(n)\gamma_\nu\psi(n) \qquad\qquad J_\nu^{\rm cons} \sim \bar\psi(n)\gamma_\nu U_\nu(n)\psi(n+\hat\nu)$$

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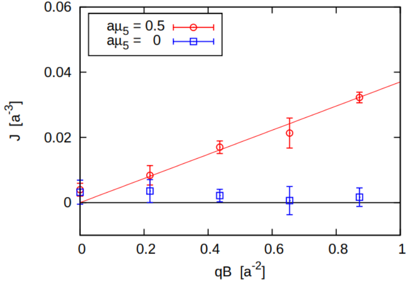

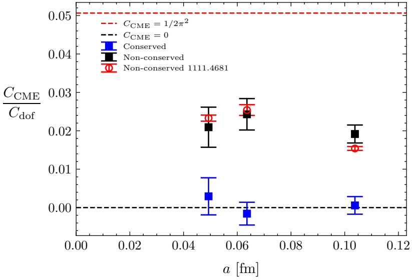

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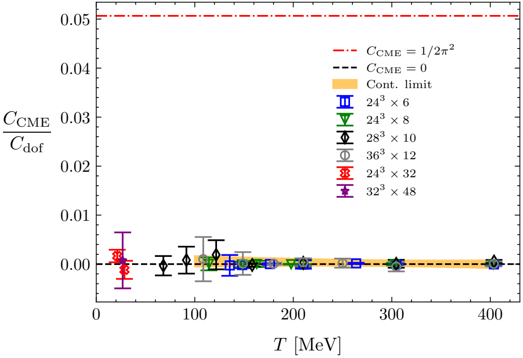
- conserved current: $C_{\rm CME} = 0$ ✓ Brandt, Endrődi, Garnacho, Markó, JHEP 09 (2024)

CME in equilibrium – final result

- full QCD simulations with staggered quarks at physical quark masses, extrapolated to the continuum limit employing the conserved electric current
- global CME current vanishes in equilibrium
  Brandt, Endrődi, Garnacho, Markó, JHEP 09 (2024)



Chiral separation effect in equilibrium

Chiral separation effect

- axial current due to magnetic field and baryon density
  Son, Zhitnitsky, PRD 70 (2004) Metlitski, Zhitnitsky, PRD 72 (2005)
- parameterize baryon density n by chemical potential μ
- CSE for small density ($\boldsymbol{B} = B\boldsymbol{e}_3$)

$$\langle J_{35} \rangle = \sigma_{\mathrm{CSE}}\, B = C_{\mathrm{CSE}}\, \mu B + \mathcal{O}(\mu^3)$$

- Bloch's theorem allows in-equilibrium CSE $(\partial_\nu J_{\nu 5} \neq 0)$
- regularization less intricate, but conserved vector current on lattice is important
- previous lattice efforts Puhr, Buividovich, PRL 118 (2017)
  Buividovich, Smith, von Smekal, PRD 104 (2021)

CSE in equilibrium – final result

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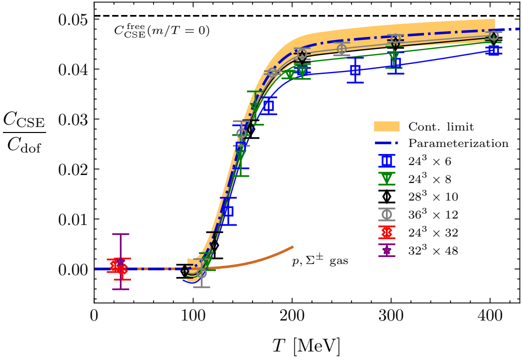
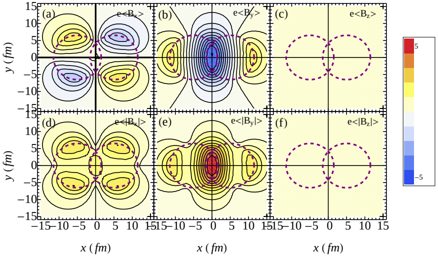
  Brandt, Endrődi, Garnacho, Markó, JHEP 02 (2024)



Local chiral magnetic effect

Inhomogeneous magnetic fields

- up to now: homogeneous magnetic background
- off-central heavy-ion collisions: inhomogeneous fields Deng et al., PRC 85 (2012)



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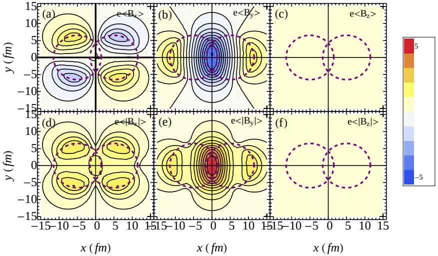

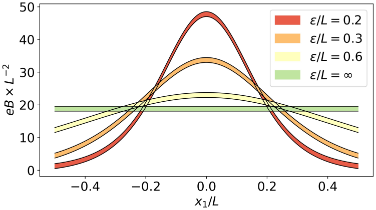

- consider profile $B(x) = B\cosh^{-2}(x/\epsilon)$ Dunne, hep-th/0406216
  with $\epsilon \sim 0.6$ fm

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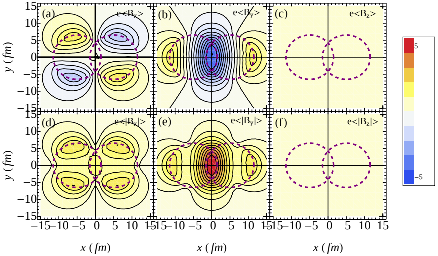

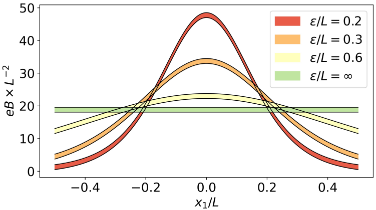

- consider profile $B(x) = B \cosh^{-2}(x/\epsilon)$ Dunne, hep-th/0406216
  with $\epsilon \sim 0.6$ fm
- impact on thermodynamic observables in QCD and phase diagram
  Brandt, Endrődi, Markó, Valois, JHEP 07 (2024)

Local currents

- response for weak μ_5 for homogeneous B

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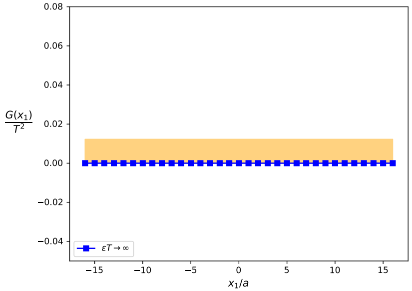

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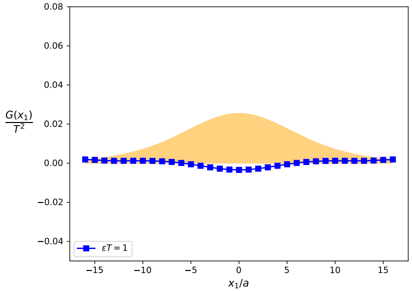

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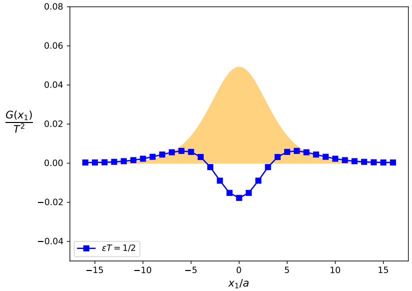

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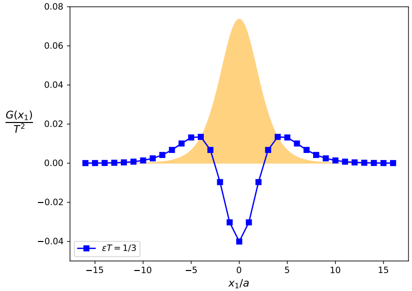

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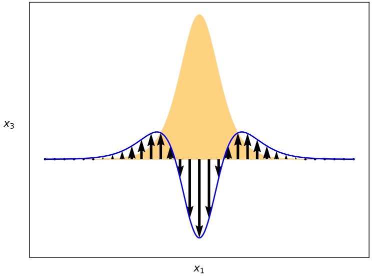
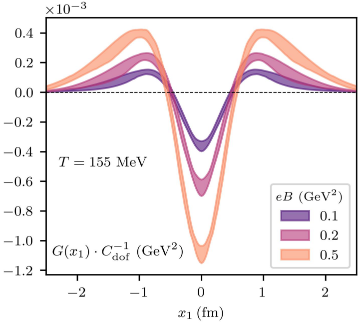


Local currents in QCD

- full QCD simulations with staggered quarks
  at physical quark masses, extrapolated to the continuum limit
  employing the conserved electric current
- non-trivial localized CME signal 🔗 Brandt, Endrődi, Garnacho, Markó, Valois, 2409.17616



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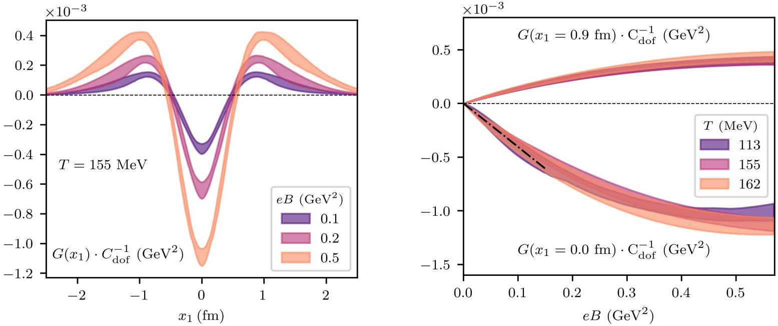

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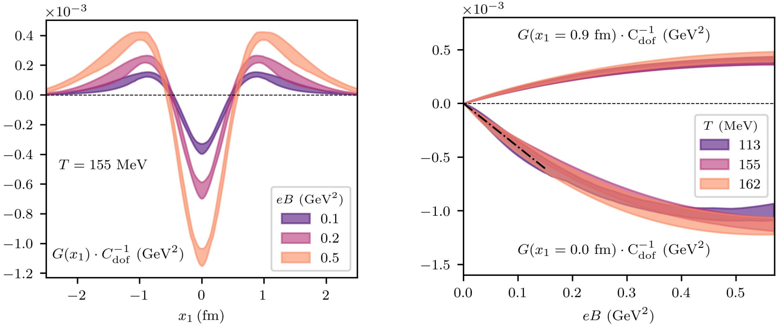


- may guide experimental efforts to detect CME

Out-of-equilibrium phenomena

Out-of-equilibrium transport

- Kubo formula: transport coefficients from spectral functions

$$\xi \sim \lim_{\omega \to 0} \frac{\rho(\omega)}{\omega}$$

- spectral function from Euclidean correlators on the lattice

$$G(x_4) = \int \mathrm{d}\omega\, \rho(\omega)\, \underbrace{K(\omega, x_4)}_{\text{known kernel}}$$

ill-posed problem, may be studied using various strategies

- Euclidean correlators

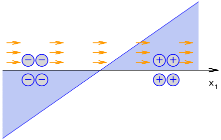
$$G_{\mathrm{CME}}(x_4) = \langle J_3(0) J_{45}(x_4) \rangle \qquad G_{\mathrm{CSE}}(x_4) = \langle J_{35}(0) J_4(x_4) \rangle$$

first results for CME Buividovich, PRD 110 (2024)

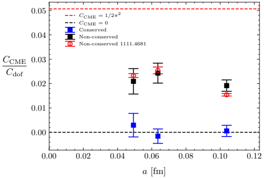
Summary

Summary

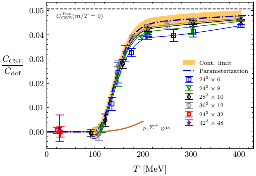
- CME subtleties:
  in- / out-of-equilibrium



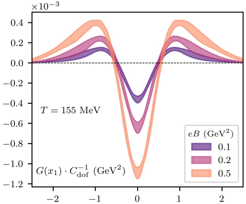
- careful regularization crucial
  in-equilibrium global CME vanishes



- in-equilibrium CSE in full QCD



- in-equilibrium local CME in QCD

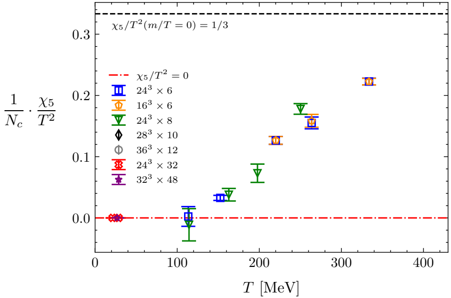


Backup

Chiral density

- chiral density n_5 is parameterized by chiral chemical potential μ_5

$$n_5(\mu_5) = \chi_5\,\mu_5 + \mathcal{O}(\mu_5^3), \qquad \chi_5 = \frac{T}{V}\,\frac{\partial^2 \log \mathcal{Z}}{\partial \mu_5^2}\bigg|_{\mu_5=0}$$



Inhomogeneous chiral imbalance

- inhomogeneous $B(x_1)$ *and* inhomogeneous $\mu_5(x_1)$

$$\langle J_3(x_1)\rangle = \int \mathrm{d}x_1' \underbrace{\mathrm{d}x_1'' \, \chi_{\mathrm{CME}}(x_1 - x_1', x_1 - x_1'') \, B(x_1'')}_{H(x_1, x_1')} \, \mu_5(x_1')$$