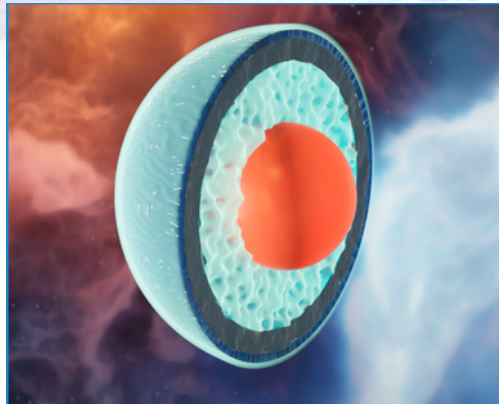


Quark Matter and Nuclear Astrophysics: recent developments



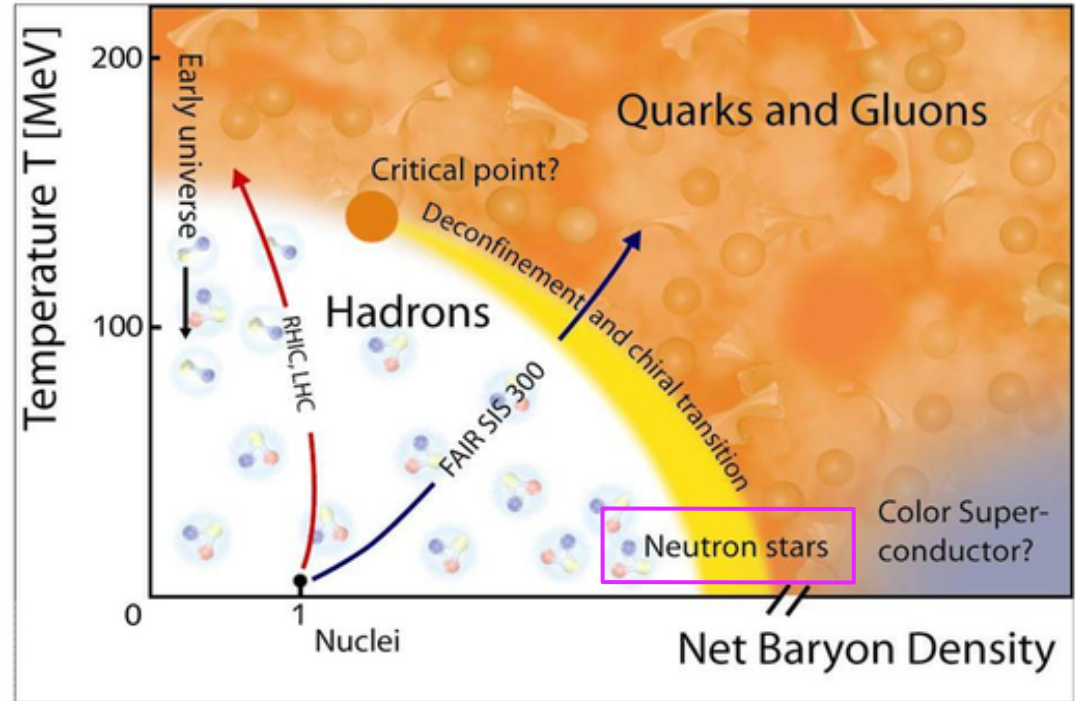
Zimányi Winter school, Budapest, Hungary

03 December 2024

Tyler Gorda

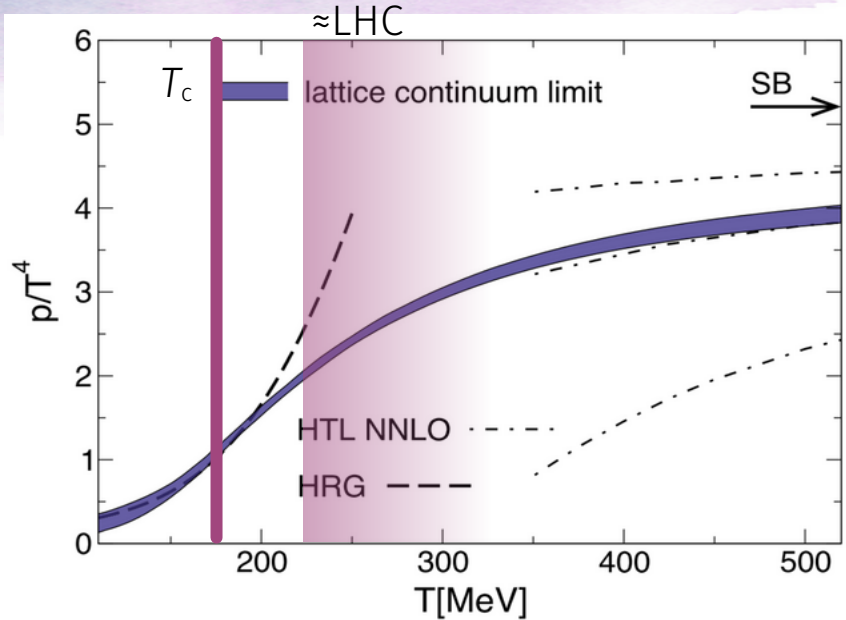
Goethe University Frankfurt

Understanding the phase diagram of QCD



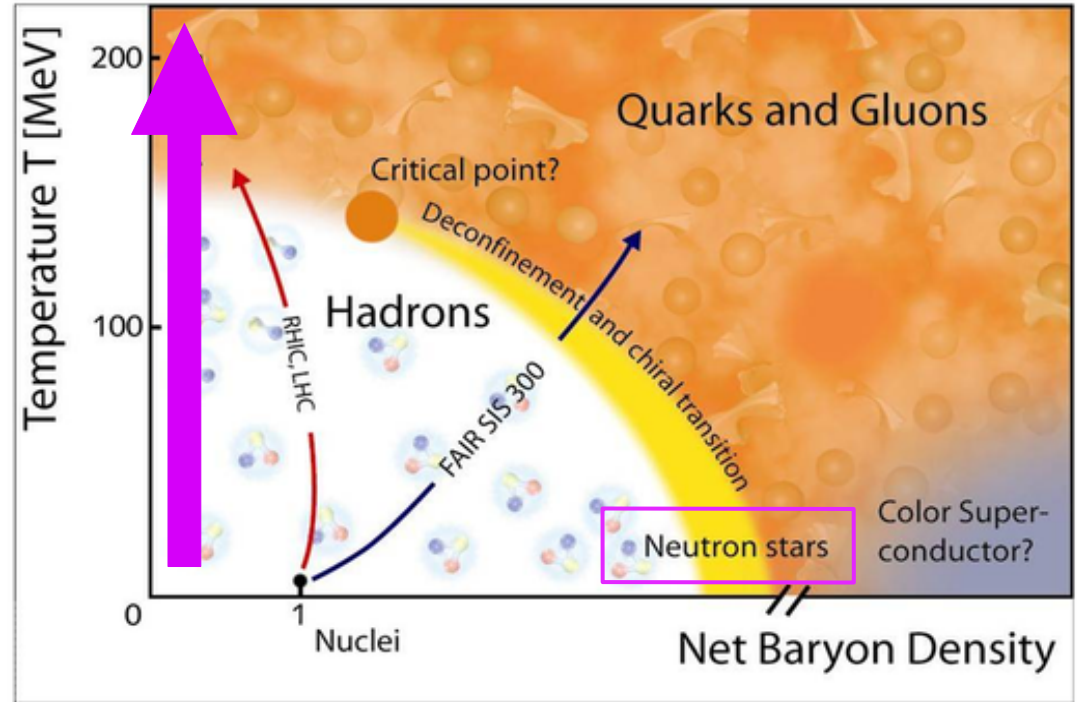
Compressed Baryonic Matter (CBM) experiment

Understanding the phase diagram of QCD



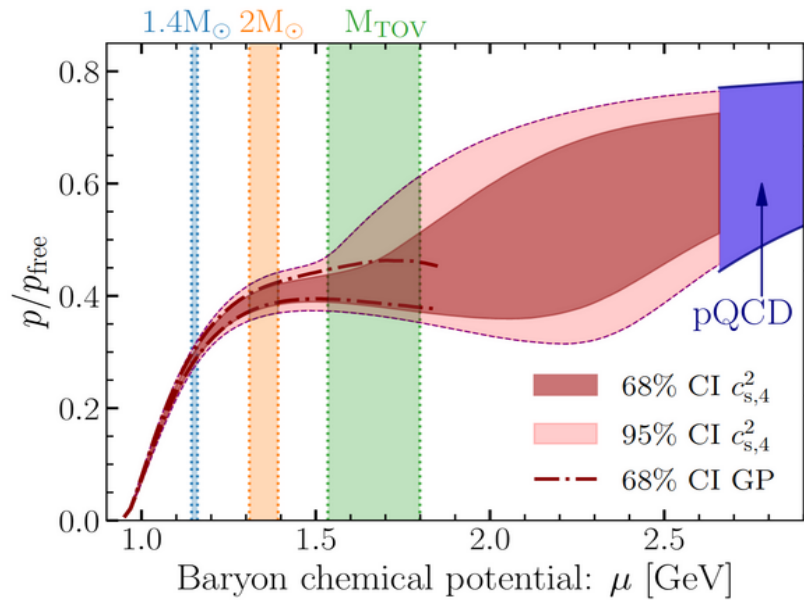
Borsanyi, Fodor, Hoelbling, Katz, Krieg, Szabo Phys. Lett. B 370 (2014), Gardim, Giacalone, Luzum, Ollitrault, Nature Physics 16, 615-619 (2020)

Lattice calculations at high- T directly tell us the EoS; can compare to hadronic/partonic calculations



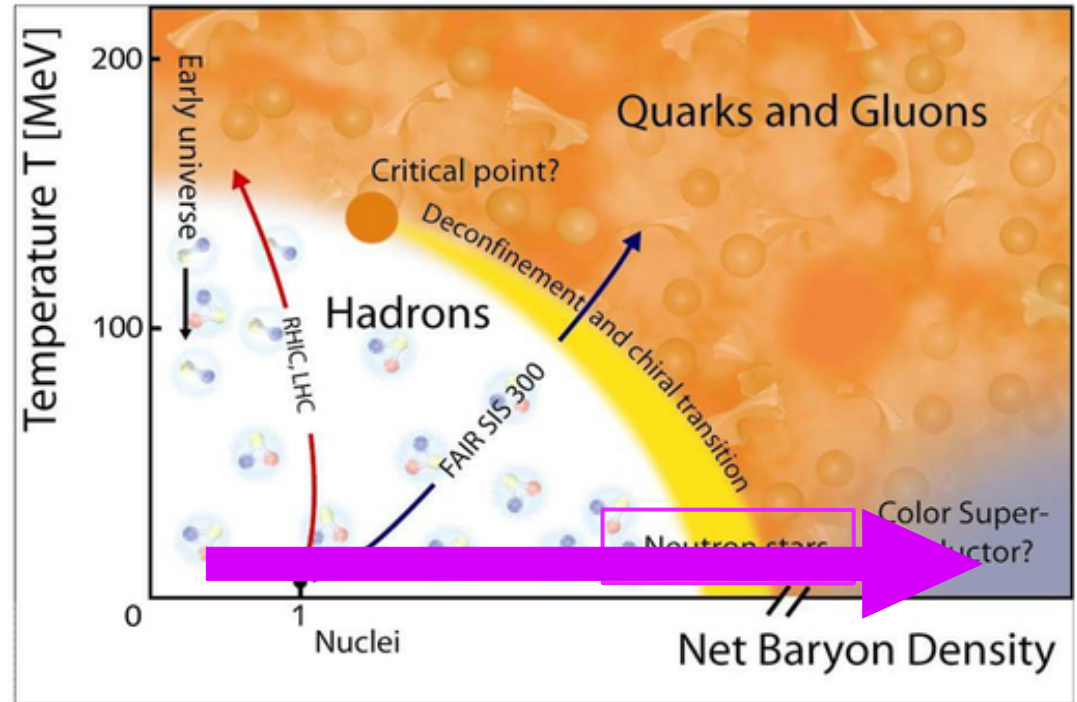
Compressed Baryonic Matter (CBM) experiment

Understanding the phase diagram of QCD



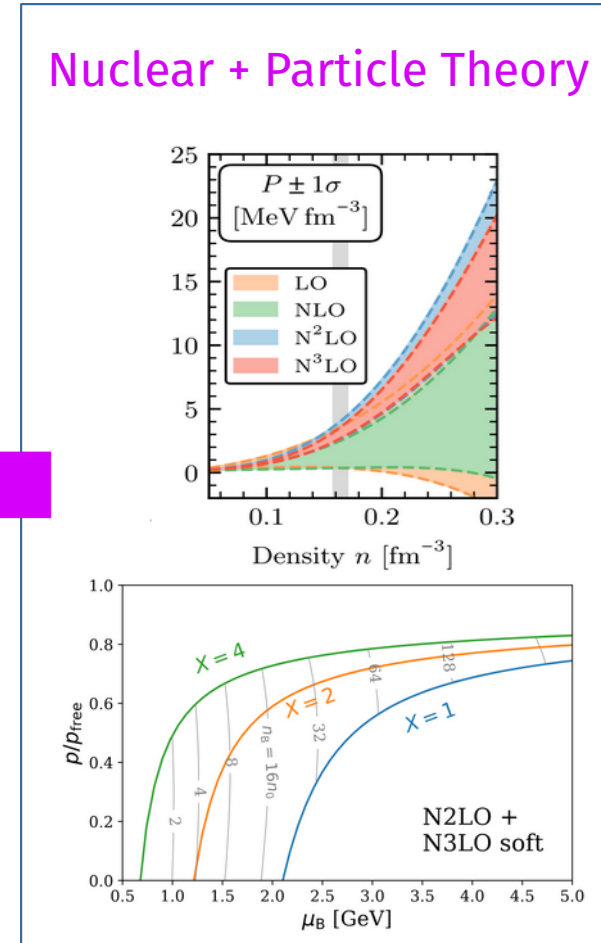
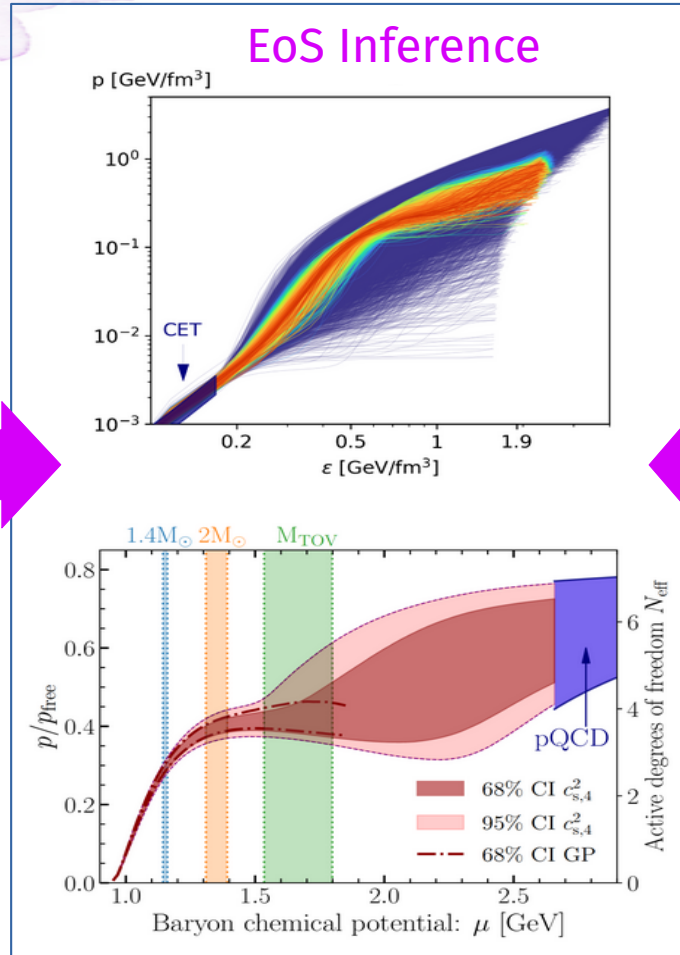
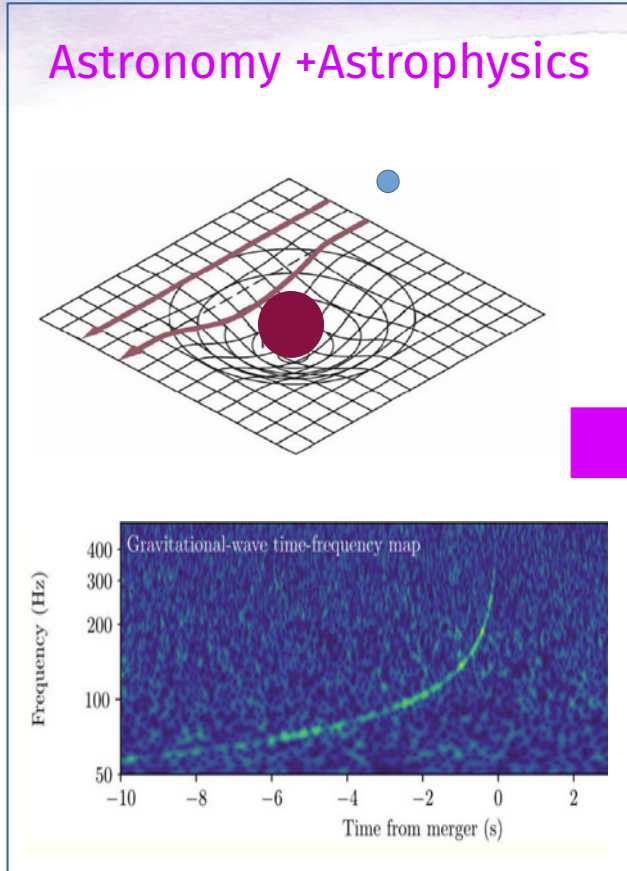
Annala, TG, Hirvonen, Komoltsev, Kurkela, Nättilä, Vuorinen Nat. Comm. 14 (2023)

At $T = 0$, no lattice, but we have **astrophysics**; and calculations in **nuclear and particle theory**

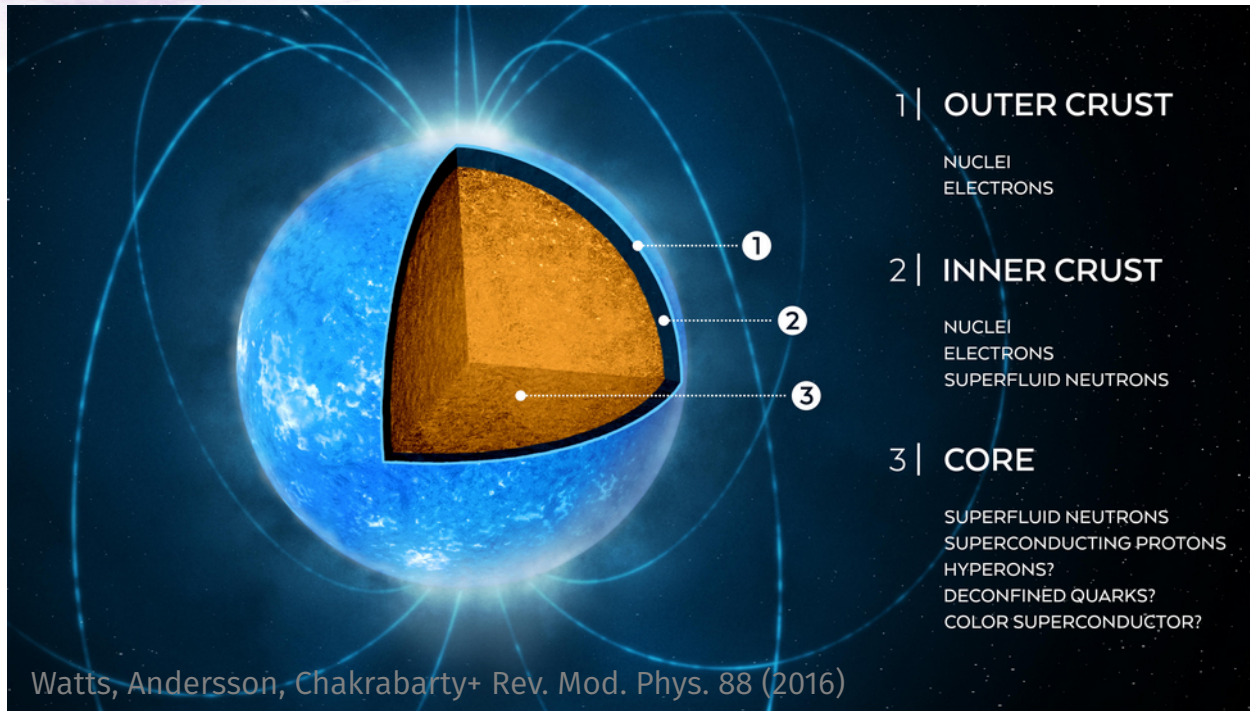


Compressed Baryonic Matter (CBM) experiment

$T = 0$ is a synthesis of theory and experiment



What is a Neutron Star?

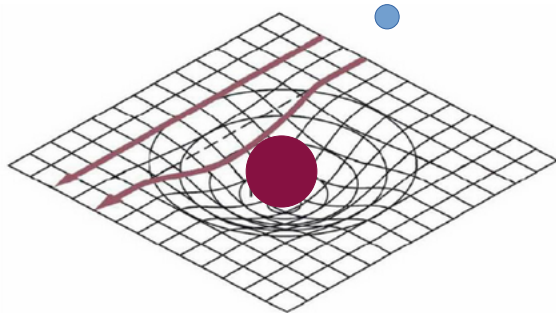


Collapsed remnant of dead stars, held from collapse by **repulsive** nuclear/QCD forces

Cleanest probes of structure/bulk properties when they are in **binary system with a companion**

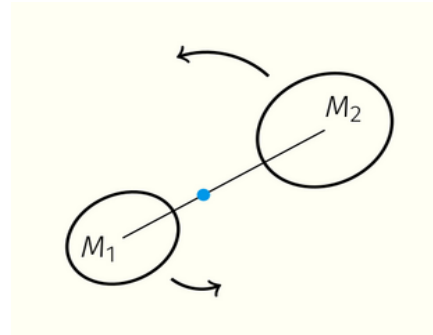
$$\text{Mass} \lesssim 2M_{\odot}, \quad 11 \text{ km} \lesssim R \lesssim 13 \text{ km}, \quad T \lesssim \text{keV} \sim 10^7 \text{ K}.$$

Neutron stars in binaries tell us about structure



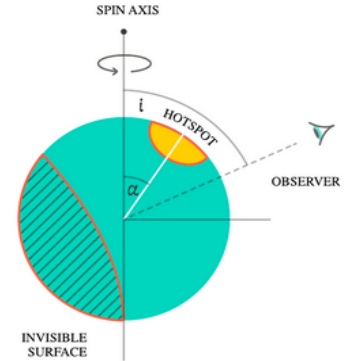
Masses

Demorest, Pennucci, Ransom, Roberts, Hessels. *Nature* 467 (2010) pp. 1081-1083;
Antoniadis+ *Science* 340 (2013) p. 6131;
Cromartie+ (NANOGrav). *Nature Astron.* 4.1 (2019).
E. Fonseca+ *Astrophys. J. Lett.* 915.1 (2021)



Deformabilities

Abbott+ (LIGO Scientific, Virgo) *PRL* 119 (2017); *PRL* 121 (2018); *PRX* 9 (2019).

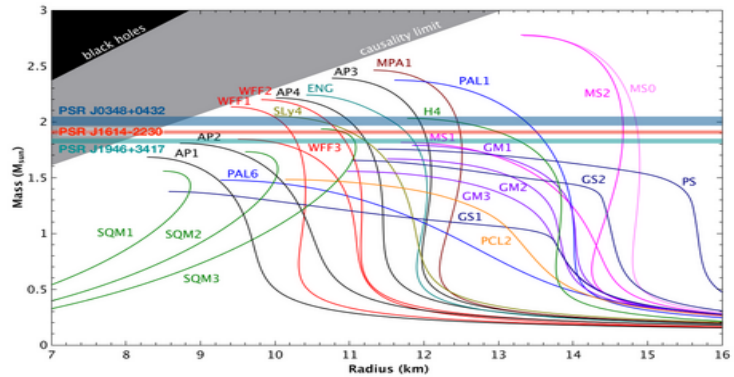


Radii, compactness

Miller+ *Astrophys. J. Lett.* 918.2 (2021), p. L28.
Riley+ *Astrophys. J. Lett.* 918.2 (2021) p. L27.

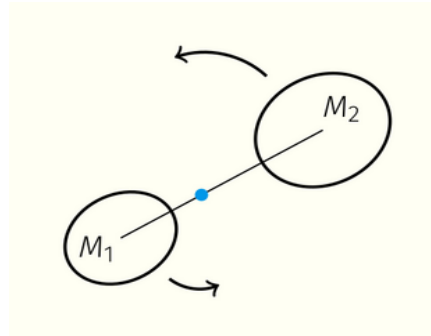
Neutron stars in binaries tell us about structure

$$M_{\text{TOV}} \geq \begin{cases} 1.97 \pm 0.04 M_{\odot} & \text{PSR J1614-2230} \\ 2.01 \pm 0.04 M_{\odot} & \text{PSR J0348+0432} \\ 2.08 \pm 0.07 M_{\odot} & \text{PSR J0740+6620} \end{cases}$$

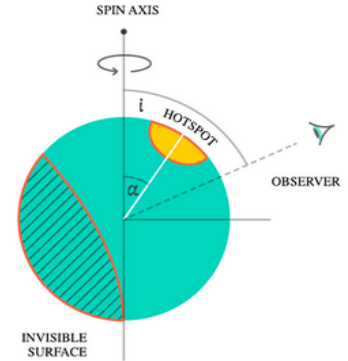


Norbert Wex

Masses



Deformabilities



Radii, compactness

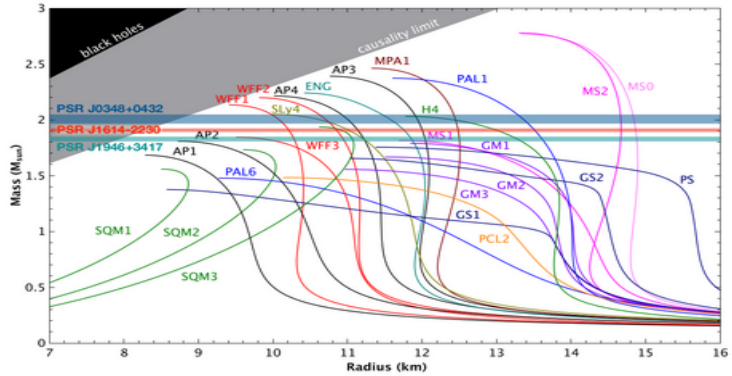
Demorest, Pennucci, Ransom, Roberts, Hessels. *Nature* 467 (2010) pp. 1081-1083;
 Antoniadis+ *Science* 340 (2013) p. 6131;
 Cromartie+ (NANOGrav). *Nature Astron.* 4.1 (2019).
 E. Fonseca+ *Astrophys. J. Lett.* 915.1 (2021)

Abbott+ (LIGO Scientific, Virgo) *PRL* 119 (2017); *PRL* 121 (2018); *PRX* 9 (2019).

Miller+ *Astrophys. J. Lett.* 918.2 (2021), p. L28.
 Riley+ *Astrophys. J. Lett.* 918.2 (2021) p. L27.

Neutron stars in binaries tell us about structure

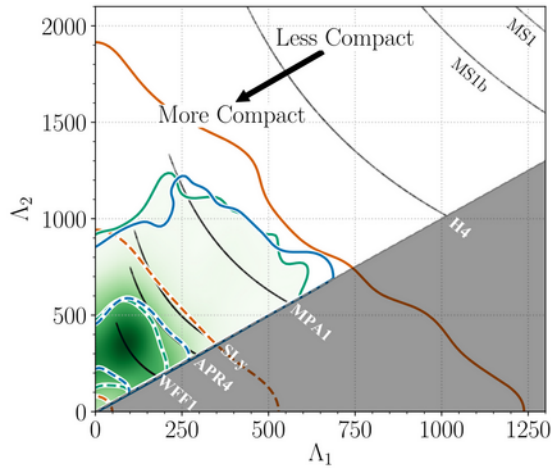
$$M_{\text{TOV}} \geq \begin{cases} 1.97 \pm 0.04 M_{\odot} & \text{PSR J1614-2230} \\ 2.01 \pm 0.04 M_{\odot} & \text{PSR J0348+0432} \\ 2.08 \pm 0.07 M_{\odot} & \text{PSR J0740+6620} \end{cases}$$



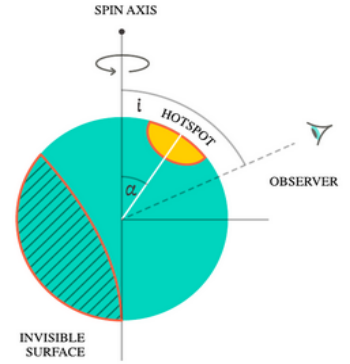
Norbert Wex

Masses

$$\Lambda(M) \equiv |Q_{ij}/\mathcal{E}_{ij}|M^5$$



Deformabilities



Radii, compactness

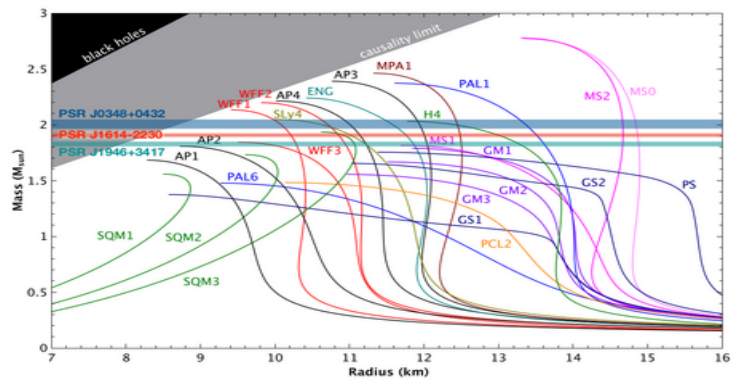
Demorest, Pennucci, Ransom, Roberts, Hessels. Nature 467 (2010) pp. 1081-1083;
 Antoniadis+ Science 340 (2013) p. 6131;
 Cromartie+ (NANOGrav). Nature Astron. 4.1 (2019).
 E. Fonseca+ Astrophys. J. Lett. 915.1 (2021)

Abbott+ (LIGO Scientific, Virgo) PRL 119 (2017); PRL 121 (2018); PRX 9 (2019).

Miller+ Astrophys. J. Lett. 918.2 (2021), p. L28.
 Riley+ Astrophys. J. Lett. 918.2 (2021) p. L27.

Neutron stars in binaries tell us about structure

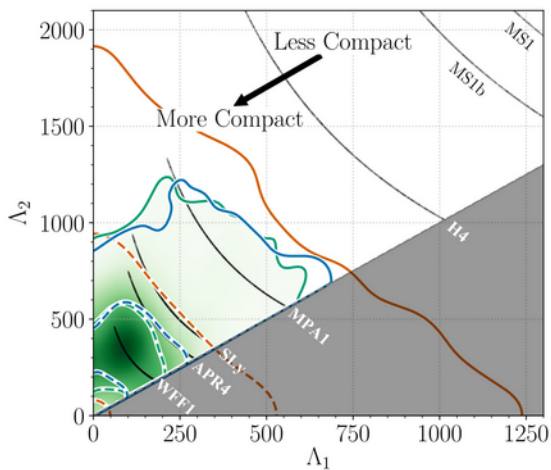
$$M_{\text{TOV}} \geq \begin{cases} 1.97 \pm 0.04 M_{\odot} & \text{PSR J1614-2230} \\ 2.01 \pm 0.04 M_{\odot} & \text{PSR J0348+0432} \\ 2.08 \pm 0.07 M_{\odot} & \text{PSR J0740+6620} \end{cases}$$



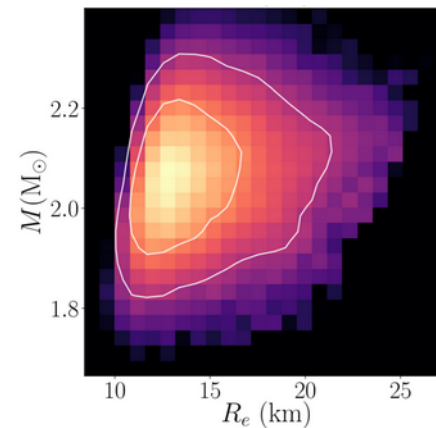
Norbert Wex

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Deformabilities



Radii, compactness

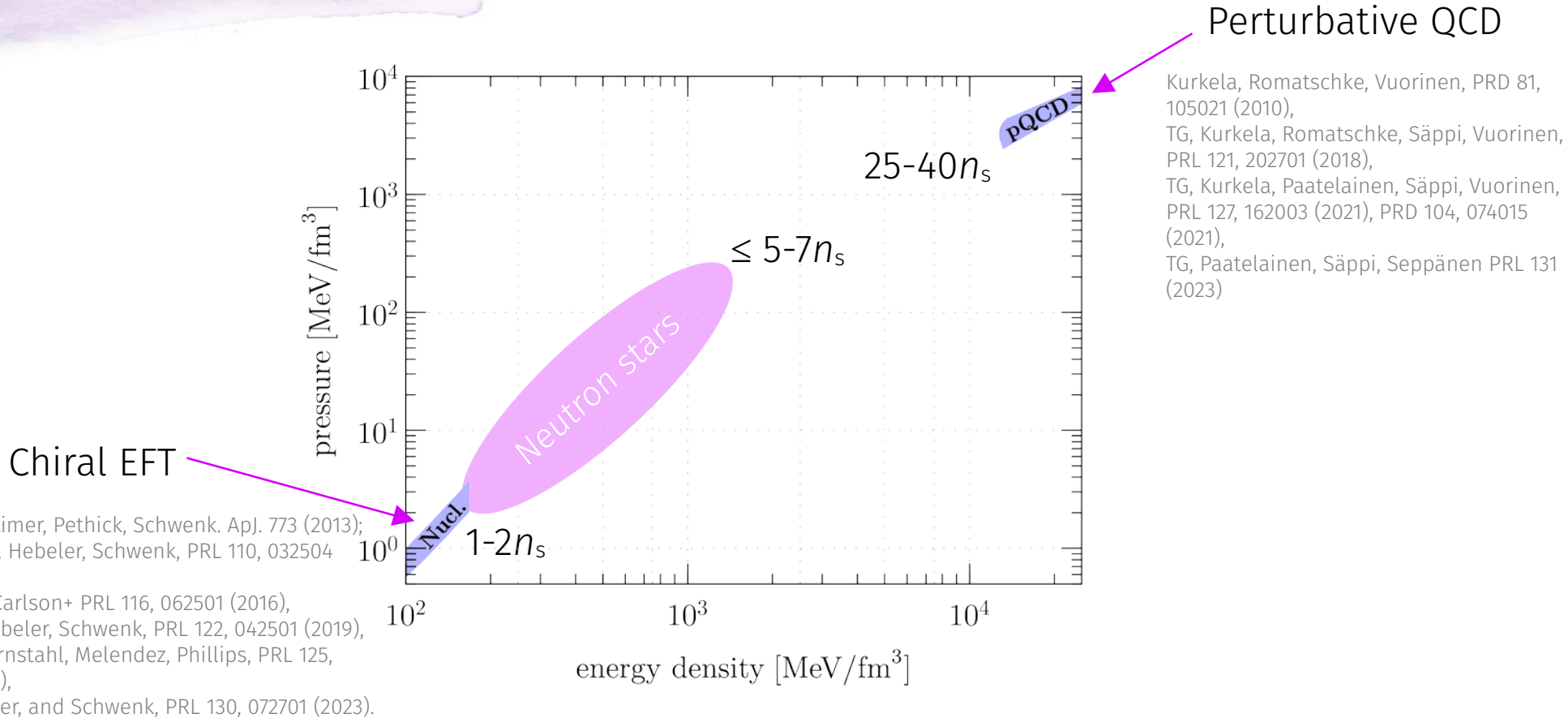
Demorest, Pennucci, Ransom, Roberts, Hessels. *Nature* 467 (2010) pp. 1081-1083;
 Antoniadis+ *Science* 340 (2013) p. 6131;
 Cromartie+ (NANOGrav). *Nature Astron.* 4.1 (2019).
 E. Fonseca+ *Astrophys. J. Lett.* 915.1 (2021)

Abbott+ (LIGO Scientific, Virgo) *PRL* 119 (2017); *PRL* 121 (2018); *PRX* 9 (2019).

Miller+ *Astrophys. J. Lett.* 918.2 (2021), p. L28.
 Riley+ *Astrophys. J. Lett.* 918.2 (2021) p. L27.

Use these data in place of lattice results

Different inputs constrain different parts of the EOS

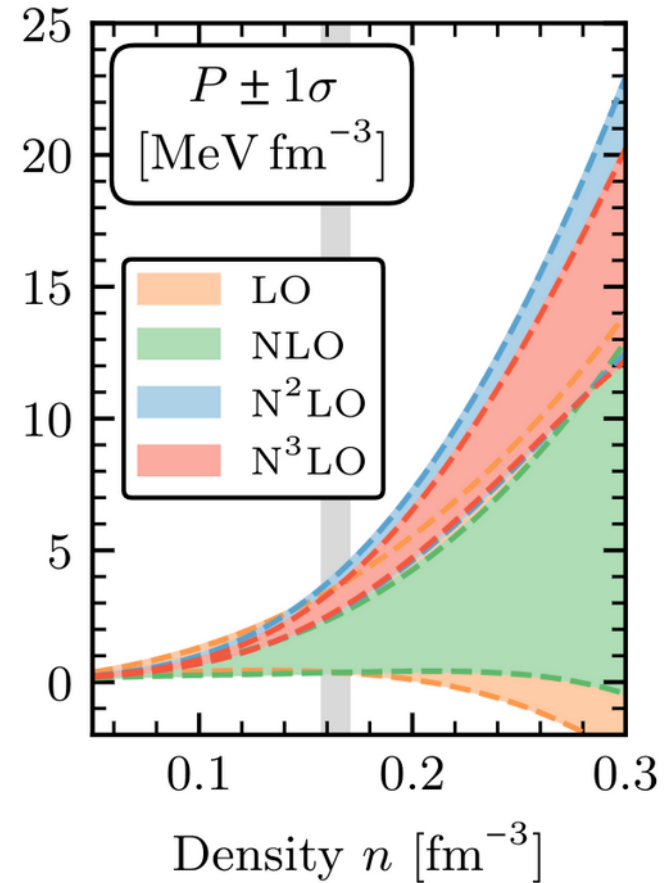
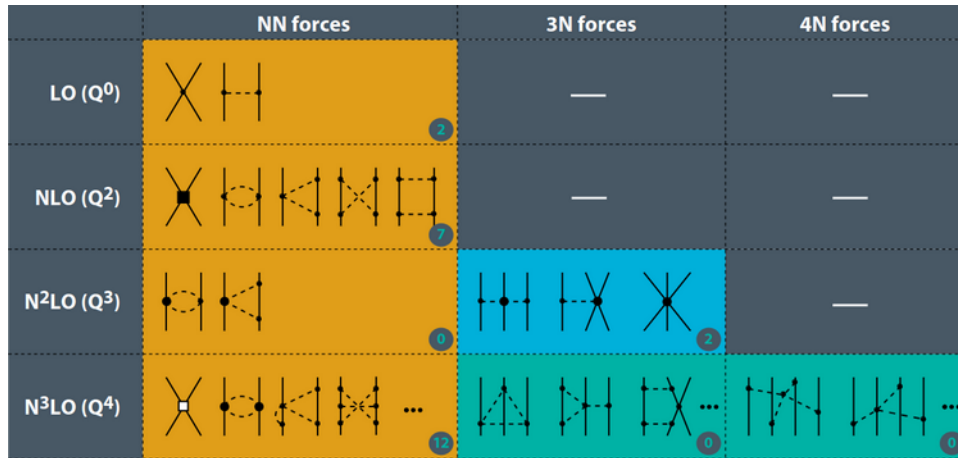


Hadronic matter: Chiral EFT

Describes **massive nucleons** interacting via pion exchange and contact interactions. EFT terms dictated by **chiral symmetry**

Calibrated by **nuclear data**

Uncertainty estimates by e.g. Gaussian process regression and naturalness arguments



Drischler, Holt, Wellenhofer, Ann.Rev.Nucl.Part.Sci. 71 (2021) 403-432
 Drischler, Furnstahl, Melendez, Phillips PRL 125 (2020) 20, 202702

Quark matter: perturbative QCD (+recent developments)

Describes (nearly) massless quarks, gluons interacting. Quarks are approximately free, up to [O(20%)] perturbative corrections

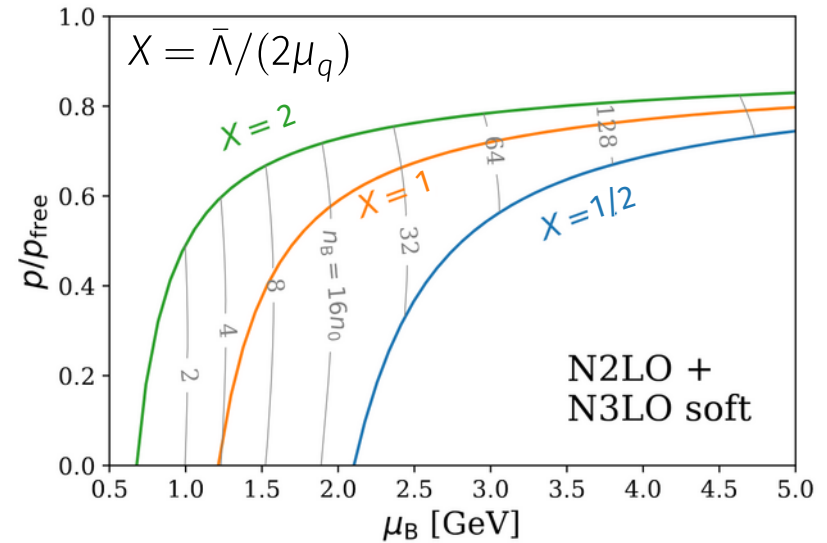
$$\frac{p}{p_0} = 1 + a_1\alpha_s(\bar{\Lambda}) + a_2\alpha_s^2(\bar{\Lambda}) + a_3\alpha_s^3(\bar{\Lambda}) + \dots$$

*free quark gas**

Freedman, McLerran, PRD 16 (1977)

TG, Kurkela, Romatschke, Säppi, Vuorinen, PRL 121 (2018) – ongoing

Calibrated by collider data.



TG, Kurkela, Paatelainen, Säppi, Vuorinen, PRL 127, 162003 (2021), PRD 104, 074015 (2021)

Quark matter: perturbative QCD (+recent developments)

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$$\frac{p}{p_0} = 1 + a_1\alpha_s(\bar{\Lambda}) + a_2\alpha_s^2(\bar{\Lambda}) + a_3\alpha_s^3(\bar{\Lambda}) + \dots$$

Free quark gas* (pointing to p/p_0)

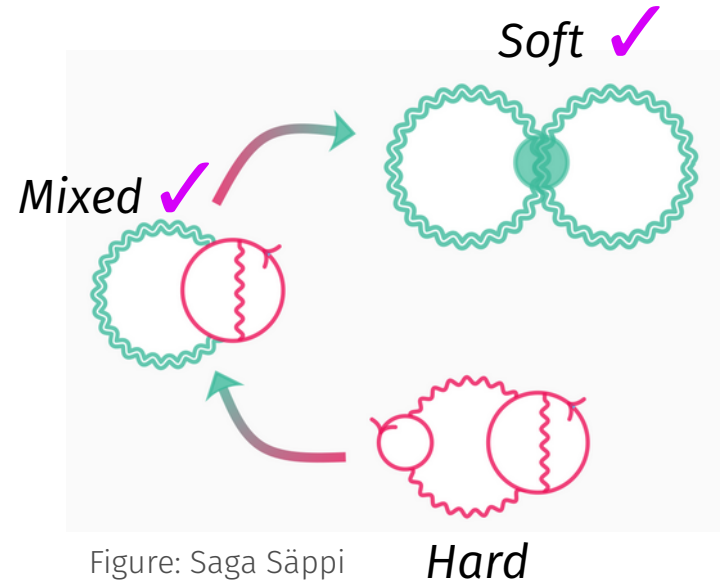
Freedman, McLerran, PRD 16 (1977) (pointing to $a_1\alpha_s(\bar{\Lambda})$)

TG, Kurkela, Romatschke, Säppi, Vuorinen, PRL 121 (2018) – ongoing (pointing to $a_2\alpha_s^2(\bar{\Lambda})$)

Calibrated by collider data.

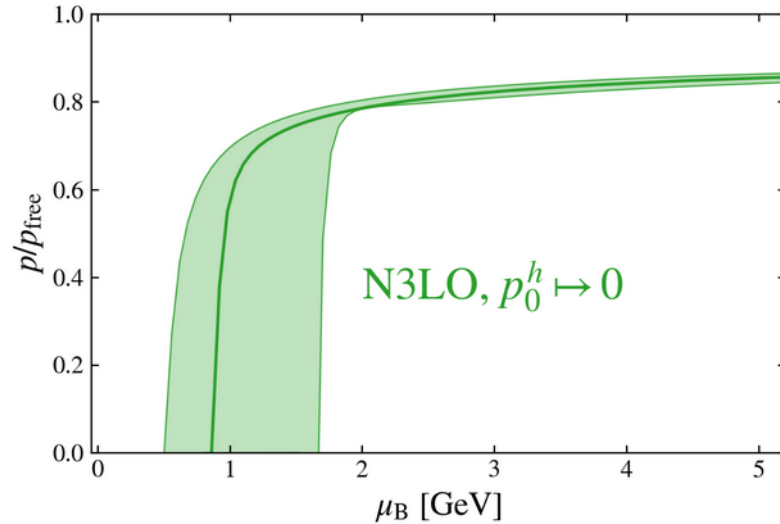
In last few years, the full structure of the N3LO pressure computation has been made clear

TG, Kurkela, Paatelainen, Säppi, Vuorinen, PRL 127, 162003 (2021), PRD 104, 074015 (2021), TG, Paatelainen, Säppi, Seppänen PRL 131 (2023)



Soft: 2 interacting gluons screened at LO
Mixed: 1 gluon screened at NLO
Hard: gluons are unscreened

Quark matter: perturbative QCD (recent developments)



TG, Paatelainen, Säppi, Seppänen PRL 131 (2023)

Soft and **Mixed** contributions now computed.
Result is extremely **well converged**

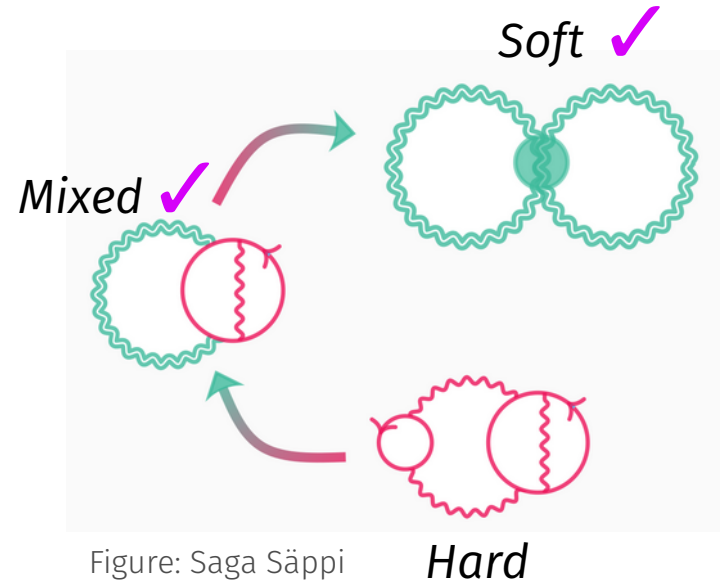


Figure: Saga Säppi

Soft: 2 interacting gluons screened at LO
Mixed: 1 gluon screened at NLO
Hard: gluons are unscreened

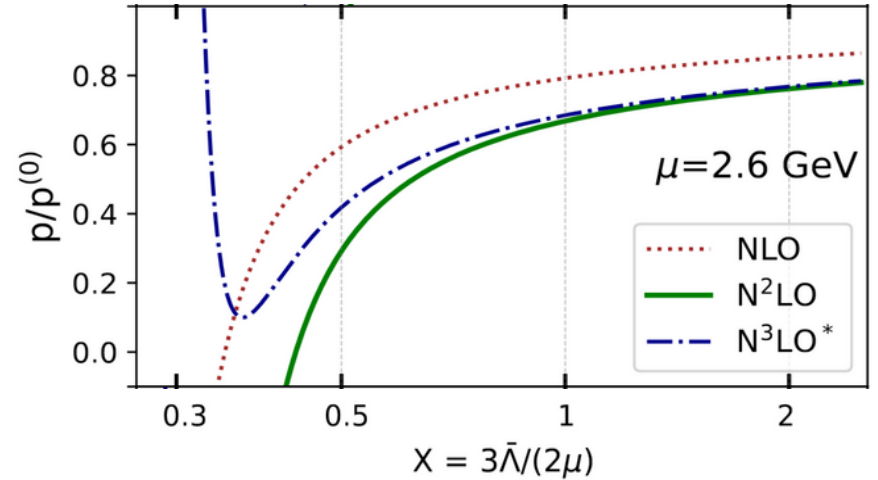
Quark matter: perturbative QCD (recent developments)

Recent **Bayesian Uncertainty estimation**

$$\frac{\rho}{\rho_0} = 1 + a_1\alpha_s(\bar{\Lambda}) + a_2\alpha_s^2(\bar{\Lambda}) + a_3\alpha_s^3(\bar{\Lambda}) + \dots$$

Scale-variation
uncertainty

Missing higher-order
(truncation) uncertainty



Quark matter: perturbative QCD (recent developments)

Recent **Bayesian Uncertainty estimation**

$$\frac{p}{p_0} = 1 + a_1\alpha_s(\bar{\Lambda}) + a_2\alpha_s^2(\bar{\Lambda}) + a_3\alpha_s^3(\bar{\Lambda}) + \dots$$

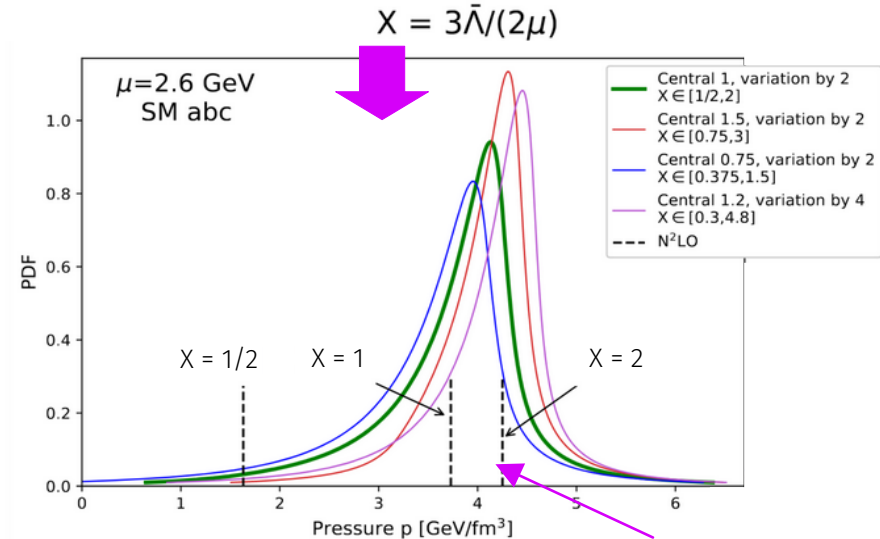
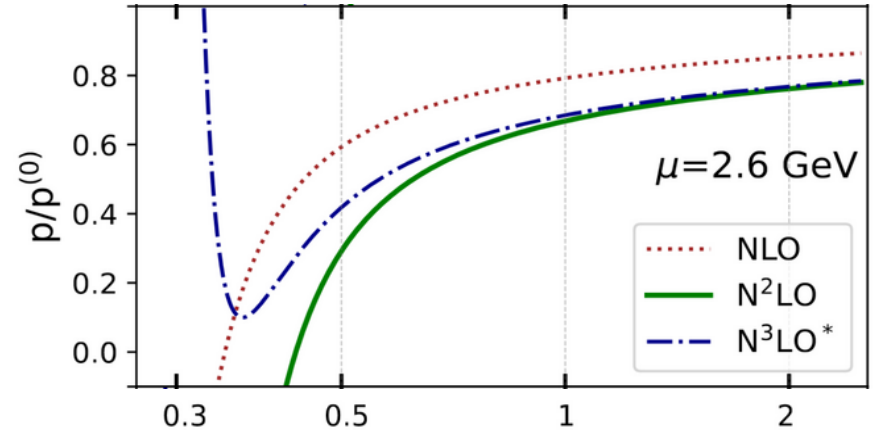
Scale-variation uncertainty

Missing higher-order (truncation) uncertainty

Machine-learning based Bayesian interpretation of these uncertainties. Perturbative series modeled as draws from a statistical model of convergent series, trained with available terms

Cacciari & Houdeau, JHEP 09, (2011), M. Bonvini, Eur. Phys. J. C 80, 989 (2020), Duhr, Huss, Mazeliauskas, Szafron, JHEP 122, (2021)

TG, Komoltsev, Kurkela, Mazeliauskas, JHEP 06 (2023)



Favors **better converged X**

Chiral EFT + pQCD + Thermodynamics constrain extreme EoSs

1. Stability

$$\partial_\mu^2 \Omega(\mu) \leq 0 \implies \partial_\mu n(\mu) \geq 0$$

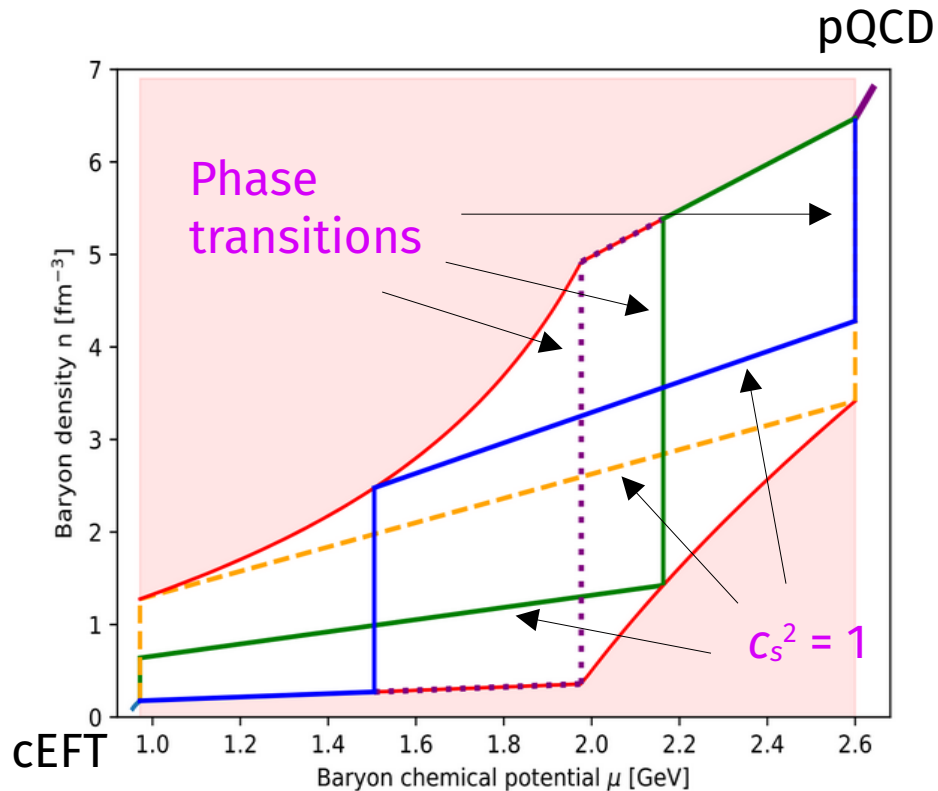
2. Causality

$$c_s^{-2} = \frac{\mu}{n} \frac{\partial n}{\partial \mu} \geq 1 \implies \partial_\mu n(\mu) \geq \frac{n}{\mu}$$

3. Consistency

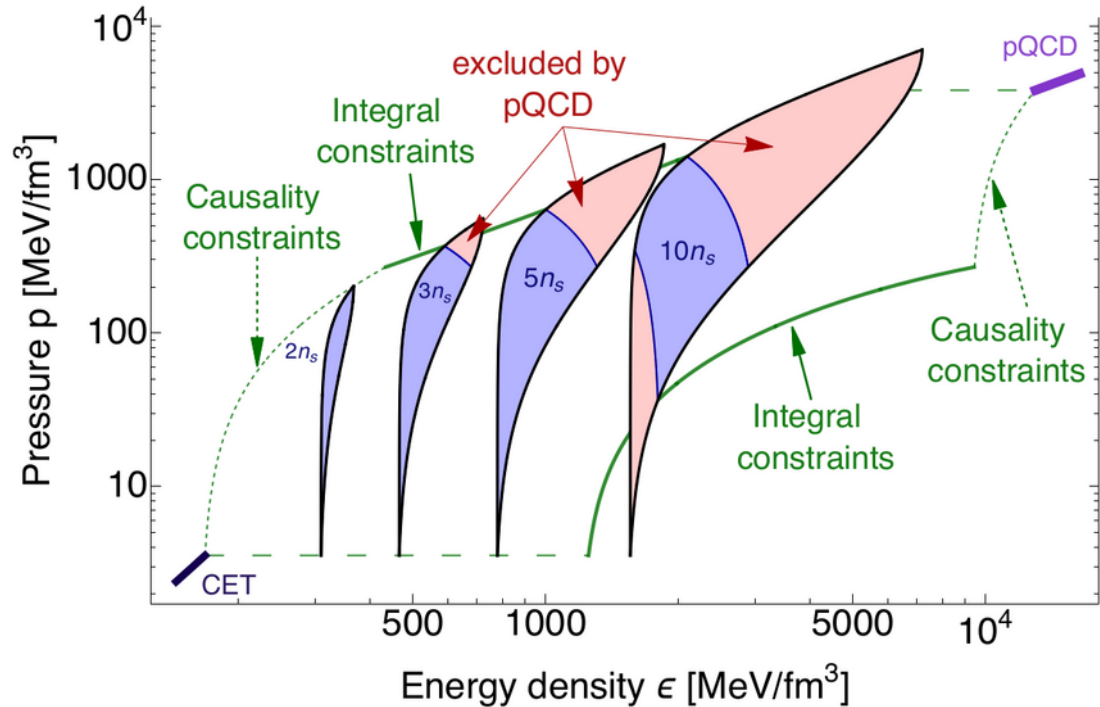
$$\int_{\mu_{\text{CET}}}^{\mu_{\text{QCD}}} d\mu n(\mu) = p_{\text{QCD}} - p_{\text{CET}} \quad \text{Fixed!}$$

“integral constraints”



Komoltsev and Kurkela, PRL 128 (2022)

Chiral EFT + pQCD + Thermodynamics constrain extreme EoSs

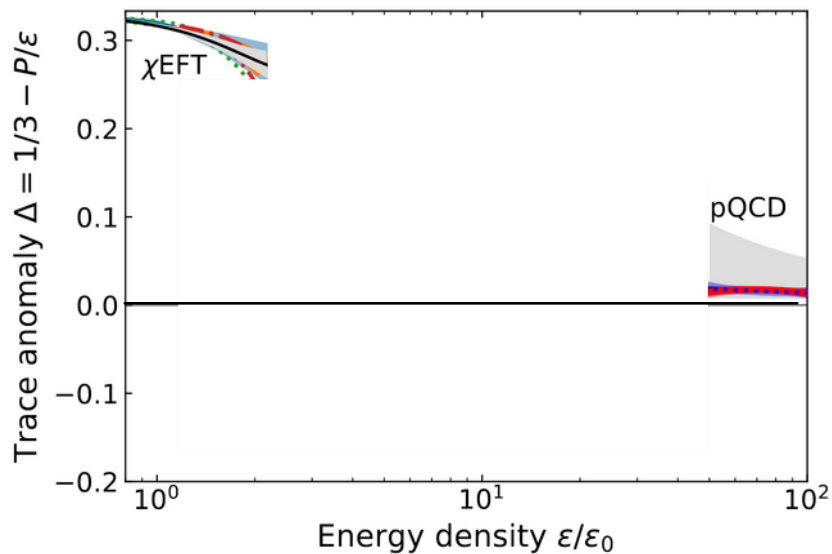


Region of (ϵ, p) at fixed n constrained by **general principles**

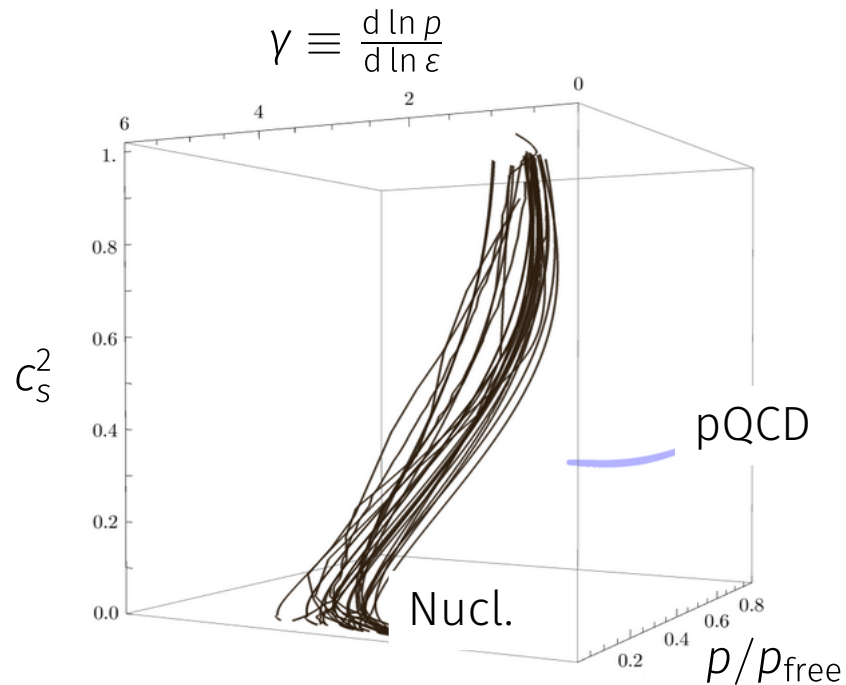
Komoltsev and Kurkela, PRL 128 (2022)

Hadronic and Quark matter are different

Quark matter is **approximately conformal (scale-invariant)**, hadronic matter is **non-conformal**. This leads to different thermodynamics:



ADAPTED from Fujimoto, Fukushima, McLerran, Praszalowicz, PRL 129 (2022) 25, 252702

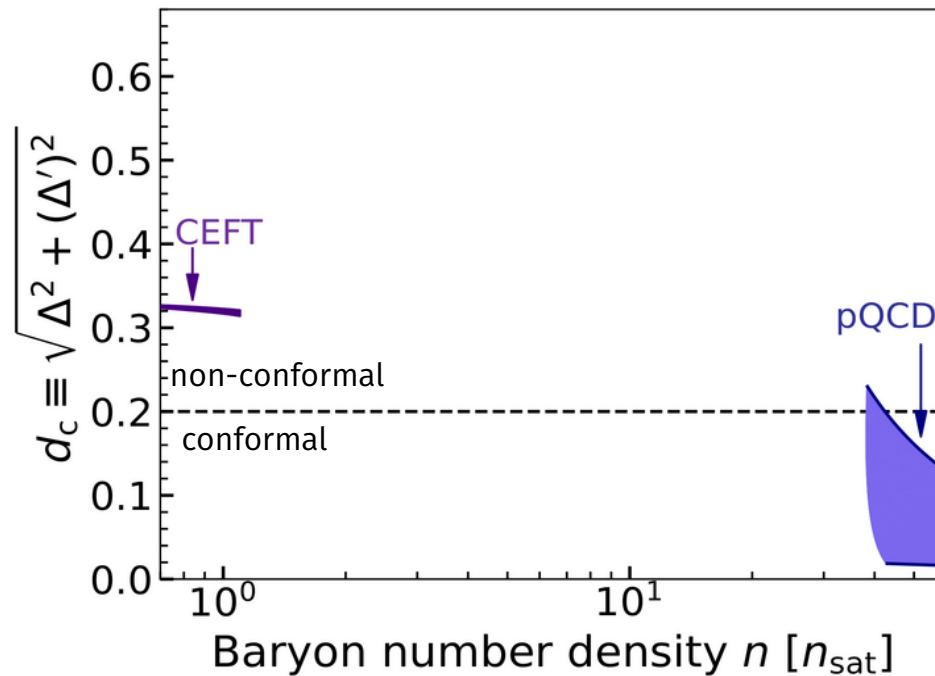


Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020)

A new measure of conformality

Δ and γ are both fixed for a conformal EoS, but for a non-conformal EOS they **can still briefly pass through** their conformal values.

Combine **trace anomaly** Δ and **its rate of change** $\Delta' \equiv d \ln \Delta / d \ln \varepsilon$, so not purely local measure



Average of chiral EFT and pQCD as dividing line

Annala, TG, Hirvonen, Komoltsev, Kurkela, Nättilä, Vuorinen Nat. Comm. 14 (2023)



Putting everything together...

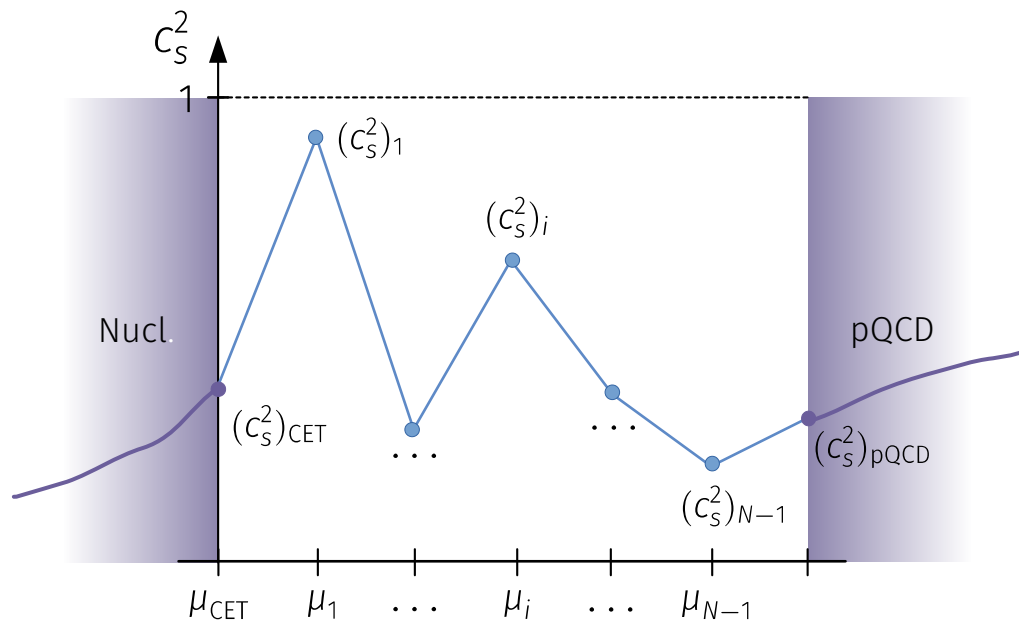
Bayesian EoS inference setup

Posterior

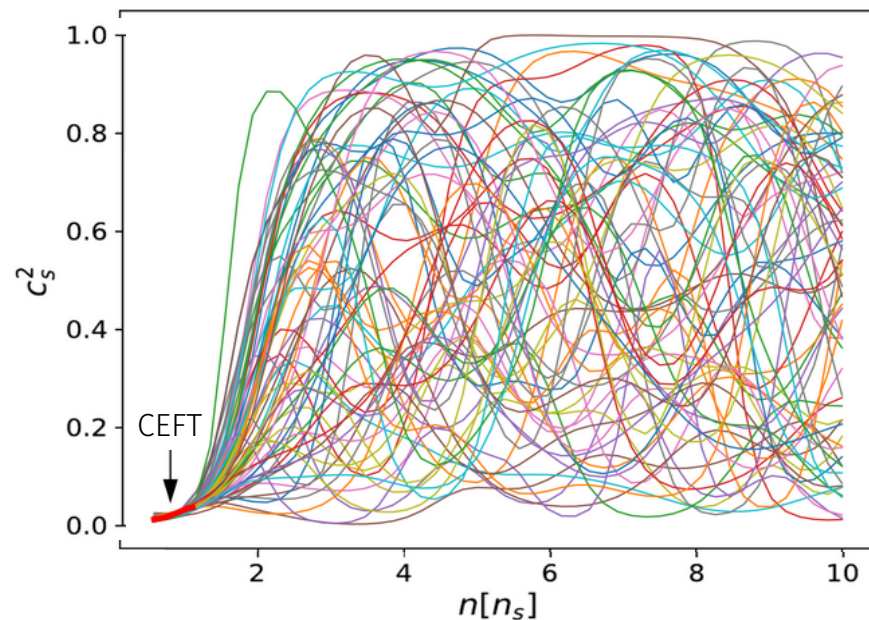
Prior

Likelihood

$$P(\text{EoS} | \text{data}) = \frac{P(\text{EoS})P(\text{data} | \text{EoS})}{P(\text{data})}$$



Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020)



TG, Komoltsev, Kurkela, ApJ 950 (2023)

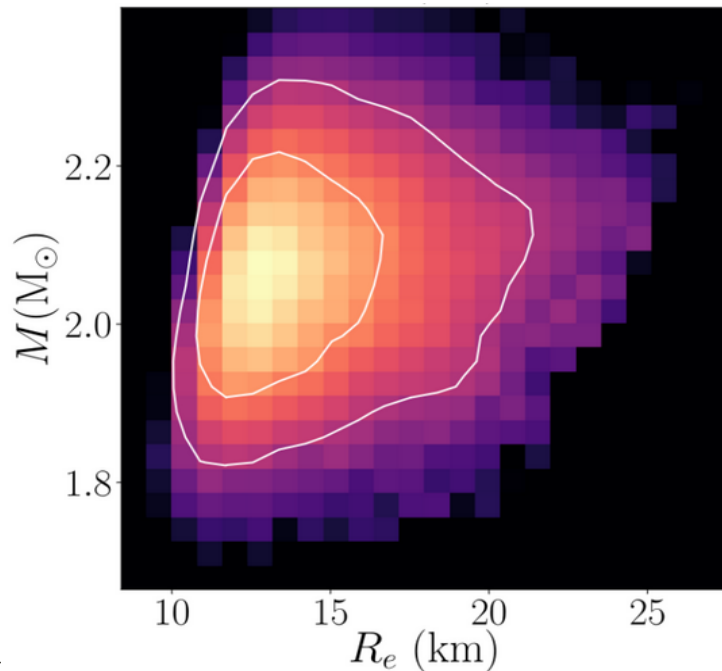
Bayesian EoS inference setup

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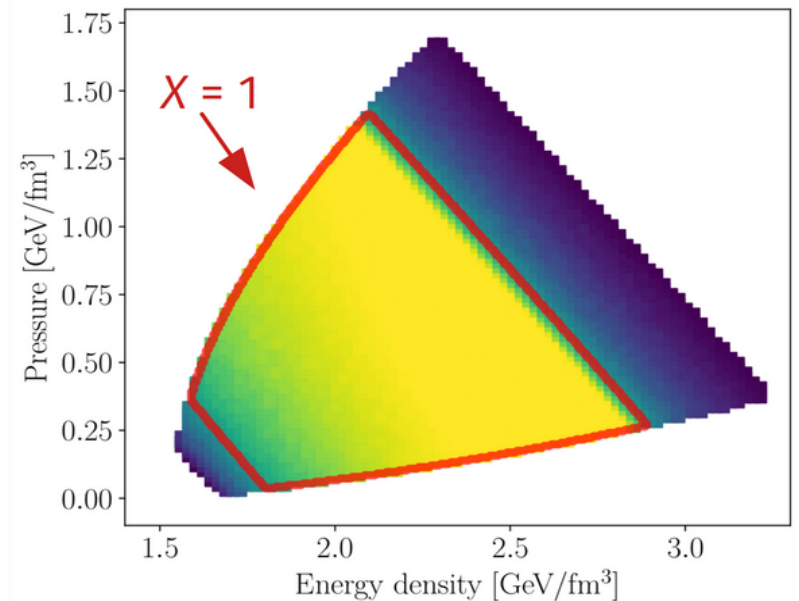
Prior

Likelihood

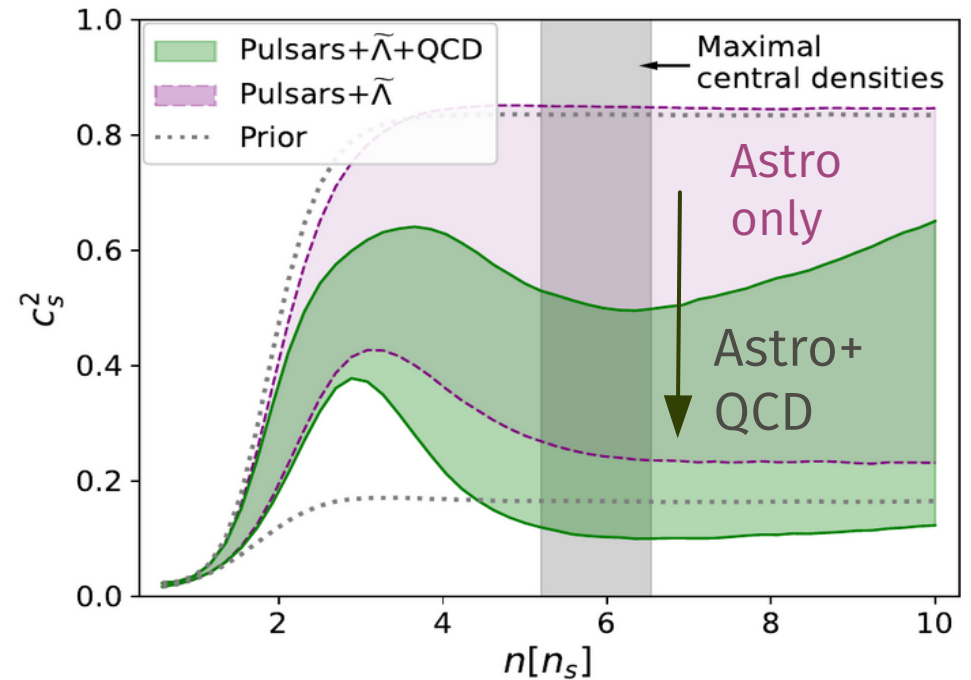
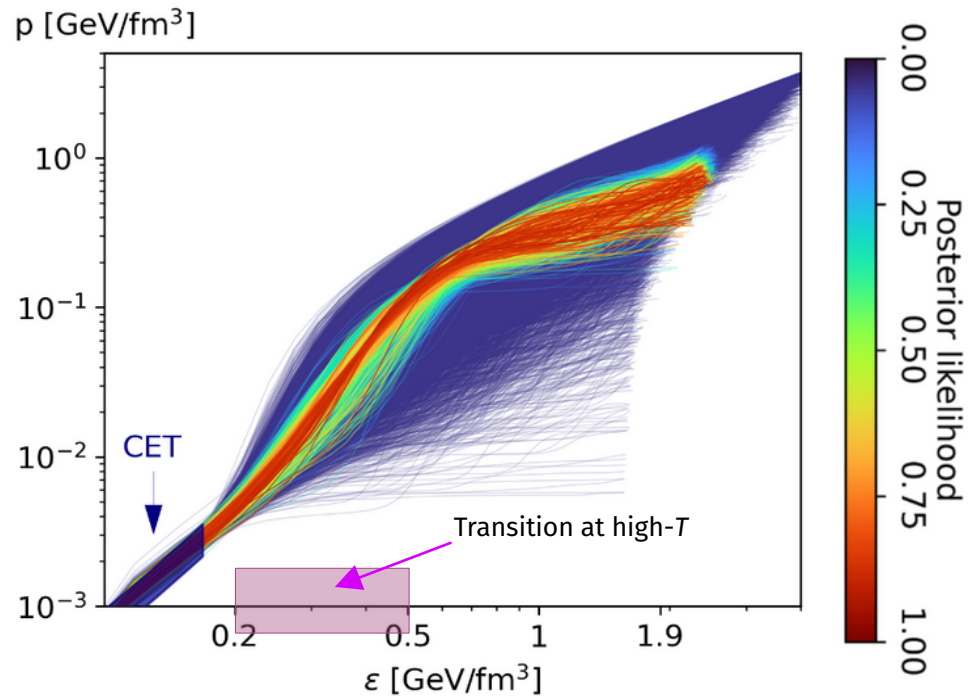
$$P(\text{EoS} \mid \text{data}) = \frac{P(\text{EoS})P(\text{data} \mid \text{EoS})}{P(\text{data})}$$



Weight of (ϵ, p) points at $n = 10n_s$



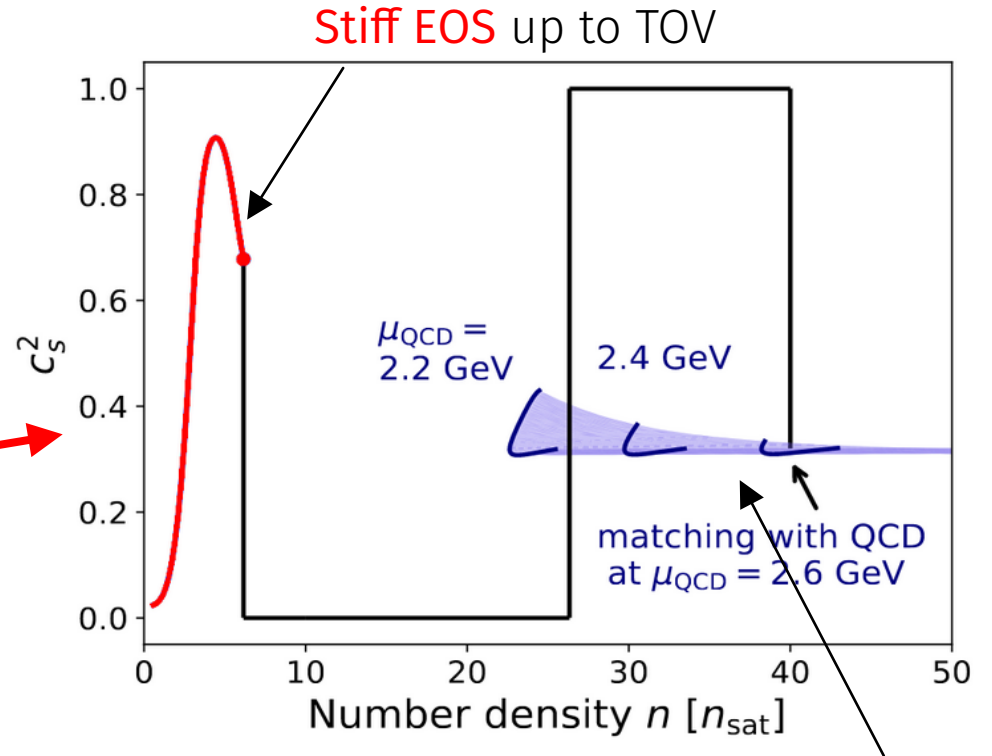
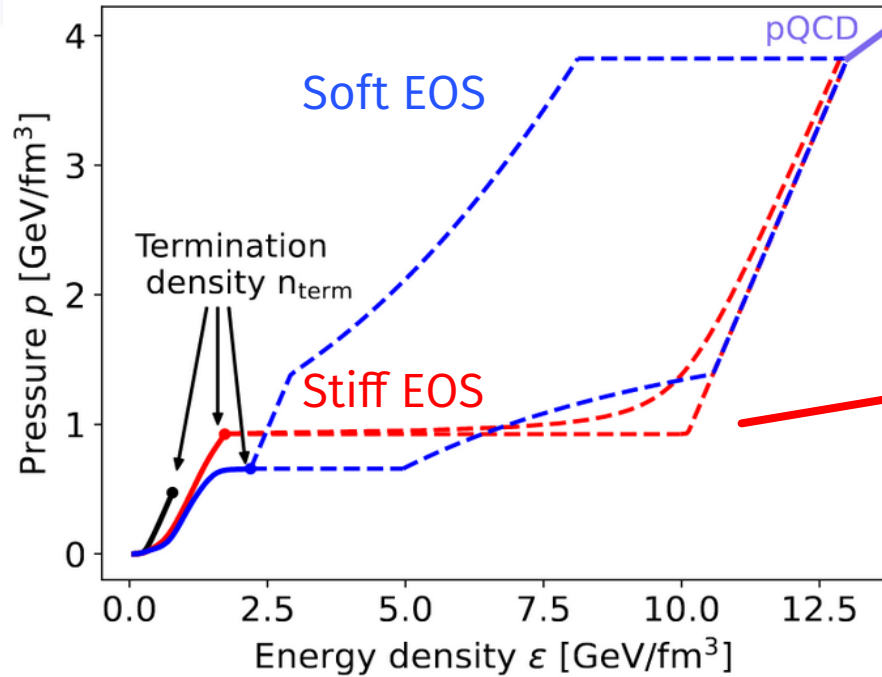
Stiff \rightarrow soft transition, with softening driven by QCD input



TG, Komoltsev, Kurkela, ApJ 950 (2023)

See also
 Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020),
 Altiparmak, Ecker, Rezzolla, ApJ.Lett. 939 (2022);
 Ecker & Rezzolla, ApJ.Lett. 939 (2022); Fujimoto, Fukushima, McLerran,
 Praszalowicz, PRL 129 (2022); Marczenko, McLerran, Redlich, Sasaki, PRC
 107 (2023);

Stiff EOSs at TOV inconsistent with pQCD at higher densities



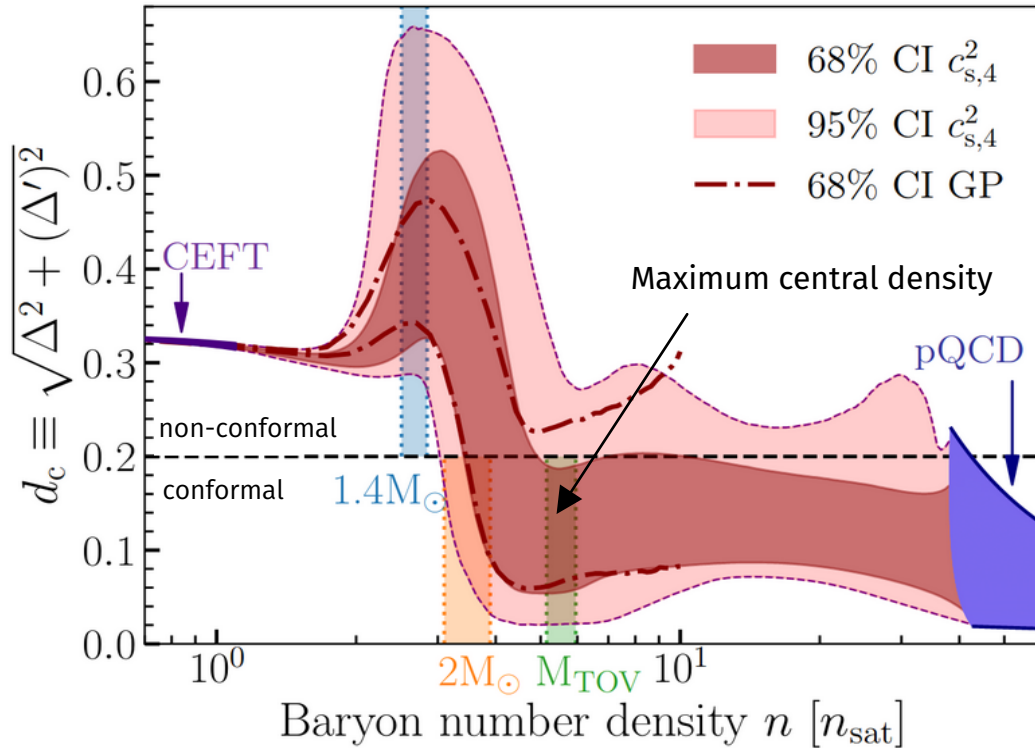
More informed QCD input yields robust posteriors

TG, Komaltsev, Kurkela, Margueron, Somasundaram, Tews PRD 109 (2024);

Public script: 10.5281/zenodo.10412734

Inconsistent with
pQCD at large densities

Clear non-conformal \rightarrow conformal transition within the cores of stable neutron stars



$P(\text{conformal}) = 88\%$ (75%) for the parametric (GP) approach, for maximally massive stars

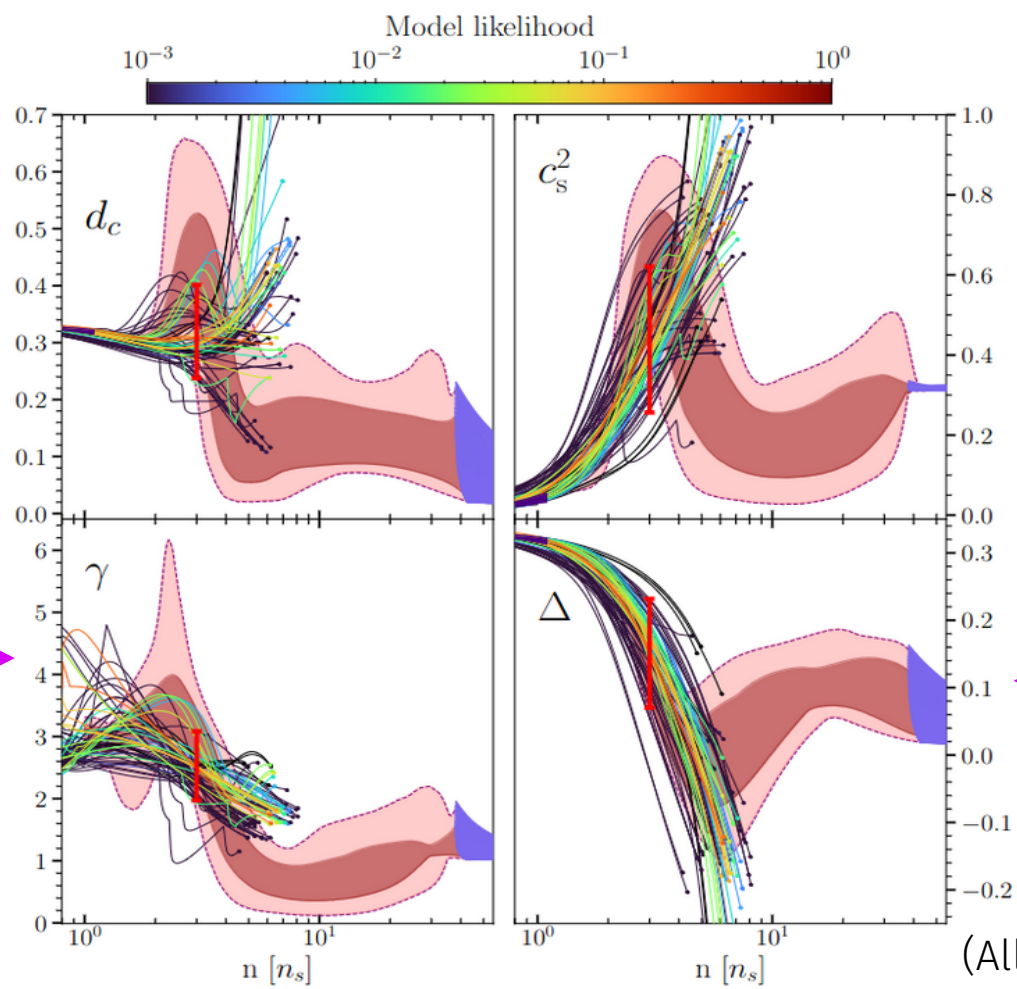
Criterion is much stricter than older γ analysis. (Would have found 99.8% previously.)

See also: Han, Huang, Tang, Fan, Science Bulletin, 68, 9 (2023)

See also
 Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020),
 Altiparmak, Ecker, Rezzolla, ApJ.Lett. 939 (2022);
 Ecker & Rezzolla, ApJ.Lett. 939 (2022); Fujimoto, Fukushima,
 McLerran, Praszalowicz, PRL 129 (2022); Marczenko, McLerran,
 Redlich, Sasaki, PRC 107 (2023);

Annala, TG, Hirvonen, Komoltsev, Kurkela, Nättilä, Vuorinen Nat. Comm. 14 (2023)

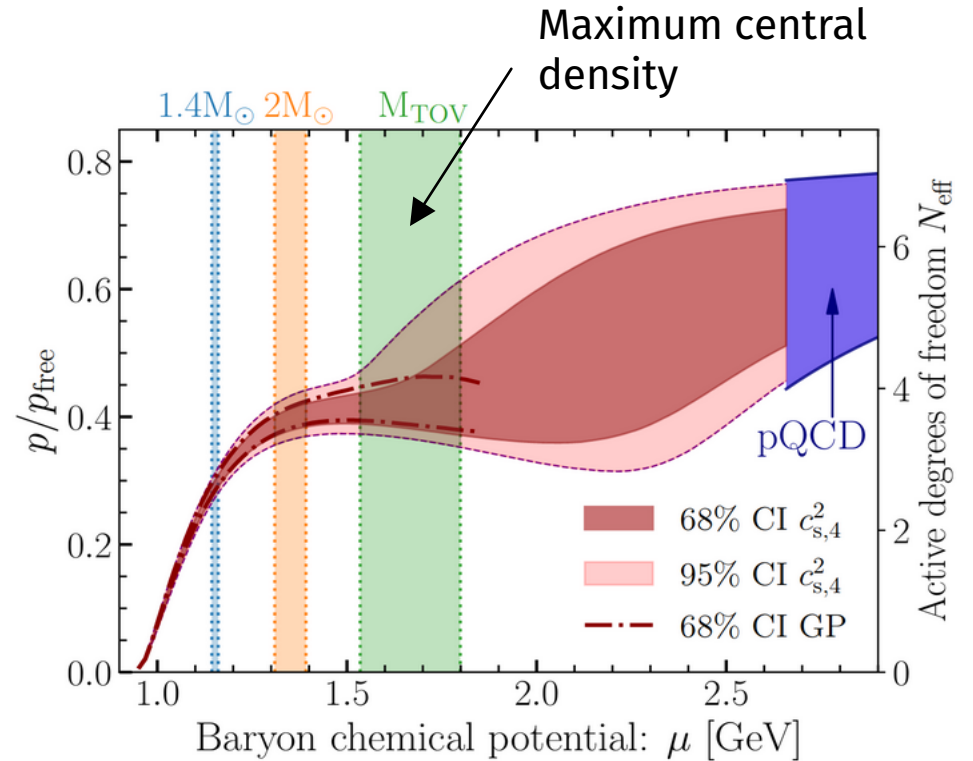
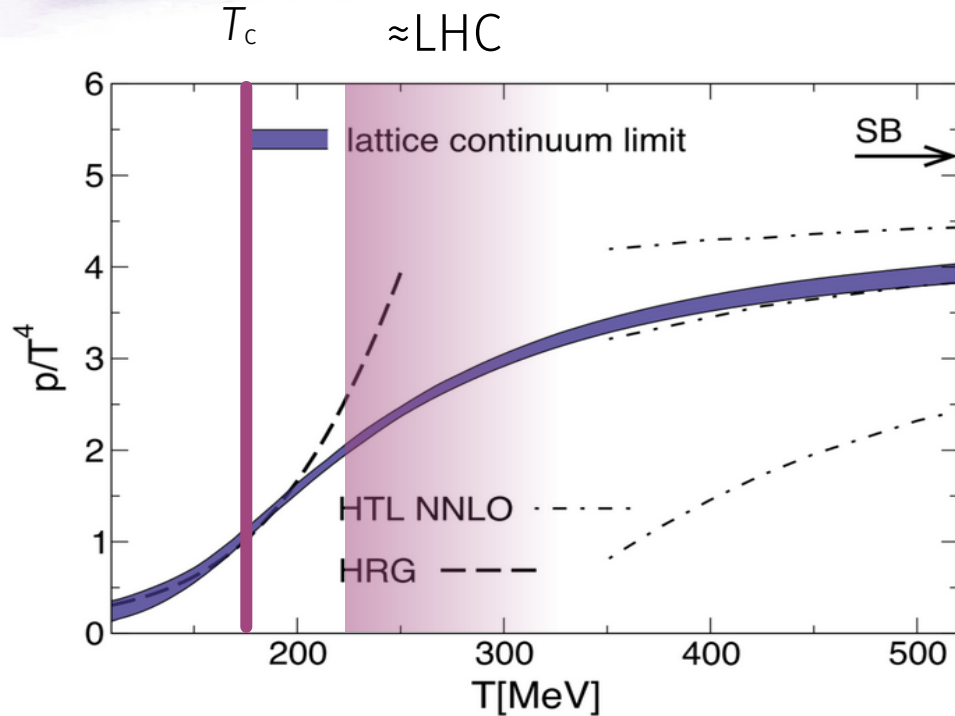
Nuclear models don't show conformal transition



Δ , γ analyses alone not so clear!

(All $T = 0$, beta-equilibrated hadronic models from CompOSE)

Number of degrees of freedom consistent with deconfined quark matter



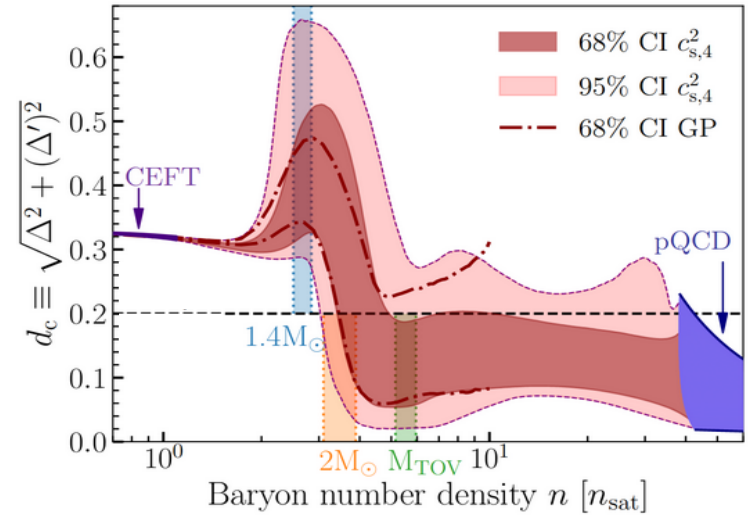
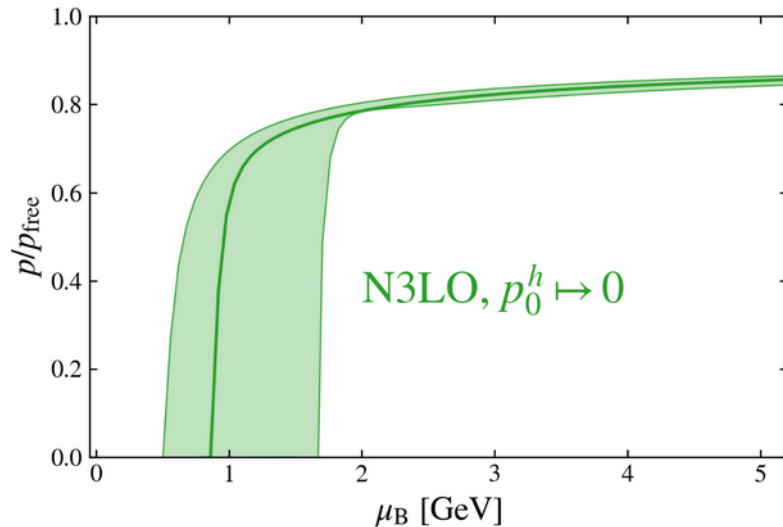
Annala, TG, Hirvonen, Komoltsev, Kurkela, Nättilä, Vuorinen
 Nat. Comm. 14 (2023)
 Annala, TG, Kurkela, Nättilä, Vuorinen Nat. Phys. 16 (2020)

Takeaways

- ✓ Robust thermodynamic evidence for **hadronic→quark transition probed within the most massive neutron stars.**

Evidence for cold quark matter created in nature.

- ✓ **Astrophysics + nuclear-theory + particle-theory input** are all essential in this conclusion



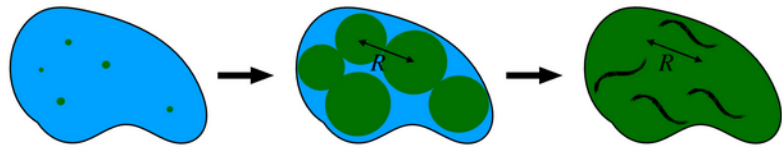
- ✓ Improvements in pQCD calculations, analysis, and input in EOS inference
 - Statistical analysis of pQCD truncation errors favors **higher pressures**, which are **better converged**
 - **Soft + mixed** pQCD EOS tightly converged; hard sector remains
 - Using **additional pQCD $c_{s,4}^2$ information** yields **robust EOS posteriors**

What's next?

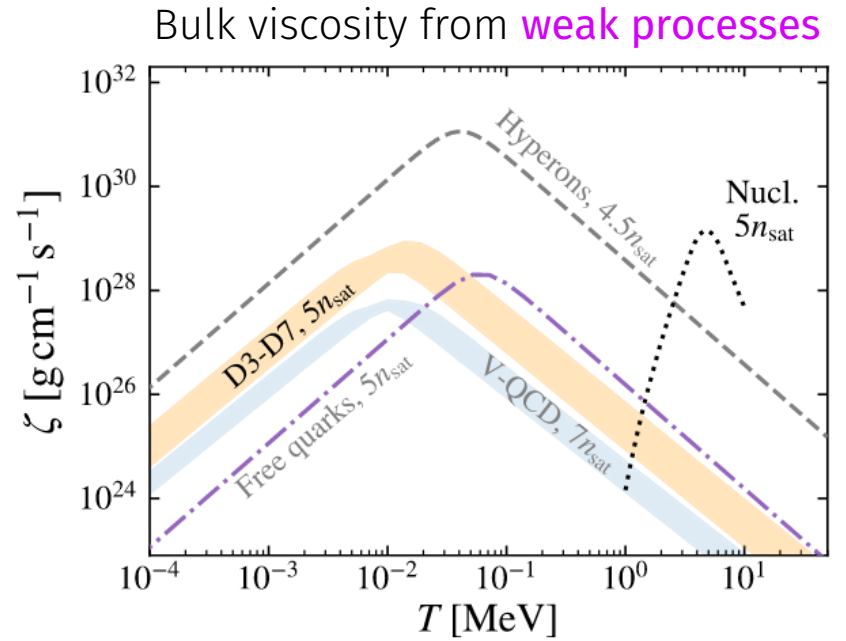
? Other lines of evidence of deconfined behavior?

- At high- T , thermodynamics is just one approach
- Recently improved transport calculations

Cruz Rojas, TG, Hoyos, Jokela, Järvinen, Kurkela, Paatelainen, Säppi, Vuorinen PRL 133 (2024); Hernandez, Manuel, Tolos PRD 109 (2024)



Casalderrey-Solana, Mateos, Sanchez-Garitaonandia 2210.03171



? Robust (post-)merger signals of deconfinement?

- MHz gravitation waves from bubble nucleation in remnant?

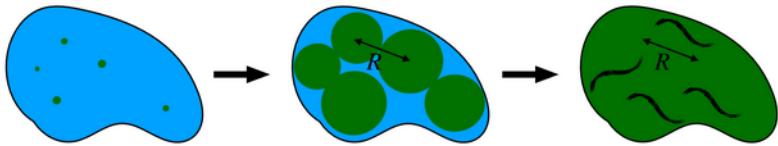
? Can similar analyses be performed at nonzero T , to apply statistical EoS inference throughout the phase diagram?

What's next?

? Other lines of evidence of deconfined behavior?

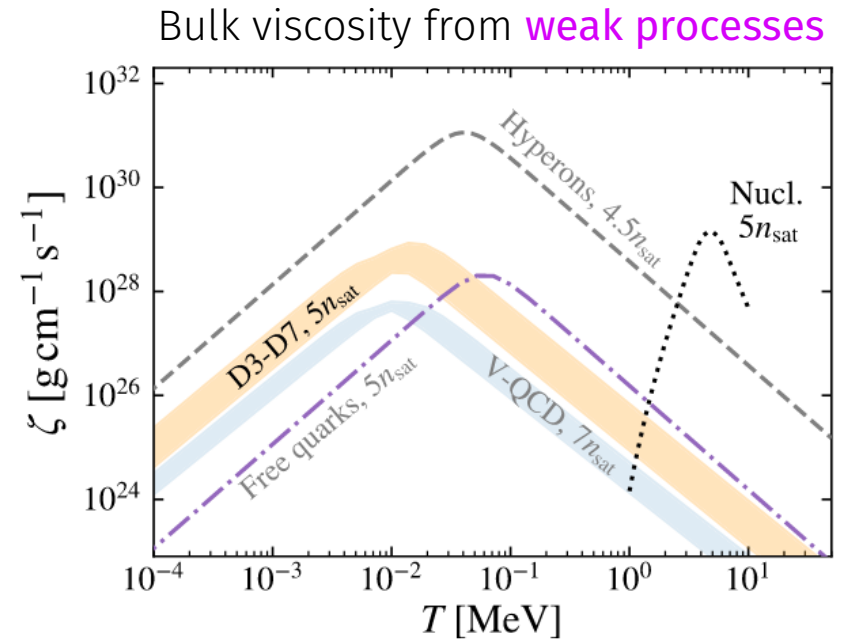
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Casalderrey-Solana, Mateos, Sanchez-Garitaonandia 2210.03171

? Can **similar analyses** be performed at **nonzero T** , to apply statistical EoS inference throughout the phase diagram?



? Robust **(post-)merger signals** of deconfinement?

- MHz gravitation waves from bubble nucleation in remnant?

Thanks for your attention!



Here there be details...

Robust to interpolants

Piecewise speed of sound

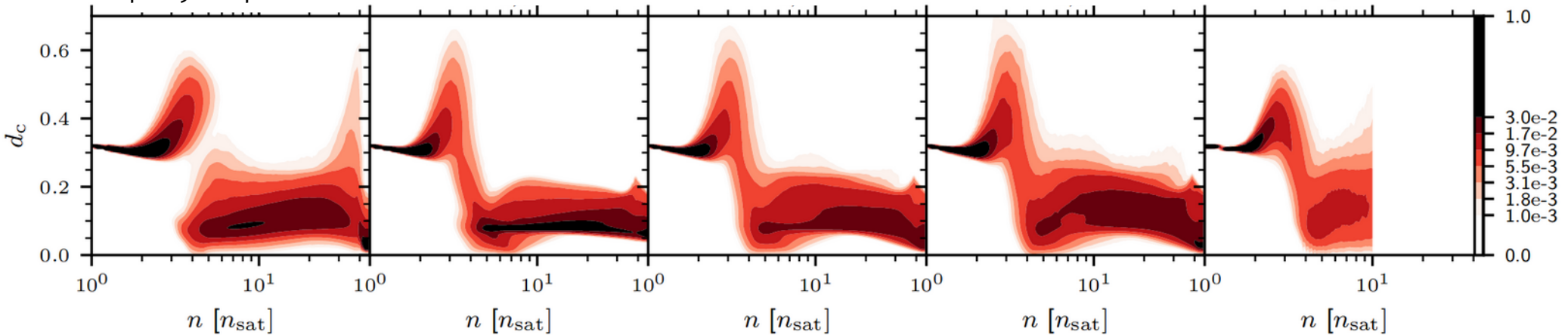
Piecewise
polytropes

3-segments

4-segments

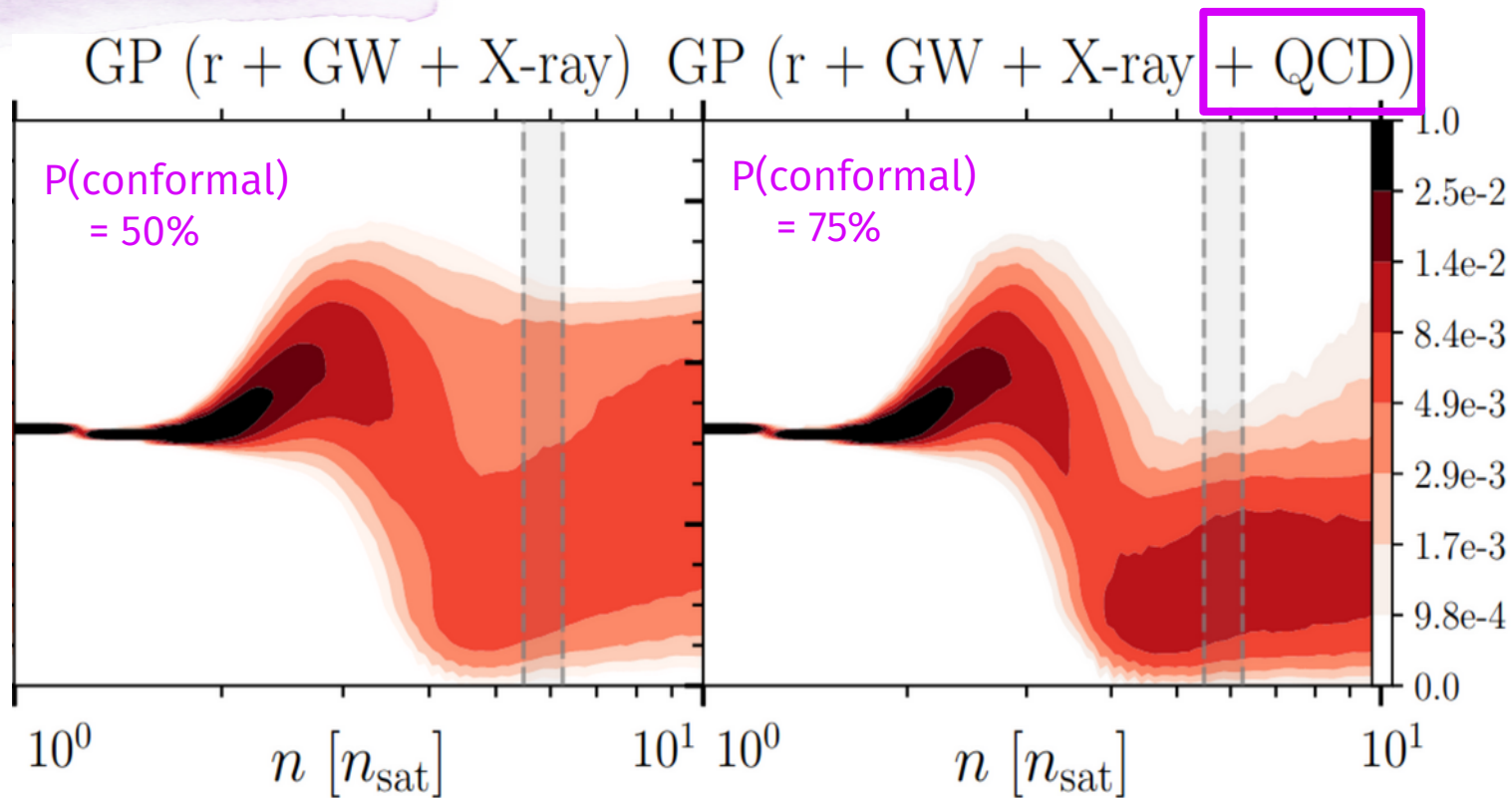
5-segments

GP



Annala, TG, Hirvonen, Komoltsev, Kurkela, Nättilä, Vuorinen
Nat. Comm. 14 (2023)

Effect from combined X-ray, QCD input



Annala, TG, Hirvonen, Komoltsev, Kurkela, Nätilä, Vuorinen
Nat. Comm. 14 (2023)

The *abc* model of convergent series

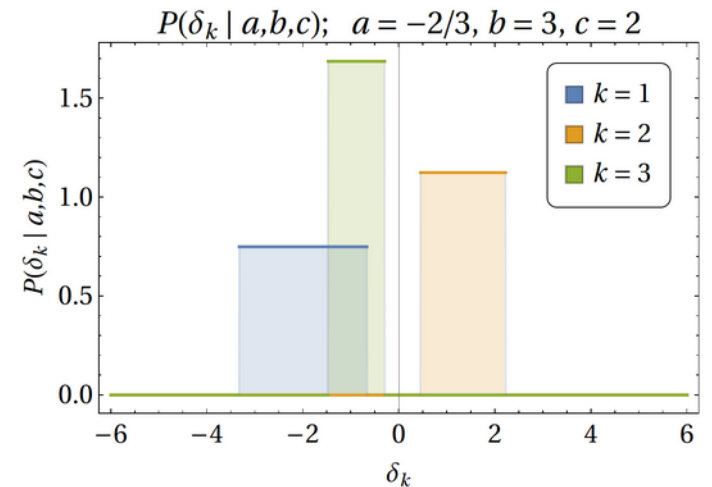
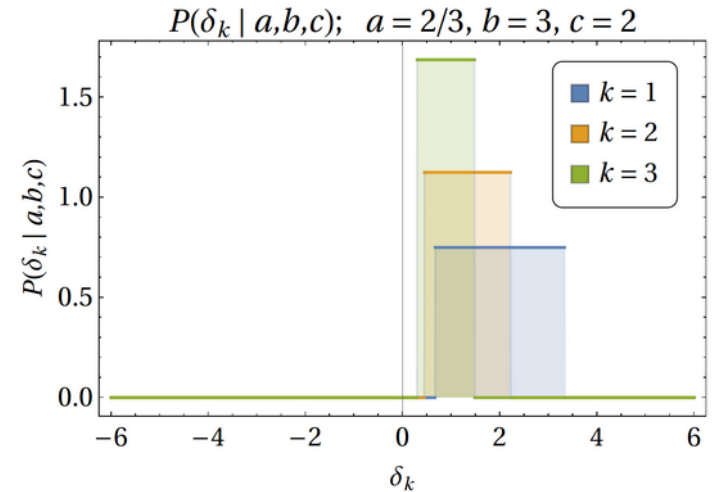
Normalize sequence by LO term

$$\delta_k \equiv \frac{\alpha^k O_k}{O_0} \quad (\delta_0 = 1)$$

Model assumes δ_k is bounded by some geometric series defined by (a, b, c)

$$(-c + b)a^k < |\delta_k| < (c + b)a^k,$$

Flat likelihoods taken for δ_k satisfying this inequality. (*Model also specifies a prior for a, b, c which favor smaller values of $|a|, b, c$)



The HARD diagrams

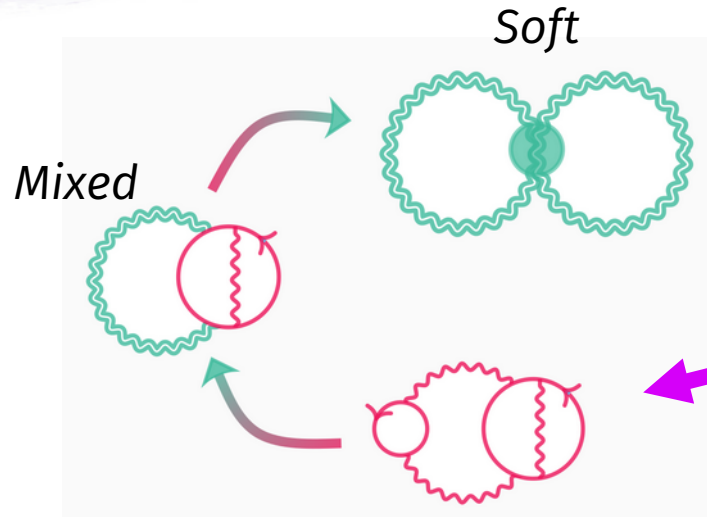
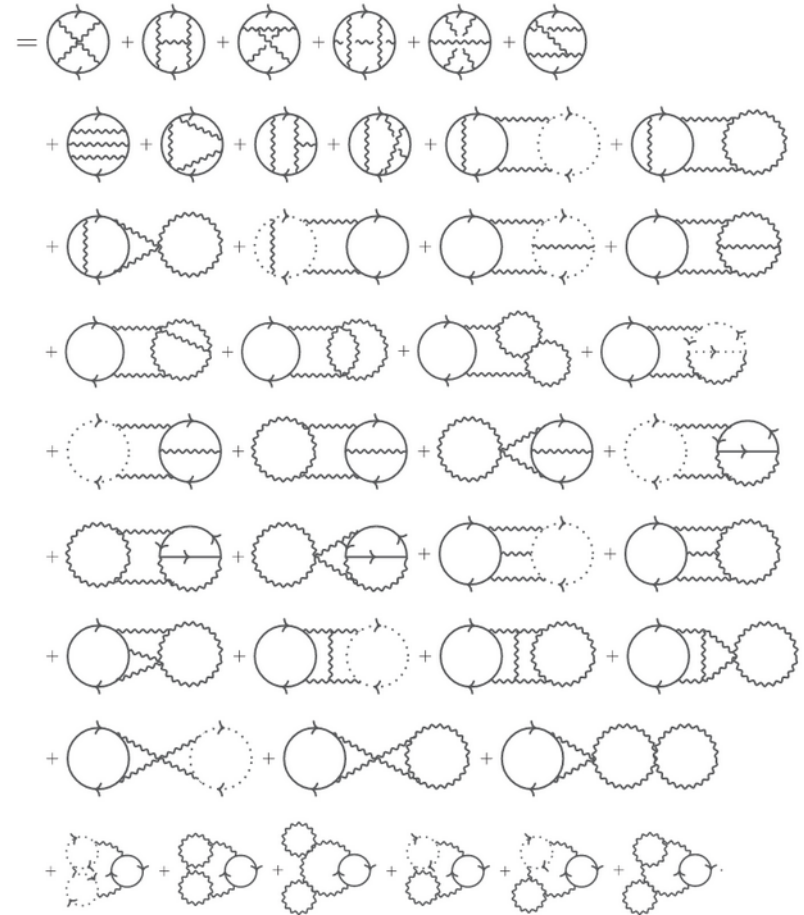


Figure: Saga Säppi

Soft: 2 interacting gluons screened at LO
Mixed: 1 gluon screened at NLO
Hard: gluons are unscreened



Gaussian Process regression priors

- Follow Landry & Essick Phys. Rev. D 99 (2019) and implement a **Gaussian Process Regression** in an auxiliary variable $\varphi(n) = -\ln(c_s^{-2}(n) - 1)$, **but as function of n**
- Use hierarchical model, for wide range of behavior

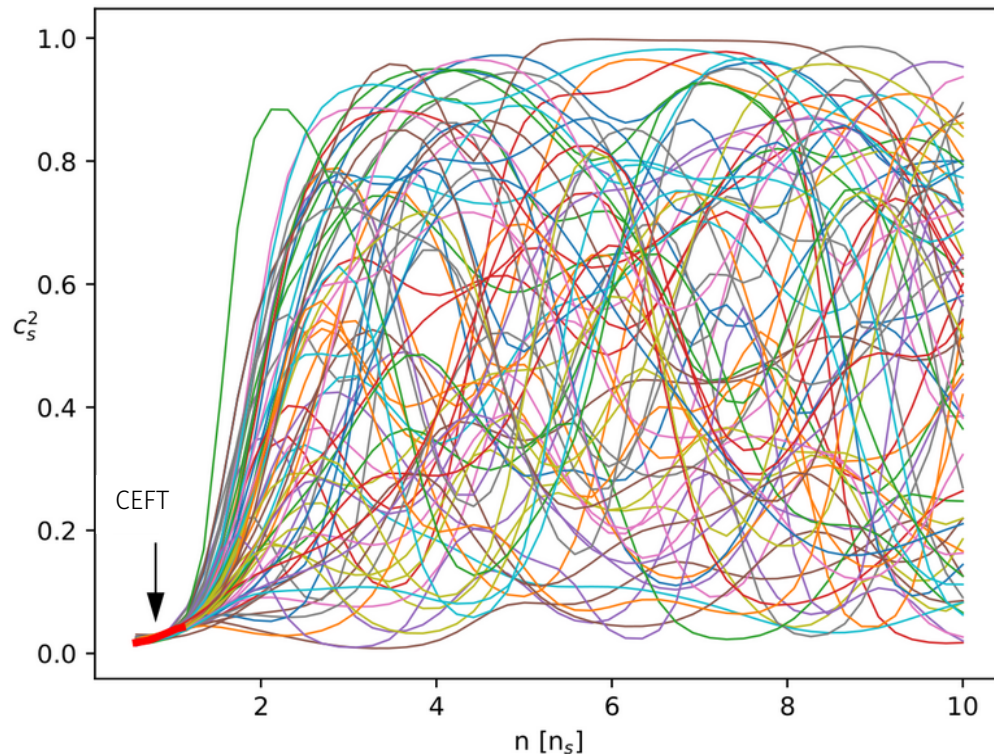
$$\varphi(n) \sim \mathcal{N}\left(-\ln(\bar{c}_s^{-2} - 1), K(n, n')\right),$$

$$K(n, n') = \eta e^{-(n-n')^2/2\ell^2}$$

$$\bar{c}_s^2 \sim \mathcal{N}(0.5, 0.25^2),$$

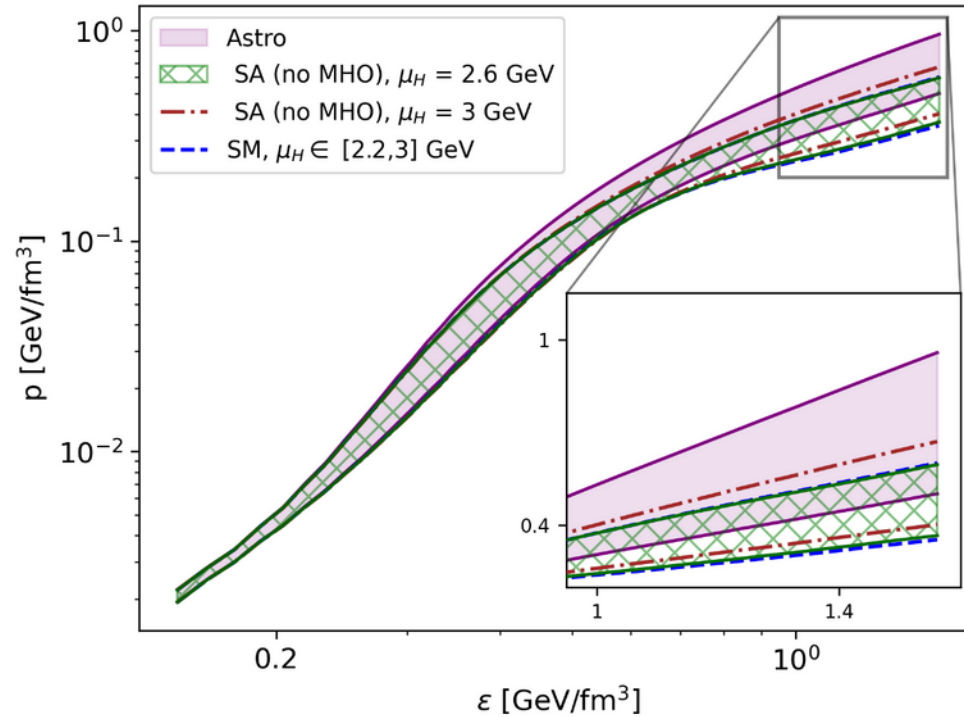
$$\ell \sim \mathcal{N}(1.0n_s, (0.2n_s)^2),$$

$$\eta \sim \mathcal{N}(1.25, 0.25^2).$$



TG, Komoltsev, Kurkela, ApJ 950 (2023)
<https://github.com/OKomoltsev/QCD-likelihood-function>

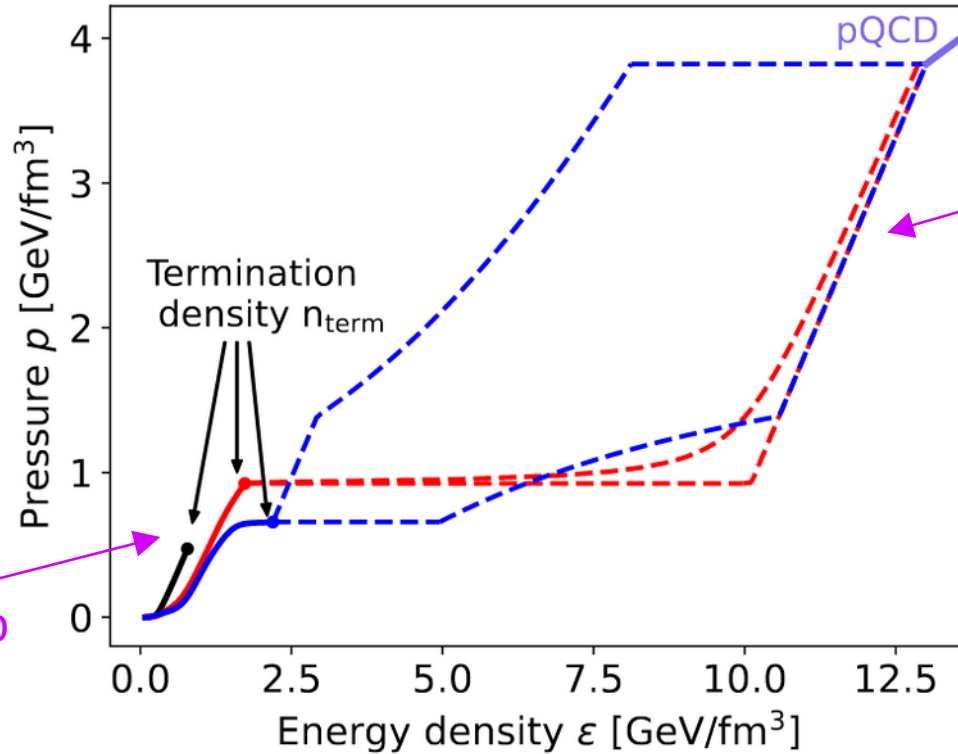
Details of statistical pQCD treatment don't affect NS inference



TG, Komoltsev, Kurkela, Mazeliauskas JHEP 06 (2023)

Quantifying the *tension* with perturbative QCD

Komoltsev, Somasundaram, TG, Kurkela, Margueron, Tews PRD 109 (2024)

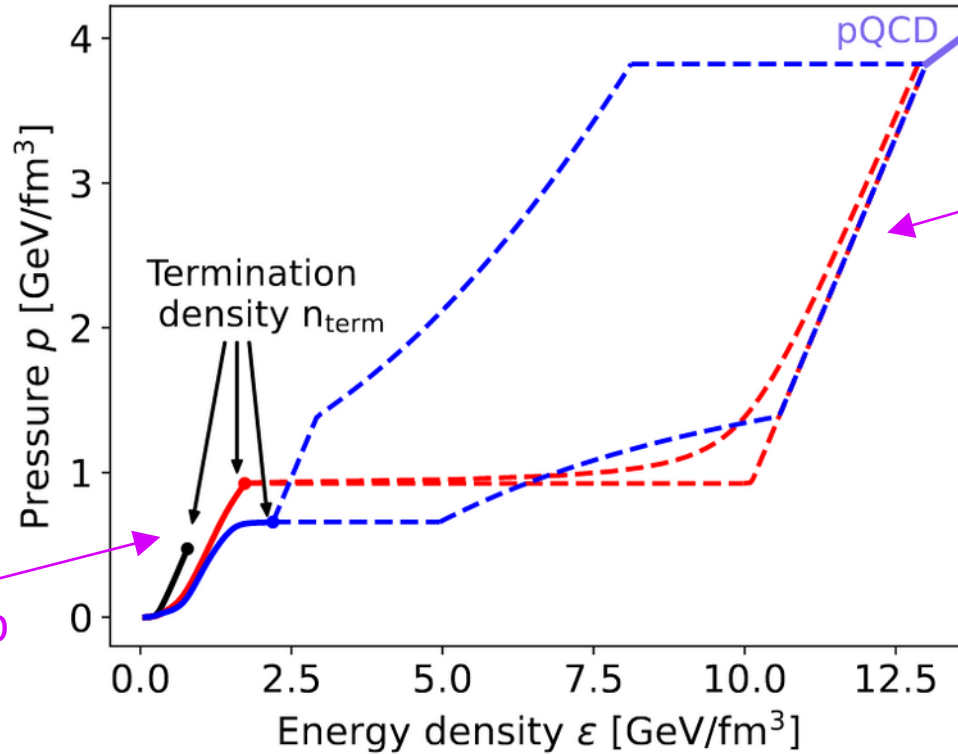


EoS modeled up to n_{term} using:

1. Gaussian process for c_s^2 TG, Komoltsev, Kurkela, ApJ 950 (2023),
2. Piecewise linear c_s^2 Somasundaram, Tews, Margueron PRC 107, L052801 (2023).

Quantifying the *tension* with perturbative QCD

Komoltsev, Somasundaram, TG, Kurkela, Margueron, Tews PRD 109 (2024)



Thermodynamics restricts EoS behavior beyond n_{term}

Introduce **pQCD tension index** I_{pQCD} to quantify how close to excluded a point is:

$$0 \leq I_{\text{pQCD}} \equiv \frac{\Delta p - \Delta p_{\text{min}}}{\Delta p_{\text{max}} - \Delta p_{\text{min}}} \leq 1$$

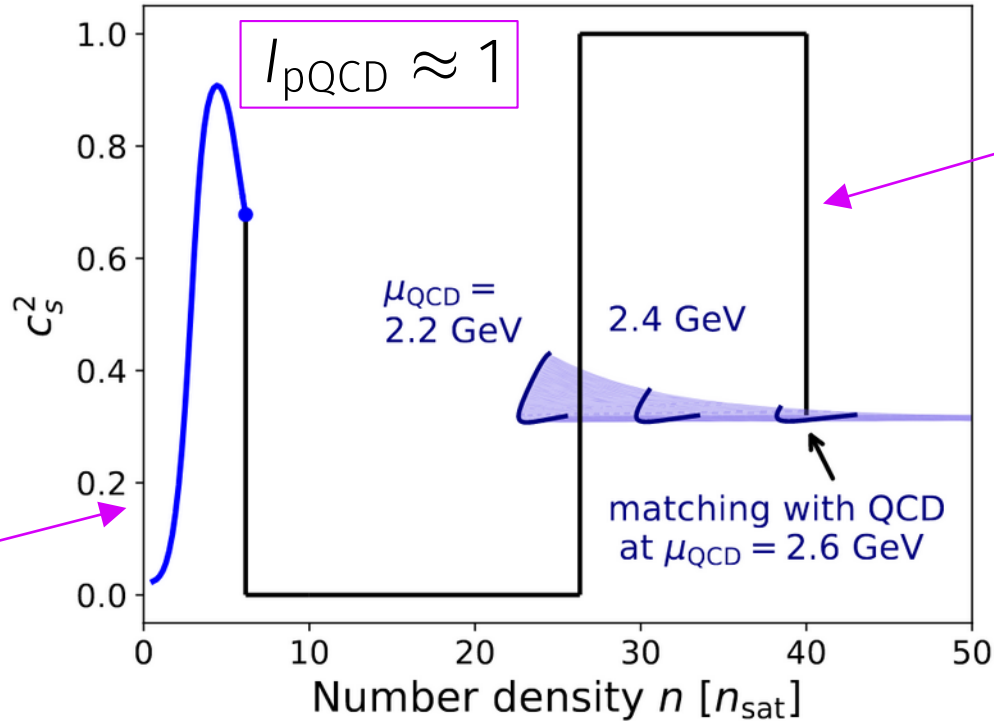
(stiffer EOSs have larger I_{pQCD})

EoS modeled up to n_{term} using:

1. Gaussian process for c_s^2 TG, Komoltsev, Kurkela, ApJ 950 (2023),
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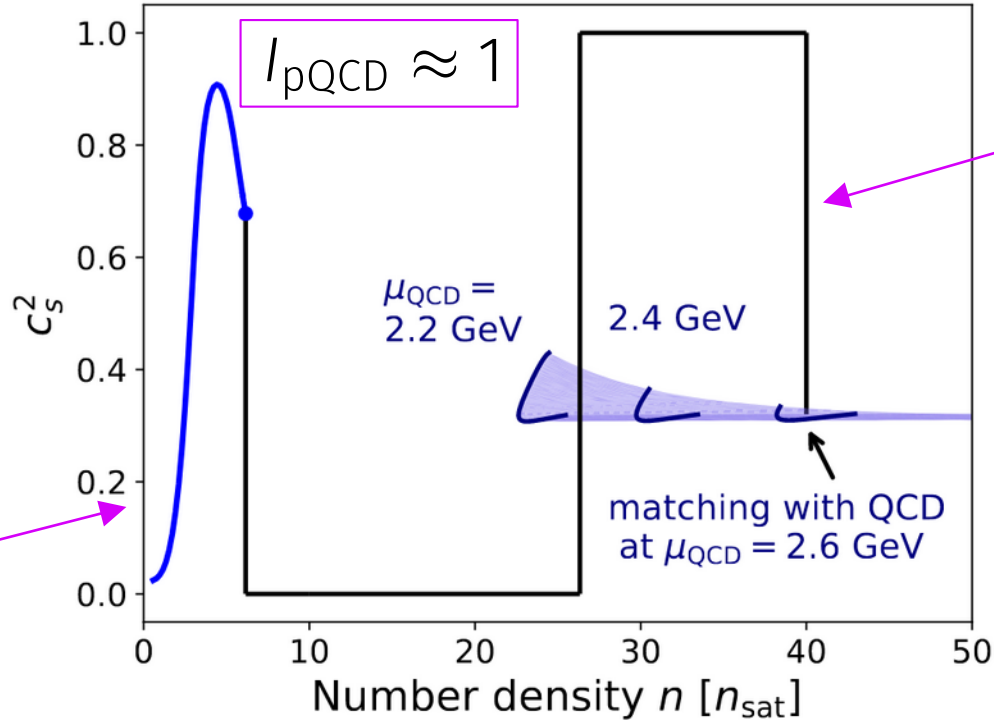


EoS modeled up
to n_{term}

Thermodynamics restricts
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Quantifying the *tension* with perturbative QCD

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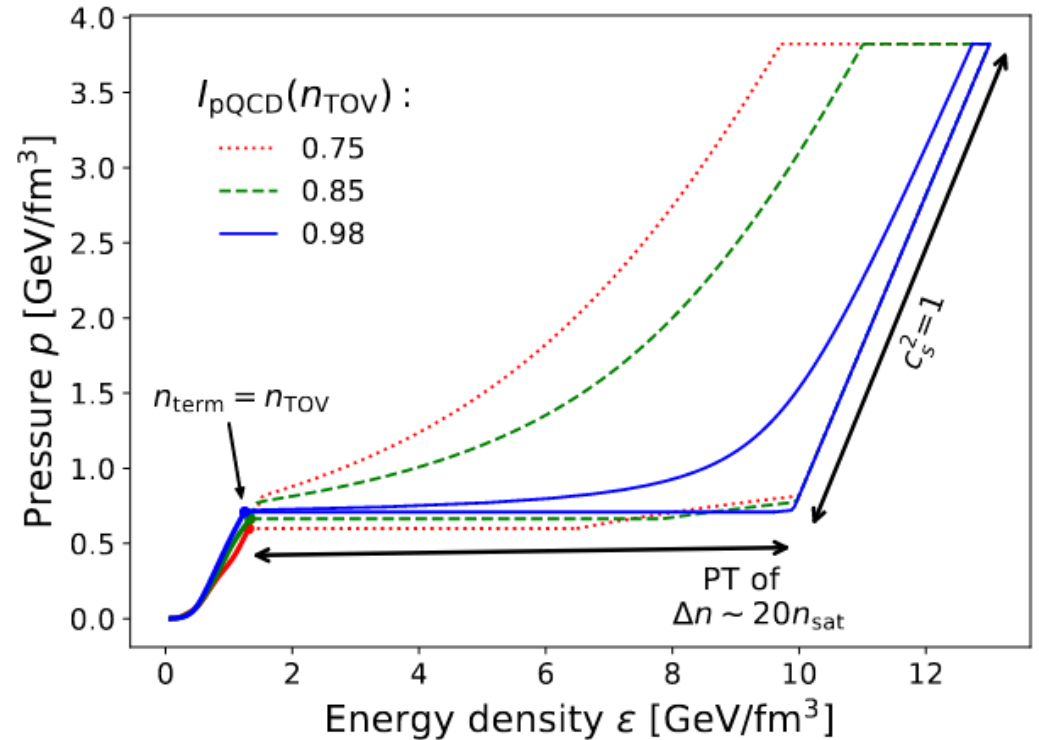
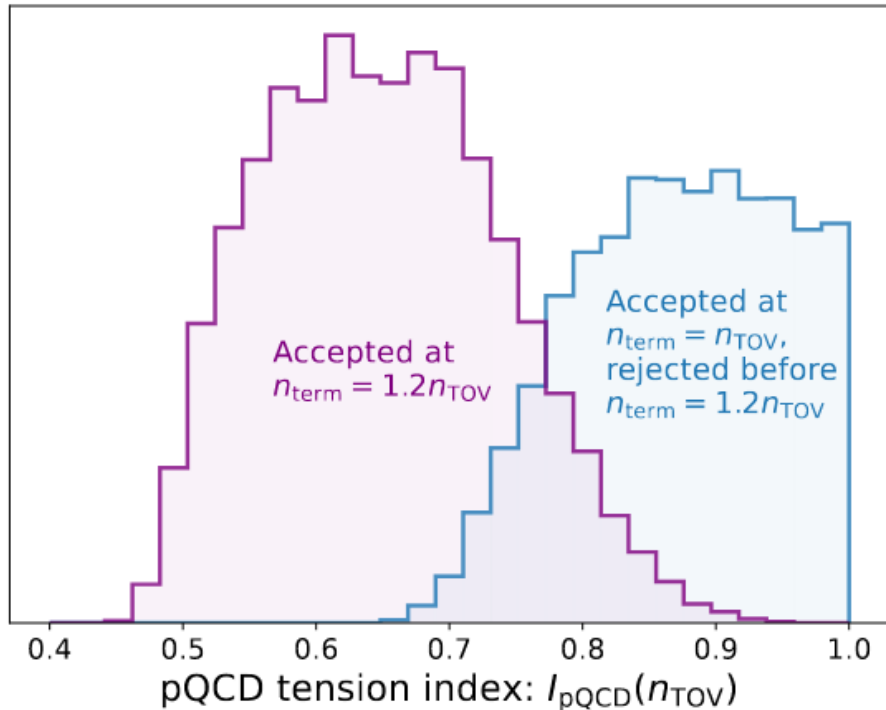
EoS modeled up to n_{term}

Thermodynamics restricts EoS behavior beyond n_{term}

Observation: High pQCD tension index also inconsistent with well-converged pQCD c_s^2

Examining $n_{\text{term}} = n_{\text{TOV}}$: Many EOSs have high I_{pQCD}

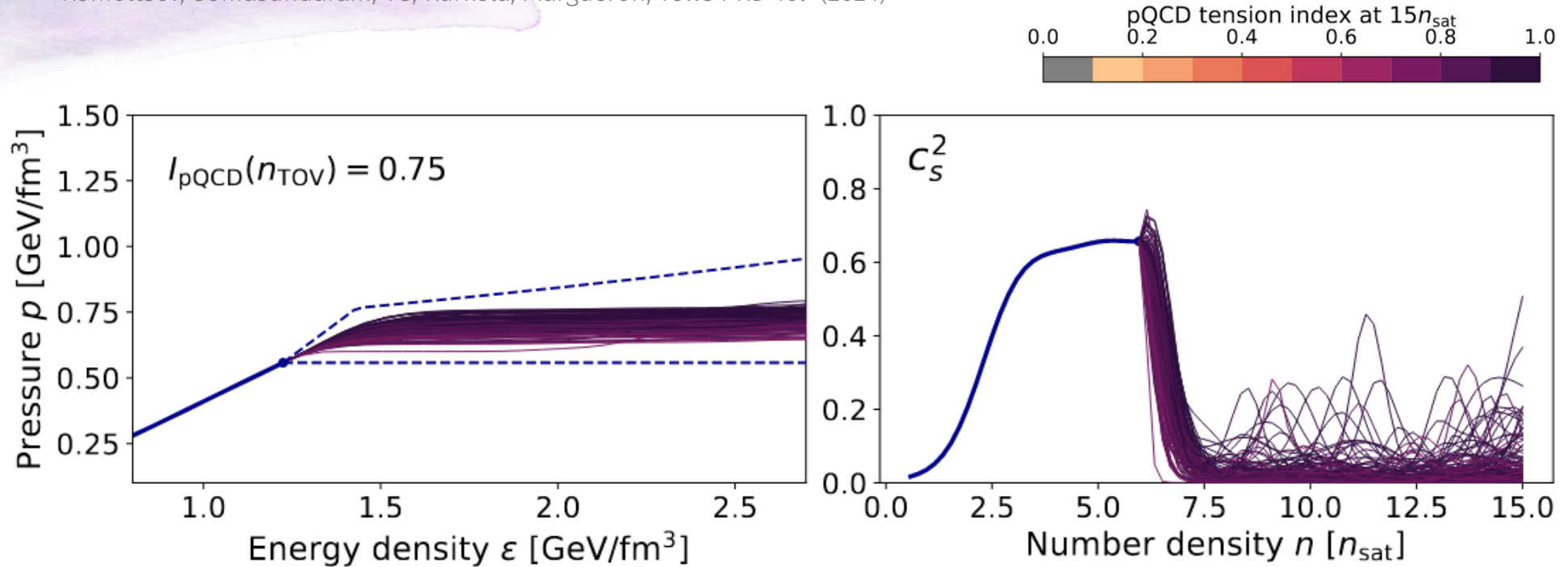
Komoltsev, Somasundaram, TG, Kurkela, Margueron, Tews PRD 109 (2024)



Allowed extensions for stiff EoSs have **tightly constrained region beyond n_{TOV}**

Extending stiff EoSs indeed show strong, prolonged softening

Komoltsev, Somasundaram, TG, Kurkela, Margueron, Tews PRD 109 (2024)



Takeaway: Softening before n_{TOV} OR Strong, prolonged softening just after, followed by $c_s^2 \approx 1$

EoS extensions for less extreme pressures

Komoltsev, Somasundaram, TG, Kurkela, Margueron, Tews PRD 109 (2024)

