

# Search for the critical point via intermittency analysis in NA61/SHINE at CERN

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for NA61/SHINE collaboration

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# Outline

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1. Introduction
2. Results for proton intermittency for Ar+Sc energy scan
3. Results for negatively charged hadrons intermittency in Xe+La collisions at  $150A$  GeV/c

# Introduction

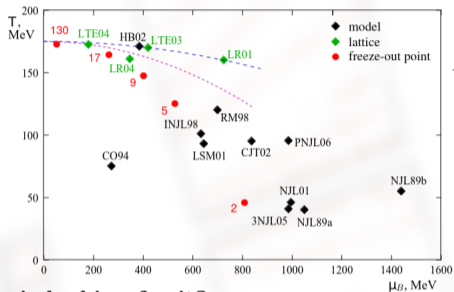
# The search for the Critical Point

## What is it?

- Critical Point (CP): Hypothetical endpoint of the first-order QGP-HM transition, with second-order phase properties.
- 2nd Order Transition: Scale-invariant power-law correlations; CP existence, location, and fluctuations are model-dependent.

Asakawa, Yazaki NPA 504 (1989) 668  
Barducci, Casalbuoni, De Curtis, Gatto,  
Pettini, PLB 231 (1989) 463

Stephanov, PoS LAT2006 (2006) 024



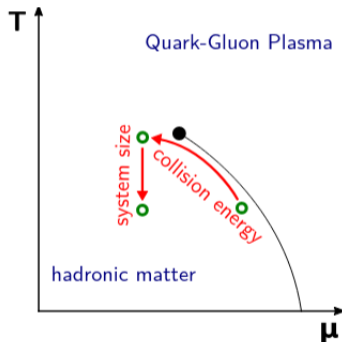
## Who is looking for it?

- *Past*: NA49 experiment at CERN. proton intermittency, pion intermittency
- *Present*
  - NA61/SHINE at CERN: proton intermittency,  $h^-$  intermittency
  - STAR Collaboration at RHIC:  $h^\pm$  intermittency

# Intermittency analysis in high energy physics

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- QCD Critical Point Search: Explored via collision energy and nuclear mass adjustments, affecting freeze-out parameters  $T_c$  and  $\mu_c$ .
- Intermittency Analysis: Examines momentum distribution fluctuations using scaled factorial moments to probe second-order phase transitions.

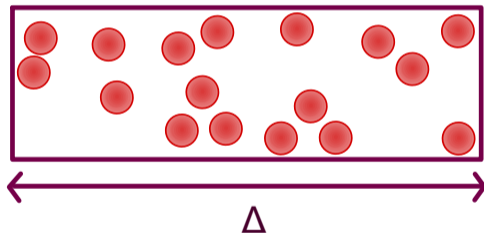


# Intermittency

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**Intermittency:** random deviations from smooth or regular behavior.

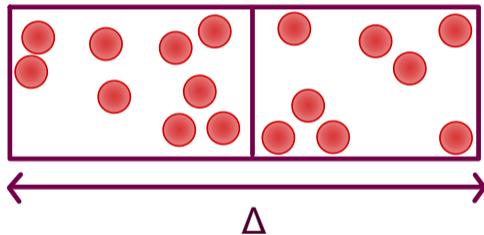
If we consider a container, a box of size  $\Delta$ , and we fill it with  $N$  balls:



# Intermittency

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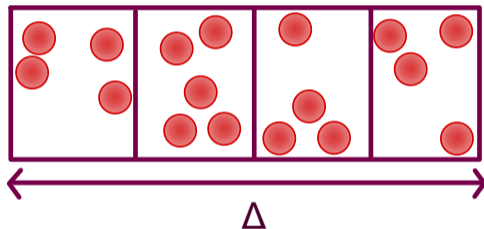
What can happen when we vary the number of cells ( $M$ ) but the length of the box  $\Delta$  and the total number of balls ( $N$ ) remain fixed?



# Intermittency

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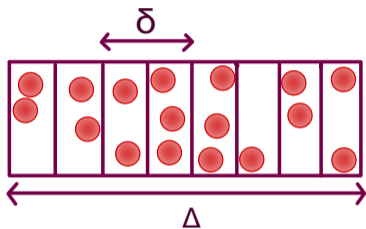


The idea of intermittency is repeating this scenario: divide the box into smaller cells of the same size  $\delta$ , keeping the main size of the box and the number of balls fixed.



## Intermittency: random deviations from smooth or regular behaviour

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$N$  is the total number of balls

$\Delta$ : is the size of the box

$\delta$  is the size of each cell

$M = (\Delta/\delta)$ : is the number of cells

$n_i$ : number of balls put into  $i^{\text{th}}$  cell such that

$$\sum_i n_i = N$$

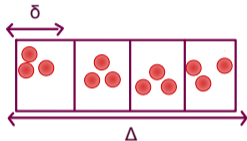
The  $r$ -order factorial moment for a given configuration (a given distribution of the balls) is defined as:

$$f_r(M) = \frac{\left[ \frac{1}{M} \sum_{i=1}^M n_i^r \right]}{\left[ \frac{1}{M} \sum_{i=1}^M n_i \right]^r} = M^{r-1} N^{-r} \sum_{i=1}^M n_i^r$$

## Intermittency: example of two extreme cases

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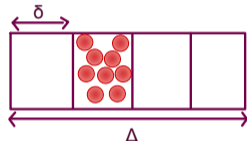
Case 1: equidistribution (N/M balls in each box)



$$n_i = N/M, \quad \forall i$$

$$f_r(M) = 1, \quad \forall i$$

Case 2: all balls in one cell (for an extreme fluctuation from a thermodynamic point of view)



$$n_i = N \quad \text{for } i = 2$$

$$n_i = 0 \quad \text{for } i \neq 2$$

$$f_r(M) = M^{r-1}$$

We say that it is intermittent behaviour if:  $\log(f_r(M))$  varies linearly with  $\log(\delta)$

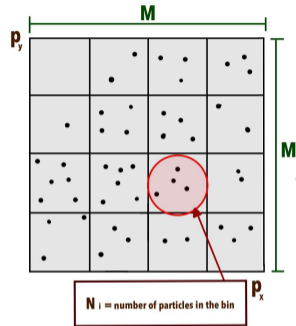
# Scaled factorial moments

- In NA61/SHINE at CERN SPS, intermittency analysis is performed in 2D transverse momentum plane.
- At the second order phase transition (critical point), the system becomes scale invariant.
- This phenomenon leads to enhanced multiplicity fluctuations with special properties that can be revealed by scaled factorial moments:

$$F_r(M) = \frac{\left\langle \frac{1}{M} \sum_{i=1}^M N_i \dots (N_i - r + 1) \right\rangle}{\left\langle \frac{1}{M} \sum_{i=1}^M N_i \right\rangle^r}$$

$M$  – sub-division intervals

$N_i$  – number of particles in  $i$ -th bin



When the system is a simple fractal,  $F_q(M)$  follows a power law dependence:  $F_q(M) = F_q(\Delta)(M^2)^{\phi_q}$  where the critical exponent or intermittency indices  $\phi_q$  obey the relation:  $\phi_q = (q - 1)d_q$  where the anomalous fractal dimension  $d_q$  is independent of  $q$ .

Wosiek, APPB 19 (1988) 863

Satz, NPB 326 (1989) 613 Bialas, Peschanski, NPB 273 (1986) 703

# Current methodologies

$F_r(M)$  depends on the shape of inclusive single particle  $p_T$  distribution. In order to eliminate this dependence we have two approaches

## $p_T$ binning

Instead of studying  $F_2(M)$  we study  $\Delta F_2$ . The quantity defined as:<sup>1</sup>

$$\Delta F_2(M) = F_2^{data}(M) - F_2^{mixed}(M)$$

<sup>1</sup>NA49 collaboration. In: Eur. Phys. J. C 75.2 (2015), p. 587m

<sup>2</sup>Bialas; Gazdzicki. In: Physics Letters B 252.3 (1990), pp. 483–486

<sup>3</sup>Antoniou; Diakonou. url: <https://indico.cern.ch/event/818624>

## Cumulative $p_T$ binning

Instead of using  $p_x, p_y$  we use cumulative quantities  $Q_x, Q_y$ :

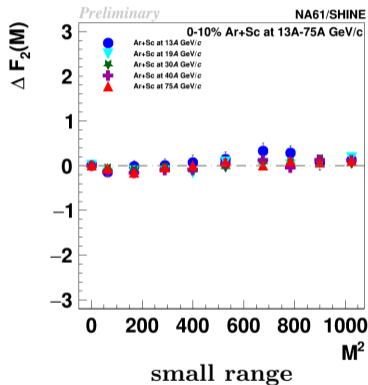
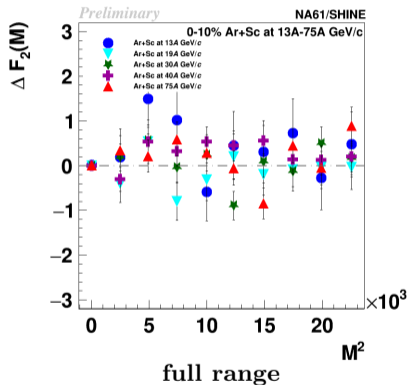
$$Q_x = \int_{x_{min}}^x \rho(x) dx / \int_{x_{min}}^{x_{max}} \rho(x) dx$$
$$Q_y = \int_{y_{min}}^y P(x, y) dy / P(x)$$

- Transforms any distribution into uniform <sup>2</sup> and removes the dependence of  $F_r$  on the shape of single particle distribution.
- Intermittency index of an ideal power law correlation function remain invariant <sup>3</sup>
- Results are displayed in:
  - $\Delta F_r(M)_c = F_r(M) - F_r(1)$  (where  $F_r(M)$  and  $F_r(1)$  by employing the cumulative  $p_T$  binning.  
 $F_r(1) = F_r(M)$  for uncorrelated particles in  $p_T$ )

Results for proton intermittency for Ar+Sc energy scan

# Results for proton intermittency for Ar+Sc energy scan

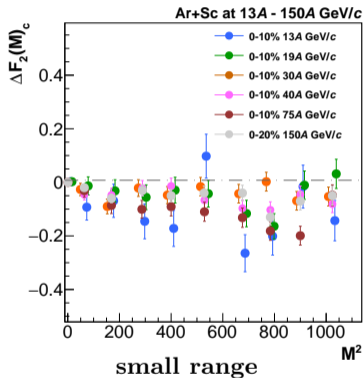
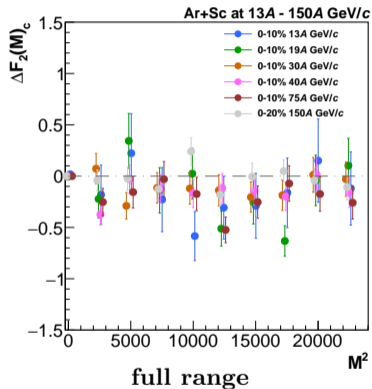
$p_T$  binning



No indication of power law increase with bin size

# Results for proton intermittency for Ar+Sc energy scan

cumulative  $p_T$  binning



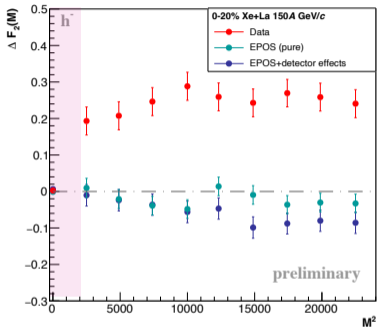
No indication of power law increase with bin size

H. Adhikary et.al (NA61/SHINE Collaboration), Eur.Phys.J.C 83 (2023) 9, 881  
H. Adhikary (for NA61/SHINE Collaboration), EPJ Web Conf. 274 (2022) 06008

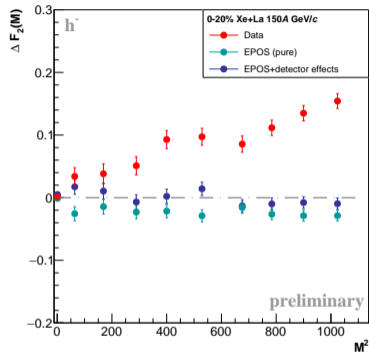
Results for negatively charged hadrons intermittency in  
Xe+La collisions at  $150A$  GeV/c



# $h^-$ intermittency: an unexpected increase



full range –  $p_T$  binning



small range –  $p_T$  binning

An unexpected increase was found when analyzing negatively charged hadrons in central Xe+La collisions at 150A GeV/c. (A similar increase was reported by STAR collaboration in 2023, but the physics form was not explained)

# Short range correlations

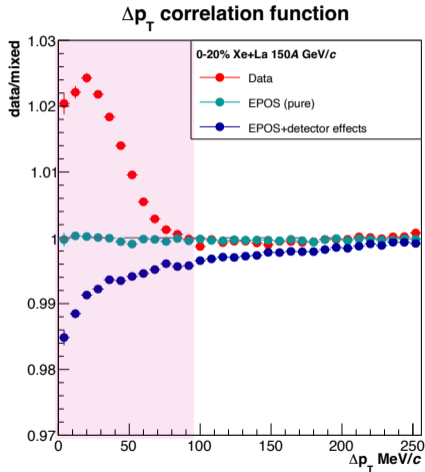
- A strong increase of  $\Delta F_2$  with  $M$  is observed for results in  $p_T$  binning.
- EPOS model does not show this increase
- In the case of protons, no increase of  $\Delta F_2$  results, neither in data nor in EPOS, was observed.

Is there a physics correlation which is present for  $h^-$  data, and absent in protons and EPOS  $h^-$ , that can explain this behavior?

Yes, short-range correlations of Bose-Einstein type

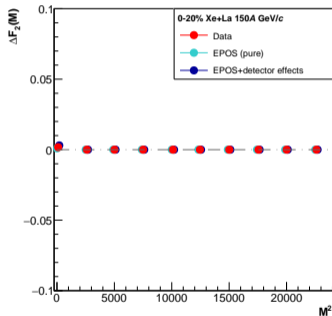
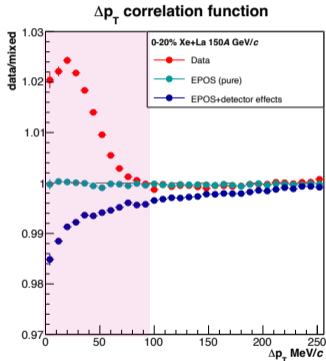
So we studied the  $h^-$ - $h^-$  correlation.

$$\Delta p_T = \sqrt{(p_{2,x} - p_{1,x})^2 + (p_{2,y} - p_{1,y})^2}$$



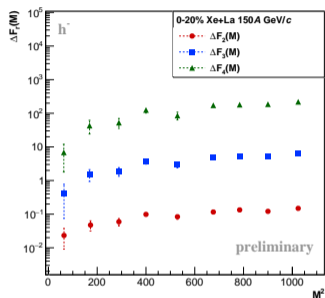
# Short range correlations

If we remove the region with  $\Delta p_T < 100$  MeV/c, the  $\Delta F_2$  is independent of  $M^2$  for both **experimental data** and EPOS.

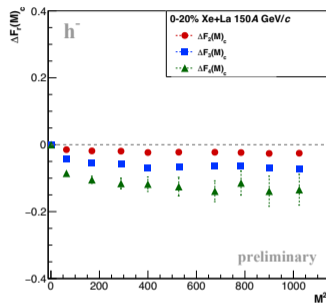


This suggests that the increase of  $\Delta F_2$  of the data was caused by these short range correlations. However, this removal of  $\Delta p_T$  region may also affect possible correlations due to CP.

# $h^-$ intermittency results for Xe+La 150A GeV/c



p<sub>T</sub> binning



cumulative p<sub>T</sub> binning

- The removal of these short-range correlations might be too drastic. Cumulative p<sub>T</sub> transformation preserves the scale-invariant power-law correlations and destroys other types of non-scale invariant correlations.
- Statistics of Xe+La at 150A GeV/c allow us to explore higher order moments of intermittency.

# Summary

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The results of intermittency analysis in NA61/SHINE were discussed

- Proton intermittency analysis for central Ar+Sc collisions (13A-150A GeV/c)
  - ✓ No signs of a power-law increase with bin size were found in proton intermittency analysis, nor using  $p_T$  or cumulative  $p_T$
- Negatively charged hadrons intermittency for central Xe+La collisions at 150A GeV/c
  - ✓ The experimental data on  $\Delta F_2$  for  $p_T$  binning exhibits an increase, but it does not follow a power law, the increase can be explained by short range correlations (HBT).
  - ✓ No indication of power law increase with bin size observed in cumulative  $p_T$  distribution. Since cumulative transformation preserves the scale-invariant power-law correlations but destroys other types of non-scale invariant correlations.
  - ✓ This might be a potential explanation to the increase seen by other collaborations.

Thanks

# Appendix

# History of intermittency analysis in high energy physics

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- The concept of intermittency was originally developed in the study of turbulent flow. (Ya.B. Zeldovich et al., Usp. Fiz. Nauk 152 3)
- Bialas and Peschanski, introduced after that intermittency analysis could be used to study fluctuations in high energy physics. It was proposed to study the scaled factorial moments of rapidity distribution of particles produced in high energy collisions as a function of the size of the rapidity interval. (Bialas, A. and Peschanski, R. (1986) Nuclear Physics B, 273, 703-718. )
- After, Wosiek's work showed indications of intermittent behavior in the critical region of the 2D Ising model. This raised the general question of whether or not intermittency and critical behavior were related.
- Satz showed that the critical behavior of the Ising model indeed leads to intermittency, with indices determined by the critical exponents (Satz: Nucl. Phys. B 326 (1989), pp. 613-618)
- And Bialas and Hwa reported that intermittency parameters could serve as a signal of second-order phase transition using scaled factorial moments (Bialas and Hwa: Phys.Lett. B253 (1991), pp. 436-438)
- And at this point, experiments begin to use this approach...



# Intermittency analysis in high energy physics

- The **search for the QCD Critical Point** in heavy ion collisions is performed by scan in the parameter controlled in the laboratory (collision energy and nuclear mass number, by changing them we change freeze-out parameters ( $T_c$ ,  $\mu_c$ ))
- Intermittency analysis was introduced in to study fluctuations in particle production by examining the scaled factorial moments of momentum distributions. These moments, dependent on the momentum interval size, may indicate a second-order phase transition.

