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Hydrodynamization and thermalization in heavy ion collisions

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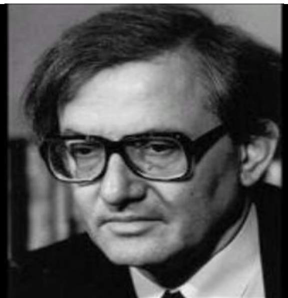
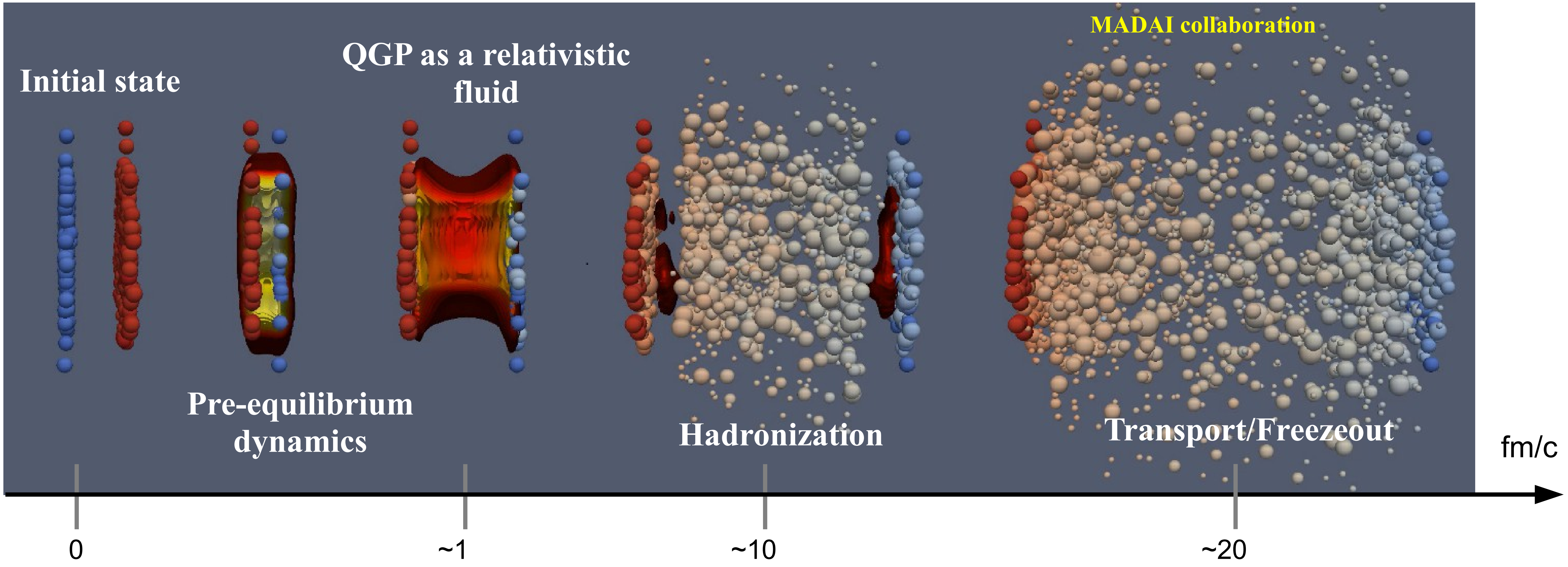
with C. Brito, D. Wagner, D. Rischke, see arXiv:2401.10098

Zimányi School 2024



Current picture of a heavy ion collision

Empirical: Fluid-dynamical modeling of heavy ion collisions works well at RHIC and LHC energies

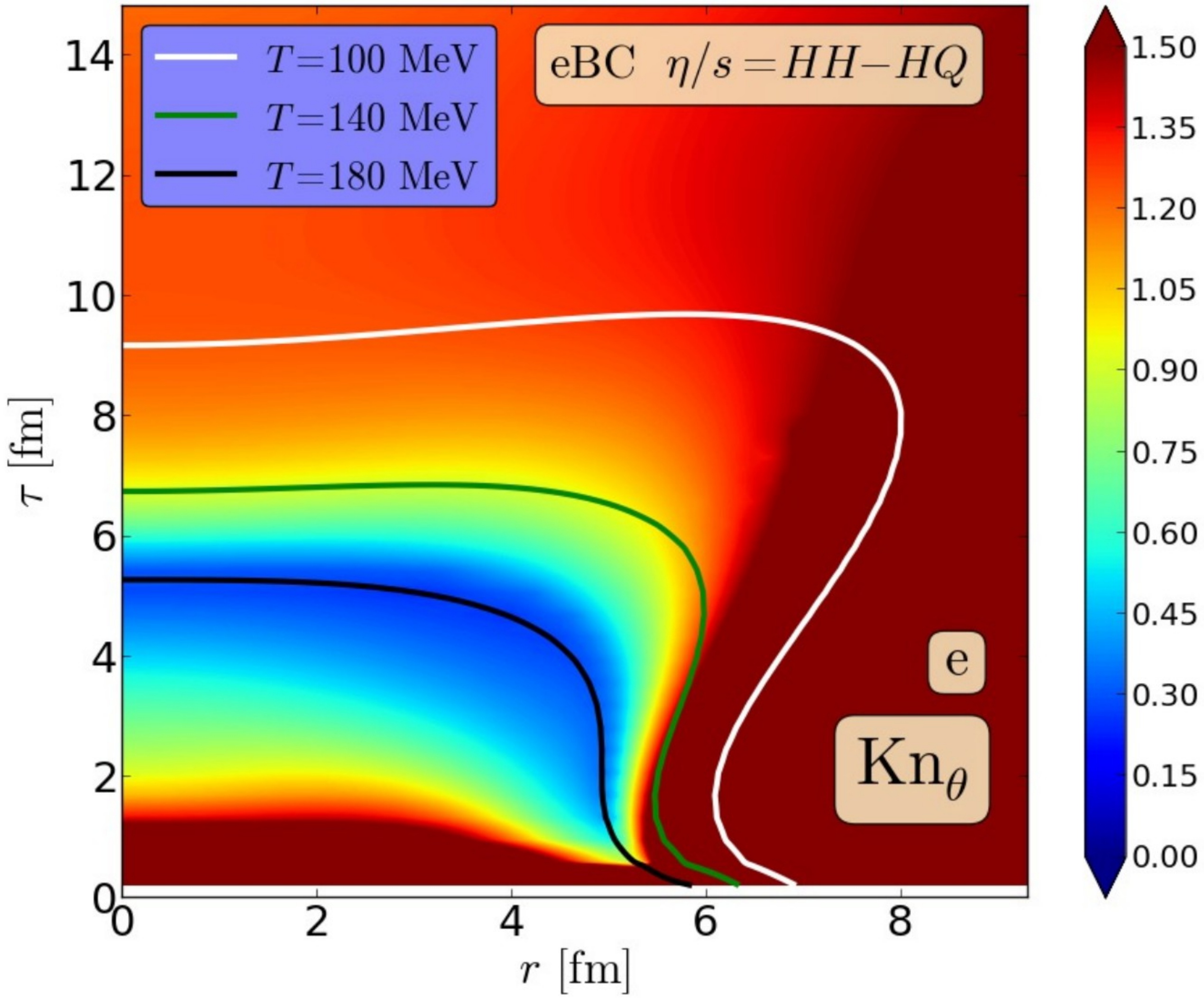


Main assumption: transient fluid dynamics can be appliedat very early times ~ 0.1--1 fm

Validity of fluid dynamics
- proximity to (local) equilibrium
- "small" gradients

Do these things happen in heavy-ion collisions?

arXiv:1404.7327



Separation of scales

Knudsen number: $K_N \sim \frac{\ell}{L} \ll 1$

microscopic (ℓ)

macroscopic (L)

$$\mathrm{Kn} = \tau_\pi \nabla_\mu u^\mu$$

Can this system be close to equilibrium? Hydrodynamization?

Basics of fluid dynamics

Conservation laws

energy-momentum conservation

$$\partial_\mu T^{\mu\nu} = 0$$

net-charge conservation

$$\begin{aligned} \partial_\mu N_s^\mu &= 0 \\ \partial_\mu N_e^\mu &= 0 \\ \partial_\mu N_b^\mu &= 0 \end{aligned}$$

strangeness

electric charge

Baryon number

Tensor decomposition

$$N^\mu = n u^\mu + q^\mu$$

$$T^{\mu\nu} = \epsilon\, u^\mu u^\nu - (P + \Pi)\, \Delta^{\mu\nu} + \pi^{\mu\nu}$$

net-charge diffusion 4-current (q^μ)

Bulk viscous pressure (Π)

Shear stress tensor ($\pi^{\mu\nu}$)

Projection operator:

$$\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$$

Closure?

Navier-Stokes theory: constitutive relations

Shear Viscosity

(Resistance to deformation)

$$\pi^{\mu\nu} = 2\eta\nabla^{\langle\mu}u^{\nu\rangle}$$

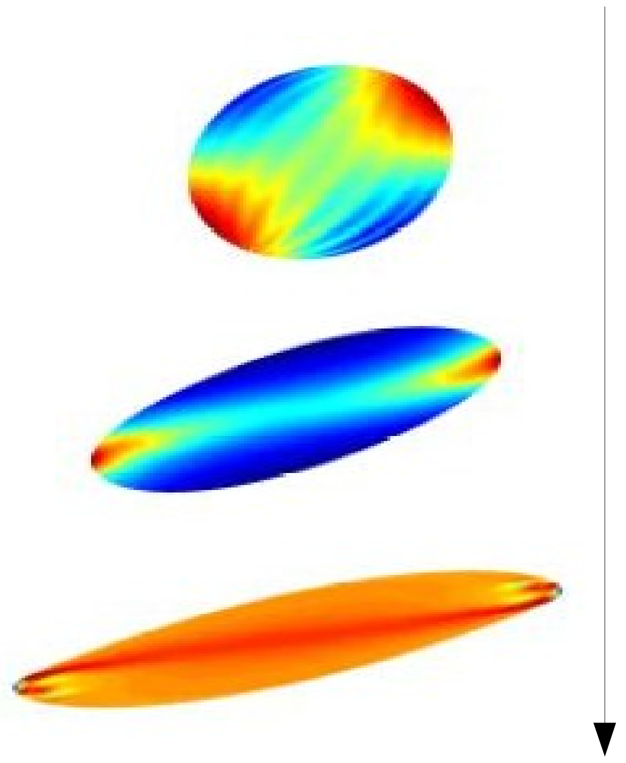
Bulk Viscosity

(Resistance to expansion)

$$\Pi = -\zeta\nabla_{\mu}u^{\mu}$$

Net-Charge Diffusion

$$q^{\mu} = \kappa\nabla^{\mu}\frac{\mu_B}{T}$$



Acausal and unstable: cannot describe relativistic fluids

What we solve is not "traditional" fluid dynamics

Causality: constitutive relations for the dissipative currents *cannot* be imposed

Dynamical equations, e.g. Israel-Stewart theory Annals Phys. 118 (1979) 341-372

$$\tau_\Pi \dot{\Pi} + \Pi = -\zeta\theta + \quad \dots$$

$$\tau_\pi \dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \quad \dots$$

relaxation times (τ_Π, τ_π)   higher-order terms ($\dots$)

<u>non-perturbative theory in gradients!</u> Heller&Spalinski, Phys.Rev.Lett. 115 (2015) 7, 072501

How can we derive/understand these theories?

We can study this problem in kinetic theory

$$k^{\mu}\partial_{\mu}f_{\mathbf{k}} = C\left[f\right]$$

Relativistic Boltzmann eq.

?

Method of moments

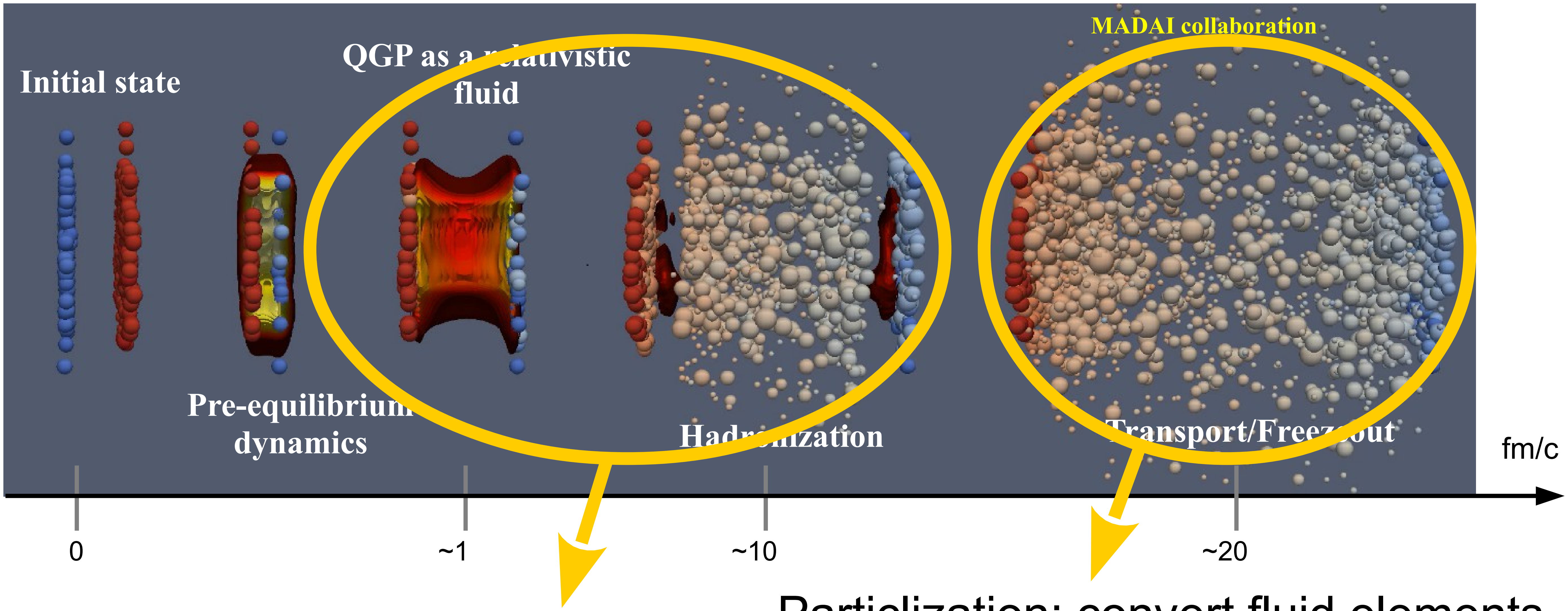
$$\tau_{\Pi}\dot{\Pi} + \Pi = -\zeta\theta + \ \ldots$$

$$\tau_{\pi}\dot{\pi}^{\langle\mu\nu\rangle} + \pi^{\mu\nu} = 2\eta\sigma^{\mu\nu} + \ \ldots$$

Relativistic Hydrodynamics

Why does this matter? Need *microscopic* description of the medium

fluid variables $T^{\mu\nu}$ → *particles* $f(\mathbf{x},\mathbf{p})$ $\qquad \delta f(k) \sim \frac{\pi^{\mu\nu}}{\varepsilon_0 + p_0} k_\mu k_\nu$



Photon emission, jet quenching

Particlization: convert fluid elements to particles

Paquet et al, PRC 93 (2016) 4, 044906

What you will see today

We solve the Boltzmann equation for an *ultrarelativistic* gas in *Bjorken flow* using the *method of moments*

Outline:

- Boltzmann equation in *Bjorken flow and* the method of moments
- Divergence of the moment expansion
- Numerical solutions of the Boltzmann equation
- Discussion and conclusions

Bjorken flow (toy model of a heavy ion collision) J. D. Bjorken, Phys. Rev. D27, 140 (1983)

Simple model for *boost invariant* longitudinal expansion

$v_z = z/t$

Homogeneous fluid in hyperbolic coordinates (τ, x, y, ς)

$$\tau = \sqrt{t^2 - z^2} \qquad \varsigma = \tanh^{-1}(z/t)$$

Static velocity

$$u^\mu = (1,0,0,0)$$

Gradients ~ 1/τ

$$\sigma^\mu_\nu = \mathrm{diag}\left(0, -\frac{1}{3\tau}, -\frac{1}{3\tau}, \frac{2}{3\tau}\right)$$

energy-momentum tensor

$$T^\mu_\nu = \mathrm{diag}\left(\varepsilon, P - \pi/2, P - \pi/2, P + \pi\right)$$

dissipative correction

Knudsen number: $K_N \sim \hat{\tau}^{-1} \equiv \tau_R/\tau$

Boltzmann equation for an ultrarelativistic gas

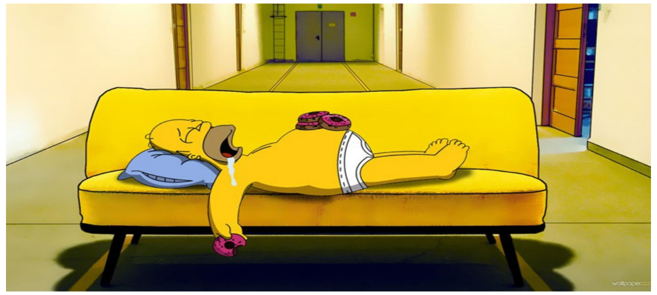
(Bjorken flow)

Single particle distribution function: $f_{\mathbf{k}} = f(\tau, k_0, k_{\eta_s})$

$$\partial_\tau f_{\mathbf{k}} = \frac{1}{E_{\mathbf{k}}} C[f_{\mathbf{k}}] \approx -\frac{1}{\tau_R}(f_{\mathbf{k}} - f_{0\mathbf{k}})$$

Relaxation time (τ_R)

Relaxation time approximation



- can be solved numerically
- used to study the fluid-dynamical limit

Florkowski&Ryblewski, Phys.Rev.C 88 (2013) 024903

Method of moments
(Bjorken flow)

H. Grad, Comm. Pure Appl. Math. 2, 331 (1949)
Israel&Stewart, Annals Phys. 118 (1979) 341-372
Denicol et al, PRD 85, 114047 (2012)

Single particle distribution function $f_{\mathbf{k}} = f(\tau, k_0, k_{\eta_s})$

$\cos\Theta \equiv k_{\eta_s}/(\tau E_{\mathbf{k}})$

Moment expansion
$$f_{\mathbf{k}} = f_{0\mathbf{k}} \sum_{\ell=0}^{\infty}\sum_{n=0}^{\infty} c_{n,\ell}(\beta E_{\mathbf{k}})^{2\ell} P_{2\ell}(\cos\Theta) L_n^{(4\ell+1)}(\beta E_{\mathbf{k}})$$

moments of *f*

$$c_{n\ell} \sim \int_k E_{\mathbf{k}}^{\ell} P_{\ell}\left(\cos\Theta_{\mathbf{k}}\right) L_n^{(2\ell+1)}\left(\beta E_{\mathbf{k}}\right) f_{\mathbf{k}}$$

Equations of motion $\dot{c}_{n\ell} = F\left(\tau, \{c_{n\ell}\}\right)$

truncation leads to hydro (no small parameter)

14-moments approx.

(Bjorken flow)

H. Grad, Comm. Pure Appl. Math. 2, 331 (1949)

Israel&Stewart, Annals Phys. 118 (1979) 341-372

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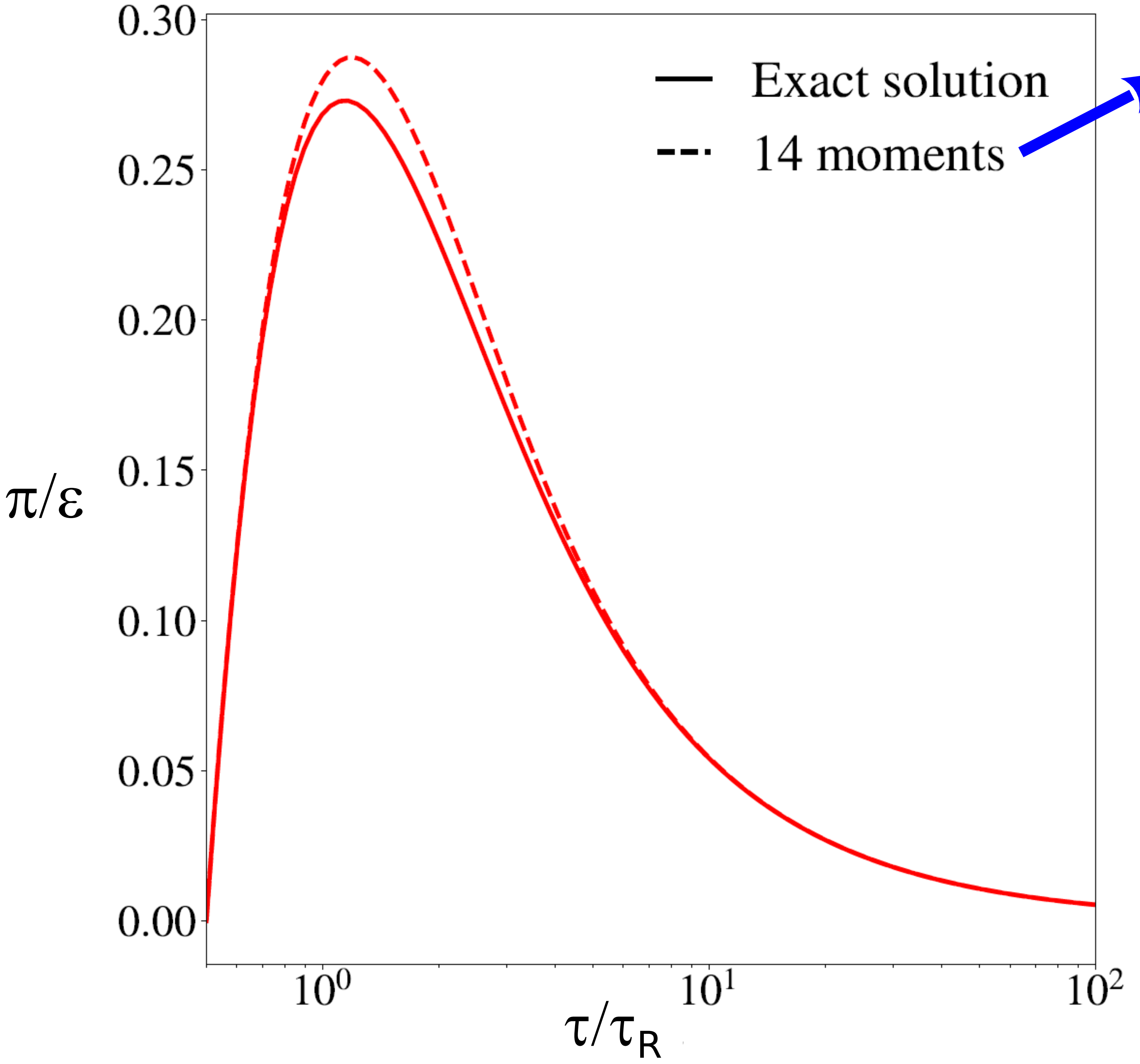


Hydro

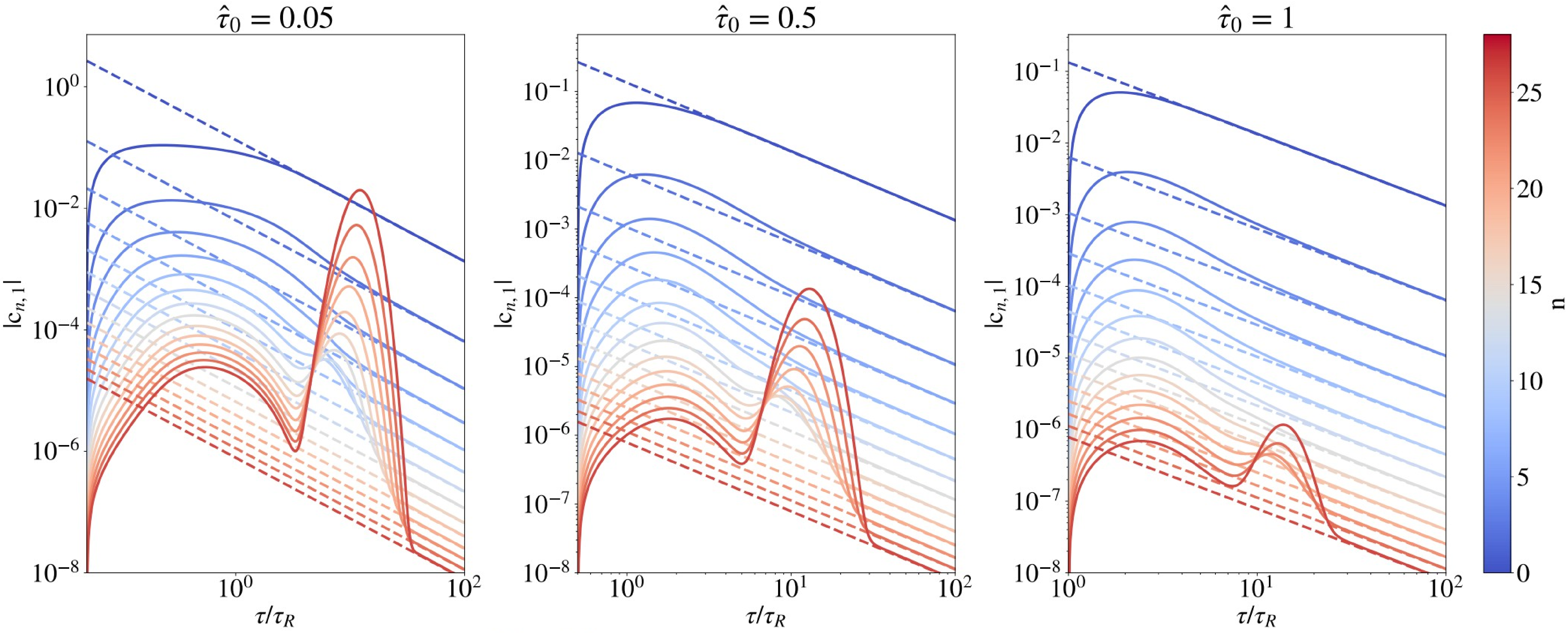
$$T^{\mu}_{\nu} = \mathrm{diag}\left(\varepsilon, P - \pi/2, P - \pi/2, P + \pi\right)$$

$$\begin{cases} \dfrac{d\varepsilon}{d\tau} + \dfrac{(\varepsilon + P)}{\tau} - \dfrac{\pi}{\tau} = 0, \\ \tau_R \dfrac{d\pi}{d\tau} + \pi + \left(\dfrac{4}{3} + \lambda\right) \tau_R \dfrac{\pi}{\tau} = \dfrac{4}{3}\dfrac{\eta}{\tau}. \end{cases}$$

Hydrodynamic solution is very close to the exact solution!

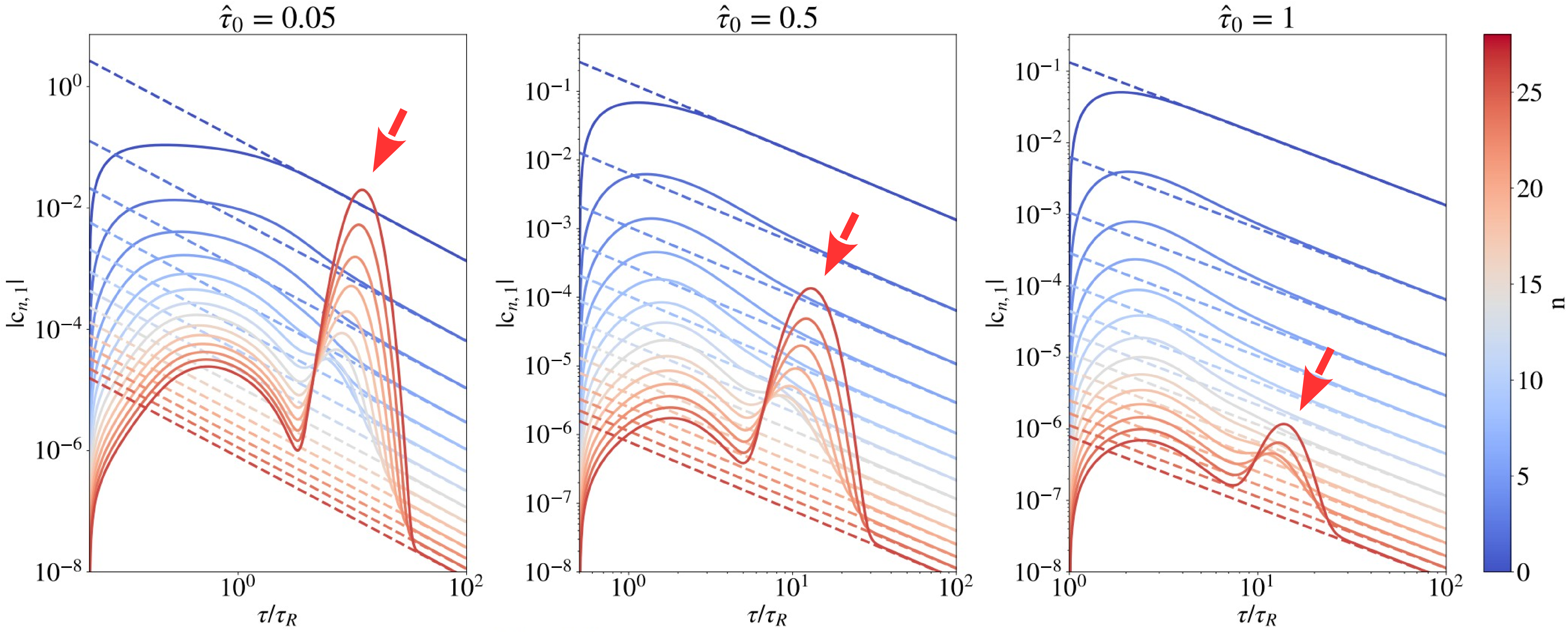
Does this imply that the system is close to equilibrium?

Moment solutions



$$f_{\mathbf{k}} = f_{0\mathbf{k}} \sum_{\ell=0}^{\infty} \sum_{n=0}^{\infty} c_{n,\ell} (\beta E_{\mathbf{k}})^{2\ell} P_{2\ell}(\cos\Theta) L_n^{(4\ell+1)}(\beta E_{\mathbf{k}})$$

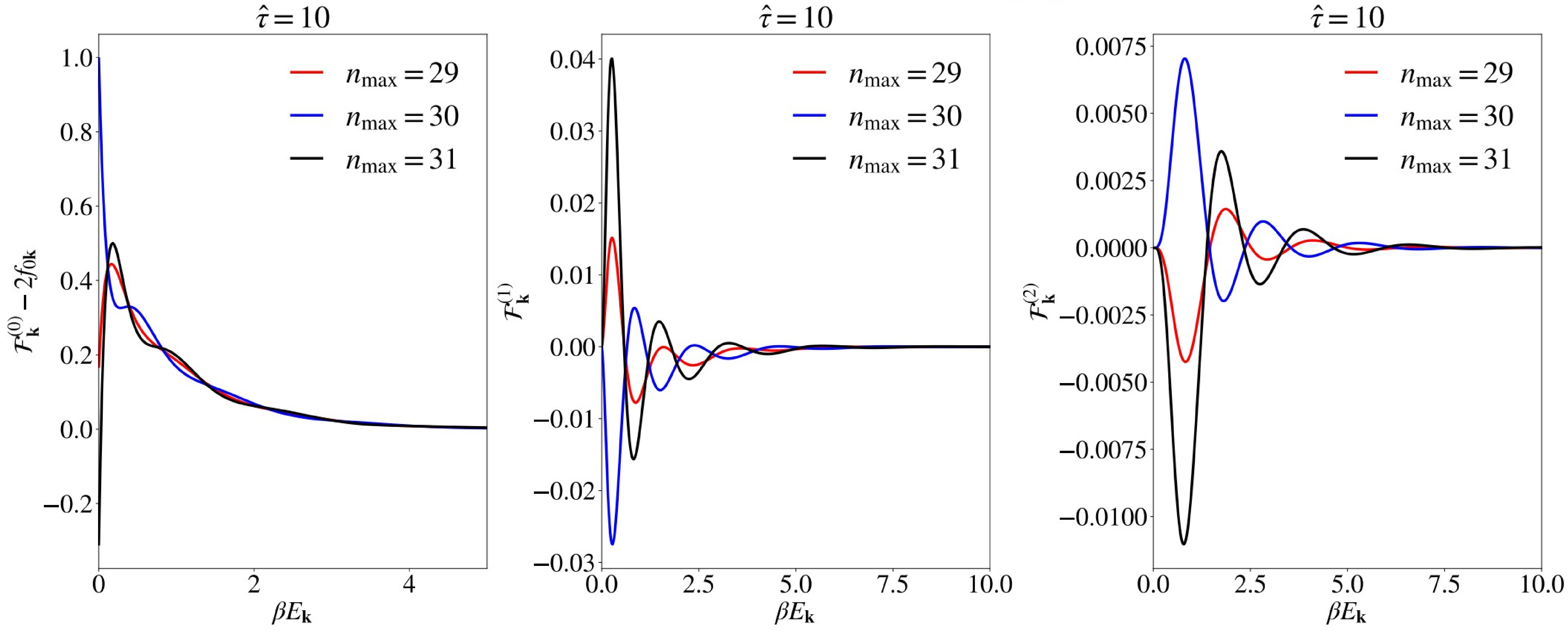
A surprise: divergence of the moment expansion



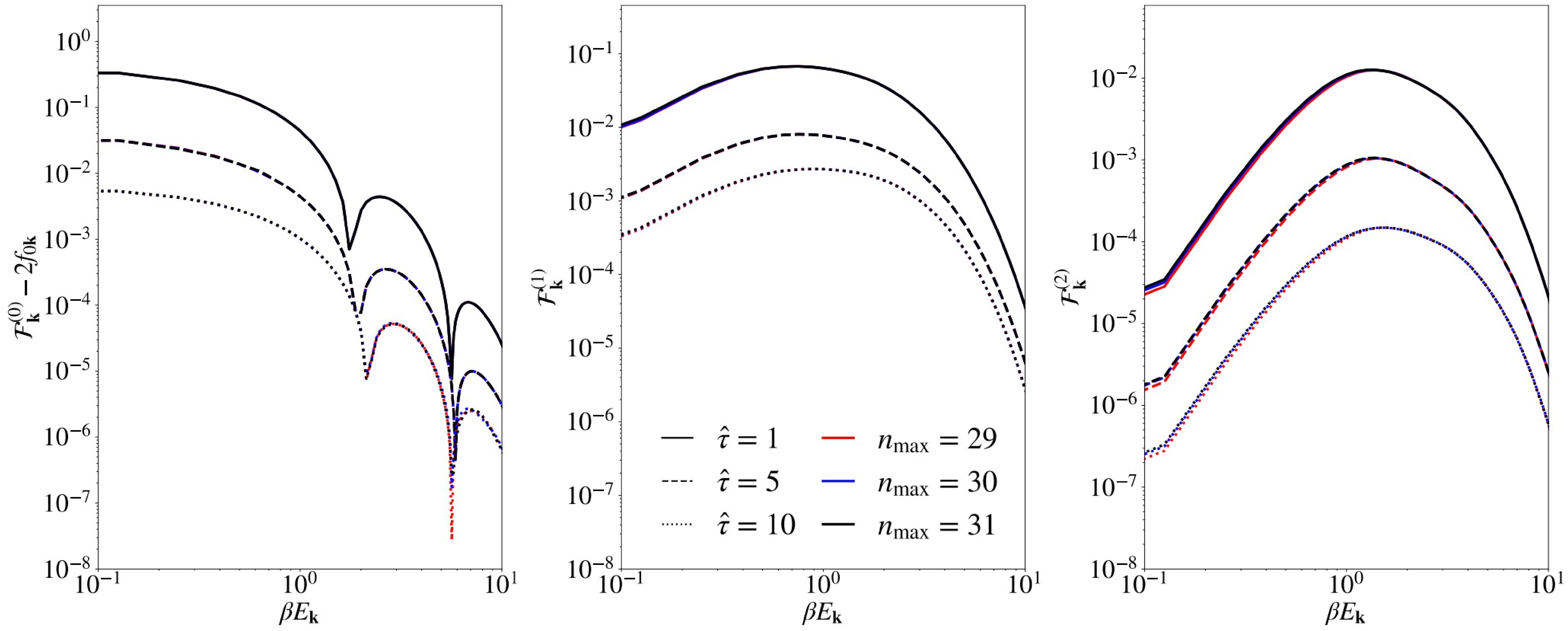
$$f_{\mathbf{k}} = f_{0\mathbf{k}} \sum_{\ell=0}^{\infty} \sum_{n=0}^{\infty} c_{n,\ell} (\beta E_{\mathbf{k}})^{2\ell} P_{2\ell}(\cos\Theta) L_n^{(4\ell+1)}(\beta E_{\mathbf{k}})$$

A surprise: divergence of the moment expansion $f_{\mathbf{k}} = \sum_{\ell=0}^{\infty} \frac{4\ell+1}{2} P_{2\ell}(\cos\Theta) \mathcal{F}_{\mathbf{k}}^{(\ell)}$

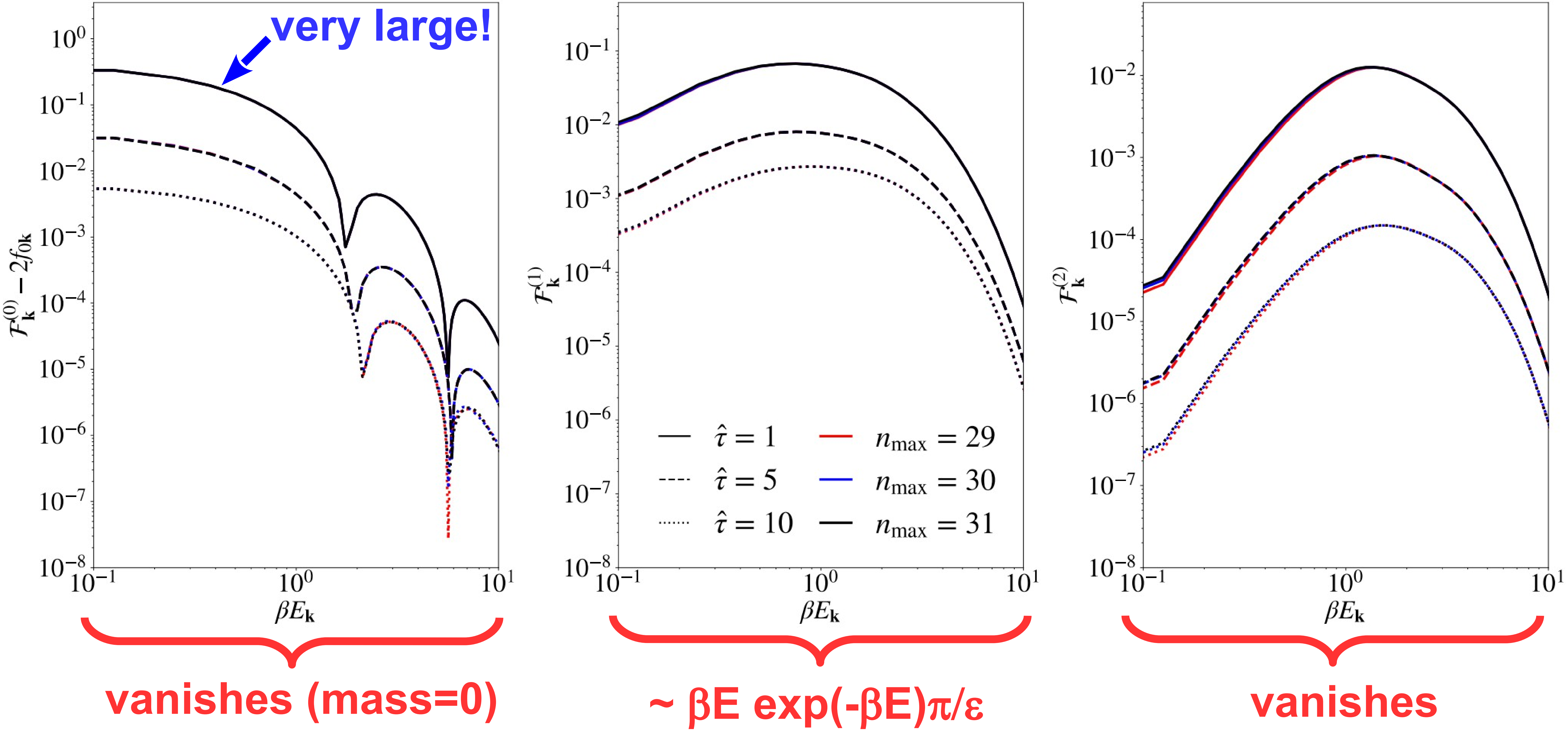
$$\mathcal{F}_{\mathbf{k}}^{(\ell)} \equiv \int_{-1}^{1} \mathrm{d}\cos\Theta P_{2\ell}(\cos\Theta)\ f_{\mathbf{k}} = \frac{2}{4\ell+1} (\beta E_{\mathbf{k}})^{2\ell} f_{0\mathbf{k}} \sum_{n=0}^{\infty} c_{n,\ell} L_n^{(4\ell+1)}(\beta E_{\mathbf{k}})$$



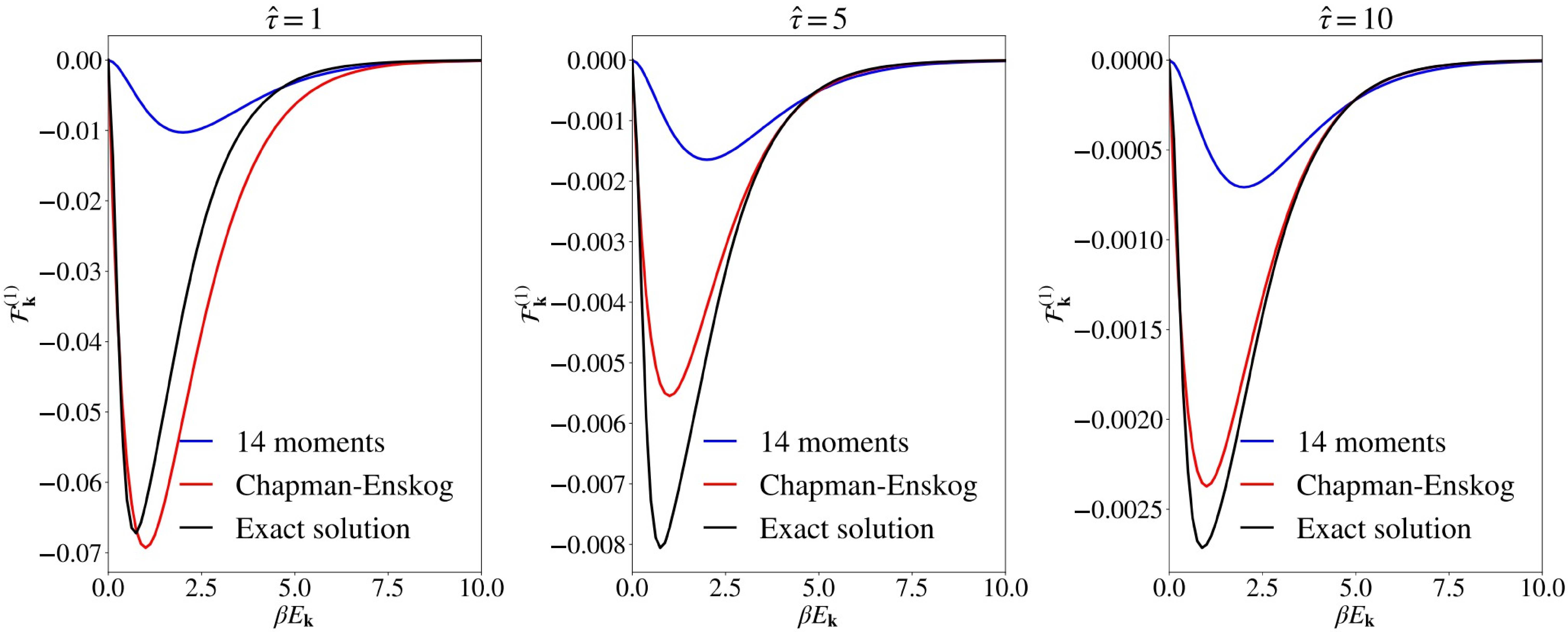
Borel-Pade resummation: results



Borel-Pade resummation: results



Comparison with hydrodynamic approximations



Hydrodynamic approximations are not in good agreement with exact δf

Summary/Conclusions

We solve the Boltzmann equation for an *ultrarelativistic* gas in *Bjorken flow* using the *method of moments*

$T^{\mu\nu}$ is well described by hydrodynamics, but δf is not.

Very large isotropic component, even without bulk viscosity

- non-hydrodynamic origin
- does not couple to $T^{\mu\nu}$ due to massless limit

Shear component of the distribution function does not match hydrodynamic approximations

→ This could be important for calculating thermal photon emission