Deuteron production in ultrarelativistic heavy-ion collisions (with a focus on elliptic flow)

Boris Tomášik, Radka Vozábová, Tomáš Poledníček

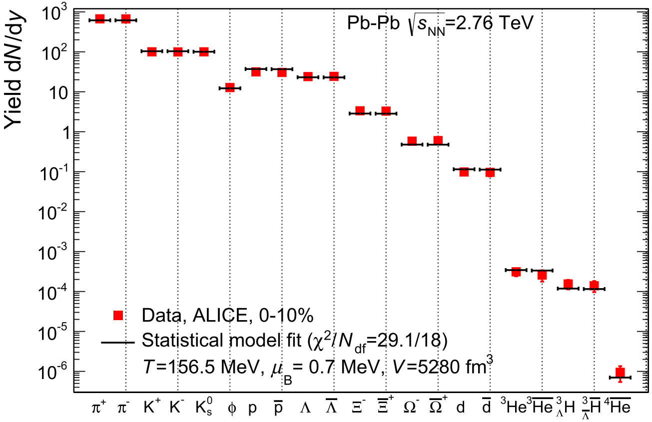
Fakulta jaderná a fyzikálně inženýrská, České vysoké učení technické, Praha, Czech Republic
and
Univerzita Mateja Bela, Banská Bystrica, Slovakia

boris.tomasik@cvut.cz

Zimányi School, Budapest, 2024

4.12.2024

Clusters and statistical model: a coincidence?



Cluster abundancies fit into a universal description with the statistical model

$T \gg E_b$!!!???

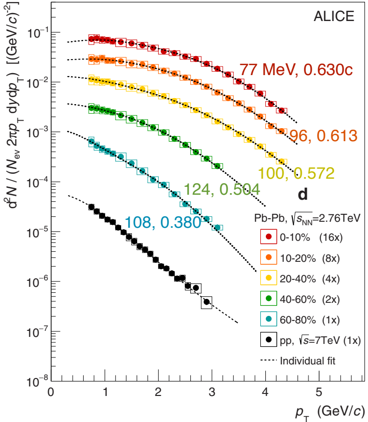
[A. Andronic *et al.*, J. Phys: Conf. Ser 779 (2017) 012012]

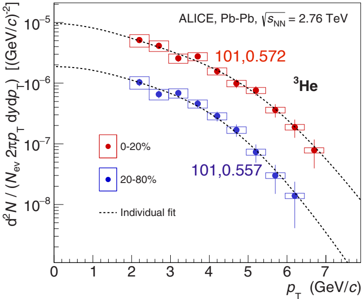
This is (a part of the) motivation to look at clusters,
although clusters actually carry *femtoscopic* information about the freeze-out.

[J. Adam *et al.* [ALICE collab], Phys. Rev. C 93 (2016) 024917]

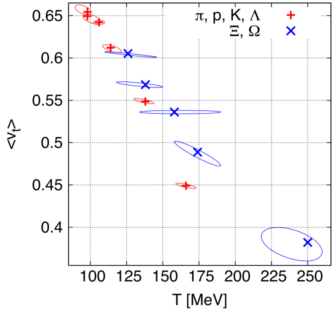
p_t spectra of d and ^{3}He fitted individually with the blast-wave formula

$$\frac{dN}{p_t\,dp_t} \propto \int_0^R r\,dr\,m_t\,I_0\left(\frac{p_t \sinh\rho(r)}{T}\right) K_1\left(\frac{m_t \cosh\rho(r)}{T}\right)\,, \qquad \rho(r) = \tanh^{-1}(\beta_s (r/R)^n)$$





Fit with MC blast-wave:



[I. Melo, B. Tomášik, J. Phys. G. 43 (2016) 015102]

Production model: coalescence

[R. Scheibl, U. Heinz, Phys. Rev. C 59 (1999) 1585]

Projection of the deuteron density matrix onto two-nucleon density matrix
Deuteron spectrum:

$$E_d \frac{dN_d}{d^3 P_d} = \frac{3}{8(2\pi)^3} \int_{\Sigma_f} P_d \cdot d\Sigma_f(R_d)\, f_p\left(R_d, \frac{P_d}{2}\right) f_n\left(R_d, \frac{P_d}{2}\right) \mathcal{C}_d(R_d, P_d)$$

QM correction factor

$$\mathcal{C}_d(R_d, P_d) \approx \int d^3 r \frac{f(R_+, P_d/2) f(R_-, P_d/2)}{f^2(R_d, P_d/2)} |\varphi_d(\vec{r})|^2$$

r relative position, R_+, R_-: positions of nucleons
approximation: narrow width of deuteron Wigner function in momentum

Analytical approximation of the (average) correction factor

[R. Scheibl, U. Heinz, Phys. Rev. C 59 (1999) 1585]

$$\langle \mathcal{C}_d \rangle (P_d) = \frac{\int_{\Sigma_f} P_d \cdot d\Sigma_f(R_d) f^2 \left(R_d, \frac{P_d}{2}\right) \mathcal{C}_d(R_d, P_d)}{\int_{\Sigma_f} P_d \cdot d\Sigma_f(R_d) f^2 \left(R_d, \frac{P_d}{2}\right)}$$

Approximations:

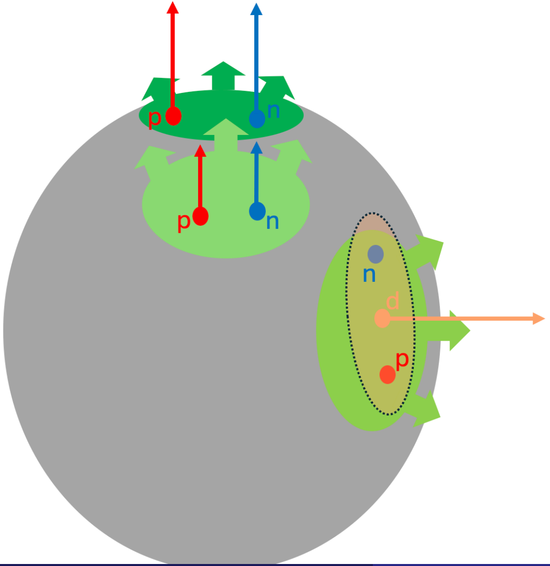
- Gaussian profile in rapidity and in the transverse direction,
- weak transverse expansion
- saddle point integration

$$\langle \mathcal{C}_d \rangle \approx \left\{ \left(1 + \left(\frac{d}{2\mathcal{R}_\perp(m)} \right)^2 \right) \sqrt{1 + \left(\frac{d}{2\mathcal{R}_\parallel(m)} \right)^2} \right\}^{-1}$$

Homogeneity lengths:

$$\mathcal{R}_\perp = \frac{\Delta\rho}{\sqrt{1 + (m_t/T)\eta_f^2}} \qquad \mathcal{R}_\parallel = \frac{\tau_0\, \Delta\eta}{\sqrt{1 + (m_t/T)(\Delta\eta)^2}}$$

Homogeneity regions, homogeneity lengths, length scales



- Particles of given momentum are produced only by a portion of fireball with similar velocity—the homogeneity region

- the size of the homogeneity region is given by flow gradients and temperature

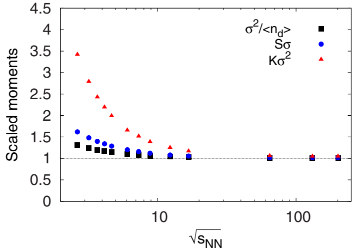
- Length scales
  - homogeneity length
  - size of the deuteron

Distinguishing coalescence: deuteron number fluctuations

[Z. Fecková, *et al.*, Phys. Rev. C 93 (2016) 054906]

Measure the fluctuations of deuteron number.

- Thermal model prediction:
  Poissonian fluctuations
- Coalescence:
  - protons and neutrons fluctuate according to Poissonian
  - deuteron number proportional to $p \cdot n$
  - Enhanced fluctuations in case of coalescence



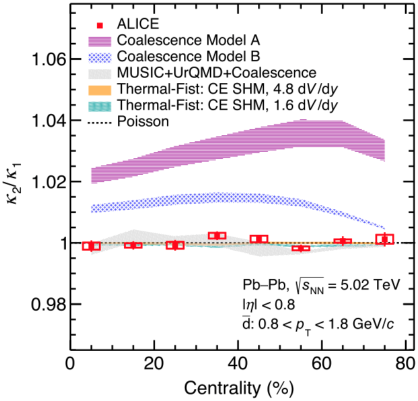
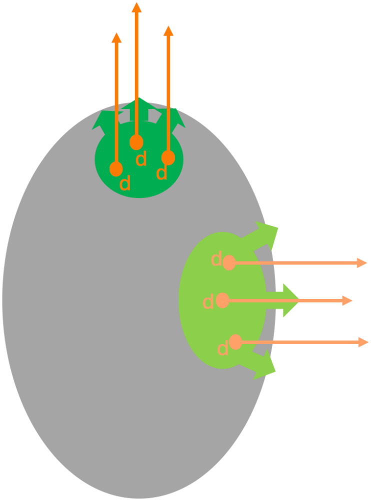


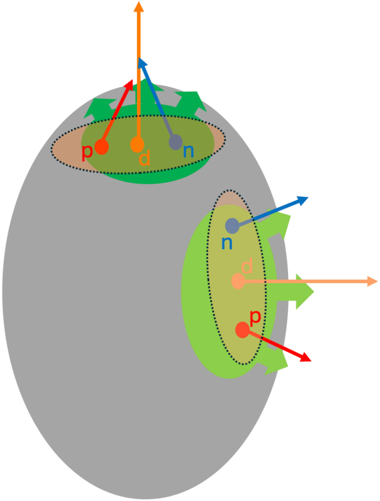
figure: [ALICE collab: Phys. Rev. Lett. 131 (2023) 037864]

Homogeneity lengths and the elliptic flow of clusters

direct thermal production of deuterons



coalescence production of deuterons



Simulate v_2 of deuterons—the strategy

- set-up Blast Wave model with azimuthal anisotropy
- assume Partial Chemical Equilibrium (lower FO temperature than T_{ch})
- the model must reproduce p_t-spectra and $v_2(p_t)$ of protons and pions
- simulate p_t spectra and $v_2(p_t)$ of deuterons in blast-wave model and in coalescence, and look for differences
- features of the model:
  - includes resonance decays
  - Monte Carlo simulation (SMASH: modified HadronSampler and decays)
  - built-in anisotropy in expansion flow and in fireball shape
  - includes modification of distribution function due to viscosity
  - freeze-out time depending on radial coordinate
- obtain T and transverse expansion from fitting p_t spectra of p and π (and K, Λ)
- then obtain anisotropy parameters from $v_2(p_t)$
- simulate thermal production of deuterons
- simulate coalescence of deuterons (by proximity in phase-space)

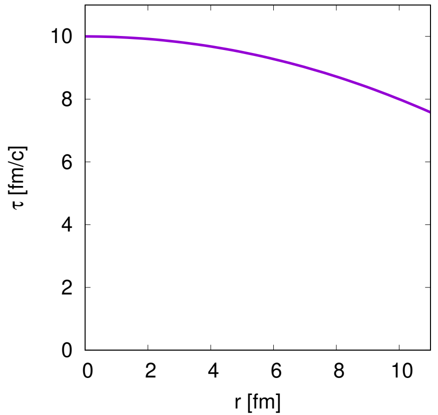
The Cooper-Frye formula:

$$E\frac{d^3N_i}{dp^3} = \int_\Sigma d^3\sigma_\mu p^\mu f(x,p)\,,$$

The freeze-out hypersurface:

$$x^\mu = (\tau(r)\cosh\eta_s, r\cos\Theta, r\sin\Theta, \tau(r)\sinh\eta_s)$$

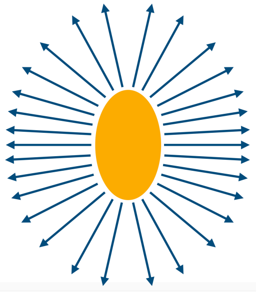
$$\tau(r) = s_0 + s_2 r^2\,, \qquad \eta_s = \frac{1}{2}\ln\left(\frac{t+z}{t-z}\right)$$



$$d^3\sigma^\mu = (\cosh\eta_s, 2s_2 r\cos\Theta, 2s_2 r\sin\Theta, \sinh\eta_s)\, r\tau_f(r) d\eta_s dr d\Theta\,,$$

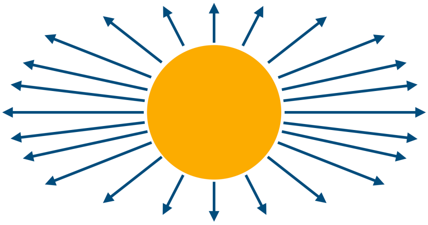
Extended Monte Carlo Blast-Wave model: azimuthal anisotropies

Shape anisotropy:



$$R(\Theta) = R_0\,(1 - a_2 \cos(2\Theta))$$

Flow anisotropy:



$$u^\mu = (\cosh\eta_s \cosh\rho(r),\, \sinh\rho(r)\cos\Theta_b,\, \sinh\rho(r)\sin\Theta_b,\, \sinh\eta_s\cosh\rho(r))$$

$$\bar{r} = r/R(\Theta)$$

$$\rho(\bar{r},\Theta_b) = \bar{r}\rho_0\,(1 + 2\rho_2\cos(2\Theta_b))$$

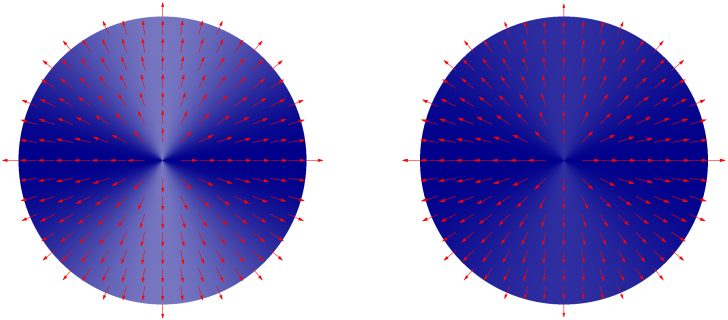
Identified $v_2(p_t)$ for different species allows resolving them.

Detour comment: the (STAR parametrisation) blast-wave formula for v_2

This is sometimes referred to as "the blast-wave formula":

$$v_2 = \frac{\int_0^{2\pi} d\phi \, (1 + 2s_2 \cos(2\phi)) \, \cos(2\phi) \, I_2\left(\frac{p_t \sinh \eta_t(\phi)}{T}\right) K_1\left(\frac{m_t \cosh \eta_t(\phi)}{T}\right)}{\int_0^{2\pi} d\phi \, (1 + 2s_2 \cos(2\phi)) \, I_0\left(\frac{p_t \sinh \eta_t(\phi)}{T}\right) K_1\left(\frac{m_t \cosh \eta_t(\phi)}{T}\right)}$$

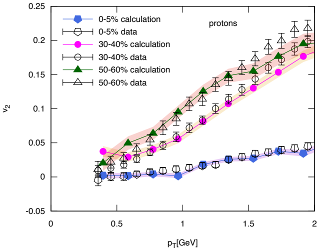
azimuthal variation of *the average* transverse velocity: $\eta_t = \bar{\rho}_0(1 + 2\rho_2 \cos(2\phi))$

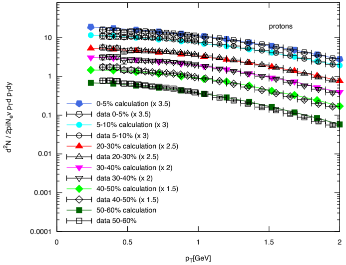


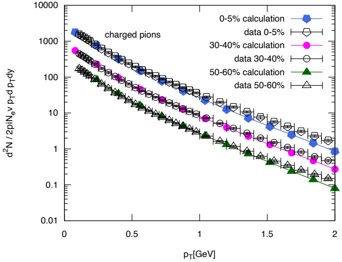
Model calibration

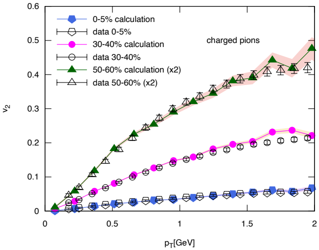
centrality	T[MeV]	ρ_0	R_0[fm]	s_0[fm/c]	a_2	ρ_2
0-5%	95	0.98	15.0	21 ± 2	0.016	0.008
30-40%	106	0.91	10.0	9 ± 1	0.085	0.03
50-60%	118	0.80	6.0	6 ± 0.5	0.15	0.02

$s_2 = -0.02\ \mathrm{fm}^{-1}$

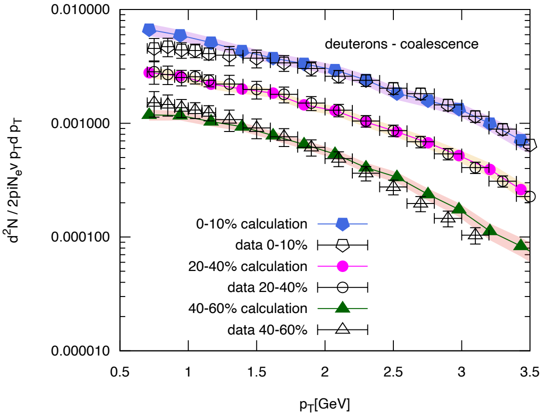


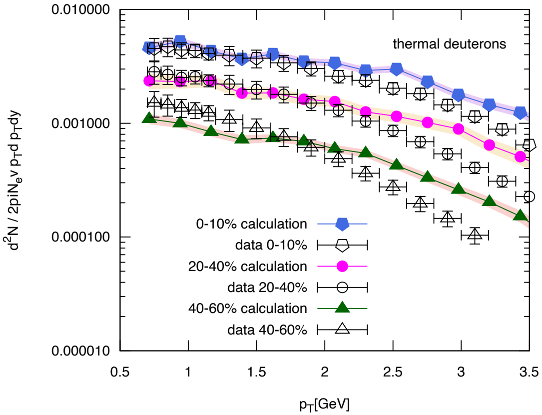




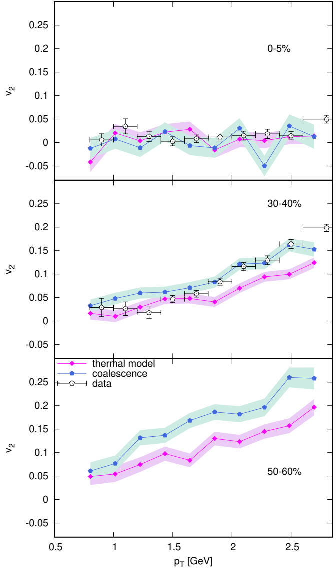


Results for deuterons: p_t spectra

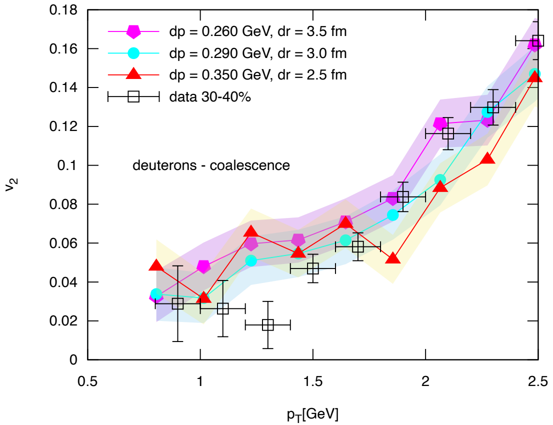




Results for deuterons: v_2



Coalescence for $\Delta p < \Delta p_{max}$ and $\Delta r < \Delta r_{max}$.



A hybrid simulation

Pb+Pb collisions, $\sqrt{s_{NN}} = 2.76$ TeV

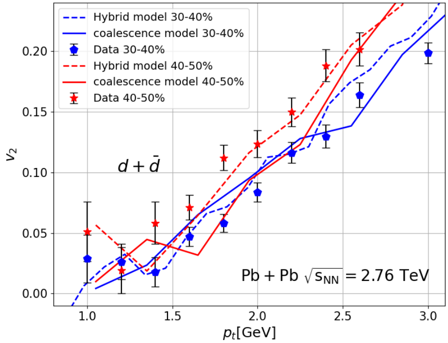
Model setup

- initial condition: TRENTO 3D
  [W. Ke, *et al.*, Phys. Rev. C **96**, 044912 (2017)]
- hydrodynamic simulation: vHLLE
  [I. Karpenko, *et al.*, Comput. Phys. Commun **185**, 3016–3027 (2014)]
- transport afterburner; SMASH
  [J. Weil *et al.* [SMASH collab], Phys.Rev.C 94,, 054905 (2016)]

Tuned to reproduce light hadron p_t spectra and v_2
Simulated 500×3000=1.5×10^6 events

more details
→ flash talk and poster by Tomáš Poledníček

Elliptic flow of deuterons



Conclusions and outlook

- Deuteron (and cluster) production is a femtoscopic probe

- Elliptic flow: can it resolve he mechanism of cluster production?
  - Extended blast-wave model: YES
  - Hybrid simulation: probably not (why?)
  - Simulations of larger clusters? (statistically hungry)
  - Simulations at lower energies—more clusters produced

[R. Vozábová, B. Tomášik, Phys. Rev. C 109, 064908 (2024), arXiv:2402.06327]