Gábor Kasza

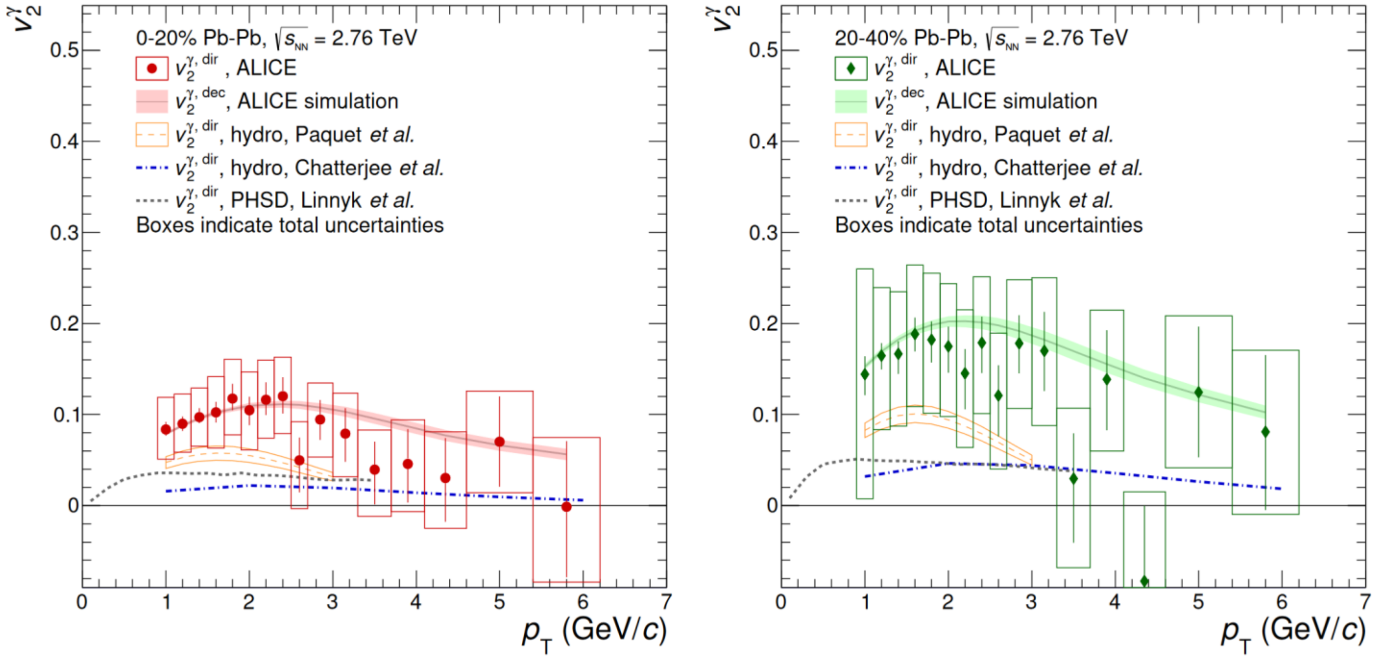
24th Zimányi Winter School

Budapest, 04/12/2024

Describing the thermal radiation with a two-component hydrodynamic model

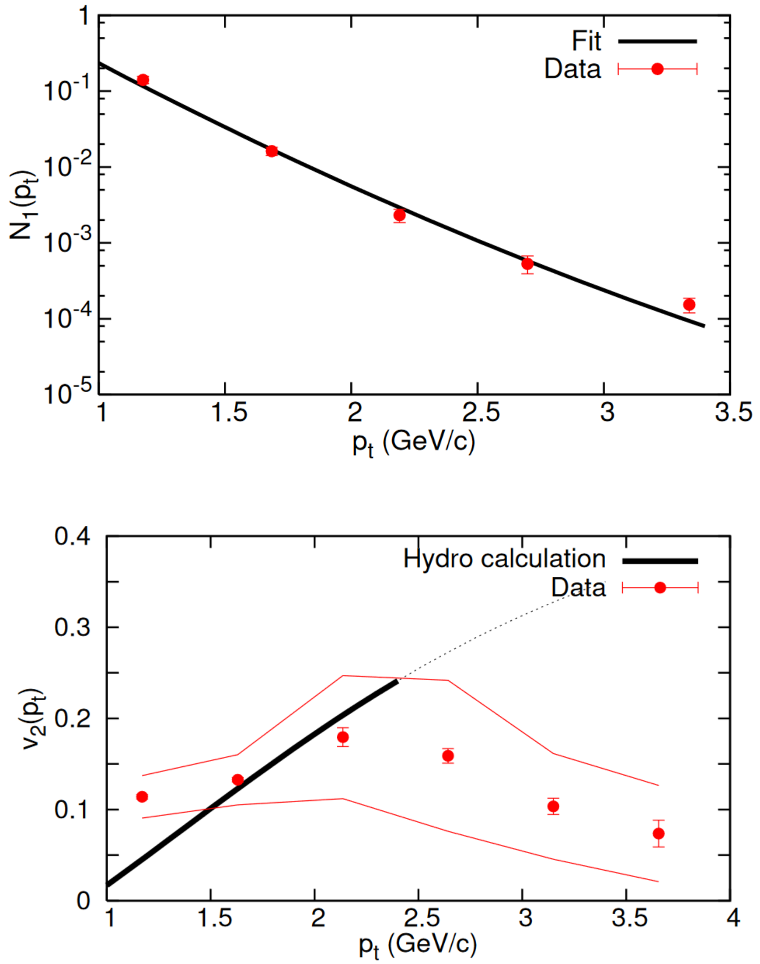
Motivation

- ***Direct photon puzzle***: the measured v_2 of direct photons is of the same order of magnitude as for hadrons.
- v_2 cannot be described simultaneously with direct photon spectra using the theoretical models known so far.



Phys.Lett.B 789 (2019) 308-322

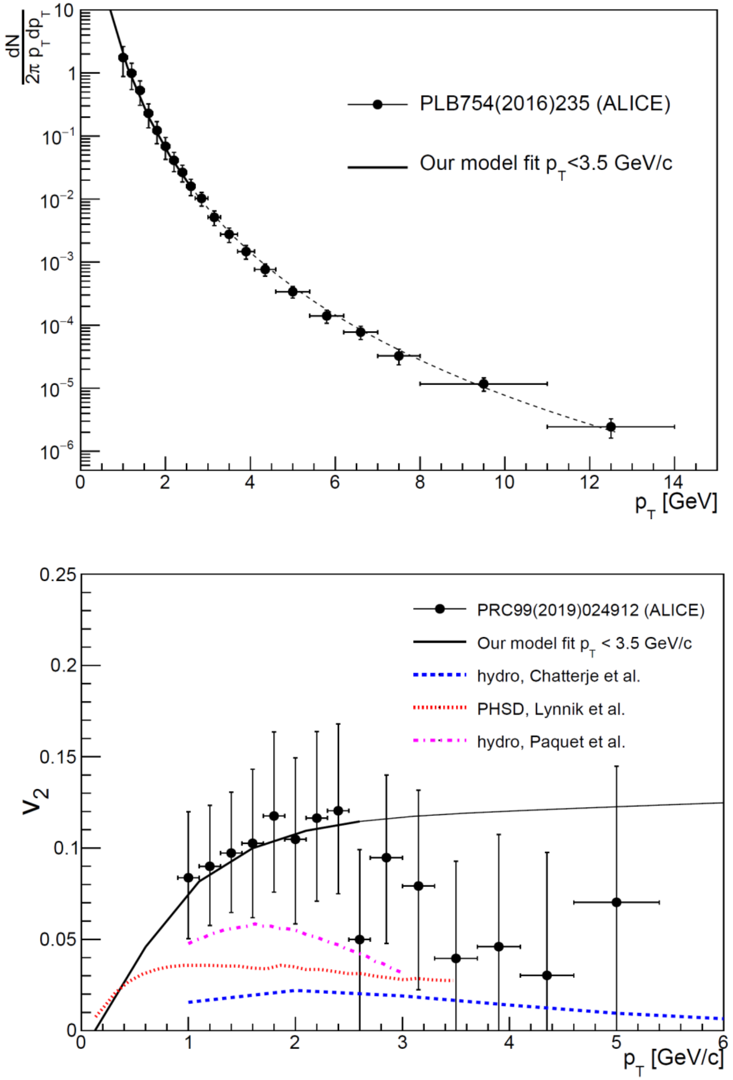
Earlier success of analytic hydro



- Based on the Csörgő-Csernai-Hama-Kodama solution of relativistic hydro.
  Acta Phys.Hung.A 21 (2004) 73-84
- No acceleration, but 1+3d.
- Gaussian temperature profile.
- ***Analytic calculation*** of spectrum and v_2 using second-order saddle-point approximation.
- Fitted to PHENIX Au+Au @ 200 GeV data.
- $T_0 > 507 \pm 12$ MeV

M. Csanád and I. Májer: Central Eur.J.Phys. 10 (2012) 850-857

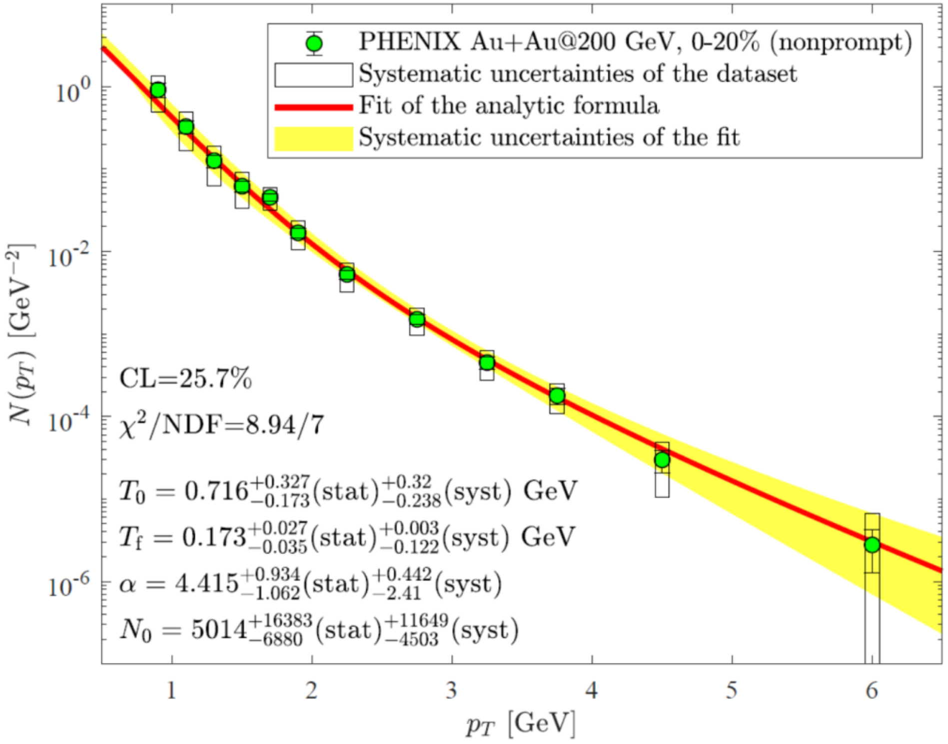
Recent successes of analytic hydro 1



- Same model (based on CCHK solution, Gaussian temperature, 1+3d, no acceleration), but:
- ***Numeric calculation*** of observables to avoid analytic approximations.
- Fitted to ALICE Pb+Pb @ 2.76 GeV data.
- ***v_2 and spectrum were fitted simultaneously.***
- T_0 = 418 ± 31 MeV

S. Lökös and G. K.: submitted to EPJA

Recent successes of analytic hydro 2



- *Scaling behaviour of data* has been found.

- Based on the relativistic hydrodynamic solution of Csörgő, Kasza, Csanád and Jiang.

T. Csörgő, G.K., M. Csanád, Z. Jiang: *Universe* 4 (2018) 6, 69

- ***Locally accelerating velocity field***, inhomogeneous temperature, but only 1+1d.
- ***Analytic calculation*** of spectrum using saddle-point approximation.
- Fitted to the non-prompt component of PHENIX Au+Au @ 200 GeV data.
- ***Non-prompt component***: dominated by hydrodynamic evolution.

G. K.: *Int.J.Mod.Phys.A* 39 (2024) 01, 2450006

New 1+1d model with generalized EoS

- Same 1+1d model (based on the CKCJ solution, accelerating velocity field, inhomogeneous temperature), but:
- Generalized for ***a broadened class of EoS that contains lQCD EoS***.
- The spectrum is embedded to the 1+3d space, but v_2 ***cannot be calculated***.
- The spectrum has a
  - low temperature component ($T < T_c$ let's call it hadronic component),
  - high temperature component ($T > T_c$ let's call it QGP component).

Equation of State

- EoS: $\mu=0$, $\boldsymbol{\varepsilon=\kappa(T)p}$
- ***Requiring temperature inhomogeneity***: strong constraint for $\kappa(T)$:

$$\frac{d}{dT}\left[\frac{\kappa(T)T}{1+\kappa(T)}\right]=\frac{c_Q}{1+\kappa(T)} \qquad T>T_c,\ \kappa(T)=\kappa_Q(T)$$

$$\frac{d}{dT}\left[\frac{\kappa(T)T}{1+\kappa(T)}\right]=\frac{c_H}{1+\kappa(T)} \qquad T<T_c,\ \kappa(T)=\kappa_H(T)$$

- Solutions for $\kappa(T)$:

$$\kappa_Q(T)=\frac{c_Q\left(\frac{T}{T_c}\right)^{1+c_Q}+\frac{\kappa_c-c_Q}{\kappa_c+1}}{\left(\frac{T}{T_c}\right)^{1+c_Q}-\frac{\kappa_c-c_Q}{\kappa_c+1}}$$

→ $c_Q = \kappa(T \gg T_c)$

c_H controls the peak of $\kappa(T)$ ←

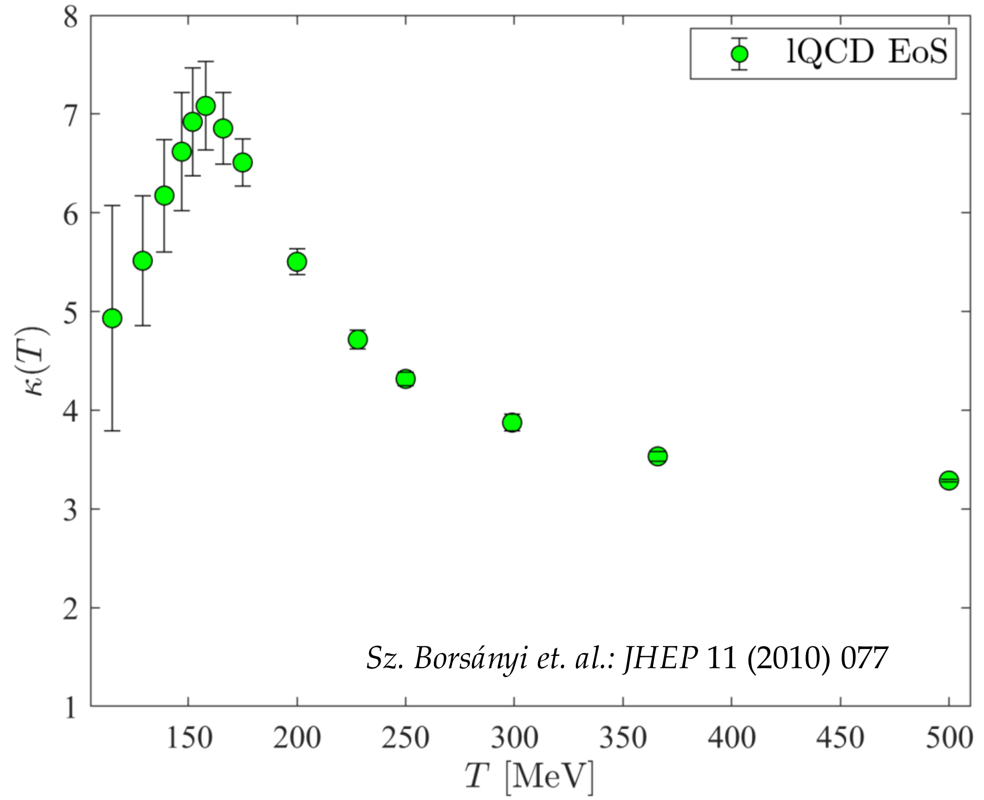
$$\kappa_H(T)=\frac{c_H\left(\frac{T}{T_f}\right)^{1+c_H}-\frac{c_H-\kappa_f}{\kappa_f+1}}{\left(\frac{T}{T_f}\right)^{1+c_H}+\frac{c_H-\kappa_f}{\kappa_f+1}}$$

Parametrization of EoS

- $\kappa_Q(T)$ and $\kappa_H(T)$ are matched at T_c:

$$\kappa(T) = \Theta(T - T_c)\kappa_H(T) + \Theta(T_c - T)\kappa_Q(T)$$

- Main goal: ***mimic the lQCD EoS***
- $T_f = 124$ MeV: extracted from slopes of hadronic p_T spectra
- $\kappa_f = \kappa(T_f) \approx 5.5$
- $T_c \approx 160$ MeV
- $\kappa_c = \kappa(T_c) \approx 7$
- $\kappa(T >> T_c) \approx 3.25$



Temperature and Source Function

- Analytic solutions for the temperature:

$$T(\tau,\eta_z) = T_0 \left(\frac{\tau_0}{\tau}\right)^{\frac{\lambda}{c_H}} \left[1 + \frac{c_H - 1}{\lambda - 1}\sinh^2\left(\Omega - \eta_z\right)\right]^{-\frac{\lambda}{2c_H}} \qquad T < T_c$$

$$T(\tau,\eta_z) = T_0 \left(\frac{\tau_0}{\tau}\right)^{\frac{\lambda}{c_Q}} \left[1 + \frac{c_Q - 1}{\lambda - 1}\sinh^2\left(\Omega - \eta_z\right)\right]^{-\frac{\lambda}{2c_Q}} \qquad T > T_c$$

- Source function:

$$S\left(x^\mu, p^\mu\right) d^4x = \frac{g}{(2\pi\hbar)^3} \frac{H(\tau)}{\tau_{\mathrm{R}}} \frac{p_\mu d\Sigma^\mu}{\exp\left(\frac{p^\mu u_\mu}{T}\right) - 1}$$

- H(τ): opacity for photons
(step-function, photons do not participate in strong interaction)

Spectrum

- Two-component direct photon spectrum:

$$N(p_T) = \frac{dN}{2\pi p_T dp_T} = \underbrace{\int_{\tau_c}^{\tau_f} N(p_T,\tau,c_H)d\tau}_{\text{Low- T (hadronic) component}} + \underbrace{\int_{\tau_0}^{\tau_c} N(p_T,\tau,c_Q)d\tau}_{\text{High-T (QGP) component}}$$

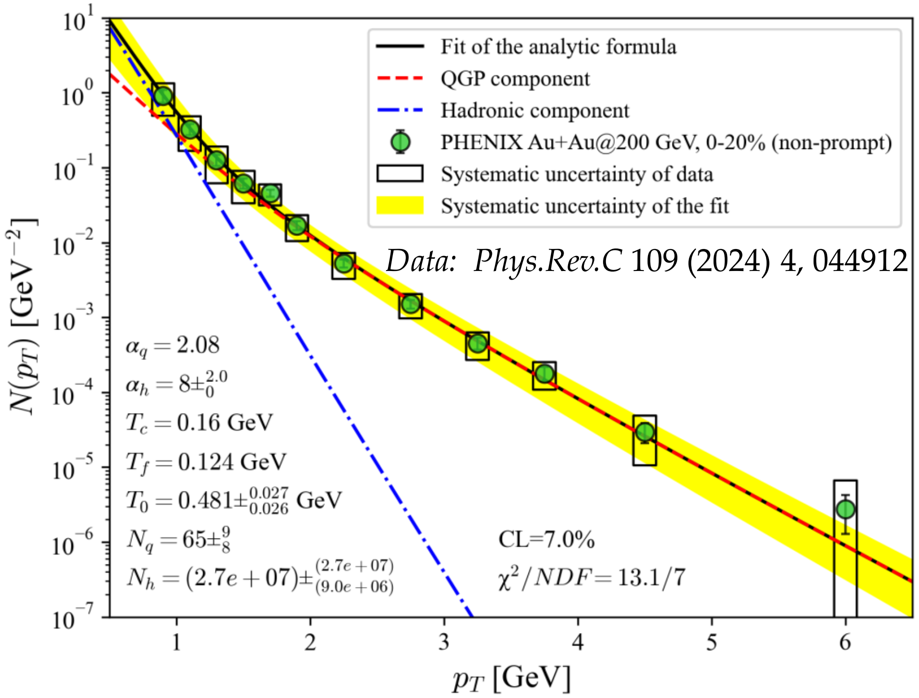
- ***Low-T component*** (where α_H=2c_H/λ-3 and λ is fixed):

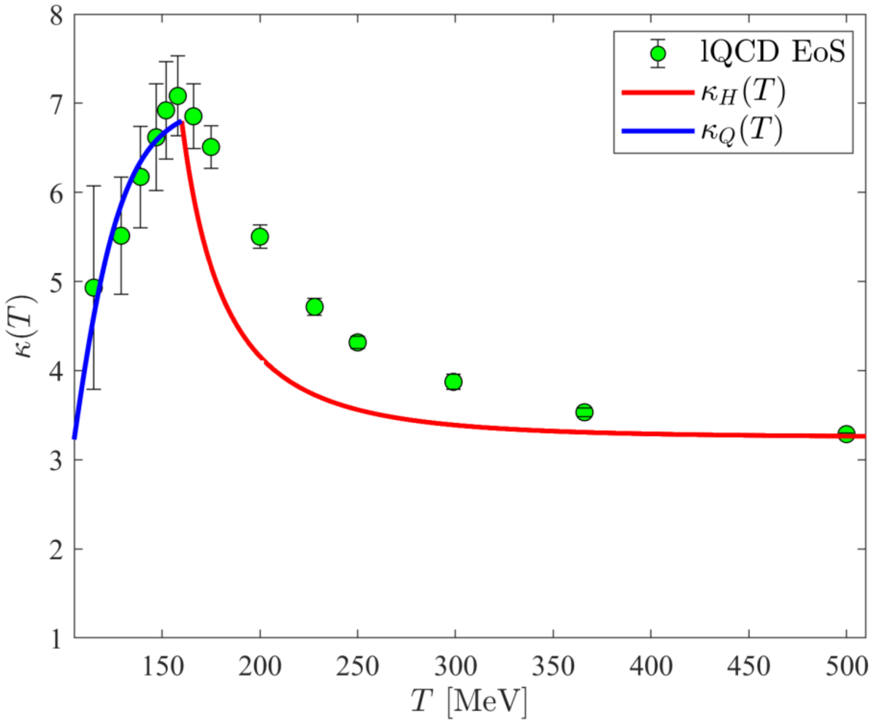
$$\left.\frac{d^2N_H}{2\pi p_\mathrm{T} dp_\mathrm{T} dy}\right|_{y=0} = N_{0,H}\,\frac{2\alpha_H}{3\pi^{3/2}}\left[\frac{1}{T_\mathrm{f}^{\alpha_H}} - \frac{1}{T_c^{\alpha_H}}\right]^{-1} p_\mathrm{T}^{-\alpha_H-2}\,\Gamma\left(\alpha_H+\frac{5}{2},\frac{p_\mathrm{T}}{T}\right)\Bigg|_{T=T_\mathrm{f}}^{T=T_c}$$

- ***High-T component*** (where α_Q=2c_Q/λ-3 and λ fixed):

$$\left.\frac{d^2N_Q}{2\pi p_\mathrm{T} dp_\mathrm{T} dy}\right|_{y=0} = N_{0,Q}\,\frac{2\alpha_Q}{3\pi^{3/2}}\left[\frac{1}{T_\mathrm{c}^{\alpha_Q}} - \frac{1}{T_0^{\alpha_Q}}\right]^{-1} p_\mathrm{T}^{-\alpha_Q-2}\,\Gamma\left(\alpha_Q+\frac{5}{2},\frac{p_\mathrm{T}}{T}\right)\Bigg|_{T=T_c}^{T=T_0}$$

Results

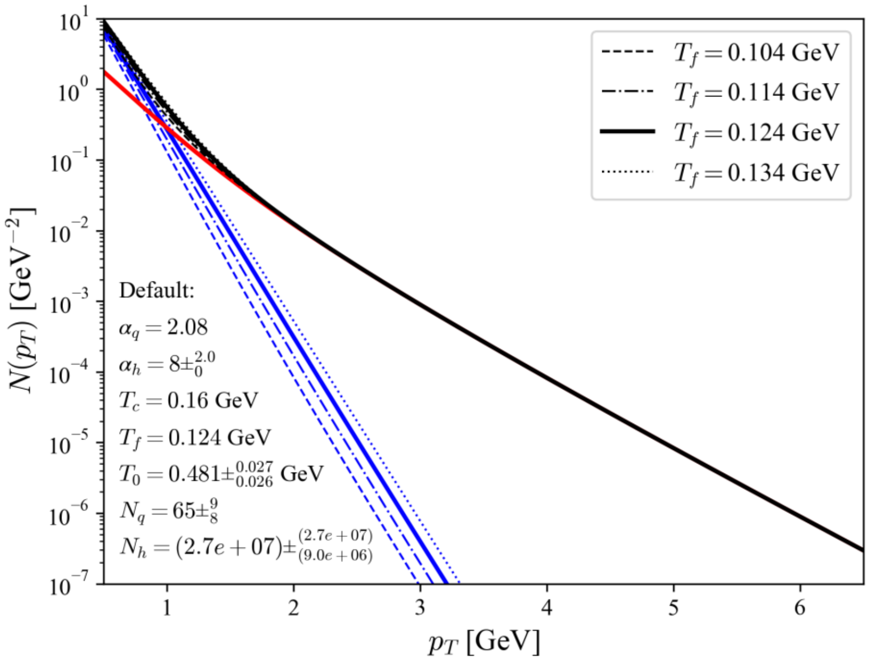


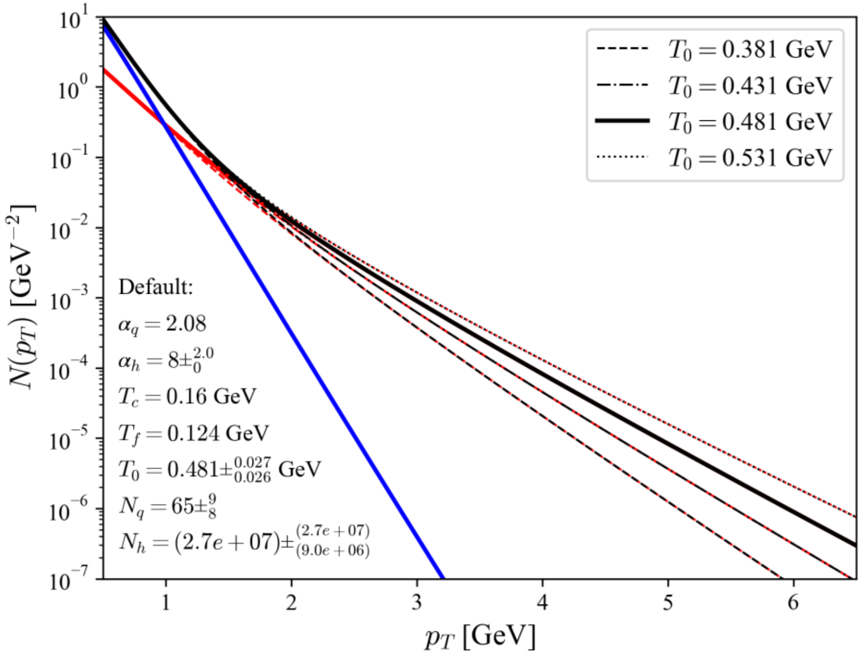


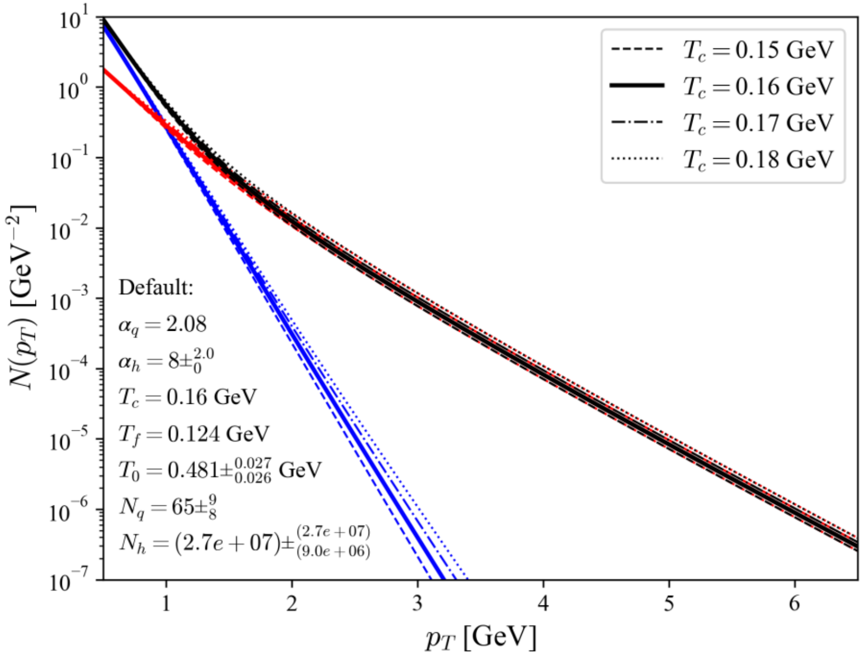
- EoS constrained by hydro eqs.: ***qualitatively similar to lQCD EoS.***
- Two component spectrum: ***quantitavely good description of data*** (CL=7.0%).
- Realistic value is obtained for the initial temperature.

Results

- Varying T_f (upper panel)
- Varying T_0 (lower left panel)
- Varying T_c (lower right panel)







Open questions

- Where simple analytic hydrodynamic models succeed, more complex models struggle.
  - → *Hydro ensures the thermalization of the system.*
- Will the 1+3d hydrodynamic model be successful in describing the PHENIX data as well? *According to our preliminary results, yes.*
- Will the 1+1d two-component hydrodynamic model be successful in describing the ALICE data as well?
- Perfect fluid models works well.
  - → *Is the effect of viscosity negligible?*

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Thank you for your attention!

Backup slides

Simple 1+3d model (L&K)

- Source function:

$$S(x,p)d^4x = \frac{g}{(2\pi\hbar)^3}\frac{\Theta(\tau-\tau_0)-\Theta(\tau-\tau_{\rm f})}{\tau_{\rm R}}p^\mu u_\mu \exp\left(\frac{p^\mu u_\mu}{T}\right) dt\, d^3x$$

- Using the CCHK solution:

$$u_\mu = \gamma\,(1,\mathbf{v}) = \gamma\left(1,\frac{\mathbf{r}}{t}\right)$$ *Hubble-type velocity field*

$$T(\tau,s) = T_{\rm f}\left(\frac{\tau_{\rm f}}{\tau}\right)^{3/\kappa}\mathcal{T}(s)$$ *Inhomogeneous temperature profile*

$$\mathcal{T}(s) = \exp(-bs/2)$$ *Scale function is chosen to be Gaussian*

- Scale variable:

$$s = \frac{r^2}{R^2}\left(1+\epsilon_2\cos(2\alpha)\right)+\frac{r_z^2}{Z^2}$$

$$\epsilon_2 = \frac{Y-X}{Y+X}$$

$$\frac{1}{R^2} = \left(\frac{1}{X^2}+\frac{1}{Y^2}\right)$$

Simple 1+3d model: observables

- Invariant transverse momentum spectrum:

$$N_1(p_\mathrm{T},\phi) = E\left.\frac{d^3N}{dp^3}\right|_{p_z=0} = \left.\frac{d^3N}{d\phi dp_\mathrm{T} dy}\right|_{y=0} = \int S(t,r,\alpha,r_z,p_\mathrm{T},\phi)dt\, r d\alpha\, dr\, dr_z$$

$$\left.\frac{d^2N}{p_\mathrm{T} dp_\mathrm{T} dy}\right|_{y=0} = \int_0^{2\pi} d\phi N_1(p_\mathrm{T},\phi)$$

- Elliptic flow:

$$v_2(p_\mathrm{T}) = \frac{\int_0^{2\pi} d\phi \cos(2\phi) N_1(p_\mathrm{T},\phi)}{\int_0^{2\pi} d\phi N_1(p_\mathrm{T},\phi)}$$

Simultaneous fit

- Dataset: ALICE Pb+Pb@2.76 TeV, 0-20%

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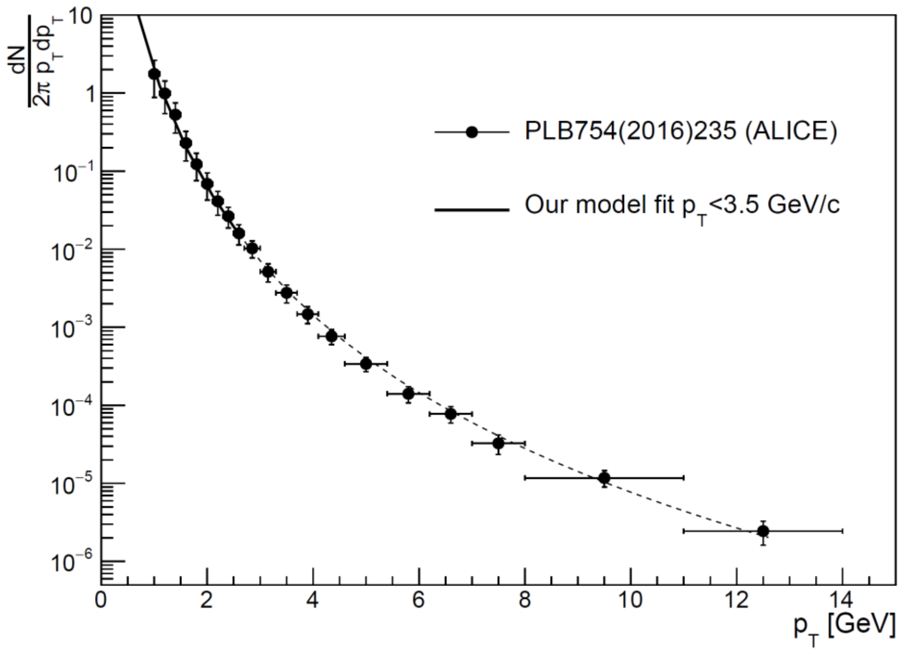

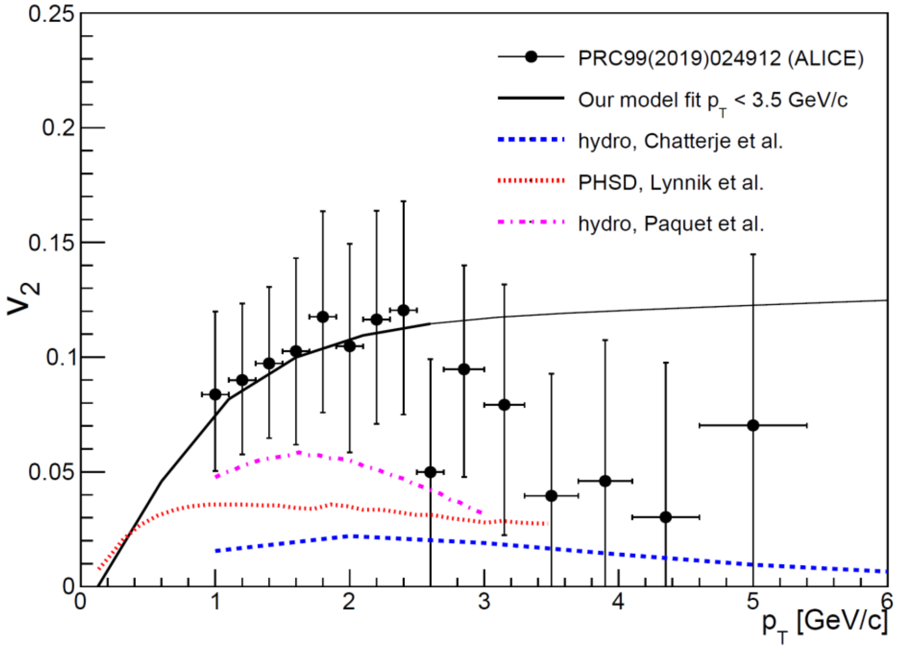

- Our model works better than the more complex microscopic models.
- Problem: why our model works on the whole p_T-range?