

Early time nonequilibrium heavy quark diffusion and energy loss



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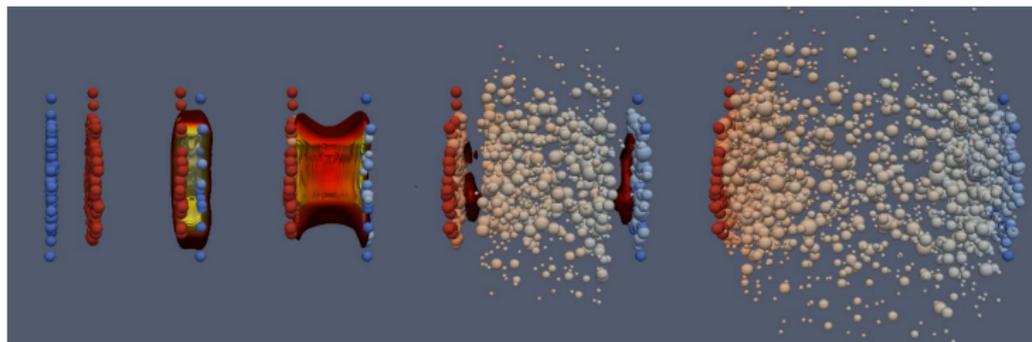
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- 2 Initial stages
- 3 Hard probes and transport coefficients
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Stages in heavy-ion collisions



MADAI collaboration

- High-energy collisions \Rightarrow **QGP** created
- Cooling during evolution, go through different **phases**
 - \Rightarrow pre-equilibrium QGP (initial stages) \rightarrow fluid QGP \rightarrow hadrons
- **Pre-equilibrium QGP**: testing the very nature of quantum physics
 - \Rightarrow Gluons first as (classical) waves \rightarrow scatterings of (quasi-)particles

Goals

Learn about **real-time** properties of QCD in **extreme conditions**

Non-equilibrium QCD

What are the initial stages of the quark-gluon plasma (QGP)?

- Significant progress from QCD calculations over the past decade(s)
- Interplay of different methods and models

Experimental traces

How can we probe them experimentally? What are their signatures?

- What are the medium properties of the pre-equilibrium QGP?
- How do they affect hard probes? What do we learn?

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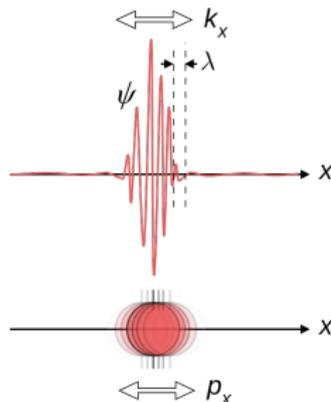
Pre-equilibrium QGP: descriptions (for couplings $g^2 \ll 1$)

- initially: **classical-statistical simulations**, 'Glasma'
 - \Rightarrow large gluon fields $A \sim 1/g$, full initial conditions
 - \Rightarrow nonlinear dynamics of interacting classical waves
 - \Rightarrow Valid while occupancies large $f(t, p) \approx \langle AA \rangle p \gg 1$
- then, as energy decreases (dilution): **kinetic theory**
 - \Rightarrow Boltzmann equation for f

$$(\partial_t + \mathbf{v} \cdot \nabla) f = \left| \begin{array}{c} \text{---} \\ \diagdown \\ \text{---} \\ \text{---} \\ \diagup \\ \text{---} \end{array} \right|^2 + \left| \begin{array}{c} \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \\ \text{---} \end{array} \right|^2$$

$$\frac{\partial f_{\vec{p}}}{\partial \tau} - \frac{p_z}{\tau} \frac{\partial f_{\vec{p}}}{\partial p_z} = -C^{2 \leftrightarrow 2}[f_{\vec{p}}] - C^{1 \leftrightarrow 2}[f_{\vec{p}}]$$

Arnold, Moore, Yaffe, JHEP 01, 030 (2003)



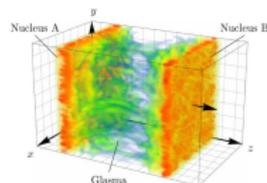
Quantum wave particle duality, approximative descriptions

classical fields $A(t, \vec{x})$ ('waves') \rightarrow interacting particle distribution $f(t, \vec{p})$

Strong initial fields: classical-statistical lattice simulations

- Initial state: **Glasma** – large longitudinal fields

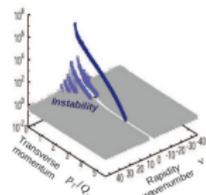
McLerran, Venugopalan (1999); Krasnitz, Venugopalan (1999, 2000, 2001); Krasnitz, Nara, Venugopalan (2001, 2003); Lappi (2003, 2006, 2011); Lappi, McLerran (2006); Schenke, Tribedy, Venugopalan (2012); Gelfand, Ipp, Müller (2016, 2017); ...



Ipp, Müller (2017)

- **Plasma instabilities** – from boost-invariant Glasma to highly occupied (mainly gluonic) plasma

Mrowczynski (1993); Arnold, Lenaghan, Moore (2003); Romatschke, Strickland (2003); Romatschke, Venugopalan (2006); Attems, Rebhan, Strickland (2012); Fukushima, Gelis (2012); Berges, KB, Schlichting, (2012, 2013); Epelbaum, Gelis (2013); ...



Berges, Schenke, Schlichting, Venugopalan (2014)

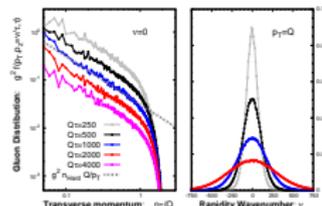
- Classical **self-similar attractor** far from equilibrium – universal dynamics of over-occupied plasma

⇒ agrees with 1. stage of 'bottom-up' scenario

Berges, KB, Schlichting, Venugopalan (2013, 2014); Kurkela, Zhu (2015); ...

⇒ Far-from-equilibrium universality class with scalars

Berges, KB, Schlichting, Venugopalan (2015); ...



Berges, KB, Schlichting, Venugopalan (2013)

Bottom-up thermalization: QCD kinetic theory

- When **quasiparticles** have formed:
Kinetic theory becomes applicable

Note: Assumes narrow excitations in spectral functions, which may not be true at low momenta for strong anisotropy
KB, Kurkela, Lappi, Peuron (2018, 2019, 2021)

- Bottom-up** thermalization: Baier, Mueller, Schiff, Son (2001)

- 1 Classical attractor (see above)
- 2 Anisotropy freezes
- 3 Radiational breakup

- QCD effective **kinetic theory** (EKT) simulations

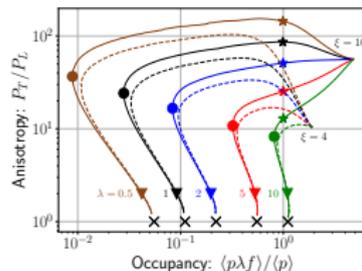
Arnold, Moore, Yaffe (2003); Kurkela, Zhu (2015); Kurkela, Mazeliauskas (2019);

$$-\frac{\partial f_{\vec{p}}}{\partial \tau} = C^{1\leftrightarrow 2}[f_{\vec{p}}] + C^{2\leftrightarrow 2}[f_{\vec{p}}] - \frac{p_z}{\tau} \frac{\partial f_{\vec{p}}}{\partial p_z}$$

- EKT: smooth transition to **hydrodynamics**;
KoMPoST: EKT + $\delta T^{\mu\nu}(\tau, \vec{x})$ perturbations

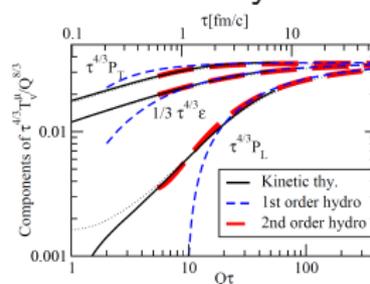
Kurkela, Zhu (2015); Kurkela, Mazeliauskas, Paquet, Schlichting, Teaney (2018)

Bottom-up evolution



Kurkela, Zhu (2015); KB, Kurkela, Lappi, Lindenbauer, Peuron (2023)

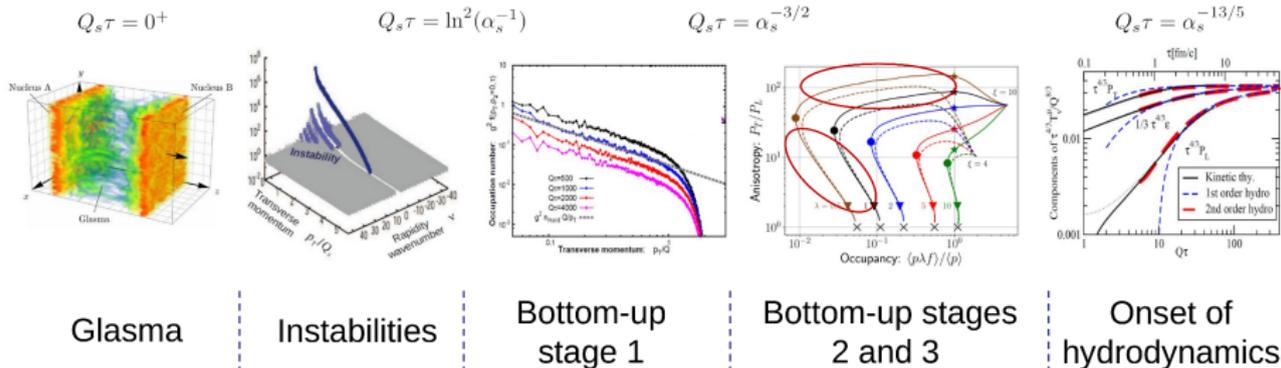
Onset of hydro



Kurkela, Zhu (2015)

⇒ Talk by G. Denicol

Initial stages in heavy-ion collisions (weak- g^2 perspective)



$$D_\mu F^{\mu\nu} = J^\nu$$

classical-statistical simulations

$$-\frac{\partial f_{\vec{p}}}{\partial \tau} = \mathcal{C}^{1 \leftrightarrow 2}[f_{\vec{p}}] + \mathcal{C}^{2 \leftrightarrow 2}[f_{\vec{p}}] - \frac{p_z}{\tau} \frac{\partial f_{\vec{p}}}{\partial p_z}$$

QCD effective kinetic theory simulations

hydrodynamics ...

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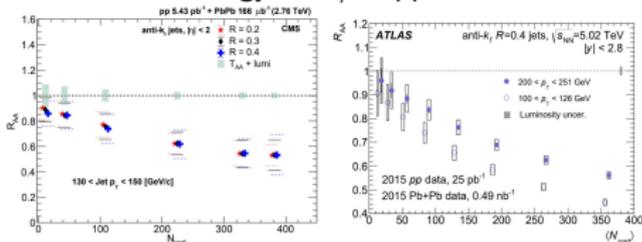
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Hard probes show signatures of QGP

Hard probes are modified while traversing the QGP

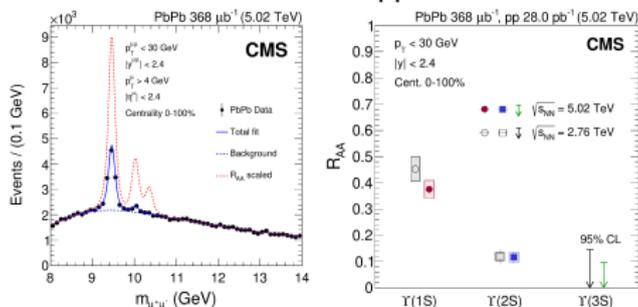
Examples: heavy quarks, quarkonia ($q\bar{q}$), jets ($p \gg T$)

Jet energy loss / suppression



CMS Collaboration, PRC (2017) ; ATLAS Collaboration, PLB (2019)

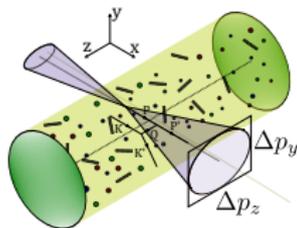
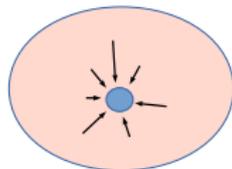
Bottomonium suppression



Transport coefficients from pre-equilibrium QGP

Jets, heavy (c, b) quarks: potential for signatures of initial stages
medium interactions \Rightarrow QGP properties encoded in observables

- Quarks/jets get 'kicks' $\dot{p}_i(\tau) = \mathcal{F}_i(\tau)$
- **Heavy-quark diffusion** coefficient $\kappa_i = \frac{d}{d\tau} \langle p_i^2 \rangle$
 \Rightarrow heavy quark (c, b), small momentum $p \ll M$
- κ enters Lindblad eq. for **quarkonium dynamics**
Brambilla, Escobedo, [Soto], [Strickland], Vairo, [v.d. Griend, Weber] (2016, 2021)
 \Rightarrow describe suppression of bottomonium ($b\bar{b}$ states)
- **Jet quenching** parameter $\hat{q}_i = \frac{d}{d\tau} \langle p_{\perp,i}^2 \rangle$
 \Rightarrow jet with high momentum $p \gg Q_s, T$
- They **encode** also pre-equilibrium dynamics



KB, Kurkela, Lappi, Lindenbauer, Peuron (2023)

Impact of initial stages on hard probes

Transport coefficients encode pre-equilibrium dynamics

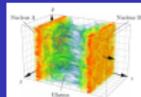
- Impact on jet energy loss, substructure? \Rightarrow Talk by Joao Barata
- Pheno. impact of $\kappa_i(\tau)$ on heavy quarks? \Rightarrow Talk by Pooja

Some recent studies:

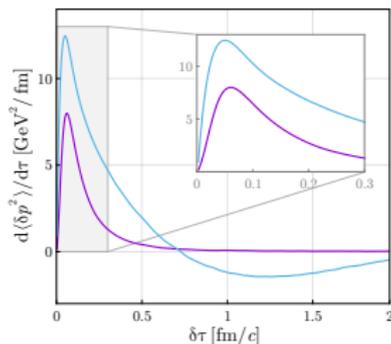
Transport coefficients during initial stages + impact on hard probes

Mrowczynski (2018); Ruggieri, Das (2018); Sun, Coci, Das, Plumari, Ruggieri, Greco (2019); Ipp, Müller, Schuh (2020); KB, Kurkela, Lappi, Peuron (2020); Khowal, Das, Oliva, Ruggieri (2022); Carrington, Czajka, Mrowczynski (2020, 2022); Avramescu, Baran, Greco, Ipp, Müller, Ruggieri (2023); KB, Kurkela, Lappi, Lindenbauer, Peuron (2023); Du (2023); Barata, Sadofyev, Wang (2023); Andres, Apolinário, Dominguez, Martinez, Salgado (2023); Pandey, Schlichting, Sharma (2024); Zhou, Brewer, Mazeliauskas (2024); Barata, Hauksson, Lopez, Sadofyev (2024); Priyam Adhya, Tywoniuk (2024); ...

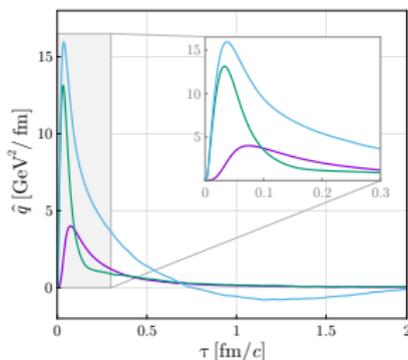
κ and \hat{q} during Glasma phase



κ_i of beauty quarks



\hat{q}_i of jets



Avramescu, Baran, Greco, Ipp, Müller, Ruggieri PRD 107, 114021 (2023); 2307.07999

- Classical-statistical simulations of hard probes in the **Glasma** phase

- Extraction of κ_i and \hat{q}_i

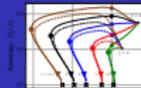
Ipp, Müller, Schuh (2020); KB, Kurkela, Lappi, Peuron (2020); Carrington, Czajka, Mrowczynski (2022); Khowal, Das, Oliva, Ruggieri (2022); Avramescu et al. (2023); Pandey, Schlichting, Sharma (2024); ...

- Often using $\kappa, \hat{q} \sim \int d\tau \langle \mathcal{FF} \rangle$, here via particle-in-cell method

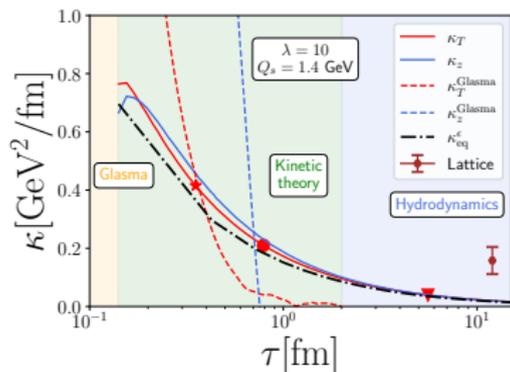
- **Large** values, **anisotropic** $\kappa_z > \kappa_T$ and $\hat{q}_z > \hat{q}_y$ (z is beam direction)

- Why are they large? Why do κ_z and \hat{q}_z become negative?

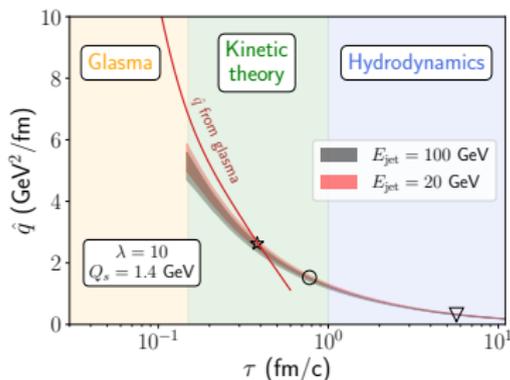
κ and \hat{q} during kinetic regime



κ of heavy quarks



\hat{q} for jet quenching



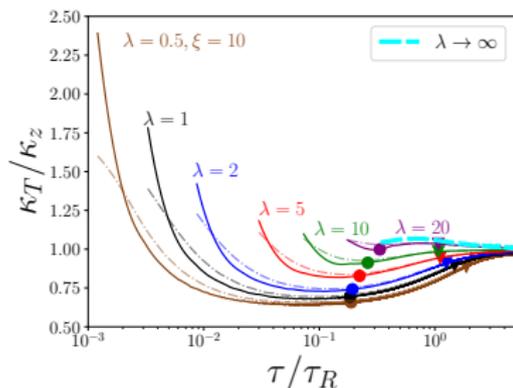
KB, Kurkela, Lappi, Lindenbauer, Peuron, for κ PRD [2303.12520];

for \hat{q} : Phys. Lett. B (2024) [2303.12595], PRD [2312.00447]

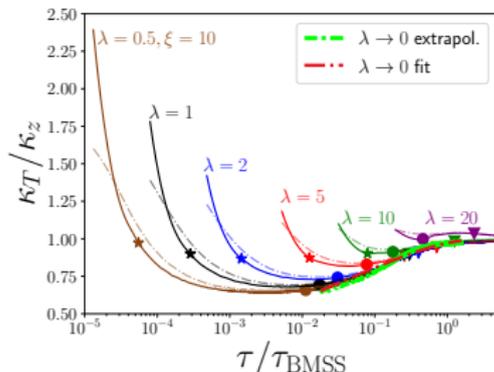
- \hat{q} smoothly connects Glasma and hydro, κ not so much
- Mostly the same ordering $\kappa_z > \kappa_T$ and $\hat{q}_z > \hat{q}_y$
- Evolution in kinetic regime understood, what about Glasma? (later)

Limiting attractors

Rescaling with $\tau_R = \frac{4\pi\eta/s(\alpha_s)}{T_\epsilon(\tau)}$



Rescaling with $\tau_{\text{BMSS}} = \alpha_s^{-13/5}/Q$



KB, Kurkela, Lappi, Lindembauer, Peuron, Phys. Lett. B (2024) [2312.11252]

- Limiting attractors from extrapolating coupling $\lambda = 4\pi N_c \alpha_s$
 - Hydrodynamic lim. attr. ($\lambda \rightarrow \infty$): very good description of P_L/P_T
 - Bottom-up lim. attr. ($\lambda \rightarrow 0$): early description of $\hat{q}^{yy}/\hat{q}^{zz}$, κ_T/κ_z
- ⇒ some 'ratios' better described with bottom-up even at $\lambda \sim \mathcal{O}(10)$
- Also: HQ drag and diffusion coeff. **scale at hydro attractor** (Du, PRC (2024))

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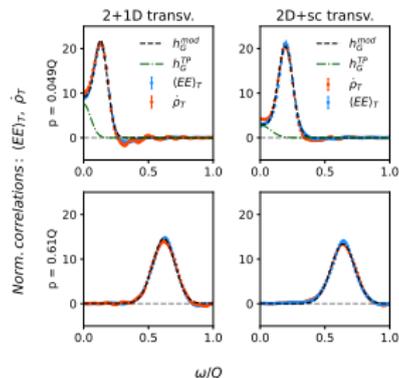
Towards understanding κ_i in the Glasma

L. Backfried, KB, P. Hotzy, 2408.12646

Connect to collective excitations in the pre-equilibrium QGP

- Spectral functions $\rho(t, \omega, p) \sim \langle [\hat{A}, \hat{A}] \rangle$ encode excitation spectrum!
- Compute $\langle EE \rangle$ in class.-stat. + algorithm for ρ (KB, Kurkela, Lappi, Peuron (2018))

Transv. polarization (w.r.t. \vec{p})



Models (non-exp. geometry)

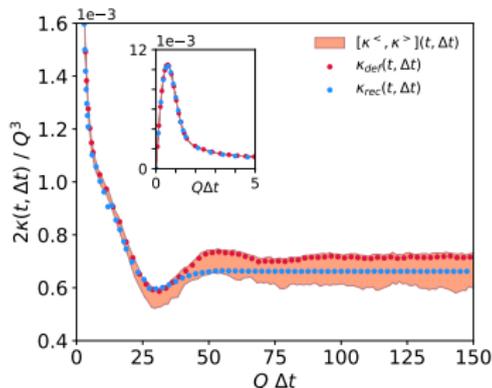
- 2+1D: Yang-Mills S_{YM}^{2D}
 - 2D+sc: S_{YM}^{2D} + adj. scalar A_z
- ⇒ Glasma-like but at classical attractor + Minkowski

extending [KB, Kurkela, Lappi, Peuron (2019, 2021)]

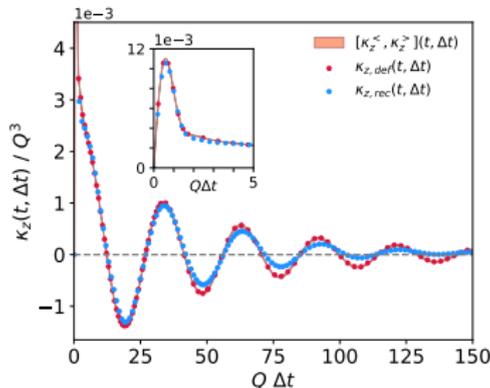
- HTL perturbation theory **breaks down** \Rightarrow broad Gaussian excitations
- New **transport peak** h_G^{TP} at $\omega = 0$ for $p \lesssim m_D \Rightarrow$ nonperturbative!

Heavy-quark diffusion coefficients in 2+1D plasmas

2+1D gluonic 2κ



Glasma-like scalar κ_z

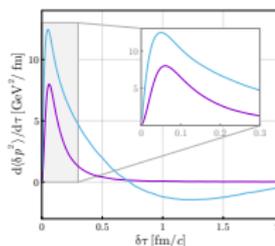


$$2\kappa(t, \Delta t) \approx \frac{g^2}{N_c} \int_t^{t+\Delta t} dt' \langle EE \rangle(t, t', \Delta \vec{x}=0), \quad \Rightarrow \text{gauge invariant}$$

$$\approx \frac{g^2}{N_c} \int_{\vec{p}} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\sin(\omega \Delta t)}{\omega} \sum_{\alpha=T,L} \langle EE \rangle_{\alpha}(t, \omega, p)$$

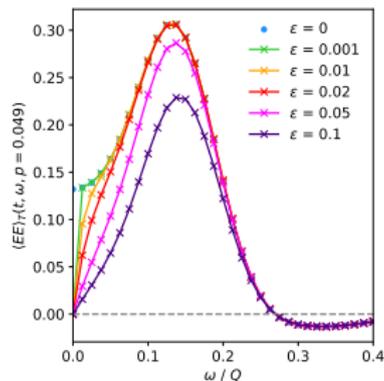
- Initial **linear** rise $\kappa_i \sim \Delta t \langle EE \rangle_i(t, t)$ (KB, Kurkela, Lappi, Peuron ('20))
- Qualitatively **similar to Glasma**: 2κ finite (diffusive), κ_z around 0
- Gauge-fixed **correlators** $\langle EE \rangle_{\alpha}(t, \omega, p)$ reconstruct evolution

Glasma reminder

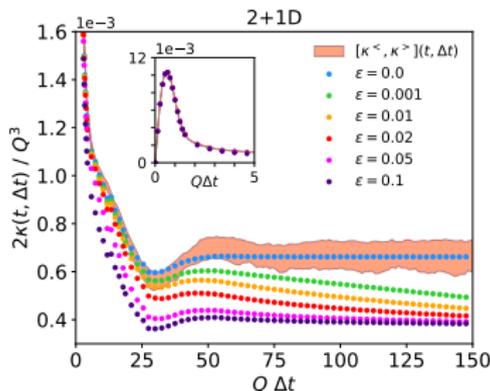


Manipulate correlations \Rightarrow study impact

Suppress low ω of $\langle EE \rangle_T$



2+1D gluonic 2κ

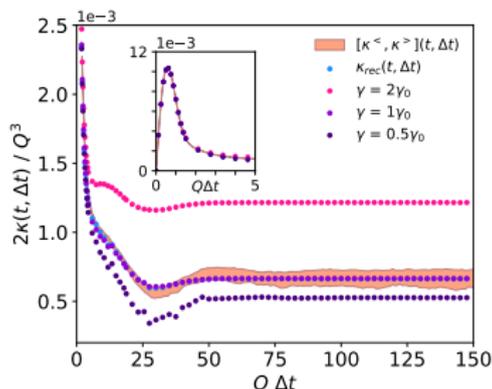


$$2\kappa(t, \Delta t) \approx \frac{g^2}{N_c} \int_{\vec{p}} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\sin(\omega\Delta t)}{\omega} \sum_{\alpha=T,L} \langle EE \rangle_{\alpha}(t, \omega, p)$$

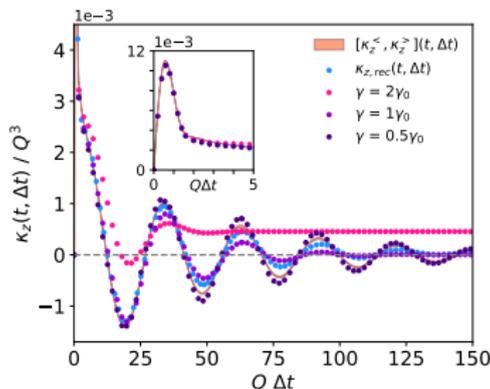
- Significant impact on late- Δt evolution
 - \Rightarrow Evidence of a new transport peak in Glasma-like systems!
- Preliminary: transport peak also in Glasma (KB, Hotzy, Müller, *in progress*)
 - \Rightarrow Enhanced transport coefficients, relevance for initial stages?

Manipulate correlations \Rightarrow study impact II

2+1D gluonic 2κ



Glasma-like scalar κ_z



$$2\kappa(t, \Delta t) \approx \frac{g^2}{N_c} \int_{\vec{p}} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\sin(\omega\Delta t)}{\omega} \sum_{\alpha=T,L} \langle EE \rangle_{\alpha}(t, \omega, p)$$

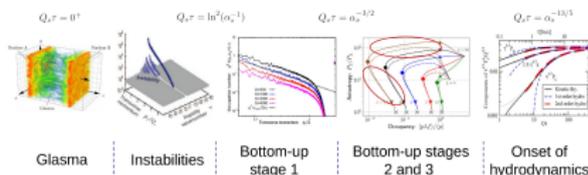
- Change peak width $\gamma \Rightarrow$ mismatch with simulations
 $\Rightarrow 2\kappa$ requires **broad** $\langle EE \rangle_T$ and κ_z **narrow** $\langle EE \rangle_z$
- We also demonstrate: scalars are **enhanced at low** $p \lesssim m_D$ (Backup)

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- **Initial stages** of the QGP

⇒ waves vs. particles



- **Hard probes** (jets, heavy quarks, quarkonia): medium interactions

⇒ Coefficients κ , \hat{q} in Glasma and bottom-up → large, anisotropic

⇒ Kinetics: hydrodynamic and limiting attractors useful

⇒ Phenomenology: Impact/signatures of pre-equilibrium dynamics?

- **Collective excitations** of pre-equilibrium QGP influence hard probes

⇒ Evidence of new transport peak in Glasma-like plasmas

⇒ Significant impact on transport coefficients

Thank you for your attention!

Backup slides

Kinetic theory \Rightarrow Bottom-up thermalization scenario

- **Bottom-up scenario**

Baier, Mueller, Schiff, Son, PLB (2001)

- Consists of **three stages**

- ① Classical attractor
- ② Anisotropy freezes
- ③ Radiational breakup

- Different bottom-up stages **separated by markers** ($\lambda = g^2 N_c$)

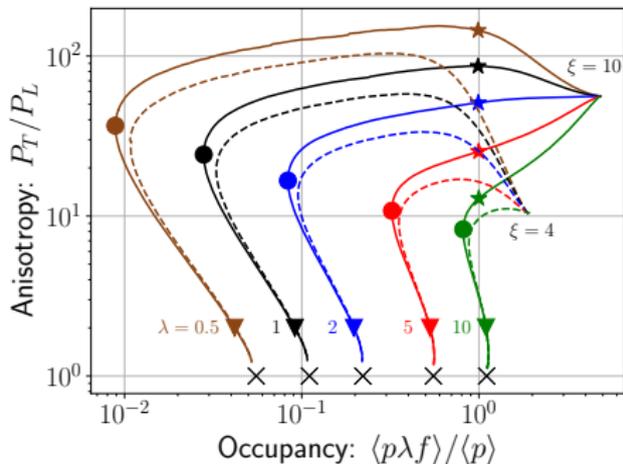
- ★ large pressure anisotropy

$P_T \gg P_L$, occupancy $f \sim 1/\lambda$

- minimum (mean) occupancy f

▽ close to isotropic $P_T/P_L = 2$

- Thermalization time scale $\tau_{\text{BMSS}} = \alpha_s^{-13/5}/Q_s$, initial momentum Q_s



Kurkela, Zhu (2015); version from:
KB, Kurkela, Lappi, Lindenbauer, Peuron (2023)

- Pressure $P_{T,L} \sim \int d^3p \frac{p_{\perp,z}^2}{p} f$
- Mean $\langle O \rangle = \int d^3p f(p) O(p)$

QGP description: effective kinetic theory (EKT)

- When quasiparticles have formed: Kinetic theory applicable

Note: Assumes narrow excitations in spectral functions, which may not be true at low momenta for strong anisotropy (Backup in 'ρ': 'Gluonic 2+1D')

KB, Kurkela, Lappi, Peuron (2018, 2019, 2021)

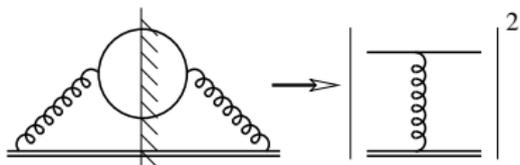
- Time evolution described by Boltzmann equation at LO

$$(\partial_t + \mathbf{v} \cdot \nabla) f = \left| \text{[Red vertex diagram]} \right|^2 + \left| \text{[Blue vertex diagram]} \right|^2$$

$$\frac{\partial f_{\vec{p}}}{\partial \tau} - \frac{p_z}{\tau} \frac{\partial f_{\vec{p}}}{\partial p_z} = -C^{2 \leftrightarrow 2}[f_{\vec{p}}] - C^{1 \leftrightarrow 2}[f_{\vec{p}}]$$

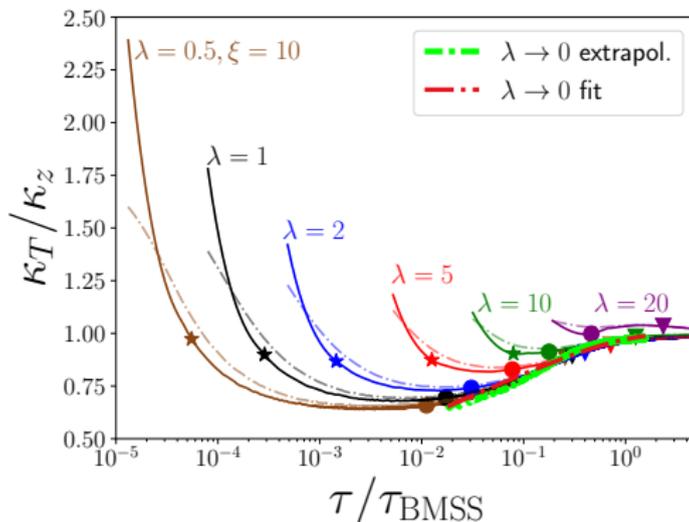
Arnold, Moore, Yaffe, JHEP 01, 030 (2003)

- Heavy-quark coefficient κ_i : Moore, Teaney (2005); Caron-Huot, Moore (2008)



Transverse vs. longitudinal diffusion

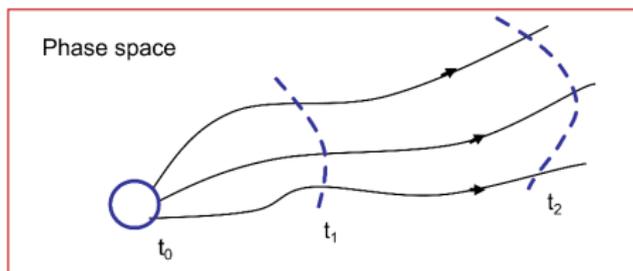
KB, Kurkela, Lappi, Lindenbauer, Peuron, 2303.12520



- $\kappa_T > \kappa_z$ during **over-occupied** stage (z is longitudinal/beam axis)
- $\kappa_T < \kappa_z$ after \star due to momentum anisotropy and **low occupancy**
- Most of the time $\kappa_T < \kappa_z$, anisotropy larger at weak coupling

Classical-statistical simulations

- At initial time t_0 set (quantum) **initial conditions** (IC):
⇒ Choose $\langle AA \rangle$, $\langle EE \rangle$ in x or p space
- Approximate quantum dynamics with **classical EOMs** $D_\mu F^{\mu\nu} = 0$
⇒ Gauge co-variant lattice formulation using links $U_j(x) = e^{ig a_j A_j(x)}$
- Obtain observables at t by **averaging** over trajectories (same IC)

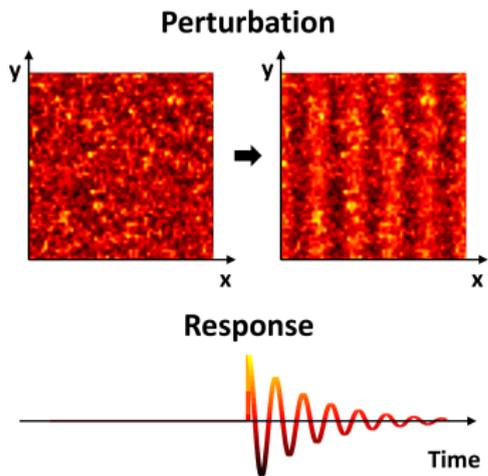


- Valid if occupancies large $f(t, p) \approx \langle AA \rangle p \approx \langle EE \rangle / p \gg 1$
- Applicability limited to earliest times!

Quasiparticles? Extract gluon spectral function ρ

Classical-statistical $SU(N_c)$ simulations + linear response theory

KB, Kurkela, Lappi, Peuron, *PRD 98, 014006 (2018)*



- Similar algorithm for fermions
- Split $A(t, \vec{x}) \mapsto A(t, \vec{x}) + \delta A(t, \vec{x})$ at t , perturb with plane wave $j_0(\vec{p}) \delta(t' - t)$
- Response $\langle \delta A(t', \vec{p}) \rangle = G_R(t', t, \vec{p}) j_0(\vec{p})$
- Linearized EOM for $\delta A(t, \vec{x})$ such that Gauss law conserved (also in gauge-cov. formulation)

Kurkela, Lappi, Peuron, *EJJC 76 (2016) 688*

- $\theta(t' - t) \rho(t', t, p) = G_R(t', t, p)$
- Fourier transform $\rho(\bar{t}, \omega, p)$ ($\bar{t} = \frac{1}{2}(t + t')$)

Very similar methods for scalars:

Aarts (2001); Piñeiro Orioli, Berges (2019); Schlichting, Smith, von Smekal (2020); KB, Piñeiro Orioli (2020); ...

Spectral and statistical correlation functions

- Equal-time correlator $\langle \{\hat{E}(t), \hat{E}(t)\} \rangle \propto f(t, \rho)$ is distribution
 \Rightarrow But what are the relevant **excitations**?
- Knowledge of **spectral function** needed ($\dot{\rho} = \partial_t \rho$, $E = \partial_t A$)

$$\dot{\rho}(x, x') = \frac{i}{N_c^2 - 1} \left\langle \left[\hat{E}(x), \hat{A}(x') \right] \right\rangle$$

- **Statistical correlator** $\langle EE \rangle$ ($\equiv \ddot{F}$) in general independent of $\dot{\rho}$

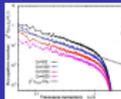
$$\langle EE \rangle(x, x') = \frac{1}{2(N_c^2 - 1)} \left\langle \left\{ \hat{E}(x), \hat{E}(x') \right\} \right\rangle$$

- Fourier transf. in $t - t'$ and $\vec{x} - \vec{x}'$ to frequency ω and momentum \vec{p}
Approximation: normally at fixed $\bar{t} = \frac{1}{2}(t + t')$, we hold $t \approx \bar{t}$
- In **classical-statistical** simulations

$$\langle EE \rangle(t, t', \rho) = \frac{1}{N_c^2 - 1} \langle E(t, \vec{p}) E^*(t', \vec{p}) \rangle$$

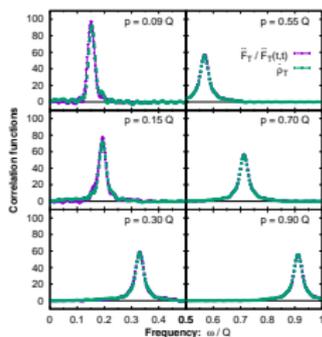
- Gauge: temporal $A_0 = 0$ + Coulomb-type $\partial^j A_j|_t = 0$

What excitations drive the dynamics in the QGP?



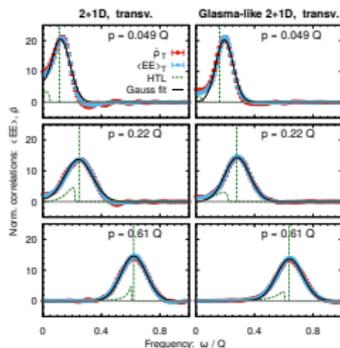
Study microscopics of the Quark-Gluon plasma

- Spectral functions $\rho(t, \omega, p) \sim \langle [\hat{A}, \hat{A}] \rangle$ encode excitation spectrum!
- Compute $\langle EE \rangle$ in class.-stat. + algorithm for ρ (KB, Kurkela, Lappi, Peuron (2018))
- Generalized FDR observed $\langle EE \rangle \sim \omega \rho$

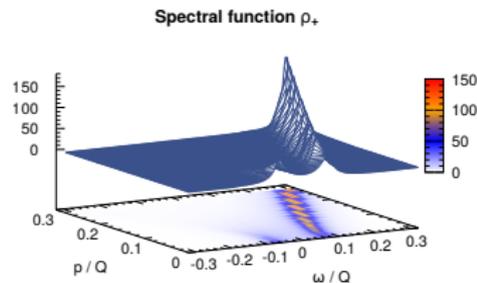


Gluonic 3+1D

KB, Kurkela, Lappi, Peuron (2018, 2019, 2021)



Gluonic 2+1D



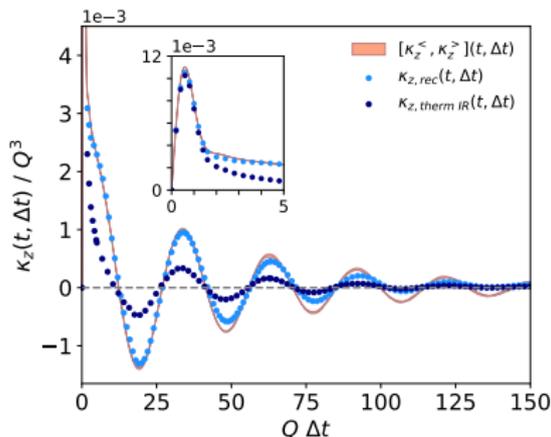
Fermionic 3+1D

KB, Lappi, Mace, Schlichting (2022)

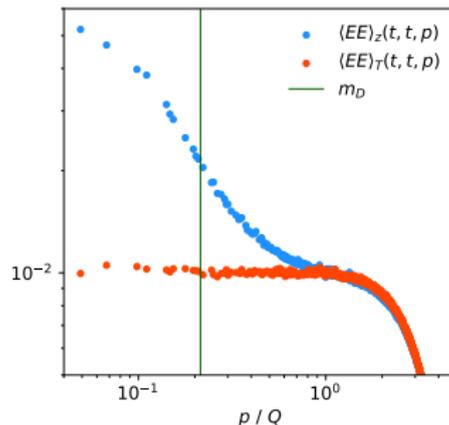
- Gauge fixed: temporal $A_0 = 0$ + Coulomb-type $\partial^j A_j|_t = 0$

2+1D: Manipulate correlations \Rightarrow study impact III

Glasma-like scalar κ_z



Equal-time correlators $\langle EE \rangle_\alpha(t, t, p)$



$$\kappa_z(t, \Delta t) \approx \frac{g^2}{N_c} \int_{\vec{p}} \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{\sin(\omega \Delta t)}{\omega} \langle EE \rangle_z(t, t, p) \frac{\dot{\rho}_z(t, \omega, p)}{\dot{\rho}_z(t, t, p)}$$

- If no infrared excess of scalars, smaller oscillations \Rightarrow evidence of infrared enhancement!