

Fluctuations of conserved charges from lattice and HRG and the freeze-out in RHIC

Peter Petreczky



- Introduction: Fluctuations of conserved charges and hadron resonance gas
- Hadron resonance gas with repulsive interactions and fluctuations of baryon number
- Chiral condensate in HRG and chiral crossover line at large baryon density
- Baryon number fluctuations and baryon-strangeness correlation along the crossover line and freeze-out condition in RHIC
- Summary

QCD thermodynamics at non-zero chemical potential

Taylor expansion :

$$\frac{p(T, \mu_B, \mu_Q, \mu_S)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \cdot \left(\frac{\mu_B}{T}\right)^i \cdot \left(\frac{\mu_Q}{T}\right)^j \cdot \left(\frac{\mu_S}{T}\right)^k \quad \text{hadronic}$$

$$\frac{p(T, \mu_u, \mu_d, \mu_s)}{T^4} = \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{ijk}^{uds} \cdot \left(\frac{\mu_u}{T}\right)^i \cdot \left(\frac{\mu_d}{T}\right)^j \cdot \left(\frac{\mu_s}{T}\right)^k \quad \text{quark}$$

$$\chi_{ijk}^{abc} = T^{i+j+k} \frac{\partial^i}{\partial \mu_a^i} \frac{\partial^j}{\partial \mu_b^j} \frac{\partial^k}{\partial \mu_c^k} \frac{1}{VT^3} \ln Z(T, V, \mu_a, \mu_b, \mu_c) \Big|_{\mu_a=\mu_b=\mu_c=0}$$

Taylor expansion coefficients give the fluctuations and correlations of conserved charges, e.g.

$$\chi_2^X = \chi_X = \frac{1}{VT^3} (\langle X^2 \rangle - \langle X \rangle^2) \qquad \chi_{11}^{XY} = \frac{1}{VT^3} (\langle XY \rangle - \langle X \rangle \langle Y \rangle)$$



information about carriers of the conserved charges (hadrons or quarks)

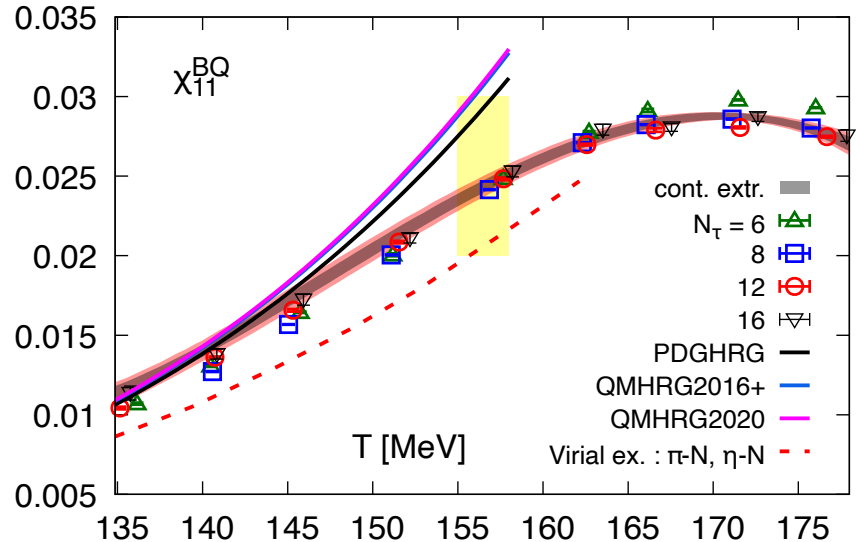
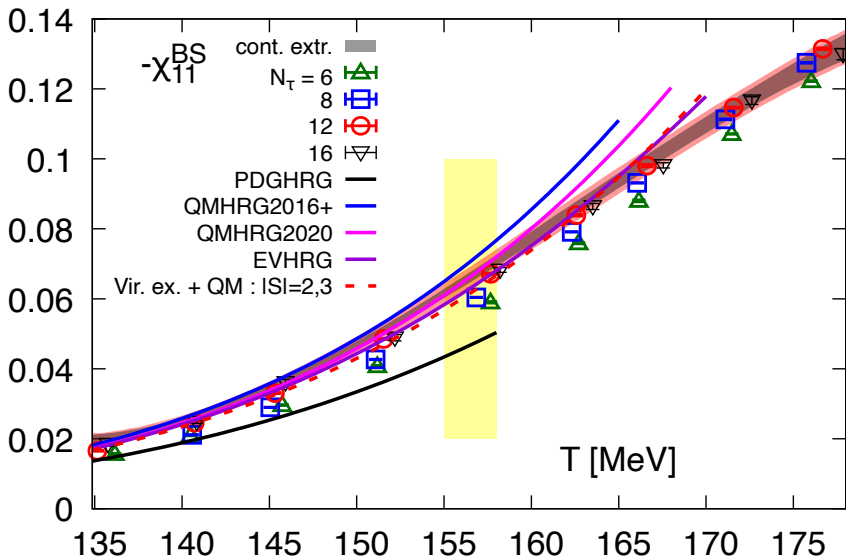
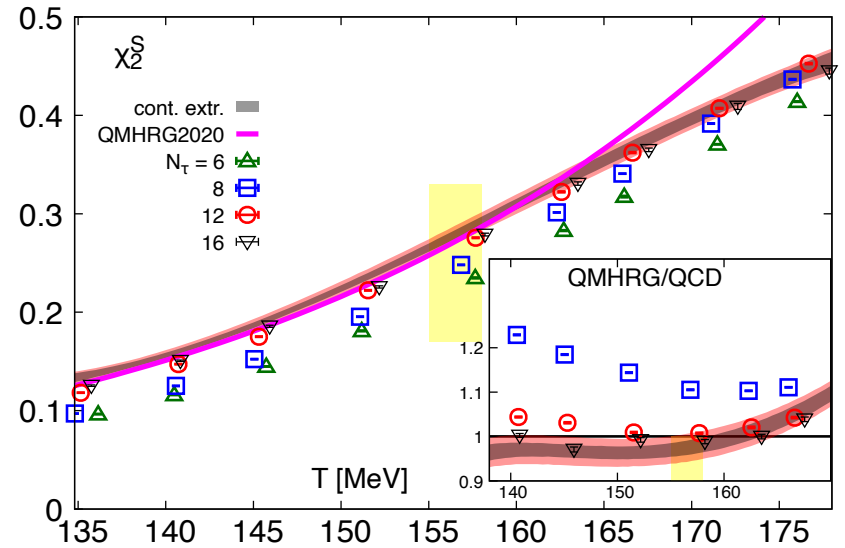
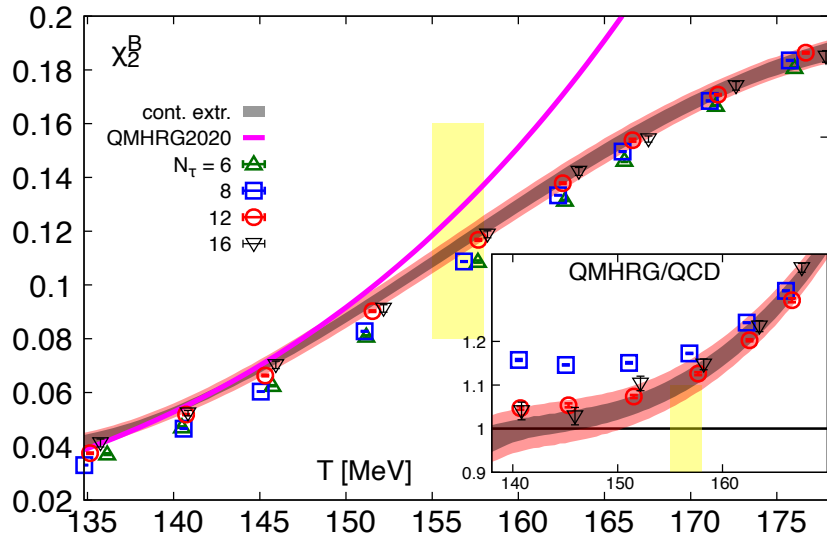


probes of deconfinement

Second order Taylor expansion coefficients and HRG

HISQ, m_{π}^{phys} , $a = 1/(TN_{\tau})$

Bollweg et al (HotQCD), PRD 104 (2021) 074512

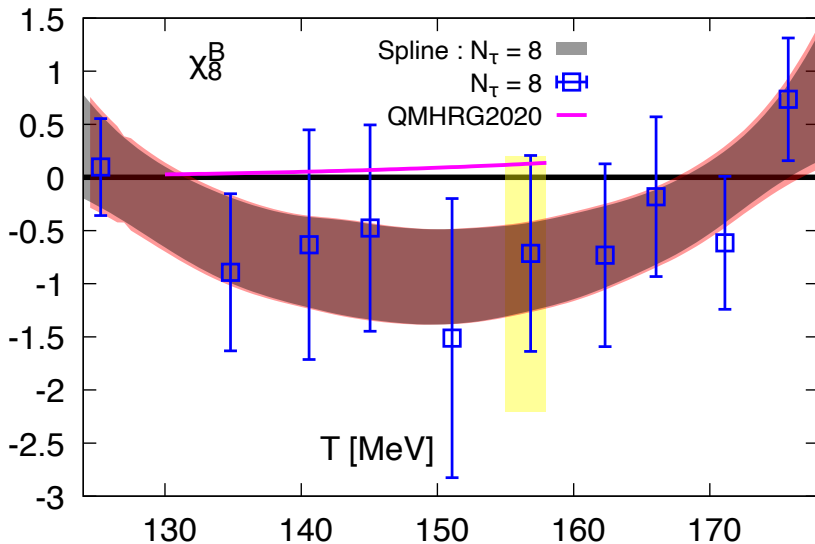
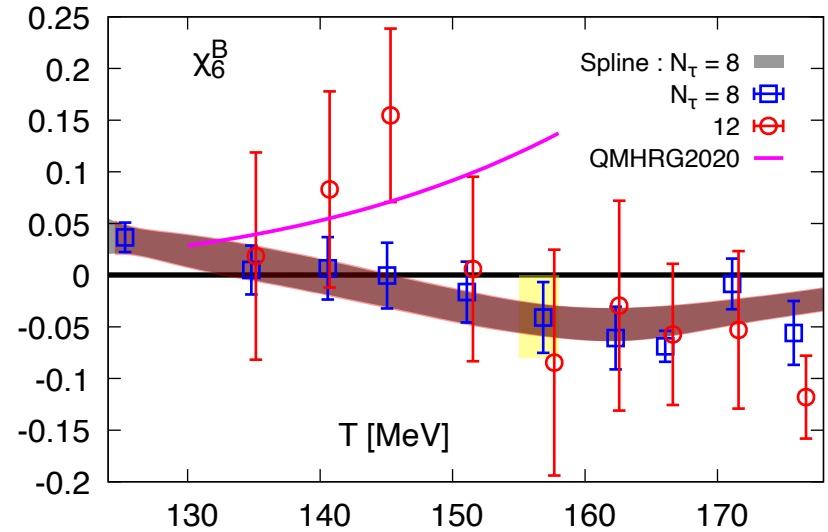
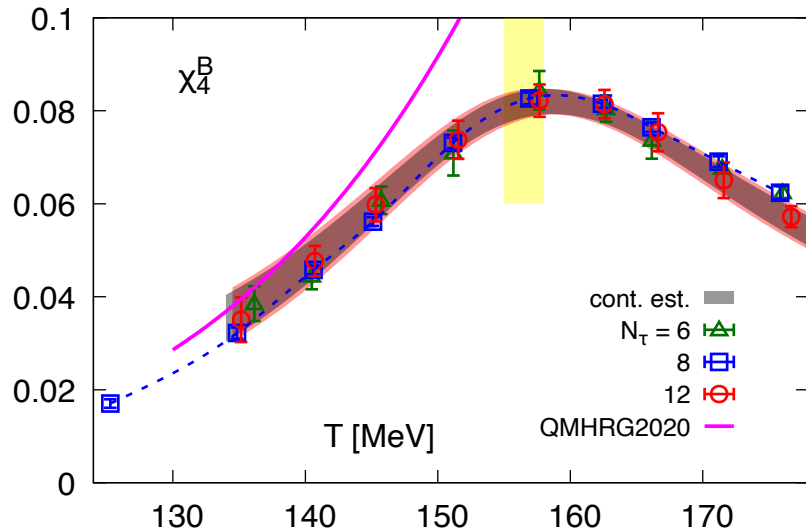


Quark Model HRG (QM-HRG) works up to temperatures $\approx 145-150$ MeV

Higher order Taylor expansion coefficients and HRG

HISQ, m_π^{phys} , $a = 1/(TN_\tau)$

Bollweg et al (HotQCD), PRD 105 (2022) 074511



For 4th order expansion coefficient QM-HRG may work only for $T < 140$ MeV

For 6th and 8th order expansion coefficients turn negative around T_c QM-HRG, only works for $T < 135$ MeV

HRG with repulsive mean field

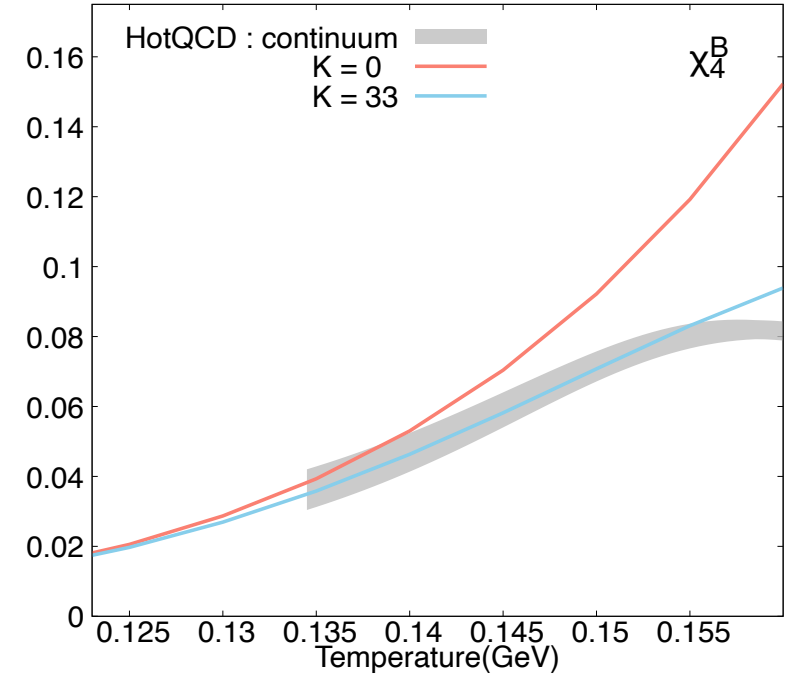
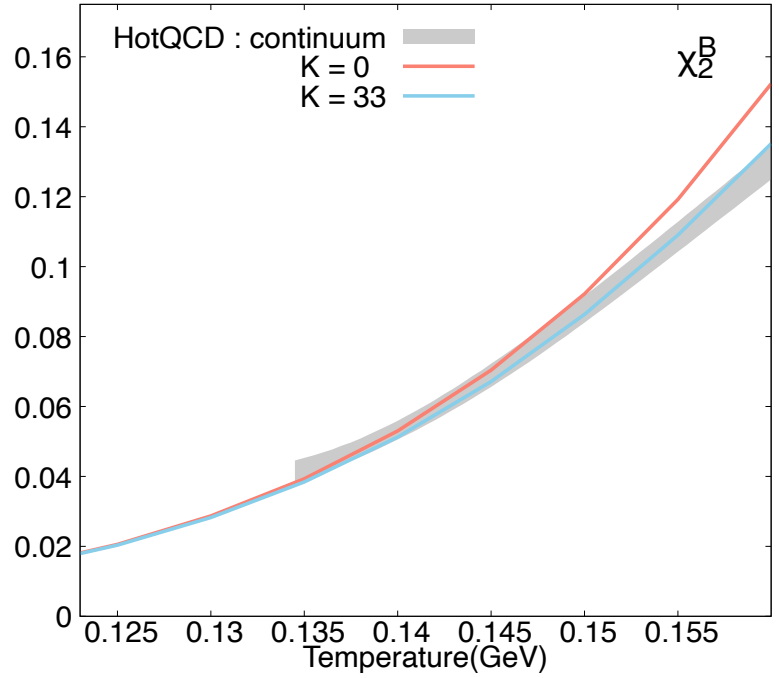
Repulsive B-B interactions are not present in usual HRG

$$p_{B,\bar{B}} = T \sum_{i \in B\{\bar{B}\}} \int g_i \frac{d^3p}{(2\pi)^3} \ln \left[1 + e^{-\beta(E_i - \mu_B + K n_{b,\bar{b}})} \right] + \frac{K}{2} n_{b\{\bar{b}\}}^2$$

$$n_{b,\bar{b}} = \sum_{i \in B} \int g_i \frac{d^3p}{(2\pi)^3} \frac{1}{e^{\beta(E_i \mp \mu_B + K n_{b,\bar{b}})} + 1}$$

↑
Repulsive mean field

Ellis-Kapusta, Prakash-Venugopalan, Huovinen et al, ...

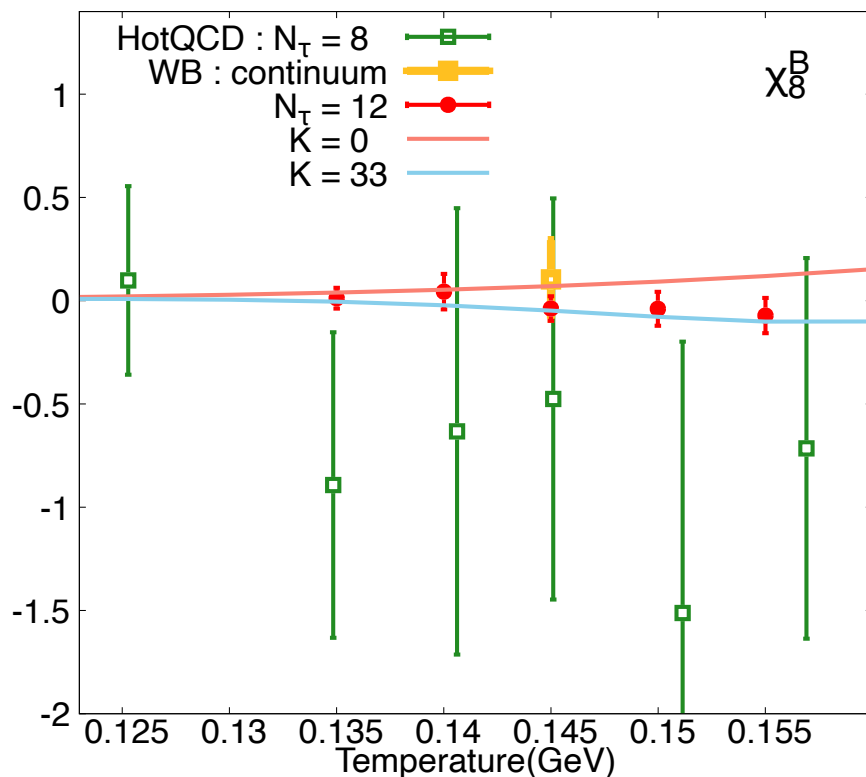
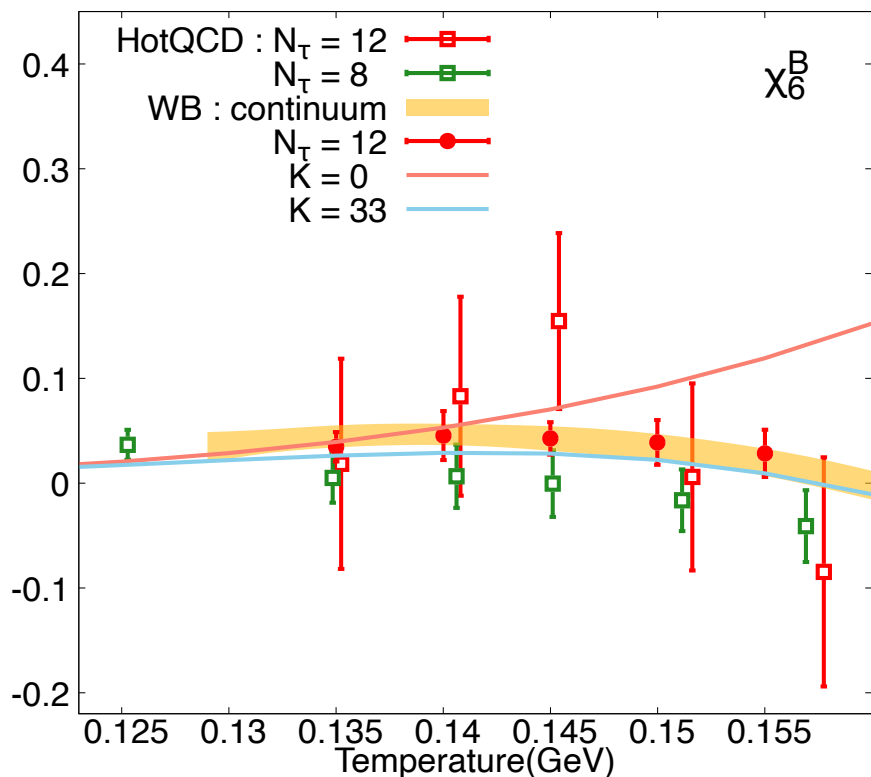


Biswas, PP, Sharma, PRC 109 (2024) 055206

Improved agreement between lattice and QM-HRG

Baryon number fluctuations in HRG with repulsive mean field

Biswas, PP, Sharma, PRC 109 (2024) 055206



WB continuum correspond to small box $LT=2$

QM-HRG with repulsive mean field can explain the lattice result on 6th and 8th order baryon number fluctuation up to $T=155$ MeV

Chiral condensate in HRG

$$\langle \bar{\psi}\psi \rangle_{l,T} = \langle \bar{\psi}\psi \rangle_{l,0} + \frac{\partial P}{\partial m_l} \quad \frac{\partial P}{\partial m_l} = - \sum_{i=H} g_i M_i \frac{\partial M_i}{\partial m_l} \int \frac{d^3 p}{(2\pi)^3} \frac{f_i(T, \mu_{eff})}{E_i}$$

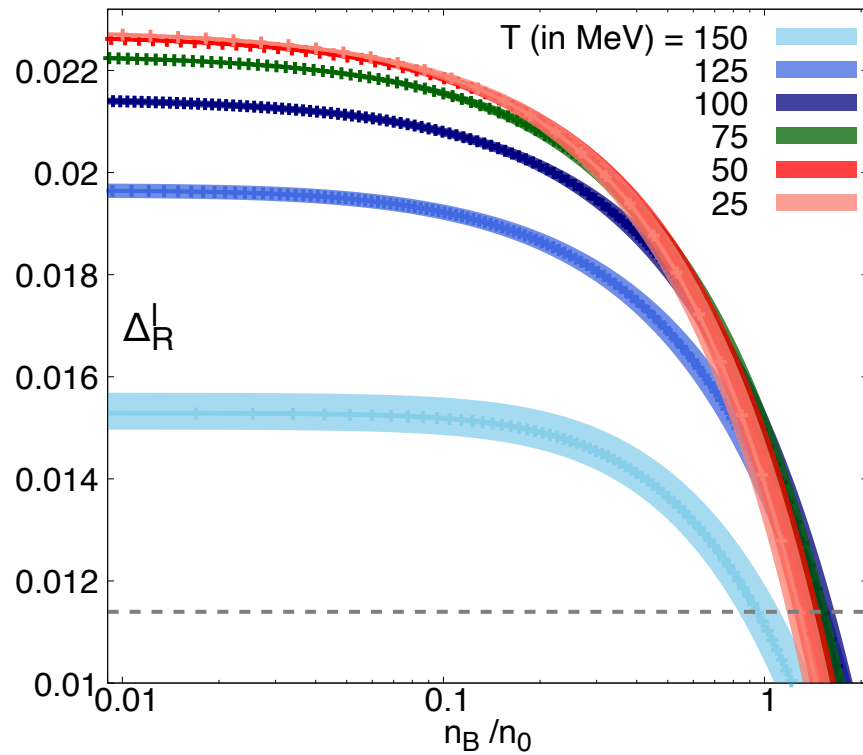
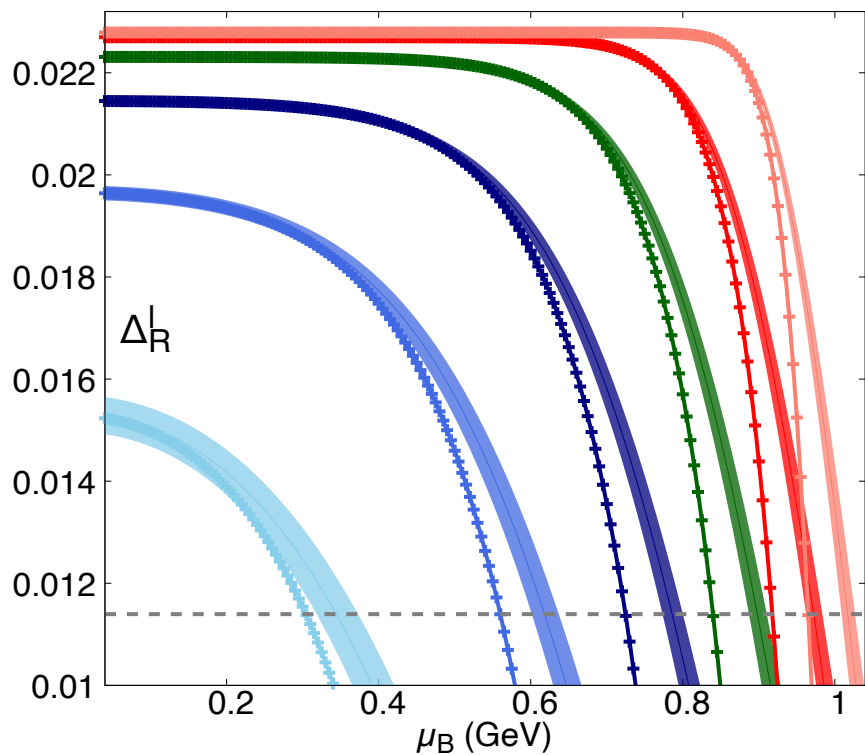
$$\Delta_R^l = d + m_s r_1^4 [\langle \bar{\psi}\psi \rangle_{l,T} - \langle \bar{\psi}\psi \rangle_{l,0}],$$

$$d = r_1^4 m_s \left(\lim_{m_l \rightarrow 0} \langle \bar{\psi}\psi \rangle_{l,0} \right)^R$$

↑
from lattice QCD,

Biswas, PP, Sharma, PRC 106 (2022) 045203

$$\mu_{eff} = B_i \mu_B + K n_b$$

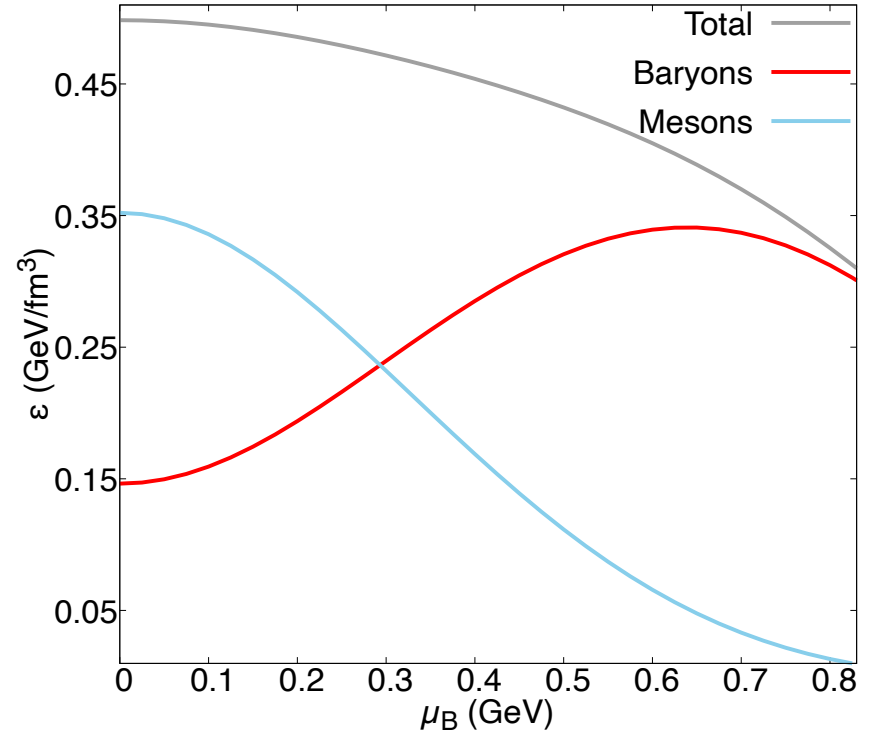
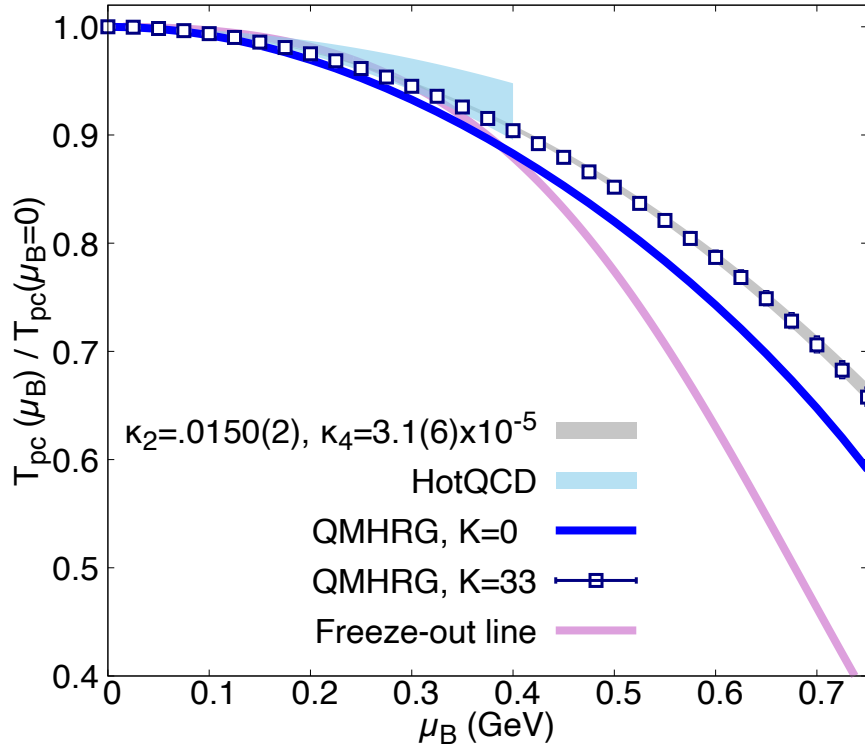


$$\Delta_R^l(T_c) \simeq \frac{1}{2} \Delta_R^l(T=0)$$

Chiral crossover line at large baryon density

$$\Delta_R^l(T, \mu_B) \simeq \frac{1}{2} \Delta_R^l(T=0) \Rightarrow T_{pc}(\mu_B)$$

Biswas, PP, Sharma, PRC 109 (2024) 055206



$$T_c(\mu_B = 0) \simeq 160 \text{ MeV} \quad T_{pc}(\mu_B)/T_{pc}(0) = 1 - \kappa_2(\mu_B/T_{pc}(0))^2 - \kappa_4(\mu_B/T_{pc}(0))^4$$

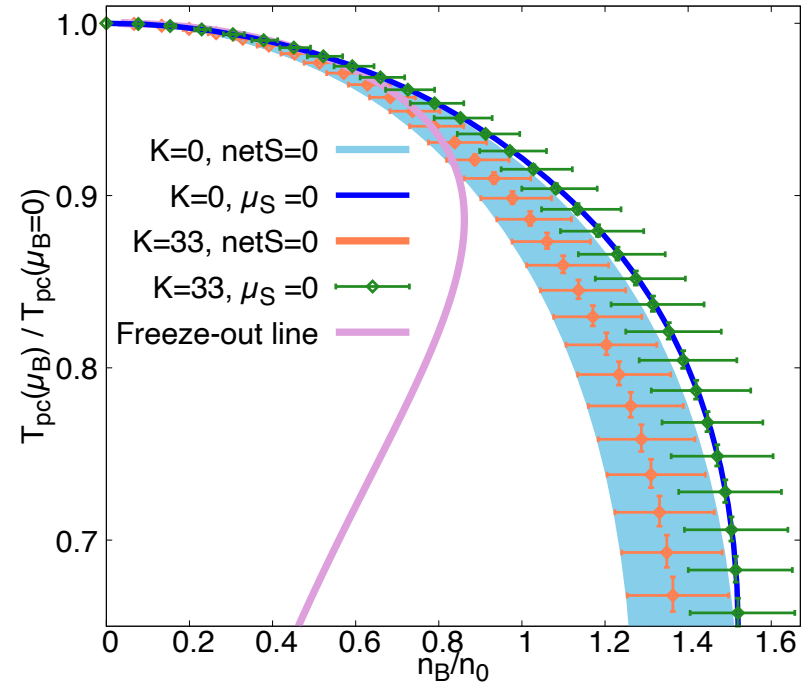
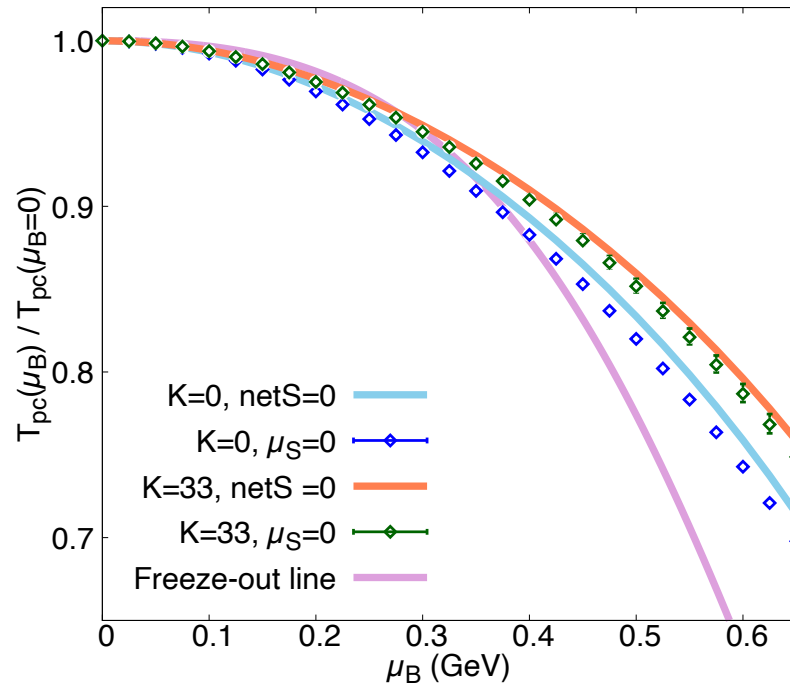
$$\kappa_2 = 0.0150(2) \text{ and } \kappa_4 = 3.1(6) \times 10^{-5}$$

in agreement with lattice QCD

For $\mu_B > 400 \text{ MeV}$ the chiral crossover line is above the freeze-out line

Chiral crossover line for strangeness neutral case

Biswas, PP, Sharma, PRC 109 (2024) 055206



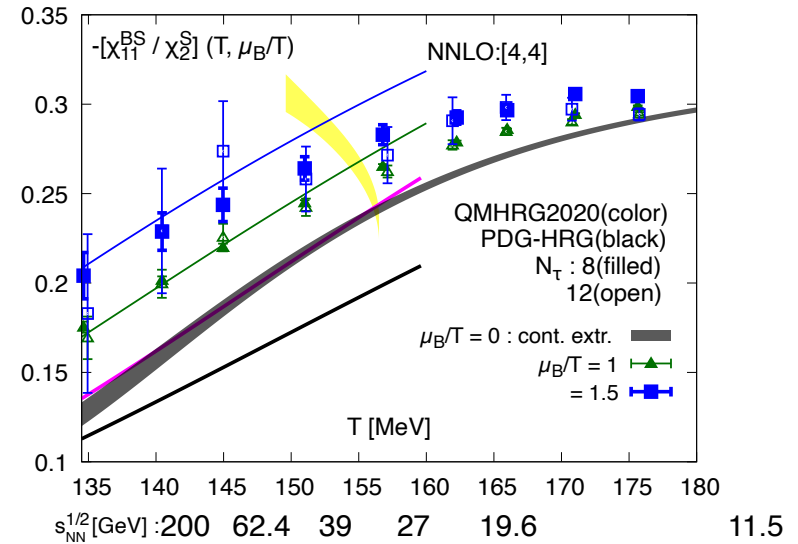
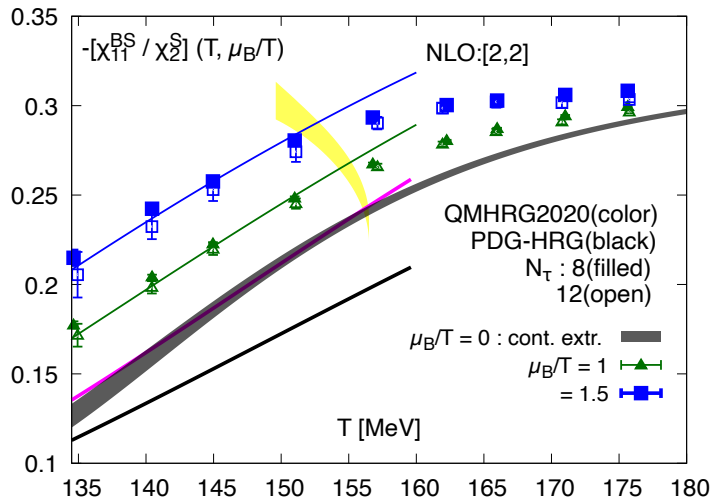
The effects of repulsive interactions lead to larger chiral crossover temperature

Imposing strangeness neutrality results only in small shift of the chiral transition line, smaller than the effects of the repulsive interactions.

Generalized susceptibilities for strangeness neutral case

$$R_{nm}^{XY}(T, \mu_B) = \frac{\chi_n^X(T, \mu_B)}{\chi_m^Y(T, \mu_B)} \quad \text{can probe freeze-out conditions in RHIC}$$

Bazavov et al, PRD 101 (2020) 074502; Bollweg et al, PRD 110 (2024) 054519

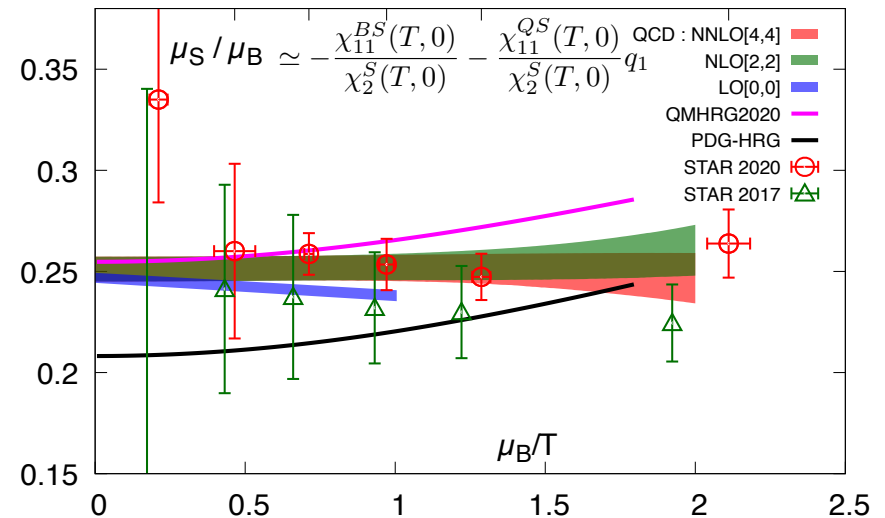


Conditions in RHIC: $n_S = 0$, $n_Q/n_B = 0.4 \Rightarrow$

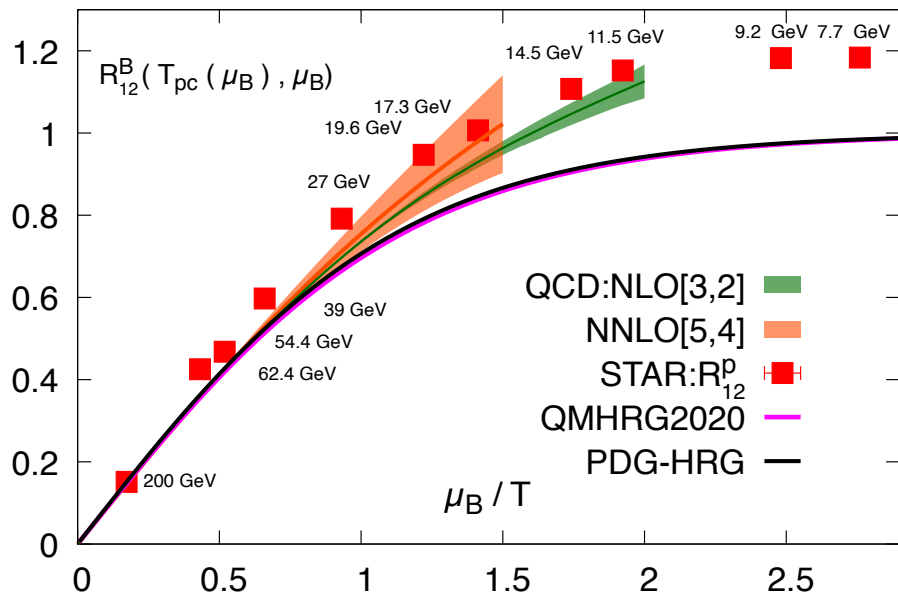
$$\mu_S / \mu_B = \sum_{i=0}^{\infty} s_{2i+1}(T) \hat{\mu}_B^{2i+1}$$

$$\mu_Q / \mu_B = \sum_{i=0}^{\infty} q_{2i+1}(T) \hat{\mu}_B^{2i+1}$$

$$\hat{\mu}_B = \mu_B / T$$



Baryon number fluctuations along the freeze-out line

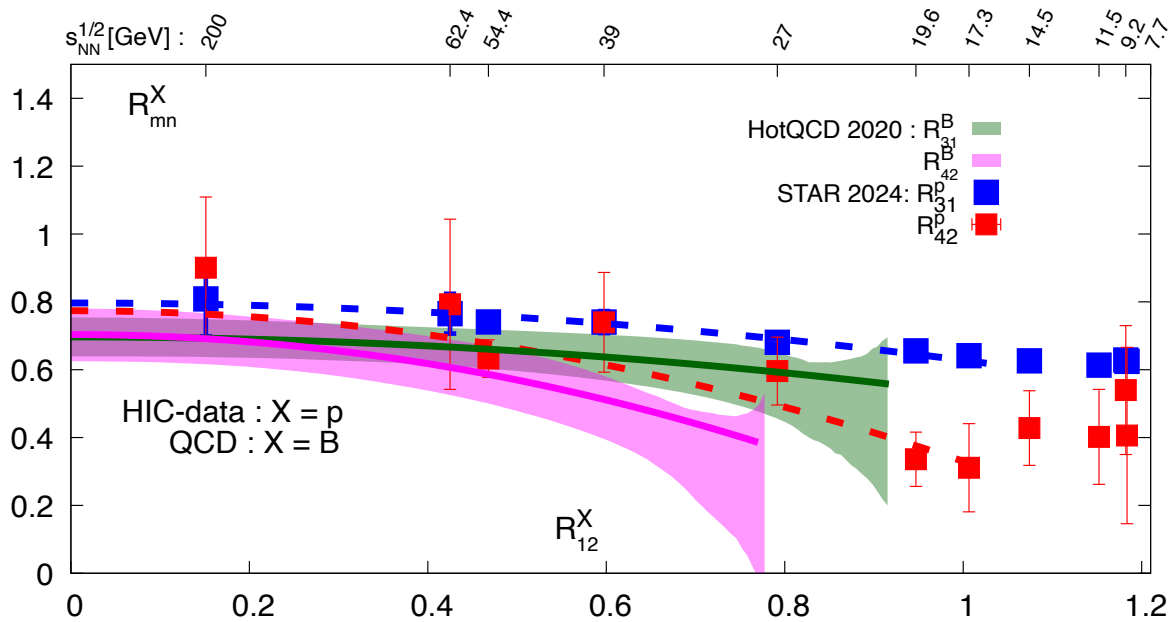


Bollweg et al, PRD 110 (2024) 054519

$$R_{12}^B(T_{pc}(\mu_B), \mu_B) = R_{12}^p(\sqrt{s_{NN}})$$

$$\Rightarrow \mu_B^f = \mu_B^f(\sqrt{s_{NN}})$$

agreement with $(T^f(\sqrt{s_{NN}}), \mu_B^f(\sqrt{s_{NN}}))$ obtained from statistical model fits of particle yields



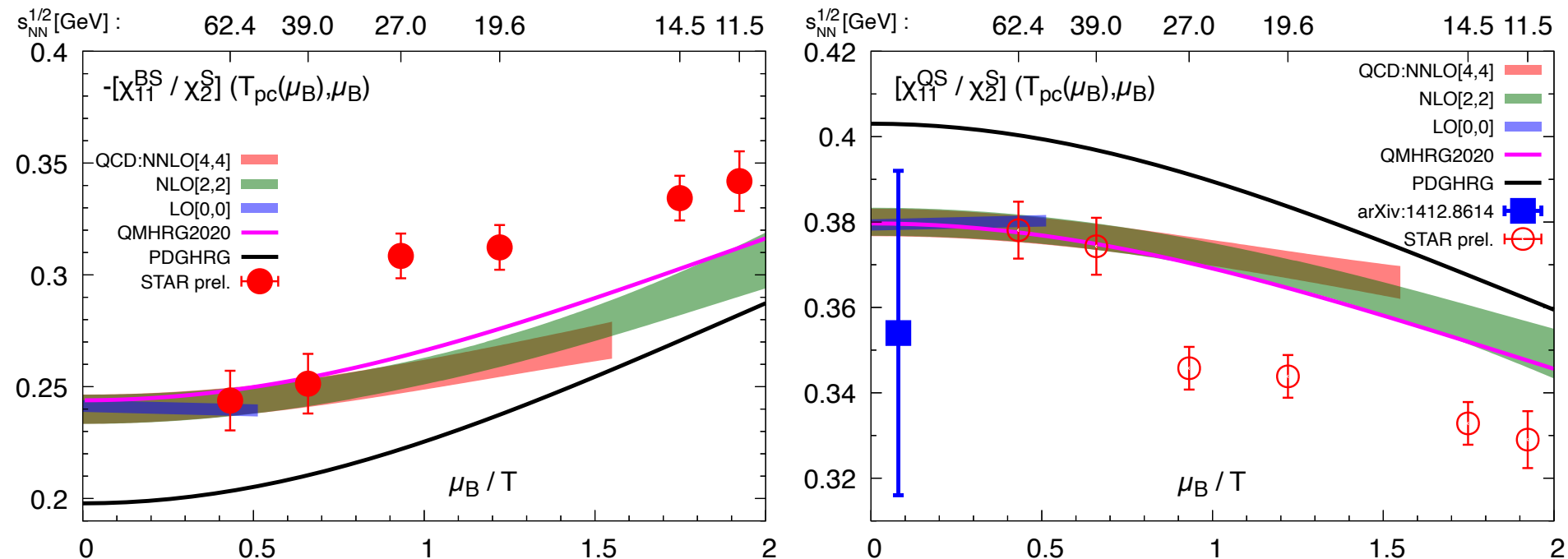
Bazavov et al, PRD 101 (2020) 074502

$$R_{nm}^B < 1$$

$$R_{42}^B < R_{31}^B \text{ for large } \mu_B$$

in agreement with LQCD

Correlations of strangeness with baryon number and strangeness along the freeze-out line



Bollweg et al, PRD 110 (2024) 054519

Tension between LQCD and STAR BES-II for $\sqrt{s} \leq 39$ GeV

Underestimated feed-down from excited states ?

Summary

- Fluctuations and correlations of conserved charges can be understood in terms of HRG if missing baryons are included (QM-HRG)
- For higher order baryon number fluctuations repulsive interactions play an important role
- For baryon chemical potential >400 MeV the estimated chiral crossover temperature is larger than the chemical freeze-out temperature
- Lattice results and the experimental results on fluctuations and correlations of conserved charges indicate that interactions in hadron gas just below the transition temperature are significant
- Fluctuations and correlations of conserved charges can be used to determine the freeze-out conditions in RHIC