QCD deconfinement transition up to μ_B=400MeV

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Based on [2410.06216] (accepted by PRD)

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Outline

1. Review of definitions of the crossover temperature

2. QCD in the grand canonical ensemble

3. The phase diagram for $\mu_B \leq 400\mathrm{MeV}$ [2410.06216]

4. Summary (big picture?)

„Chiral symmetry restoration”

Chiral symmetry:

- $SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$ for $\mathrm{m_q} \rightarrow 0$ (chiral limit)
- spontaneously broken at low T (likely 2nd order for N_f=2, unclear for N_f=3)

Chiral condensate: order parameter for chiral symmetry breaking in the two-flavour chiral limit

$$\langle \bar{\psi}\psi \rangle = \frac{T}{V}\frac{\partial \ln Z}{\partial m_{ud}} \qquad \langle \bar{\psi}\psi \rangle_R = -\left[\langle \bar{\psi}\psi \rangle_T - \langle \bar{\psi}\psi \rangle_0\right]\frac{m_{ud}}{f_\pi^4}$$

Chiral susceptibility:

$$\chi = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_{ud}^2} \qquad \chi_R = \left[\chi_T - \chi_0\right]\frac{m_{ud}^2}{f_\pi^4}$$

Peak postition: crossover temperature
Peak height and/or inverse width: strength of the crossover

„Deconfinement, center symmetry”

Center symmetry:

- discrete Z_3 symmetry for $\mathrm{m_q} \to \infty$ (pure gauge theory)
- spontaneously broken at high T (weak 1st order)

Polyakov-loop: $\mathrm{P} = \frac{1}{\mathrm{V}} \frac{1}{\mathrm{N_c}} \sum_{\vec{x}} \mathrm{Tr} \prod_{t=0}^{N_t-1} U_4(\vec{x},\ t)$

An order parameter for the deconfinement transition in pure gauge theory

$$|\langle P \rangle| = e^{-F_Q/T} \;\Rightarrow\; |\langle P \rangle| = 0 \leftrightarrow F_Q = \infty$$

Static quark free energy $F_Q = -T \log |\langle P \rangle|$

needs additive renormalization, e.g. $F_Q^R = F_Q(T) - F_Q(T_0)$

Static quark entropy $S_Q = -\frac{\partial F_Q}{\partial T}$ peak position renorm. scheme independent

[Bazavov et al, 1603.06637]:
- peak position of S_Q very close to peak position of chiral susceptibility
- smooth, monotonically decreasing function at higher T

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QCD in the grand canonical ensemble

$$\hat{p} \coloneqq \frac{p}{T^4} = \frac{1}{(LT)^3} \log \mathrm{Tr}\left(e^{-(H-\mu_B B-\mu_S S-\mu_Q Q)/T}\right) \quad \text{(pressure)}$$

We set $\mu_Q = 0$ in what follows.

$$\chi_{ij}^{BS} = \frac{\partial^{i+j}\hat{p}}{\partial\hat{\mu}_B^i \partial\hat{\mu}_S^j} \qquad \left(\hat{\mu}_{B/S} \coloneqq \frac{\mu_{B/S}}{T}\right) \quad \text{(susceptibilities)}$$

DERIVATIVES ⇔ FLUCTUATIONS/CORRELATIONS:

$\chi_1^B \propto \langle B \rangle \propto n_B$;

$\chi_2^B \propto \langle B^2 \rangle - \langle B \rangle^2$; $\chi_{11}^{BS} \propto \langle BS \rangle - \langle B \rangle \langle S \rangle$;

$\chi_3^B \propto \langle B^3 \rangle - 3\langle B^2 \rangle \langle B \rangle + 2\langle B \rangle^3$; ...

The QCD path integral

$$Z = \int DA_\mu D\bar{\psi} D\psi e^{-S_{YM} - \bar{\psi} M(A_\mu, m, \mu)\psi} = \int DA_\mu \mathrm{det} M(A_\mu, m, \mu)\, e^{-S_{YM}}$$

Can be simulated with Monte Carlo if $\mathrm{det}M\, e^{-S_{YM}}$ is real and positive:

- zero chemical potential $\mu = 0$
- purely imaginary chemical potential $\mathrm{R}e(\mu) = 0$
- isospin chemical potential $\mu_u = -\mu_d$

Otherwise: complex action/sign problem

$\Rightarrow$ desperate times, desperate measures

Lattice QCD at nonzero baryon density

Analytic continuation (ver. 1): Imaginary chemical potential

Calculate $\langle O \rangle$ at $\mathrm{Im}\mu_B$ ($\mu_B^2 < 0$), extrapolate to $\mu_B^2 > 0$

Analytic continuation (ver. 2): Taylor

TODAY

Calculate $\frac{\partial^n}{\partial \mu_B^n} \langle O \rangle$ at $\mu_B = 0$, extrapolate

Reweighting:

Simulate a different theory, correct the Boltzmann weight in observable

C O S T

What to do with strangeness?

Sketches of the phase diagram usually have only two axes.

1. Zero strangeness chemical potential (simpler):

$$\mu_S = 0$$

2. Zero strangeness density (more realistic):

Tune $\mu_S(T, \mu_B)$ such that $\chi_1^S(T) = 0$

If this is done order by order in μ_B, one can write:

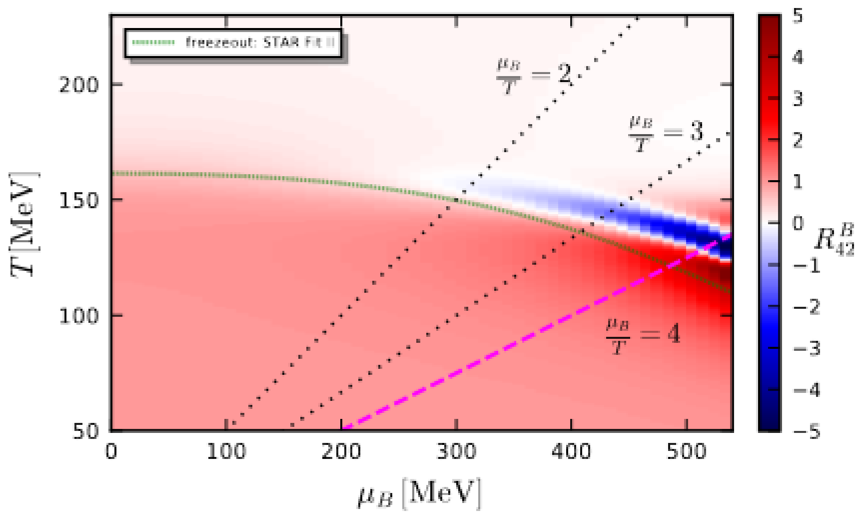
$$\mu_{S(T,\mu_B)} = s_1(T)\mu_B + s_3(T)\mu_B^3 + \cdots$$

Later, I will compare the phase diagram with these two conditions.

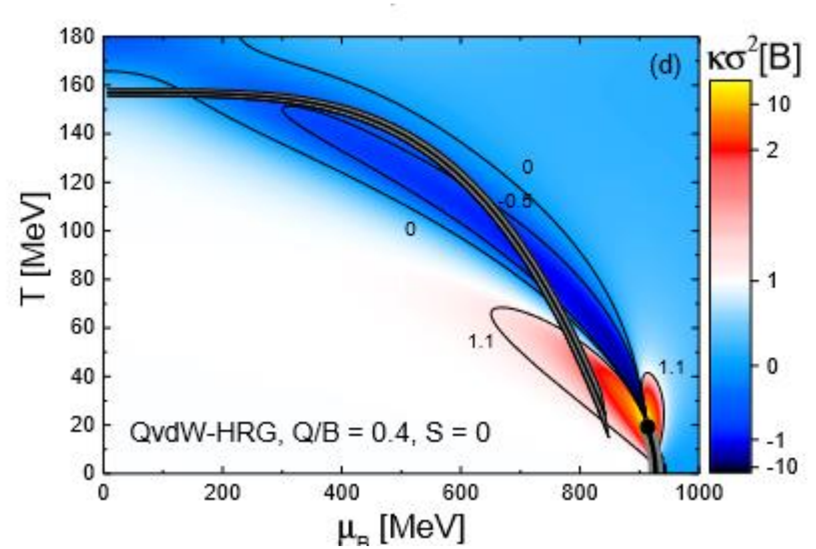
Possible scenarios for $\mu_B > 0$?

[Stephanov, PRL 107 (2011)] **Non-monotonic χ_4^B/χ_2^B near critical point**

FRG on quark-meson model
(lattice assisted model)
Chiral CEP = (93 MeV, 672 MeV)
[Fu et al, 2101.06035]



Quantum VdW gas
(Non-ideal version of HRG)
Liquid-gas CEP = (19.7 MeV, 922 MeV)
[Vovchenko et al, 1609.03975]



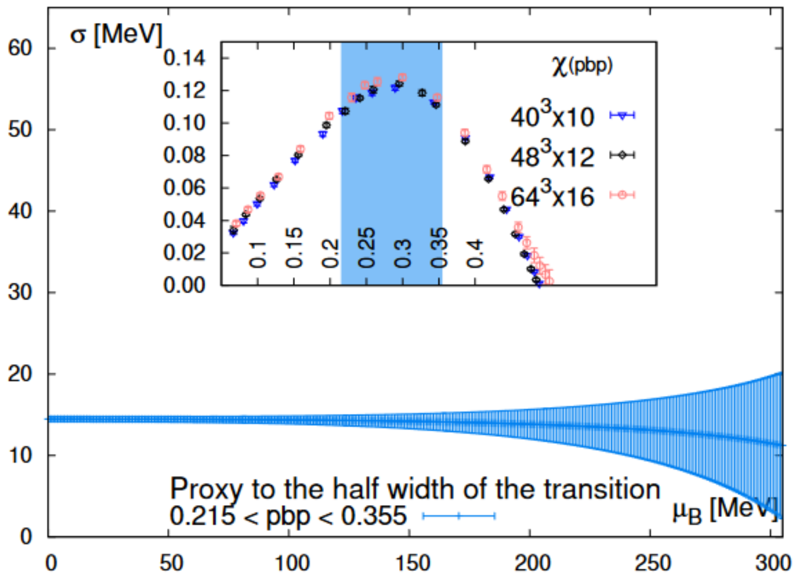
In both models, χ_4^B/χ_2^B is non-monotonic quite far away from the respective CEP.

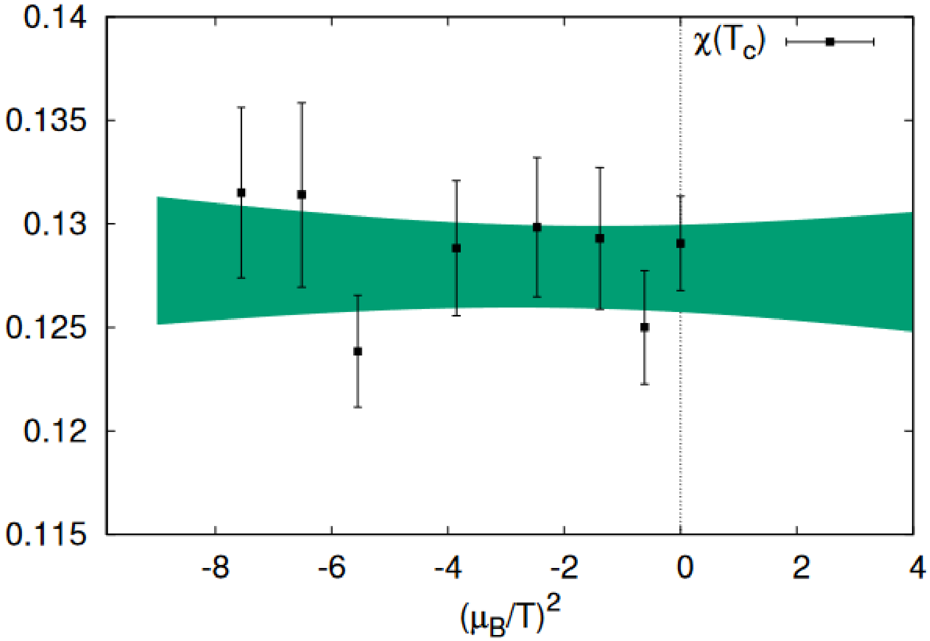
The two critical points

In the simplest scenario, where we assume that deconfinement and chiral symmetry restoration are also linked at $\mu_B > 0$.

Property	**Chiral/deconfinement CEP**	**Nuclear liquid-gas CEP**
Universality class	3D Ising	3D Ising
Baryon fluctations	Diverge at CEP	Diverge az CEP
χ_4^B/χ_2^B	Non-monotonic (potentially far from CEP)	Non-monotonic (potentially far from CEP)
Lee-Yang zeros	Approach real axis at CEP	Approach real axis at CEP
T	???	O(10-20 MeV)
μ_B	???	O(1 GeV)
Chiral symmetry	Restored on one side	Broken on both sides
Confinement	Only on one side	On both sides

Chiral crossover ⇒ approx. constant width and strength





$$\sigma \approx \left(\sqrt{-\chi\left(\frac{\partial^2\chi}{\partial T^2}\right)^{-1}}\right)_{T=T_c(\mu_B)}$$

Wuppertal-Budapest PRL125 (2020)

Slope near zero negligible. We need larger chemical potentials.

⇒ Need smaller volume, so the sign problem remains managable.

GOAL: LOOK AT POLYAKOV LOOP AND RELATED OBSERVABLES FOR μ_B>0

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Lattice setup

- 4HEX action (reduced taste breaking, i.e. better meson spectrum at finite a)

- Exponential definition of μ_B (physical quantization of B at finite a)

- Smeared Polakov loop (reduced noise at finite a, same continuum limit)

- 16^3X8 lattice **(FINITE BOX AND FINITE LATTICE SPACING)**

Sign problem?
- small signal/noise ratio in Taylor coefficients, concentrate most computer time into one volume and spacing
- much weakersign problem in a smaller volume

Cut-off effects? Previous work: [WB: [2312.07528]](#)

Baryon fluctuations ($\chi_2^B, \chi_4^B, \chi_6^B, \chi_8^B$) for N_t=8,10,12 and the continuum were all consistent with each other, so we expect cut-off effects to be small.

Finite volume effects? Previous work: [WB: [2410.06216]](#)

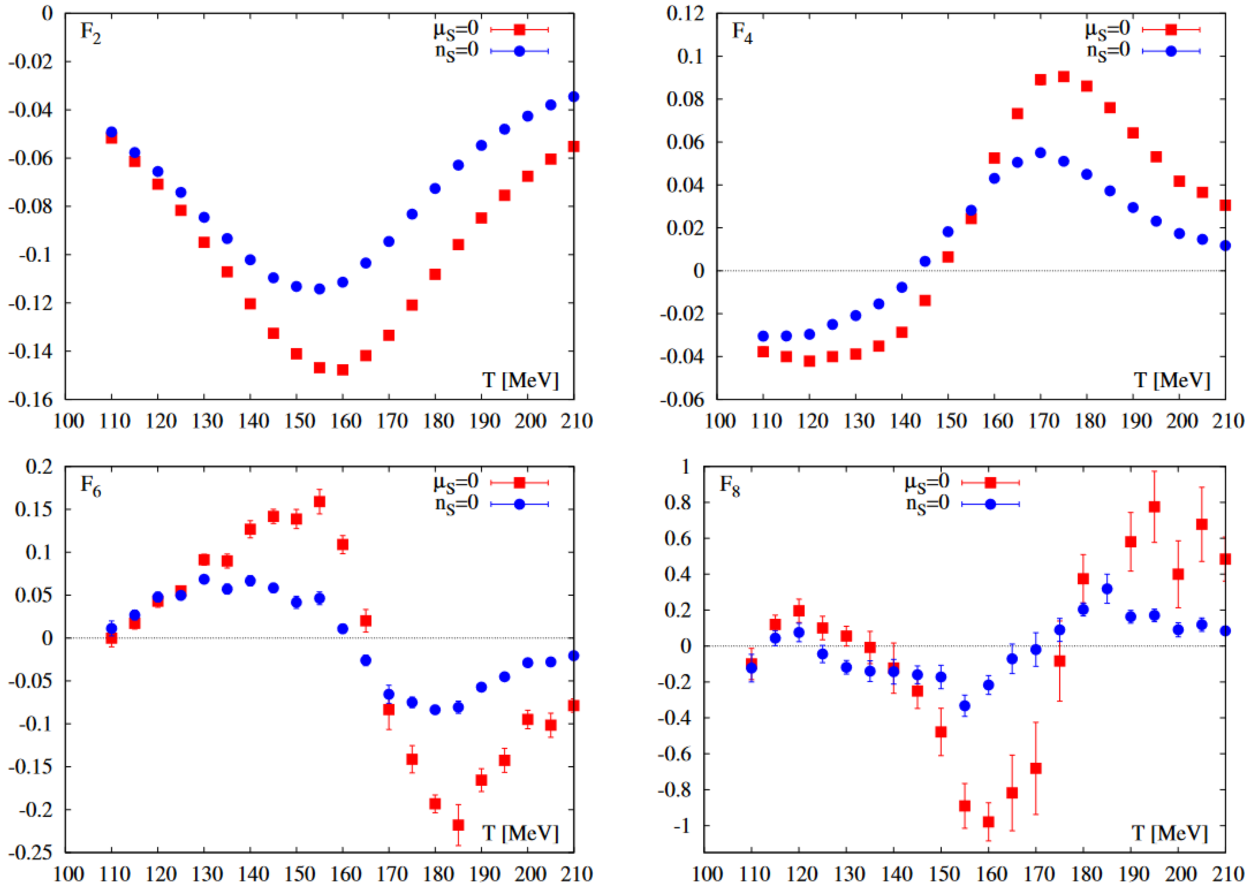
At μ_B=0 about 10MeV effect in T_c and a negligible effect on the curvature of the phase diagram in the T-μ_B plane.

Sketch of the full analysis

1. Calculate the bare free energy $F_Q^{bare}(T,\mu_B)$ with an 8th order Taylor expansion (use computer algebra to generate long formulas (e.g. $\partial^8(|\langle P\rangle|\hat{}2)/\partial\mu_u^8$ has 405 terms), evaluate the long formulas on gauge configurations)
2. The bare free energy is fitted at every μ_B separately in T, to determine its temperature derivative
3. The renormalized $S_Q(T,\mu_B)$ is calculated by subtracting the temperature derivative of the free energy counterterm/renormalization function:
$$S_Q(T,\mu_B) = S_Q^{bare}(T,\mu_B) - S_Q^{c.t.}(T)$$

SYSTEMATIC ERROR SOURCES CONSIDERED:
- 3rd or 4th order polynomial fit to the counterterm
- polynomial fit to counterterm in bare coupling or in temperature
- use T>0 data or the T=0 static potential for determining the counterterm
- (3,2) or (3,3) rational fit for the bare free energy

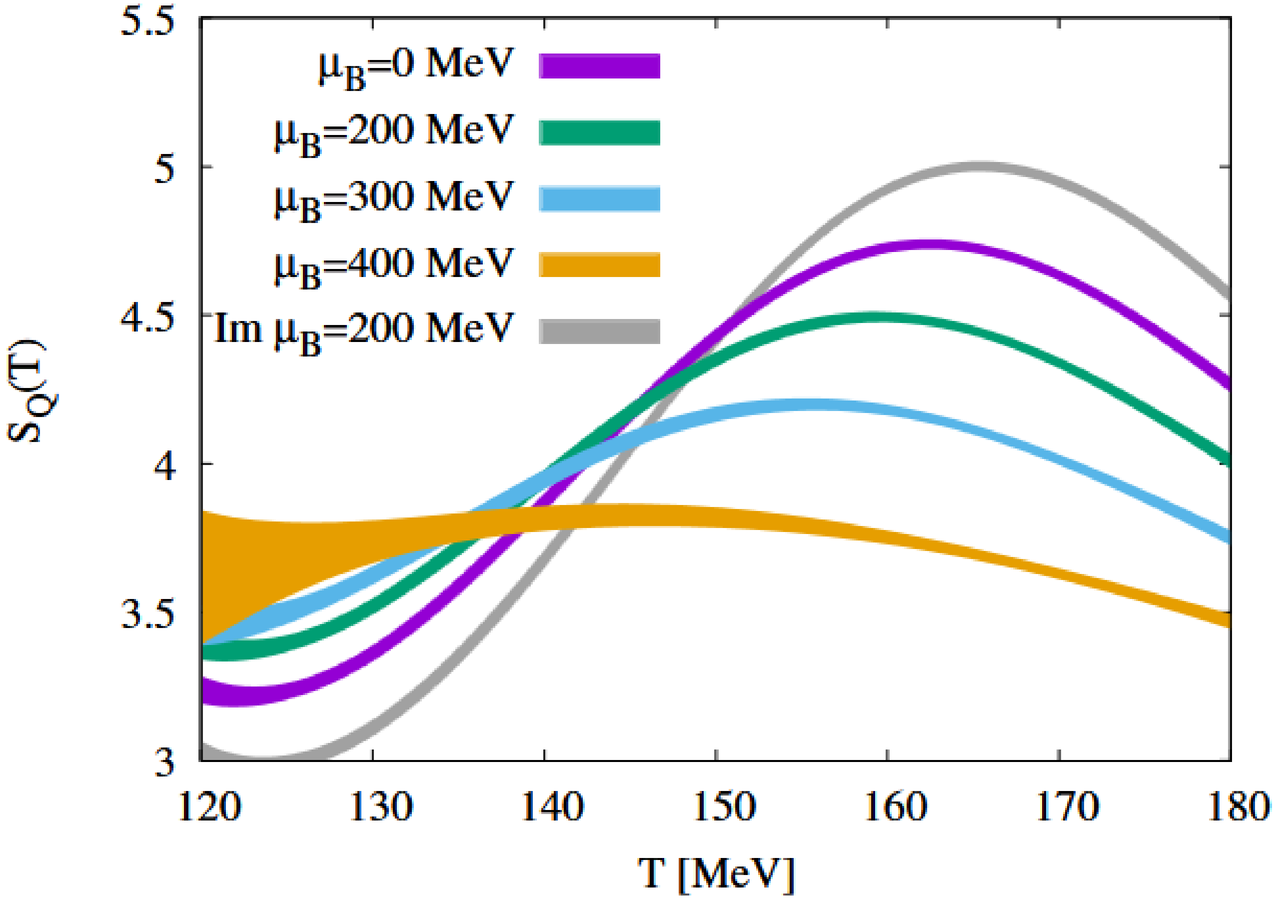
Taylor coefficients of the static quark free energy



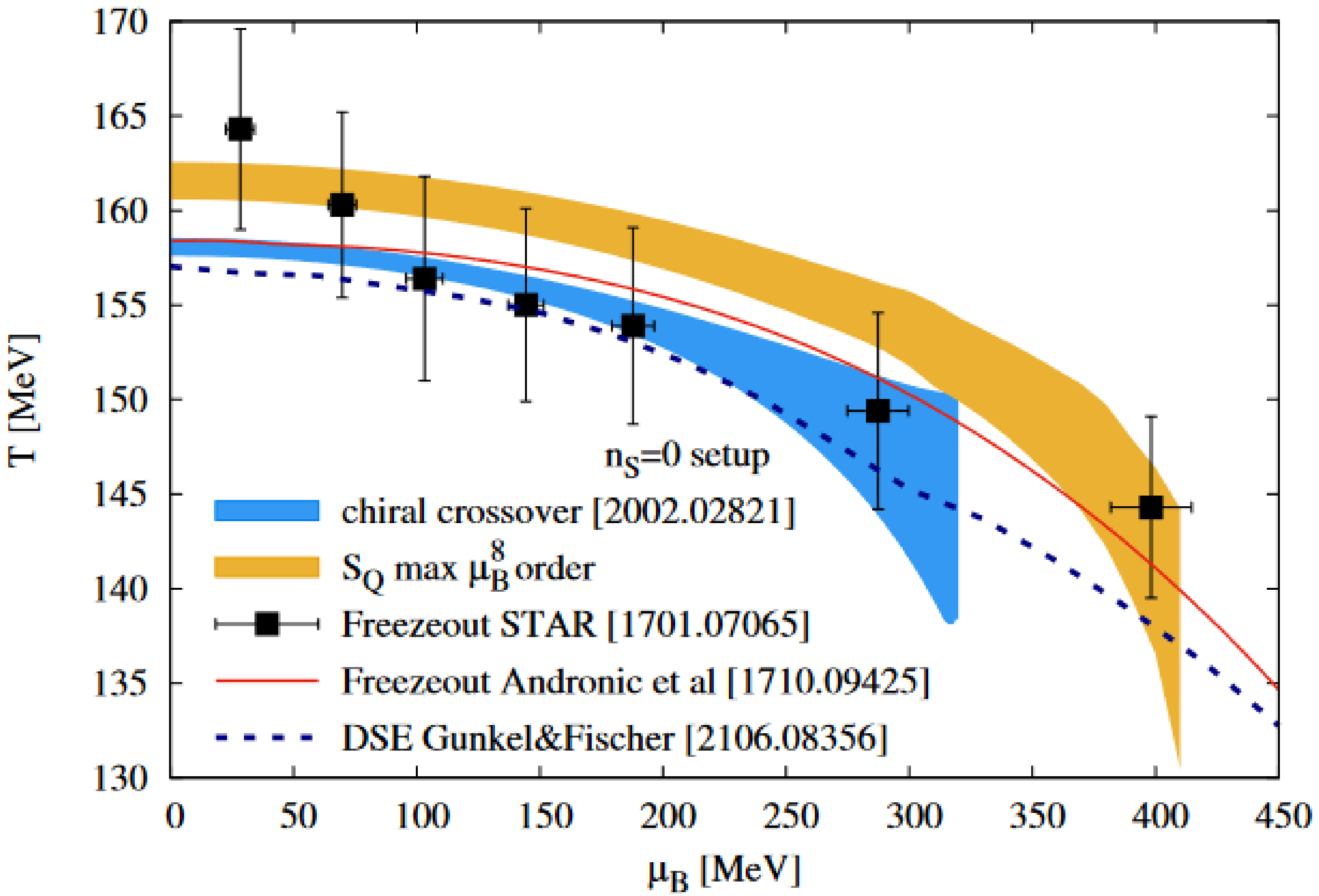
$$F_Q(T,\mu_B) = F_Q(T,0) + T \sum_{n=2,4,\ldots} \frac{F_n(T)\hat{\mu}_B^n}{n!}$$

2nd order coeff (μ_S=0)
[D'Elia, 1907.09461]

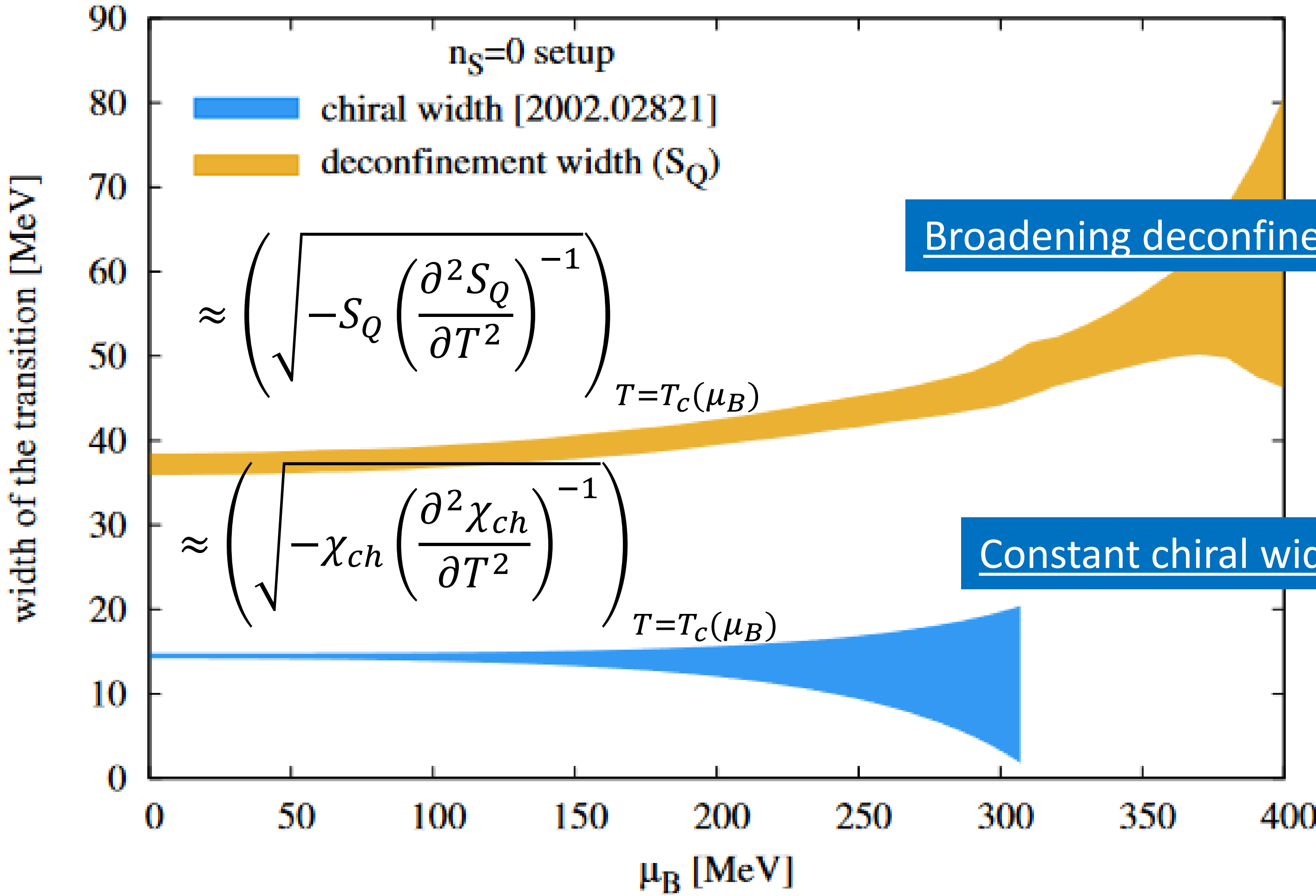
Static quark entropy (n_S=0)



The transition line on the phase diagram (n_S=0)



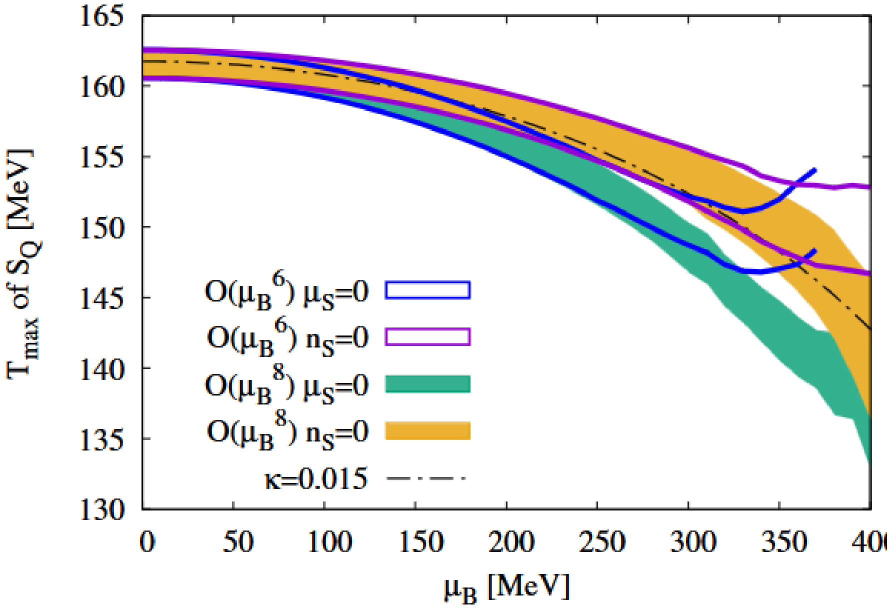
The width of the crossover ($n_S=0$)

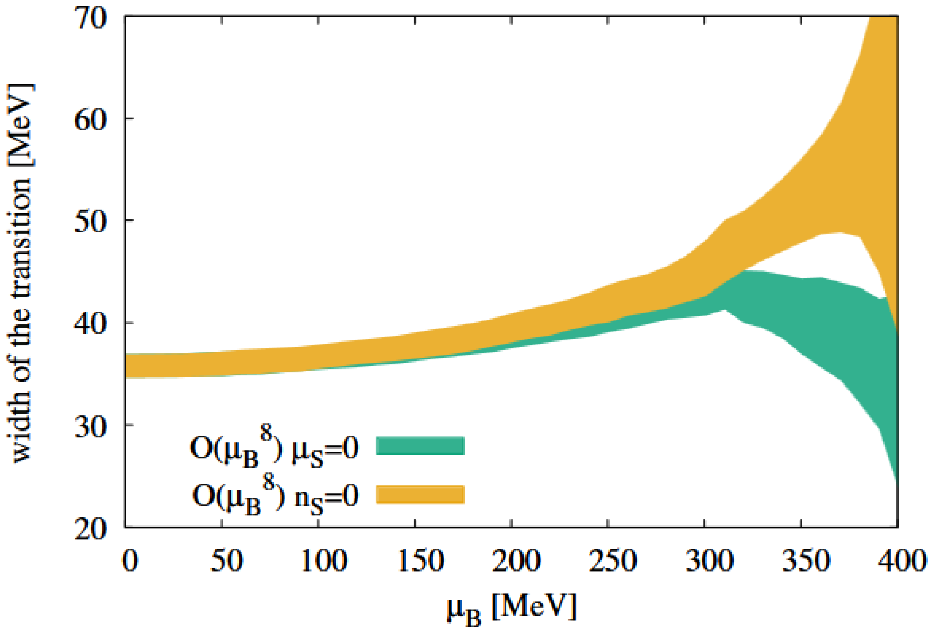


$$\approx \left(\sqrt{ -S_Q \left(\frac{\partial^2 S_Q}{\partial T^2} \right)^{-1} } \right)_{T=T_c(\mu_B)}$$

$$\approx \left(\sqrt{ -\chi_{ch} \left(\frac{\partial^2 \chi_{ch}}{\partial T^2} \right)^{-1} } \right)_{T=T_c(\mu_B)}$$

Comparing n_S=0 and μ_S=0 and 6th and 8th order





REGION OF RELIABILITY?

The 8th order coefficient is totally negligible for:

$T \leq 300$ MeV for $n_S = 0$ and $T \leq 250$ MeV for $\mu_S = 0$

The 1 sigma errorbars of the 8th and 6th order touch for:

$T \approx 400 MeV$ for $n_S = 0$ and $T \approx 330 MeV$ for $\mu_S = 0$

Summary

- Calculation of Polyakov loop and related observables at $\mu_B > 0$ using an 8th order Taylor expansion
- For $\mu_B \leq 400$MeV the crossover and chemical FO lines stay close
- Deconfinement CEP disfavored below μ_B=400MeV
- How strangeness is treated matters (μ_S=0 vs n_S=0):
  - Truncated Taylor expansion breaks down earlier for μ_S=0
  - Hint for deconfinement trend reversing for μ_S=0
  - Existing CEP estimates use μ_S=0

BIG PICTURE: DIFFERENT BEHAVIOR (FOR SMALL μ_B)

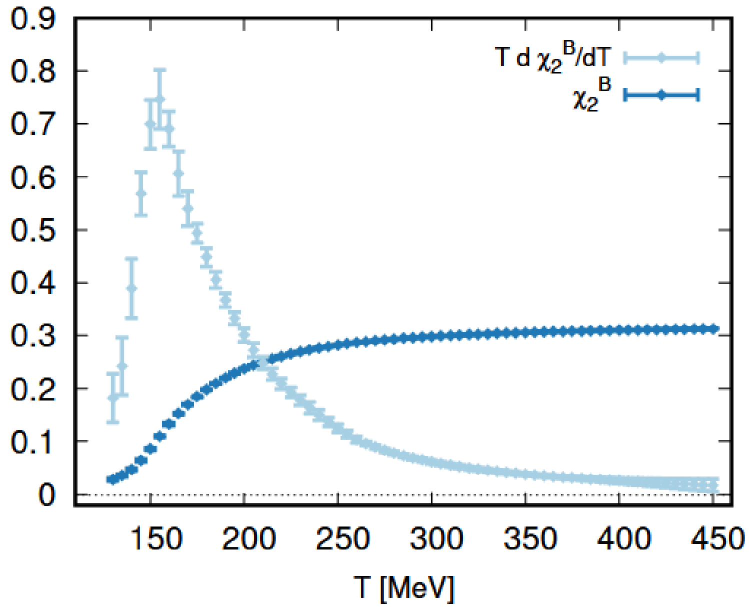
constant (chiral width WB 2020) VS

strengthening (Lee-Yang zero estimates: C. Schmidt Tue) VS

weakening transition (deconfinement width: this work)

BACKUP

„Deconfinement, degrees of freedom”



$$\chi_2^B = \left(\frac{1}{T^3}\frac{\partial^2 p}{\partial \mu_B^2}\right)_{\mu_B=0}$$

For free massless quarks:

$$\chi_2^B = \frac{N_f}{9}$$

Again, has this has same peak position, after which it approaches the free quark value.

Similarly, the temperature derivative of the entropy has a peak in the same position, etc.

Bottom line: gauge invariant bulk thermodynamic quantities that are appropriately defined (e.g. with a renormalization scheme independent peak) show a transition at roughly the same T. There is just one crossover.

The „hard part”

Log determinant:

$$\log\det M(U, m_j, \mu_j) - \log\det M(U, m_j, 0) = A_j \hat{\mu}_j + \frac{B_j \hat{\mu}_j}{2!} + \cdots$$

The A_j, B_j, … are evaluated using the reduced matrix formalism

Polyakov loop: $P = P_R + P_I$
Chain rule: $\partial_j \langle X \rangle = \langle \partial_j X \rangle + \langle A_j X \rangle - \langle A_j \rangle \langle X \rangle$

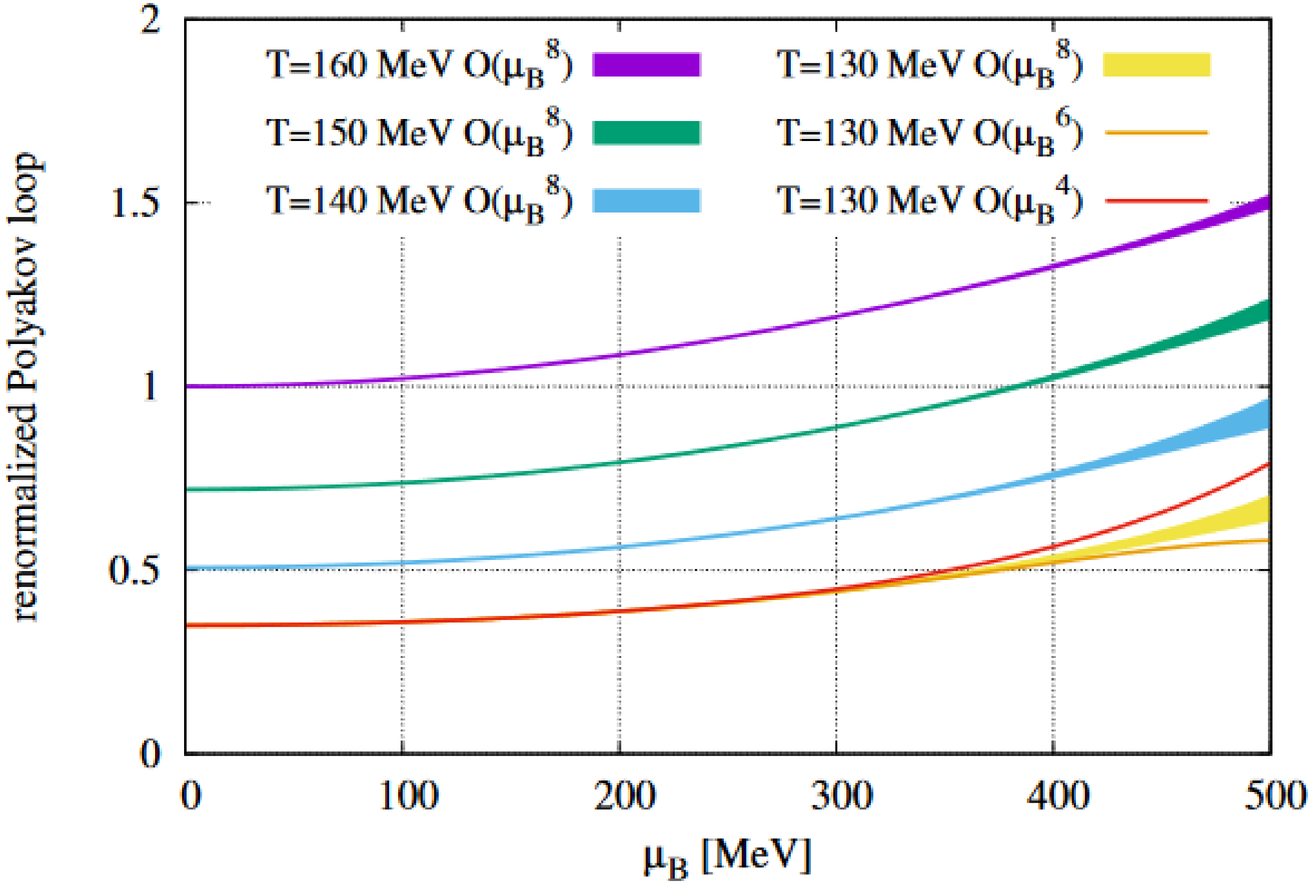
This has to be applied repeatedly to generate the terms.

$$\begin{aligned}
\partial_u^2 Q &= +2\langle P_R\rangle\langle B_u P_R\rangle + 2\langle P_R\rangle\langle A_u A_u P_R\rangle - 2\langle A_u P_I\rangle\langle A_u P_I\rangle - 2\langle B_u\rangle\langle P_R\rangle\langle P_R\rangle - 2\langle A_u A_u\rangle\langle P_R\rangle\langle P_R\rangle \\
\partial_u^4 Q &= +2\langle P_R\rangle\langle D_u P_R\rangle + 6\langle P_R\rangle\langle B_u B_u P_R\rangle + 8\langle P_R\rangle\langle A_u C_u P_R\rangle + 12\langle P_R\rangle\langle A_u A_u B_u P_R\rangle + 2\langle P_R\rangle\langle A_u A_u A_u A_u P_R\rangle \\
&\quad +6\langle B_u P_R\rangle\langle B_u P_R\rangle - 8\langle A_u P_I\rangle\langle C_u P_I\rangle - 24\langle A_u P_I\rangle\langle A_u B_u P_I\rangle - 8\langle A_u P_I\rangle\langle A_u A_u A_u P_I\rangle + 12\langle A_u A_u P_R\rangle\langle B_u P_R\rangle \\
&\quad +6\langle A_u A_u P_R\rangle\langle A_u A_u P_R\rangle - 2\langle D_u\rangle\langle P_R\rangle\langle P_R\rangle - 24\langle B_u\rangle\langle P_R\rangle\langle B_u P_R\rangle - 24\langle B_u\rangle\langle P_R\rangle\langle A_u A_u P_R\rangle \\
&\quad +24\langle B_u\rangle\langle A_u P_I\rangle\langle A_u P_I\rangle - 6\langle B_u B_u\rangle\langle P_R\rangle\langle P_R\rangle - 8\langle A_u C_u\rangle\langle P_R\rangle\langle P_R\rangle - 24\langle A_u A_u\rangle\langle P_R\rangle\langle B_u P_R\rangle \\
&\quad -24\langle A_u A_u\rangle\langle P_R\rangle\langle A_u A_u P_R\rangle + 24\langle A_u A_u\rangle\langle A_u P_I\rangle\langle A_u P_I\rangle - 12\langle A_u A_u B_u\rangle\langle P_R\rangle\langle P_R\rangle - 2\langle A_u A_u A_u A_u\rangle\langle P_R\rangle\langle P_R\rangle \\
&\quad +18\langle B_u\rangle\langle B_u\rangle\langle P_R\rangle\langle P_R\rangle + 36\langle A_u A_u\rangle\langle B_u\rangle\langle P_R\rangle\langle P_R\rangle + 18\langle A_u A_u\rangle\langle A_u A_u\rangle\langle P_R\rangle\langle P_R\rangle
\end{aligned}$$

8th order: 405 terms
+ strangeness neutrality

The renormalized Polyakov loop (n_S=0)

Wuppertal-Budapest, 2410.06216, 4HEX 16^3X8



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