Thermodynamic relations for perfect spin hydrodynamics

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02-06.12.2024, 24th Zimányi School, Budapest

based on: W. Florkowski + MH, arXiv:2405.03263 and Z. Drogosz + WF + MH, Phys.Rev.D 110 (2024) 9, 096018

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The problem to be solved

The usual Israel-Stewart approach with spin uses a phenomenological form of scalar thermodynamic relations multiplied by u^μ, where $\omega_{\alpha\beta} = \Omega_{\alpha\beta}/T$ is the spin polarization tensor, thus, from

$$\varepsilon + P = T\sigma + \mu n + \frac{1}{2}\Omega_{\alpha\beta}S^{\alpha\beta} \tag{1}$$

we obtain

$$S^\mu_{\text{eq}} = P\beta^\mu - \xi N^\mu_{\text{eq}} + \beta_\lambda T^{\lambda\mu}_{\text{eq}} - \frac{1}{2}\omega_{\alpha\beta}S^{\mu,\alpha\beta}_{\text{eq}}, \tag{2}$$

$$dS^\mu_{\text{eq}} = -\xi\, dN^\mu_{\text{eq}} + \beta_\lambda\, dT^{\lambda\mu}_{\text{eq}} - \frac{1}{2}\omega_{\alpha\beta}dS^{\mu,\alpha\beta}_{\text{eq}}, \tag{3}$$

$$d(P\beta^\mu) = N^\mu_{\text{eq}}\, d\xi - T^{\lambda\mu}_{\text{eq}}\, d\beta_\lambda + \frac{1}{2}S^{\mu,\alpha\beta}_{\text{eq}}d\omega_{\alpha\beta}. \tag{4}$$

where the spin tensor is $S^{\lambda,\mu\nu} = u^\lambda S^{\mu\nu}$. Two problems exists:
1) $S^{\lambda,\mu\nu}_{eq}$ usually has additional terms. 2) If $\omega_{\mu\nu} \sim S^{\mu\nu} \sim \mathcal{O}(\partial^1)$, the spin tensor in (1) should be neglected.

Our solution

Kinetic theory + proper counting scheme leads to:

$$S^{\mu}_{eq} = \mathcal{N}^{\mu} - \xi N^{\mu}_{eq} + \beta_{\lambda} T^{\lambda\mu}_{eq} - \tfrac{1}{2}\omega_{\alpha\beta} S^{\mu,\alpha\beta}_{eq}, \qquad \mathcal{N}^{\mu} = \coth(\xi)\, N^{\mu}_{eq} \neq P\beta^{\mu},$$

$$dS^{\mu}_{eq} = -\xi dN^{\mu}_{eq} + \beta_{\lambda} dT^{\lambda\mu}_{eq} - \tfrac{1}{2}\omega_{\alpha\beta} dS^{\mu,\alpha\beta}_{eq}, \tag{5}$$

$$d\mathcal{N}^{\mu} = N^{\mu}_{eq} d\xi - T^{\lambda\mu}_{eq} d\beta_{\lambda} + \tfrac{1}{2} S^{\mu,\alpha\beta}_{eq} d\omega_{\alpha\beta}, \tag{6}$$

$$N^{\mu}_{eq} = \bar{n} u^{\mu} + n_t t^{\mu}, \tag{7}$$

$$T^{\mu\nu}_{eq} = \bar{\varepsilon} u^{\mu} u^{\nu} - \bar{P} \Delta^{\mu\nu} + P_k\, k^{\mu} k^{\nu} + P_{\omega}\, \omega^{\mu} \omega^{\nu} + P_t \left(t^{\mu} u^{\nu} + t^{\nu} u^{\mu} \right). \tag{8}$$

And spin current tensor includes an ortogonal parts to u^{λ}.

$$S^{\lambda,\mu\nu}_{eq} = u^{\lambda} S^{\mu\nu} + smth^{\lambda,\mu\nu}. \tag{9}$$