Multiplicity distributions from maximally entangled initial state

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Multiplicity distributions

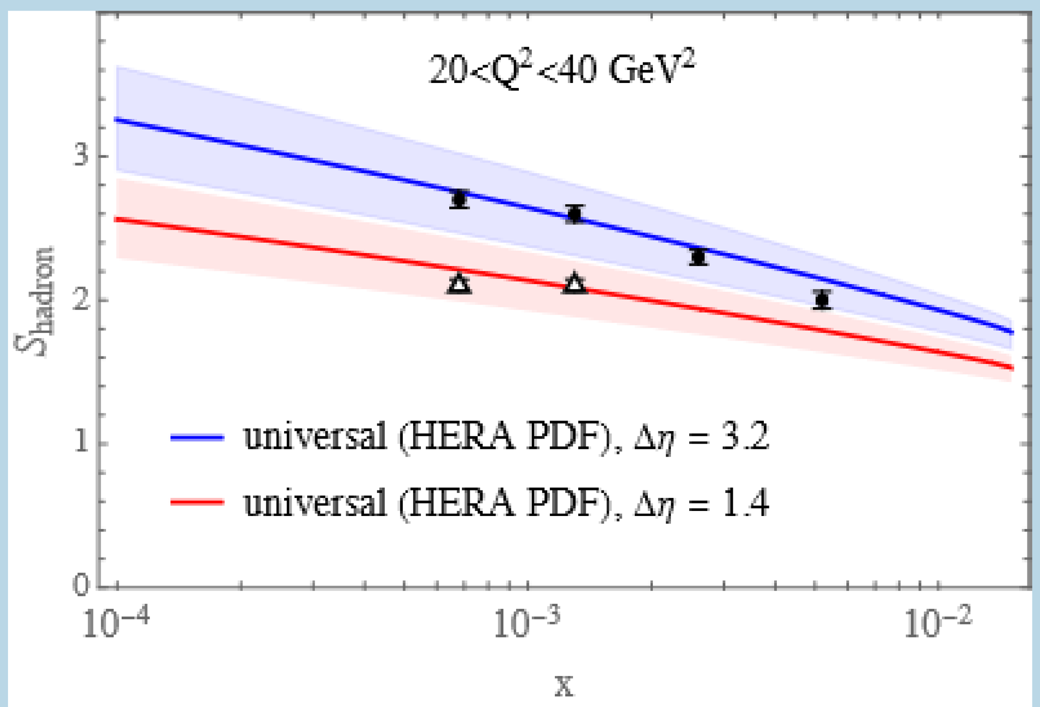
- Distribution of charged particles
- Branching and clan structure explanations [1] : $(n+1)\frac{P_{n+1}}{P_n} = a + bn$
- Data can be described by negative binomial distribution (NBD)
- "First principle" origin of NBD?

Conjecture: initial partonic state is maximally entangled [2]!

- High energy, initial state partonic microstates have equal probability
- System maximally entangled ⇒ von Neumann entropy is maximal
- If so, what is the probability distribution function?
- Parton-hadron duality ⇒ FS particle MD ∼ IS distribution
- FS MD entropy = IS entanglement entropy

Connection to pQCD

- Maximal entropy ⇒ uniform distribution: $S = \ln(\overline{n}(x,\mu))$
- pQCD calculations can be utilized [3]:
  - $\overline{n}(x,\mu) = xg(x) + \sum_f (q_f(x,\mu) + \overline{q}_f(x,\mu))$
- Good agreement with the data



Multiplicities from the principle of maximum entropy

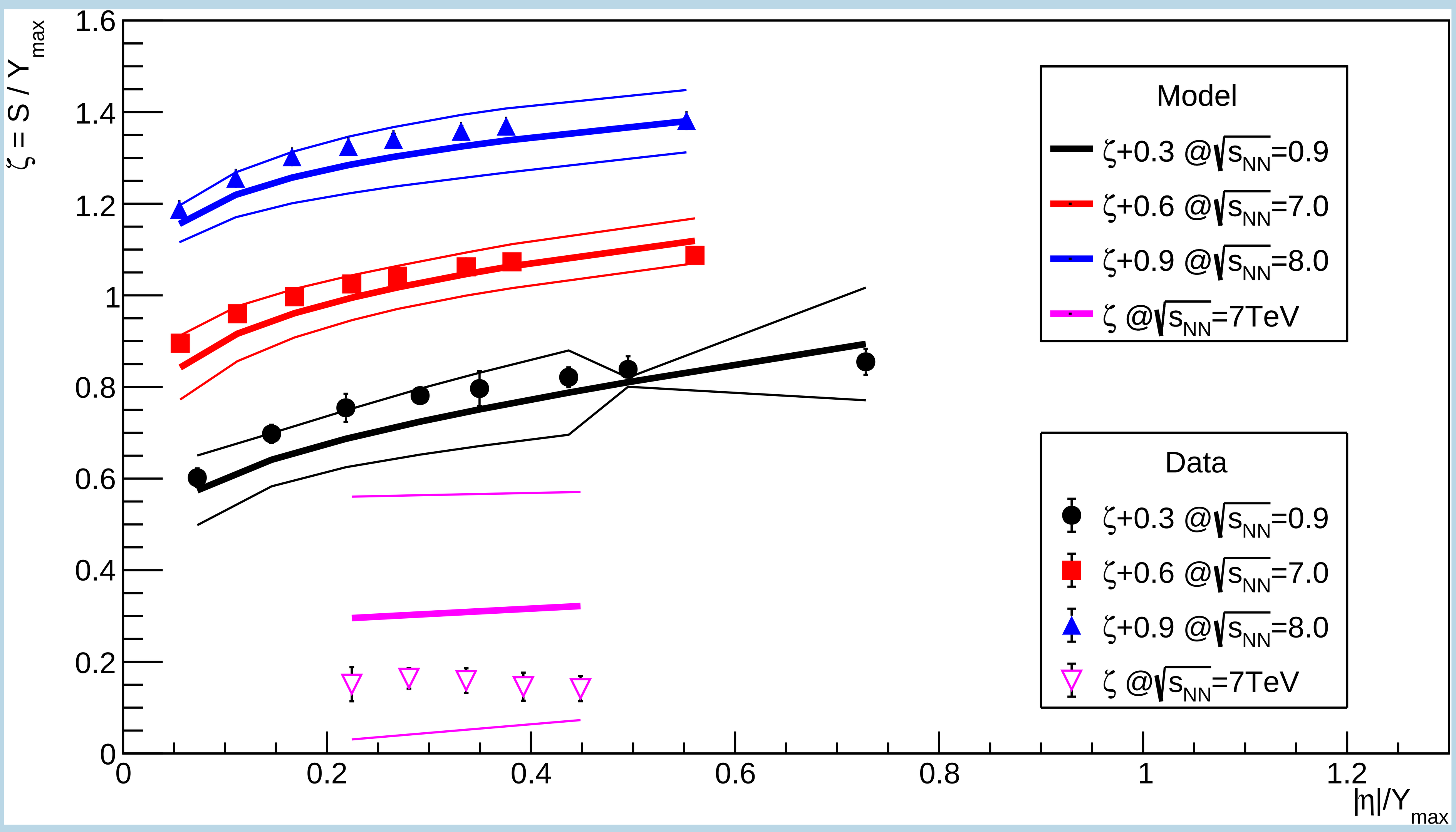
- Using mathematical statistic notions to derive hadron multiplicities
- Assumptions: maximally entangled initial state and parton-hadron duality
- Employing POME and Lagrange multipliers to derive the $p(x)$ distribution
  - with maximum entropy ⇒ $\mathcal{S}(x) = -\int_0^\infty p(x)\ln p(x)dx$
  - being probability ⇒ $P(x) = \int_0^\infty p(x)dx - 1 = 0$
  - with fixed mean ⇒ $Q(x) = \int_0^\infty xp(x)dx - \mu = 0$
  - the full Lagrangian: $\mathcal{L} = \mathcal{S} - \alpha\mathcal{P} - \beta\mathcal{Q}$
- From the maximization of the Lagrangian and the constrains ⇒ $p(x;\mu) = \frac{1}{\mu}e^{-\frac{x}{\mu}}$
- Poisson-transform ⇒ $P(n;k) = (1-p)^n p$, with $k = 0, 1, 2, \ldots$ the geometric distribution
- Usual re-parametrization with the $\overline{n}$ parameter ⇒ $P(n;\overline{n}) = \frac{1}{\overline{n}+1}\left(\frac{\overline{n}}{\overline{n}+1}\right)^n$
- Describe the measured multiplicities in moving rapidity windows (e.g. LHCb [4])
- Convolution of exponential distributions ⇒ $\Gamma(x;\alpha,\beta) = \frac{\beta^\alpha x^{\alpha-1}}{\Gamma(\alpha)}e^{-\beta x}$
- Re-parametrized Poisson-transform with $\overline{n}$ and k ⇒ $P(n;k,\overline{n}) = \frac{\Gamma(n+k)}{\Gamma(k)\Gamma(n+1)}\left(\frac{k}{\overline{n}+k}\right)^k\left(\frac{\overline{n}}{\overline{n}+k}\right)^n$
- Describe the measured multiplicities in opening rapidity windows (e.g. ALICE, CMS [5, 6])
- k=1 restores geometric distribution, if more exponential distributions were convoluted ⇒ $k > 1$

Results

- Could be the sign for gluon low-x saturation if $1/k \to 0$?
- Statistical ⇔ physical meaning of k? It appears in saturation models [7]
- Balitzky-Kovchegov equation based cascade equation describe color dipole evolution [2]

$$\frac{dP_n(\eta)}{d\eta} = -\lambda n P_n(\eta) + \lambda(n-1)P_{n-1}(\eta)$$

- The $P_n(\eta)$ number of dipoles: GD and NBD solutions ⇒ data comparison and entropy



- Scaling with maximum rapidity $Y_{\max} = \ln\left(\sqrt{s_{NN}}/m_p\right) \xrightarrow{?}$ maximum entropy
- Moving η window ⇒ constant entropy, opening $|\eta|$ window ⇒ saturating entropy
- Due to correlations, the system should de-cohere but no good data to check it

Summary

- POME ⇒ FS MD can be derived
- Physics of POME = max. entangled IS
- Relation to pQCD but more general
- Solutions of Balitzky-Kovchegov equation
- The calculations fit the data
- Different systems? Same physics?
- Related to gluon saturation?

Outlook

- Data comparison in various systems
- Dedicated pp data $\xrightarrow{?}$ better PDFs?
- UPC measurements $\xrightarrow{?}$ better nPDFs?
- Data comparison in moving and opening η
- Decoherence due to correlations?
- Difference in forward and mid-rapidity?
- POME ⇔ saturation models?

References

[1] Giovannini et al., NPB PS.(1992) 25, 115
[2] D. Kharzeev et al., PRD (2017) 95, 11
[3] M. Hentschinski et al., arXiv:2408.01259
[4] LHCb, EPJC (2012), 72, 1947
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[6] CMS, JHEP (2011), 01, 079
[7] A. Dumitru, PRC (2012) 85, 044920

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