Finanțat de Uniunea Europeană
NextGenerationEU

Planul Național de Redresare și Reziliență

Dirac fermions under imaginary rotation

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Abstract

Recent years have seen an increase in the interest to investigate the thermodynamic properties of strongly-interacting systems under rotation. Such studies are usually performed using lattice gauge techniques on the Euclidean manifold and with an imaginary angular velocity, $\Omega = i\Omega_I$. When $\nu = \beta\Omega_I/2\pi$ is a rational number, the thermodynamics of free scalar fields "fractalizes" in the large volume limit, that is, it depends only on the denominator q of the irreducible fraction $\nu = p/q$ [1].

The present study considers the same problem for free, massless, fermions at finite temperature $T = \beta^{-1}$ and chemical potential μ and confirms that the thermodynamics fractalizes when $\mu = 0$. Curiously, fractalization has no effect on the chemical potential μ, which dominates the thermodynamics when q is large. The fractal behavior is shown analytically for the fermionic condensate (this poster), the charge currents and the energy-momentum tensor [3]. For these observables, the limits on the rotation axis are validated by comparison to the results obtained in [2] for the case of real rotation. Enclosing the system in a fictitious cylinder of radius R and length L_z allows constructing averaged thermodynamic quantities that satisfy the Euler relation and fractalize (see [3]).

Rotating fermions in the macrocanonical ensemble

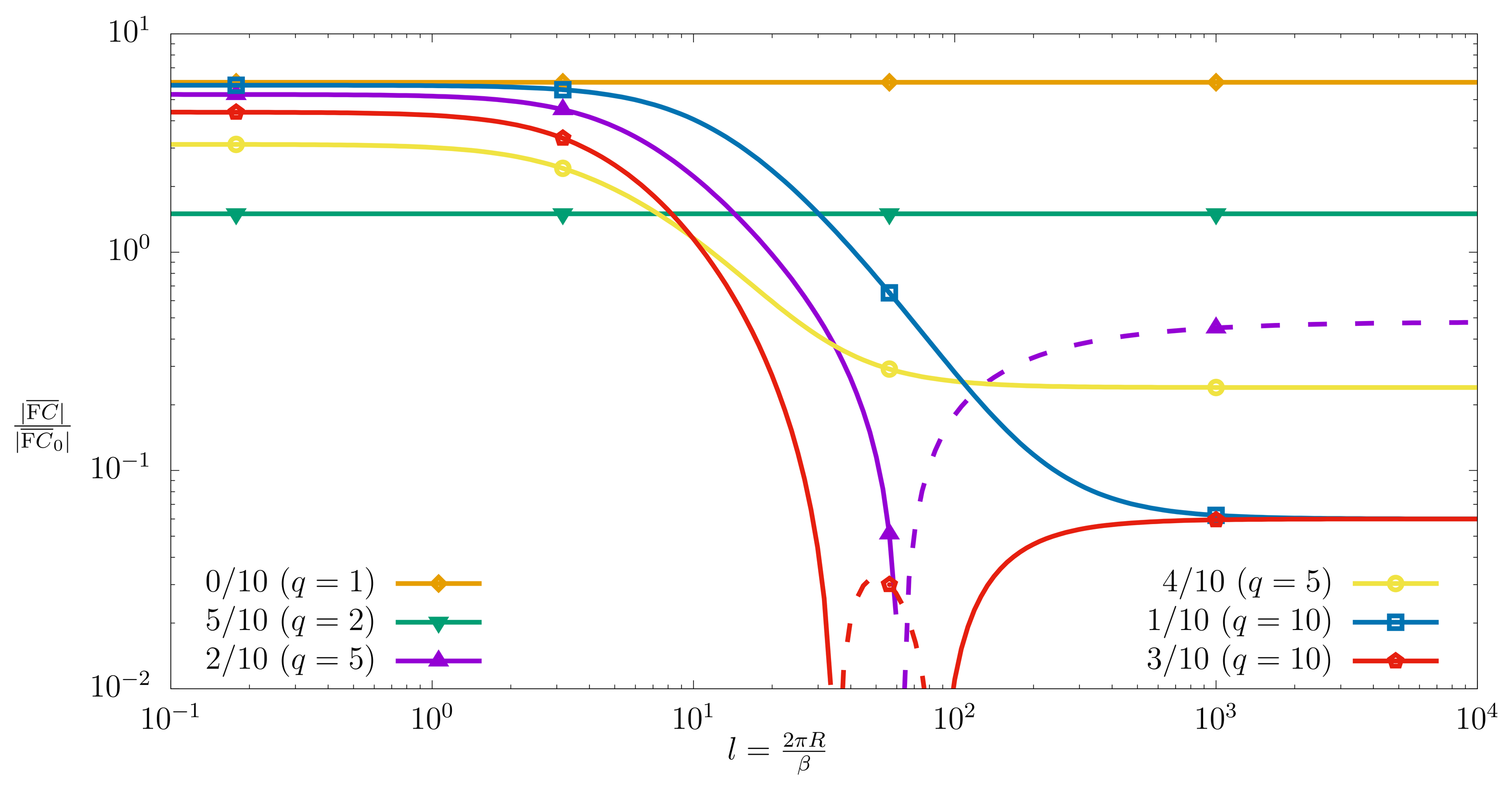
- Rotation frequency $\Omega \mapsto i\Omega_I$ and chemical potential μ.
- Free, massless fermions, with charge $\widehat{Q} = \int d^3x\, \widehat{\Psi}^\dagger \widehat{\Psi}$ (conserved).
- Quantized field $\widehat{\Psi}(x) = \sum_j \left[U_j(x)\hat{b}_j + V_j(x)\hat{d}_j^\dagger\right]$.
- Modes U_j, V_j are eigenfunctions of the Hamiltonian $H = i\partial_t = -i\gamma^0\boldsymbol{\gamma}\cdot\boldsymbol{\nabla} + M\gamma^0$, vertical momentum $P^z = -i\partial_z$, angular momentum along the z axis $J^z = -i\partial_\varphi + S^z$, and helicity operator $h = \mathbf{S}\cdot\mathbf{P}/P$, where the spin matrix $\boldsymbol{S}$ is given by $\boldsymbol{S} = \frac{1}{2}\gamma^5\gamma^0\boldsymbol{\gamma} = \frac{1}{2}\begin{pmatrix}\boldsymbol{\sigma} & 0\\ 0 & \boldsymbol{\sigma}\end{pmatrix}$, as to diagonalize the density operator.
- Density operator describind rigidly-rotating thermal states
$$\hat{\rho} = \exp\left\{-\beta\left(:\widehat{H}: -\mu:\widehat{Q}: -\Omega:\widehat{J}^z:\right)\right\}.$$
- Thermal expectation values $\langle\widehat{A}\rangle = \mathcal{Z}^{-1}\,\mathrm{Tr}(\hat{\rho}\widehat{A})$ with $\mathcal{Z} = \mathrm{Tr}(\hat{\rho})$.
- Thermal expectation value energy $\mathcal{E}_j^\sigma \equiv E_j - \sigma\mu$, with $\sigma = \pm 1$.
- Rotating frame effective energy $\tilde{\mathcal{E}}_j^\sigma \equiv \mathcal{E}_j^\sigma - \Omega m_j = \tilde{E}_j - \sigma\mu$.
- Rotating frame (comoving) energy $\tilde{E}_j \equiv E_j - \Omega m_j$.

Imaginary thermal expectation values

- Thermal expectation values become for operator $\widehat{A}$
$$A_\beta^\Omega \equiv \langle:\widehat{A}:\rangle_\beta^\Omega = \sum_{j,\sigma}\mathcal{C}_\sigma(\mathcal{A})\frac{\mathcal{A}(U_j,U_j)}{e^{\beta\tilde{\mathcal{E}}_j^\sigma}+1}.$$
- Thermal factor becomes (expansion holds for any $\mathcal{E}_j^\sigma$)
$$\frac{1}{\exp(\beta\tilde{\mathcal{E}}_j^\sigma)+1} \equiv \frac{1}{\exp[\beta(\mathcal{E}_j^\sigma - i\Omega_I m_j)]+1} = \theta(\mathcal{E}_j^\sigma)\sum_{v=1}^\infty(-1)^{v+1}e^{-v\beta\mathcal{E}_j^\sigma}e^{iv\beta\Omega_I m_j} + \theta(-\mathcal{E}_j^\sigma)\sum_{v=0}^\infty(-1)^v e^{v\beta\mathcal{E}_j^\sigma}e^{-iv\beta\Omega_I m_j}.$$
- Thermal expectation values split as
$$A_\beta^{i\Omega_I} \equiv \langle:\widehat{A}:\rangle_\beta^{i\Omega_I} \equiv A_{v=0}^{i\Omega_I} + \Delta A^{i\Omega_I}.$$

Thermodynamics inside a fictitious cylinder

- Calculate thermodynamic quantities from $\bar{\Phi}(\beta,\mu,Q) = \frac{1}{\beta}\int\bar{\mathcal{E}}_{\beta\Omega,\beta\mu}d\beta$.
- The averaged quantities over a fictitious cylinder of radius R fractalize:



Fermion condensate fractalization

- The imaginary frequency thermal expectation value for the fermion condensate:
$$\frac{FC^{i\Omega_I}}{M} = \frac{\mu^2}{2\pi^2} + \frac{2}{\pi^2}\sum_{v=1}^\infty\frac{1}{(v\beta)^2}(-1)^{v+1}\frac{c_v}{1+\alpha_v^2}\cos(v\beta\mu\alpha_v),$$
where $s_v = \sin\left(\frac{v\beta\Omega_I}{2}\right)$, $c_v = \cos\left(\frac{v\beta\Omega_I}{2}\right)$ and $\alpha_v = \frac{2\rho}{v\beta}s_v$,
- On the rotation axis
$$\left.\frac{FC^{i\Omega_I}}{M}\right|_{\rho\to 0} = \frac{\mu^2}{2\pi^2} + \frac{1}{6\beta^2} - \frac{\Omega_I^2}{8\pi^2} = \left[\frac{\mu^2}{2\pi^2} + \frac{1}{6\beta^2} + \frac{\Omega^2}{8\pi^2}\right]_{\Omega\to i\Omega_I},$$
which agrees with the results obtained for real rotation [2].
- Consider $\nu = \beta\Omega_I/2\pi = p/q$ rational frequencies (as irreducible fractions).
- Take $v = r + qQ$, denote $l = 2\pi\rho/\beta$ and $x_r = ls_r/\pi q$.
- Split $\sum_{v=1}^\infty \equiv \sum_{r+qQ=1}^\infty \equiv \sum_{Q=0}^\infty\sum_{r=1}^q$.
- Fractalized value of the fermion condensate:
$$\frac{FC^{i\Omega_I}}{M} = \frac{\mu^2}{2\pi^2} + \frac{2}{\pi^2\beta^2q^2}\sum_{r=1}^q(-1)^{r+1}c_r\cos(\beta\mu qx_r)\sum_{Q=0}^\infty\frac{(-1)^{kQ}}{\left(Q+\frac{r}{q}\right)^2+x_r^2}.$$
- When $r = q$, $c_r = (-1)^{pQ}$ and $s_r = x_r = 0 \Rightarrow$ position-independent term.
- The terms with $1 \le r < q$ vanish when $l \to \infty$.

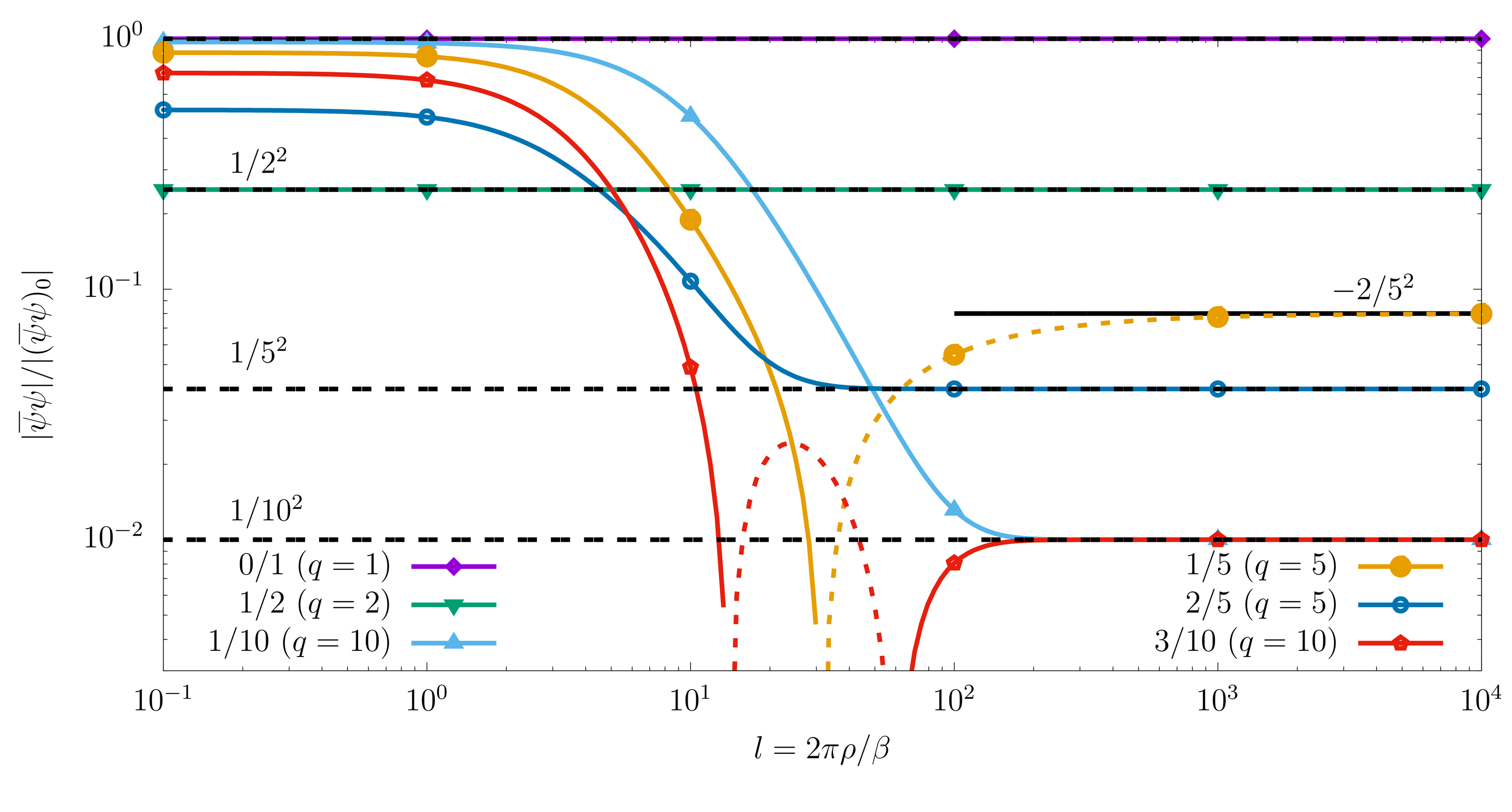
$k = p + q$ **odd case**

$$\left.\frac{FC^{i\Omega_I}}{M}\right|_{k=\mathrm{odd}} = \frac{\mu^2}{2\pi^2} + \frac{1}{6\beta^2q^2} - \sum_{r=1}^{q-1}\frac{(-1)^{r+1}c_r}{2\pi^2\beta^2q^2x_r}\cos(\beta\mu qx_r)\,\mathrm{Im}\,[\Delta\psi_r],$$
where $\Delta\psi_r = \psi\left(\frac{\frac{r}{q}+ix_r+1}{2}\right) - \psi\left(\frac{\frac{r}{q}+ix_r}{2}\right)$ with ψ the digamma function.

$k = p + q$ **even case**

$$\left.\frac{FC^{i\Omega_I}}{M}\right|_{k=\mathrm{even}} = \frac{\mu^2}{2\pi^2} - \frac{1}{3\beta^2q^2} + \sum_{r=1}^{q-1}\frac{(-1)^{r+1}c_r}{\pi^2\beta^2q^2\,x_r}\cos(\beta\mu qx_r)\,\mathrm{Im}\,\psi_r,$$
where $\psi_r = \psi(r/q + ix_r)$ with ψ the digamma function.

- The chemical potential contribution to the asymptotic term is independent of $\nu = \beta\Omega_I/2\pi$!
- The temperature dependence of the asymptotic term reveals the fractalized effective temperature $T_q = T/q$ [4]!



This work is supported by the European Union - NextGenerationEU through grant No. 760079/23.05.2023, funded by the Romanian ministry of research, innovation and digitalization through Romania's National Recovery and Resilience Plan, call no. PNRR-III-C9-2022-I8.

References

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