



UNIVERSITÀ DEGLI STUDI
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Few aspects of electroweak phenomenology at hadron colliders

Alessandro Vicini

University of Milano, INFN Milano

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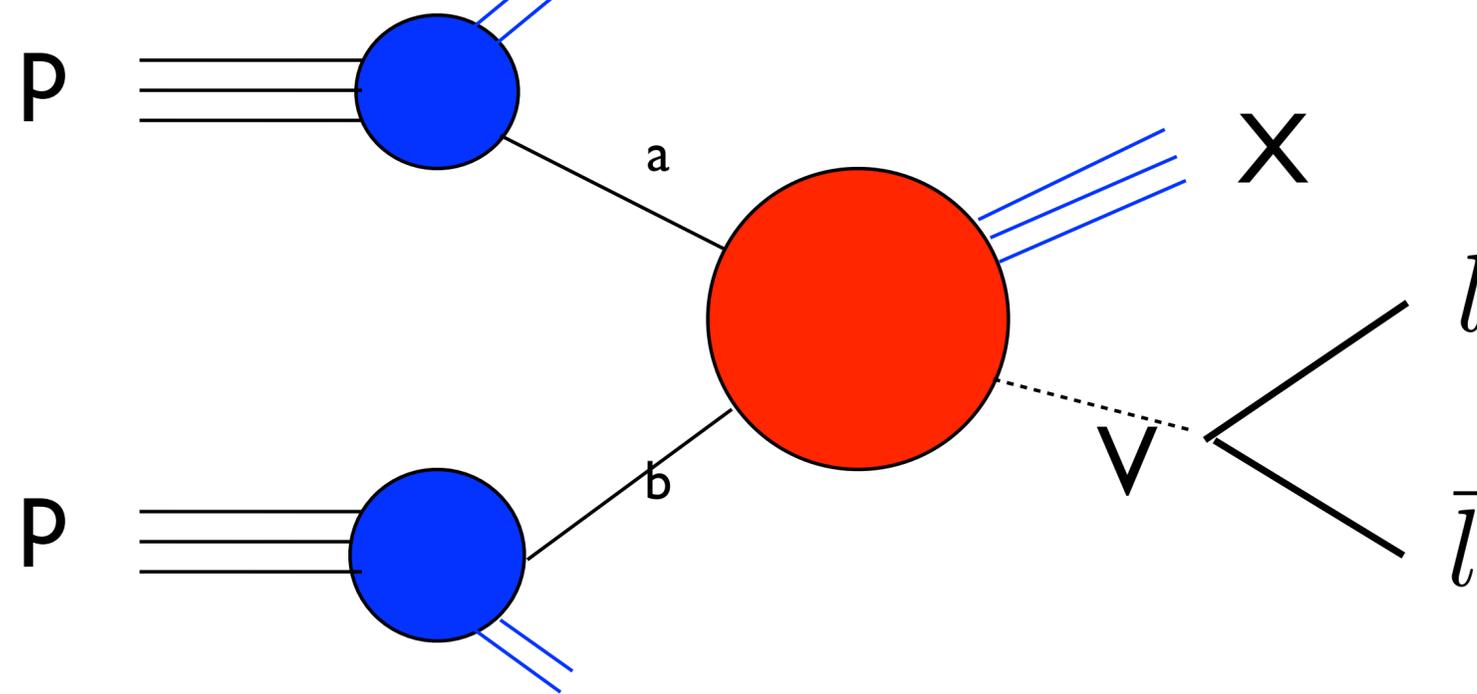
Outline of the lecture

- Phenomenology of the EW quantum corrections. QED effects, weak effects, interplay in higher orders.
- Scattering processes at a hadron collider: QCD, EW and mixed QCD-EW corrections
- Resonance-aware matching: algorithmic issues with two competing interactions
- Exact calculations with full off-shell description vs on-shell expansions.
- photon and leptons

We discuss high-precision (i.e. **sub-percent**) phenomenology → we need to identify all relevant corrections at this level

Hard scattering at hadron colliders

$$\sigma(P_1, P_2; m_V) = \sum_{a,b} \int_0^1 dx_1 dx_2 f_{h_1,a}(x_1, M_F) f_{h_2,b}(x_2, M_F) \hat{\sigma}_{ab}(x_1 P_1, x_2 P_2, \alpha_s(\mu), M_F)$$



e.g. the Drell-Yan process

The factorisation theorems guarantee the validity of the above picture up to power correction effects

The interplay of QCD and EW interactions appears both in the partonic cross section and in the proton PDFs

- the partonic cross section receives both QCD and EW perturbative corrections
- the DGLAP evolution of the proton PDFs is achieved with a QCD+QED kernel

m_W and $\sin^2 \theta_{eff}^l$ determination
via template fitting

Template fitting and its theoretical systematic uncertainties

Given one experimental kinematical distribution

- we compute the corresponding theoretical distribution for several hypotheses of one Lagrangian input parameters (e.g. m_W)
- we compute, for each $m_W^{(k)}$ hypothesis, a χ_k^2 defined in a certain interval around the jacobian peak (fitting window)
- we look for the minimum of the χ^2 distribution

The m_W value associated to the position of the minimum of the χ^2 distribution is the experimental result

Template fitting and its theoretical systematic uncertainties

Given one experimental kinematical distribution

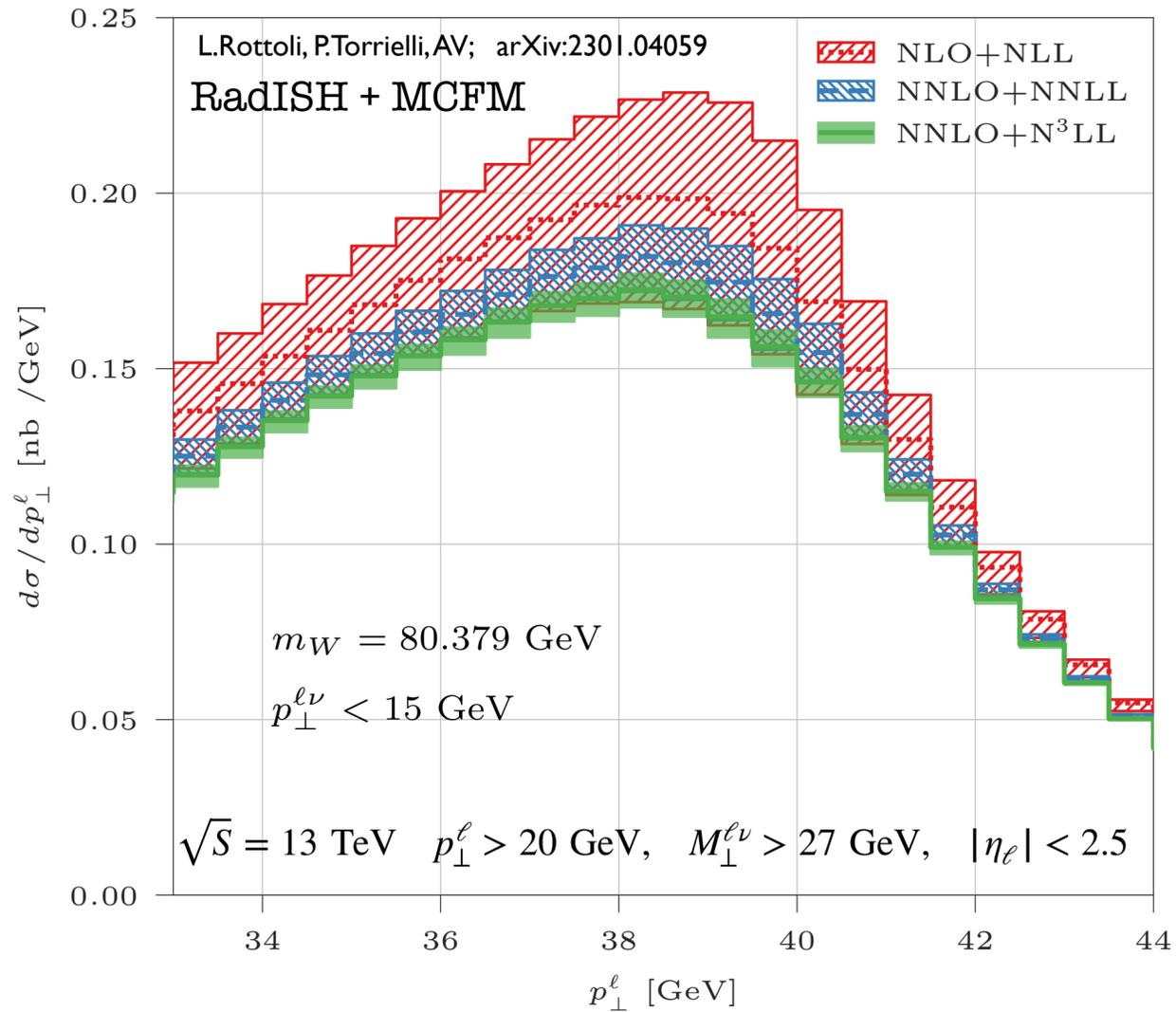
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The theoretical uncertainties of the templates contribute to the **theoretical systematic error**

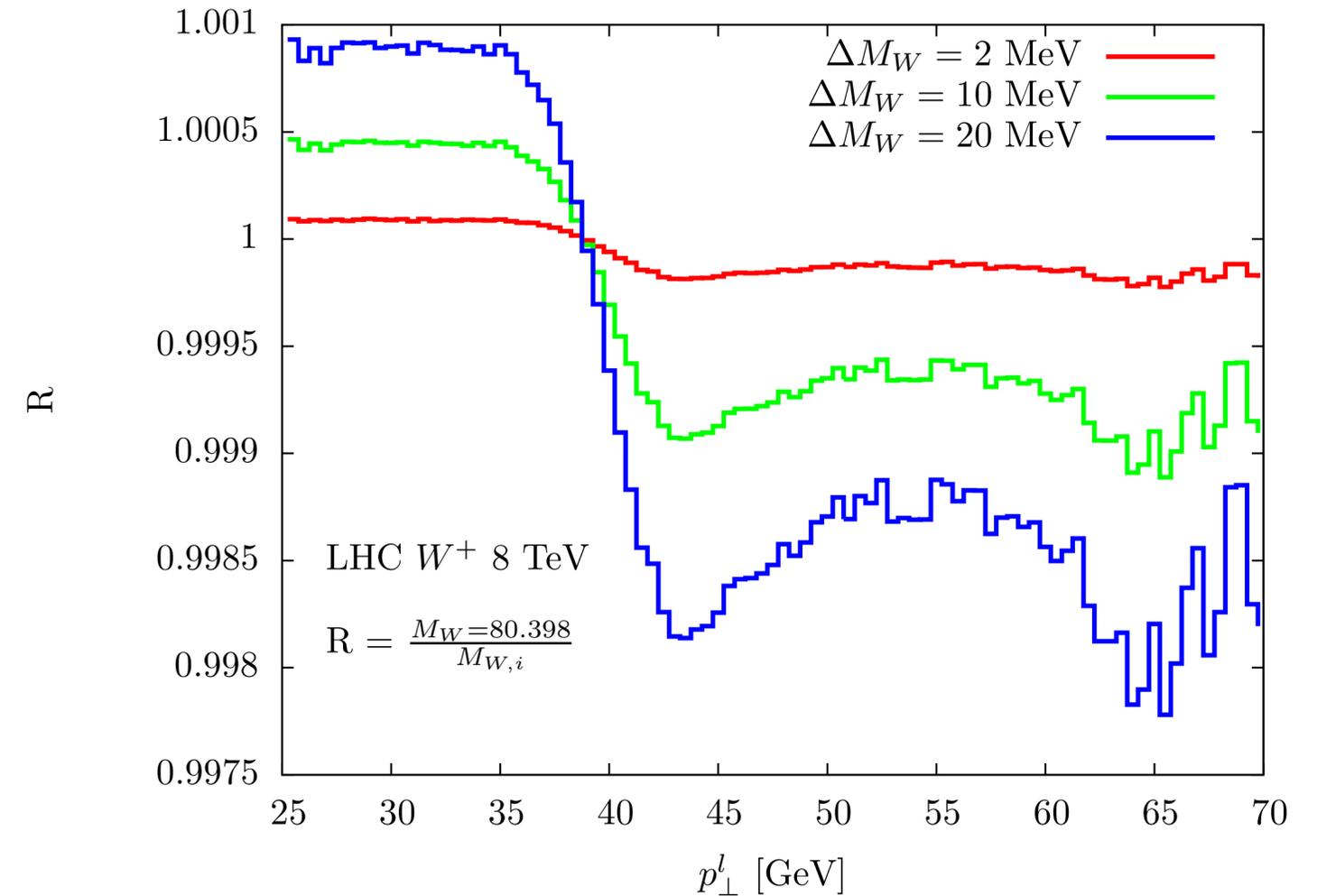
- higher-order QCD and EW
- PDF uncertainties
- heavy quarks corrections
- non-perturbative QCD

m_W determination at hadron colliders: sensitivity



The position of the jacobian peak is related to the value of m_W
 increasing m_W shifts the peak “to the right”

A change by 10 MeV is invisible at glance, we show it with a ratio

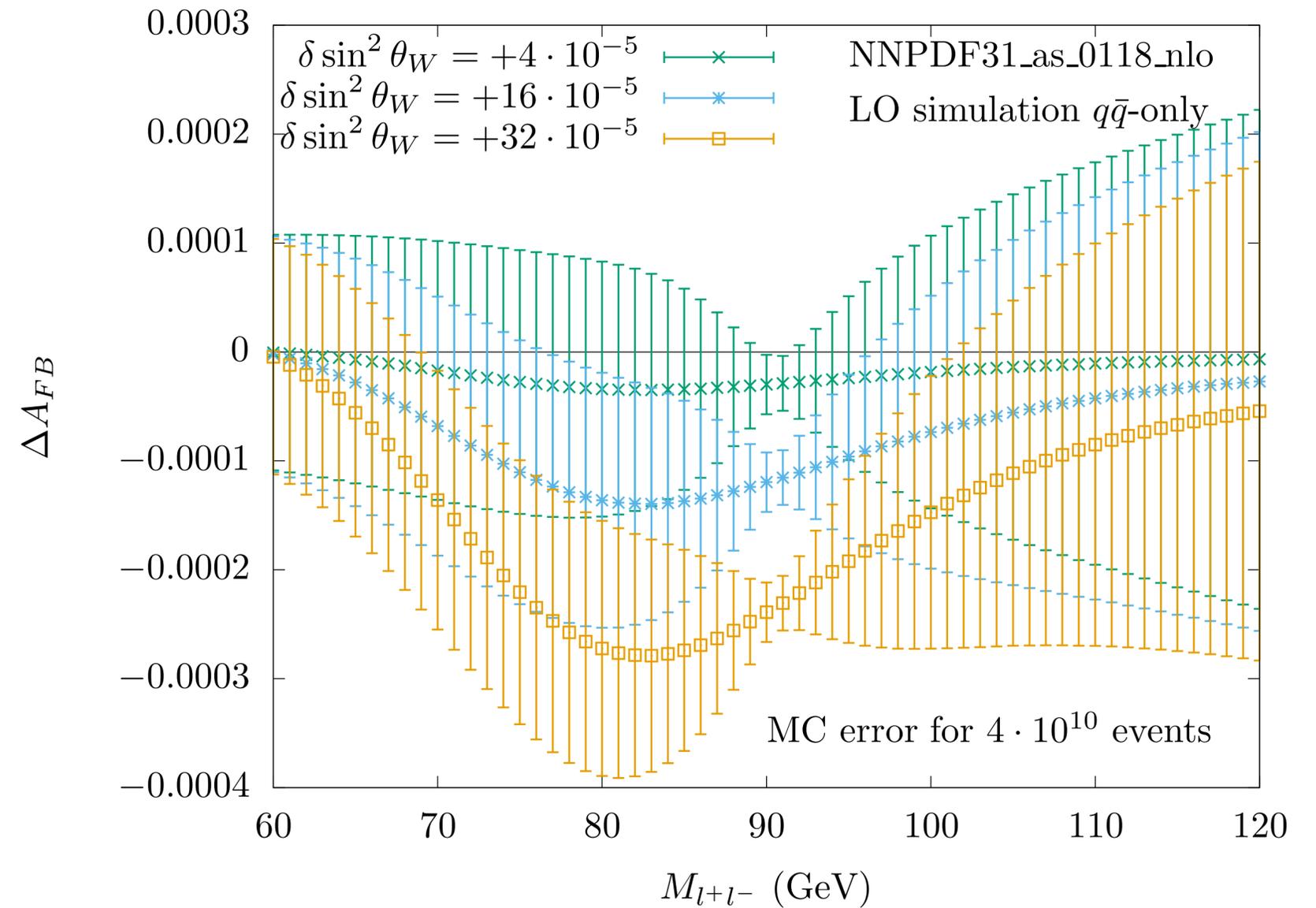
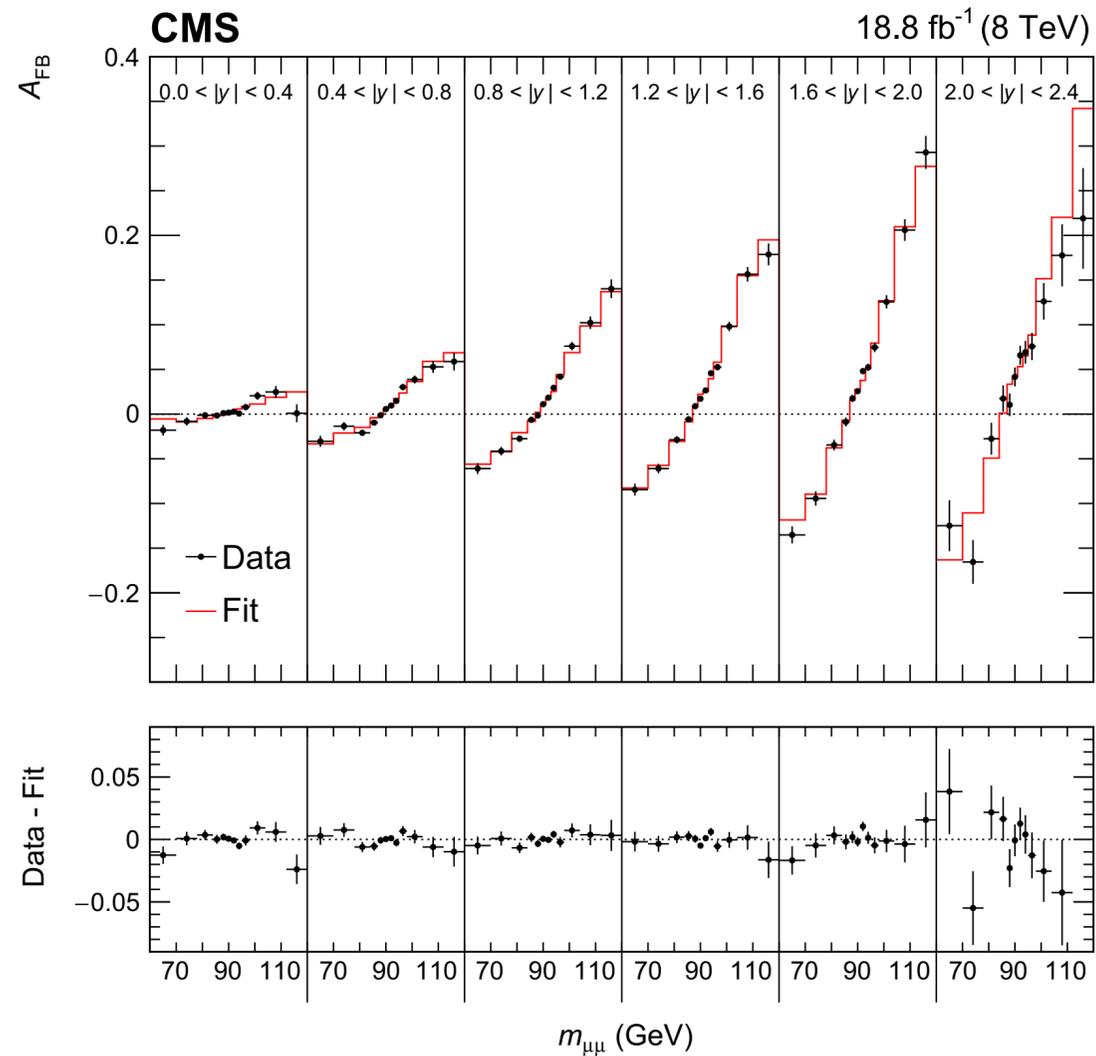


The m_W determination at 10 MeV level requires the ability to appreciate the shape of the distribution at 0.1% level

The theoretical uncertainties set a limit on our resolution

$\sin^2 \theta_{eff}^\ell$ determination at hadron colliders: sensitivity

LEP/SLD results $\sin^2 \theta_{eff}^\ell = 0.23152(16)$



The $\sin^2 \theta_{eff}^\ell$ determination at level requires the ability to appreciate $A_{FB}(m_{\ell\ell})$ distribution at the 10^{-4} level

The theoretical uncertainties set a limit on our resolution → how large are the radiative corrections to this asymmetry ?

Electroweak corrections

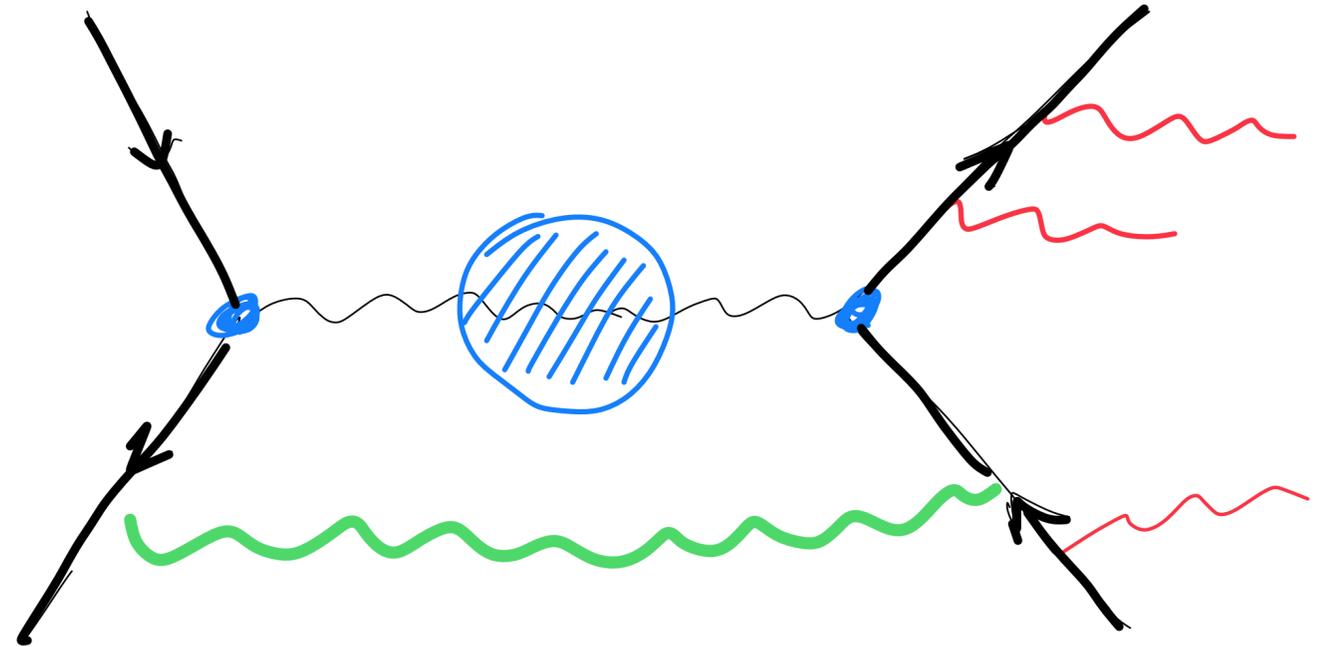
EW corrections: QED and weak effects

- Three main sources of large EW corrections:

- coupling redefinition

- QED real radiation

- EW virtual Sudakov logarithms



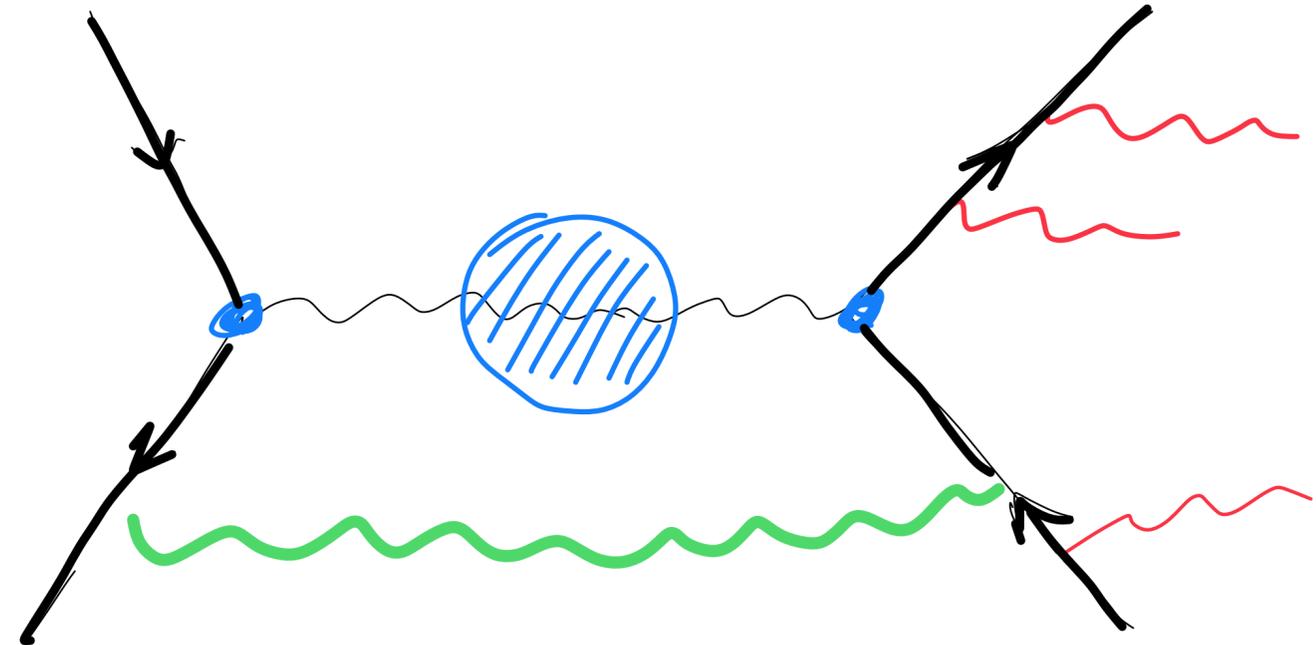
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Breakdown of the NLO-EW corrections to $q\bar{q} \rightarrow \mu^+\mu^-$

$$\sigma(q\bar{q} \rightarrow \mu^+\mu^- + X) = \sigma_0 + \frac{\alpha}{2\pi}\sigma_\alpha + \dots \quad \sigma_\alpha = \sigma_\alpha^{QED} + \sigma_\alpha^{weak}$$

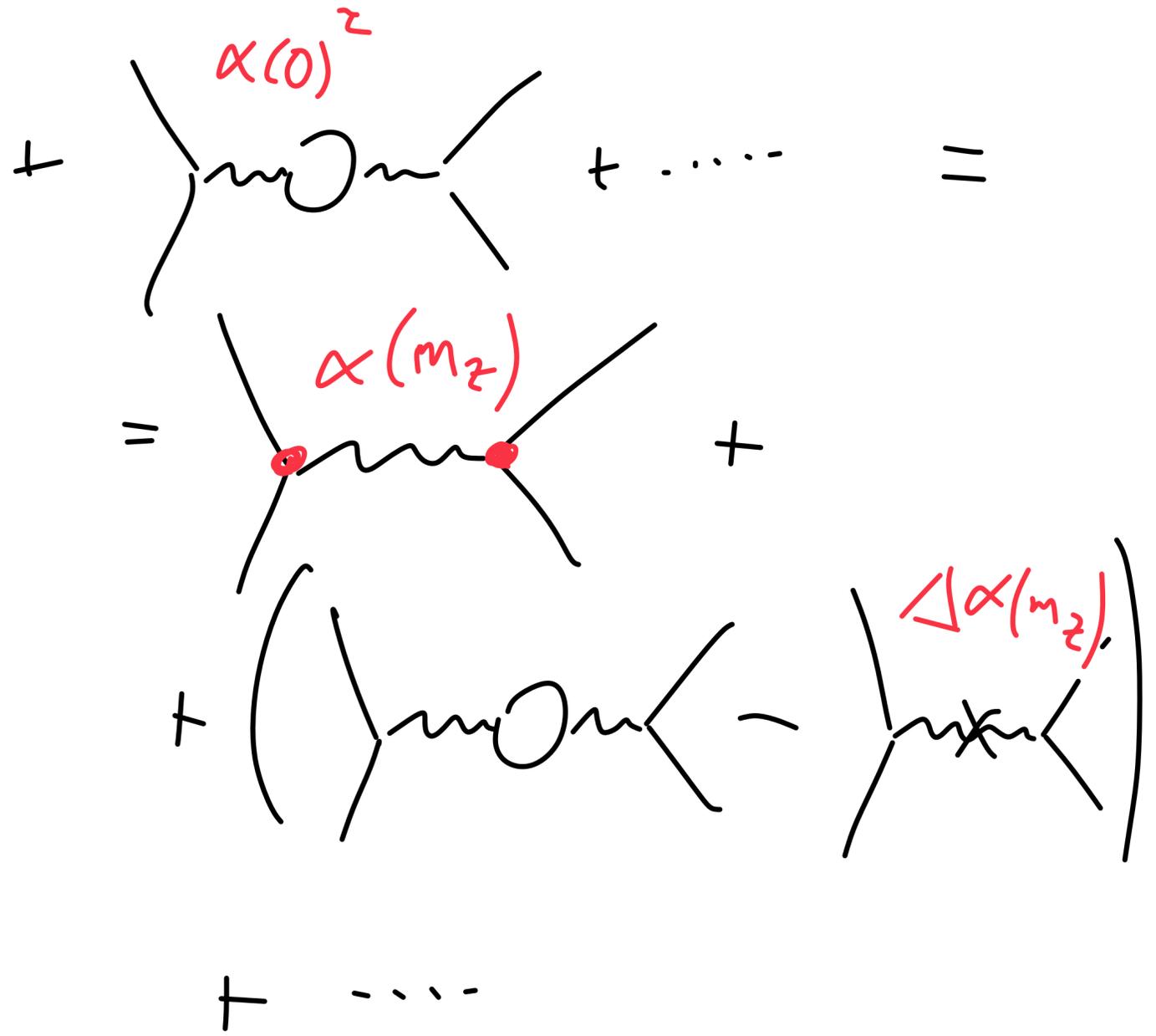
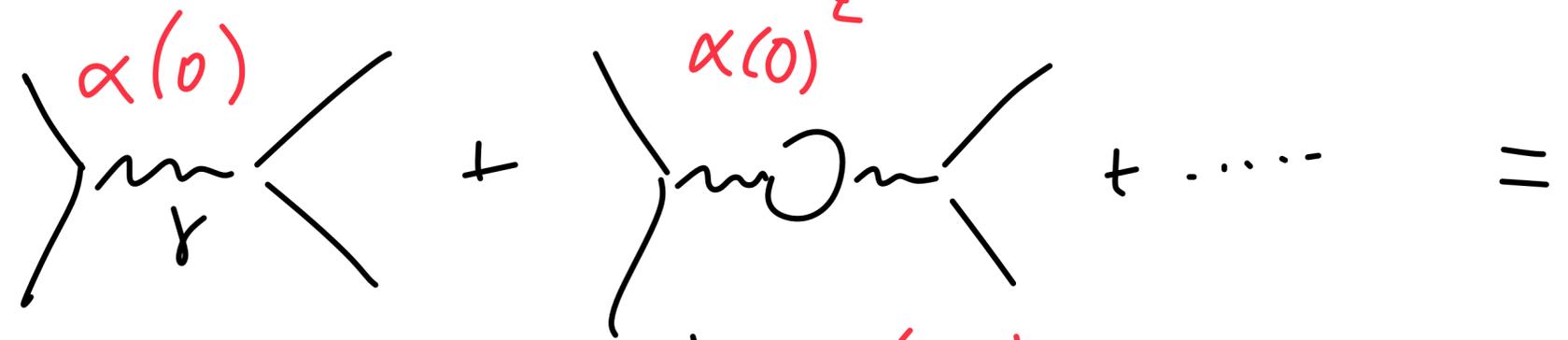
$$\sigma_\alpha^{QED} = \sigma_\alpha^{QED,real} + \sigma_\alpha^{QED,virt} = \sigma_\alpha^{QED,LL} + \sigma_\alpha^{QED,remainder}$$

$$\sigma_\alpha^{weak} = \sigma_\alpha^{weak,virt} = \sigma_\alpha^{weak,renormalization} + \sigma_\alpha^{weak,Sudakov} + \sigma_\alpha^{weak,remainder}$$

The separation of QED and weak effects is not possible in general, in presence of a charged-current interaction

EW corrections: coupling constants and higher-order universal effects

- Given a scheme with $(\alpha(0), G_\mu, m_Z, m_H)$ as input parameters (renormalisation completed) can we improve our predictions, with the systematic inclusion of higher-order corrections ?



$$\alpha(0) \rightarrow \alpha(m_Z) = \frac{\alpha(0)}{1 - \Delta\alpha(m_Z)} \quad \Delta\alpha(m_Z) \sim 0.07$$

A proper choice of the couplings can yield a more accurate description of the data already at LO

caveat: upon inclusion of (N)NLO corrections, double counting must be removed !

EW corrections: coupling constants and higher-order universal effects

- Given a scheme with $(\alpha(0), G_\mu, m_Z, m_H)$ as input parameters (renormalisation completed) can we improve our predictions, with the systematic inclusion of higher-order corrections ?

- at (N)NLO , the LO couplings receive radiative corrections, stemming from self-energy and vertex corrections

we identify in these corrections some **universal contributions**, independent of the process details (scales, external particles)

we can include up to 2-loop and possibly higher-loop contributions

$$\alpha(0) \rightarrow \alpha(m_Z) = \frac{\alpha(0)}{1 - \Delta\alpha(m_Z)} \quad \Delta\alpha(m_Z) \sim 0.07$$

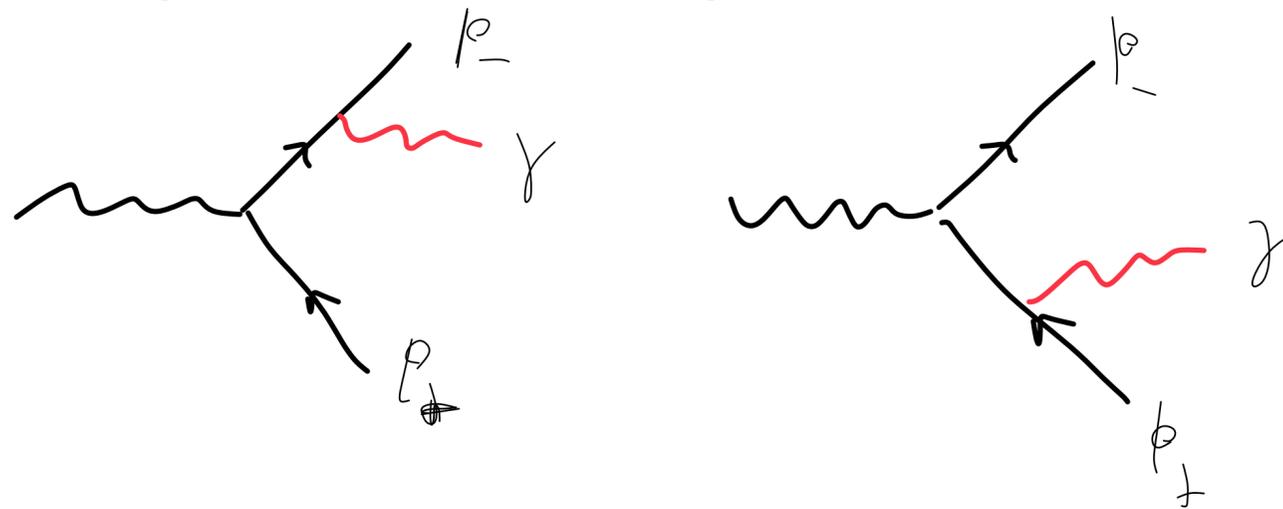
$$\sin^2 \theta_W \rightarrow \sin^2 \bar{\theta}_W \equiv \sin^2 \theta_W + \Delta\rho \cos^2 \theta_W \quad \Delta\rho = 3x_t \left[1 + \rho^{(2)}(m_H^2/m_t^2)x_t \right] \left[1 - \frac{2\alpha_s}{9\pi}(\pi^2 + 3) \right]$$

$$\text{with } 3x_t = \frac{3\sqrt{2}G_\mu m_t^2}{16\pi^2} \sim 0.01$$

EW corrections: QED effects

- in soft approximation, the amplitude for the emission of a photon is described by an eikonal current

$$j_{eik}^\mu = -ie \left[\frac{p_-^\mu}{k \cdot p_-} - \frac{p_+^\mu}{k \cdot p_+} \right]$$



- leading QED radiation corrections, simulated via Parton Shower

$$d\sigma^\infty = \Pi(Q^2, \varepsilon) \sum_{n=0}^{\infty} \frac{1}{n!} |\mathcal{M}_{n,LL}|^2 d\Phi_n$$

$$|M_{1,LL}|^2 = \frac{\alpha}{2\pi} P(z) I(k) \frac{8\pi^2}{E^2 z(1-z)} |\mathcal{M}_0|^2 \quad \text{and} \quad |\mathcal{M}_{n,LL}|^2 \text{ is obtained iterating}$$

$$\Pi(Q^2, \varepsilon) = \exp \left(-\frac{\alpha}{2\pi} I_+ \int d\Omega_\gamma I(k) \right) \quad \text{with} \quad I_+ = \int_0^{1-\varepsilon} dz \frac{1+z^2}{1-z} \quad \text{and} \quad I(k) = \sum_{i,j} Q_i Q_j \eta_i \eta_j \frac{p_i \cdot p_j}{(p_i \cdot k)(p_j \cdot k)} E_\gamma^2$$

Leptons and photons

the radiation emitted by a massive (final state) lepton develops large mass (collinear) logarithms $\propto \log \frac{s}{m_\ell^2}$

The Kinoshita-Lee-Nauenberg (KLN) theorem guarantees that when considering sufficiently inclusive observables, i.e. integrating over the whole radiation emitted by the lepton, such divergent terms cancel

A “bare” lepton is described in an exclusive way by its 4-momentum components, after radiation has been emitted

A muon momentum reconstructed in a muon chamber is close to this description

The fully exclusive description of the radiation features the large logarithmic corrections

For a recombined (“dressed”) lepton

the 4-momentum is obtained by summing

the lepton and surrounding photons momenta in a given phase-space region.

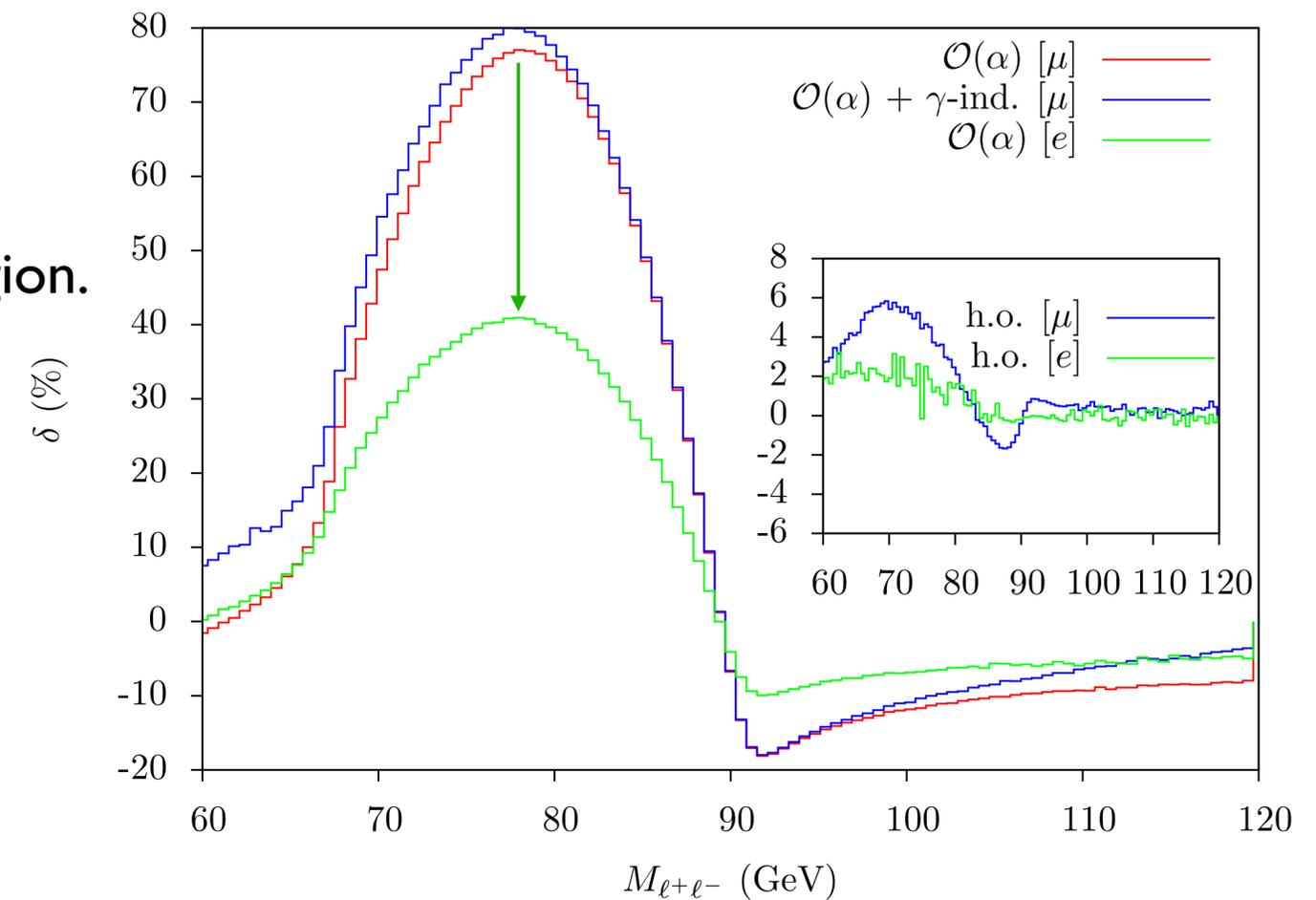
Relevant for electrons.

Including the radiation effectively implements the KLN theorem, leading to smaller radiative effects

The “dressed lepton” definition is applicable at perturbative order (cfr. the discussion about QCD jet definitions)

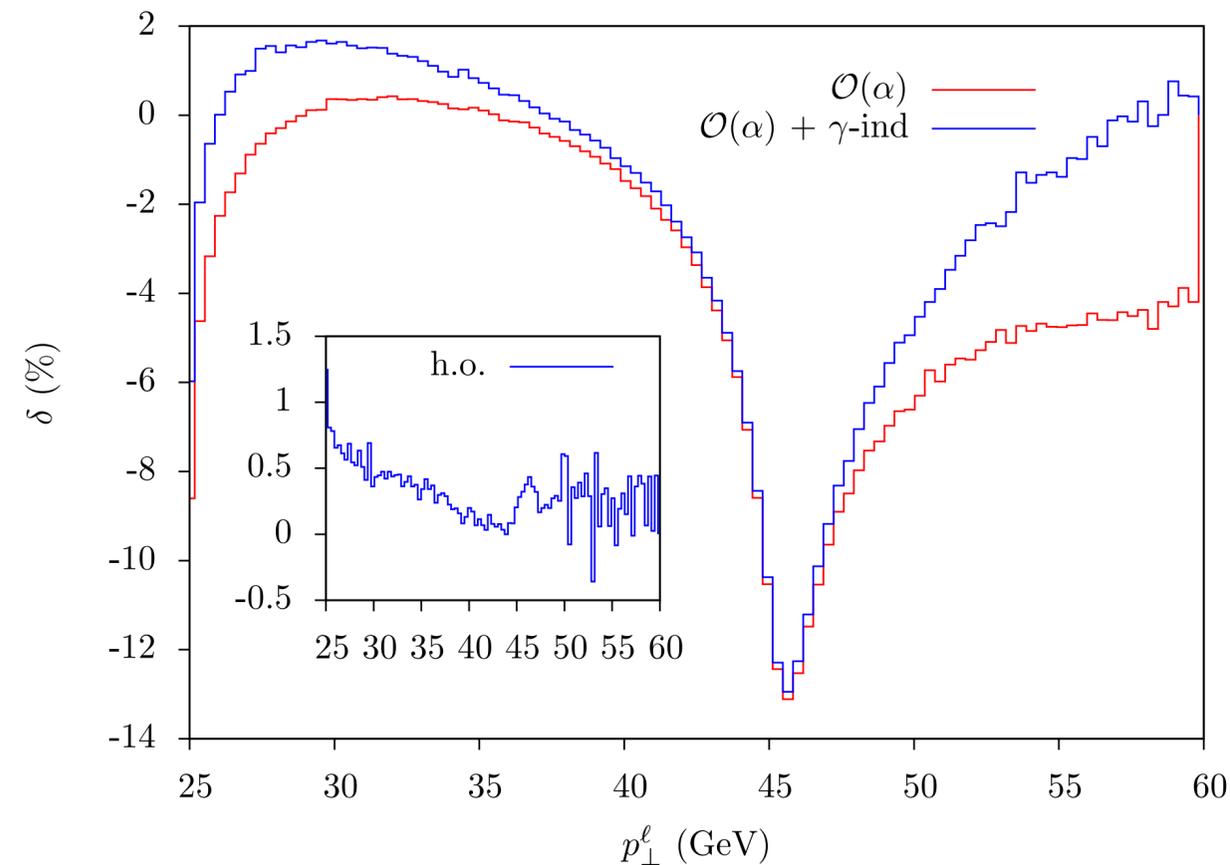
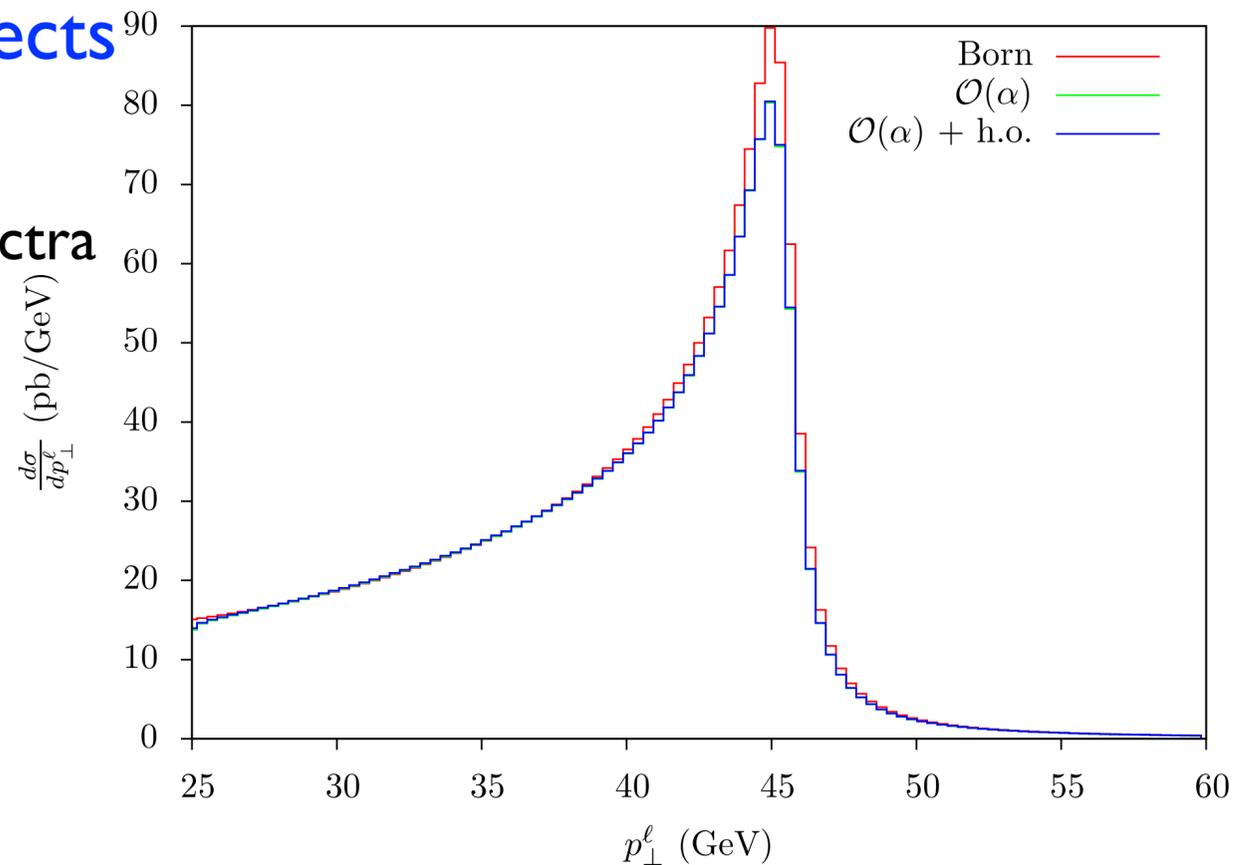
Born leptons DO NOT exist.

Unfolding QED-FSR corrections is a dangerous procedure threatening the possibility of a detailed comparison with theory at high-precision level (beyond QED-LL)

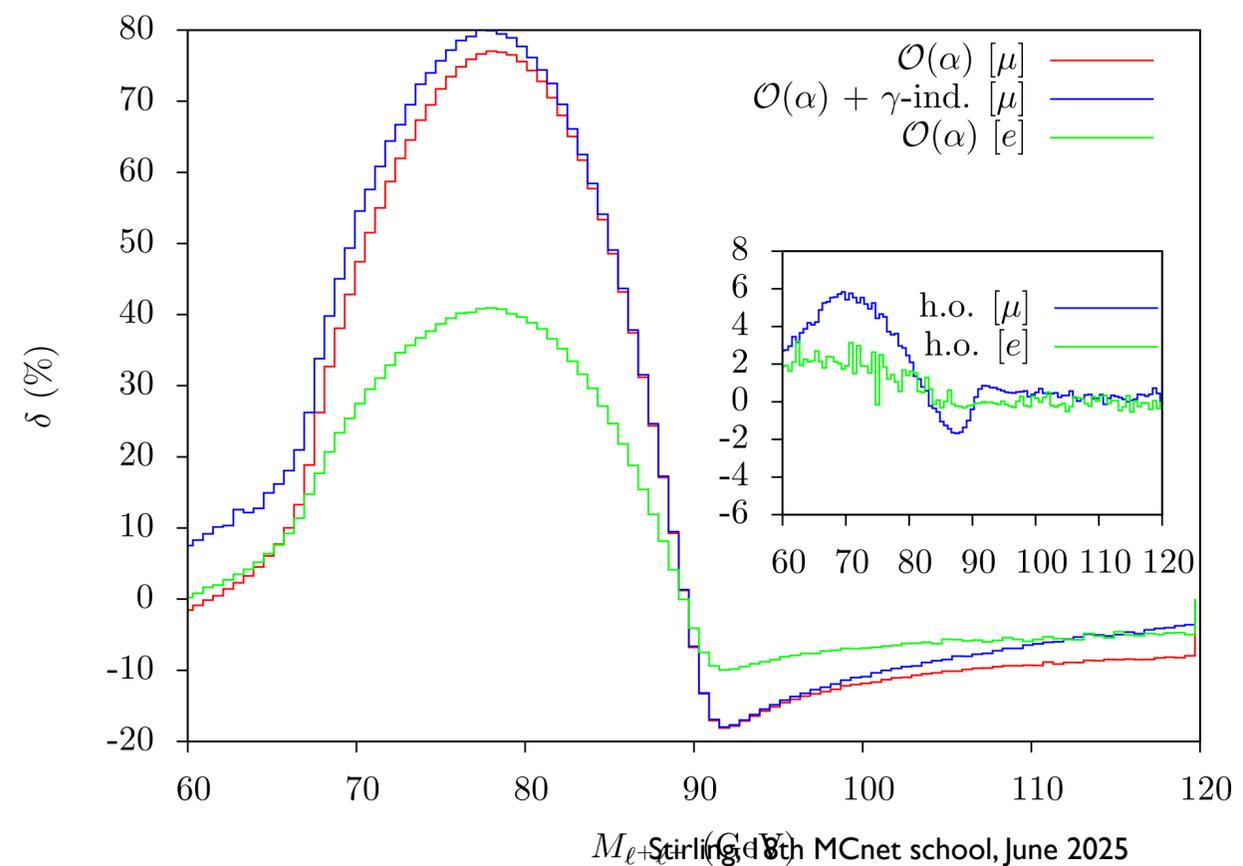
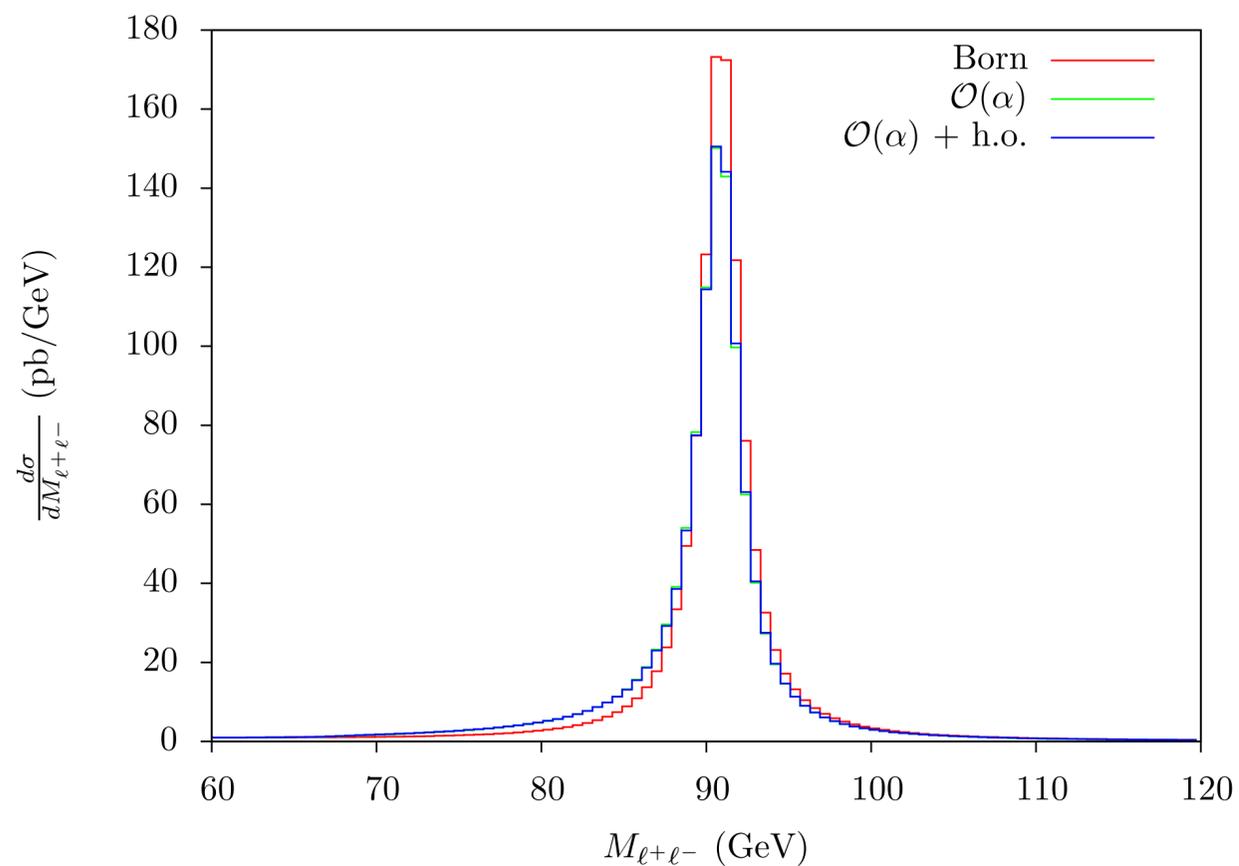


EW corrections: QED effects

- distortion of the (leptonic) spectra for bare muons



- radiative return mechanism: events generated at $\sqrt{s} \sim m_Z$ FSR radiate, so that $m_{\ell\ell} < m_Z$ filling the distribution below the peak



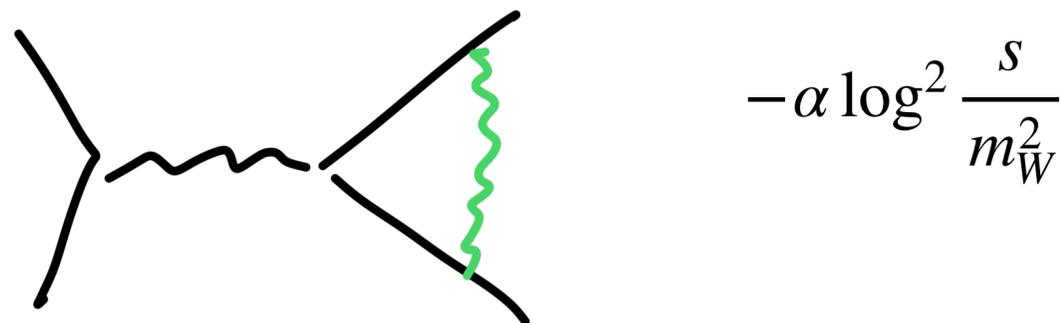
EW corrections: weak Sudakov logarithms

- in the massless photon case, large logarithmic corrections due to soft emissions appear in individual cross sections and cancel when we combine the cross sections with real and virtual corrections



- for large values of the kinematical invariants, the gauge boson masses are “soft”

→ the virtual corrections feature, for each loop a term $\alpha \log^2 \frac{s}{m_W^2} \equiv \alpha L^2 \sim 0.18$ when $s = (1 \text{ TeV})^2$



in the weak case we do not have the emission of a real weak boson emission

there is an unbalance between virtual and real weak exchanges, leaving the large negative virtual correction

EW corrections: weak Sudakov logarithms

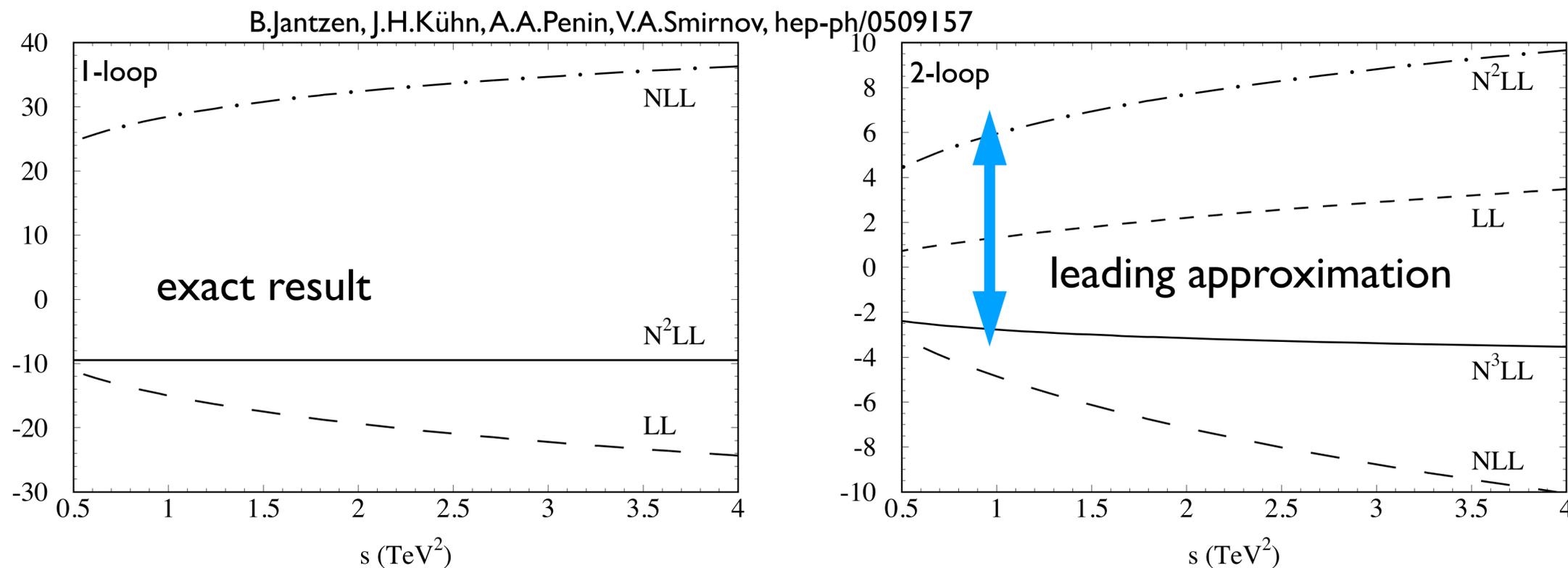
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→ the virtual corrections feature, for each loop a term $-\frac{\alpha}{\sin^2 \theta_W \pi} \log^2 \frac{s}{m_W^2} \equiv -\frac{\alpha}{\sin^2 \theta_W \pi} L^2 \sim -0.25$ when $s = (1 \text{ TeV})^2$

- $$\sigma = \sigma_0 + \frac{\alpha}{\pi} \sigma_\alpha + \left(\frac{\alpha}{\pi}\right)^2 \sigma_{\alpha^2} + \dots$$

in the high-energy limit, we can isolate the EW Sudakov logarithms

$$\sigma_\alpha^{virt,Sudakov} \sim c_{12} L^2 + c_{11} L + c_{10}, \quad \sigma_{\alpha^2}^{virt,Sudakov} \sim c_{24} L^4 + c_{23} L^3 + c_{22} L^2 + c_{21} L + c_{20}, \quad \dots$$



corrections to $e^+e^- \rightarrow q\bar{q}$
due to EW Sudakov logs

EW corrections: weak Sudakov logarithms

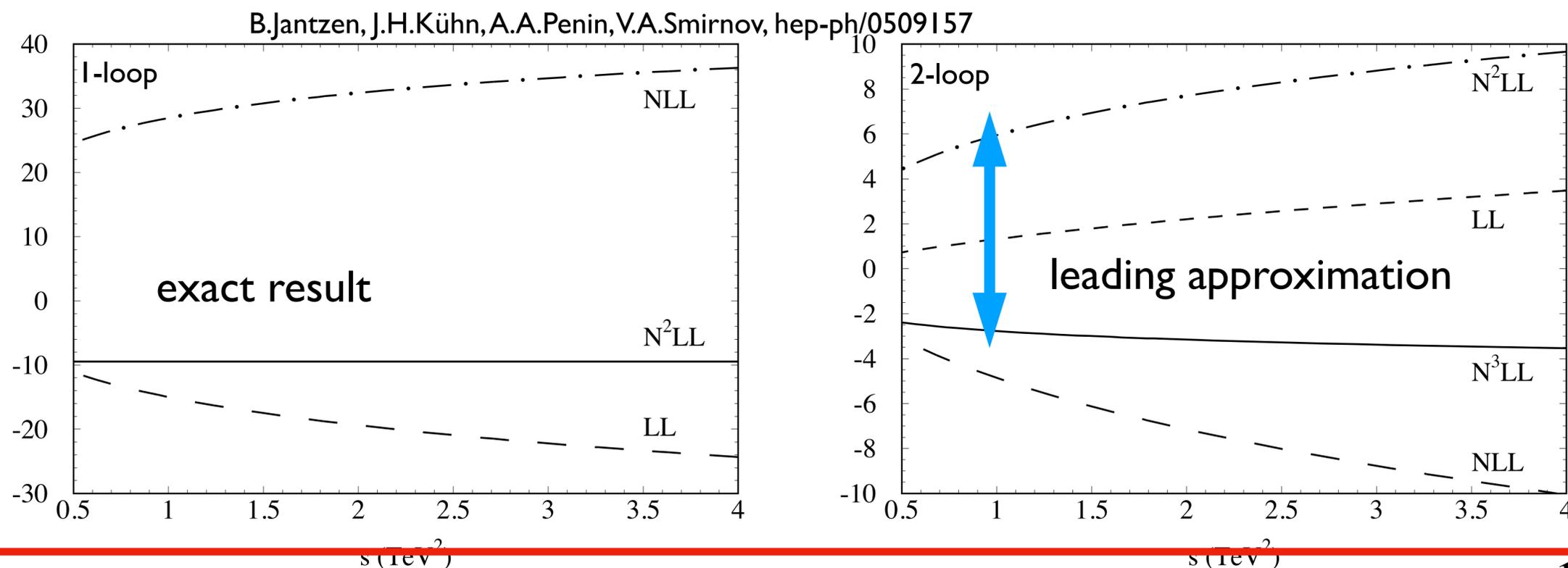
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corrections to $e^+e^- \rightarrow q\bar{q}$
due to EW Sudakov logs

Estimate of missing higher-order logs: $\delta_\alpha^{Sudakov} \rightarrow \exp(\delta_\alpha^{Sudakov}) \rightarrow$ at 2-loop estimate $\sim \frac{1}{2}(\delta_\alpha^{Sudakov})^2$

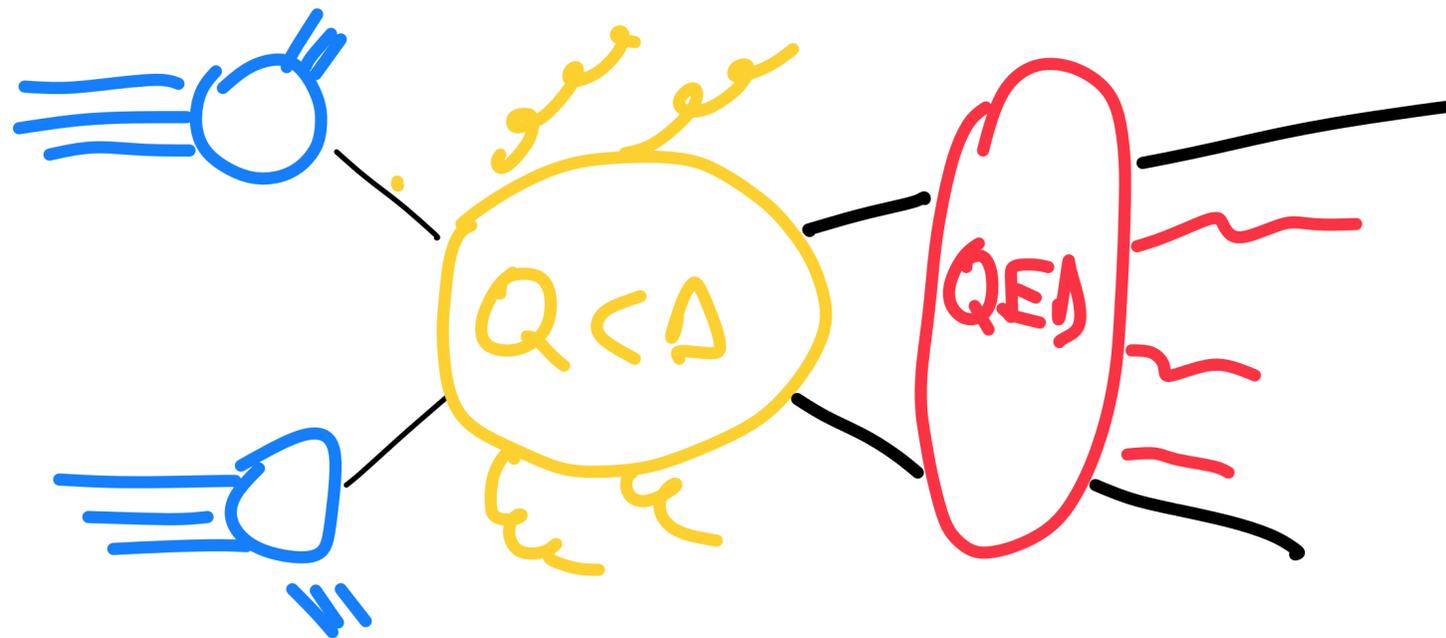
caveat: the subleading logs are large and with alternating signs

Mixed QCD-EW corrections

Mixed QCD and EW effects at hadron colliders

A standard approximated combination: QCD \times QED-FSR

- the rate and distributions of the scattering processes are computed at (N)NLO-QCD, possibly matched with QCD-PS
then the final state particles are further corrected by QED-FSR emissions (universality of LL QED corrections)
- quite accurate description of all the phase-space regions with soft and/or collinear QCD/QED radiation



Mixed QCD and EW effects at hadron colliders

Matching NLO-QCD and NLO-EW with QCD-PS and QED-PS

POWHEG NLO-(QCD+EW)

$$d\sigma = \sum_{f_b} \bar{B}^{f_b}(\Phi_n) d\Phi_n \left\{ \begin{array}{l} \text{no-emission probability} \\ \Delta^{f_b}(\Phi_n, p_T^{min}) \\ + \sum_{\alpha_r \in \{\alpha_r | f_b\}} \frac{[d\Phi_{rad} \theta(k_T - p_T^{min}) \Delta^{f_b}(\Phi_n, k_T) R(\Phi_{n+1})]_{\bar{\Phi}_n^{\alpha_r} = \Phi_n}}{B^{f_b}(\Phi_n)} \end{array} \right\}$$

first hard emission probability

The exact NLO QCD+EW normalisation is guaranteed by the \bar{B} function

QCD and QED radiation are described by $\{ \}$

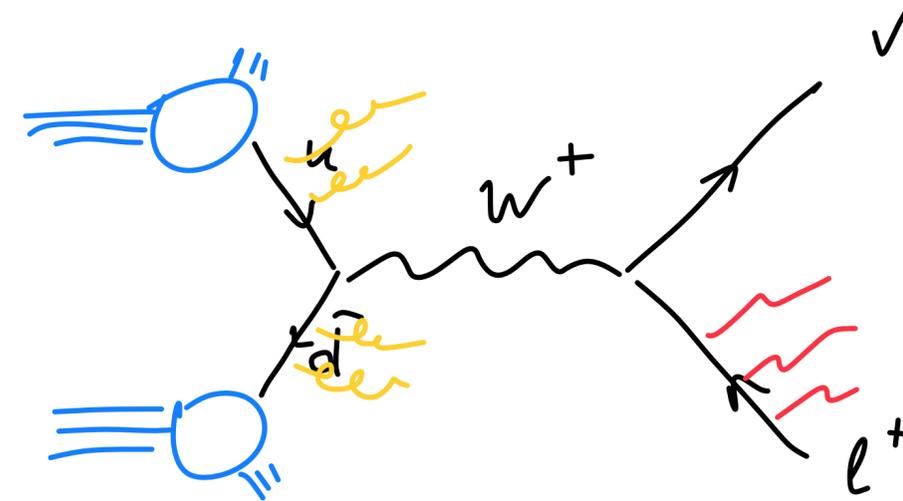
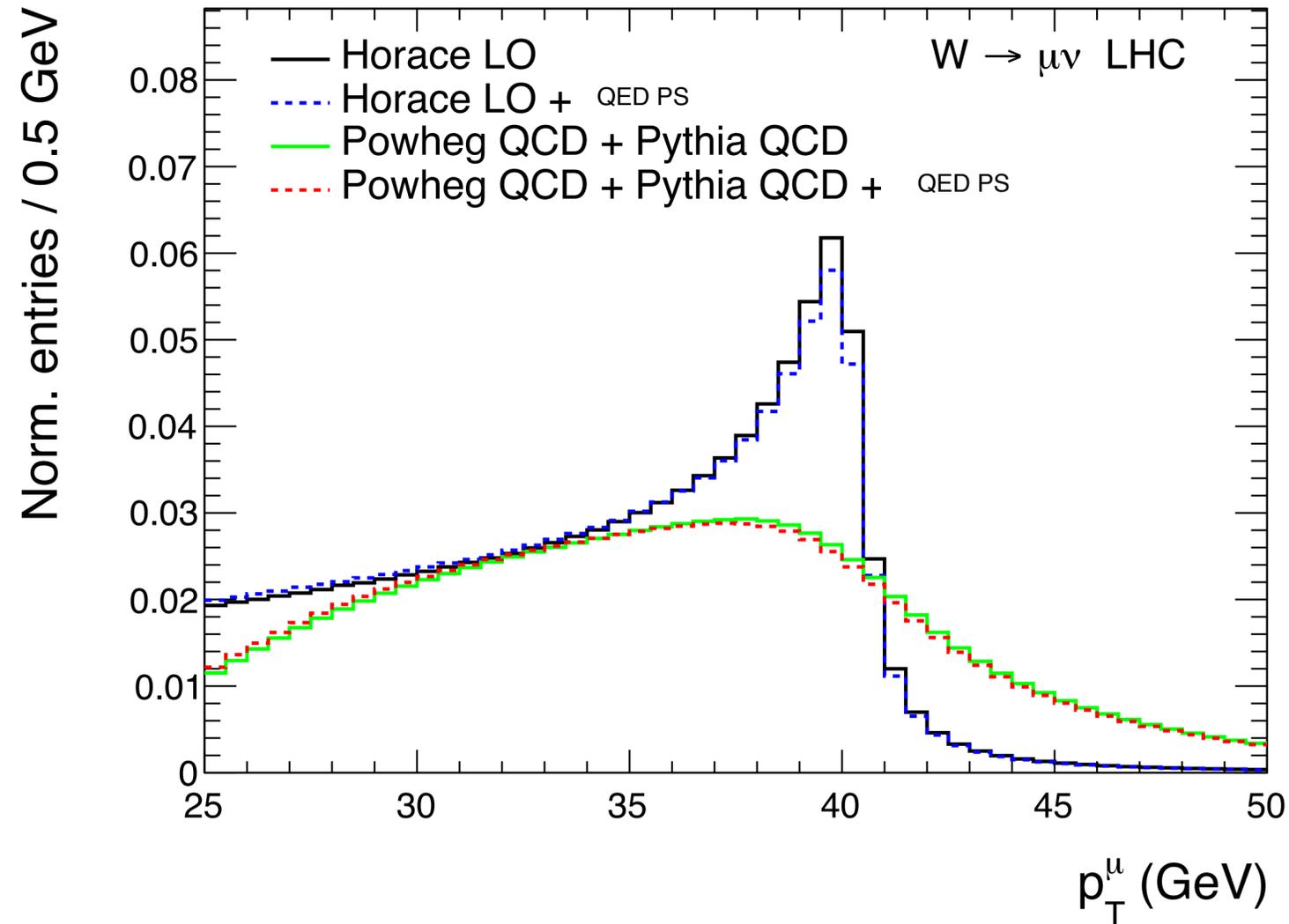
The first emission is by construction the hardest. The following emissions are handled by QCD and QED Parton Showers

The factorised formulation $[\bar{B}(QCD) + \bar{B}(EW)] \{R(QCD) + R(QED)\}$ (QCDPS + QEDPS) introduces mixed QCD-EW corrections

Good approximation of the exact results in the soft/collinear phase-space regions + one improvement in the description of hard/large angle radiation

Interplay of QCD and QED corrections: the Drell-Yan example

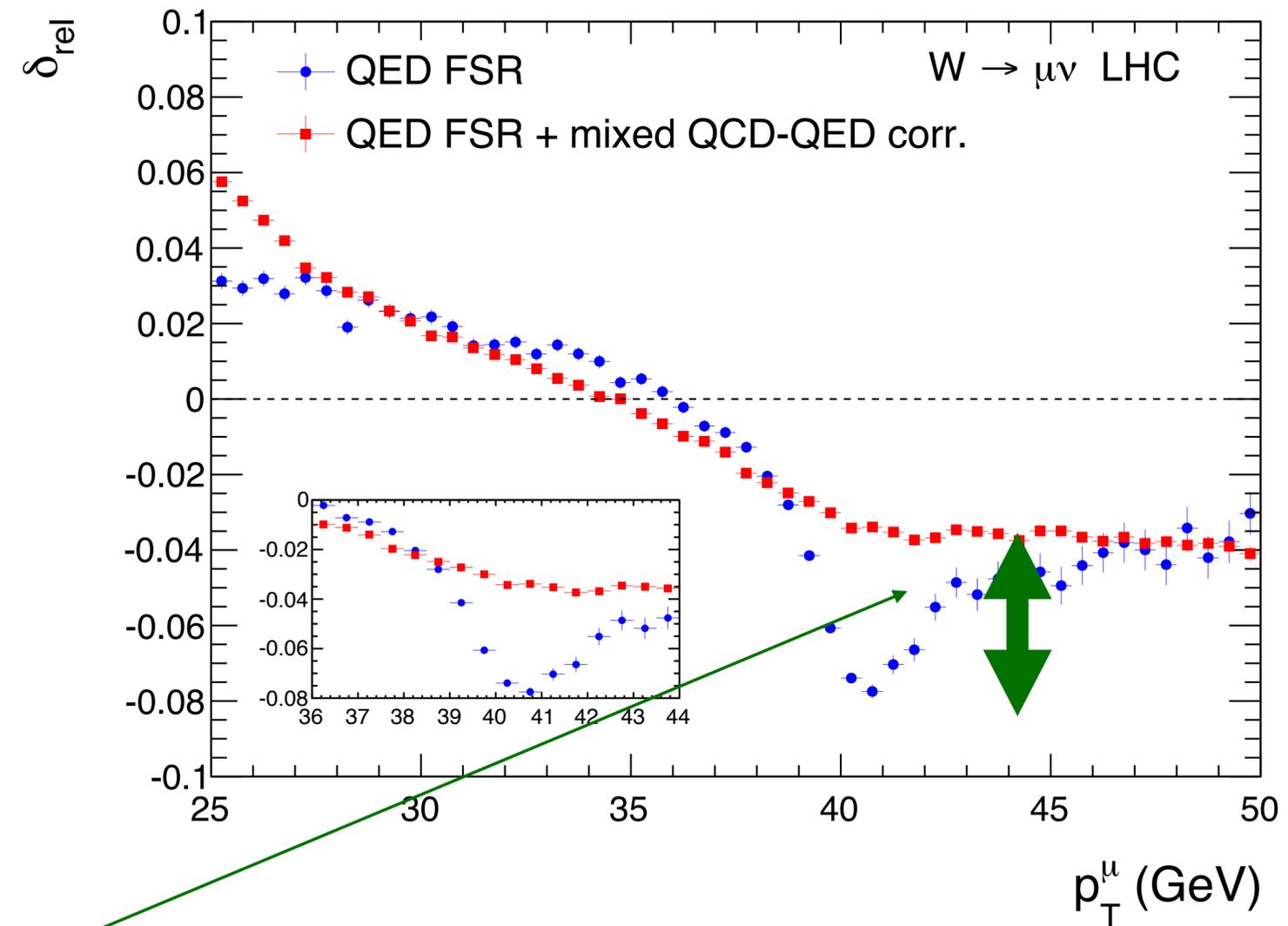
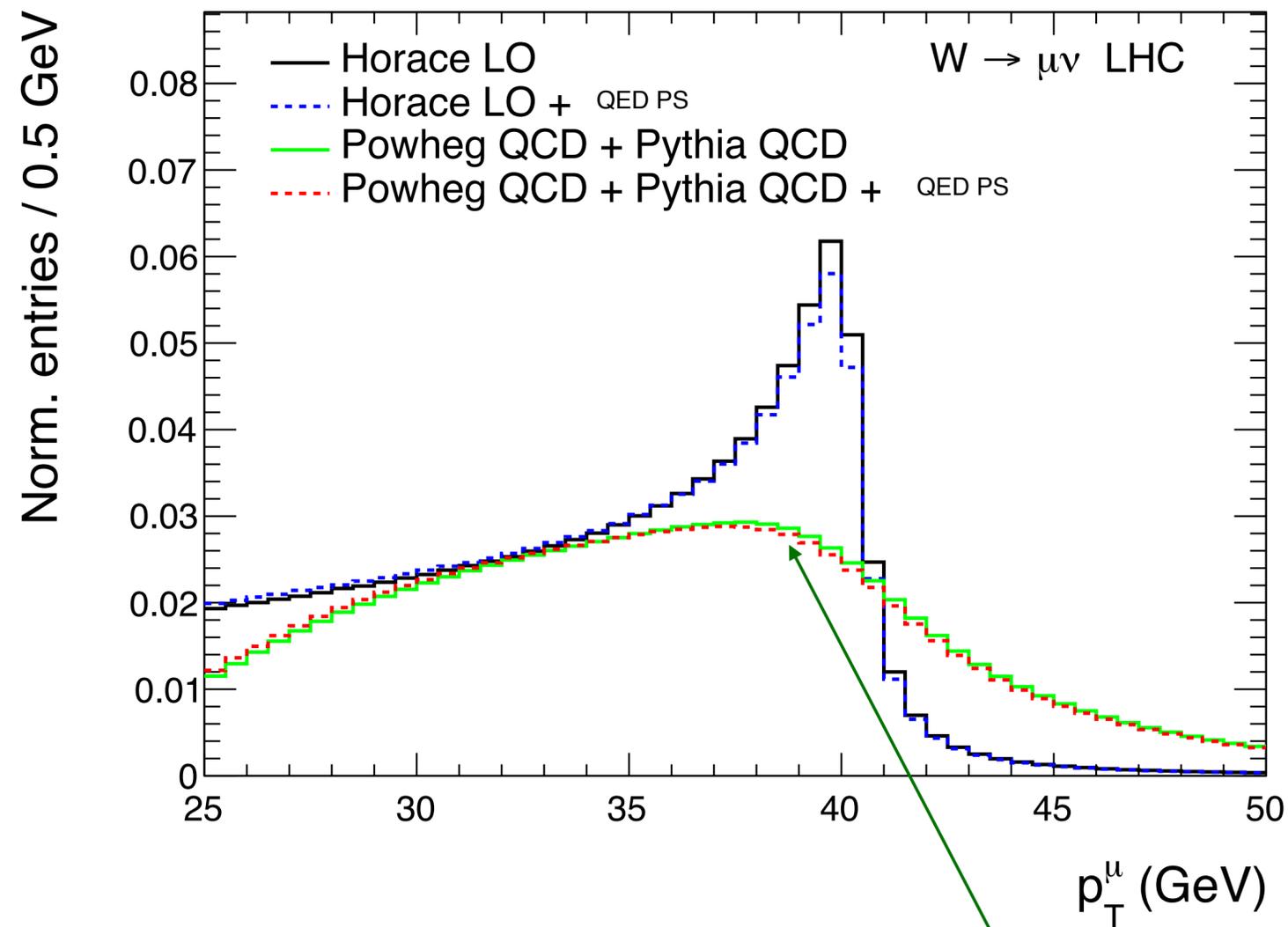
C.Carloni Calame, M.Chiesa, H.Martinez, G.Montagna, O.Nicrosini, F.Piccinini, AV, arXiv:1612.02841



- very large impact of initial-state QCD radiation on the p_{Tlep} distribution
- large radiative corrections due to QED final state radiation at the jacobian peak

Interplay of QCD and QED corrections: the Drell-Yan example

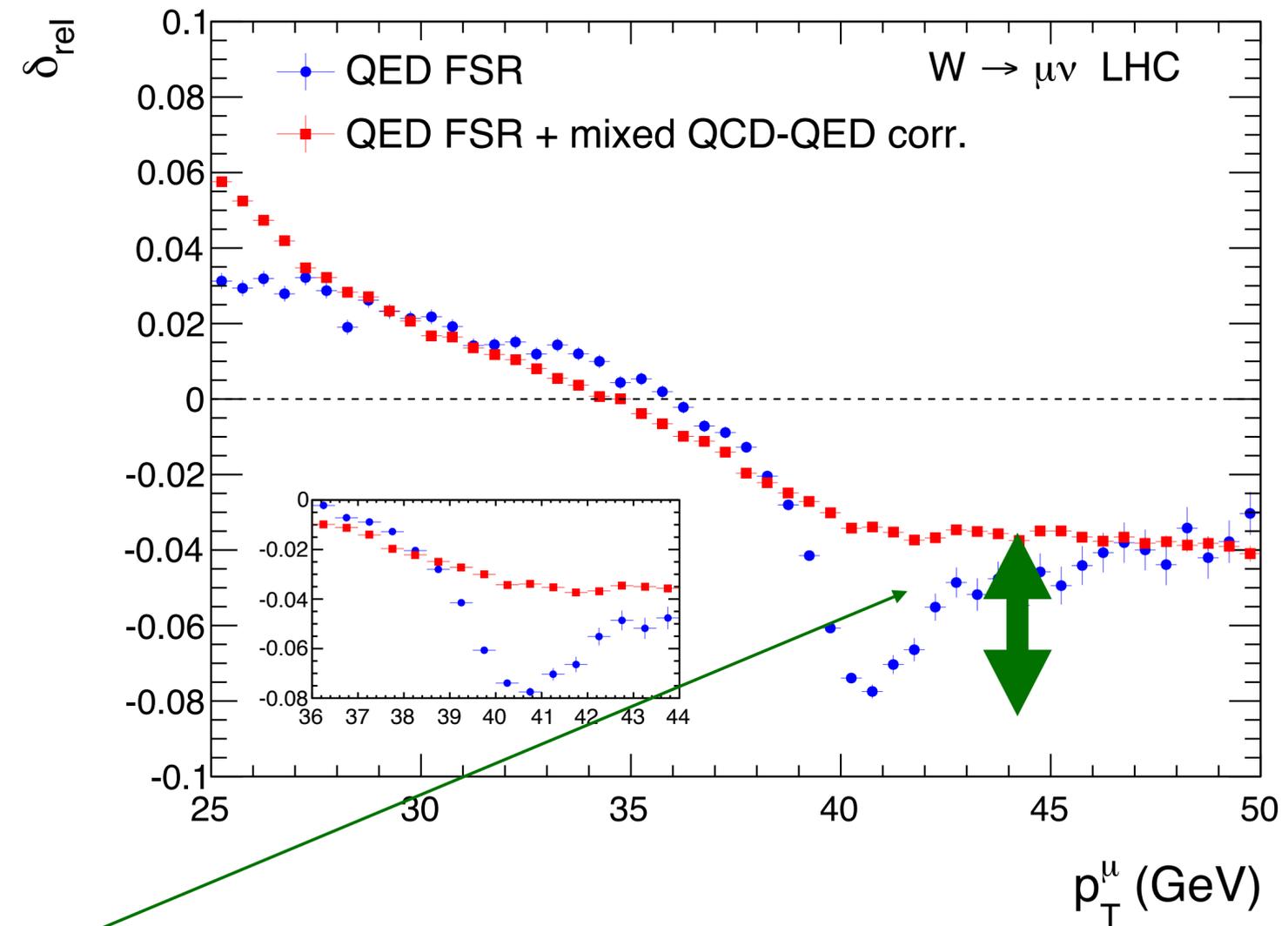
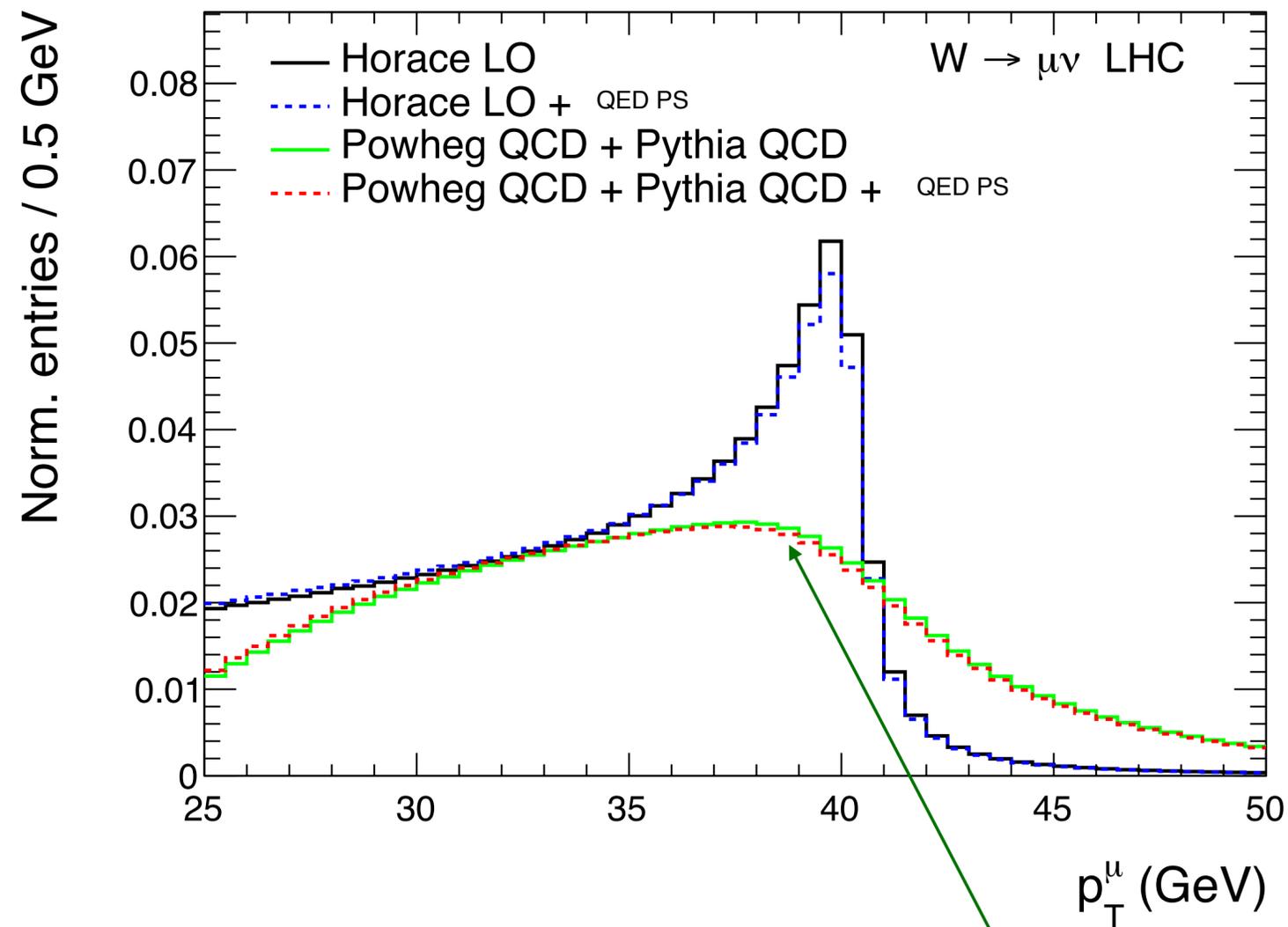
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- very large impact of initial-state QCD radiation on the $p_{T\text{lep}}$ distribution
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- very large **interplay of QCD and QED corrections** redefining the precise shape of the jacobian peak

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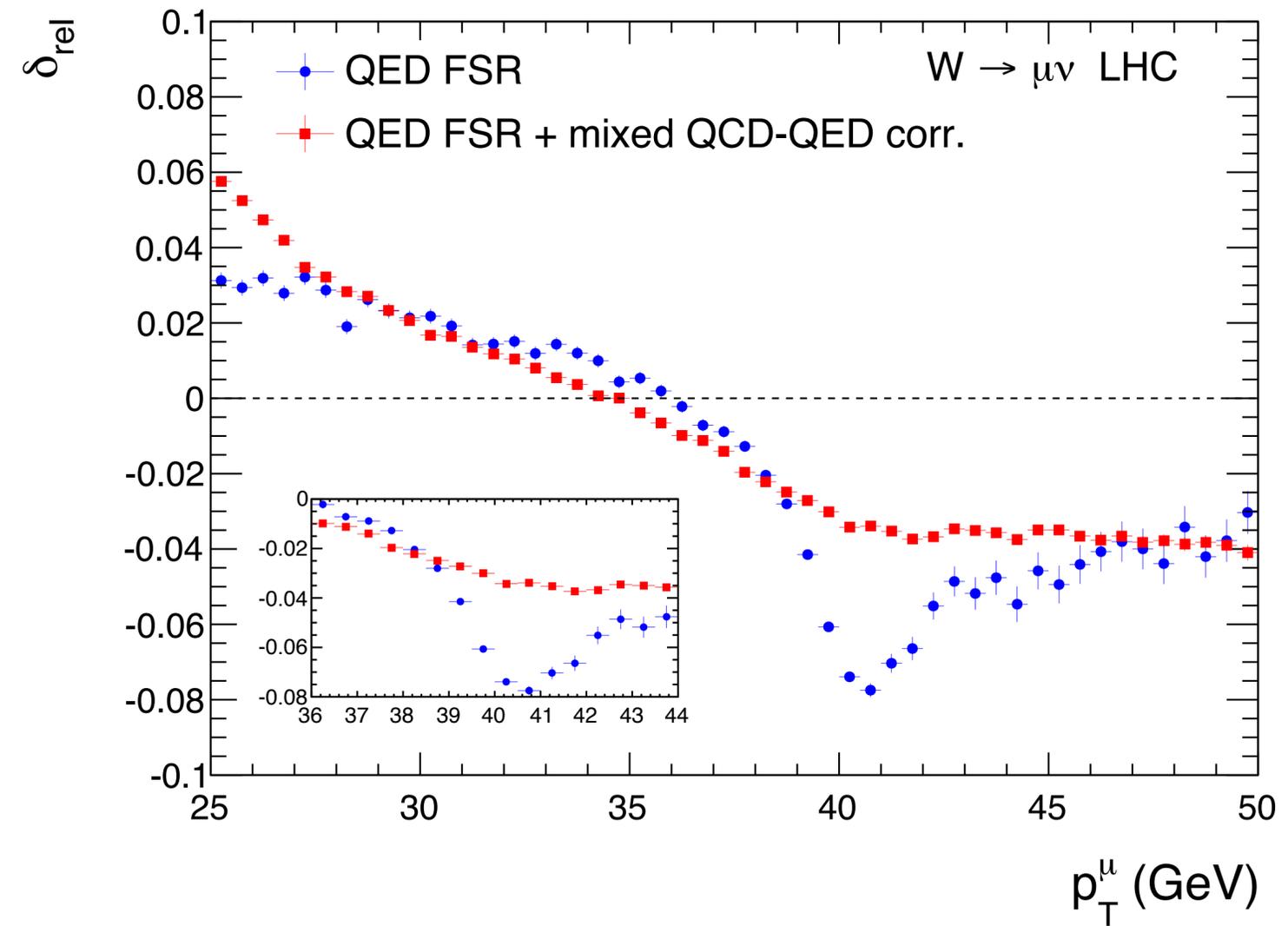
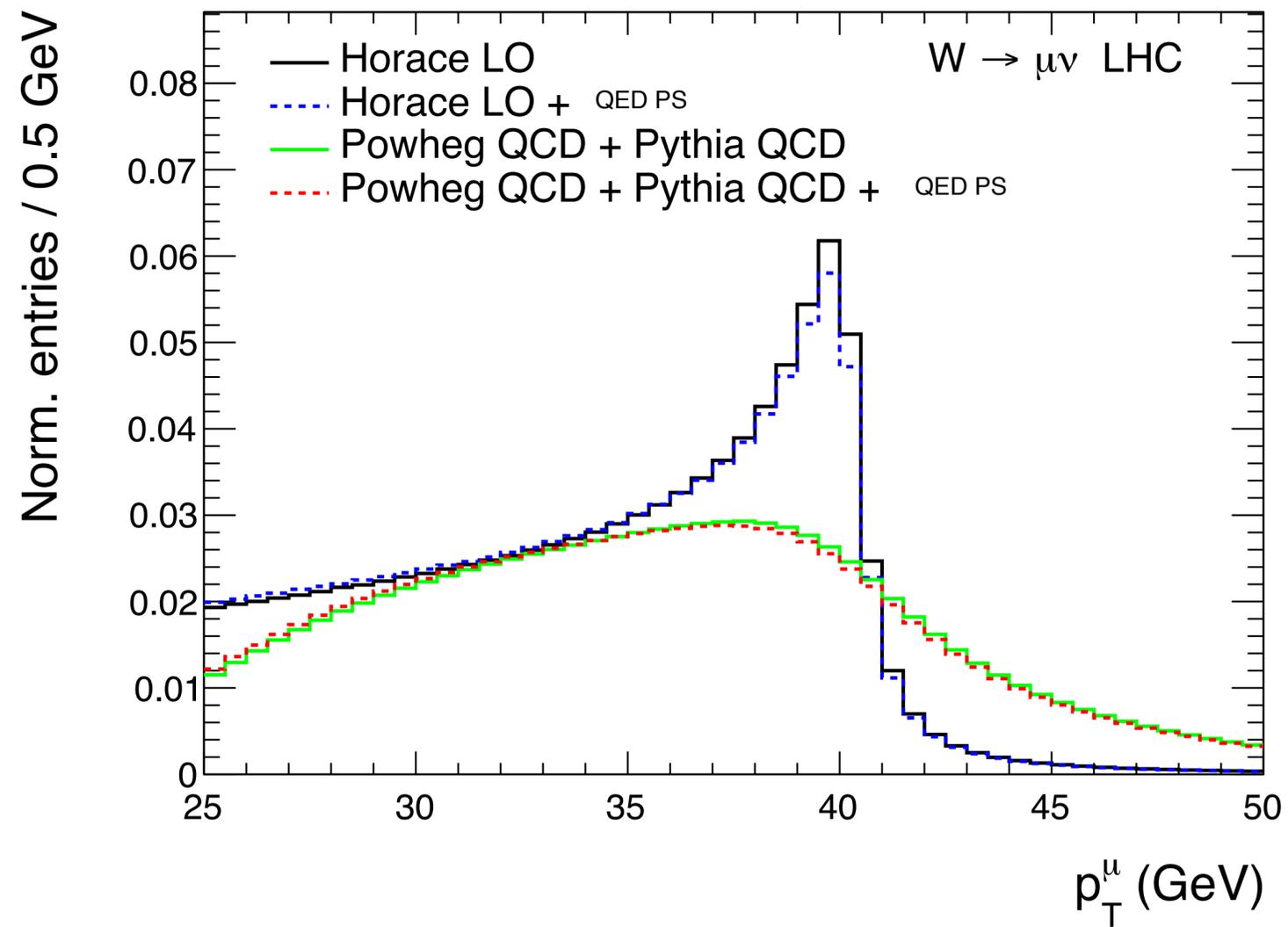
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NLO-QCD + QCDPS + QEDPS is the lowest order meaningful approximation of this observable

the precise size of the mixed QCDxQED corrections (and uncertainties) depends on the choice for the QCD modelling

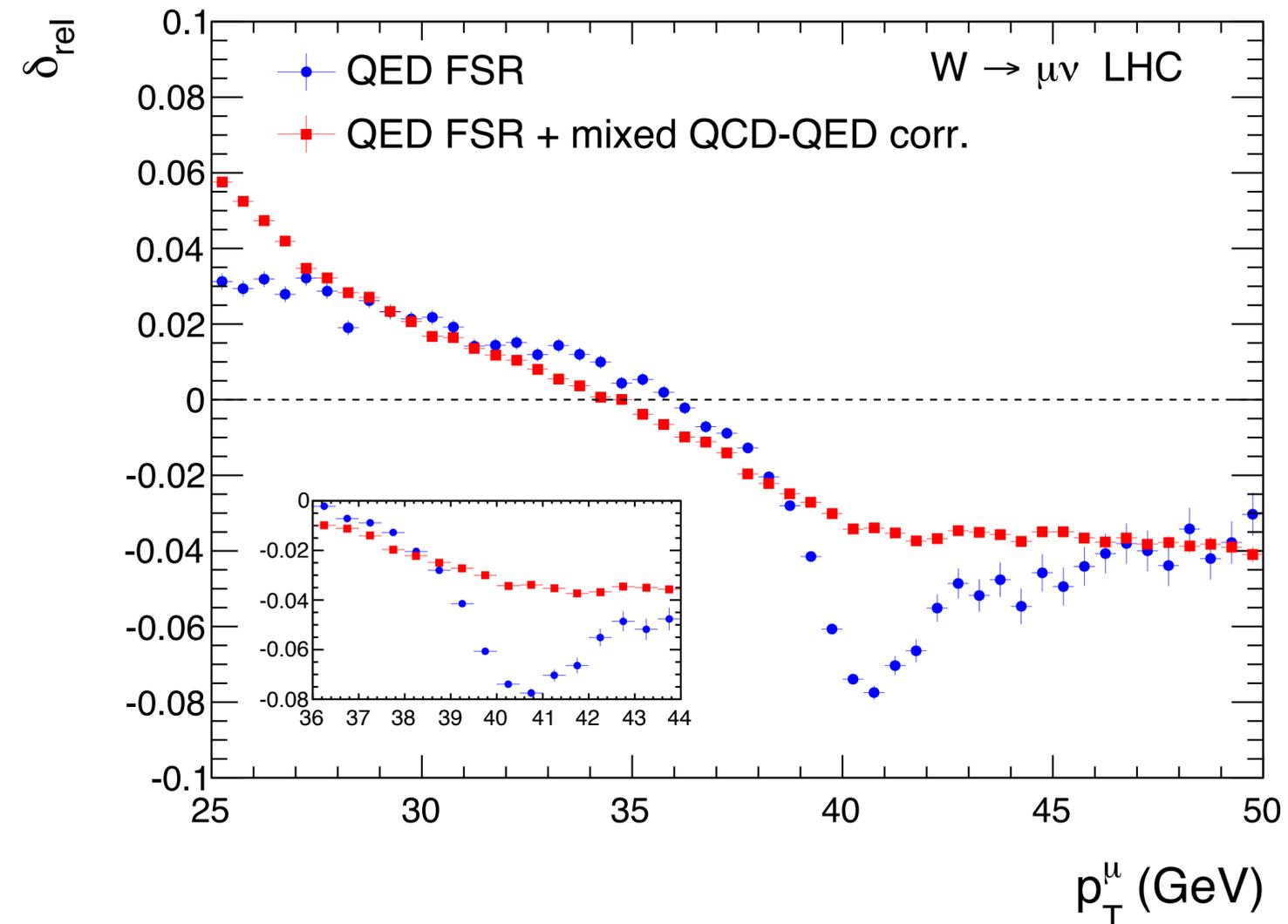
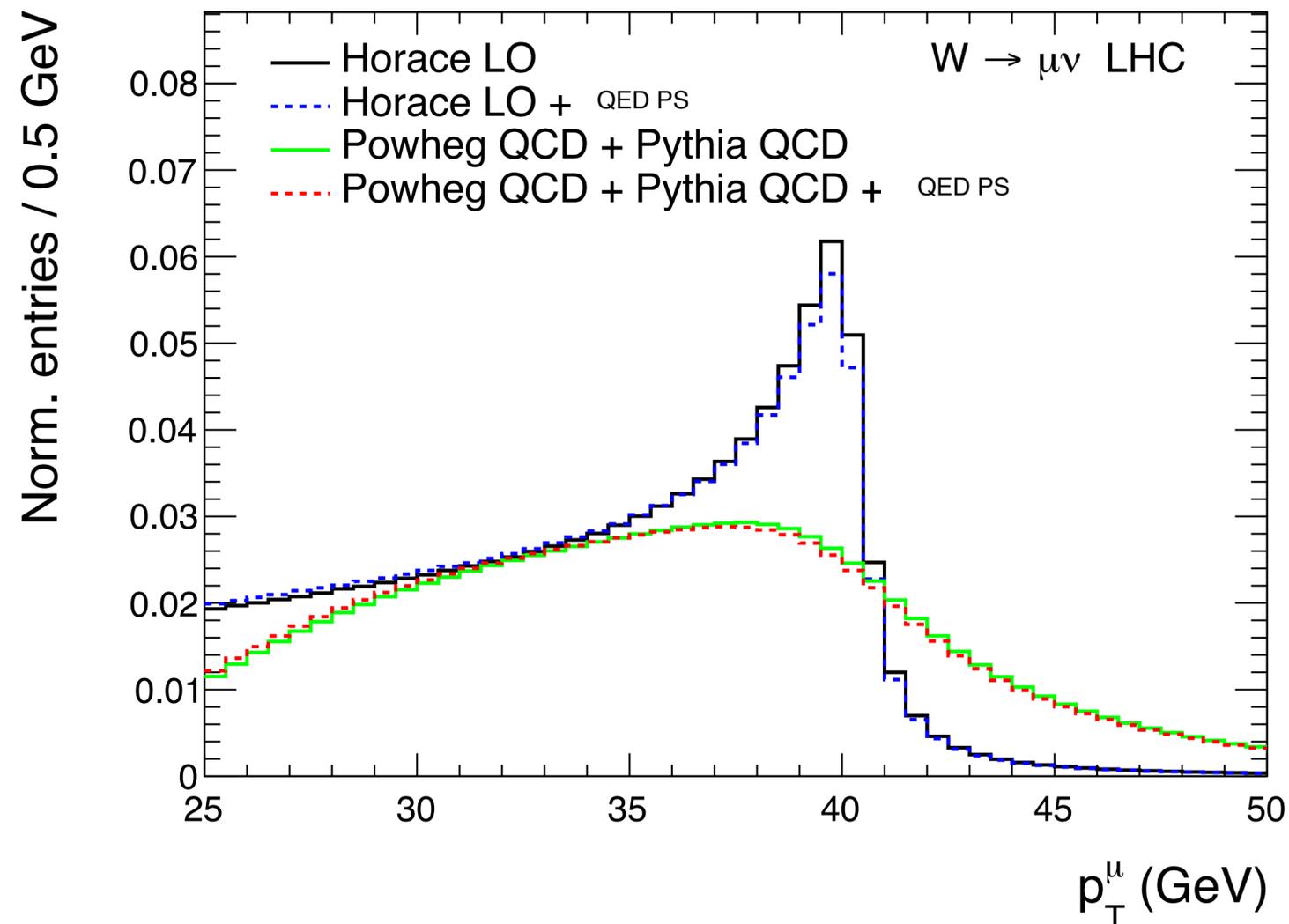
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in m_W determination we need a control over the shape of the distributions **at 0.1% level**

here we have effects of **$\mathcal{O}(\text{few } \%)$**

how can we assess a reliable estimate of **1) the size of the mixed QCD-EW effects**

2) the residual error on these effects

?

QED and QCD competing in the algorithmic simulation of real radiation

How can we appreciate the differences between a PS with or without matching with fixed-order?

POWHEG NLO-(QCD+EW)

$$d\sigma = \sum_{f_b} \bar{B}^{f_b}(\Phi_n) d\Phi_n \left\{ \Delta^{f_b}(\Phi_n, p_T^{min}) + \sum_{\alpha_r \in \{\alpha_r | f_b\}} \frac{[d\Phi_{rad} \theta(k_T - p_T^{min}) \Delta^{f_b}(\Phi_n, k_T) R(\Phi_{n+1})]_{\bar{\Phi}_n^{\alpha_r} = \Phi_n}}{B^{f_b}(\Phi_n)} \right\}$$

the original version of the POWHEG describes radiation in two steps:

- the hardest emission (in p_T) is emitted according to the above probability distribution $\{$
- all subsequent emissions are emitted via Parton Shower in the remaining available phase space

the decision about emitting a parton is taken using the Sudakov form factor as no-emission probability density

the probability that a (hard) photon successfully passes the Sudakov test is suppressed compared to the gluon $Q_f \alpha \ll C_F \alpha_s$

if we generate with POWHEG only the hardest parton, only one parton, it will be in 99.5% of the cases a gluon

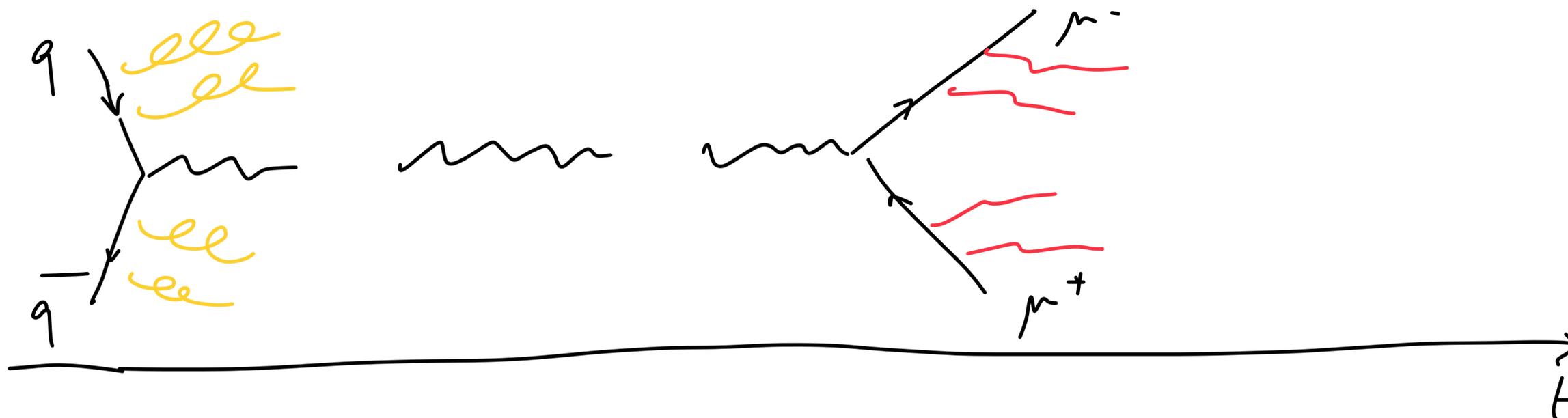
→ the photon radiation **never benefits of the exact matrix element corrections !!!** the photon spectrum is pure QED PS

→ **Solution:** use a **resonance-aware approach** to describe the gauge-boson **production** and **decay**

QED and QCD competing in the algorithmic simulation of real radiation

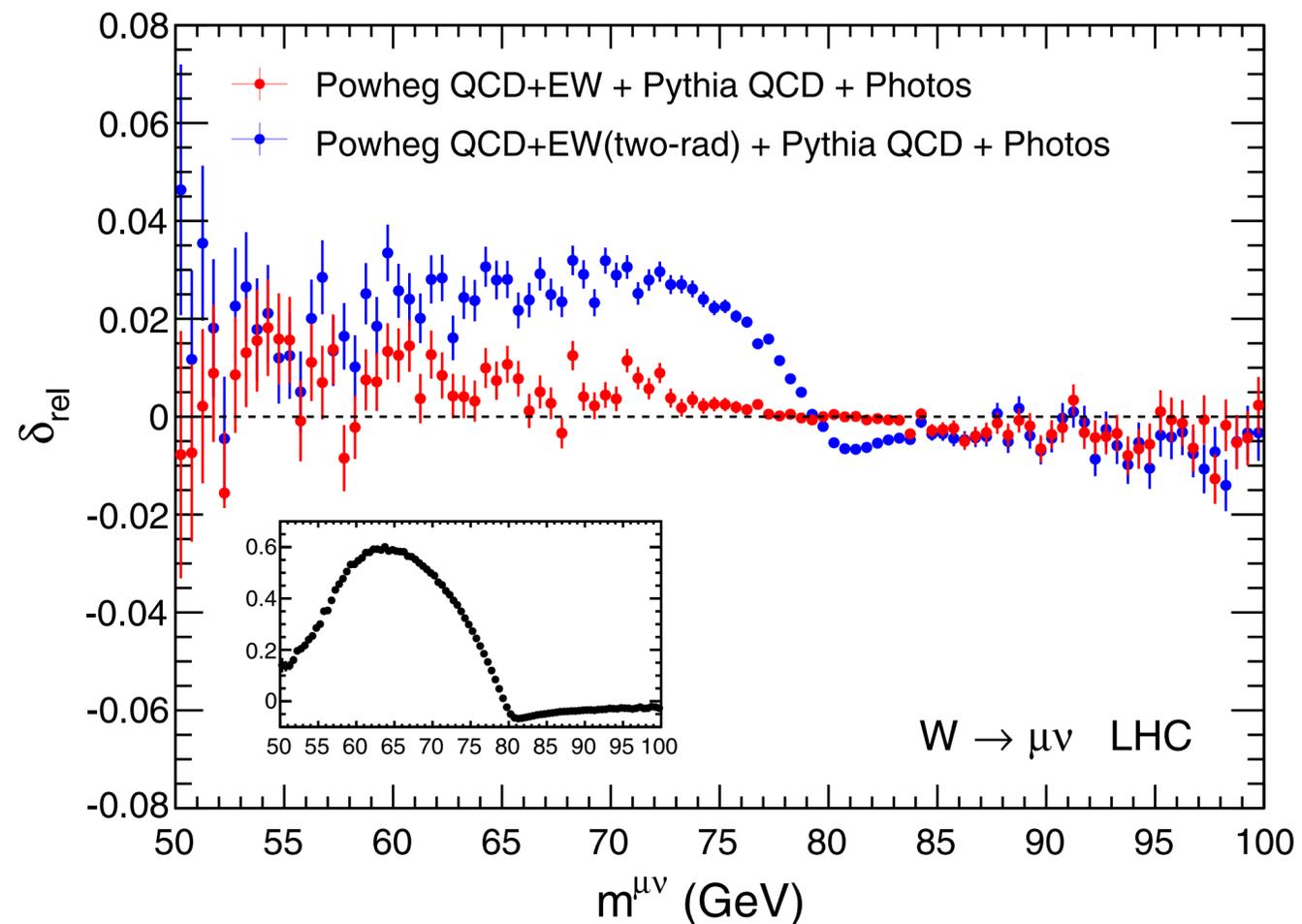
Two-rad approach to describe the gauge-boson production and decay simplified “**resonance-aware**” treatment

- 1) QCD emissions take place in the production process of the gauge boson
→ we apply the POWHEG algorithm to decide the hardest parton ; it is ok if we get almost only gluons
- 2) The typical interaction time for QCD emissions is smaller than the lifetime of the intermediate gauge boson
The subsequent gauge-boson decay has thus a distinct Monte Carlo history
- 3) We can apply, in a separate independent way, the algorithm to emit the hardest photon, without competing with gluons
→ we always get a hardest photon; the algorithm only chooses its p_T
- 4) We always use the exact matrix elements, including photon radiation from all charged legs (no approximations)
The resonance aware sampling improves the phase-space sampling, filling regions which would remain empty



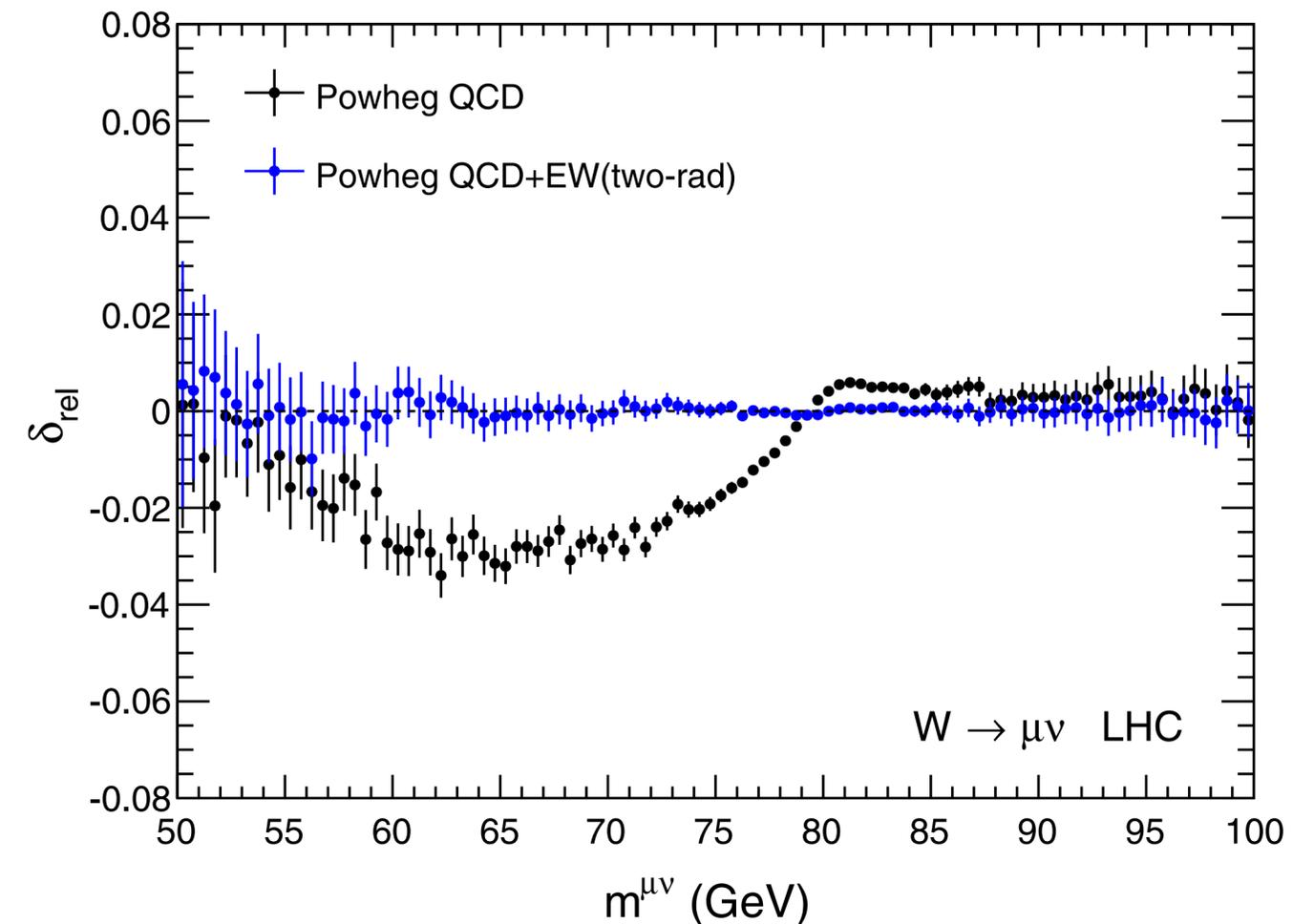
Impact of a resonance-aware treatment

Impact of **neglecting** vs **including** the matrix-element corrections



2 versions of POWHEG NLO-(QCD+EW)+(QCD+QED)-PS
in units POWHEG NLO-QCD +(QCD)-PS

Impact of the QED showering model:
PHOTOS vs Pythia-QED



(POWHEG NLOPS-QCD +PHOTOS) /
(POWHEG NLOPS-QCD +Pythia-QED)

(POWHEG NLO-(QCD+EW)+QCDPS +PHOTOS) /
(POWHEG NLO-(QCD+EW)+QCDPS +Pythia-QED)

An excursus on resonance-aware simulations

Problems in POWHEG of algorithmic nature in the simulation of processes with resonance or several different scales

Radiation can bring the configuration of momenta above a resonance to another one below, with drastic changes in the weight associated to the event

1) IR subtraction terms (FKS, CS) do not preserve the virtuality of the resonance spoiling the efficiency of IR cancellations
→ severe convergence problem

2) The Sudakov form factor $\Delta(\Phi_B, p_T) = \exp \left\{ - \sum_{\alpha} \int_{k_T > p_T} R(\Phi_{\alpha}) / B(\Phi_B) d\Phi_{rad}^{\alpha} \right\}$

may feature very different ratios R/B, because of the interplay of the radiation with the resonance leading to large distortions of the kinematical distributions

A solution is achieved by exploiting the idea of Monte Carlo history

forcing all the emissions stemming from one resonance to be generated at fixed virtuality of the resonance

→ optimisation of the IR cancellations

→ sensible way of filling the phase-space in a physically motivated way

Combining QCD and EW corrections

The POWHEG matching is one of the possible options to combine in the same generator QCD and EW effects with QCD and QED showers

Given the NLO-QCD and NLO-EW corrections:

- additive combination = no assumption about the mixed QCDxEW effects
- multiplicative combination = valid when the amplitude factorises (e.g. soft/collinear limits)

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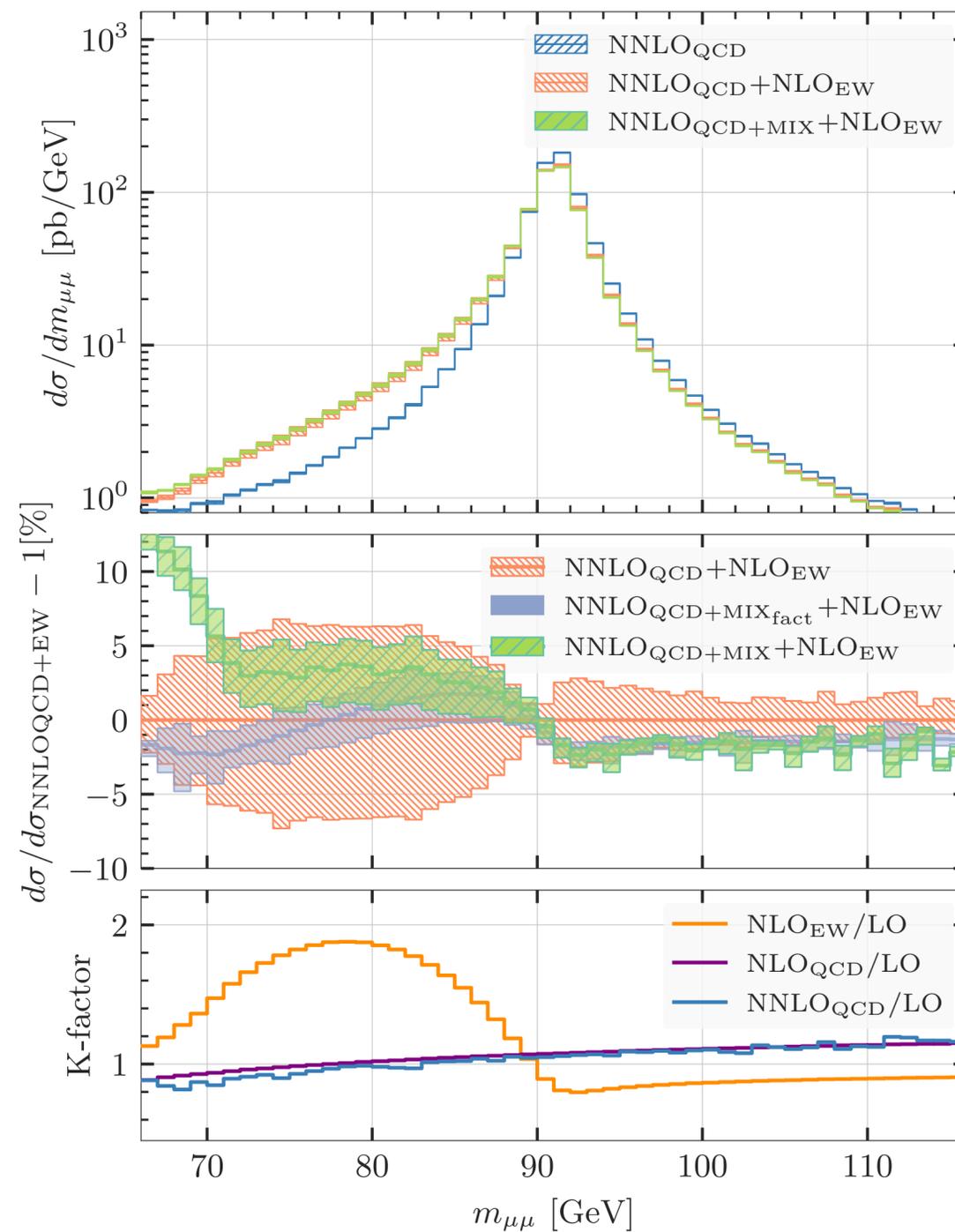
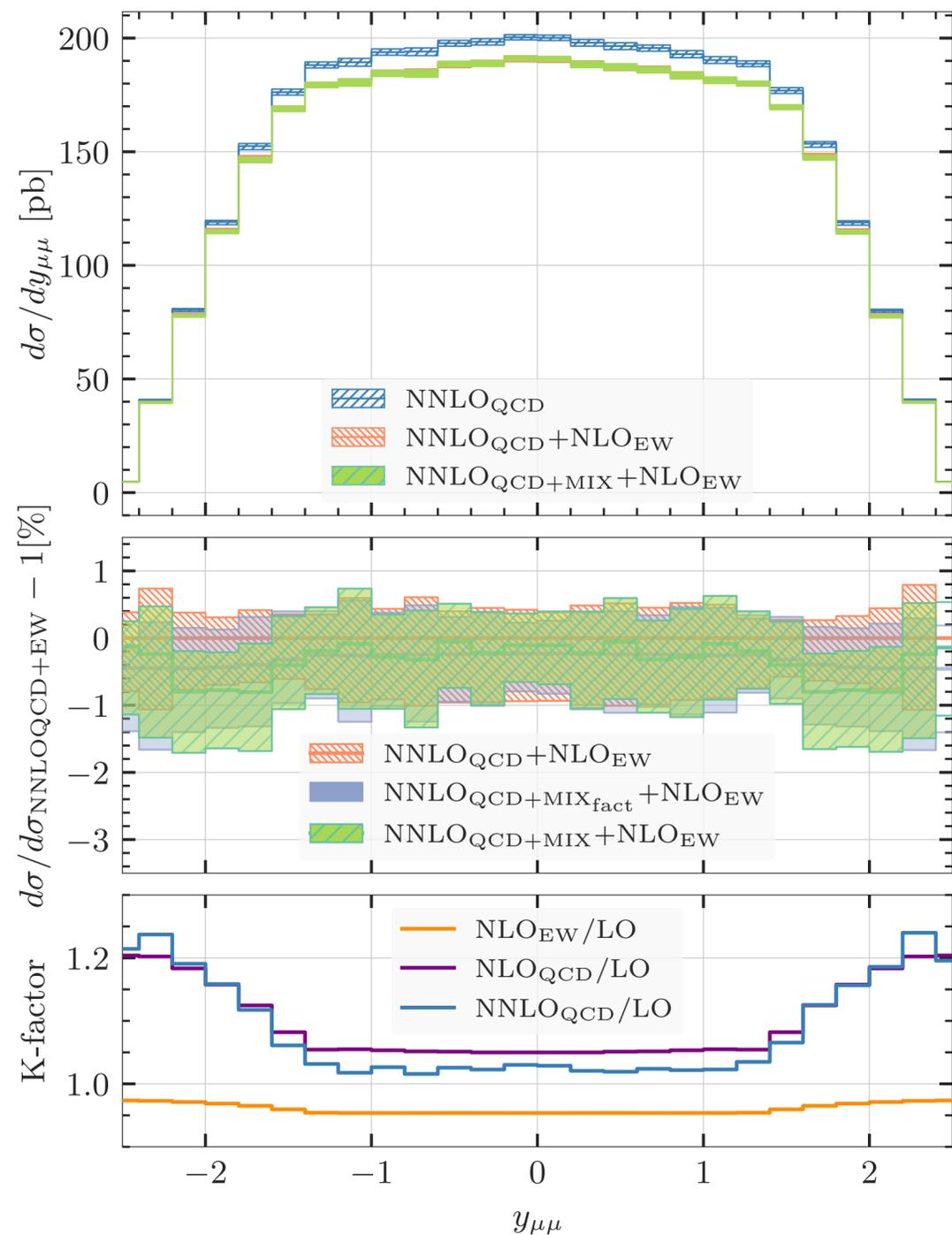
The unambiguous evaluation of mixed QCD-EW effects requires an explicit NNLO calculation

$$\begin{aligned}\sigma(h_1 h_2 \rightarrow \ell \bar{\ell} + X) = & \sigma^{(0,0)} + \\ & \alpha_s \sigma^{(1,0)} + \alpha \sigma^{(0,1)} + \\ & \alpha_s^2 \sigma^{(2,0)} + \alpha \alpha_s \sigma^{(1,1)} + \alpha^2 \sigma^{(0,2)} + \\ & \alpha_s^3 \sigma^{(3,0)} + \dots\end{aligned}$$

- first examples of mixed QCDxEW corrections in the literature available for the Drell-Yan processes ($\sigma^{(1,1)}$)
- still missing the matching with resummation / MC showers

Phenomenology of Neutral Current Drell-Yan including exact NNLO QCD-EW corrections

T.Armadillo, R.Bonciani, L.Buonocore, S.Devoto, M.Grazzini, S.Kallweit, N.Rana, F.Tramontano, AV, arXiv:2106.11953, Phys.Rev.Lett. 128 (2022) 1, 012002 and arXiv:2412.16095



Non-trivial distortion of the rapidity distribution (absent in the naive factorised approximation)

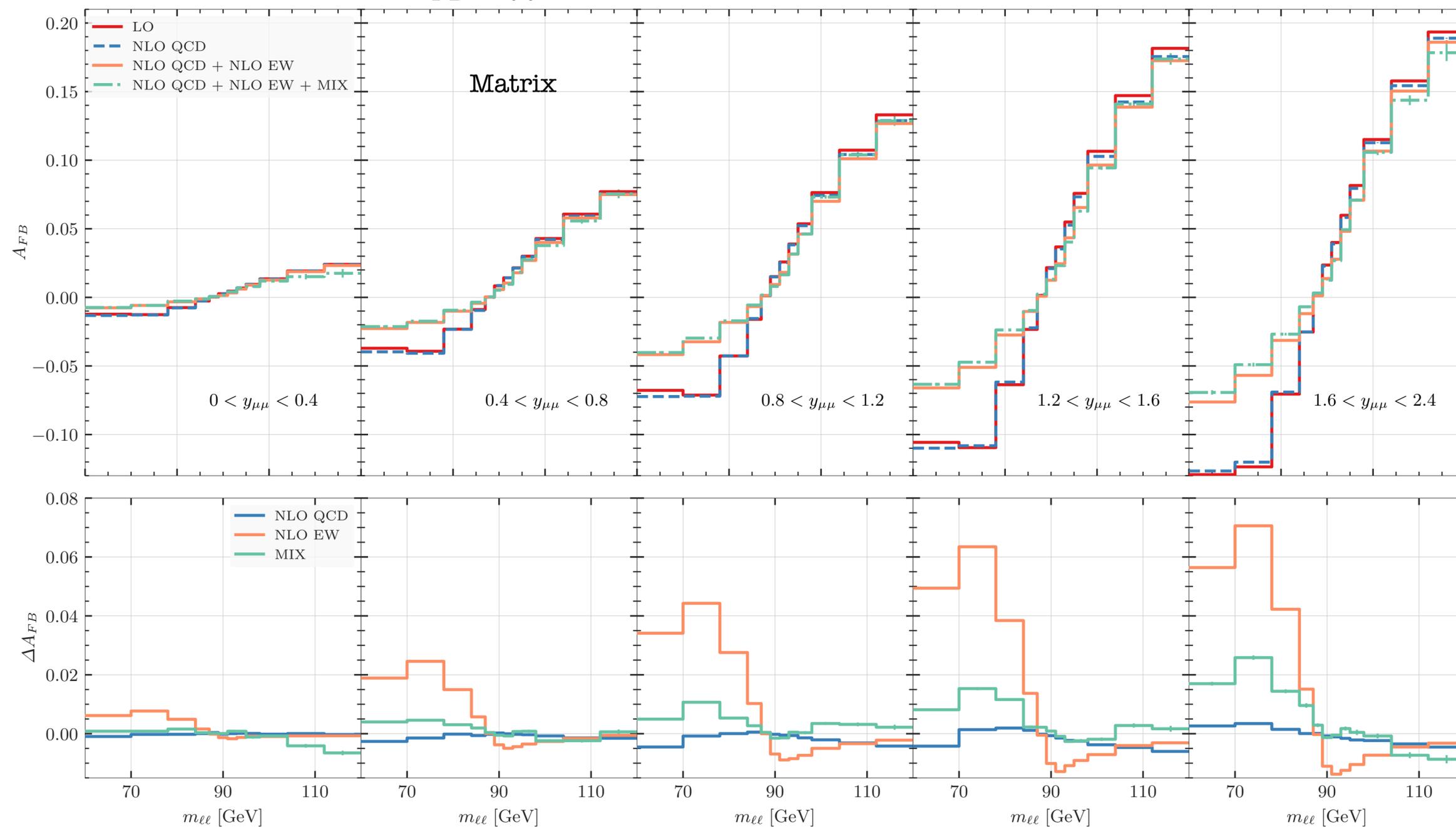
Large effects below the Z resonance (the factorised approximation fails) → impact on the $\sin^2 \theta_{\text{eff}}$ determination

O(-1.5%) effects above the resonance neglected in standard additive tools

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The forward-backward asymmetry $A_{FB}(m_{\ell\ell})$ as a function of the lepton-pair rapidity

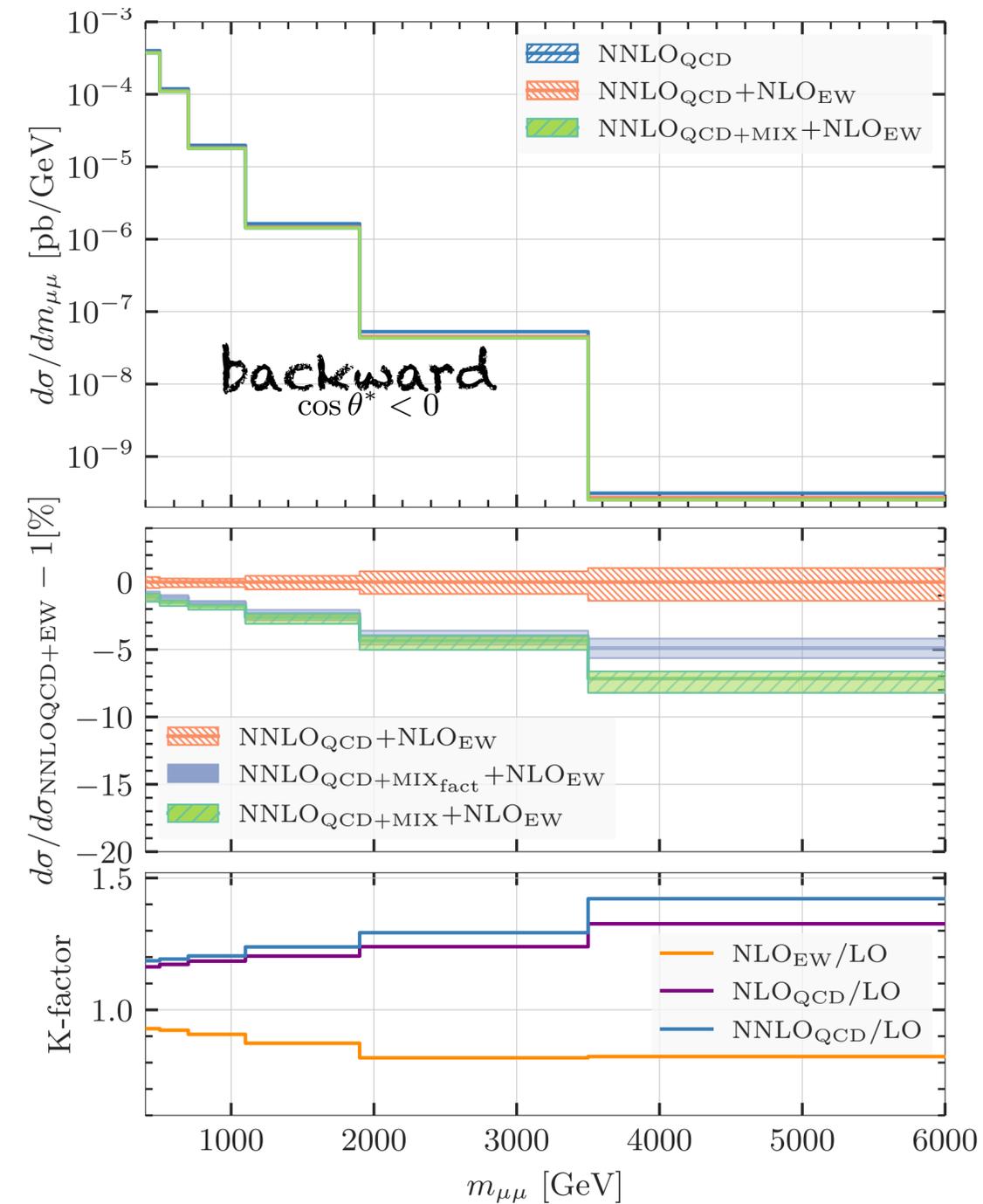
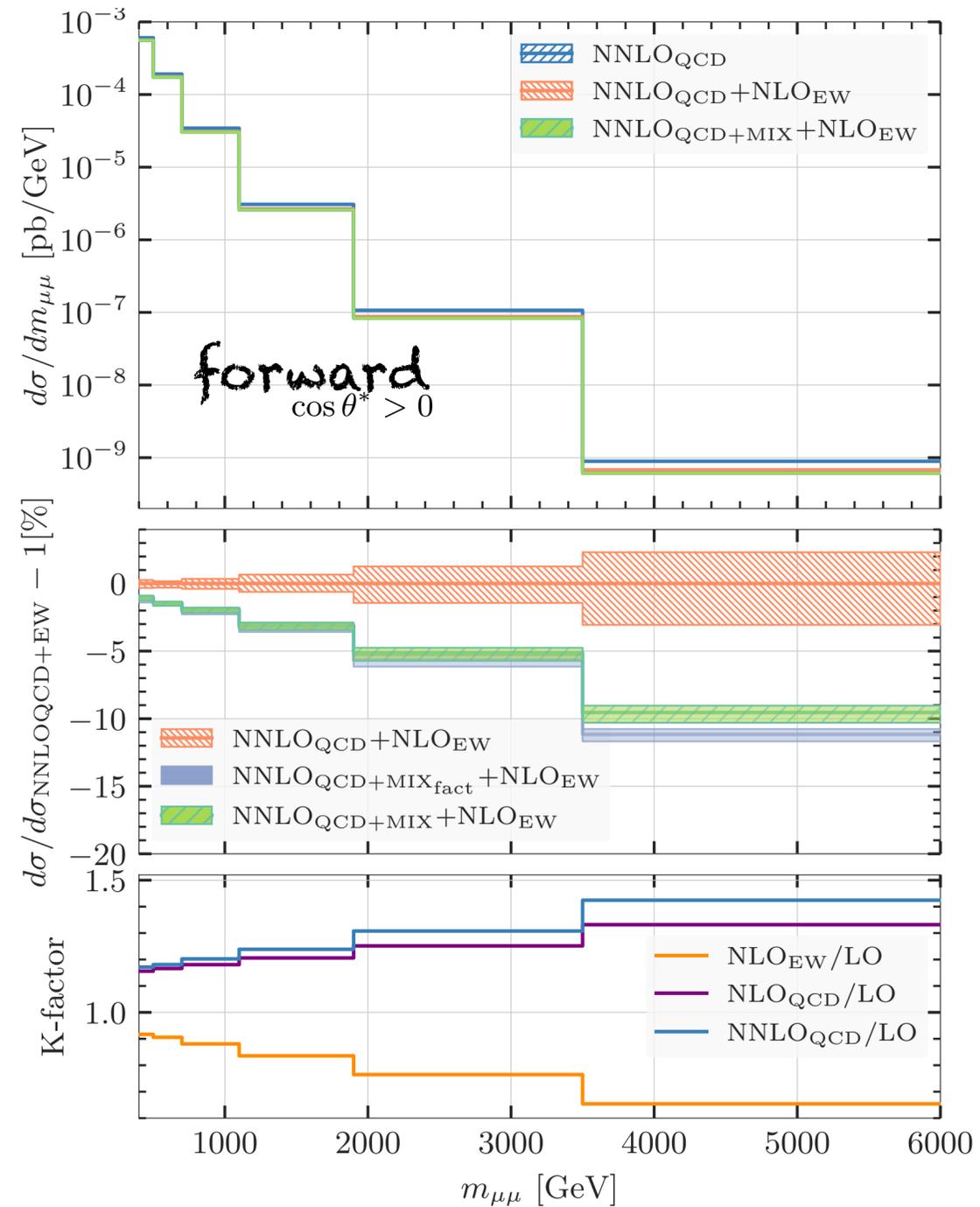


Sizeable mixed QCD-EW effects on A_{FB} below the Z resonance → impact on the $\sin^2 \theta_{eff}$ determination

A fraction is already taken into account in MC tools with factorized combinations of QCD with QED-FSR

Phenomenology of Neutral Current Drell-Yan including exact NNLO QCD-EW corrections

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Negative mixed NNLO QCD-EW effects (-3% or more) at large invariant masses,

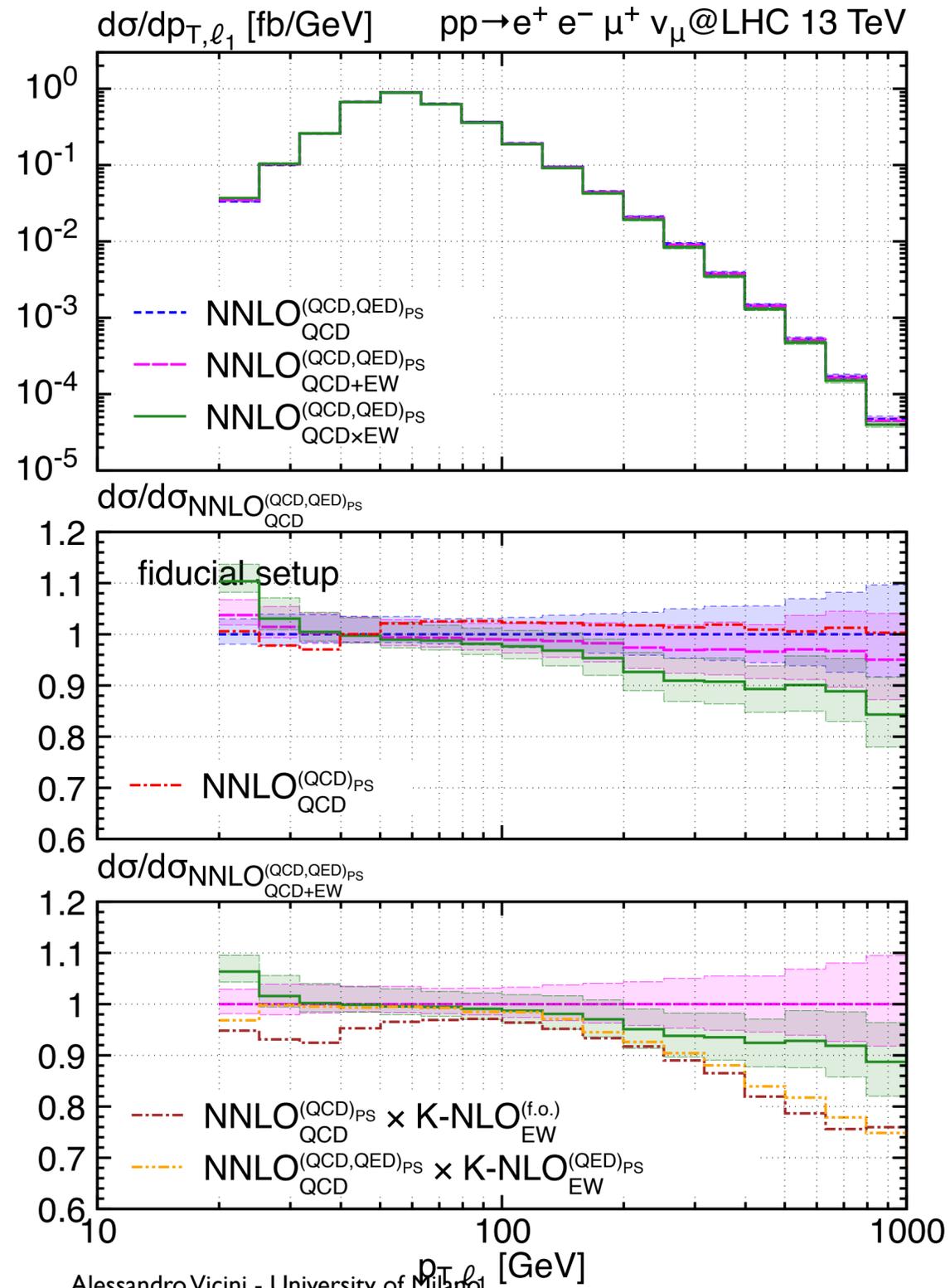
absent in any additive combination → impact on the searches for new physics and PDF determination

When
NLO QCD and EW corrections
are both large

Radiative corrections to WZ production

MINNLO_PS-QCD + NLO-EW+QEDPS corrections to $e^+e^-\mu^+\nu_\mu$ inclusive production

J.M.Lindert, D.Lombardi, M.Wiesemann, G.Zanderighi, S.Zanoli, arXiv:2208.12660



$p_{\perp}^{\ell,1}$ is a key observable for its sensitivity to the TGC

taking NNLO+PS QCD predictions as a reference,

QED showering and NLO-EW effects implemented with different recipes

QED effects at few % level in the whole momentum range

large effects relevant at low- p_{\perp}^{ℓ} , close to phase-space boundaries

EW corrections growing with p_{\perp}^{ℓ} , exceeding the -10% level at large- p_{\perp}^{ℓ}

NNLO-QCD + NLO-EW vs NNLO-QCD x NLO-EW

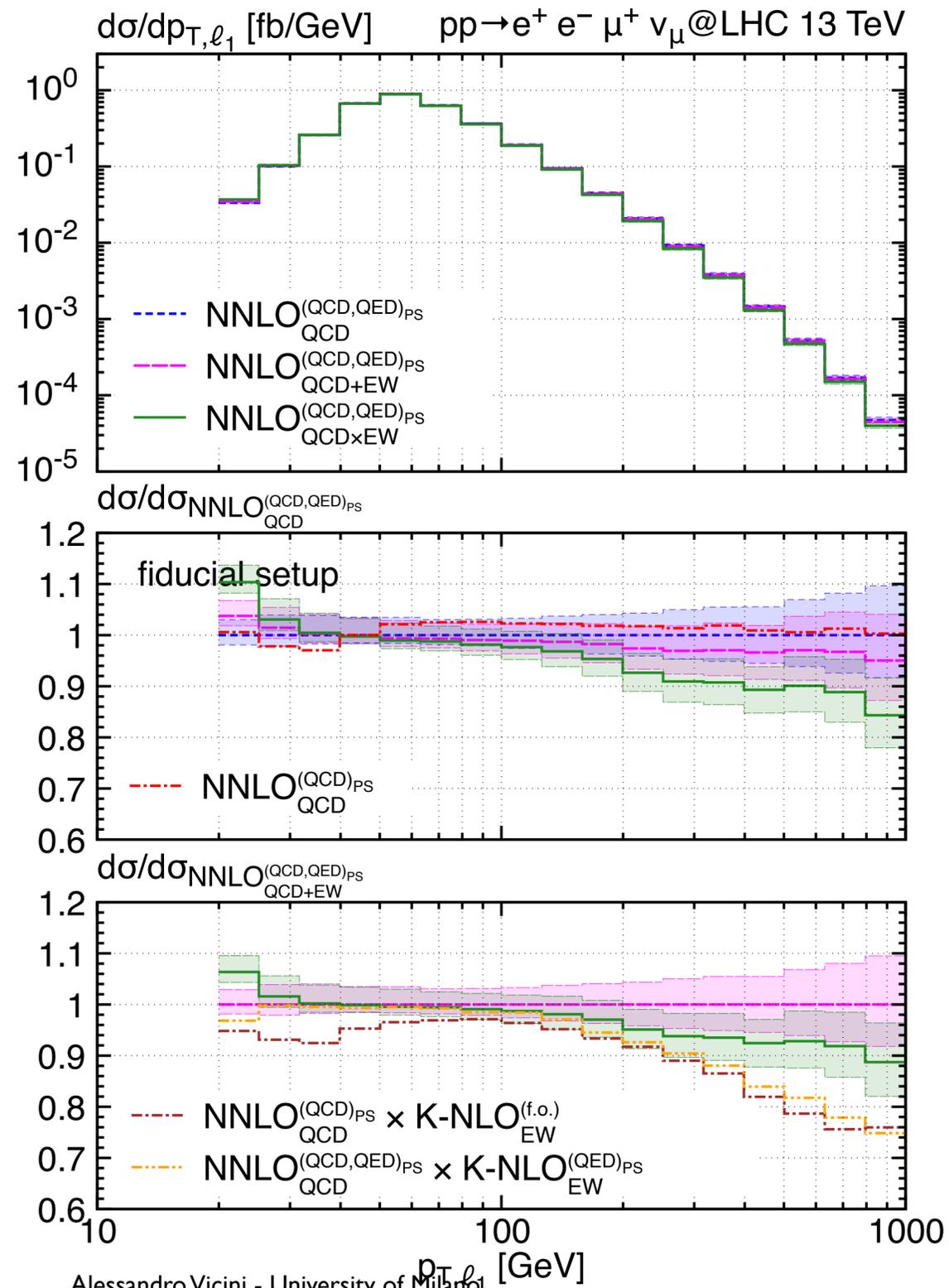
differences at the several % level (effect of giant QCD K-factor)

different QCD and EW combinations significantly differs from the exact results

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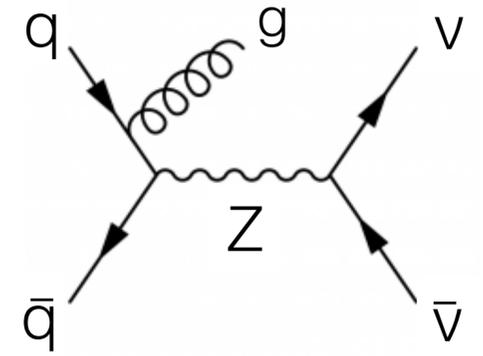
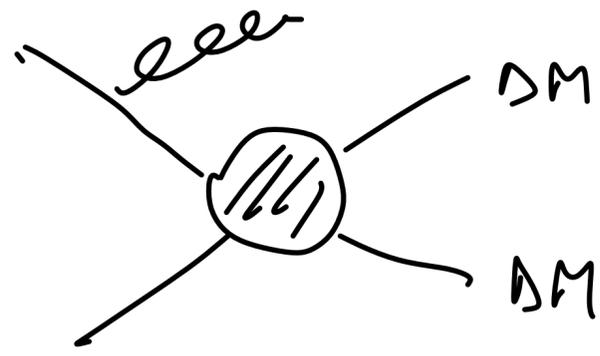
NNLO-QCD + NLO-EW vs NNLO-QCD x NLO-EW

differences at the several % level (effect of giant QCD K-factor)

different QCD and EW combinations significantly differs from the exact results

exact NNLO QCD-EW corrections needed to solve these ambiguities

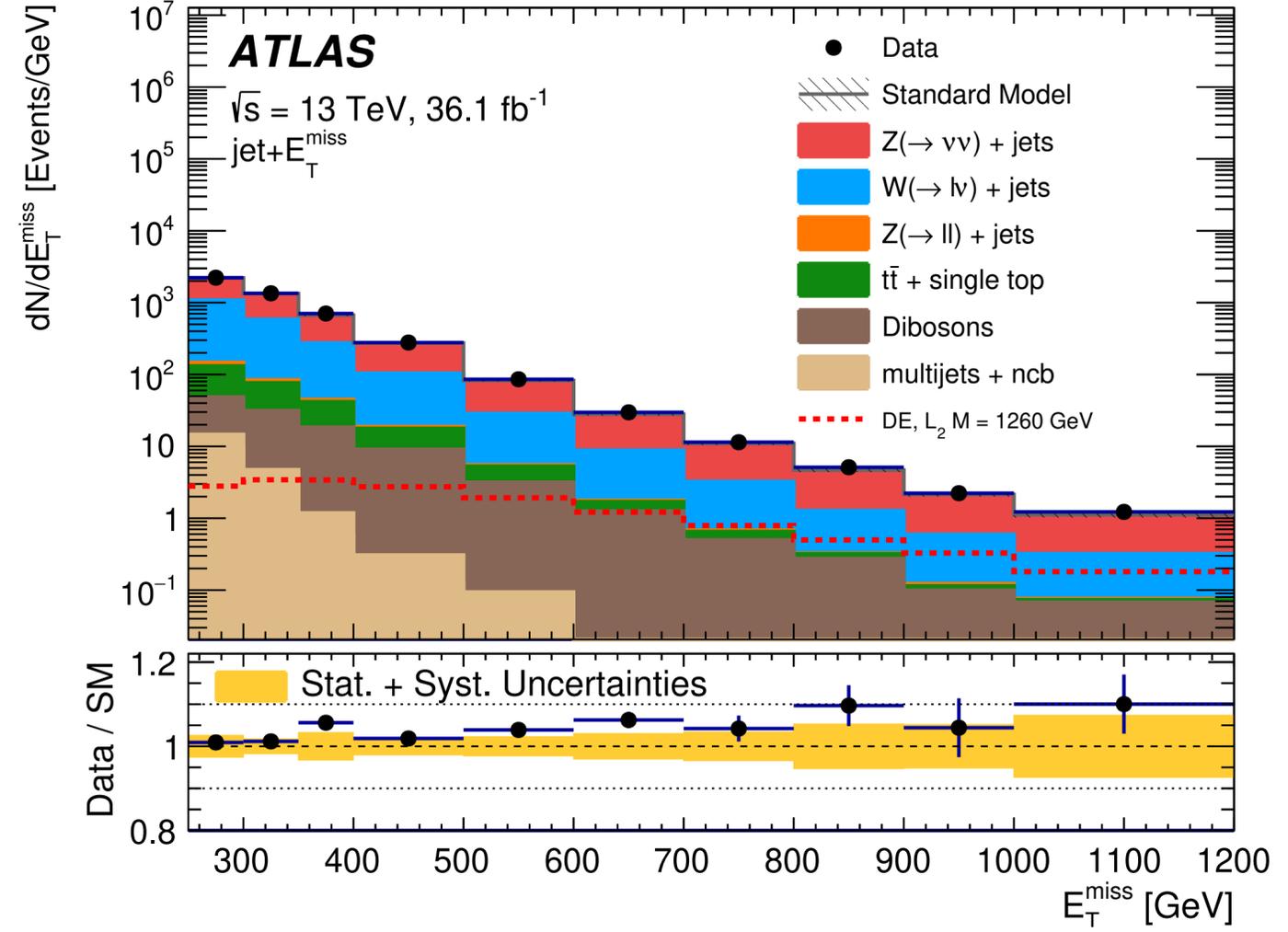
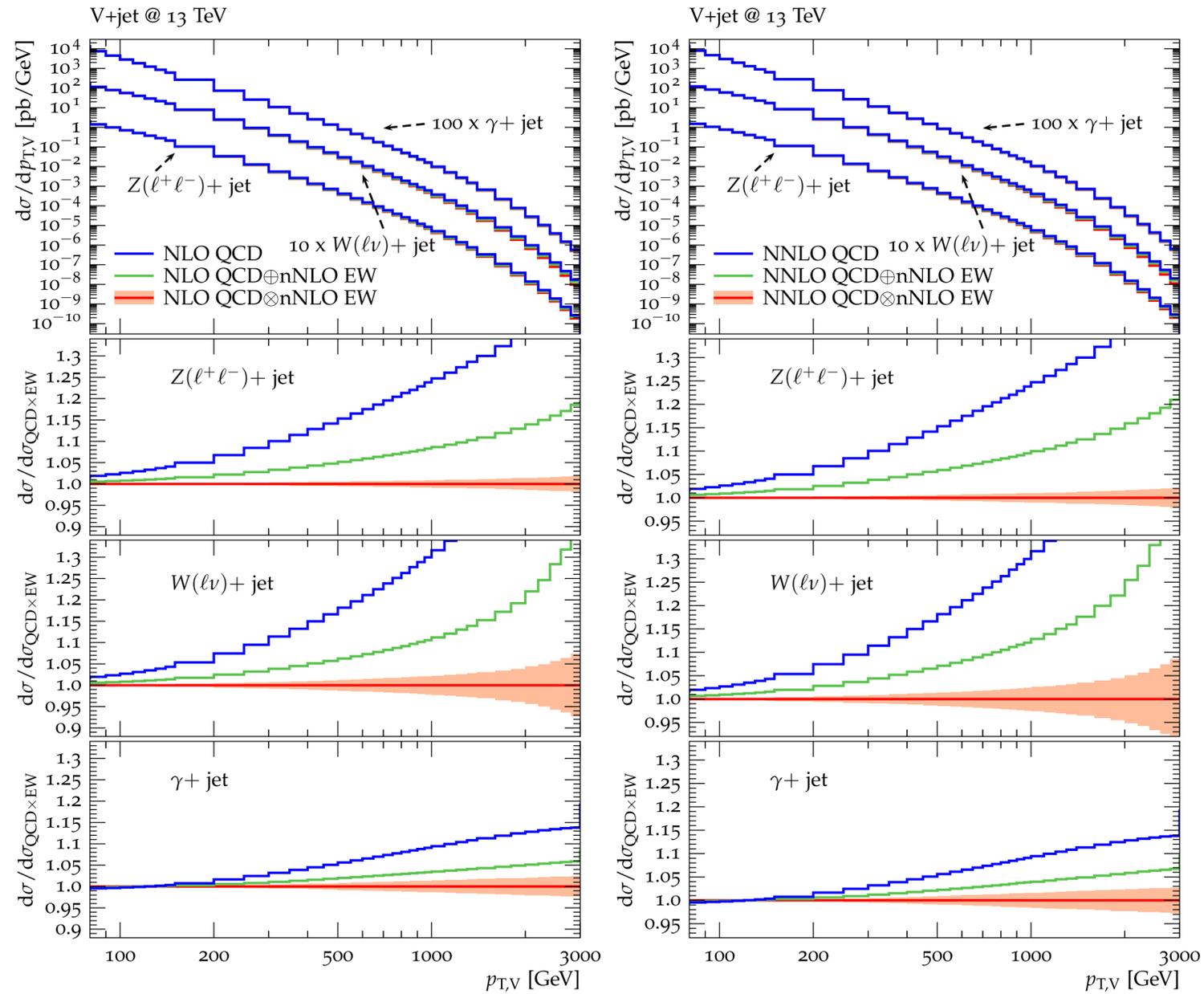
A background to dark matter searches



J.Lindert et al, arXiv:1705.04664

All possible channels of V+jet are studied as proxies to $pp \rightarrow \nu\bar{\nu} + j$ at large p_{\perp}^V

The uncertainties in the SM predictions are due to missing exact $\mathcal{O}(\alpha\alpha_s)$ results



Photons in the proton?

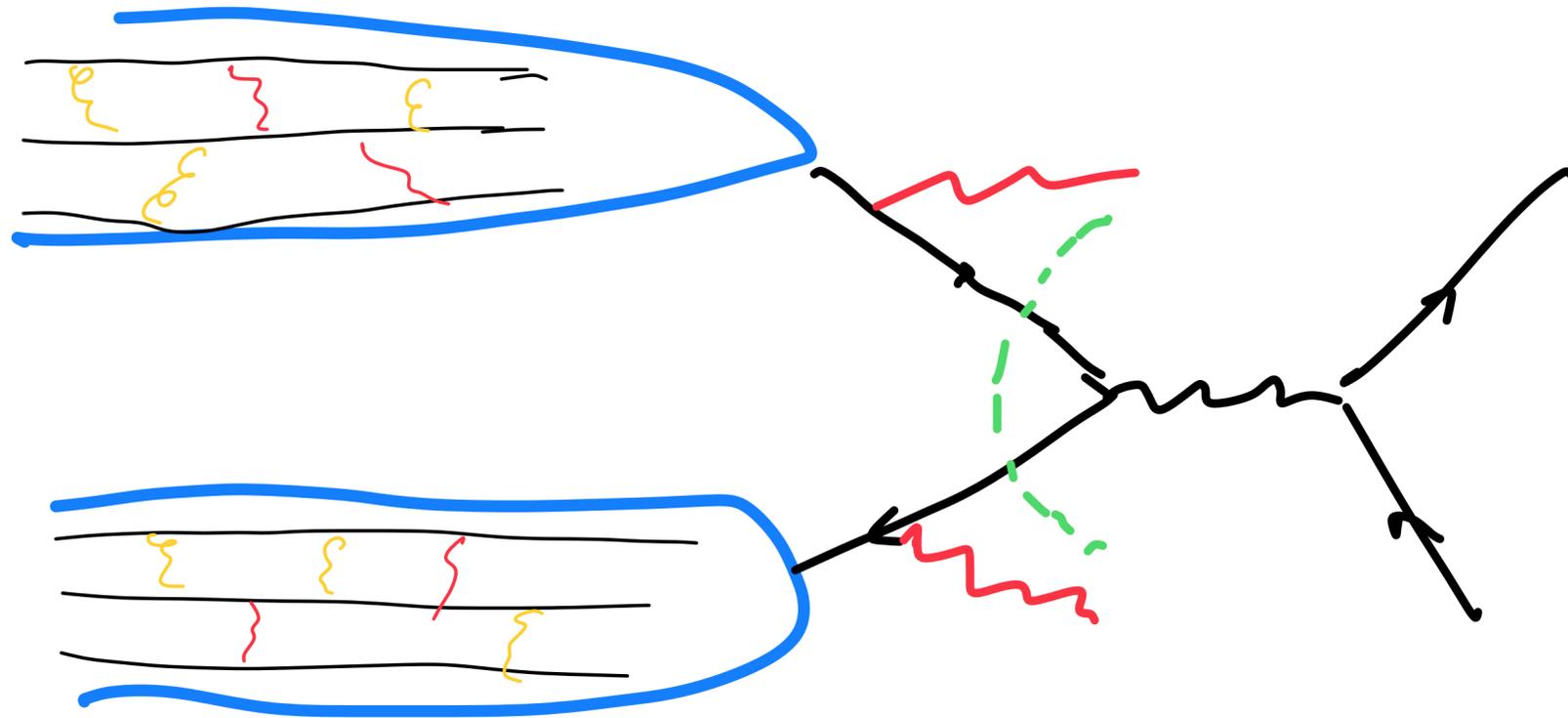
Photons in the proton

Quarks are electrically charged \rightarrow they interact inside the proton \rightarrow photon is a parton inside the proton (with its density)

Any hard scattering process with electrically charged initial state partons implies QED ISR \rightarrow collinear divergences

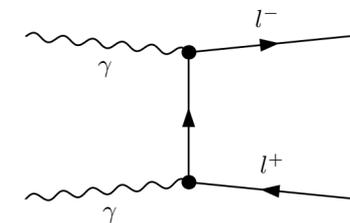
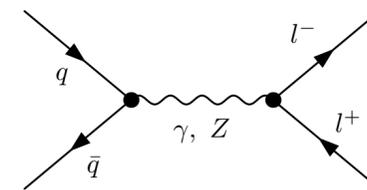
The initial state collinear divergences are factorized and reabsorbed in the physical proton PDFs

Every time we include QED ISR effects, we must use proton PDFs with QCD+QED evolution kernels



e.g. $\sigma(pp \rightarrow \mu^+ \mu^- + X)$

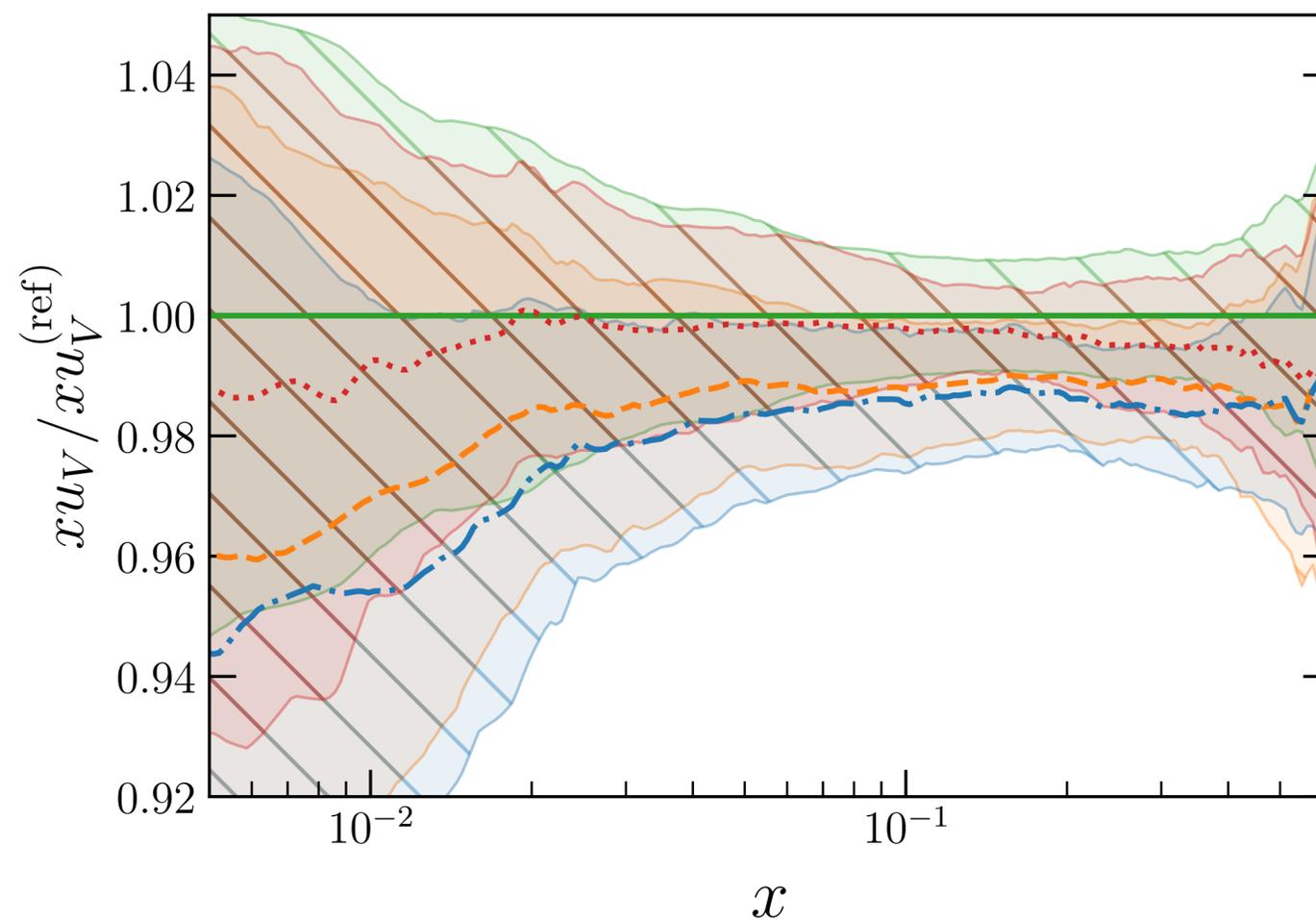
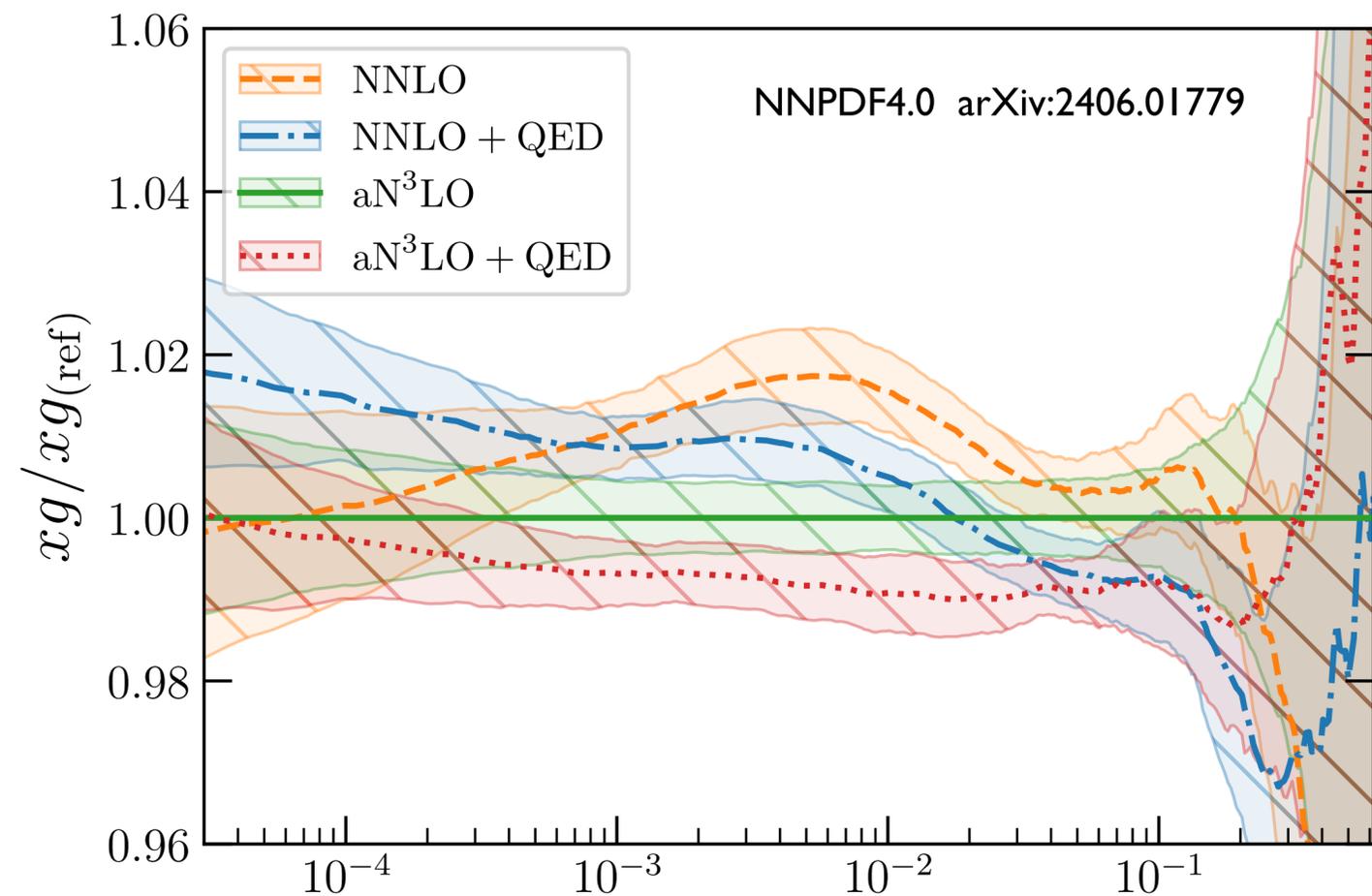
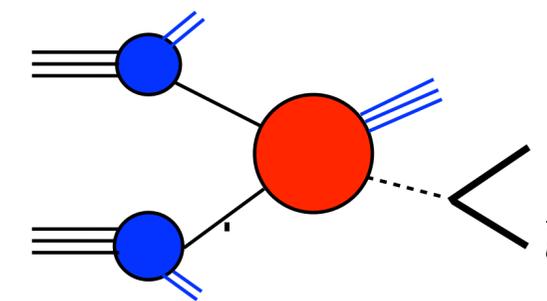
at LO contributions from $q\bar{q} \rightarrow \mu^+ \mu^-$
 $\gamma\gamma \rightarrow \mu^+ \mu^-$



both partonic cross sections are of $\mathcal{O}(\alpha^2)$
 the $\gamma\gamma$ process is suppressed by the small PDF luminosity

Mixed QCD and EW effects in proton PDFs

- two possible DGLAP evolutions of proton PDFs:
 - pure QCD
 - including QED and mixed QCD-QED kernels
- the inclusion of a photon density in the proton “subtracts” momentum to quarks and gluons
 - reduced quark-induced xsecs
 - compensated by new photon-induced partonic channels



When both QCD and EW
are present at LO

QCD and EW interfering at LO

$$\mathcal{M}_0 = \text{[QCD vertex]} + \text{[EW vertex]} = \alpha_s \mathcal{M}_{\alpha_s}^{(0)} + \alpha \mathcal{M}_{\alpha}^{(0)}$$

$$\mathcal{M}_1 = \text{[QCD loop]} + \text{[EW loop]} + \dots = \alpha_s^2 \mathcal{M}_{\alpha_s^2}^{(1)} + \alpha \alpha_s \mathcal{M}_{\alpha_s \alpha}^{(1)} + \alpha^2 \mathcal{M}_{\alpha^2}^{(1)}$$

$$|\mathcal{M}_0|^2 = \alpha_s^2 |\mathcal{M}_{\alpha_s}^{(0)}|^2 + \alpha \alpha_s 2\text{Re}(\mathcal{M}_{\alpha}^{(0)} \mathcal{M}_{\alpha_s}^{(0)\dagger}) + \alpha^2 |\mathcal{M}_{\alpha}^{(0)}|^2 \equiv \text{LO}_1 + \text{LO}_2 + \text{LO}_3$$

$$2\text{Re}(\mathcal{M}_1 \mathcal{M}_0^\dagger) = 2\text{Re} \left[\alpha_s^3 \mathcal{M}_{\alpha_s^2}^{(1)} \mathcal{M}_{\alpha_s}^{(0)\dagger} + \alpha_s^2 \alpha \left(\mathcal{M}_{\alpha_s^2}^{(1)} \mathcal{M}_{\alpha}^{(0)\dagger} + \mathcal{M}_{\alpha_s \alpha}^{(1)} \mathcal{M}_{\alpha_s}^{(0)\dagger} \right) + \alpha_s \alpha^2 \left(\mathcal{M}_{\alpha^2}^{(1)} \mathcal{M}_{\alpha_s}^{(0)\dagger} + \mathcal{M}_{\alpha_s \alpha}^{(1)} \mathcal{M}_{\alpha}^{(0)\dagger} \right) + \alpha^3 \mathcal{M}_{\alpha^2}^{(1)} \mathcal{M}_{\alpha}^{(0)\dagger} \right]$$

$$\equiv \text{NLO}_1 + \text{NLO}_2 + \text{NLO}_3 + \text{NLO}_4$$

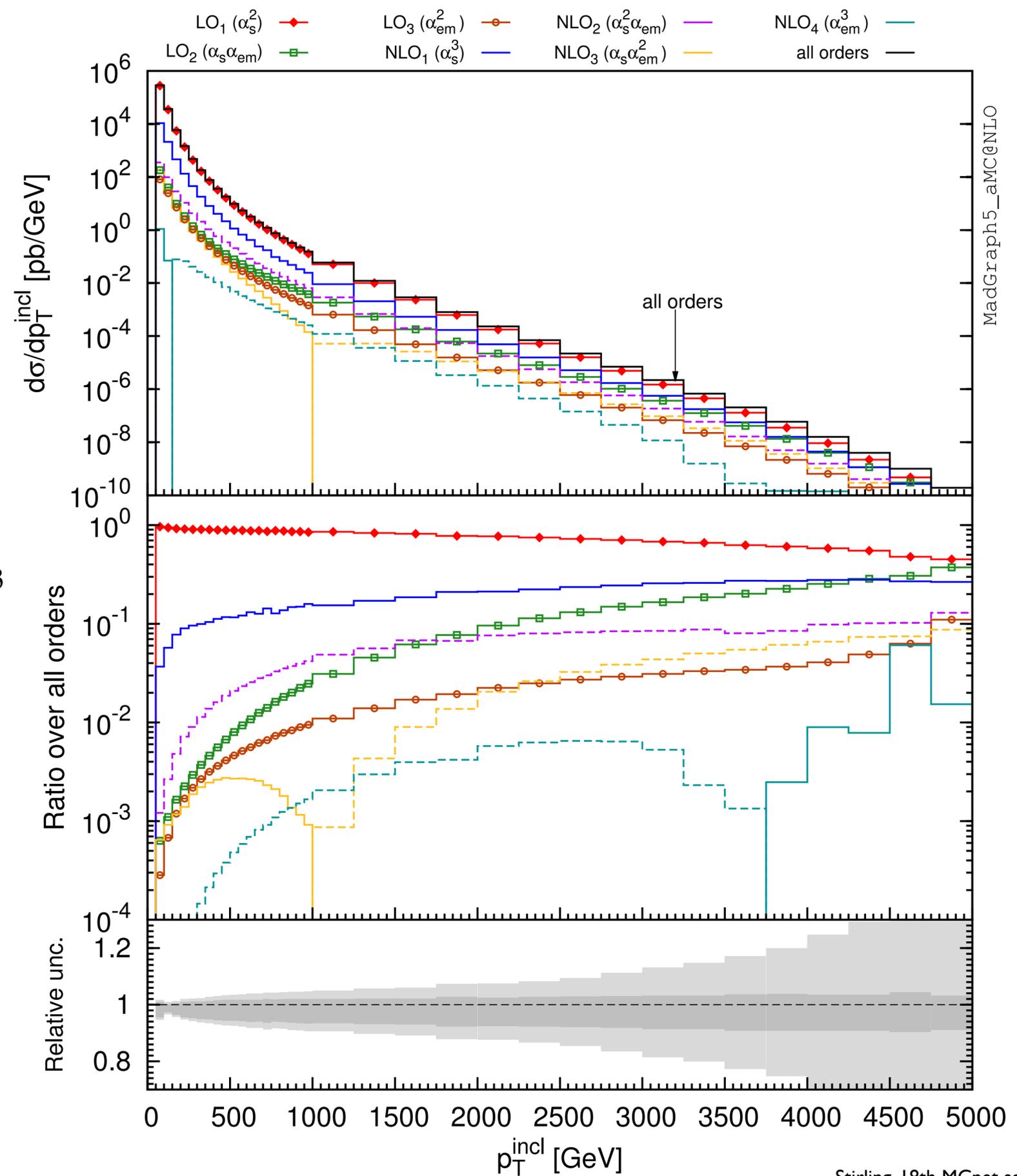
When both interactions interfere at LO, then the splitting into NLO-QCD and NLO-EW loses meaning
 orders \rightarrow loop counting
 at a given order, $\rightarrow \alpha_s^i \alpha^j$ power counting

QCD and EW interfering at LO

dijet hadroproduction

Frederix, Frixione, Hirschi, Pagani, Shah, Zaro, arXiv:1612.06548

Some NLO contributions are larger than LO_2, LO_3



Pole expansion
vs
full off-shell production

Gauge boson production and pole expansion

The presence of a resonance may help to organise the full amplitude in a hierarchical way

$$\mathcal{M}(q^2) = \frac{R^{(-1)}(\mu^2)}{q^2 - \mu^2} + R^{(0)}(\mu^2) + (q^2 - \mu^2)R^{(1)}(\mu^2) + \dots$$

At the complex mass pole, the residue $R^{(-1)}(\mu^2)$ and all the other coefficients R are gauge invariant

The expansion is expected to be accurate when $q^2 - \mu^2 \sim \# m\Gamma$

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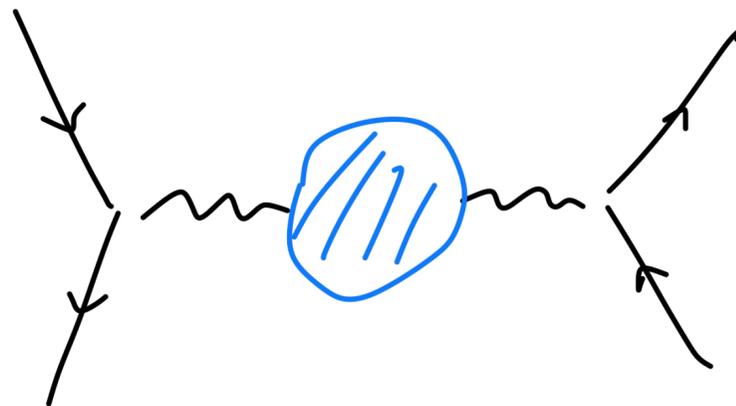
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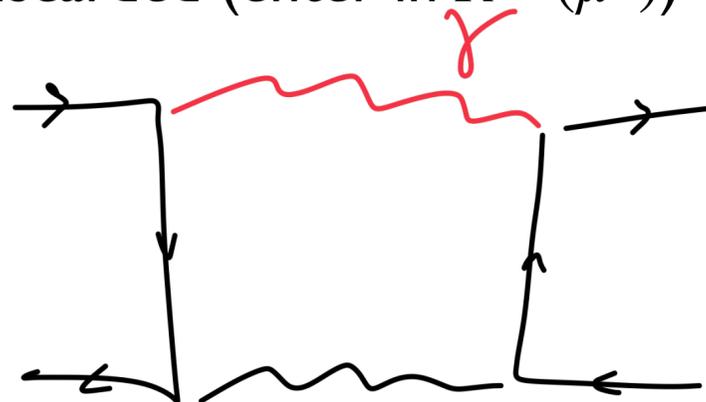
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The expansion helps to simplify loop calculations:

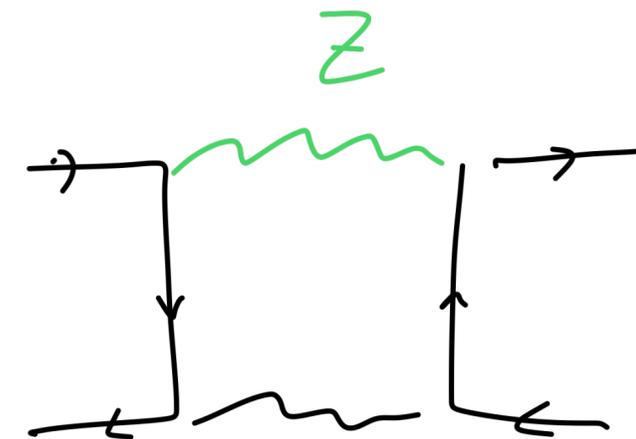
- the exchange of soft particles can be exactly computed
- difficult massive diagrams can be discarded (enter in $R^{(0)}(\mu^2)$)



resonant $\rightarrow R^{(-1)}(\mu^2)$



soft part resonant. $\rightarrow R^{(-1)}(\mu^2)$
hard part non-resonant $\rightarrow R^{(0)}(\mu^2)$



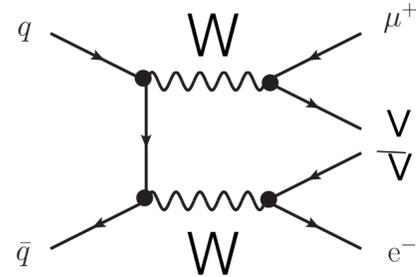
non-resonant $\rightarrow R^{(0)}(\mu^2)$

Charged current 4-fermion production and pole expansion

The pole expansion allows to separate

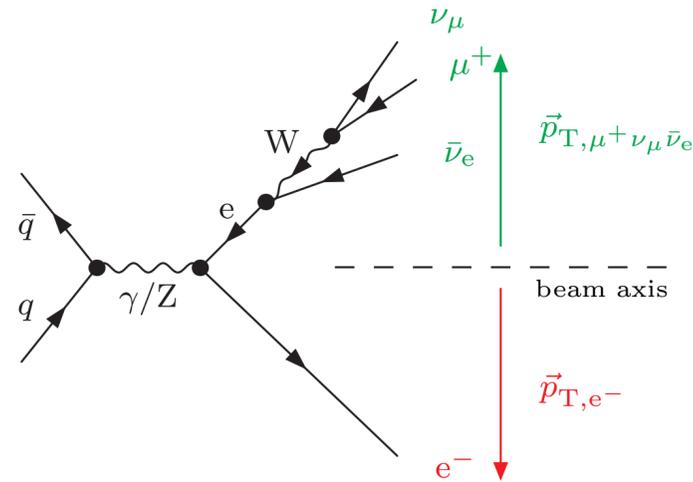
- a double pole term describing 2 quasi-on shell Ws

e.g.



- a single pole term with one resonant W

e.g.

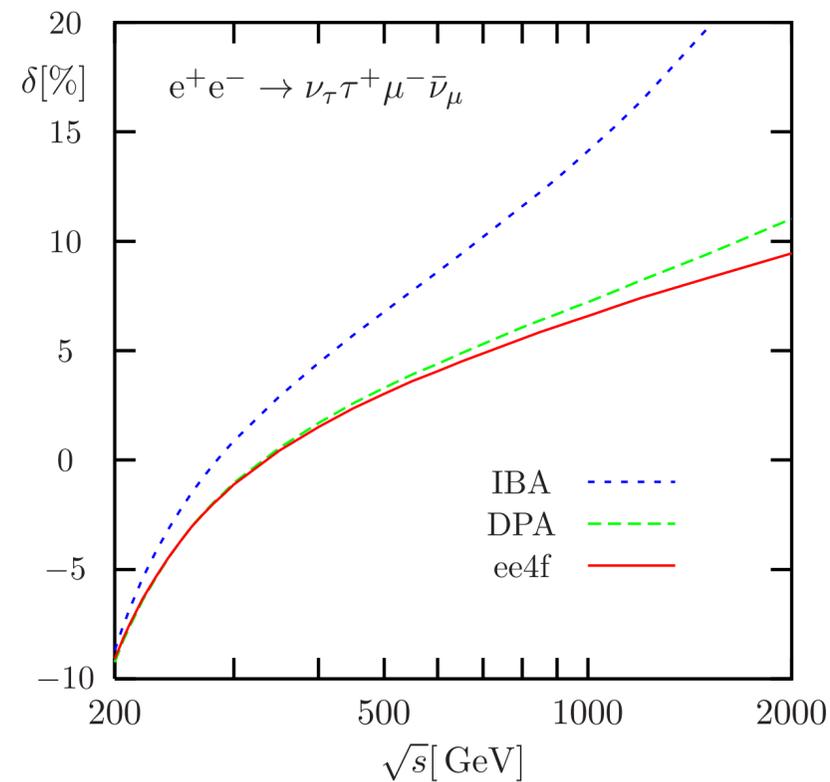
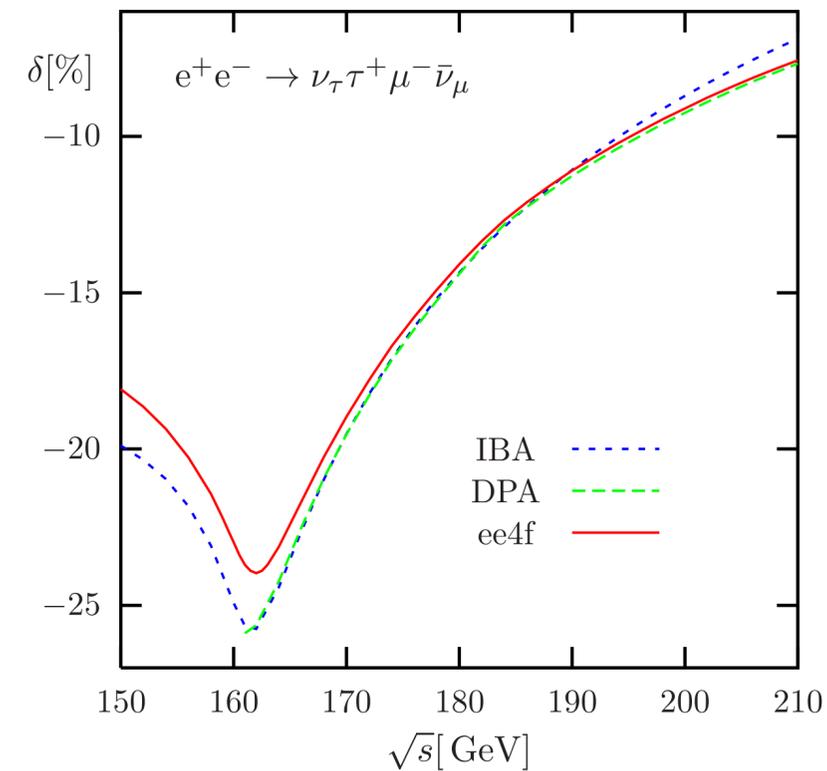
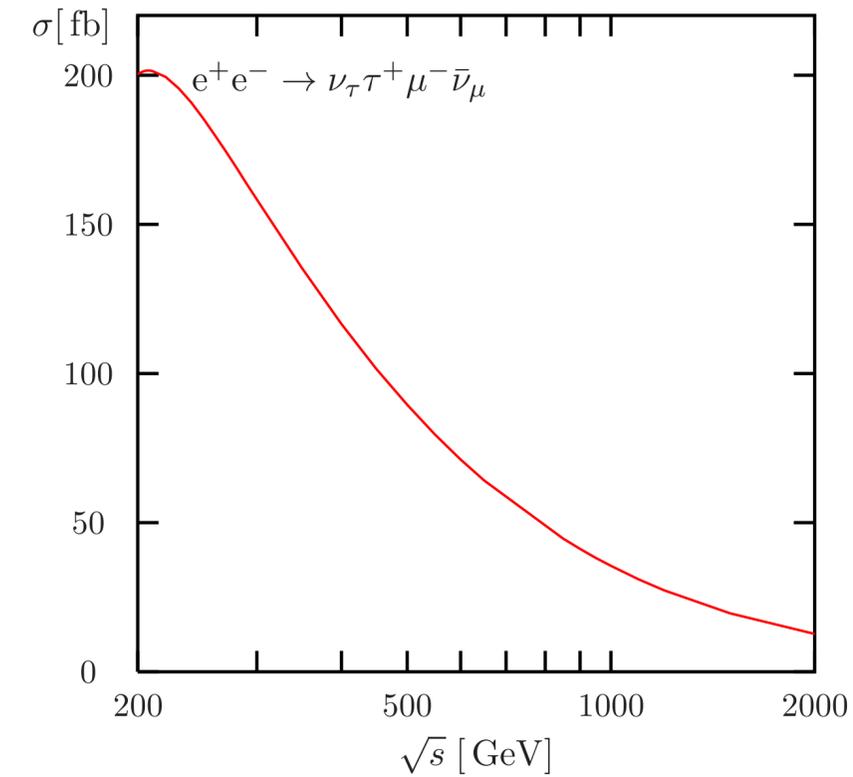
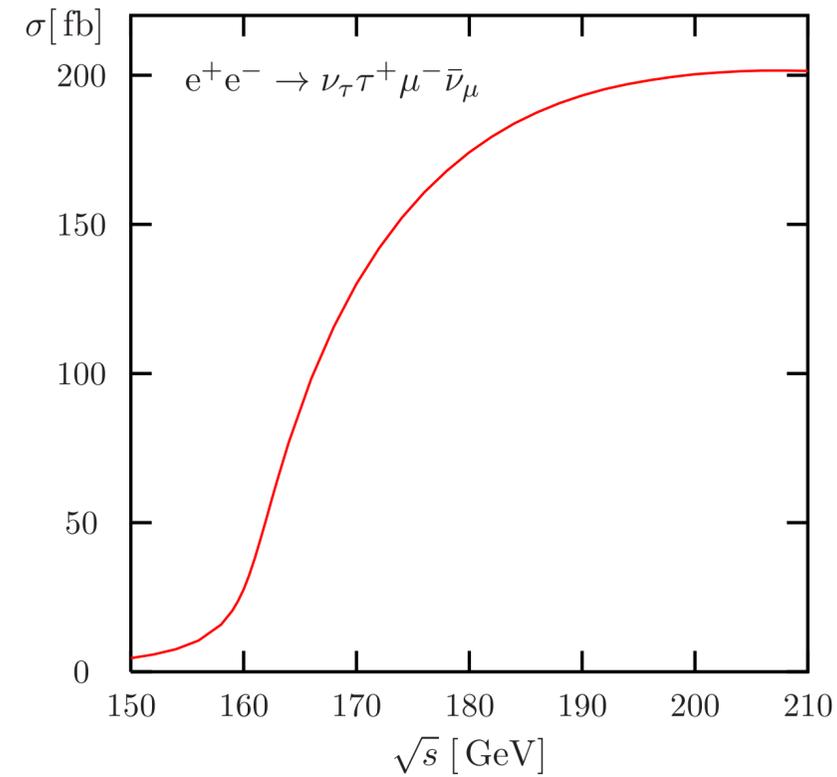


- a regular remainder

- Motivated in some phase-space regions

- Insufficient accuracy at the WW threshold, where m_W can be extracted with high precision

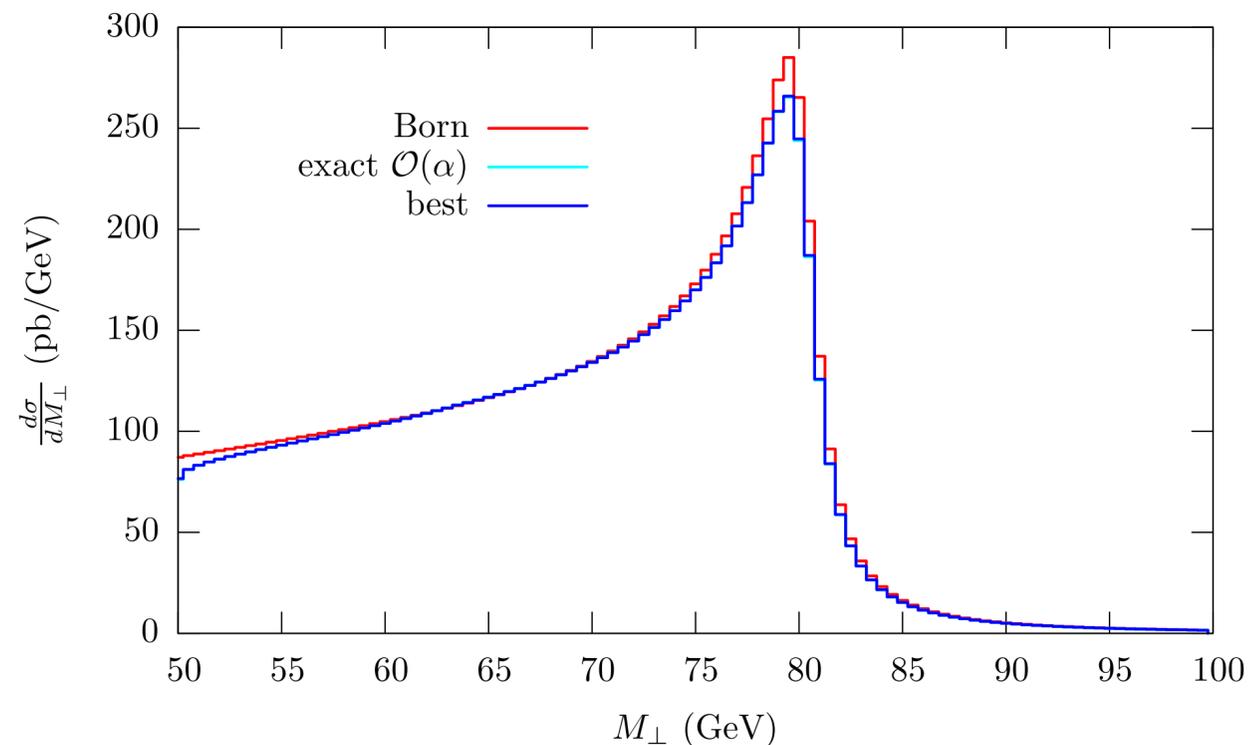
Alessandro Vicini - University of Milano



Off-shell corrections at NLO

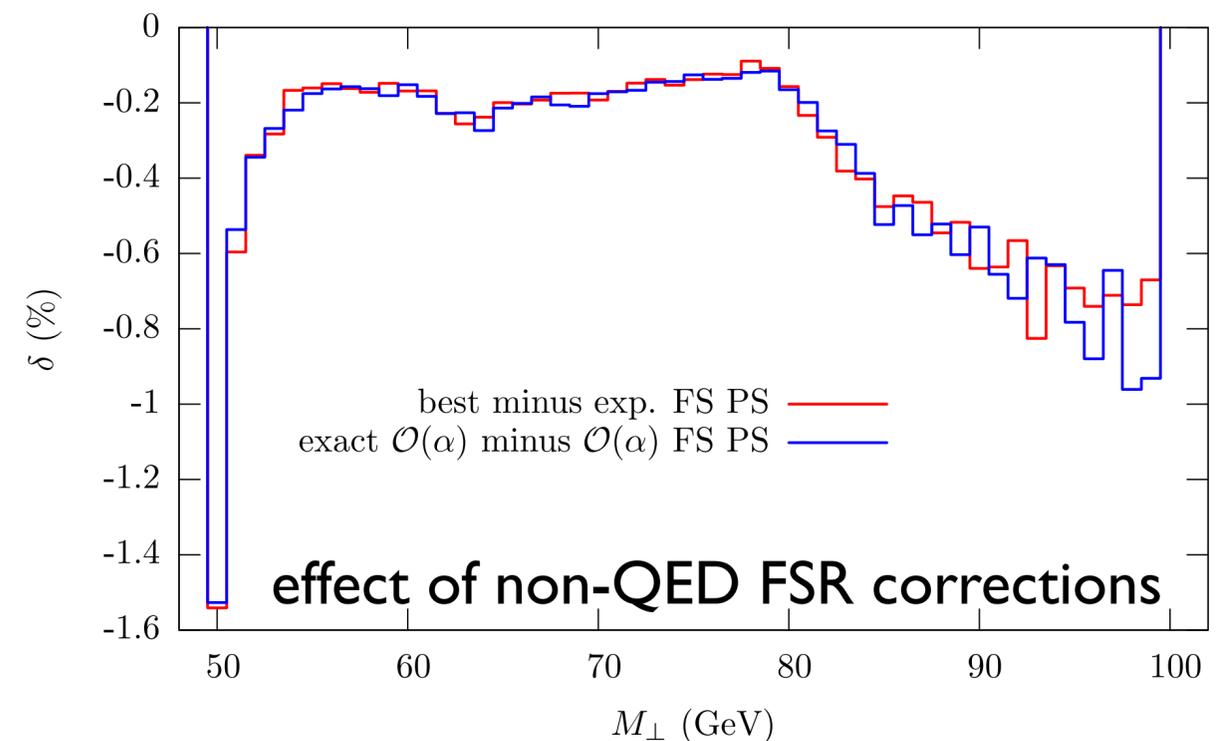
The complete inclusion of off-shell effects (i.e. the usage of exact amplitudes, without approximations) is needed, also for the radiative corrections:

- in the precision determination of the mass and width of intermediate particles because the line shape affects the outcome of the fit (performed in a not vanishing mass window)



e.g. single W production

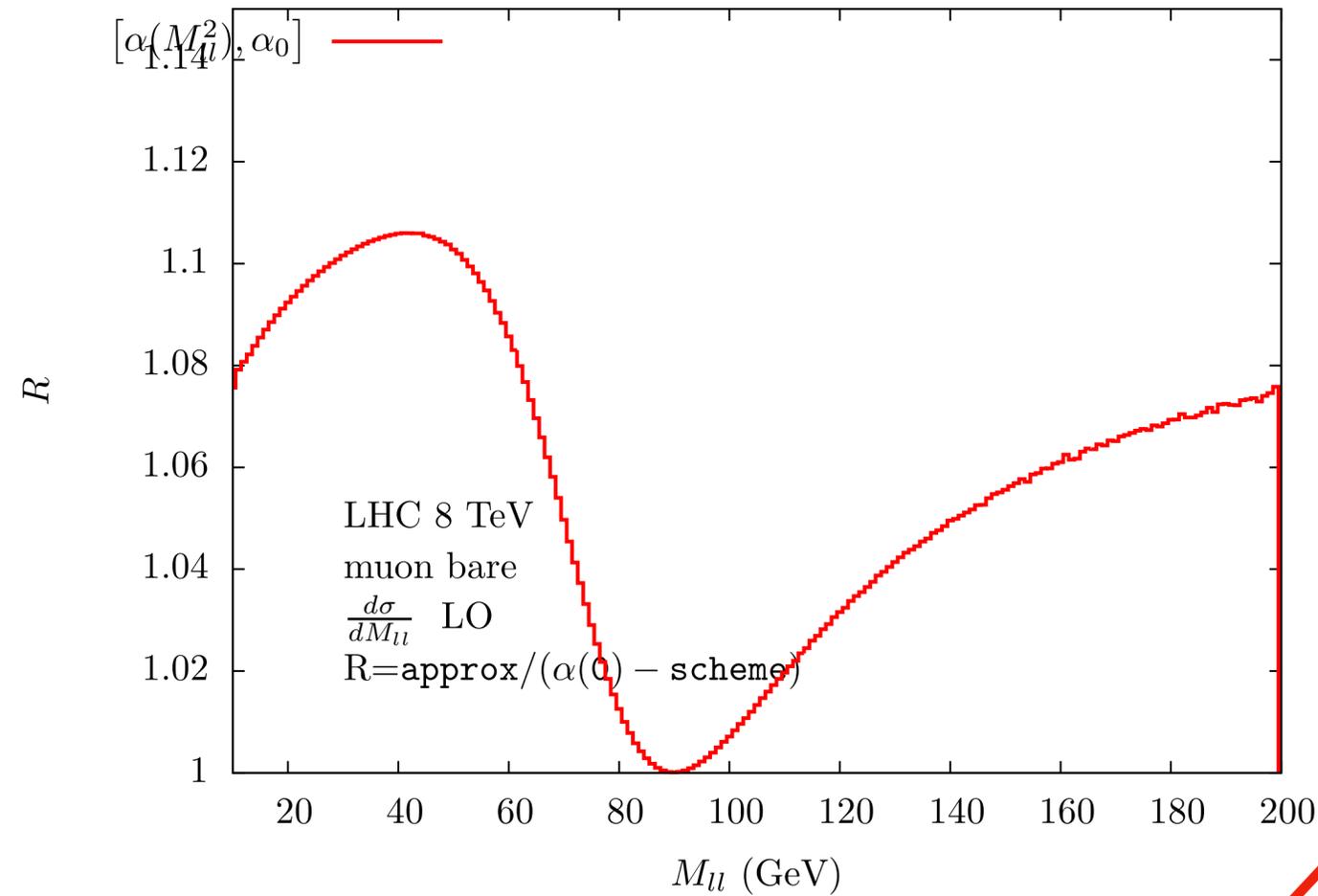
Γ_W determination from the tail of $M_{\perp}^{\ell\nu}$



- QED and EW corrections have a sizeable impact in the prediction of differential kinematical distributions
- the interplay of QCD and EW corrections is not trivial
 - combination of kinematical effects
 - presence of several partonic channels
 - large opposite sign effects
- the combined simulation of QCD and EW corrections is not trivial from algorithmic side
 - (general problem of resonant intermediate systems)
- approximation of exact EW calculations can be devised for specific phase-space regions, with limited validity

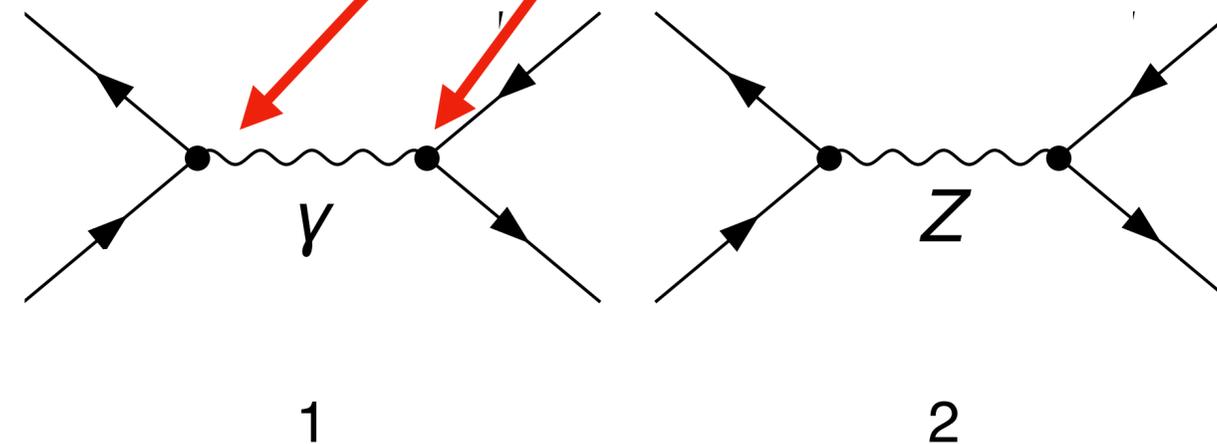
Thank you

Beyond LO approximation in neutral current Drell Yan

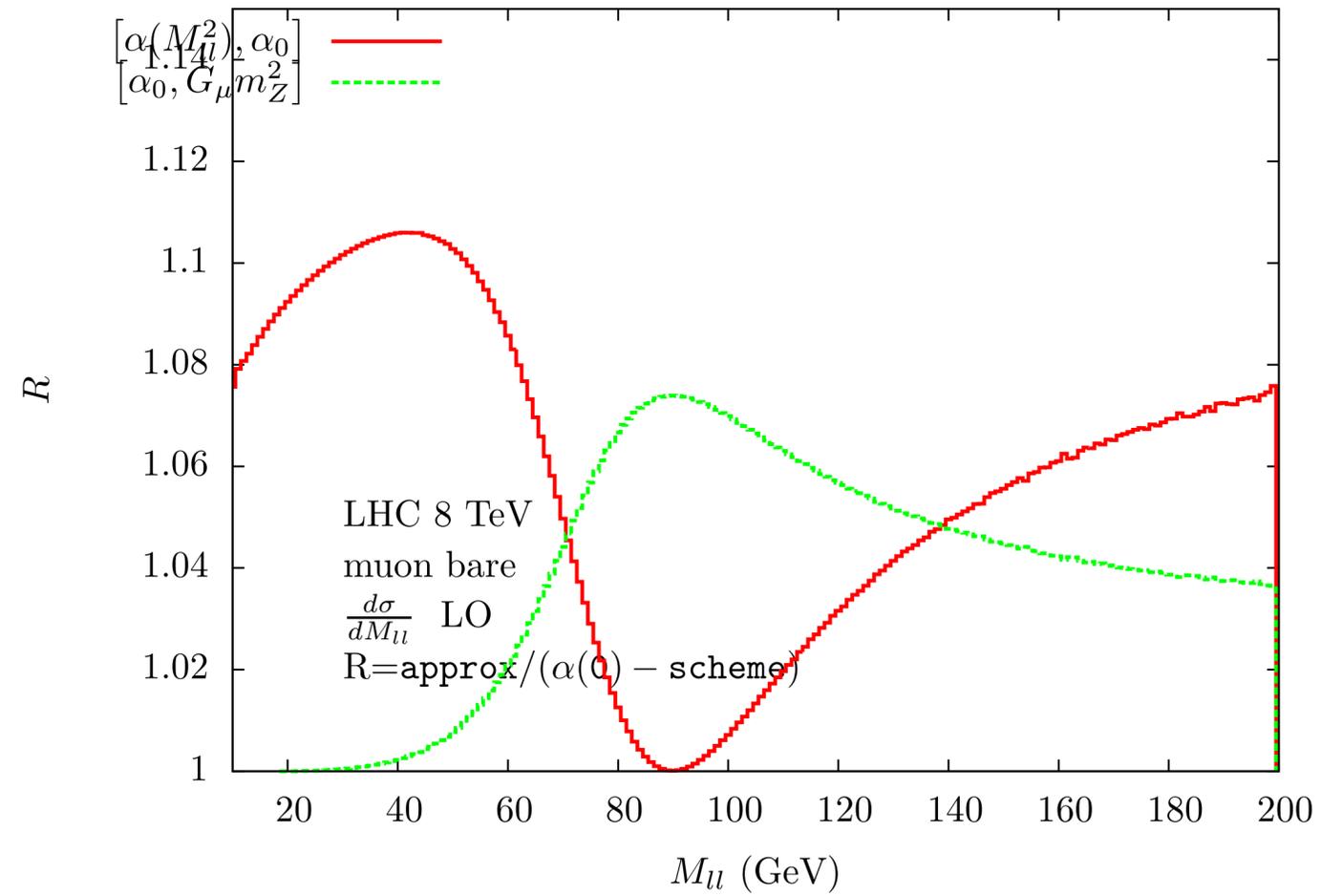


Running of α only in the photon diagram enhances the photon exchange contribution which grows with the invariant mass

$$\alpha(0) \rightarrow \alpha(M_{\ell\ell}^2)$$

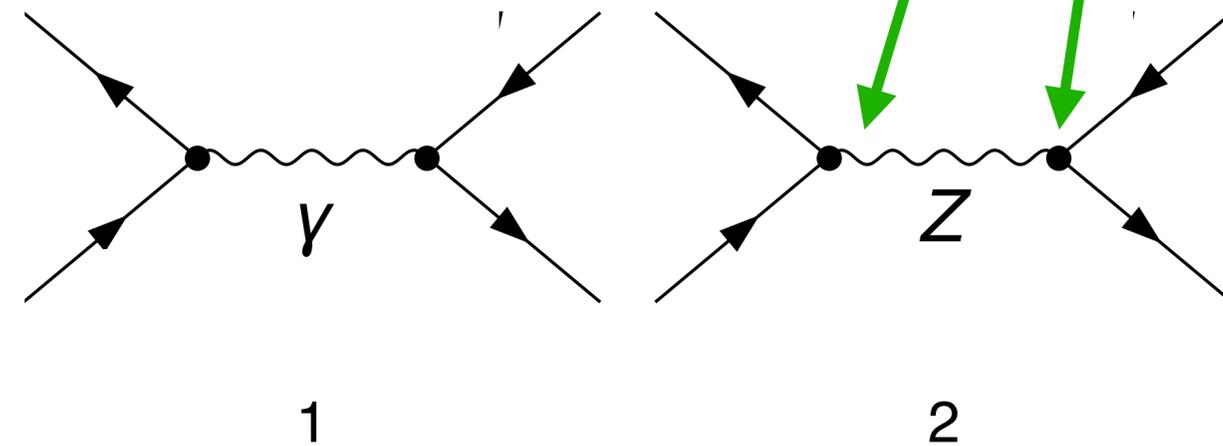


Beyond LO approximation in neutral current Drell Yan

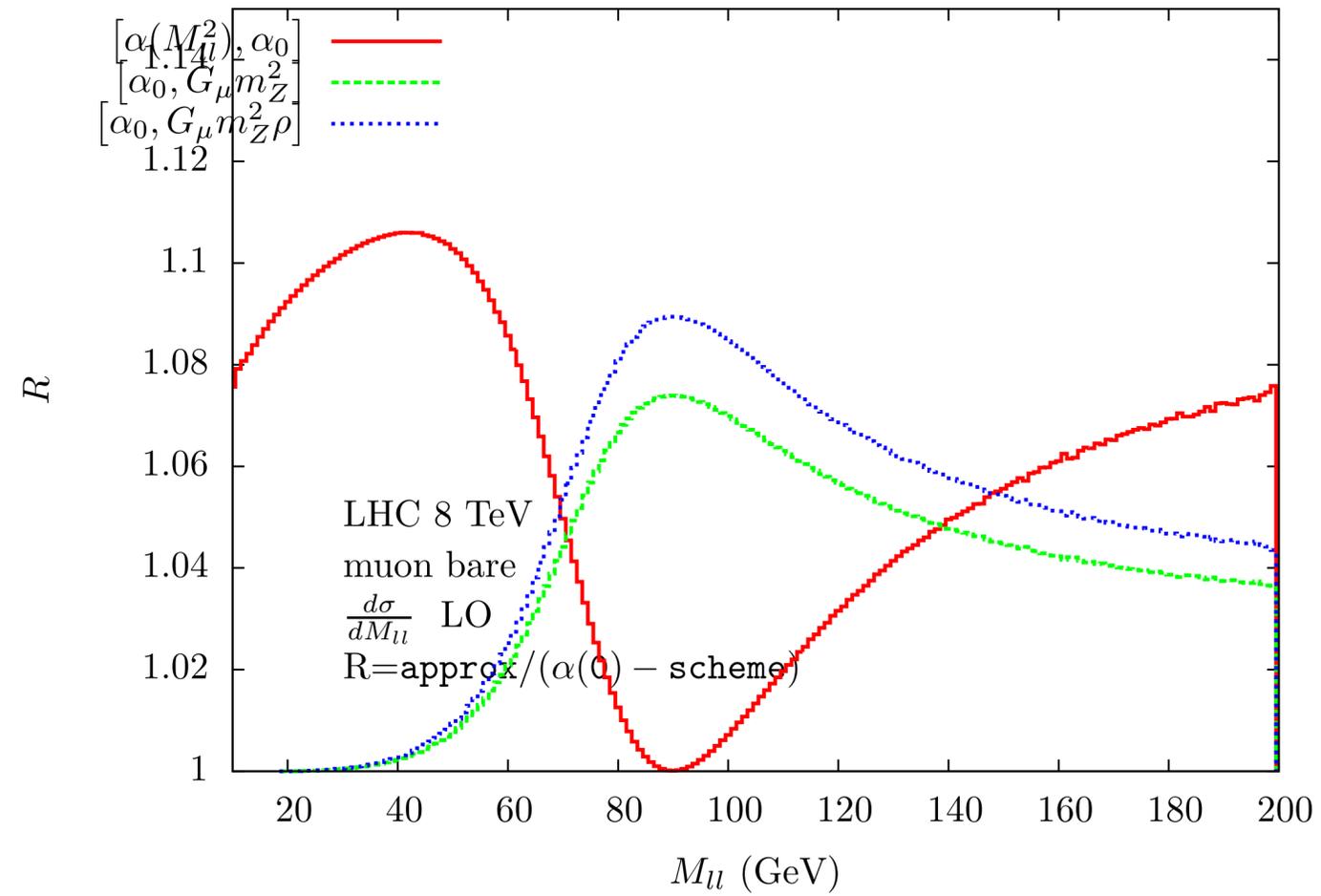


Use of G_μ **only** in the Z diagram enhances the peak of the Z resonance

$$\alpha(0) \rightarrow G_\mu$$

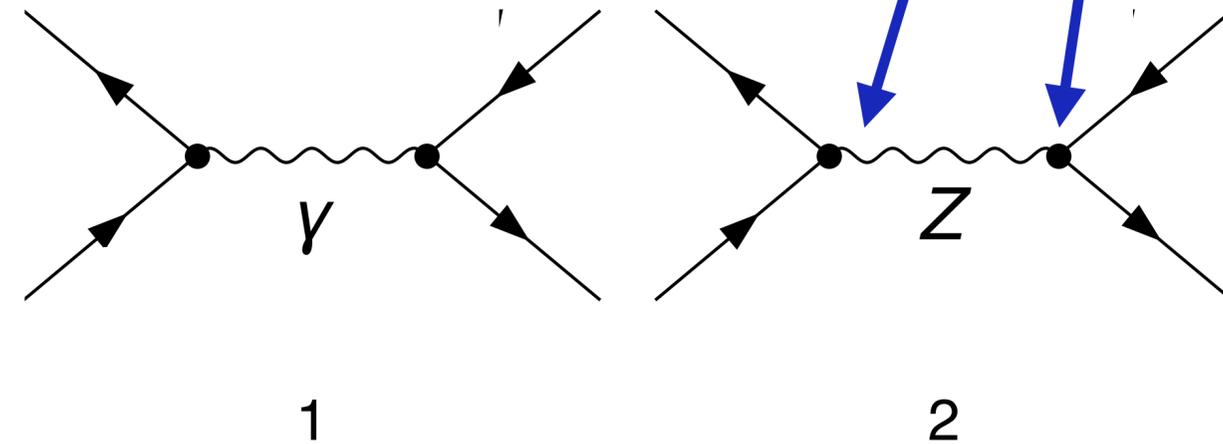


Beyond LO approximation in neutral current Drell Yan

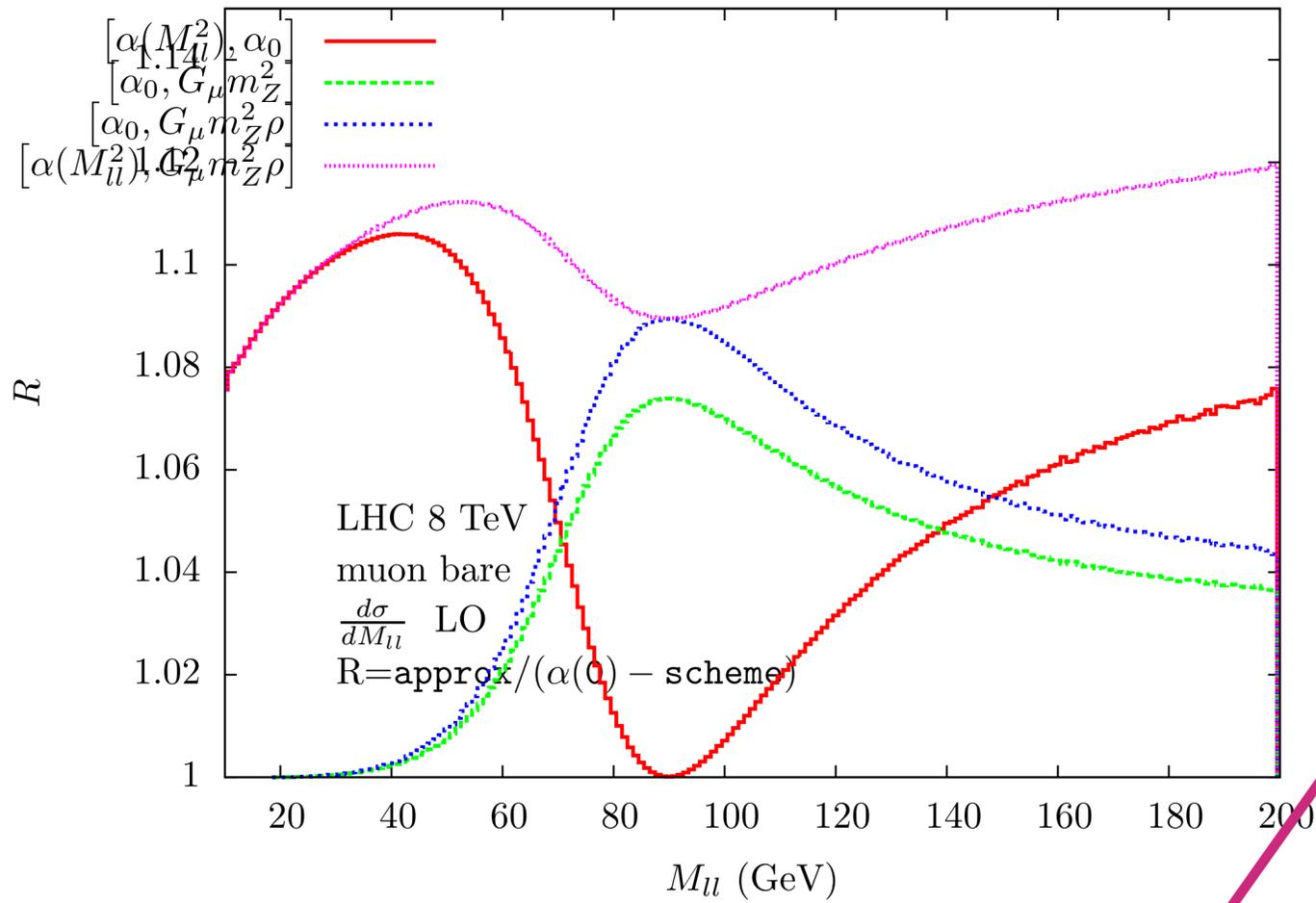


Use of G_μ and ρ **only** in the Z diagram enhances the peak of the Z resonance

$$\alpha(0) \rightarrow G_\mu \rho$$



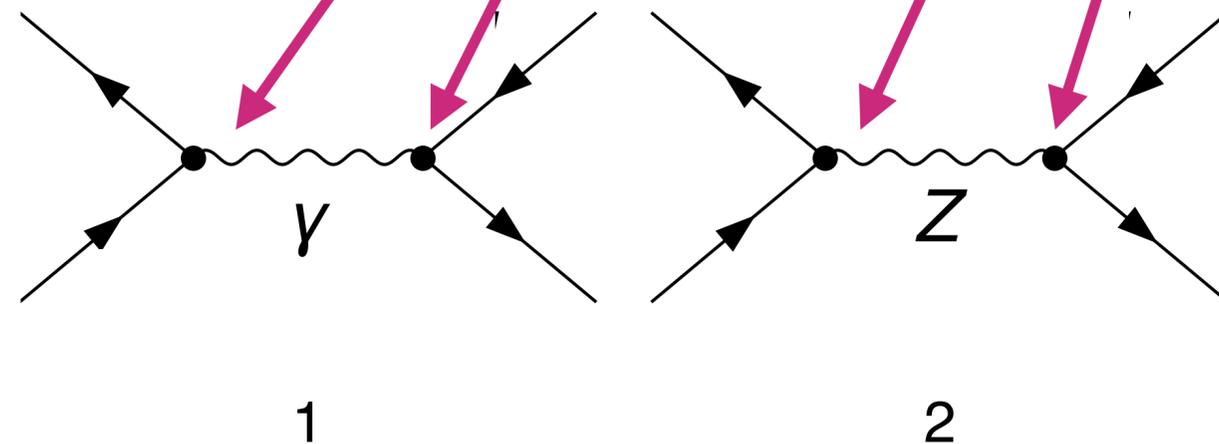
Beyond LO approximation in neutral current Drell Yan



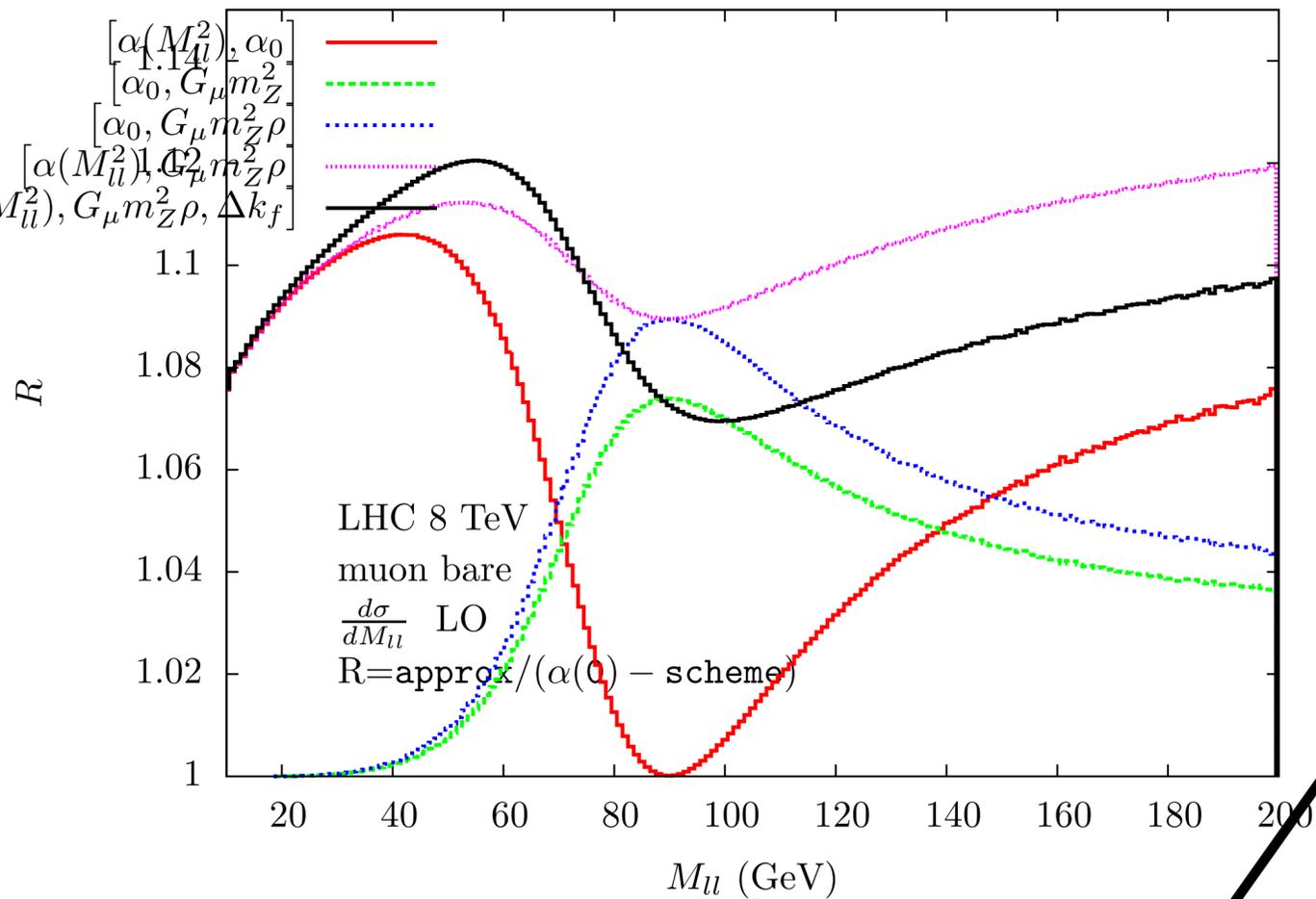
- running of α only in the photon diagram
- use of G_μ and ρ only in the Z diagram:

$$\alpha(0) \rightarrow \alpha(M_{\ell\ell}^2)$$

$$\alpha(0) \rightarrow G_\mu \rho$$



Beyond LO approximation in neutral current Drell Yan

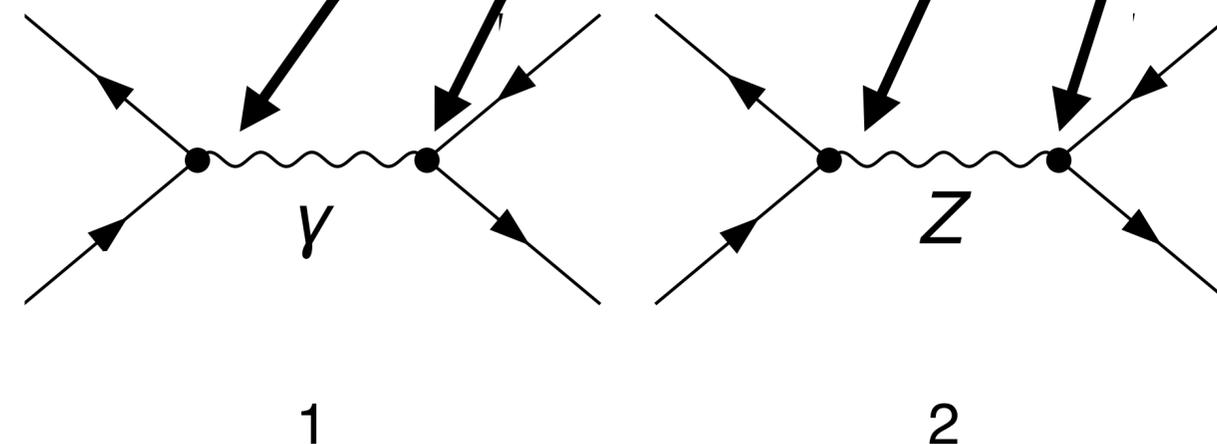


- running of α only in the photon diagram
- use of G_μ and ρ only in the Z diagram:
- rescaling of $\sin^2 \theta_W$ in the Z vector coupling by $1 + \Delta\kappa_f$

$$\alpha(0) \rightarrow \alpha(M_{\ell\ell}^2)$$

$$\text{rescaling } \alpha(0) \rightarrow G_\mu \rho$$

$$\sin^2 \theta_W \rightarrow (1 + \Delta\kappa_f) \sin^2 \theta_W$$



Several effects enter in the coupling redefinition

NLO-EW contains at first order all these effects but not the higher-order corrections

$\Delta\kappa_f$ is the only correction which modifies the precise $\sin^2 \theta_W$ value

EW corrections: matching NLO-EW with QED-PS

- leading soft and collinear enhancement factor is universal → can be easily applied to the final state of a hard scattering (see PHOTOS, or the PHOTONS module in Sherpa, or the QED-FSR showers in Pythia/Herwig)

the inclusion of subleading EW effects requires a matching procedure with exact matrix elements (see Marek's lectures)

- Matching an exact NLO-EW calculation with a full QED Parton Shower can be achieved e.g. like in HORACE

$$d\sigma^\infty = \Pi(Q^2, \varepsilon) F_{SV} \sum_{n=0}^{\infty} \frac{1}{n!} \left(\prod_{k=1}^n F_{H,k} \right) |\mathcal{M}_{n,LL}|^2 d\Phi_n$$

The expansion in α of this formula coincides with the exact fixed order calculation

Hard exact matrix element corrections are applied “democratically” to all the emitted photons

The correction factors F are by construction IR regular

$$d\sigma_\alpha = [1 + C_\alpha] |\mathcal{M}_0|^2 d\Phi_0 + |\mathcal{M}_1|^2 d\Phi_1$$

$$F_{SV} = 1 + (C_\alpha - C_{\alpha,LL})$$

$$F_H = 1 + \frac{|\mathcal{M}_1|^2 - |\mathcal{M}_{1,LL}|^2}{|\mathcal{M}_{1,LL}|^2}$$

Mixed QCD and EW effects at hadron colliders

Matching NLO-QCD and NLO-EW with QCD-PS and QED-PS

POWHEG NLO-(QCD+EW)

$$d\sigma = \sum_{f_b} \bar{B}^{f_b}(\Phi_n) d\Phi_n \left\{ \Delta^{f_b}(\Phi_n, p_T^{min}) + \sum_{\alpha_r \in \{\alpha_r | f_b\}} \frac{[d\Phi_{rad} \theta(k_T - p_T^{min}) \Delta^{f_b}(\Phi_n, k_T) R(\Phi_{n+1})]_{\alpha_r}^{\bar{\Phi}_n^{\alpha_r} = \Phi_n}}{B^{f_b}(\Phi_n)} \right\}$$

the difference between QCDxQED and QCDxEW approximations starts at $O(\alpha_s)$

POWHEG NLO-QCD x (QCD+QED)-PS $\alpha_s \alpha (c_2 L_{QCD}^2 + c_1 L_{QCD} + c_0) (c_{11} L_{QED} l_{QED} + c_{10} L_{QED} + c_{01} l_{QED})$

POWHEG NLO-(QCD+EW) x (QCD+QED)-PS $\alpha_s \alpha (c_2 L_{QCD}^2 + c_1 L_{QCD} + c_0) (c_{11} L_{QED} l_{QED} + c_{10} L_{QED} + c_{01} l_{QED} + c_{00})$

the difference $\alpha_s \alpha c_{00} (c_2 L_{QCD}^2 + c_1 L_{QCD} + c_0)$ important when c_{00} is large

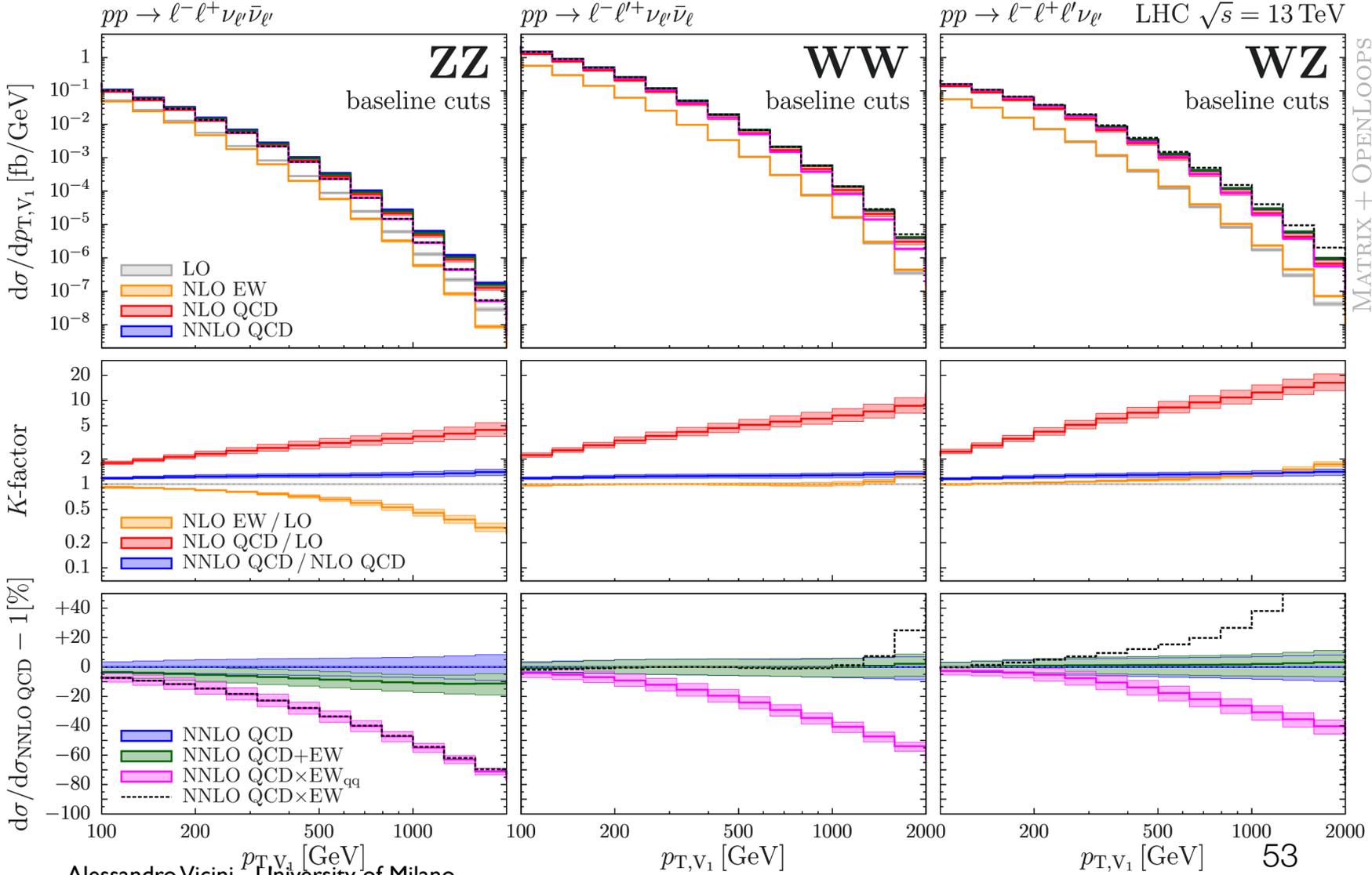
c_{00} does not contain QED logs, but Sudakov EW logs $c_{00} \propto -\frac{\alpha}{4\pi \sin^2 \theta_W} \log^2 \frac{s}{m_W^2}$

Diboson production: NNLO-QCD + NLO-EW corrections

M.Grazzini, S.Kallweit, J.Lindert, S.Pozzorini, M.Wiesemann, arXiv:1912.00068

- large QCD and EW corrections need a consistent combination to achieve O(1%) precision → Matrix+OpenLoops
 - comparison of additive vs multiplicative combinations of QCD and EW effects, to estimate mixed QCD-EW missing corrections
 - differences between 1) hard-hard boson regions and 2) (hard boson, hard jet, soft boson) regions
 - in 1) good convergence of the QCD expansion and factorisation of the EW Sudakov logs
 - in 2) “giant” K-factors, large EW Sudakov logs, large photon-induced contributions compete to the final result
- non-trivial estimate of the remaining uncertainties

jet-vetoes milden the “giant” K-factor and enhance the sensitivity to tri- and quadri-linear couplings



p_{T,V_1} is a “worst-case” observable stressing all potential issues

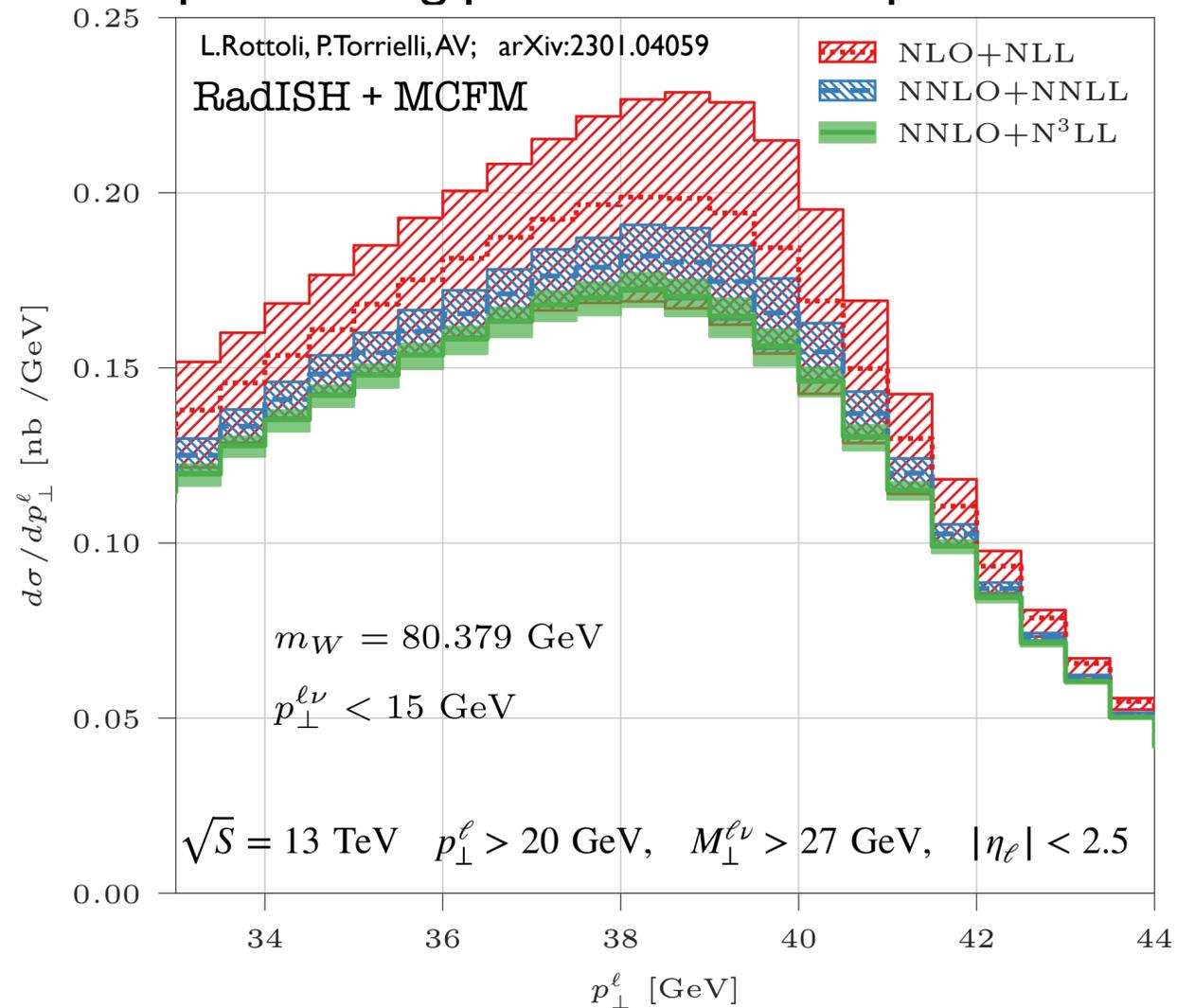
$$d\sigma_{NNLO QCD+EW} = d\sigma_{LO} (1 + \delta_{QCD} + \delta_{EW}) + d\sigma_{LO}^{gg}$$

$$d\sigma_{NNLO QCD \times EW} = d\sigma_{NNLO QCD+EW} + d\sigma_{LO} \delta_{QCD} \delta_{EW}$$

$$d\sigma_{NNLO QCD \times EW_{qq}} = d\sigma_{LO}^{q\bar{q}} (1 + \delta_{QCD}^{q\bar{q}}) (1 + \delta_{EW}^{q\bar{q}}) + d\sigma_{LO}^{\gamma\gamma} (1 + \delta_{EW}^{\gamma\gamma/q\gamma}) + d\sigma_{LO}^{gg}$$

Template fitting: description of the single lepton transverse momentum distribution

The template fitting procedure is acceptable if the data are described by the theoretical distribution with high quality

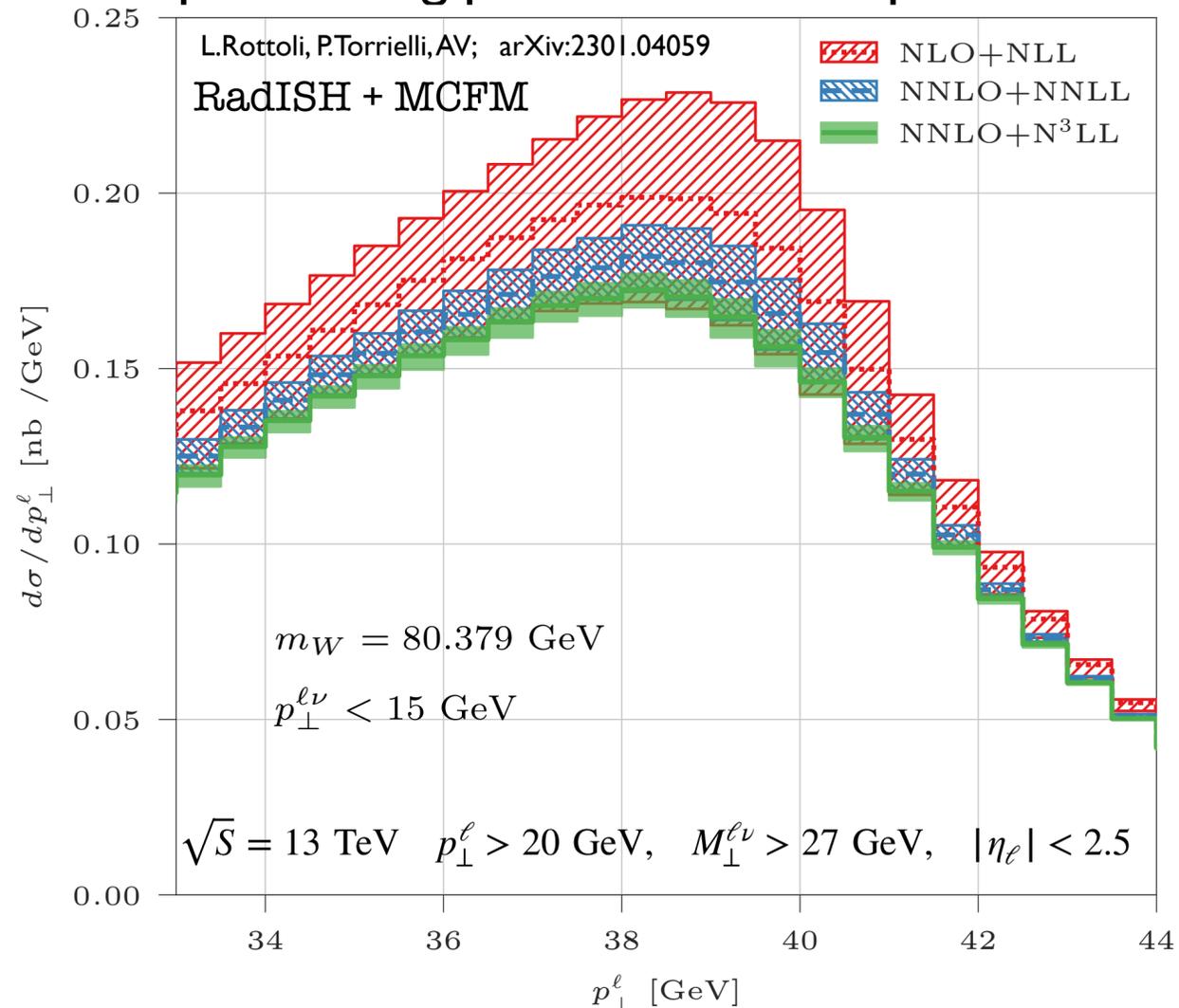


Scale variation of the NNLO+N³LL prediction for p_{\perp}^{ℓ} provides a set of equally good templates but the width of the uncertainty band is at the few percent level **a factor 10 larger** than the naive estimate would require !

- **data driven** approach
- a Monte Carlo event generator is tuned to the data in NCDY (p_{\perp}^Z)
- for one **QCD scale choice**
- ↓
- the same parameters are then used to prepare the CCDY templates

Template fitting: description of the single lepton transverse momentum distribution

The template fitting procedure is acceptable if the data are described by the theoretical distribution with high quality

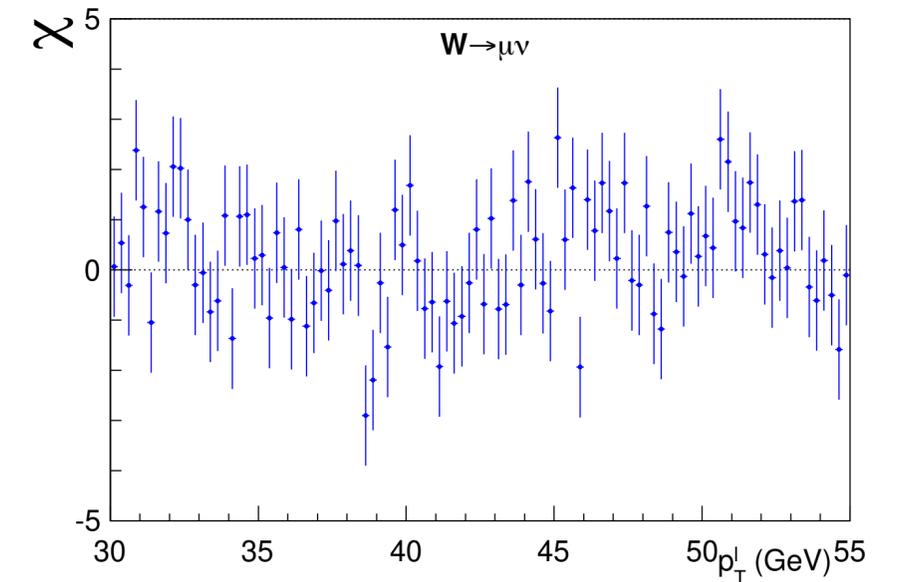
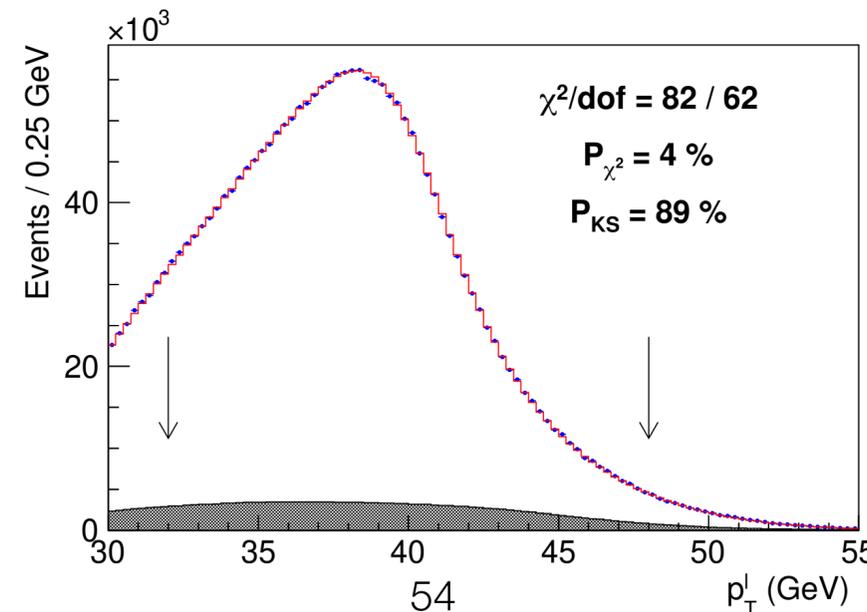
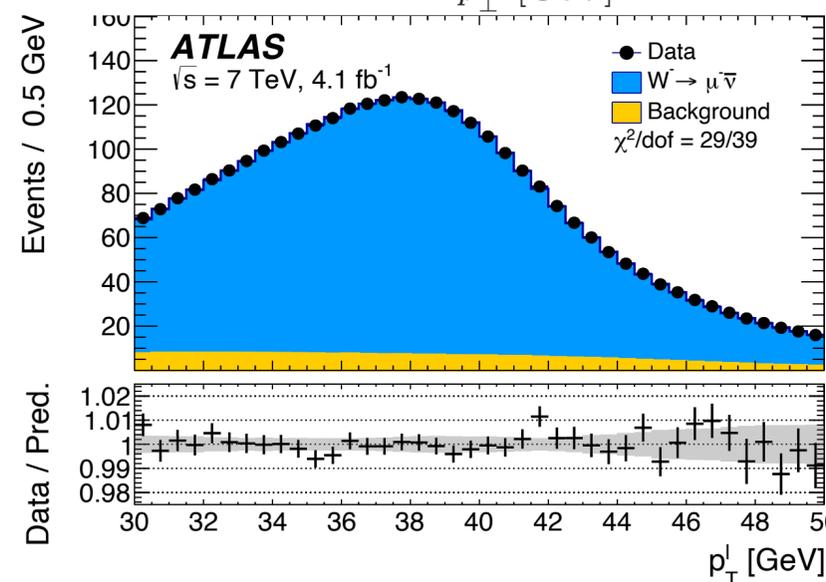


Scale variation of the NNLO+N³LL prediction for $p_{T\ell}$ provides a set of equally good templates but the width of the uncertainty band is at the few percent level **a factor 10 larger** than the naive estimate would require !

→ **data driven** approach
 a Monte Carlo event generator is tuned to the data in NCDY ($p_{T\ell}^Z$)
 for one **QCD scale choice**

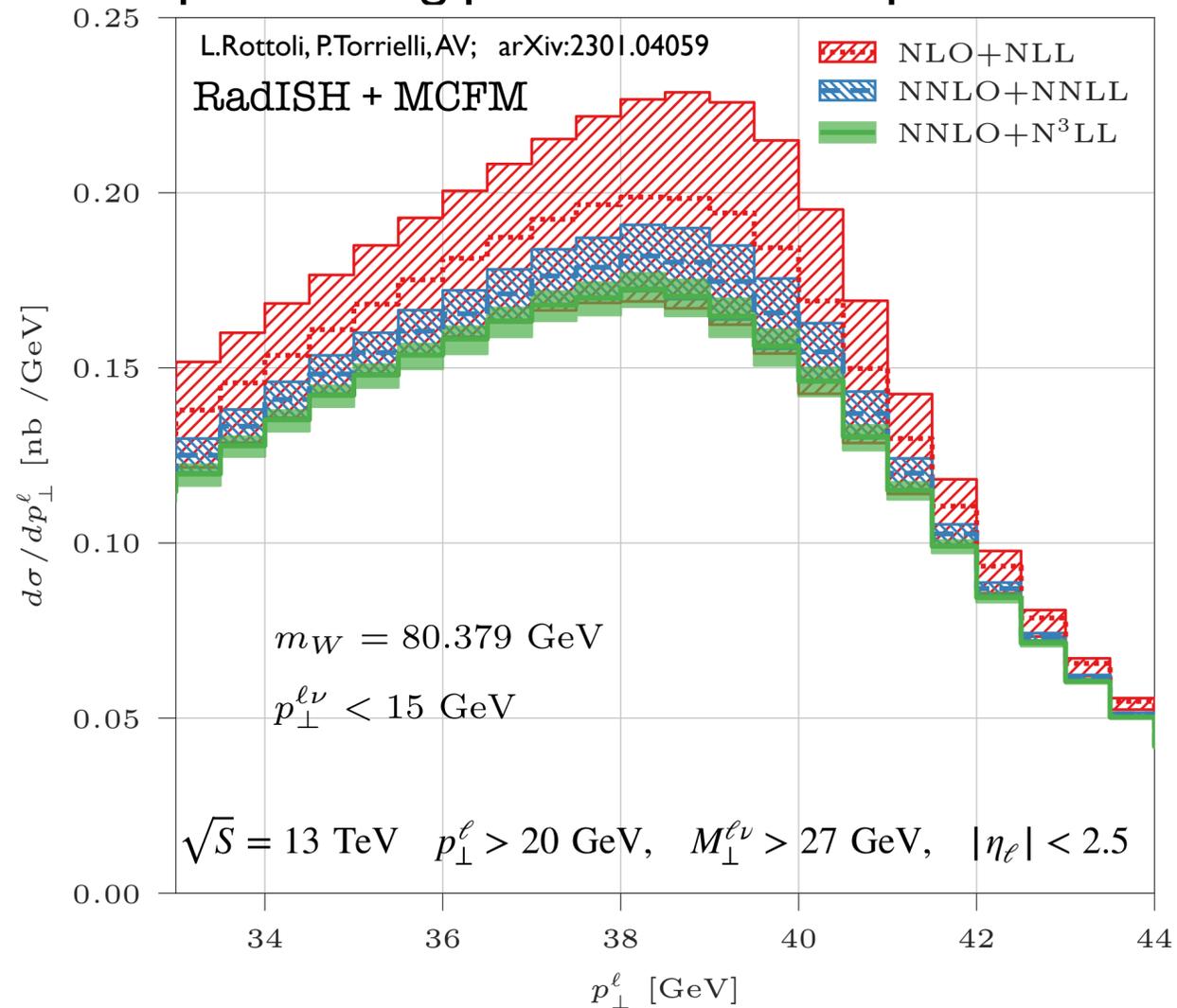
↓
 the same parameters are then used to prepare the CCDY templates

CDF collaboration, Science 376, 170-176 (2022)



Template fitting: description of the single lepton transverse momentum distribution

The template fitting procedure is acceptable if the data are described by the theoretical distribution with high quality



Scale variation of the NNLO+N³LL prediction for p_{\perp}^{ℓ} provides a set of equally good templates but the width of the uncertainty band is at the few percent level **a factor 10 larger** than the naive estimate would require !

→ **data driven** approach
 a Monte Carlo event generator is tuned to the data in NCDY (p_{\perp}^Z)
for one QCD scale choice

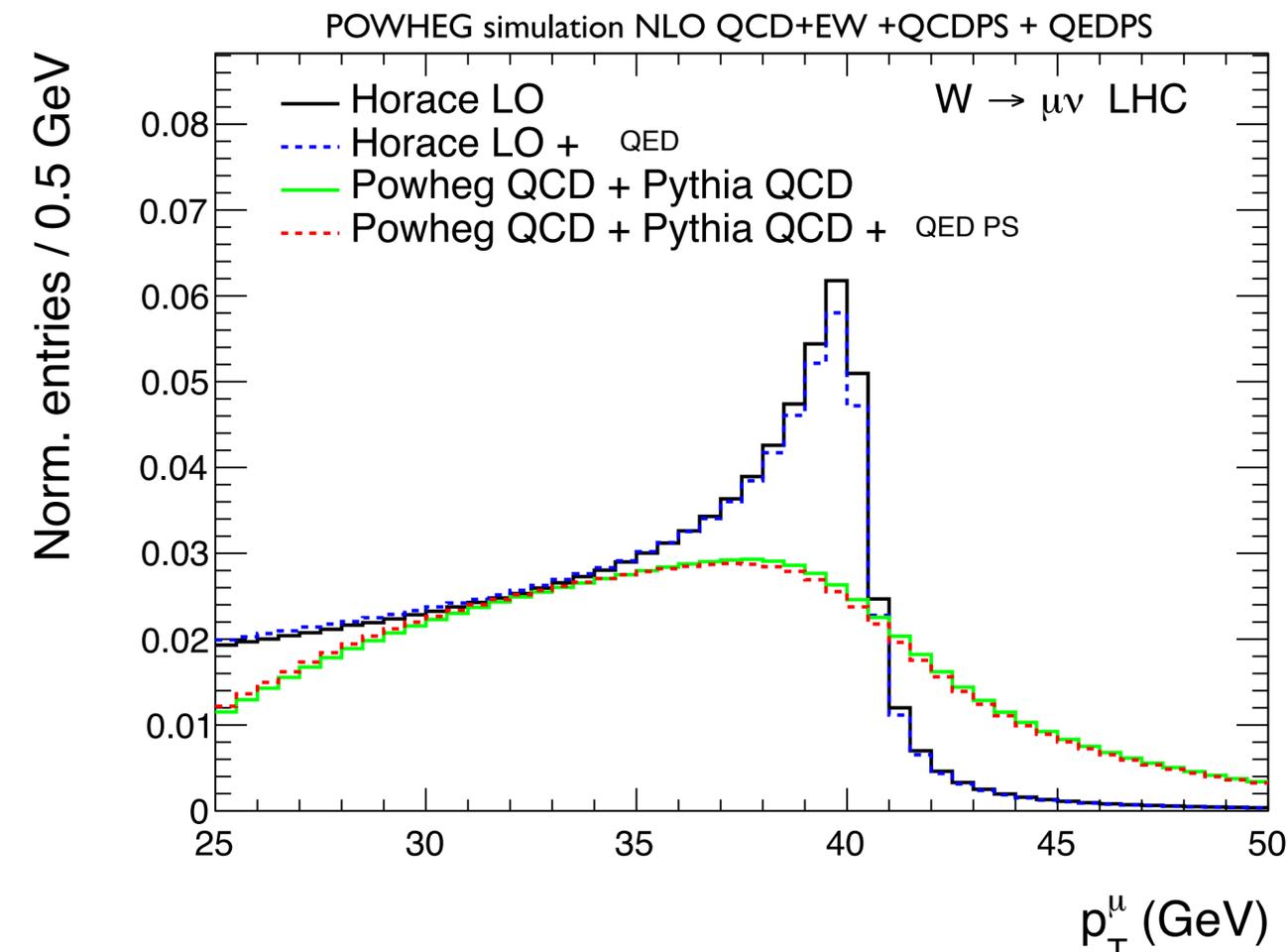
↓

the same parameters are then used to prepare the CCDY templates

A data driven approach improves the accuracy of the model (i.e. its ability to describe the data)
 does not improve the precision of the model (the intrinsic ambiguities in the model formulation)

What are the limitations of the transfer of information from NCDY to CCDY ?

Impact of mixed QCD-QED corrections in the m_W determination



Huge impact of **QED** and **mixed QCD-QED** corrections in the m_W determination

What is the theoretical uncertainty on this estimated shift ?

$pp \rightarrow W^+$, $\sqrt{s} = 14$ TeV			M_W shifts (MeV)			
Templates accuracy: NLO-QCD+QCD _{PS}			$W^+ \rightarrow \mu^+ \nu$		$W^+ \rightarrow e^+ \nu(\text{dres})$	
Pseudodata accuracy			M_T	p_T^ℓ	M_T	p_T^ℓ
		QED FSR				
1	NLO-QCD+(QCD+QED) _{PS}	PYTHIA	-95.2±0.6	-400±3	-38.0±0.6	-149±2
2	NLO-QCD+(QCD+QED) _{PS}	PHOTOS	-88.0±0.6	-368±2	-38.4±0.6	-150±3
3	NLO-(QCD+EW)+(QCD+QED) _{PS} two-rad	PYTHIA	-89.0±0.6	-371±3	-38.8±0.6	-157±3
4	NLO-(QCD+EW)+(QCD+QED) _{PS} two-rad	PHOTOS	-88.6±0.6	-370±3	-39.2±0.6	-159±2