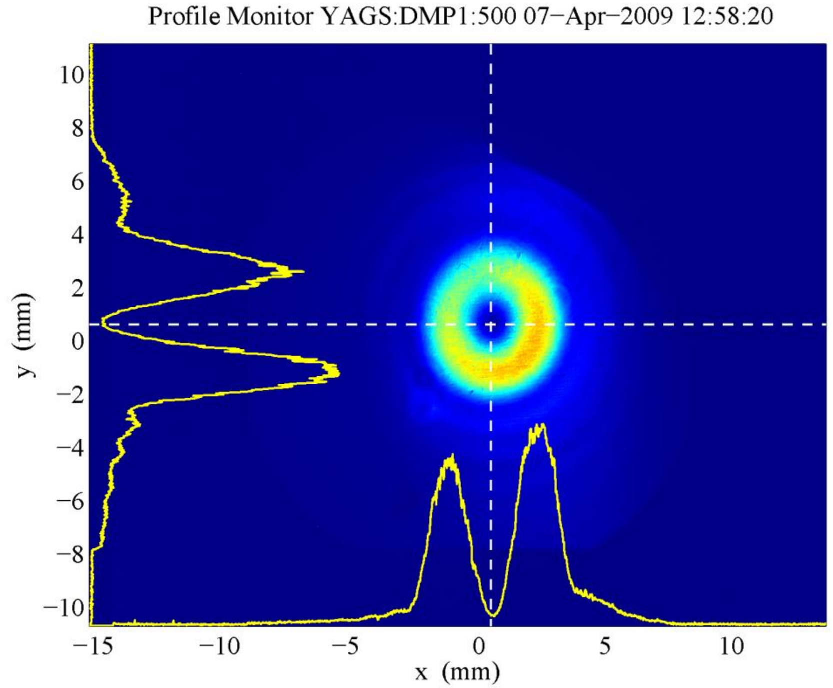
Coherent synchrotron radiation: theory and simulations.

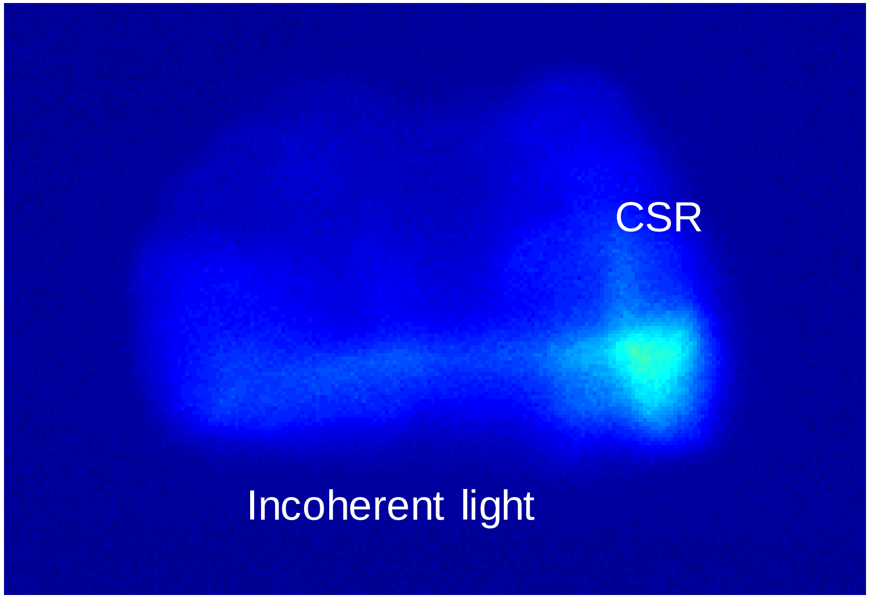
Sasha Novokhatski

ICFA Beam Dynamics Mini Workshop on
Low Emittance Rings 2011

October 3-5, 2011 Heraklion, Crete, Greece

CSR images





LCLS: CSR light on the YAG screen, when the electron bunch is bent down

Radiation from a horizontal bend

Theory of CSR

In 1949 J.Schwinger published a paper "On the Classical radiation of Accelerated Electrons" (*Phys. Rev. v. 75, Num 12, 1920),* where he presented his approximation for the spectrum of the synchrotron radiation for the circular trajectory of an electron

$$P(\omega)=\frac{3^{2/3}}{4\pi}\frac{e^2}{\rho}\left(\frac{E}{mc^2}\right)^4\frac{\omega_0\omega}{\omega_c^2}\int_{\omega/\omega_c}^{\infty}K_{5/3}(\eta)\,d\eta$$

introducing the critical frequency

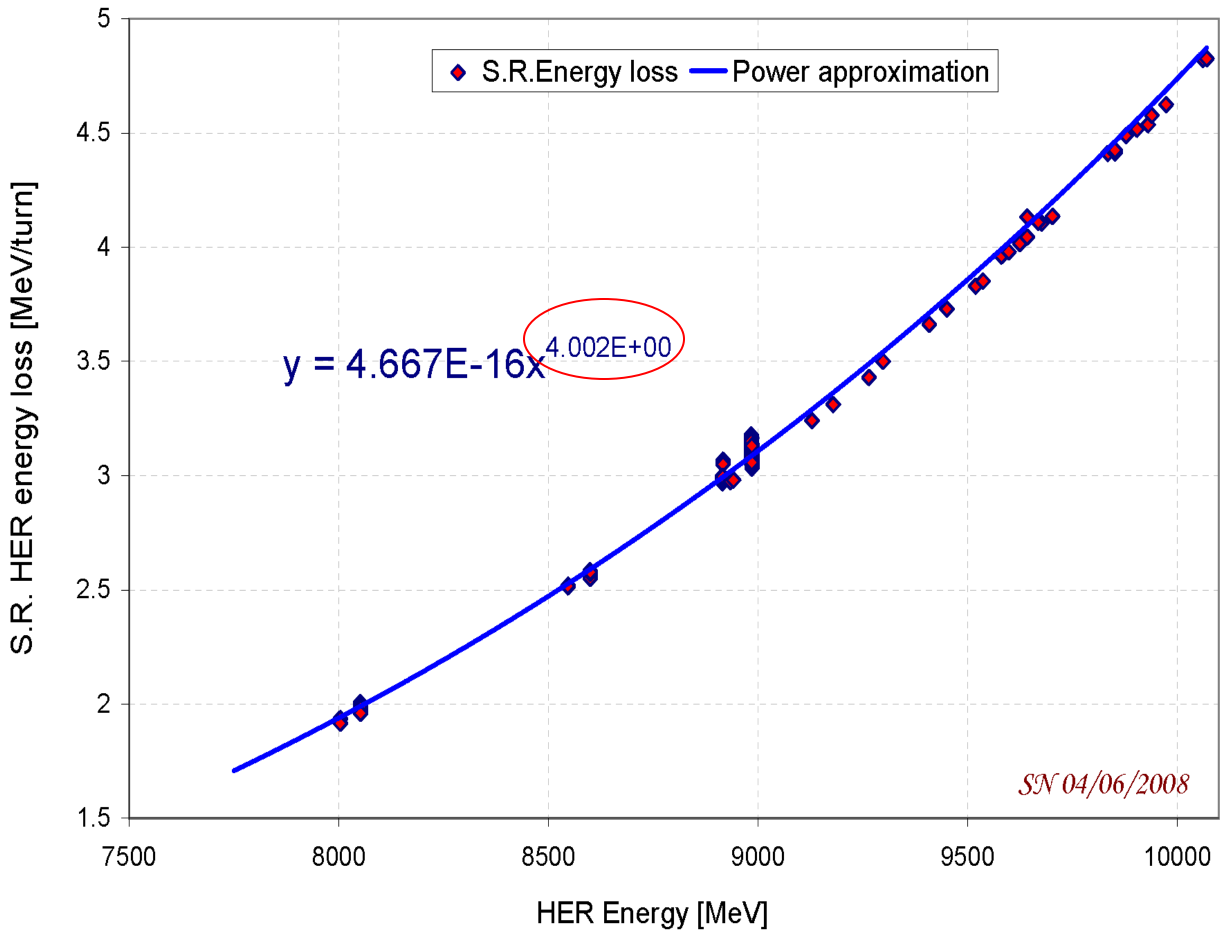
$$\omega_c=\frac{3}{2}\omega_0\left(\frac{E}{mc^2}\right)^3=\frac{3}{2}\frac{c}{\rho}\left(\frac{E}{mc^2}\right)^3$$

He verified that his approximation gives the total power

$$P(t)=\int_0^{\infty}P(\omega)\,d\omega=\frac{2}{3}\omega_0\frac{e^2}{\rho}\left(\frac{E}{mc^2}\right)^4$$

I had a chance to check E^4

Measurement at PEP-II HER during the energy scan in 2008



SR spectrum

For small frequencies power is independent of the electron energy

$$\omega << \omega_c$$

$$P(\omega) \approx \frac{3^{1/6}\Gamma\left(\frac{2}{3}\right)}{4\pi}\frac{e^2}{\rho}\left(\frac{\omega}{\omega_0}\right)^{1/3}$$

CSR-impedance ?

For high frequencies power exponentially decreases

$$\omega >> \omega_c$$

$$P(\omega) \approx \frac{3}{4}\left(\frac{3}{2\pi}\right)^{1/2}\frac{e^2}{\rho}\left(\frac{E}{mc^2}\right)^4\frac{\omega_0}{\omega_c}\left(\frac{\omega}{\omega_c}\right)^{1/2}\exp\left(-\frac{\omega}{\omega_c}\right)$$

Critical frequency and power loss

We know that a bunch of electrons with a finite length may coherently excite electromagnetic fields if their frequencies less than a bunch (critical) frequency

$$\omega << \omega_b = \frac{c}{\sigma}$$

Comparing the critical frequencies we may introduce an equivalent bunch length for SR fields

$$\omega_c = \omega_b \qquad \sigma_{SR} = \frac{3}{2} \frac{\rho}{\left(\frac{E}{mc^2}\right)^3}$$

Now we rewrite the formula for the SR power

$$P(t) = \frac{2}{3}\omega_0 \frac{e^2}{\rho}\left(\frac{E}{mc^2}\right)^4 = \frac{2}{3}\omega_0 \frac{Q^2}{\rho}\left(\frac{3}{2}\frac{\rho}{\sigma_{SE}}\right)^{4/3} = 2\cdot 3^{1/3}\frac{Q^2}{\rho^2}\left(\frac{\rho}{2\sigma_{SE}}\right)^{4/3}$$

Schiff, Nordvick, Saxon, Murphy, Derbenev, …

CSR shielding in 1954

PHYSICAL REVIEW VOLUME 96, NUMBER 1 OCTOBER 1, 1954

Suppression of Coherent Radiation by Electrons in a Synchrotron*

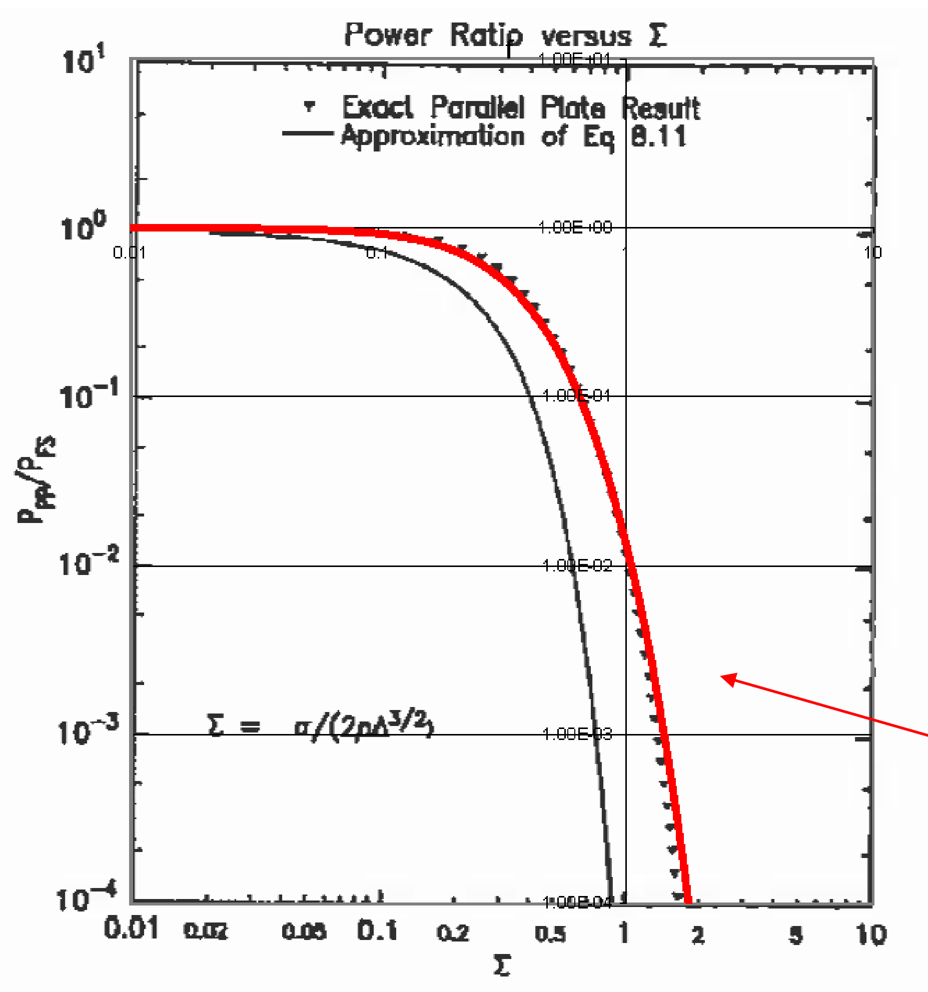
JOHN S. NODVICK† AND DAVID S. SAXON
University of California, Los Angeles, California
(Received May 25, 1954)

$$K_{pp}(\rho,h,\sigma)=\frac{2}{3}\frac{Z_0c}{\pi\rho}\times\frac{\rho}{h}\sum_{n=1}\left(\frac{\sin n\sqrt{3}\frac{\sigma}{\rho}}{n\sqrt{3}\frac{\sigma}{\rho}}\right)^2\times\sum_{j=1,3,..}^{\gamma_j<n}\frac{\gamma_j^4}{n^3}\times\left[K_{1/3}^2\left(\frac{\gamma_j^3}{3n^2}\right)+K_{2/3}^2\left(\frac{\gamma_j^3}{3n^2}\right)\right]$$

$$\gamma_j=j\pi\frac{\rho}{h}$$

Murphy, Warnock, …

It can be a very strong shielding



LONGITUDINAL WAKEFIELD FOR AN ELECTRON MOVING ON A CIRCULAR ORBIT

J.B. MURPHY [a,*], S. KRINSKY [a] and R.L. GLUCKSTERN [b]

[a] *National Synchrotron Light Source, Brookhaven National Laboratory, Upton, NY 11973, USA;* [b] *Physics Department, University of Maryland, College Park, MD 20742, USA*

(Received 6 May 1996; Revised 28 January 1997; In final form 10 March 1997)

$$\tilde{\Sigma} = 2\sqrt{12} \times \Sigma = \frac{\sqrt{12}\sigma}{h}\sqrt{\frac{\rho}{h}}$$

$$\frac{P_{pp}}{P_{FS}} = \frac{\tilde{\Sigma}}{\sinh(\tilde{\Sigma})}$$

hyperbolic sine

CSR simulations

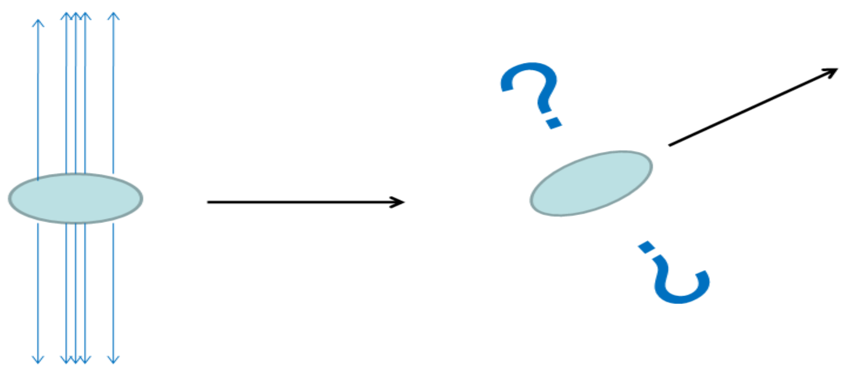
- R. Li. Nucl. Instrum. Meth. Phys. Res. A, 429, 310, 1998.
- G. Bassi et al., Nuc. Instrum. Methods Phys. Res. A, 557, pp. 189–204 (2006).
- M. Borland, Phys. Rev. ST Accel. Beams 4, 070701 (2001).
- G.V. Stupakov and I. A. Kotelnikov, Phys. Rev. ST Accel. Beams, 12, 104401 (2009).
- T.Agoh and K.Yokoya, Phys. Rev. ST Accel. Beams, 7, 054403 (2004).
- …

Time-domain presentation

A bunch and radiation are moving together for a long time. How do they separate? A chamber wall cuts the field?

A bunch retarding causes the field radiation?

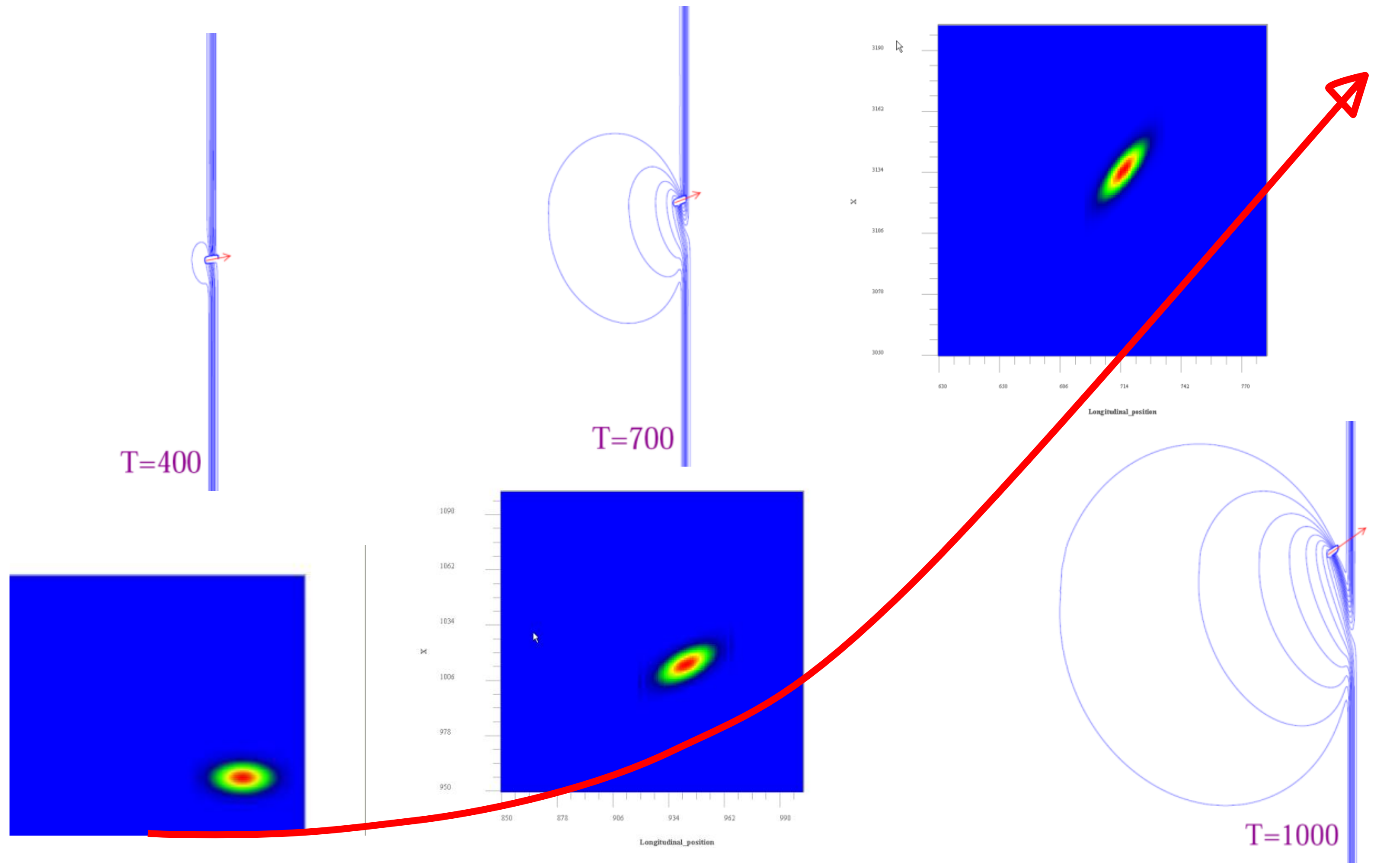
How does the bunch field change
when the bunch has been rotated in a magnetic field?



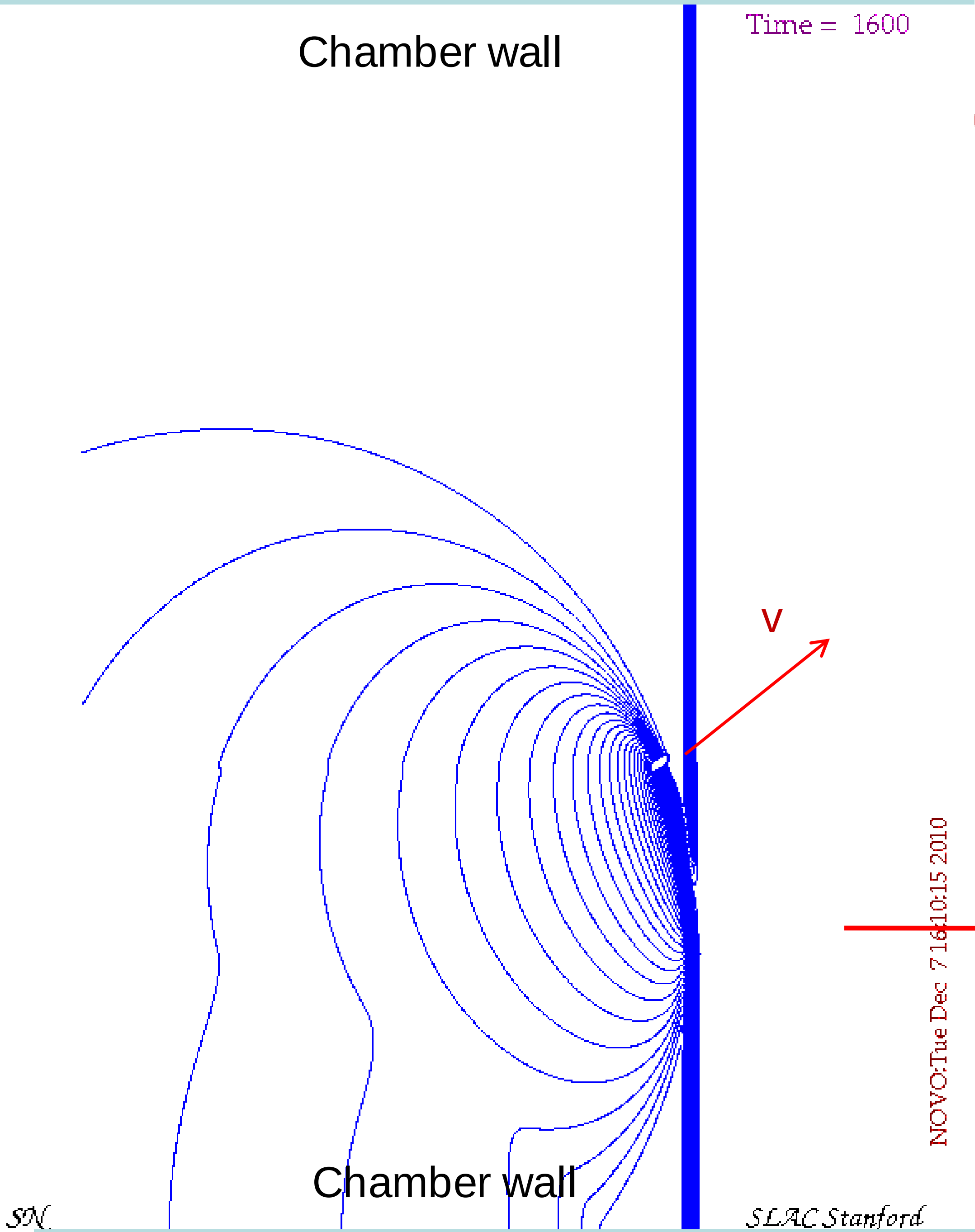
Recently a new method was developed at SLAC

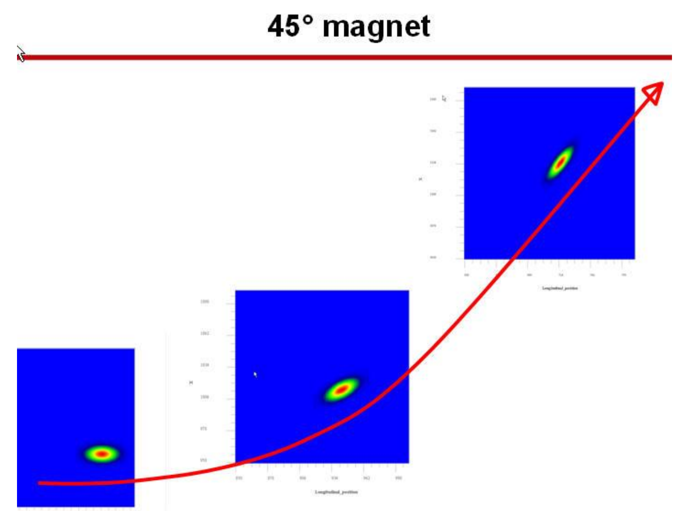
- The method is based on an implicit scheme for solving the electromagnetic equations, Maxwell's equations.
- This algorithm is free of frequency dispersion effects which means that all propagating waves will have their natural phase velocity, completely independent of simulation parameters like mesh size or time step.
- Other known methods, usually explicit, have "mesh driven" dispersion and because of this they need a much smaller mesh size which slows down calculations and can sometimes cause unstable solutions.
- An implicit scheme is a self-consistent method that allows us to calculate fields of much shorter bunches.

First results: Field dynamics in a magnet

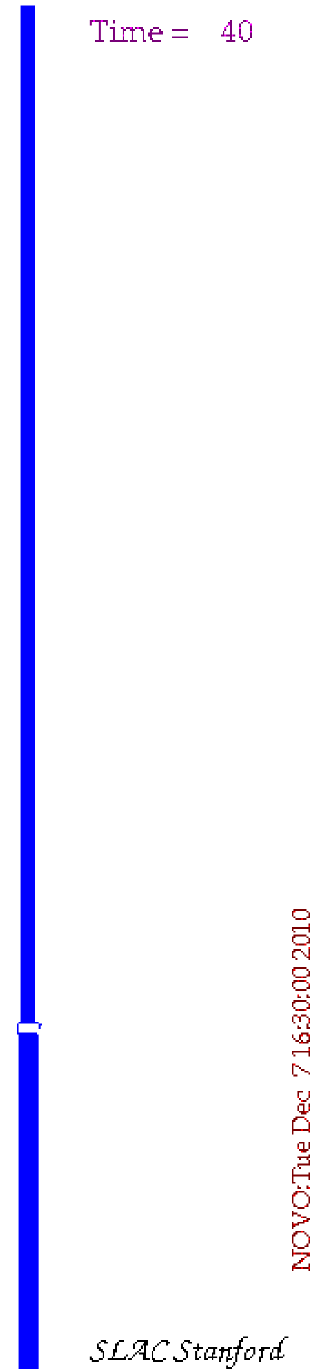


Movie show





Initial direction



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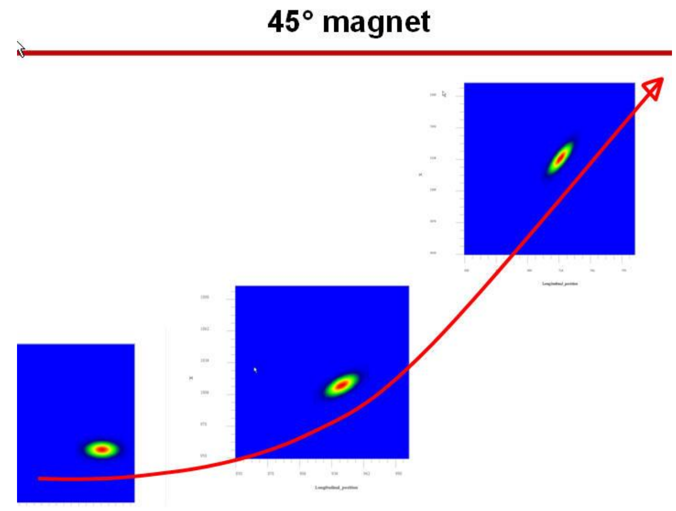
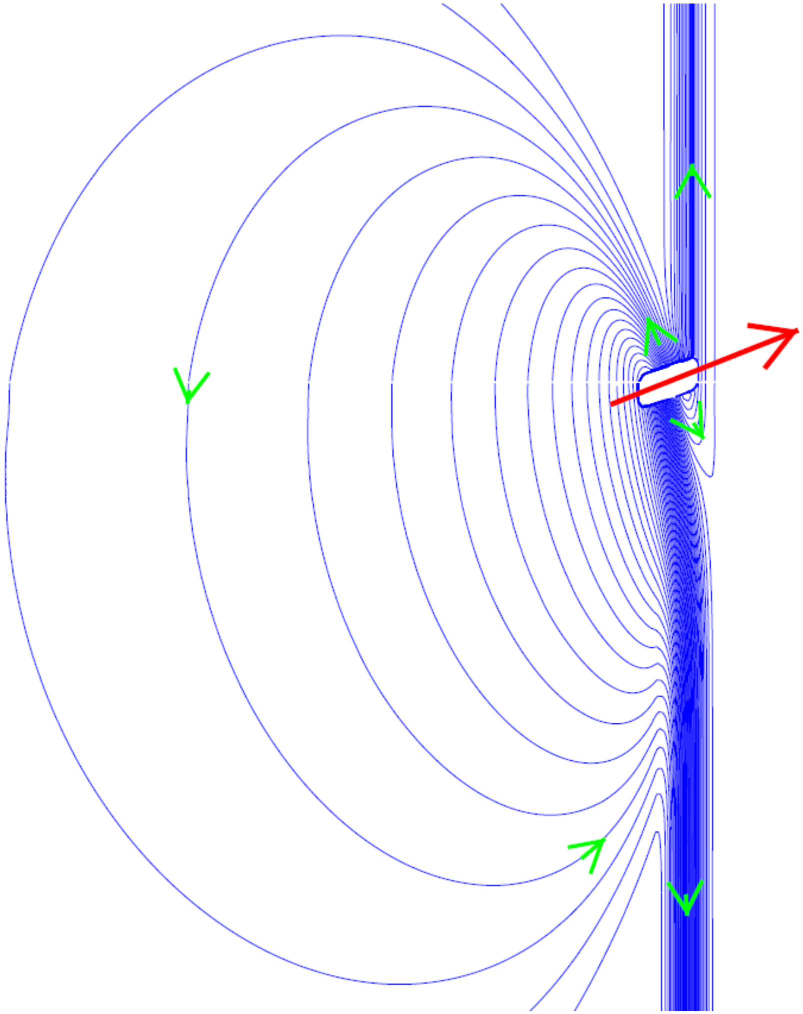
Movie show



Initial direction

Bunch self-field remakes itself moving in a magnetic field



The upper field lines take the position of the lower lines

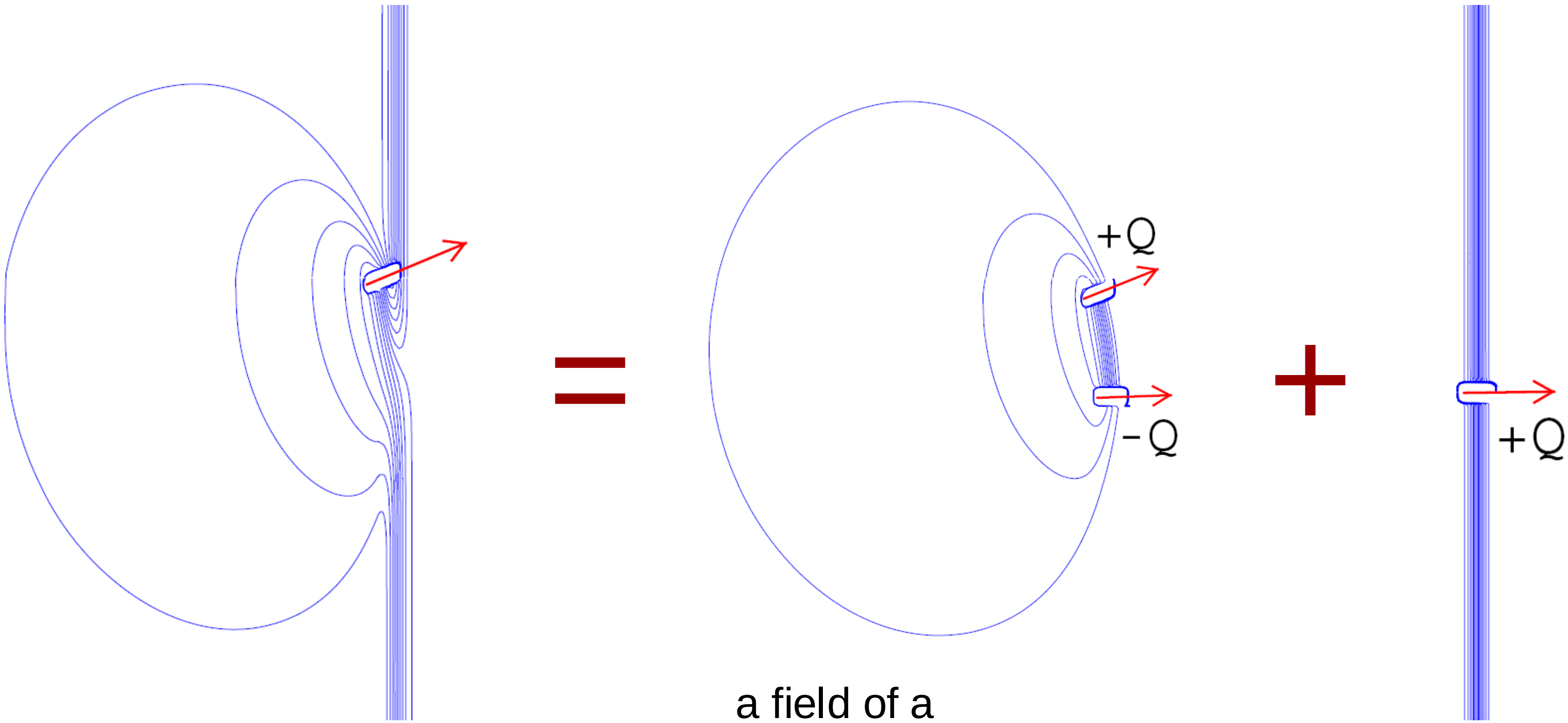
The white box shows the bunch location

The red arrow shows the bunch velocity vector

Green arrows show field line directions

The lower field lines take the place of the upper lines

The picture becomes clear if we decompose the field

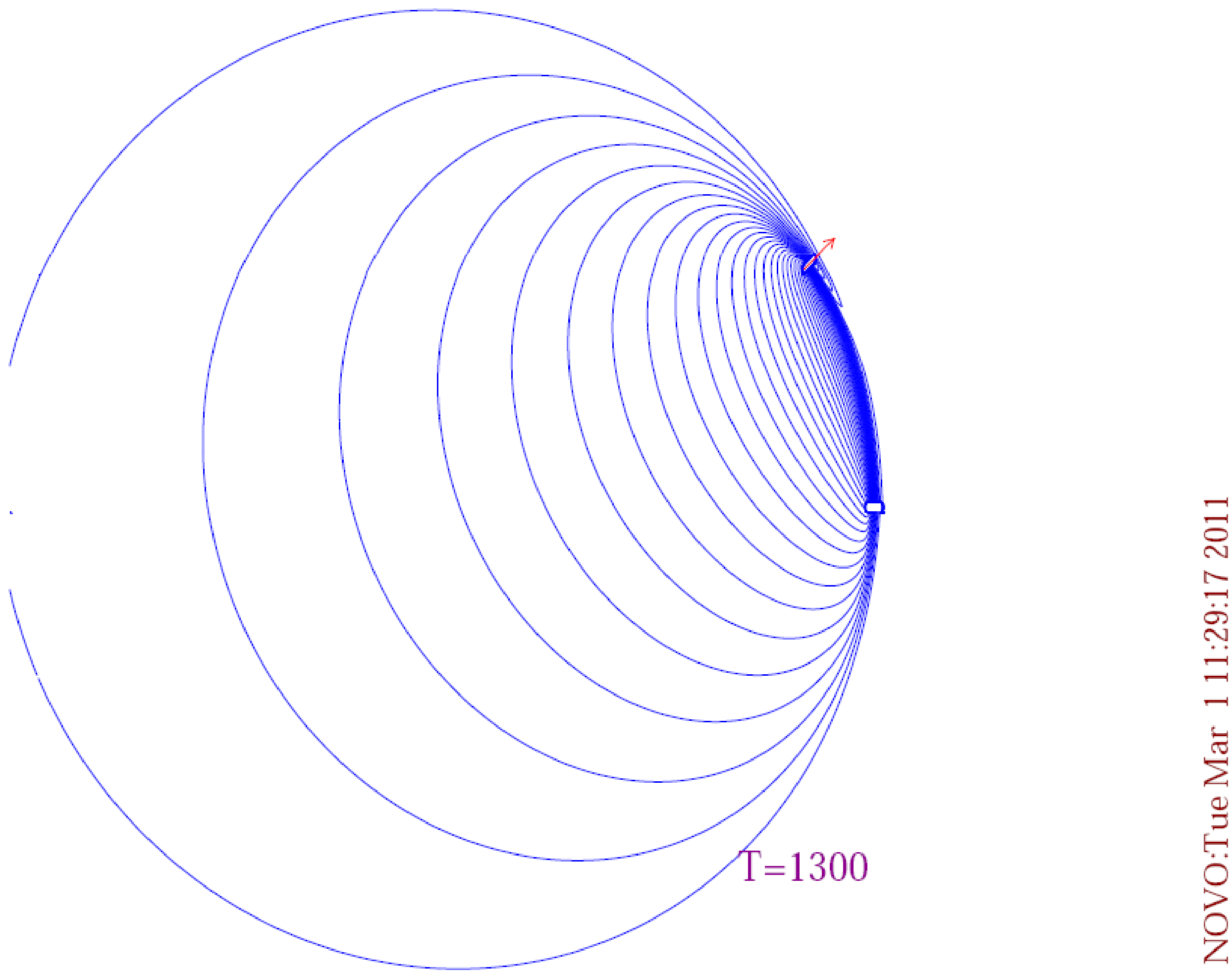


a field of a moving dipole

Decomposition of the field of a bunch moving in a magnetic field into two fields:

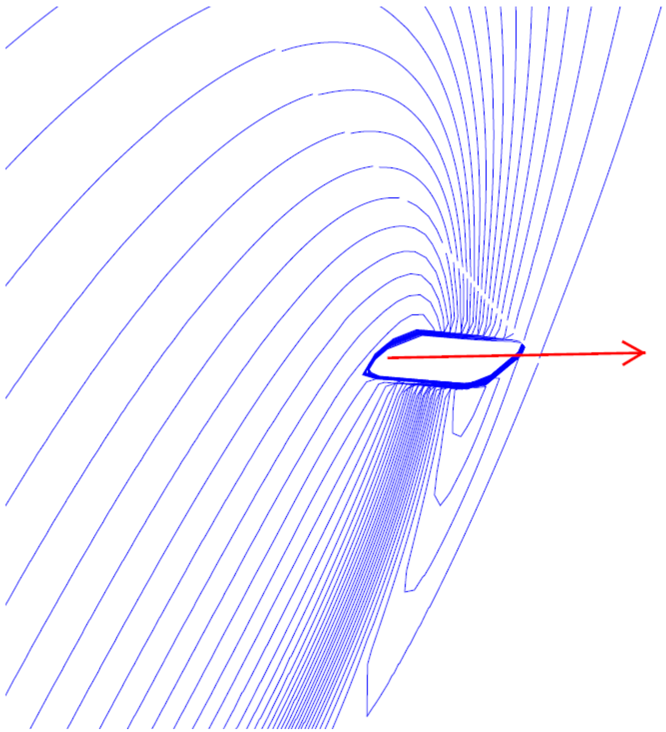
a field of a bunch straight in initial direction

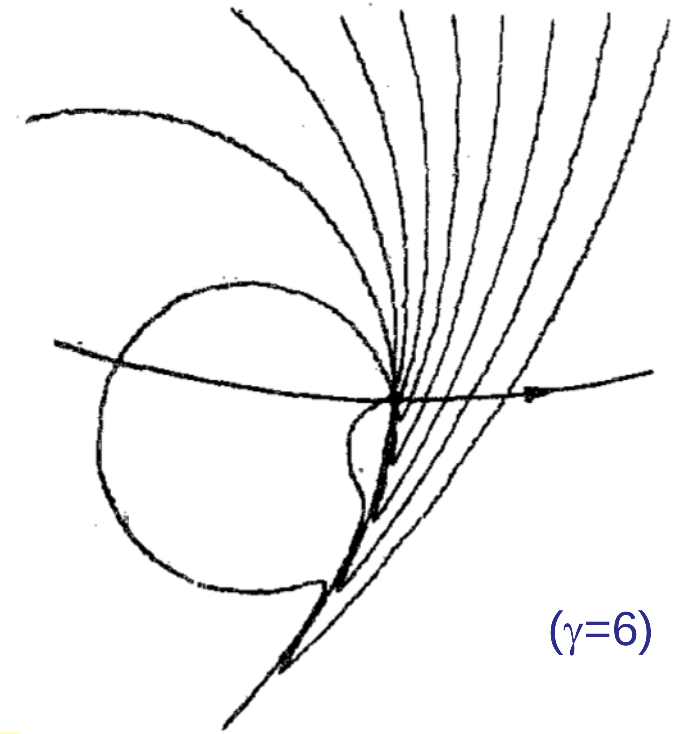
Detailed plot of a dipole field



Comparison with a classical synchrotron radiation

γ - region in front of a particle





FORCE LINES OF ELECTRIC AND MAGNETIC FIELDS
OF AN ARBITRARILY MOVING CHARGE

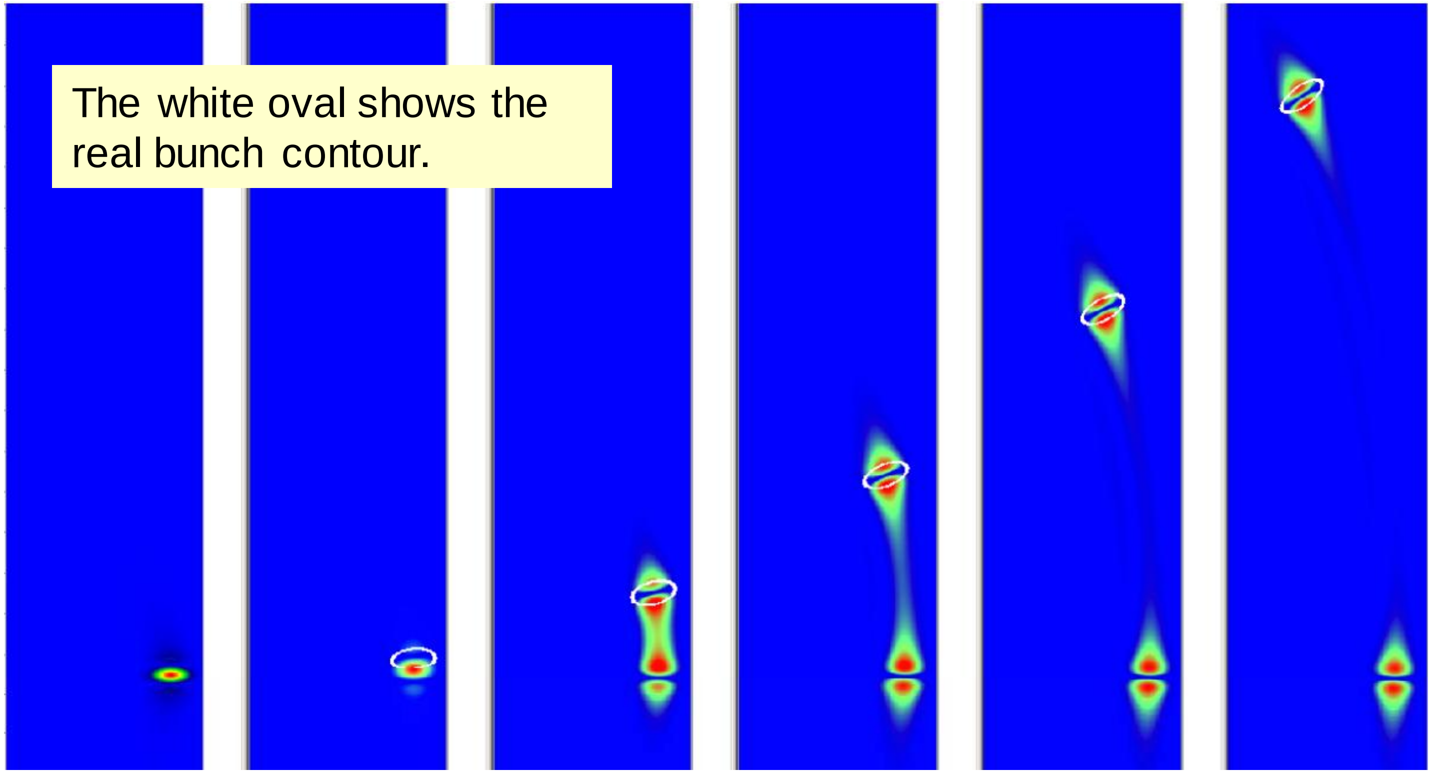
S. G. Arutyunyan

equivalent bunch length

for a bending radius ρ

$$\sigma \approx \frac{2}{3}\frac{\rho}{\gamma^3}$$

An absolute dipole electric field in time



When a dipole is created an electric field appears between a real bunch and a virtual bunch. This field increases in value and reaches a maximum value when the bunches are completely separated and then it goes down as the bunches move apart leaving fields only around the bunches.

Electrical forces inside a bunch

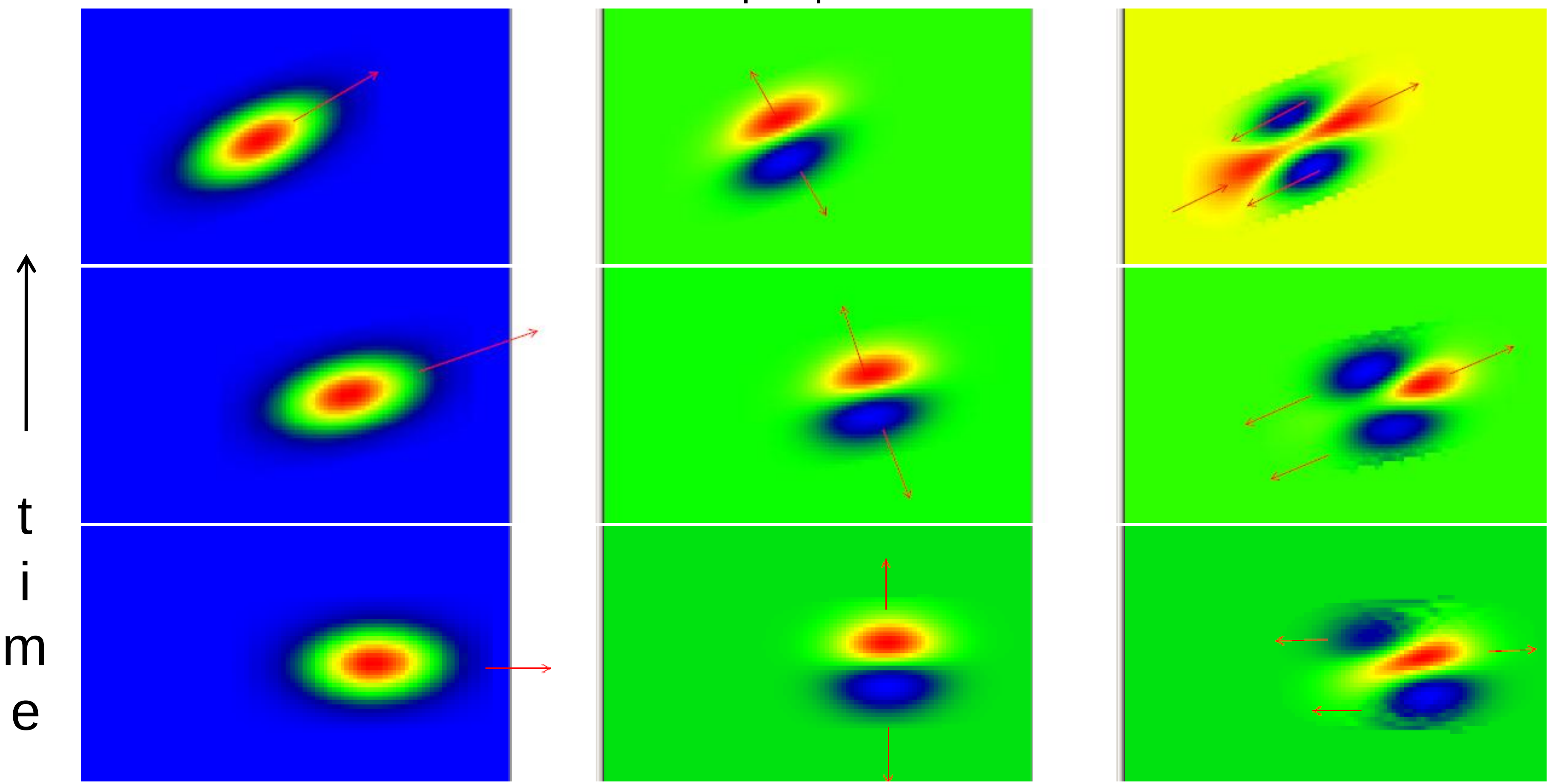
Bunch shape

transverse force

$$\vec{F}_{\perp} = \frac{\vec{J}}{|\vec{J}|} \times \left[\vec{E} \times \vec{J} \right]$$

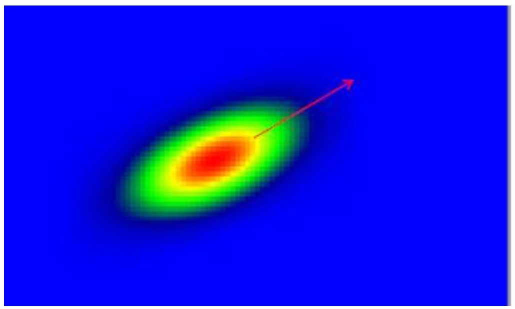
collinear force

$$F_{\parallel}^{e} = \left(\vec{J} \cdot \vec{E} \right)$$



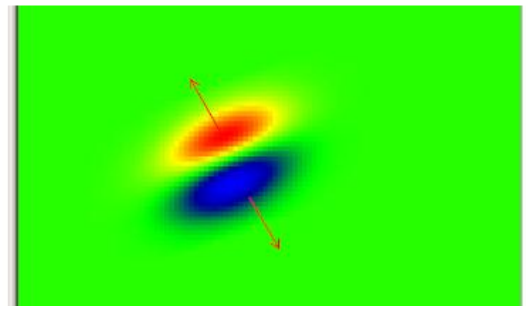
Electrical forces inside a bunch

Bunch shape



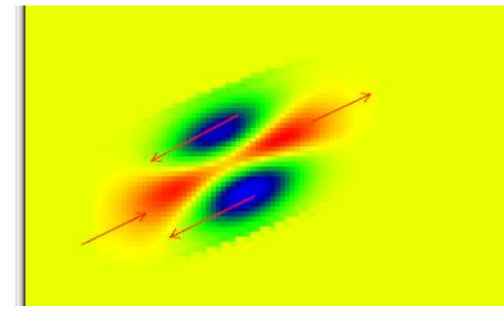
transverse force

$$\vec{F}_{\perp} = \frac{\vec{J}}{|\vec{J}|} \times \left[\vec{E} \times \vec{J} \right]$$



collinear force

$$F_{\parallel}^{e} = \left(\vec{J} \cdot \vec{E} \right)$$

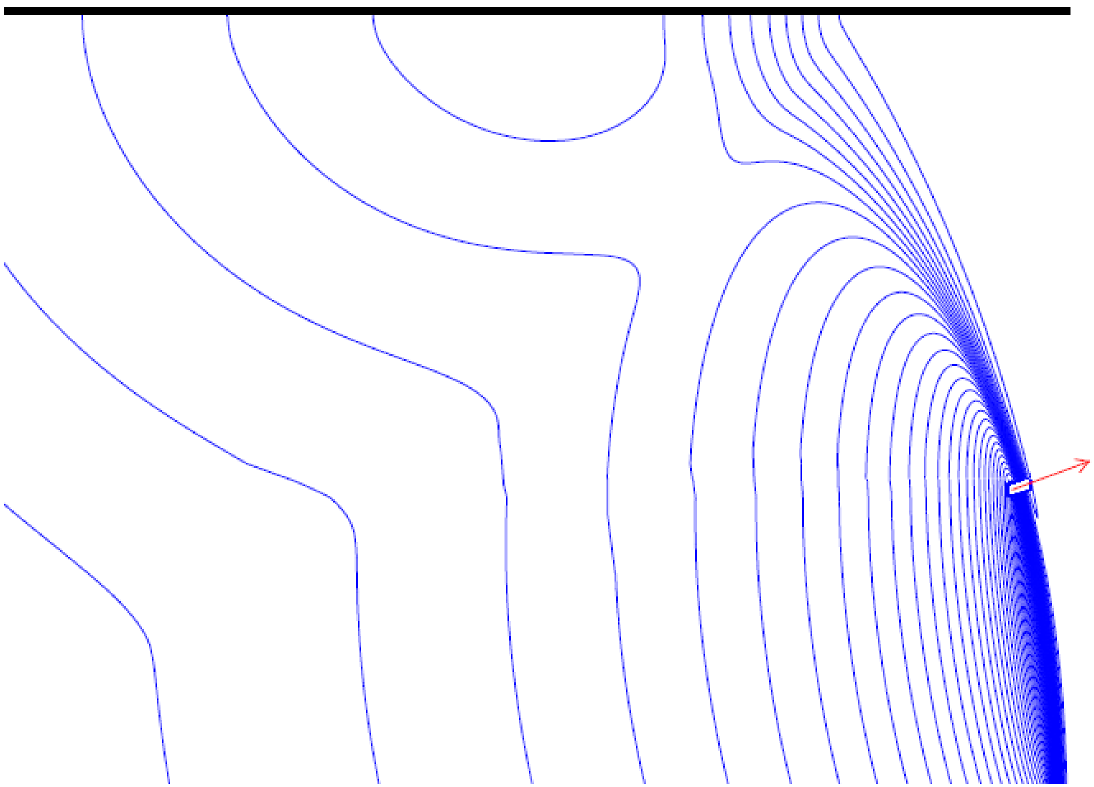


The transverse force is the well known space-charge force, which probably is compensated by a magnetic force in the ultra-relativistic case.

The collinear force is responsible for an energy gain or an energy loss. The particles, which are in the center, in front and at the end of the bunch are accelerating, whereas the particles at the boundaries are decelerating.

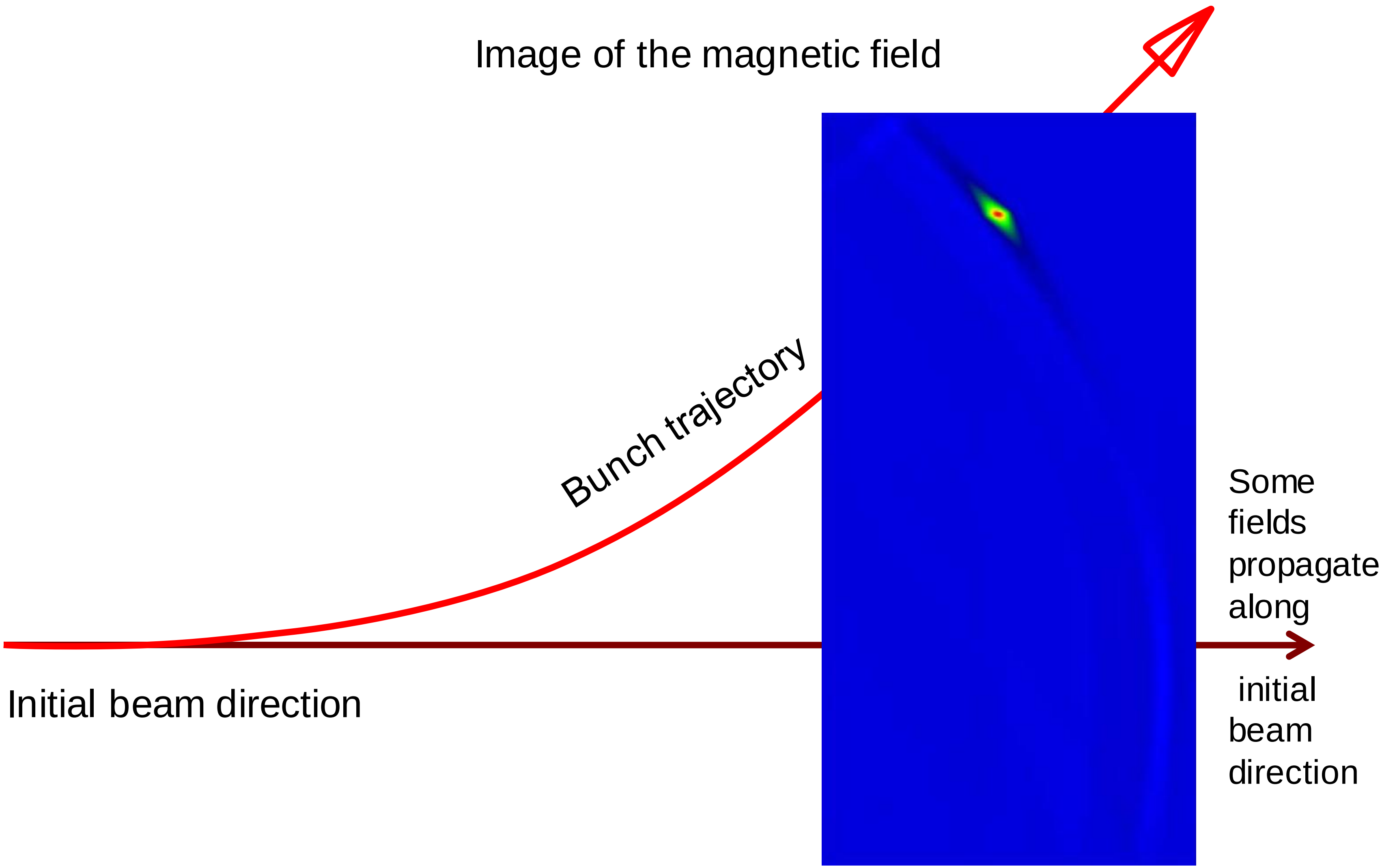
The total effect is deceleration and the bunch loses energy, however the bunch gets an additional energy spread in the transverse direction.

A new bunch field

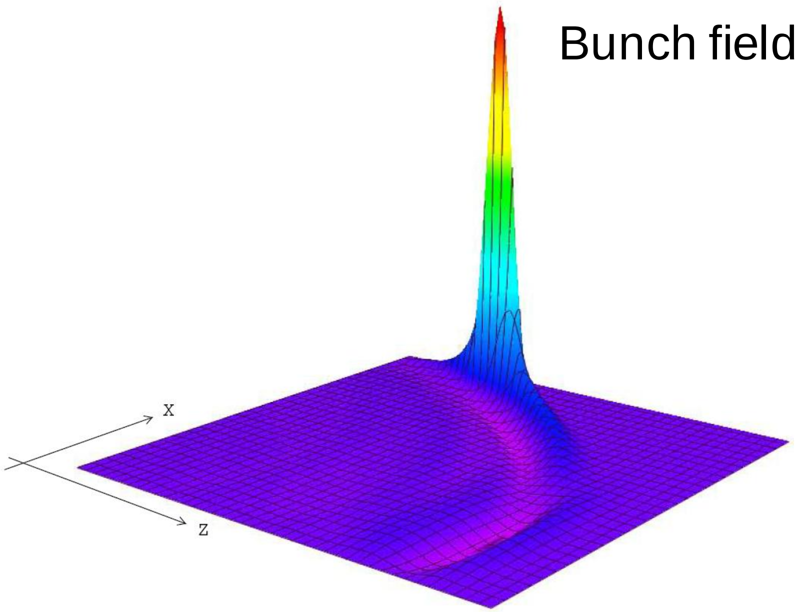


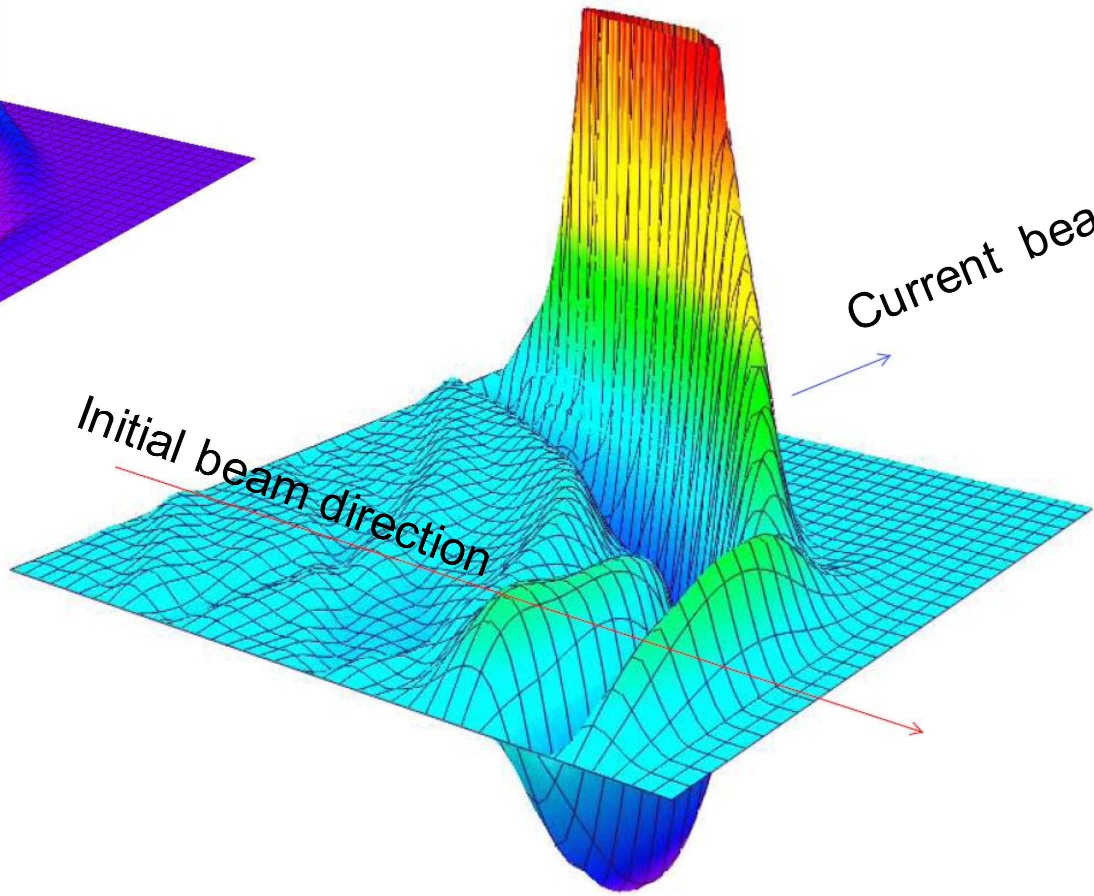
A long way to a steady-state regime

Coherent edge radiation



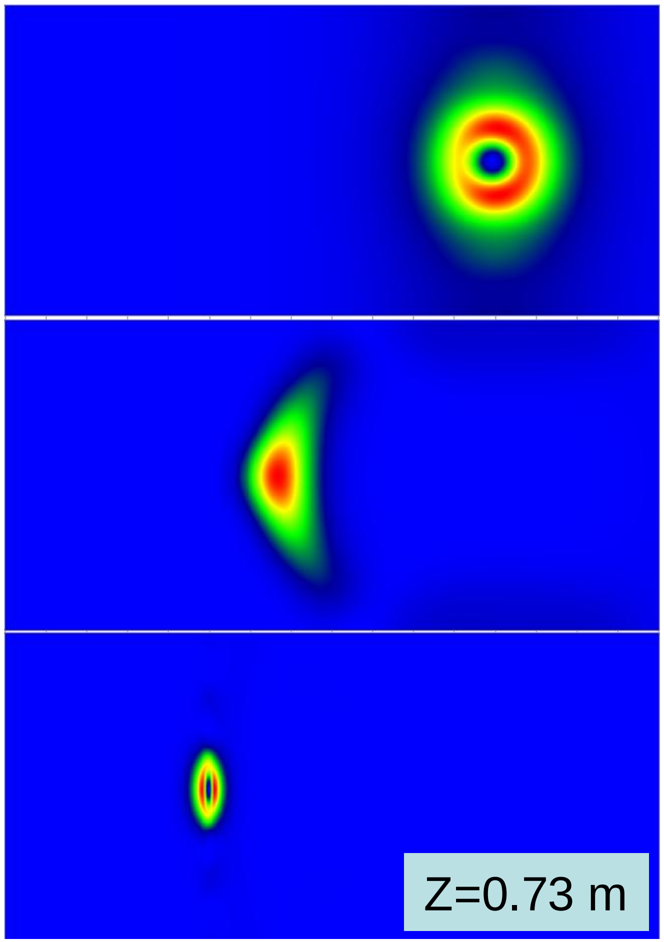
Magnetic field plots





Images of radiation (transverse magnetic field)

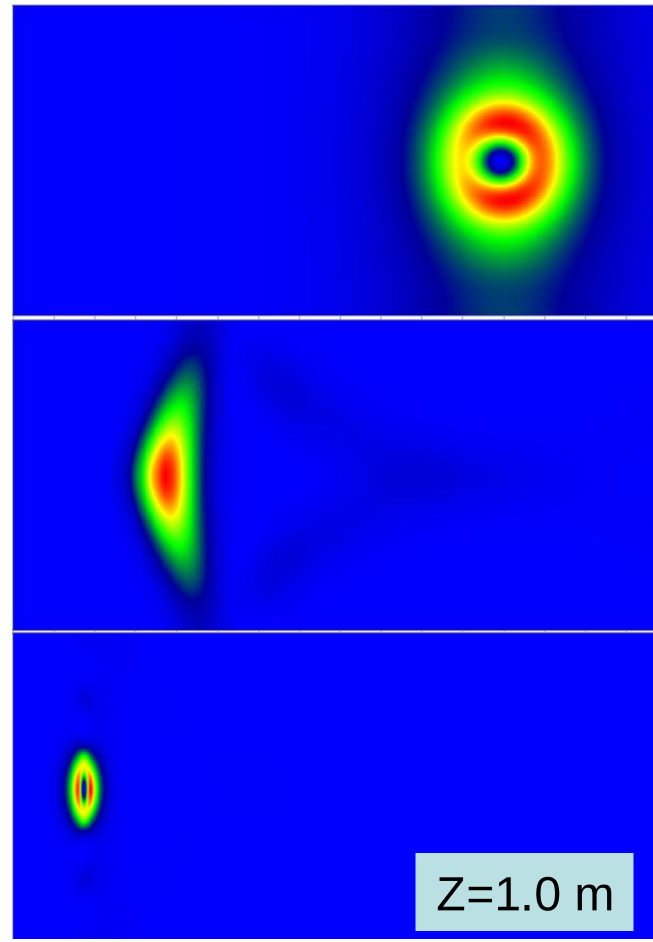
very similar to the images, which we have seen on the YAG screen after the dump magnets, which bend down the beam at LCLS.



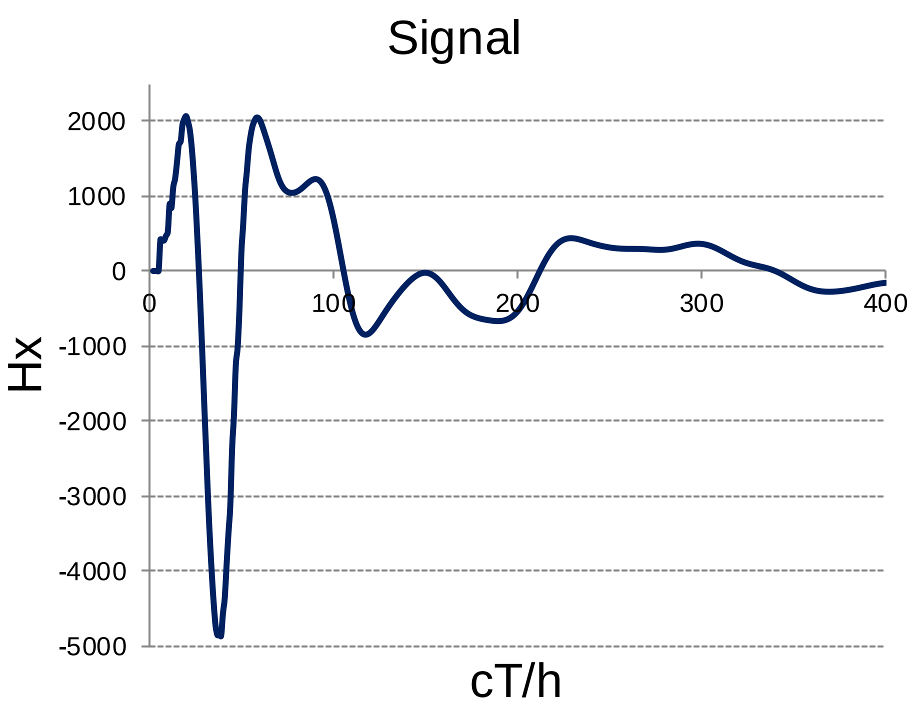
Edge radiation

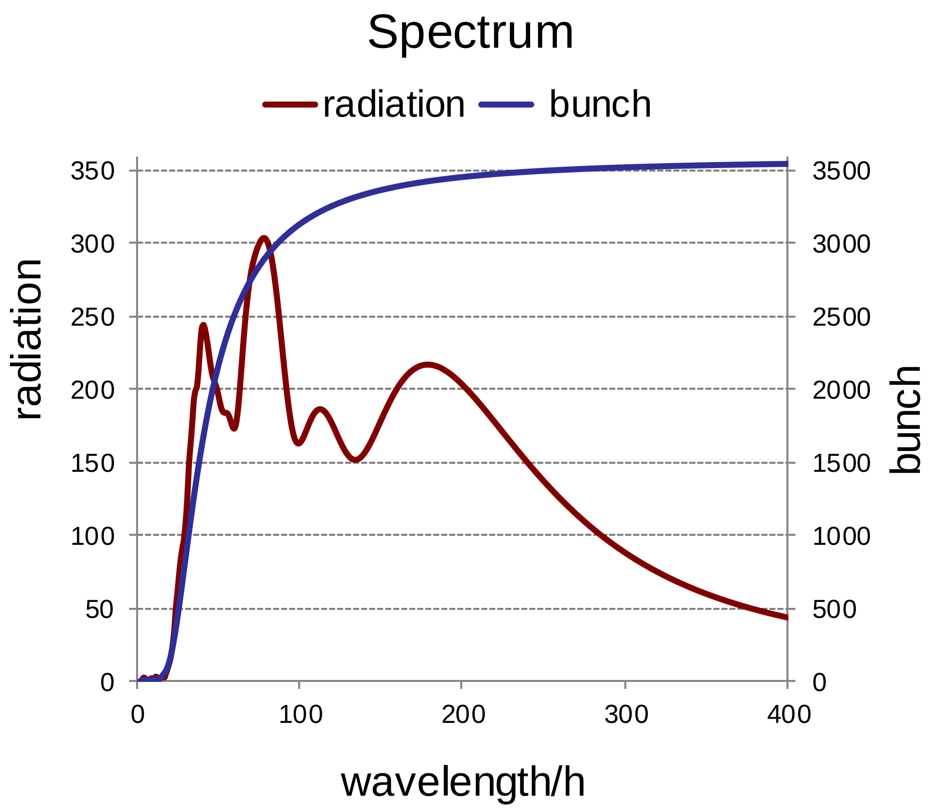
Synchrotron radiation

Bunch field



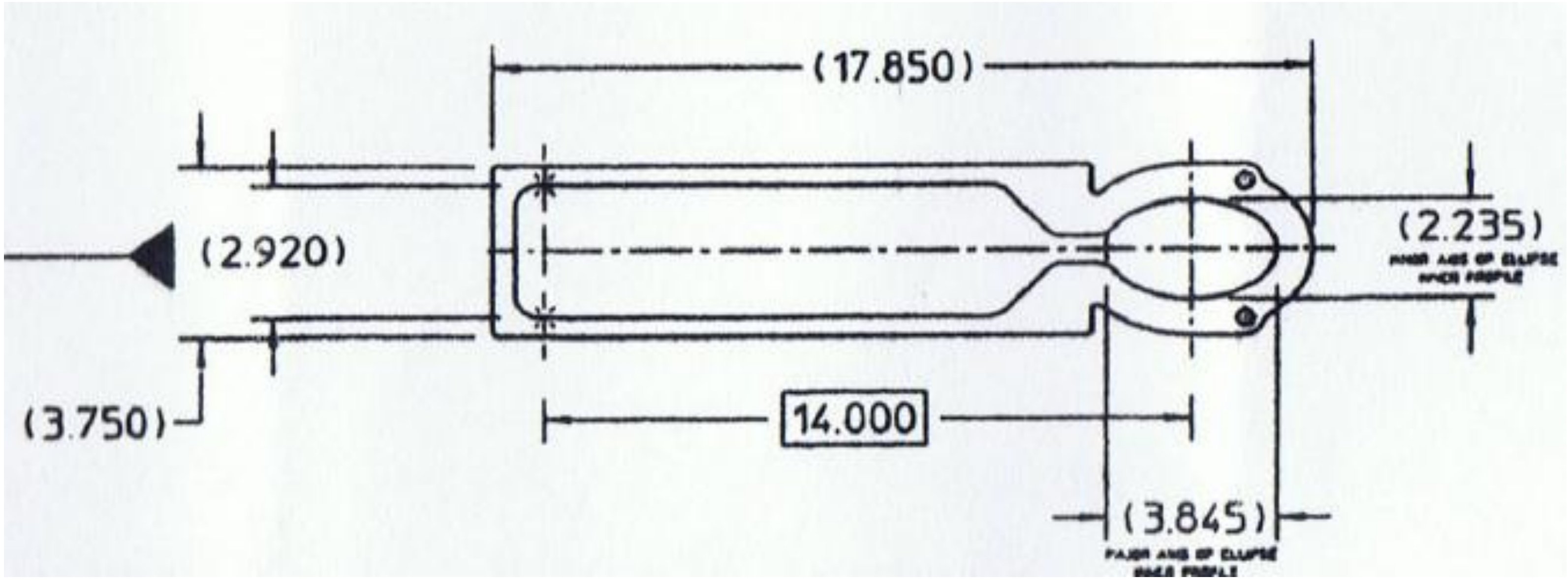
Signal and spectrum of edge radiation



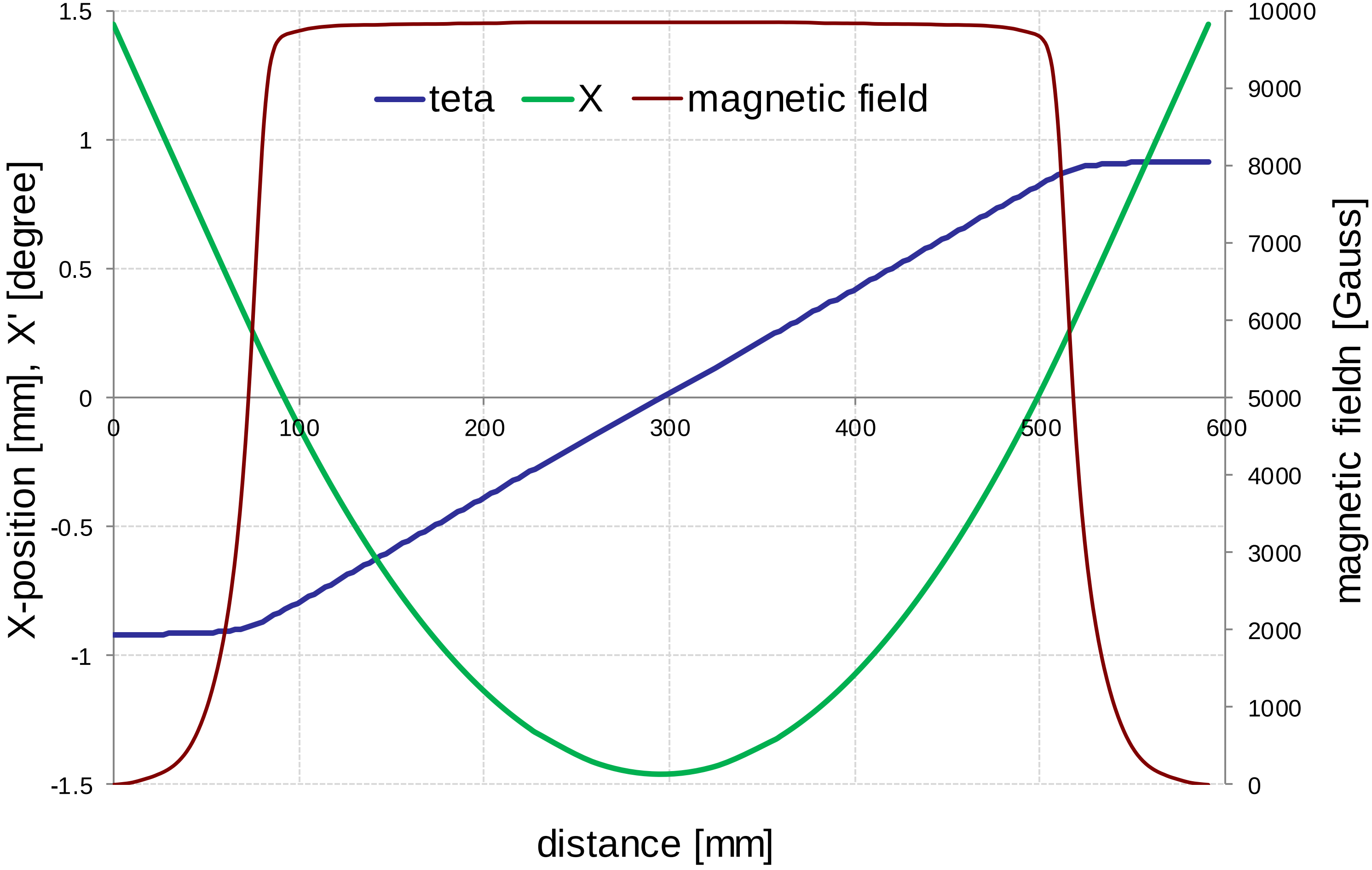


First peak in spectrum corresponds to and is a little bit less than the bunch wavelength = 2πσ

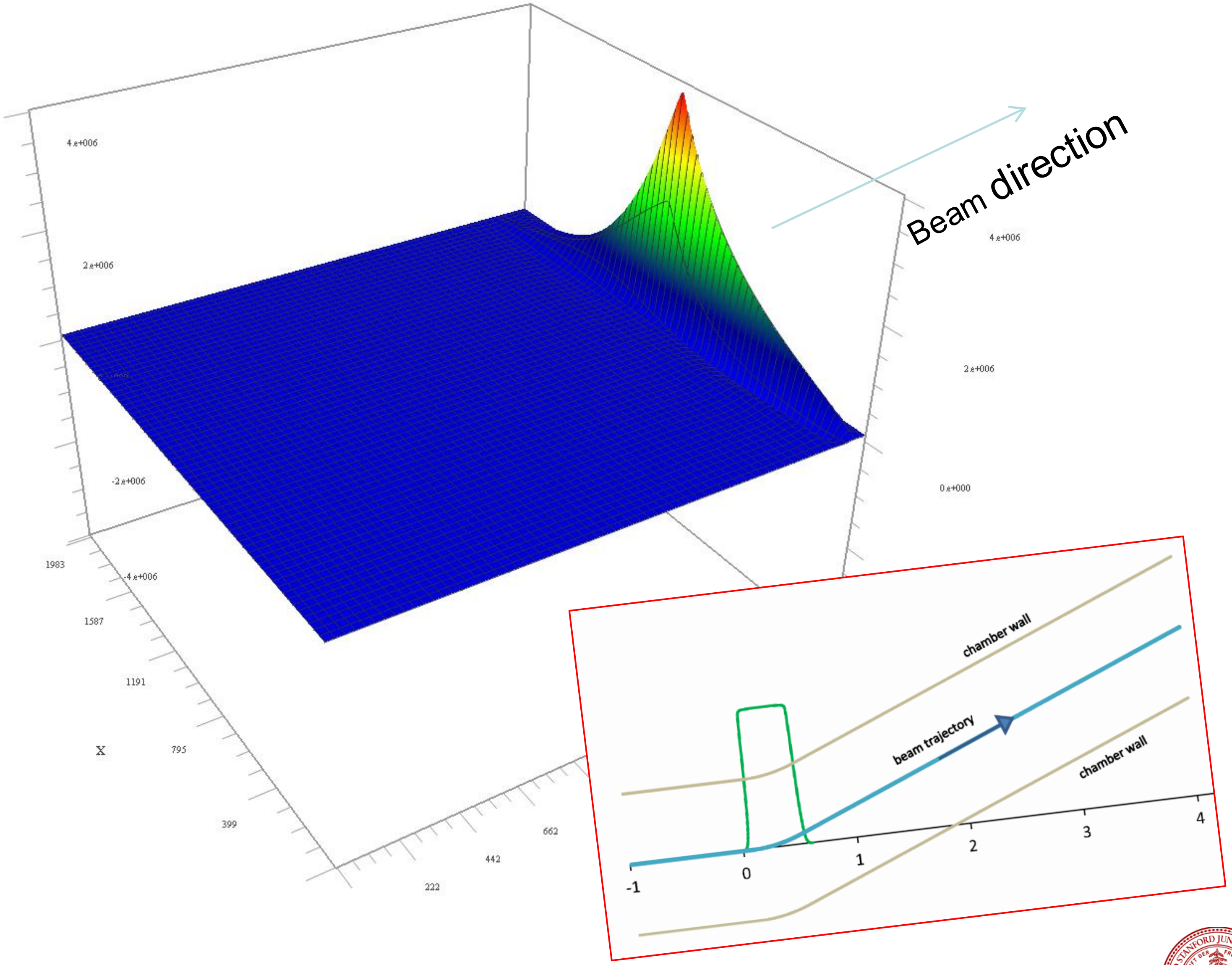
PEP-II LER pumping chamber



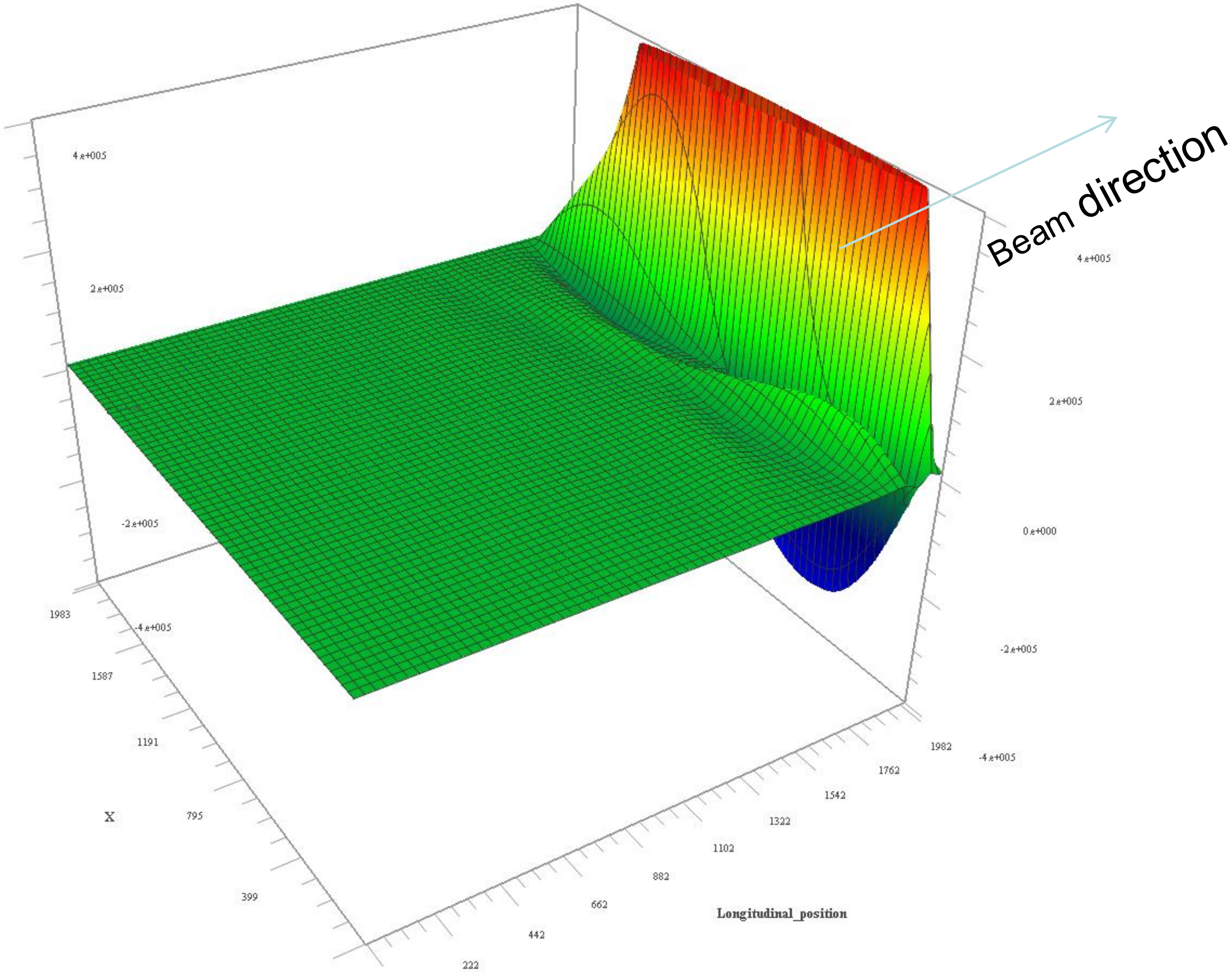
PEP-II LER dipole (a model)
X beam position and magnetic field



Beam and chamber fields

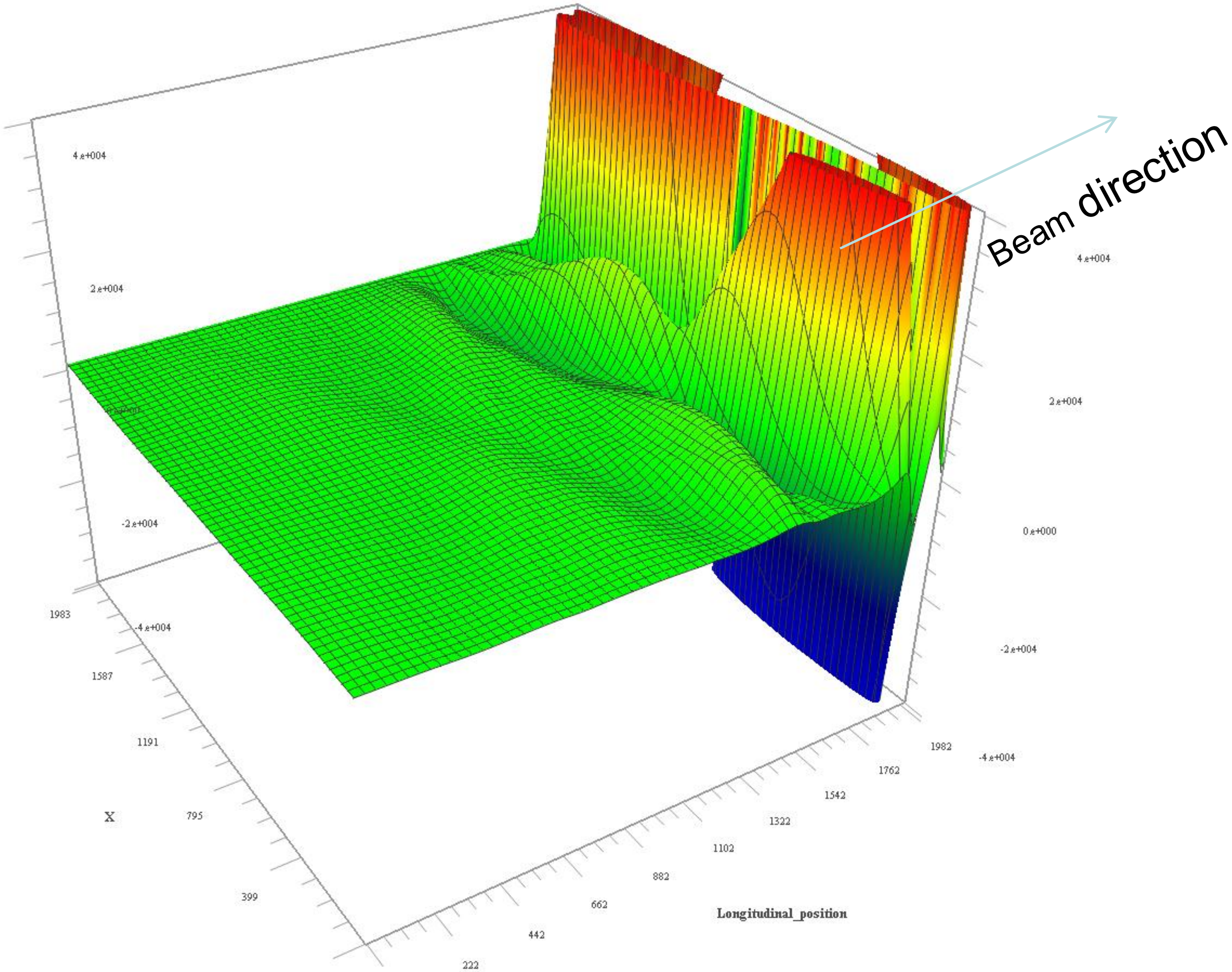


Magnified by 10 times

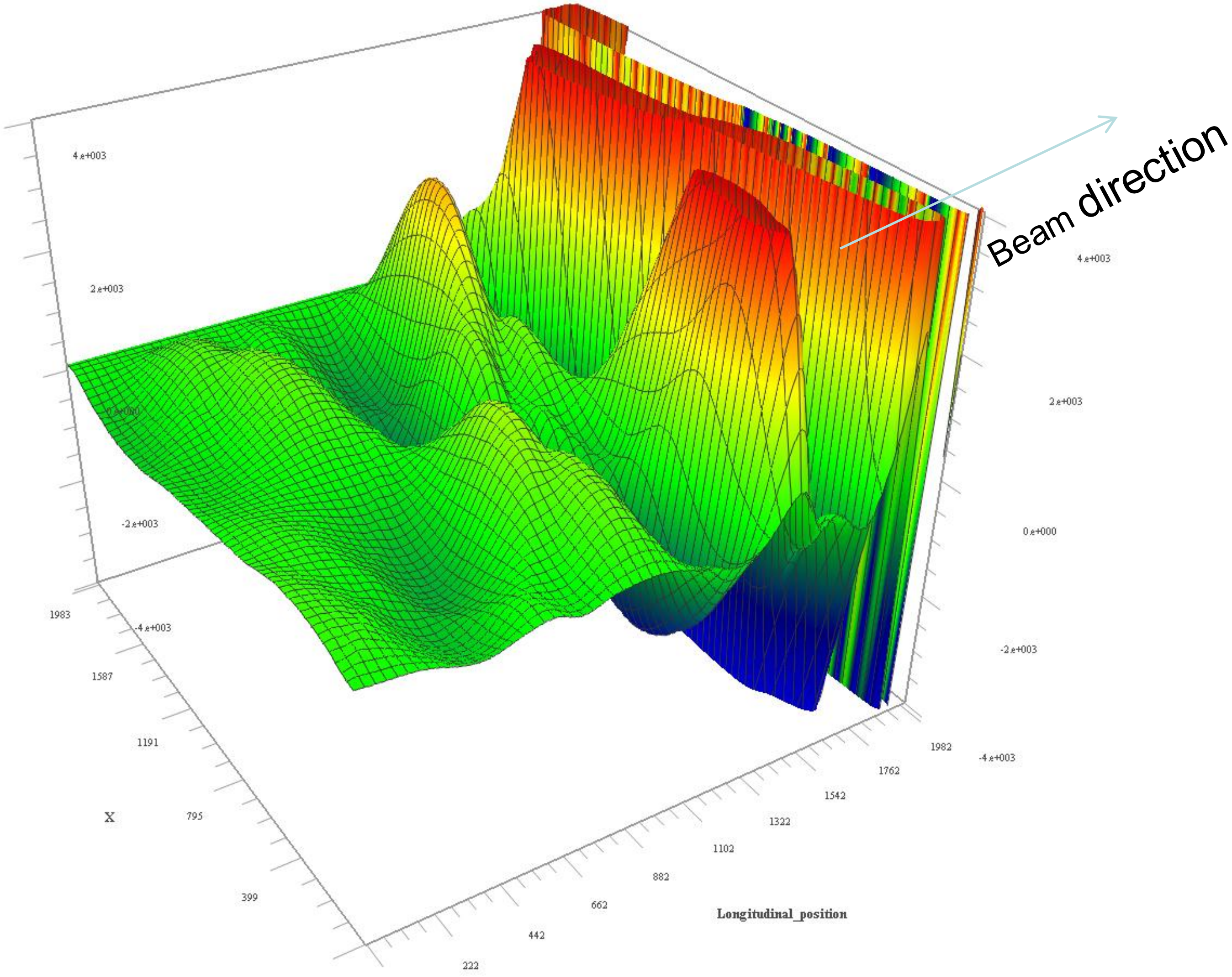


Sasha Novokhatski October 4, 2011

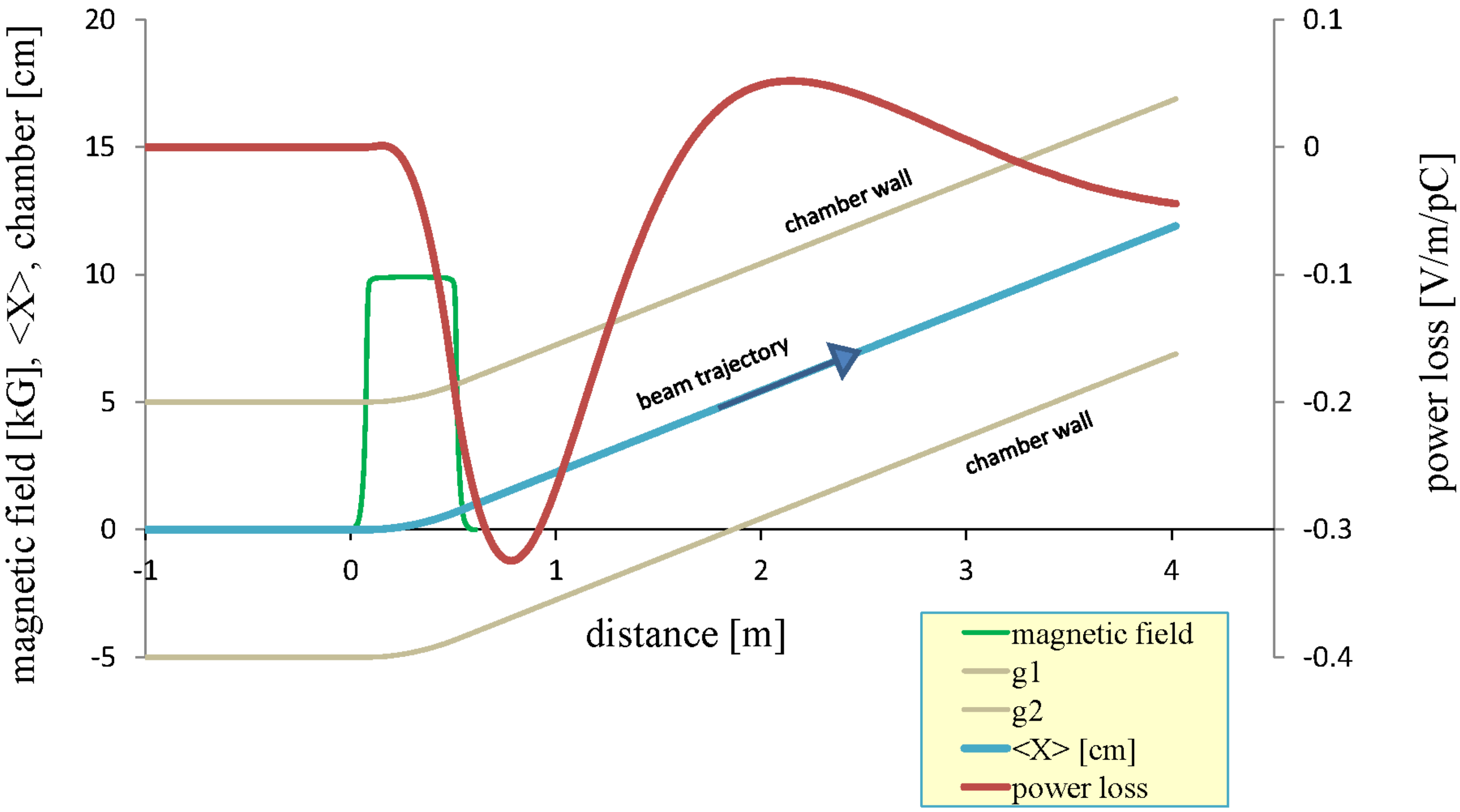
Magnified by 100 times



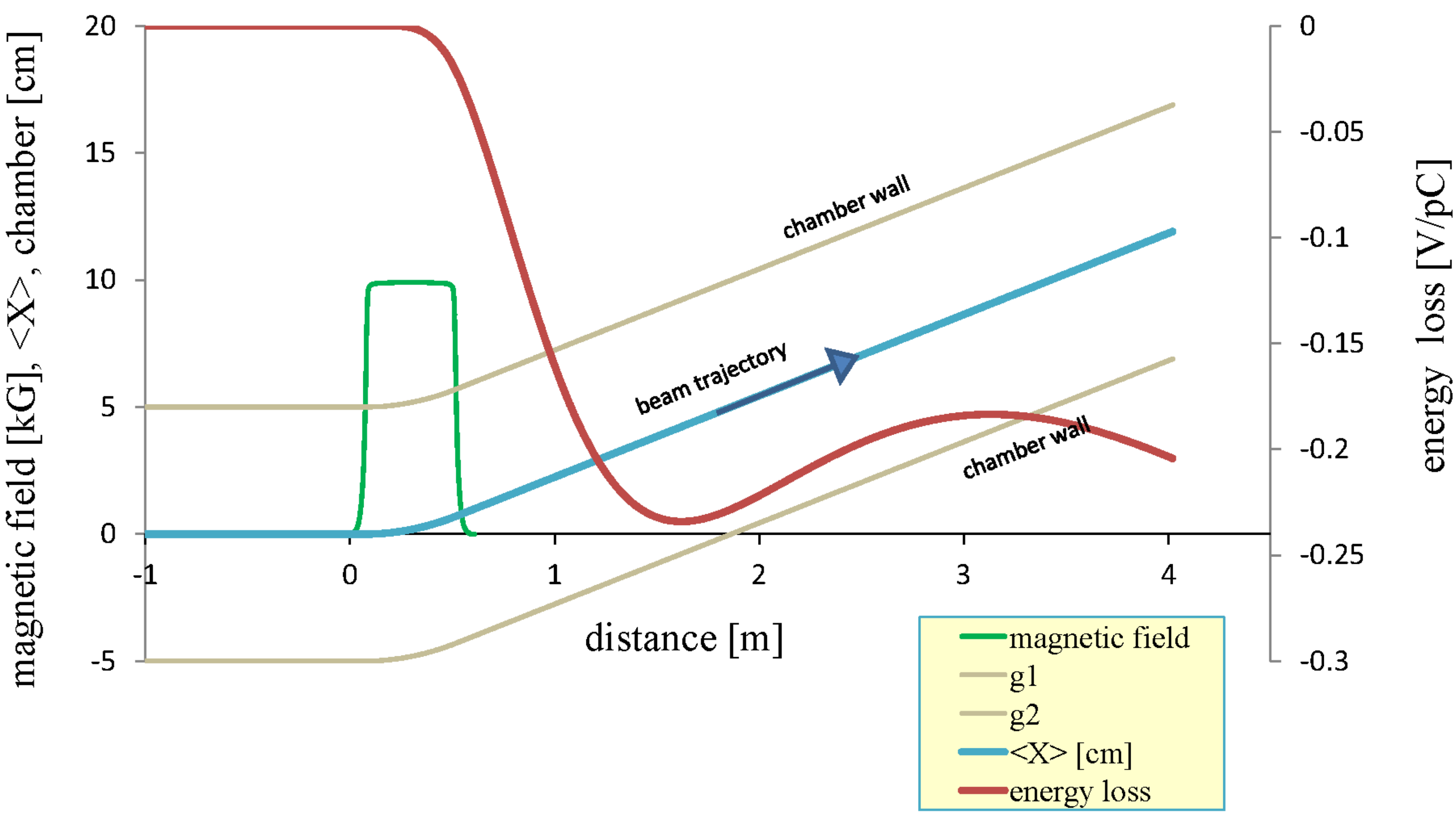
Magnified by 1000 times



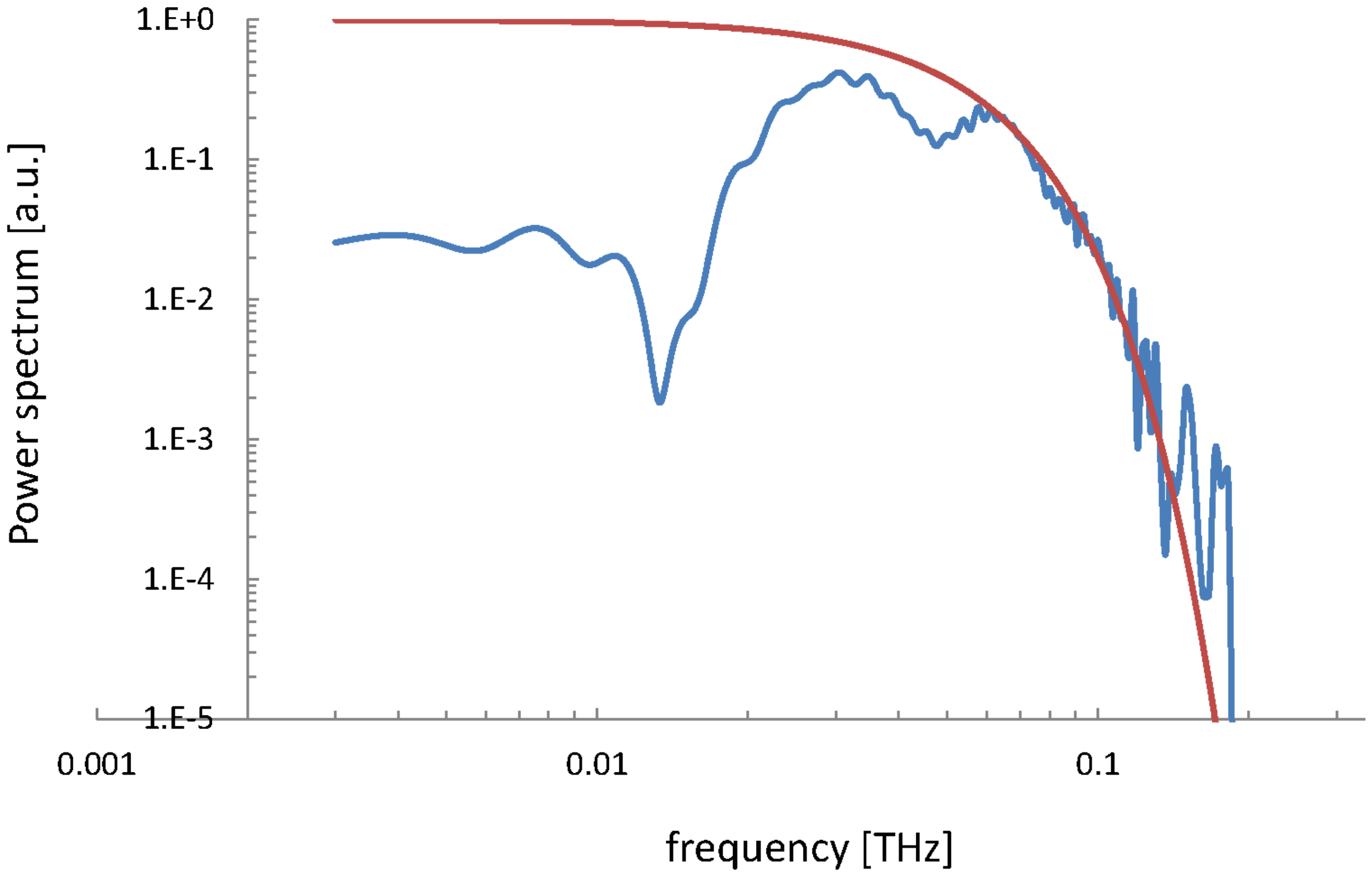
Beam power loss



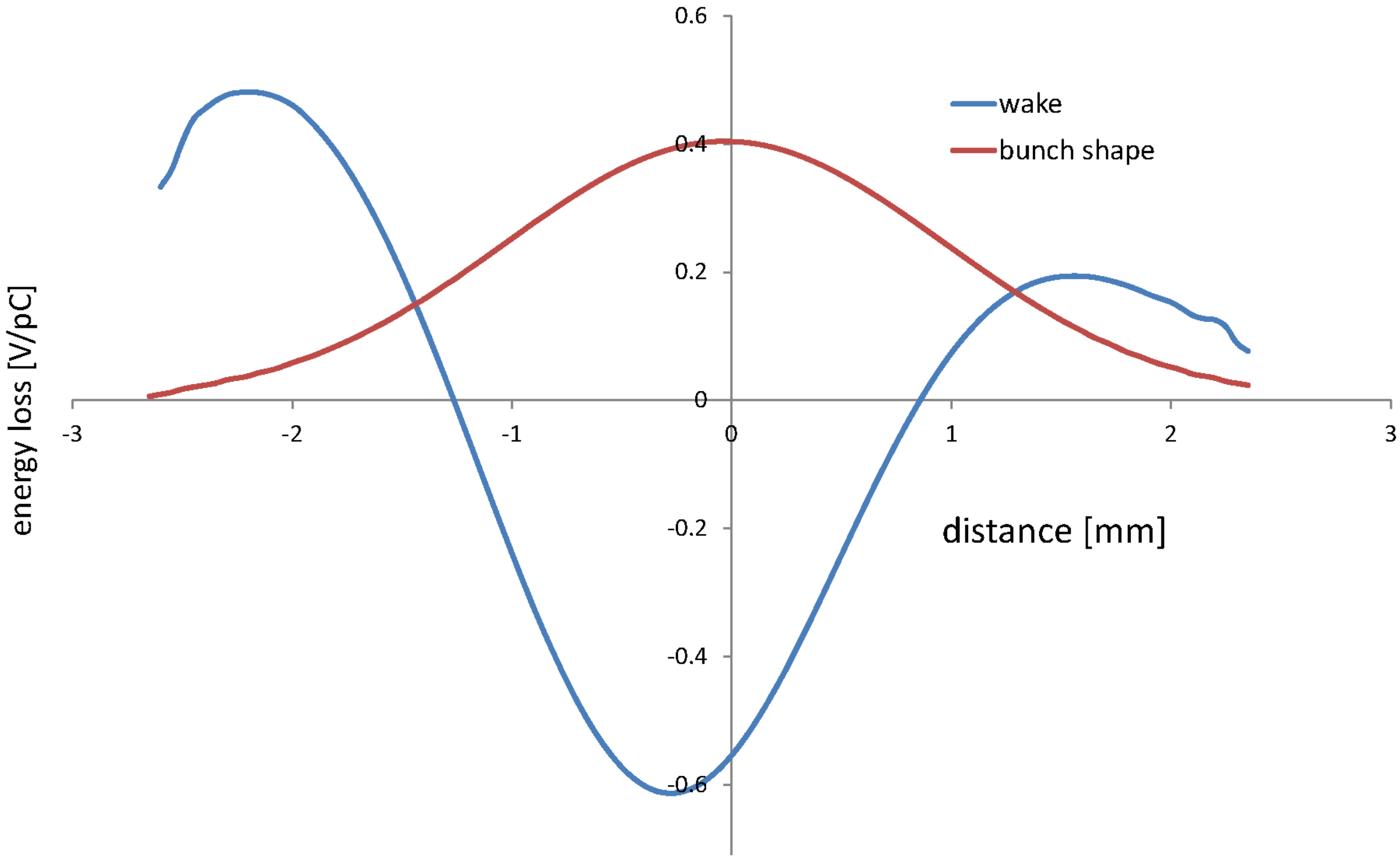
Beam energy loss



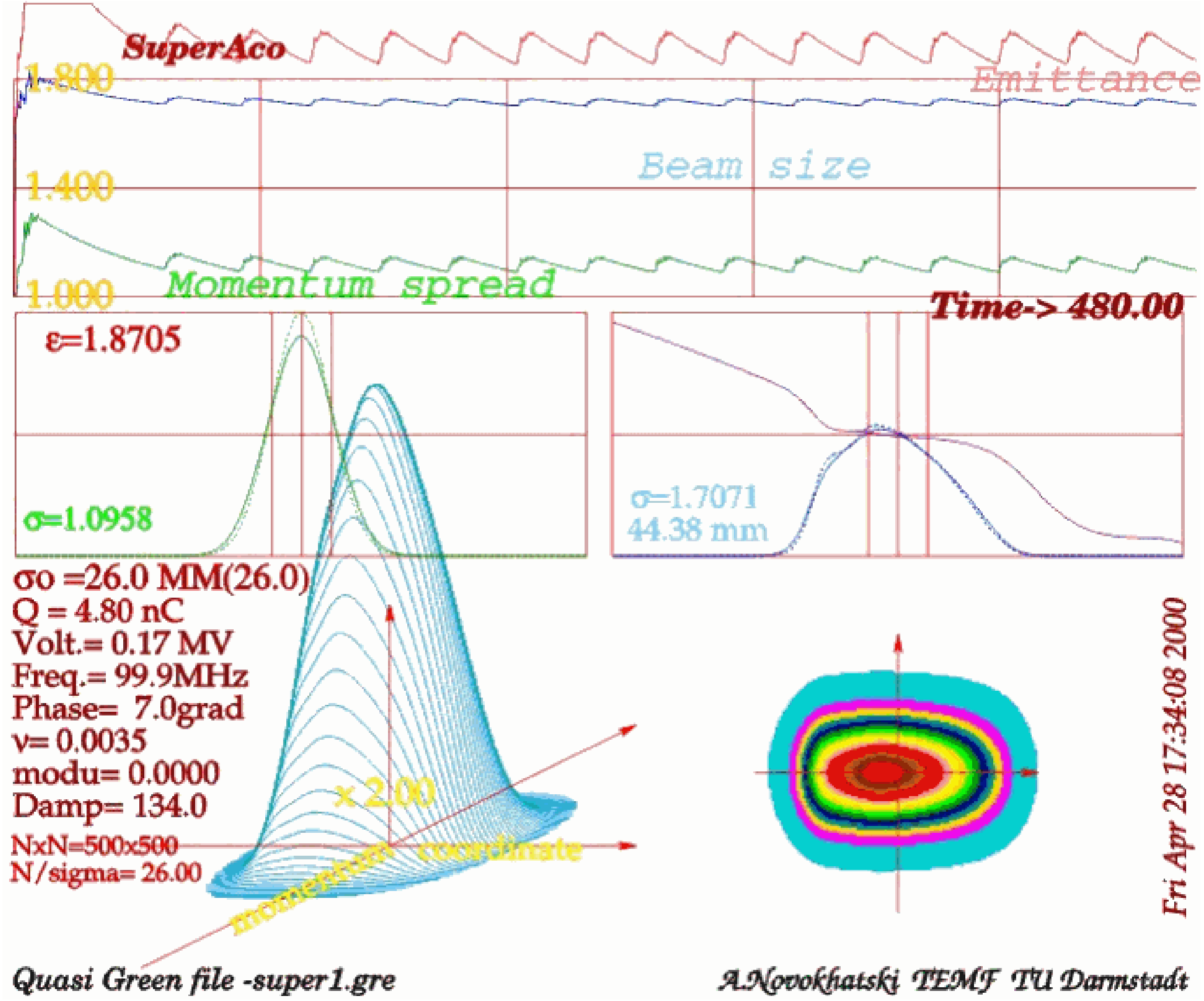
Spectrum of a 1mm bunch



Energy loss along a bunch



Next step:
combine CSR and Fokker-Planck calcultions.



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