Intrabeam scattering simulations in low emittance rings

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Thanks to:

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A. Chao, M.T.F. Pivi (SLAC).

Plan of Talk

- Introduction
- Conventional calculation of IBS
- Multi-particles codes structure
- Growth rates estimates and comparison with conventional theories
- Bunch distribution evolution
- Parallel implementation
- Conclusions and outlook

IBS Calculations procedure

1. Evaluate equilibrium emittances ε_i and radiation damping times τ_i at low bunch charge
2. Evaluate the IBS growth rates $1/T_i(\varepsilon_i)$ for the given emittances, averaged around the lattice, using K. Bane approximation*
3. Calculate the "new equilibrium" emittance from:

$$\varepsilon_i' = \frac{1}{1-\tau_i/T_i}\varepsilon_i$$

- For the vertical emittance use* :

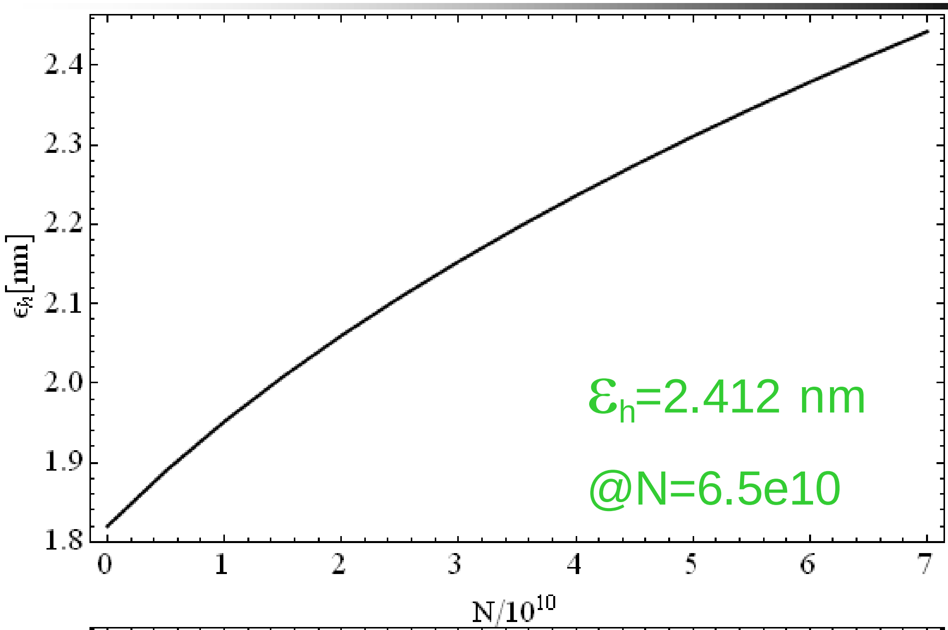
$$\varepsilon_y' = (1-r_\varepsilon)\frac{1}{1-\tau_y/T_y}\varepsilon_y + r_\varepsilon\frac{1}{1-\tau_x/T_x}\varepsilon_y$$

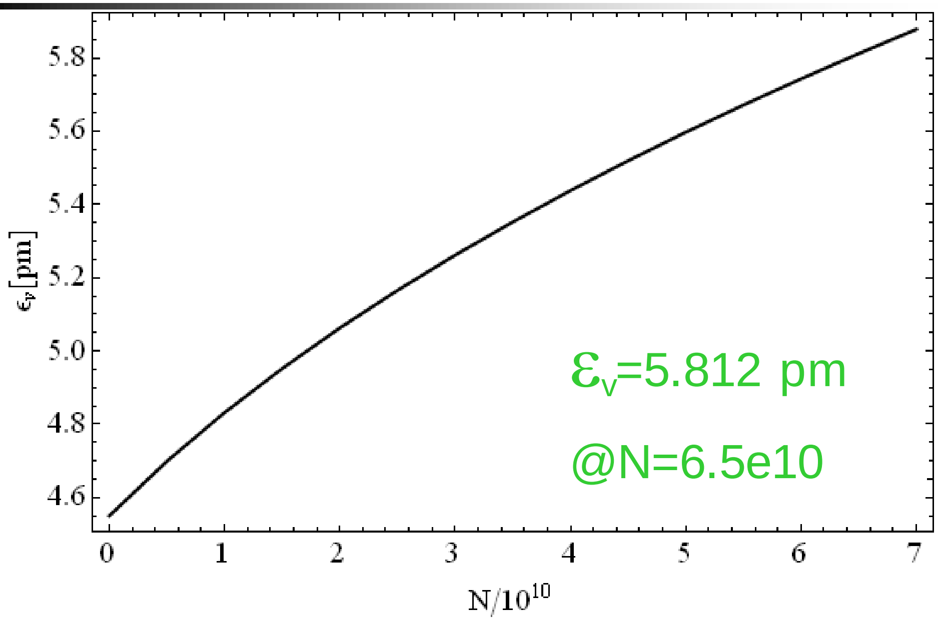
- where r_ε varies from 0 (ε_y generated from dispersion) to 1 (ε_y generated from betatron coupling)

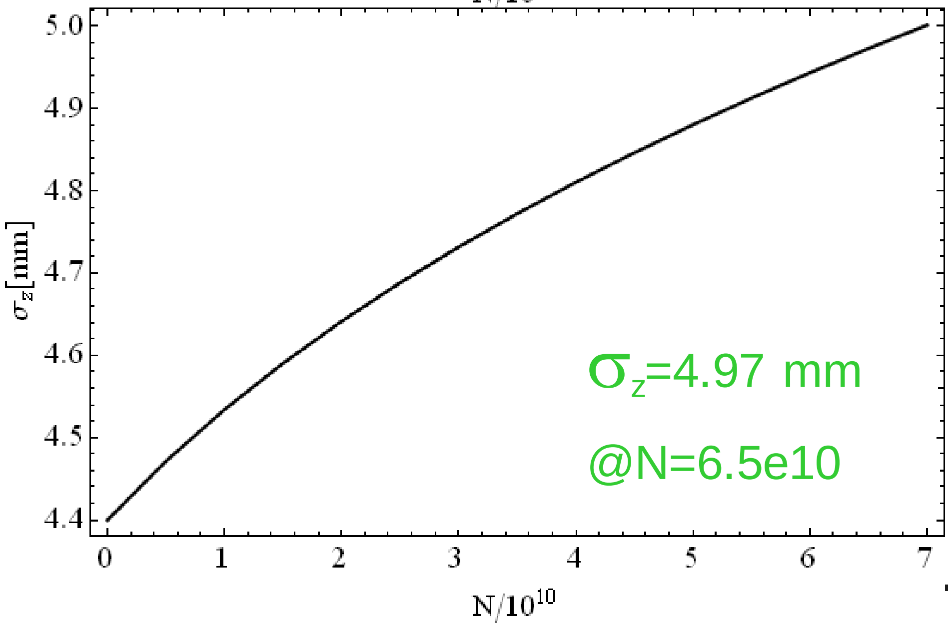
4. Iterate from step 2

* K. Kubo, S.K. Mtingwa, A. Wolski, "Intrabeam Scattering Formulas for High Energy Beams," Phys. Rev. ST Accel. Beams **8**, 081001 (2005)

IBS in SuperB LER (lattice V12)



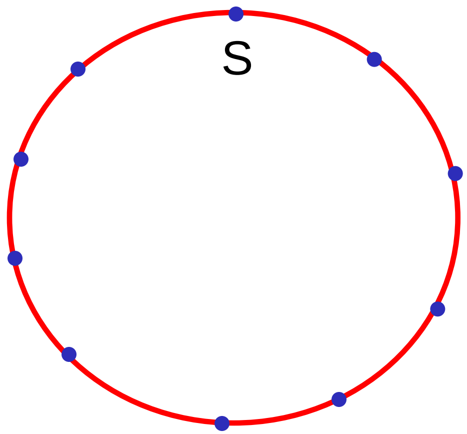


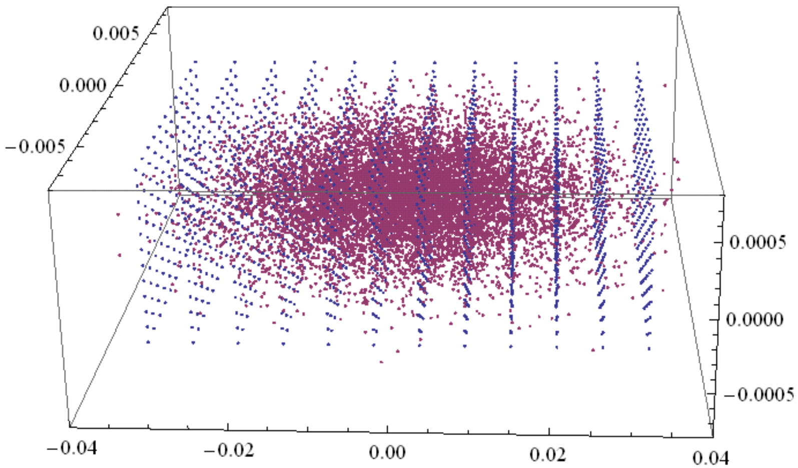


Effect is reasonably small. Nonetheless, there are some interesting questions to answer:

- What will be the impact of IBS during the damping process?
- Could IBS affect the beam distribution, perhaps generating tails?

Algorithm for Macroparticle Simulation of IBS





- The lattice is read from a MAD (X or 8) files containing the Twiss functions and R transport matrices.
- 6-dim Coordinates of particles are generated (Gaussian distribution at S).
- The scattering routine is called:
  - –Particles of the beam are grouped in cells.
  - –Particles inside a cell are coupled
  - –Momentum of particles is changed because of scattering.
  - –Invariants of particles and corresponding growth rate are recalculated.
- Particles are tracked at next elemenet a 6-dim R matrix.
- Radiation damping and excitation effects are evaluated at each turn.

Zenkevich-Bolshakov Algorithm

For two particles colliding with each other, the changes in momentum for particle 1 can be expressed as:

$$\Delta P_{1x} = \frac{P}{2}\left[\zeta\sqrt{1+\frac{\xi^2}{4\alpha^2}}\sin\phi - \frac{\xi\theta}{2\alpha}\cos\phi\right]\sin\varphi + \theta(\cos\varphi - 1)$$

$$\Delta P_{1y} = \frac{P}{2}\left[\theta\sqrt{1+\frac{\xi^2}{4\alpha^2}}\sin\phi - \frac{\xi\zeta}{2\alpha}\cos\phi\right]\sin\varphi + \zeta(\cos\varphi - 1)$$

$$\Delta P_{1s} = \frac{P}{2}[2\alpha\gamma\sin\varphi\cos\phi + \gamma\xi(\cos\varphi - 1)],$$

$$\boxed{\xi = \frac{P_1 - P_2}{\gamma P},\ \theta = x_1' - x_2',\ \zeta = y_1' - y_2',\ \alpha = \frac{\sqrt{\theta^2+\zeta^2}}{2}}$$

with the equivalent polar angle φ_{eff} and the azimuthal angle ϕ distributing uniformly in [0; 2π], the invariant changes caused by the equivalent random process are the same as that of the IBS in the time interval Δts

$$\varphi_{\text{eff}} = \frac{2r_0}{\gamma}\sqrt{\frac{\pi c\rho\Delta t_s}{\bar{\beta}^3}L_c} \Longrightarrow$$

$$\frac{dJ_{1x}}{dt} = \frac{\pi r_0^2}{4\gamma^2\bar{\beta}^3}c\rho L_c\left[-4x_1'\theta + \xi^2 + \zeta^2 + 4\frac{x_{\beta 1}D_x}{\beta_x^2}\gamma\xi + \frac{D_x^2\gamma^2}{\beta_x^2}(\theta^2+\zeta^2)\right]$$

$$\frac{dJ_{1y}}{dt} = \frac{\pi r_0^2}{4\gamma^2\bar{\beta}^3}c\rho L_c(-4y_1'\zeta + \xi^2 + \theta^2)$$

$$\frac{dJ_{1s}}{dt} = \frac{\pi r_0^2}{4\gamma^2\bar{\beta}^3}c\rho L_c\left(-4\frac{\delta_1}{\gamma}\xi + \zeta^2 + \theta^2\right),$$

Radiation Damping and Quantum Excitation

- Normalized coordinates are defined by Twiss (B) and Dispersion (H) matrix :

$$\vec{X} = B\,H\,\vec{x}$$

- Synchrotron Radiation is taken into account with the following map:

$$\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} = \lambda_x \begin{pmatrix} X_1 \\ X_2 \end{pmatrix} + \sqrt{\varepsilon_x (1-\lambda_x^{\,2})} \begin{pmatrix} \hat{r}_1 \\ \hat{r}_2 \end{pmatrix}$$

$$\begin{pmatrix} X_5 \\ X_6 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & \lambda_z^{\,2} \end{pmatrix} \begin{pmatrix} X_5 \\ X_6 \end{pmatrix} + \begin{pmatrix} 0 \\ \sqrt{\varepsilon_z (1-\lambda_z^{\,4})}\,\hat{r}_5 \end{pmatrix}$$

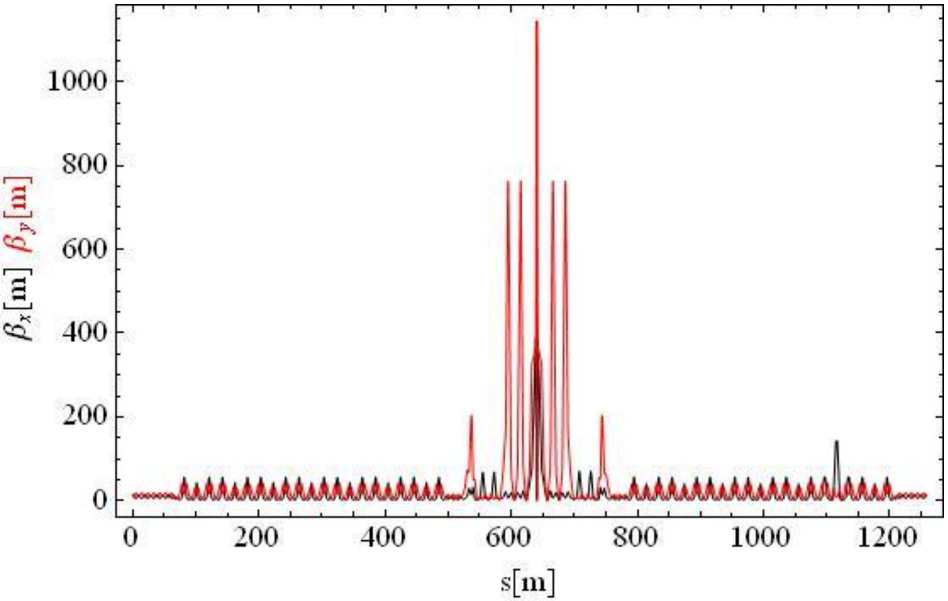
$$\lambda_i = \exp(-1/\tau_i)$$

$\hat{r}$: unit Gaussian random

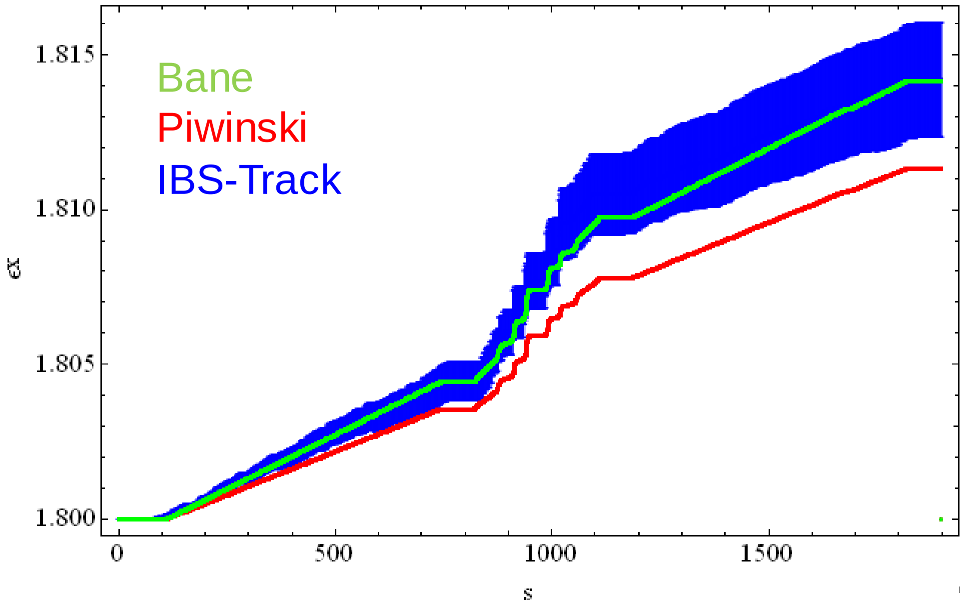
- Switch back to physical coordinates by:

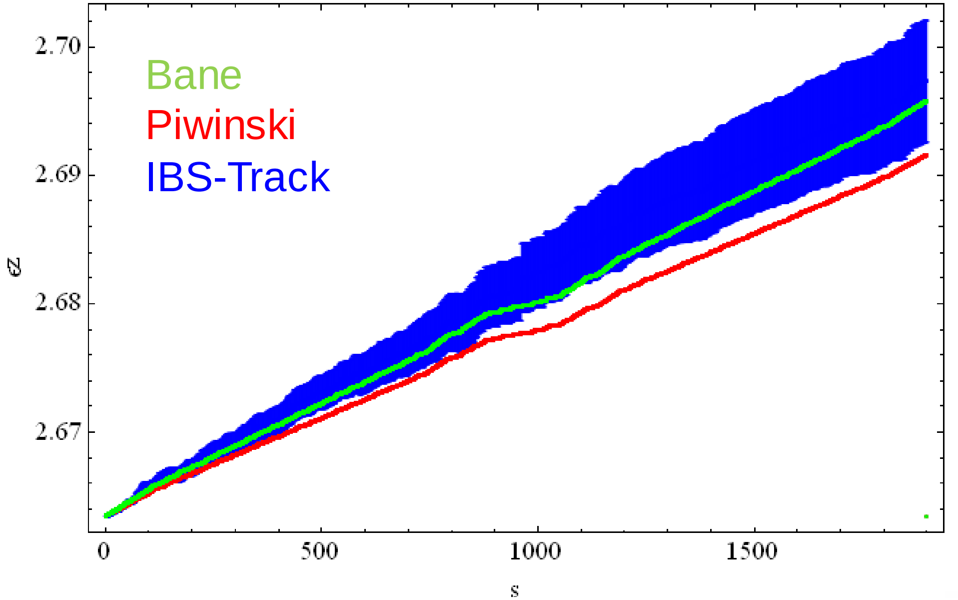
$$\vec{x} = \left(B\,H\right)^{-1} \vec{X}$$

Intrabeam Scattering in SuperB LER

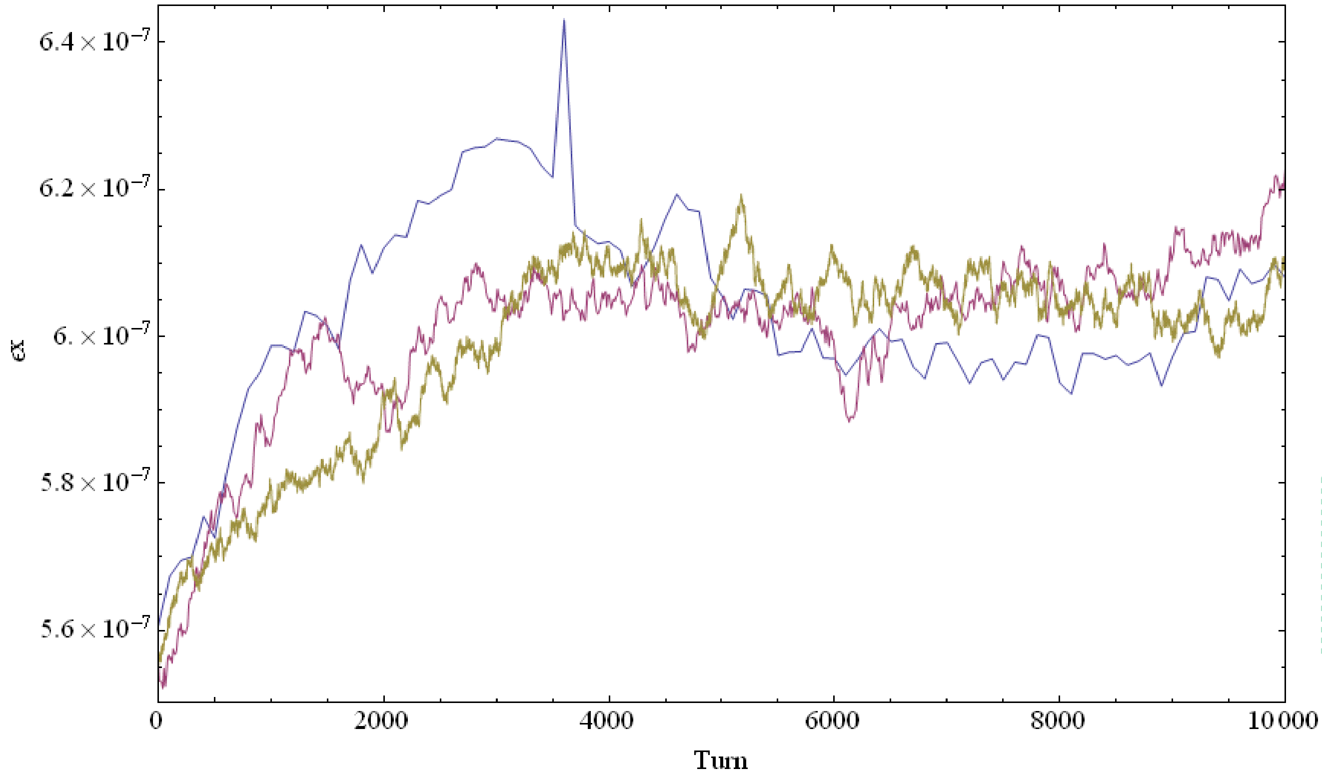


Parameter	Unit	Value
Energy	GeV	4.18
Bunch population	10^{10}	6.5
Circumference	m	1257
Emittances (H/V)	nm/pm	1.8/4.5
Bunch Length	mm	3.99
Momentum spread	%	0.0667
Damping times (H/V/L)	ms	40/40/20
N. of macroparticles	-	10^5
N. of grid cells	-	64x64x64





Scaling Law



M.P. Number=40000

NTurn≈10 damping times

σ_z=12.0*10^{-3}

δp=4.8*10^{-4}

ε_x=(5.63*10^{-4})/γ

ε_y=(3.56*10^{-5})/γ

Grid size: $6\sigma_x x 6\sigma_y x 6\sigma_z$

Cell size: $\sigma_x/2 x \sigma_y/2 x \sigma_z/2$

Blue (100*dt):

N_{bunch}=**10^5***2.1*10^{10}

$\tau_x = 10^{-4} * 42.02 * 10^{-3}$

$\tau_y = 10^{-4} * 37.16 * 10^{-3}$

$\tau_s = 10^{-4} * 17.56 * 10^{-3}$

Magenta (10*dt):

N_{bunch}=**10^4***2.1*10^{10}

$\tau_x = 10^{-3} * 42.02 * 10^{-3}$

$\tau_y = 10^{-3} * 37.16 * 10^{-3}$

$\tau_s = 10^{-3} * 17.56 * 10^{-3}$

Gold (1*dt):

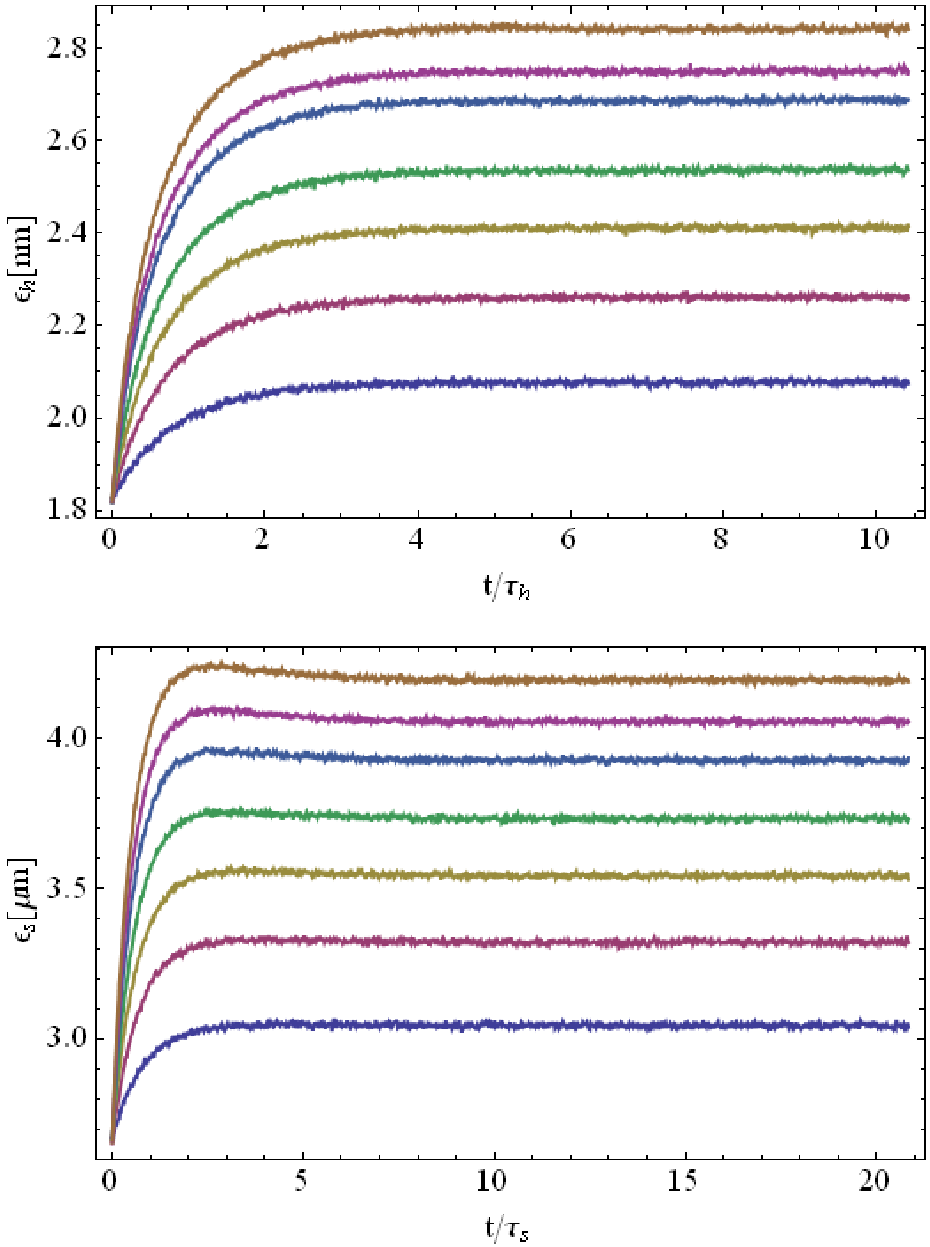
N_{bunch}=**10^3***2.1*10^{10}

$\tau_x = 10^{-3} * 42.02 * 10^{-3}$

$\tau_y = 10^{-3} * 37.16 * 10^{-3}$

$\tau_s = 10^{-3} * 17.56 * 10^{-3}$

Emittance Evolution in SuperB LER



- SuperB V12 LER

  Nb= $2\text{x}10^{10}$ - $12\text{x}10^{10}$

F=10

$\tau_x = 10^{-1}$ x 40 ms

$\tau_y = 10^{-1}$x 40 ms

$\tau_s = 10^{-1}$x 20 ms

Differential equation system for ε_x and ε_z

Radial and longitudinal emittance growths can be predicted by a model that takes the form of a coupled differential equations:

$$\begin{cases} \dot{\varepsilon}_x = -\dfrac{\varepsilon_x(t)}{\tau_x} + \dfrac{\varepsilon_x}{T_x} \\ \dot{\varepsilon}_z = -\dfrac{\varepsilon_z(t)}{\tau_z} + \dfrac{\varepsilon_z}{T_z} \end{cases} \quad \xRightarrow{\text{Bane Model}} \quad \begin{cases} \dot{\varepsilon}_x = -\dfrac{1}{\tau_x / T_{rev}}\left(\varepsilon_x(t) - \varepsilon_{xeq}\right) + \dfrac{N\ a}{\varepsilon_x^{3/4}(t)\ \varepsilon_z(t)} \\ \dot{\varepsilon}_z = -\dfrac{1}{\tau_z / T_{rev}}\left(\varepsilon_z(t) - \varepsilon_{zeq}\right) + \dfrac{N\ b}{\varepsilon_x^{3/4}(t)\ \varepsilon_z(t)} \end{cases}$$

N number of particles per bunch

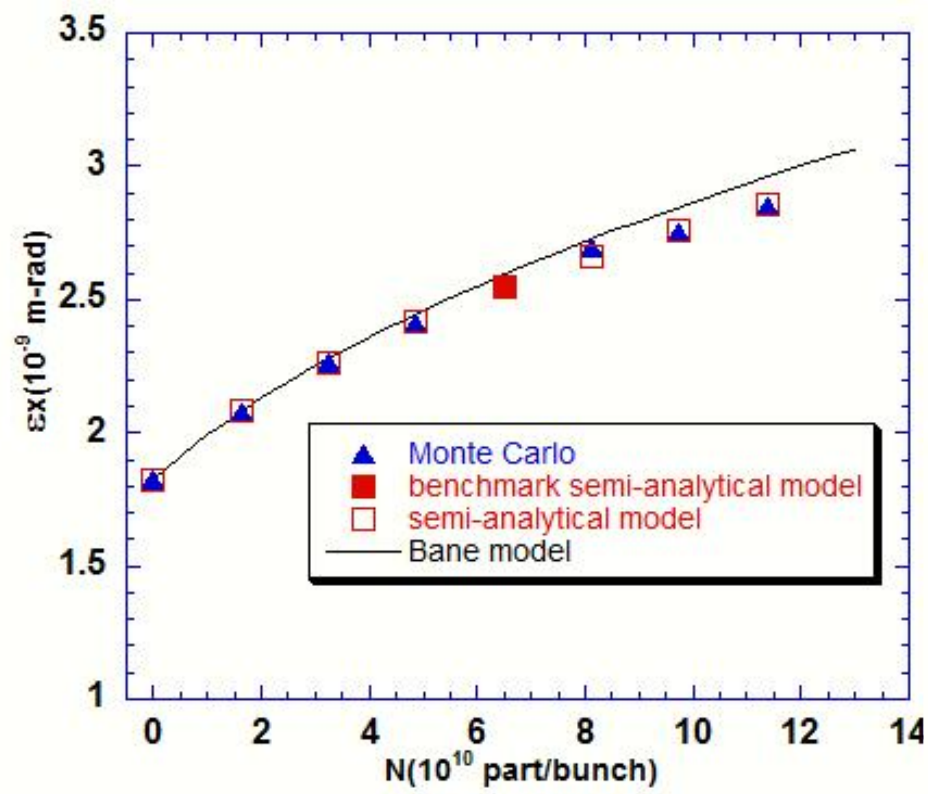
a and **b** coefficients characterizing IBS obtained once by fitting the tracking simulation data for a chosen benchmark case

$$\begin{cases} \varepsilon_x(t=0) = \varepsilon_{x0} \\ \varepsilon_z(t=0) = \varepsilon_{z0} \end{cases}$$

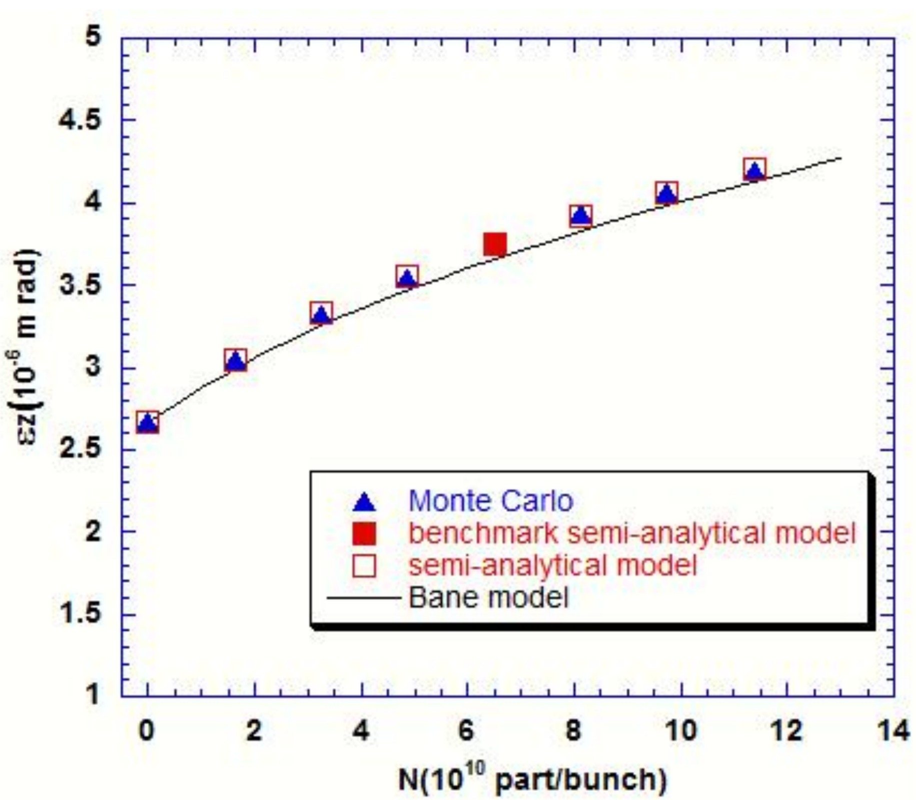
Obtained by fitting the zero bunch intensity case (IBS =0)

Summary plots: SuperB parameters

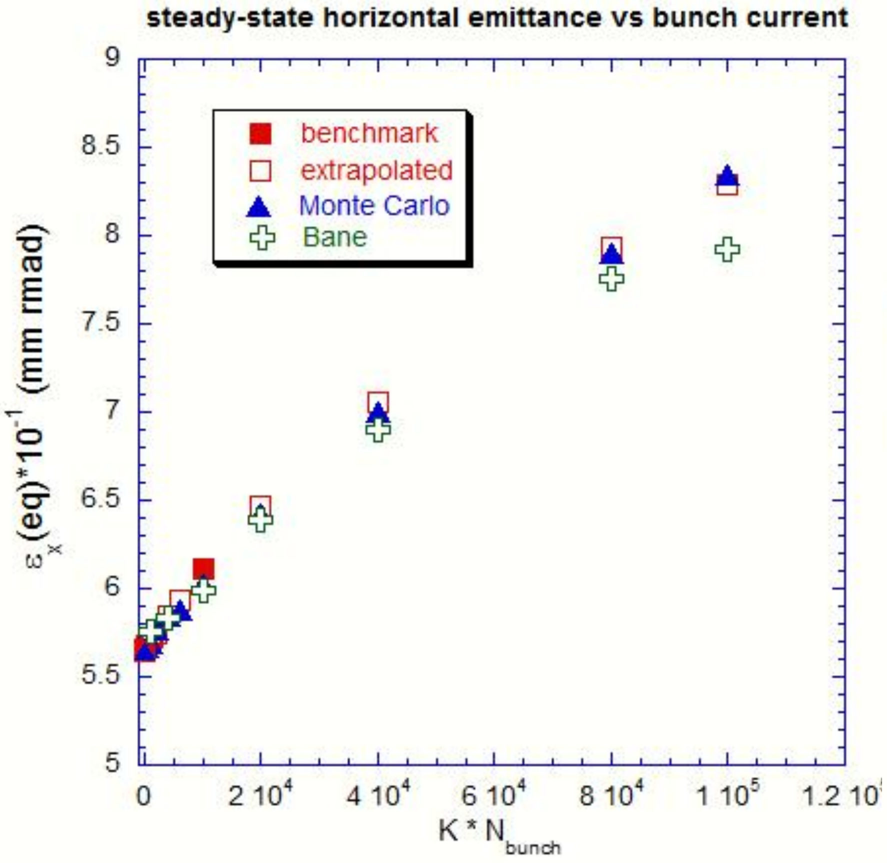
Equilibrium **horizontal emittance vs bunch current**

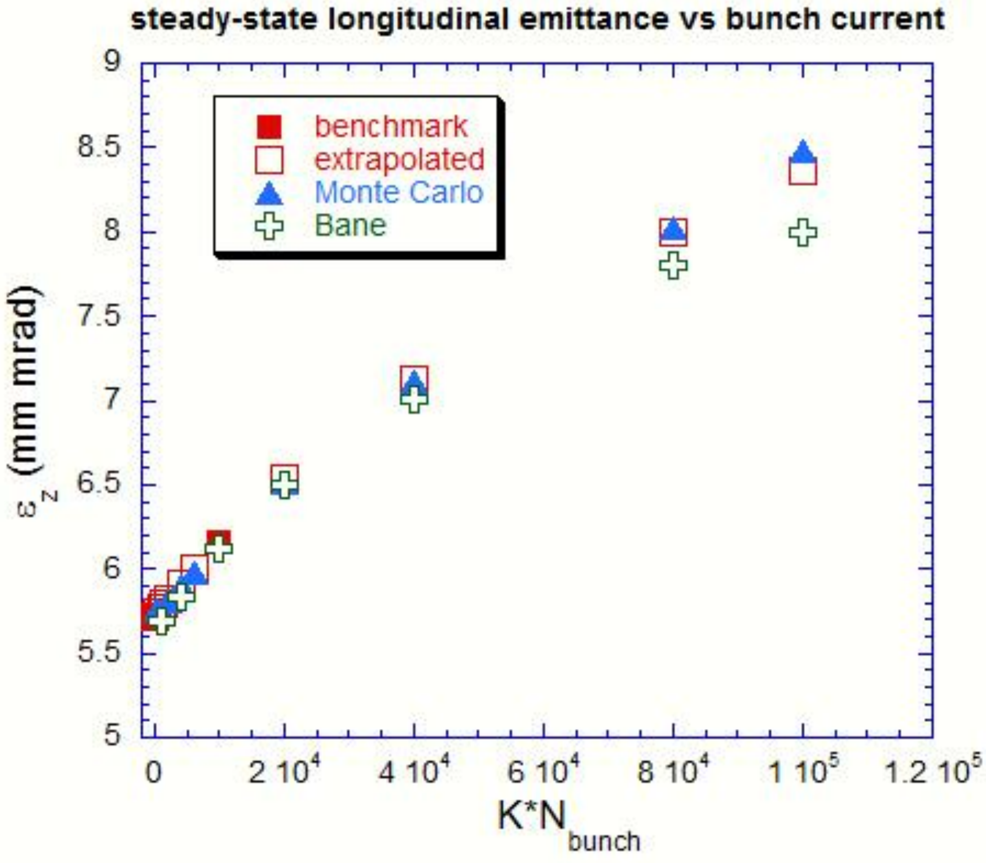


Equilibrium **longitudinal emittance vs bunch current**



Summary plots: DAFNE parameters





MacroParticleNumber=40000

σ_z=12.0*10^{-3}

δp=4.8*10^{-4}

NTurn=1000 (≈10 damping times)

ε_x=(5.63*10^{-4})/γ

ε_y=(3.56*10^{-5})/γ

τ_x = 1000^{-1} * 0.042

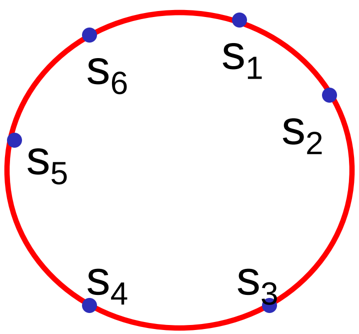
τ_y = 1000^{-1} * 0.037

τ_s = 1000^{-1} * 0.017

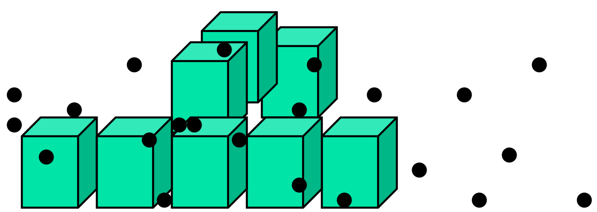
Software for IBS and Radiation Effects

Goals:

1. Follow the evolution of the particle distribution in the DR (we are not sure it remains Gaussian).
2. Calculate IBS effect for any particle distribution (in case it doesn't remain Gaussian).



- The lattice is read from a MADX file containing the Twiss functions.
- Particles are tracked from point to point in the lattice by their invariats (no phase tracking up to now).
- At each point of the lattice the scattering routine is called.



- 6-dim Coordinates of particles are calculated.
- Particles of the beam are grouped in cells.
- Momentum of particles is changed because of scattering.
- Invariants of particles are recalculated.

- Radiation damping and excitation effects are evaluated at the end of every loop.

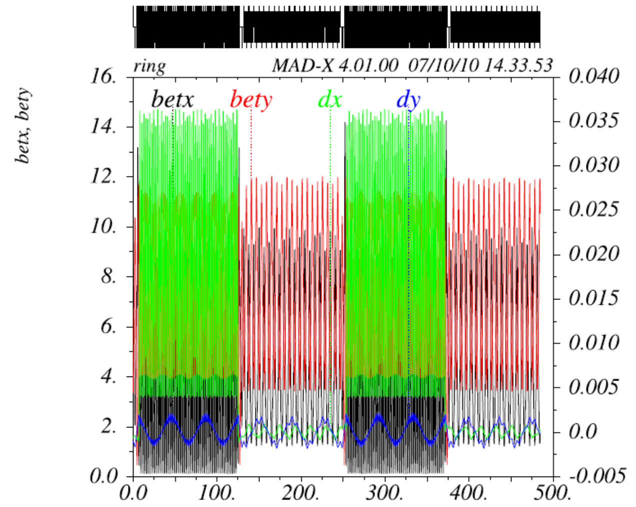
- A routine has also been implemented in order to speed up the calculation of IBS effect.

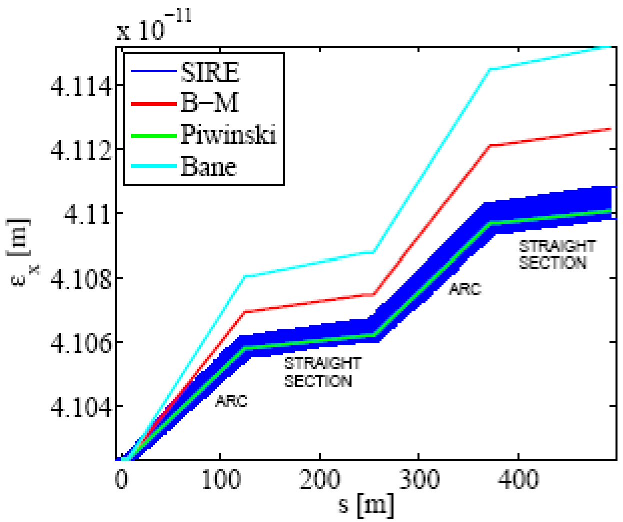
A. Vivoli

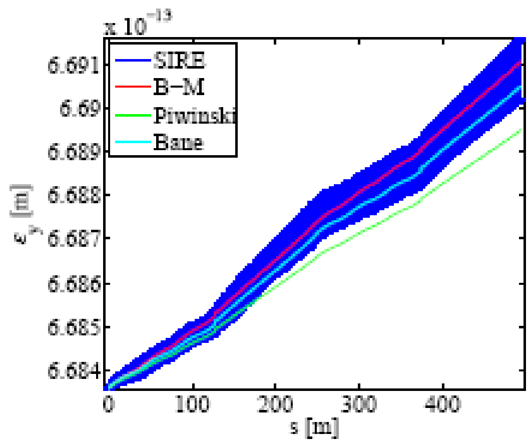
SIRE: Benchmarking (Gaussian Distribution) CLIC DR

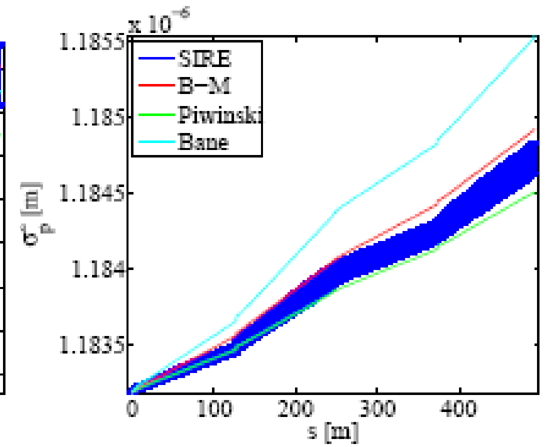
Table 1: Parameters for validation

Parameter	Unit	Value
Energy	GeV	2.86
Bunch Population	10^9	4.07
Circumference	m	493.16
Norm. Emittance H,V	nm	229.6 , 3.74
Momentum Spread	%	0.109
Bunch Length	mm	0.922
N. macro-particles	10^3	200
N. cells	10^3	200
ΔT	μs	1.645









F. Antoniou, IPAC10

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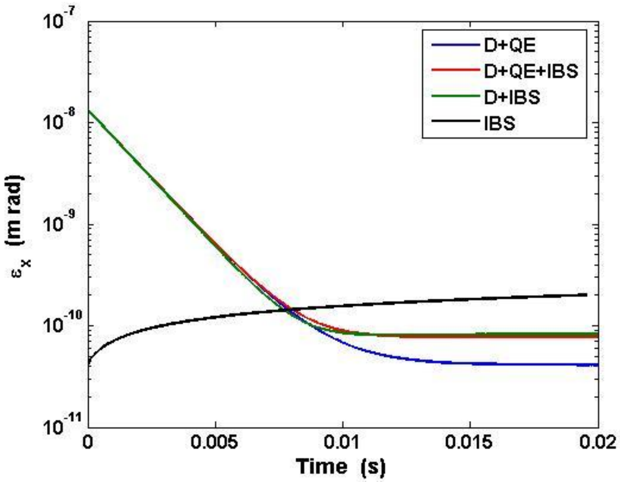
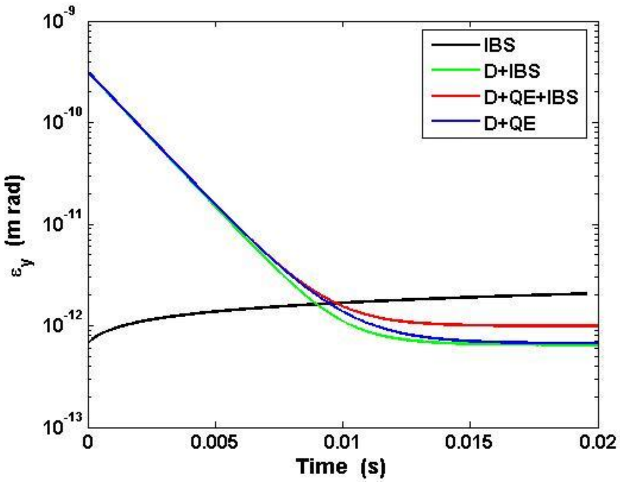
SIRE: Distribution Study

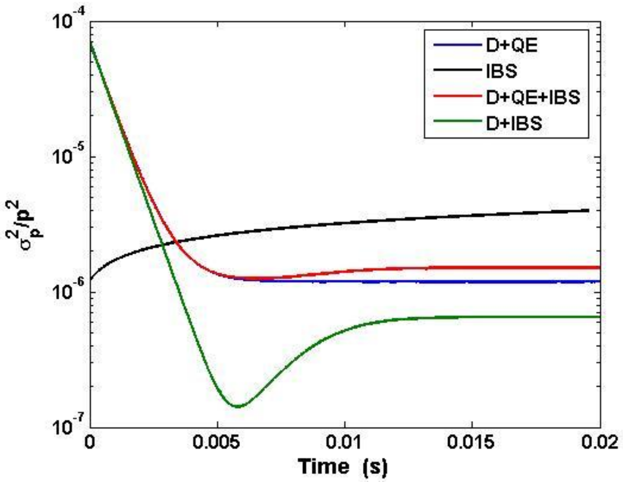
Case studies:

- A – Damping + QE
- B – Damping + IBS + QE
- C – Damping + IBS
- D – IBS

Parameter	A	B	C	D
INITIAL $\gamma\varepsilon_x$, $\gamma\varepsilon_y$, $\sigma_z\sigma_p$ (m,m,eV m)	74.3e-6,1.8e-6, 1.71e+5	74.3e-6, 1.8e-6, 1.70e+5	74.3e-6, 1.8e-6, 1.71e+5	229.7e-9,3.7e-9, 2.87e+3
FINAL $\gamma\varepsilon_x$, $\gamma\varepsilon_y$, $\sigma_z\sigma_p$ (m,m,eV m)	229.7e-9, 3.76e-9, 2.88e+3	435.6e-9, 5.54e-9, 3.65e+3	458.5e-9, 3.61e-9, 1.58e+3	1.12e-6, 1.16e-8, 9.61e+3







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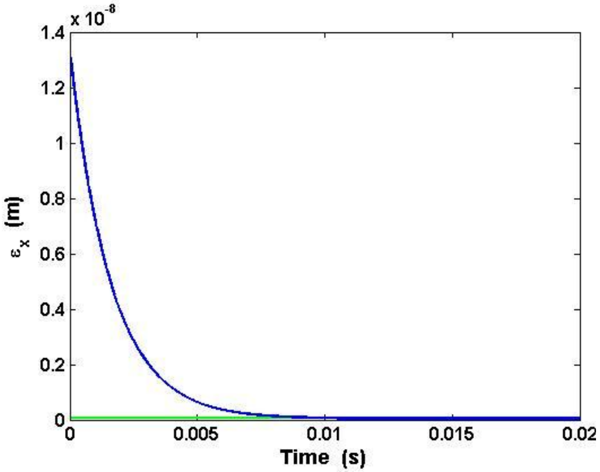
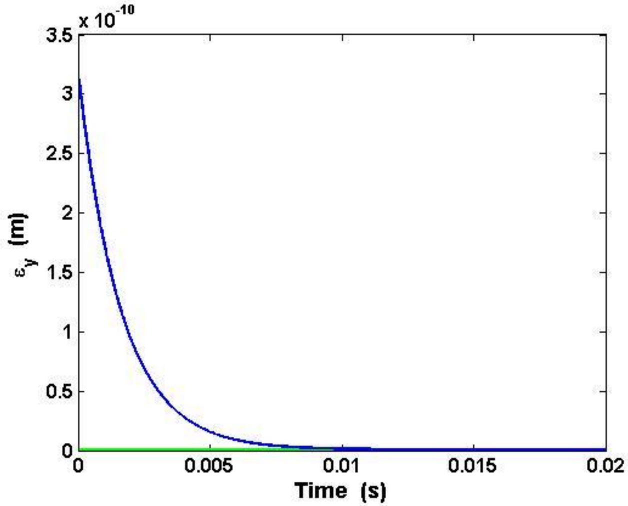
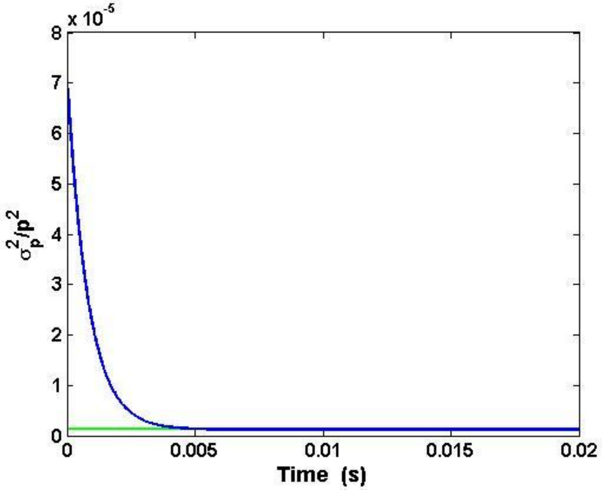
CLIC DR A: Damping + QE

Simulation of the CLIC Damping Rings case A:



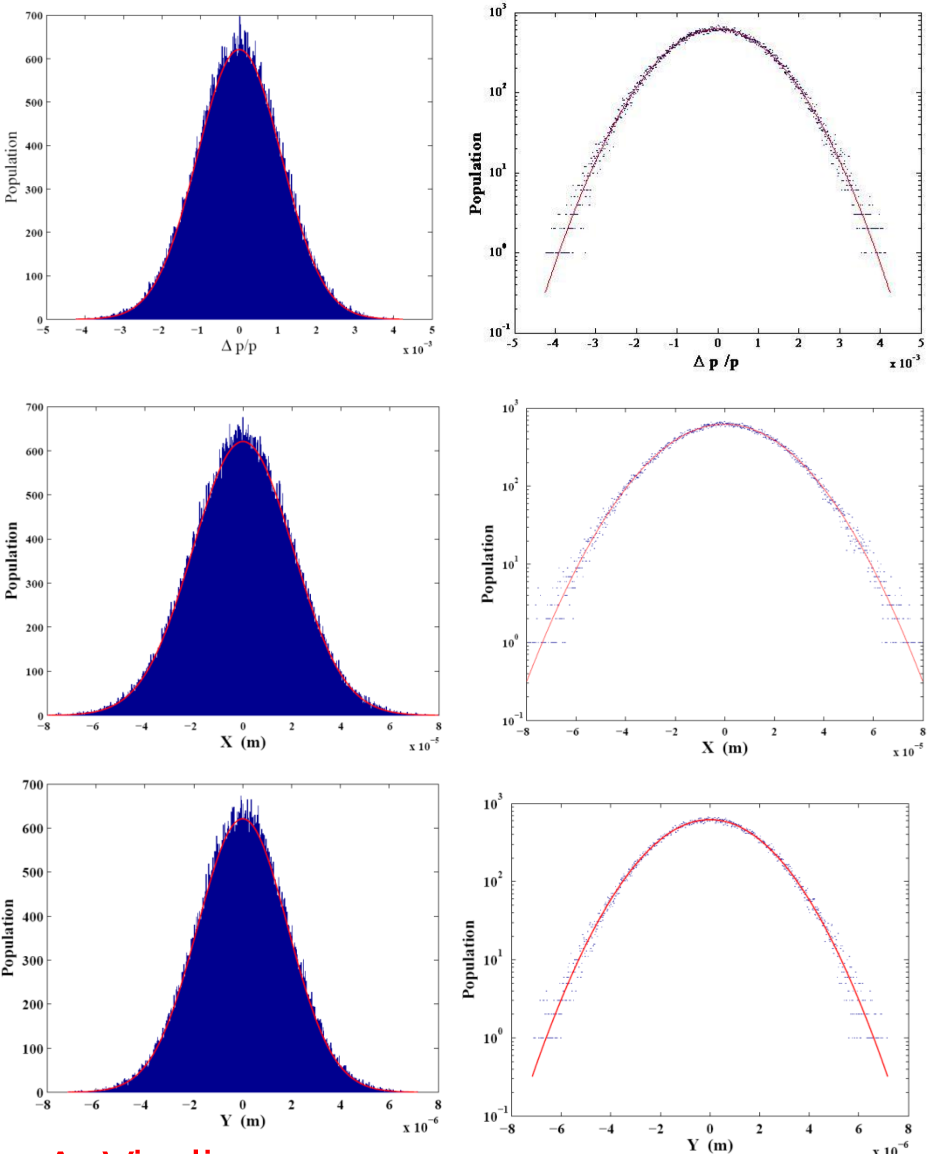




Beam parameters

	ε_x (m)	ε_y (m)	ε_z (eV m)
Injection	13.27e-9	321.6e-12	1.71e+5
Extraction (SIRE)	4.104e-11	6.72e-13	2.88e+3
Extraction (MAD-X)	4.102e-11	6.69e-13	2.87e+3

SIRE: IBS Distribution study A: Damping + QE



$$p_k(\xi_k) = \frac{1}{\sqrt{2\pi}\sigma_k} e^{-\frac{\xi_k^2}{2\sigma_k^2}}$$

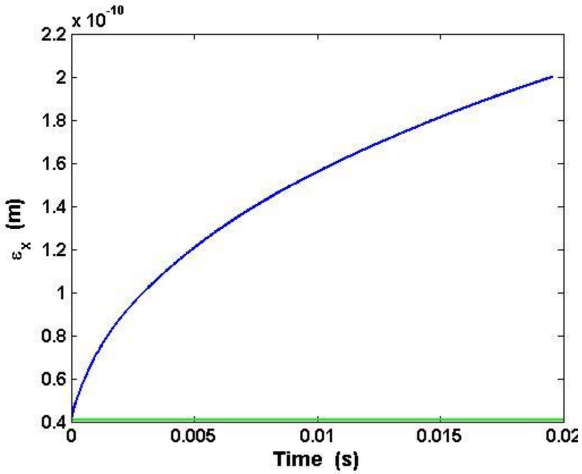
Parameter	χ^2_{999}	Confidence
ΔP/P	964.2251	0.7876
X	976.2195	0.6988
Y	957.4559	0.8290

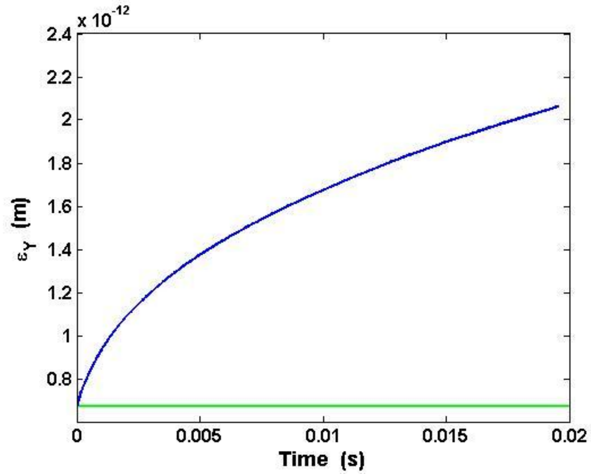
Parameter	Value
Eq. ε_x (m rad)	4.1039e-011
Eq. ε_y (m rad)	6.7113e-013
Eq. σ_δ	1.0901e-3
Eq. σ_z (m)	9.229e-4

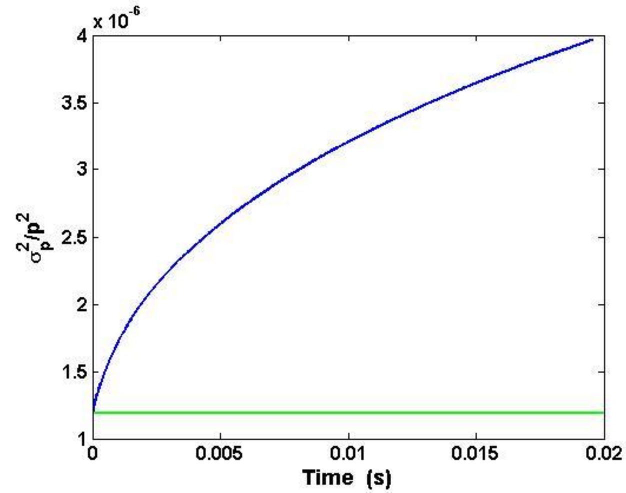
A. Vivoli

CLIC DR D: IBS

Simulation of the CLIC Damping Rings case D:







Beam parameters

	ε_x (m)	ε_y (m)	ε_z (eV m)
Injection	4.104e-11	6.663e-13	2871
Extraction (SIRE)	2.001e-010	2.064e-12	9609

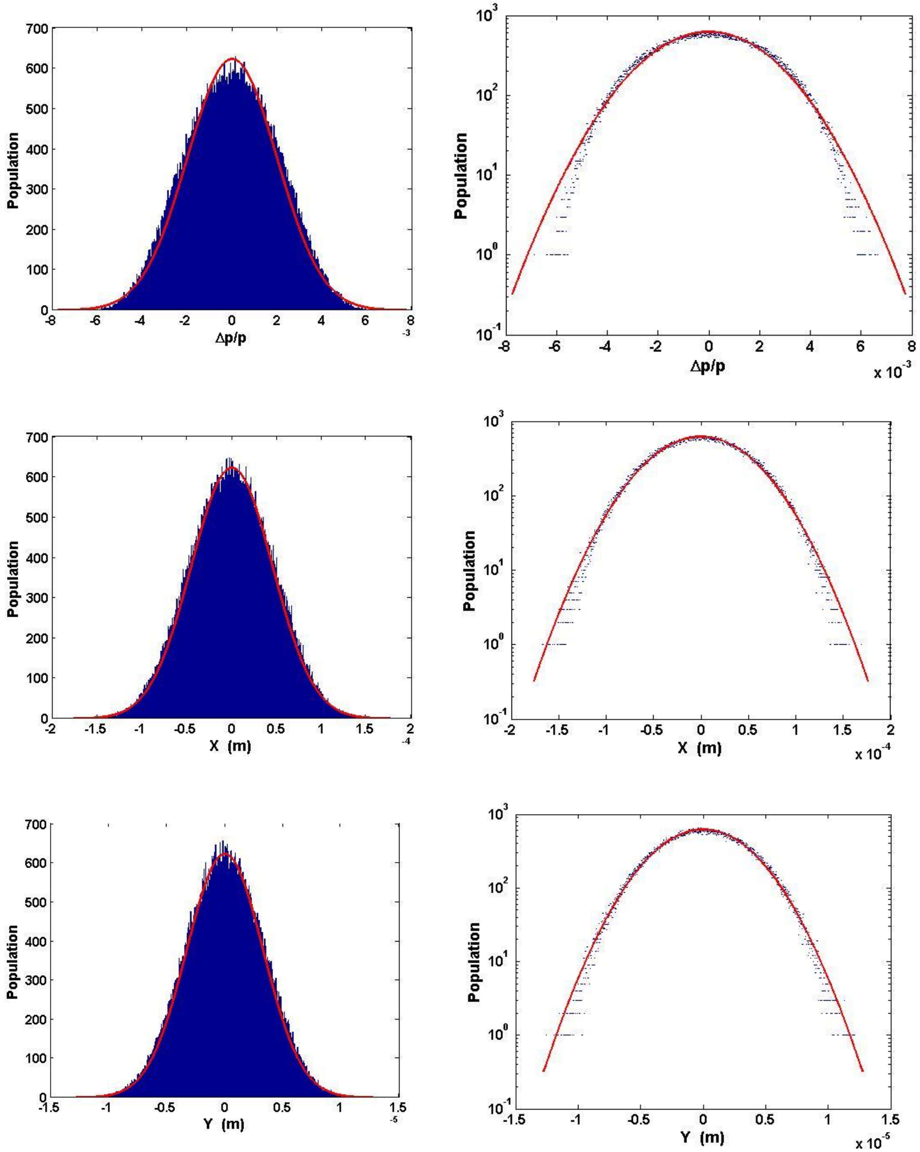
Table 2: Comparison SIRE – conventional formalisms

Formalism	$1/T_H$ (s^{-1})	$1/T_V$ (s^{-1})	$1/T_L$ (s^{-1})
Bjorken-Mtingwa	1579	739	969
SIRE	1186	665	800
SIRE (compressed)	1239	687	786
Modified Piwinski	1300	626	775

	1/Tx (s^{-1})	1/Ty (s^{-1})	1/Tz (s^{-1})
Bjorken-Mtingwa	29.6	21.0	28.9
SIRE compressed (Gauss)	21.6	17.8	20.6
SIRE not compressed (Gauss)	18.1	18.0	19.3
SIRE compressed	17.0	14.6	17.2
SIRE not compressed	18.3	15.3	16.5

A. Vivoli

SIRE: IBS Distribution study D: IBS



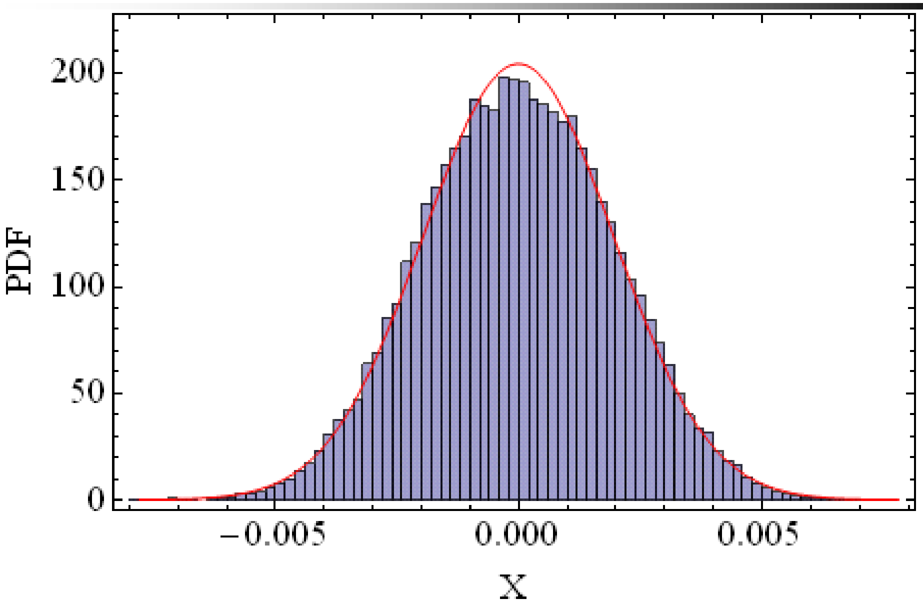
$$p_k(\xi_k) = \frac{1}{\sqrt{2\pi}\sigma_k} e^{-\frac{\xi_k^2}{2\sigma_k^2}}$$

Parameter	χ^2_{999}	Confidence
Δp/p	3048.7	<1e-15
X	1441.7	<1e-15
Y	1466.9	<1e-15

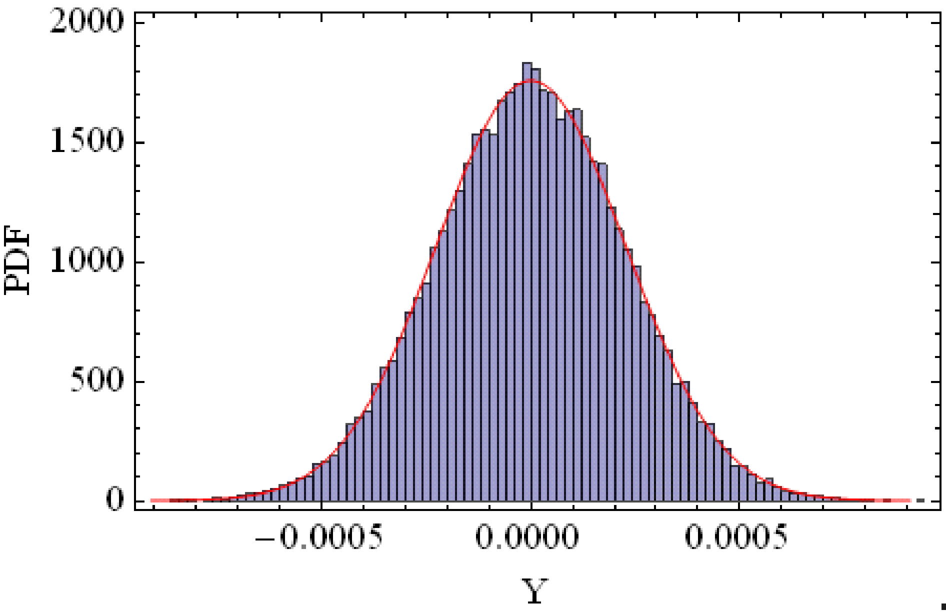
Parameter	Value
Eq. ε_x (m rad)	2.001e-10
Eq. ε_y (m rad)	2.064e-12
Eq. σ_δ	1.992e-3
Eq. σ_z (m)	1.687e-3

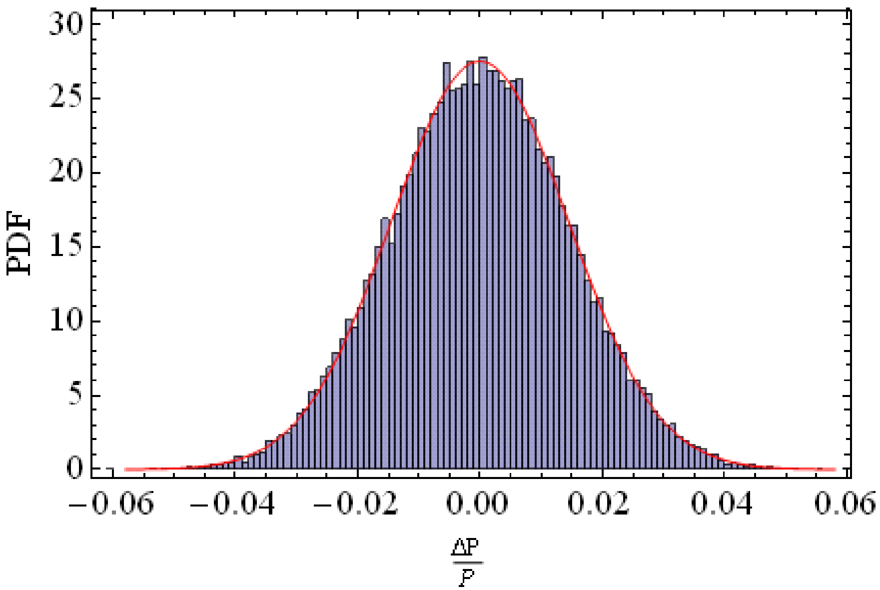
A. Vivoli

Bunch Distribution @ Last Turn: DAΦNE

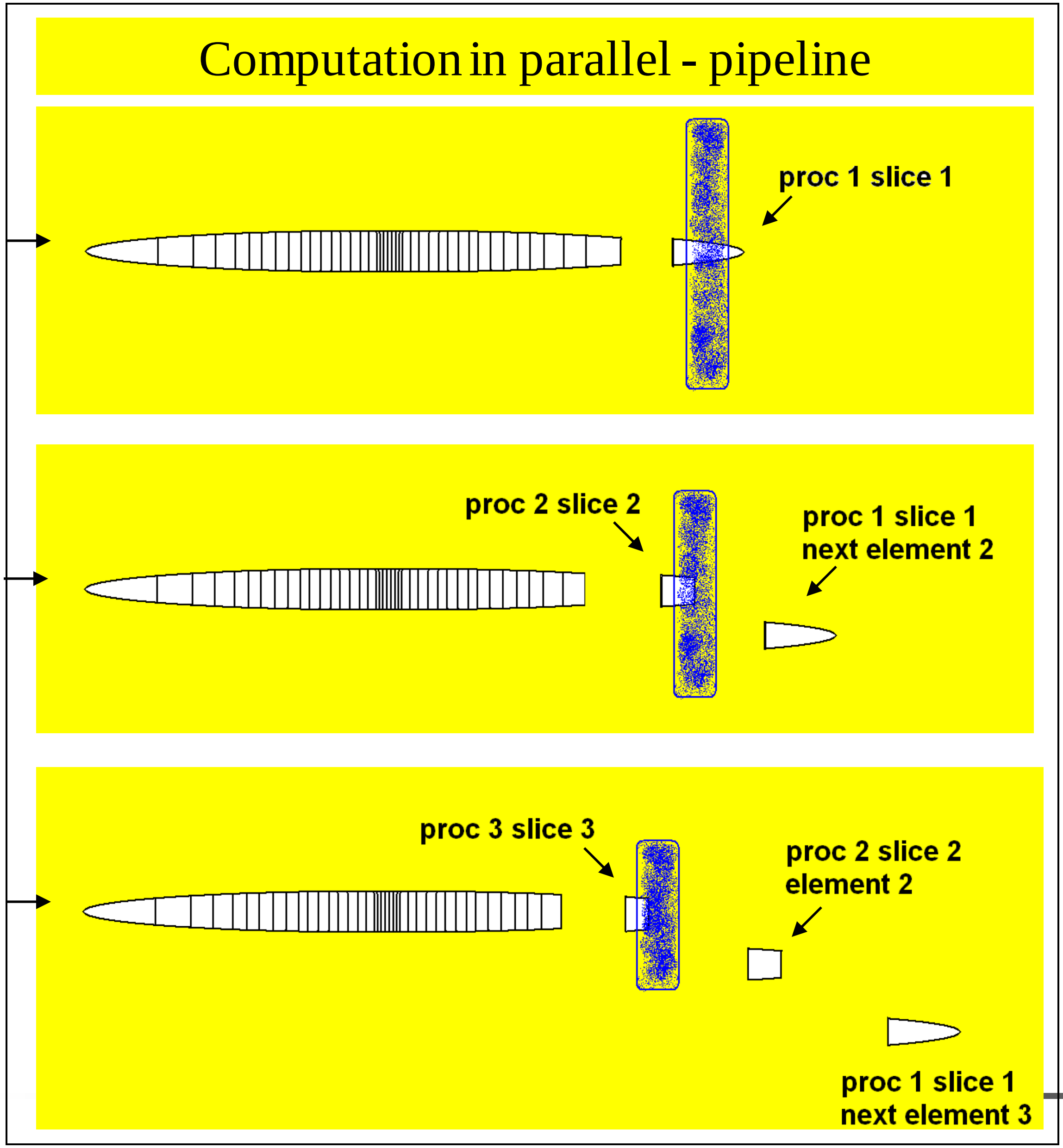


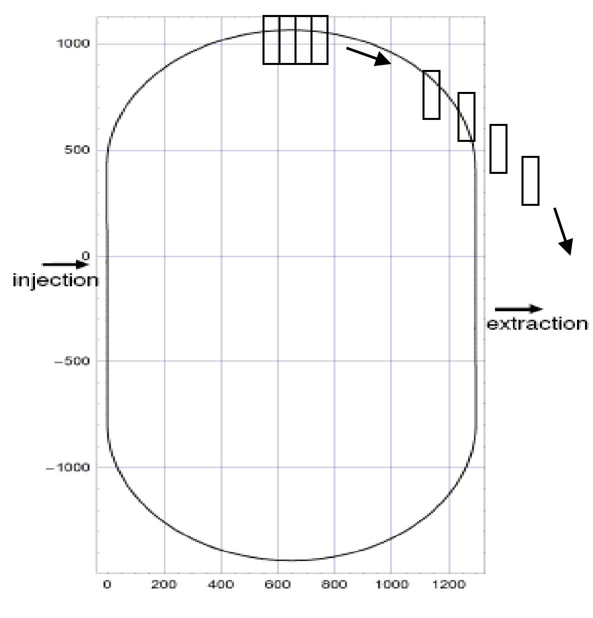
- The Kolmogorov-Smirnov Normality Test gives a confidence level >99% in all cases
- For DAFNE parm.s IBS is very small < 5% steady state emittance increase.





Bunch-slice parallel decomposition

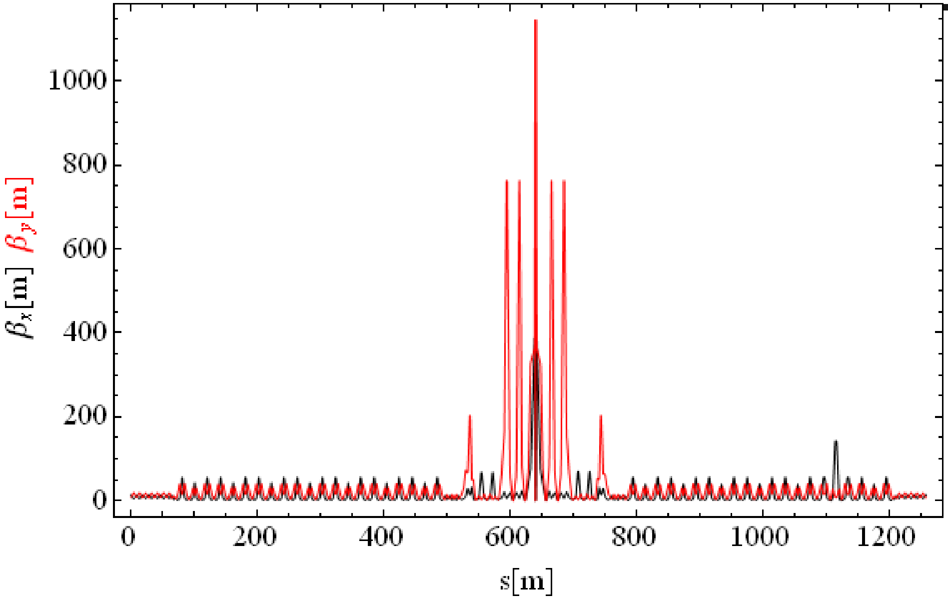


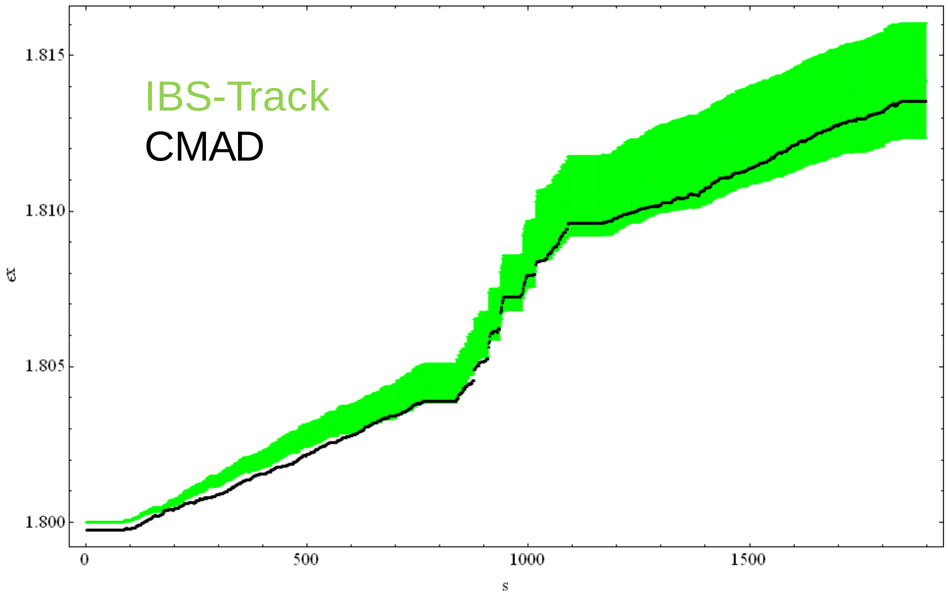


Each processor deals with the bunch-slice, then send information to the next in the pipeline. The last processor print out the beam information. At each turn, 1 processor gathers all particles and compute Radiation Damping and Quantum Excitation.

M. Pivi (SLAC)

SuperB LER (using CMAD)





SuperB V12 LER lattice (~1800IPs)

σ_z=5.0*10^{-3} m

δp=6.3*10^{-4}

ε_x=1.8*10^{-9} m

ε_y=0.25/100*ε_x

ppb= 5.7 10^{12}

MacroParticleNumber=3 x 10^5

Grid size = 10σ_y x 10σ_x

cells = 64 x 64

slices = 64

processors for this run = 64

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CPU timing studies: SuperB LER 1 turn

PRELIMINARY

Rperm	iCell	CPU Time Gain	N. of Processors
ON	1000	1	1
ON	1000	1.6	2
ON	1000	2	4
ON	1000	8.7	16
ON	1000	16.3	32
ON	1000	25	64

- Gain not linear: bunch slice decomposition not balanced
- Gain @ 64 CPU is only 25 but total execution time is well below 1 min..
- Still working on routines optimization

Summary

•Interesting aspects of the IBS such as its impact on damping process and on generation of non Gaussian tails may be investigated with a multiparticle algorithm.

•Two codes implementing the Zenkevich-Bolshakov algorithm to investigate IBS effects have been developed at LNF and at CERN:

– Benchmarking with conventional IBS theories gave good results (both codes).

–Evolution of the particle distribution shows deviations from Gaussian behaviour due to IBS effect (SIRE, CLIC-DR).

• Parallel implementation of the algorithm is ready :

–IBS routines included in CMAD (thanks to M. Pivi).

•Comparison of the code results with measurements at SLS and/or Cesr-TA would provide the possibility of

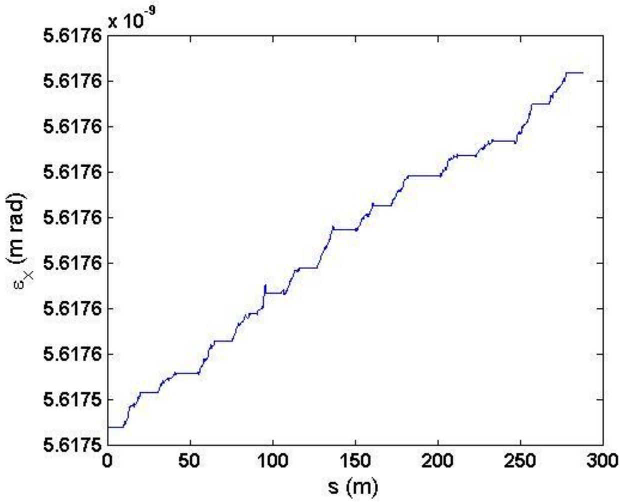
•Benchmarking with real data

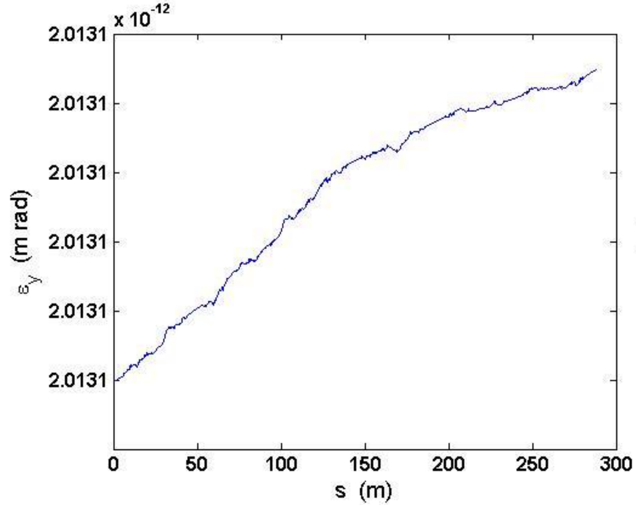
•Tuning code parameters (number of cells, number of interactions, etc.)

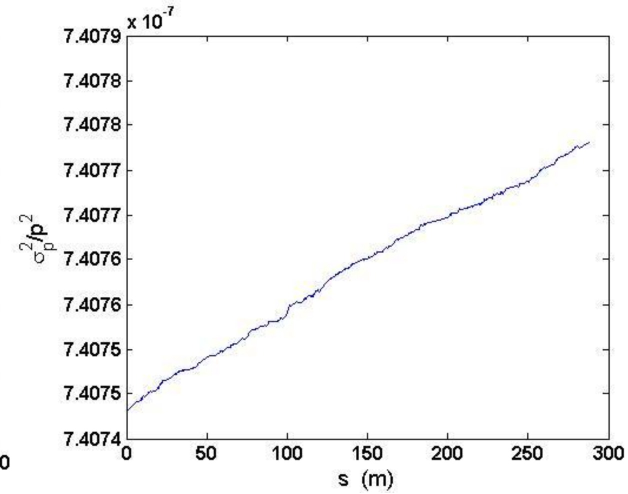
•Revision of the theory or theory parameters (Coulomb log, approximations used, etc.)

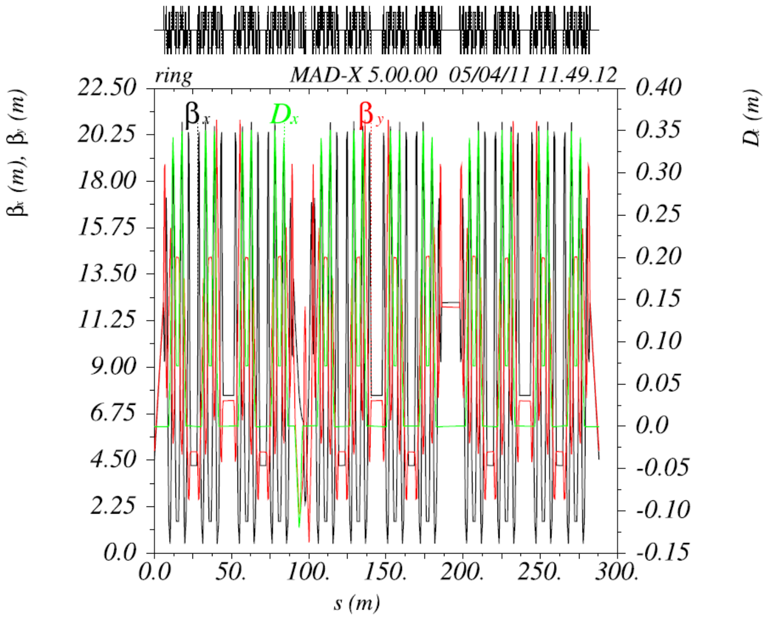
SPARES

SIRE: SLS simulations





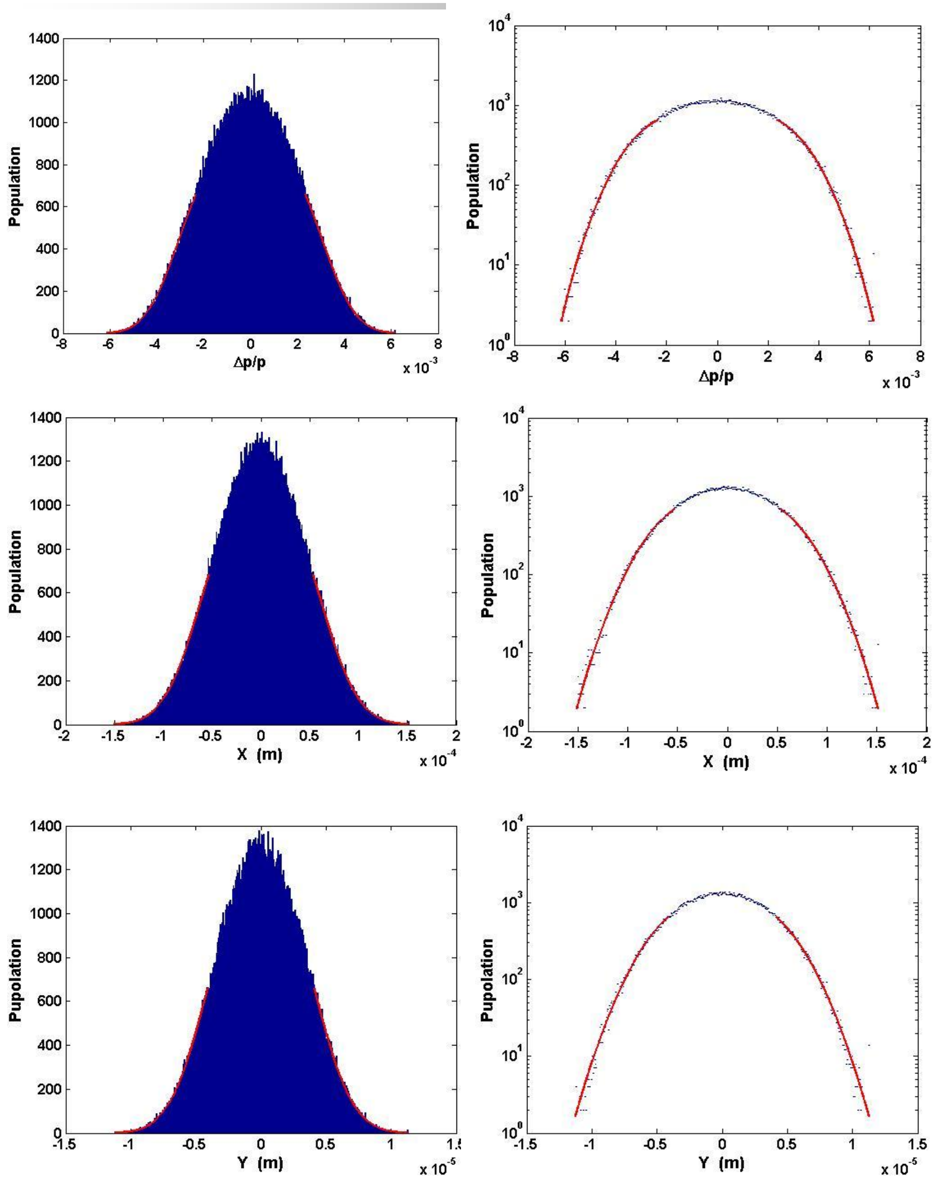




	1/Tx (s^{-1})	1/Ty (s^{-1})	1/Tz (s^{-1})
MADX (B-M)	20	37	59
SIRE (compressed)	15.6	24.5	47.2
SIRE (not compressed)	14.4	23.4	42.2

A. Vivoli, F. Antoniou

SIRE: IBS Distribution study D: IBS



$$p_k(\xi_k) = N_k e^{-\alpha_k (\xi_k^2)^{\beta_k}}$$

Parameter	χ^2_{37}	Confidence	Sample %
ΔP/P	38.81	0.39	26
X	36.73	0.48	25
Y	46.83	0.13	22

Parameter	Value
α_p	5.281e+7
β_p	1.568
α_x	3.840e+10
β_x	1.280
α_y	4.557e+12
β_y	1.196

A. Vivoli

Bjorken-Mtingwa

$$\frac{1}{T_i} = 4\pi A(\log)\left\langle \int_0^\infty \frac{\delta\lambda\lambda^{1/2}}{\left|L+\lambda I\right|^{1/2}} \left\{ TrL^{(i)} Tr\left(\frac{1}{L+\lambda I}\right) - 3TrL^{(i)}\left(\frac{1}{L+\lambda I}\right)\right\}\right\rangle$$

$$L^{(p)} = \frac{\gamma^2}{\sigma_p{}^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$L^{(x)} = \frac{\beta_x}{\varepsilon_x}\begin{pmatrix} 1 & -\gamma\phi_x & 0 \\ -\gamma\phi_x & \gamma^2 H_x/\beta_x & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$L^{(y)} = \frac{\beta_y}{\varepsilon_y}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \gamma^2 H_y/\beta_y & -\gamma\phi_y \\ 0 & -\gamma\phi_y & 1 \end{pmatrix}$$

$$L = L^{(p)} + L^{(x)} + L^{(y)}$$

$$A = \frac{r_0{}^2 cN}{64\pi^2 \overline{\beta}^3 \gamma^4 \varepsilon_x \varepsilon_y \sigma_s \sigma_p}$$

Piwinski

$$\frac{1}{T_p} = A\left\langle \frac{\sigma_H{}^2}{\sigma_p{}^2} f(a,b,q) \right\rangle$$

$$\frac{1}{T_x} = A\left\langle f\left(\frac{1}{a},\frac{b}{a},\frac{q}{a}\right) + \frac{H_x{}^2\sigma_{\mathrm{H}}{}^2}{\varepsilon_x} f(a,b,q) \right\rangle$$

$$\frac{1}{T_y} = A\left\langle f\left(\frac{1}{b},\frac{a}{b},\frac{q}{b}\right) + \frac{H_y{}^2\sigma_{\mathrm{H}}{}^2}{\varepsilon_y} f(a,b,q) \right\rangle$$

$$\frac{1}{\sigma_{\mathrm{H}}{}^2} = \frac{1}{\sigma_p{}^2} + \frac{H_x{}^2}{\varepsilon_x} + \frac{H_y{}^2}{\varepsilon_y}$$

$$a = \frac{\sigma_H}{\gamma}\sqrt{\frac{\beta_x}{\varepsilon_x}}, \qquad b = \frac{\sigma_H}{\gamma}\sqrt{\frac{\beta_y}{\varepsilon_y}}, \qquad q = \sigma_H \beta \sqrt{\frac{2d}{r_0}}$$

$$f(a,b,q) = 8\pi \int_0^1 du \frac{1-3u^2}{PQ}\left\{2\ln\left[\frac{q}{2}\left(\frac{1}{P}+\frac{1}{Q}\right)\right] - EulerGamma\right\}$$

$$P^2 = a^2 + (1-a^2)u^2, \qquad Q^2 = b^2 + (1-b^2)u^2$$

Bane's high energy approximation

- Bjorken-Mtingwa solution at high energies
- Changing the integration variable of B-M to $\lambda' = \lambda \sigma_H^2/\gamma^2$

- Approximations
  - a,b<<1 (if the beam cooler longitudinally than transversally)→ The second term in the braces small compared to the first one and can be dropped
  - Drop-off diagonal terms (let ζ=0) and then all matrices will be diagonal

$$(L+\lambda' I) = \frac{\gamma^2}{\sigma_H^2}\begin{pmatrix} a^2+\lambda' & -a\zeta_x & 0 \\ -a\zeta_x & 1+\lambda' & -b\zeta_y \\ 0 & -b\zeta_y & b^2+\lambda' \end{pmatrix}$$

$$\zeta_x = \phi_{x,y}\sigma_H\sqrt{\frac{\beta_{x,y}}{\varepsilon_{x,y}}}$$

$$\frac{1}{T_p} \approx \frac{r_0^2 cN(\log)}{16\gamma^3\varepsilon_x^{3/4}\varepsilon_y^{3/4}\sigma_s\sigma_p^3}\left\langle \sigma_H g(a/b)(\beta_x\beta_y)^{-1/4}\right\rangle$$

$$\frac{1}{T_{x,y}} \approx \frac{\sigma_p^2\langle H_{x,y}\rangle}{\varepsilon_{x,y}}\frac{1}{T_p}, \qquad g(a) = \frac{2\sqrt{a}}{\pi}\int_0^\infty \frac{du}{\sqrt{1+u^2}\sqrt{a^2+u^2}}$$