Intra-Beam Scattering Theory and Measurements

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Outline

- Intrabeam scattering theory
- Measurement—focusing on KEK’s ATF
- Recent PEP-X IBS calculations
- Conclusion

I will focus on electron machines

Intra-Beam Scattering (IBS)

The Touschek effect is a single Coulomb scattering event where energy transfer from transverse to longitudinal leads to immediate particle loss

IBS, in contrast, describes multiple scattering that leads to an increase in all bunch dimensions and in energy spread

In hadronic or heavy ion machines, IBS increases emittances with time; In low emittance e^- rings tends to increase the steady-state beam dimensions.

Theory of Touschek effect first developed by Touschek (1962), IBS by Brueck and LeDuff (1965). More systematic development of IBS theory by Piwinski (1974), Bjorken-Mtingwa (1983, using quantum mechanical scattering theory), Martini (1984, modification of Piwinski's formulation).

For a summary of theory and important references, see A. Piwinski in *Handbook of Accelerator Physics and Engineering*, 3rd Printing, Sec 2.5.9.

An interesting historical note: "Intrabeam scattering in electron and proton storage rings (a review)," A. Ruggiero, Fermilab-FN-413, Nov. 1984.

See talks at IBS07

Piwinski (P) Solution

Growth rates (in amplitude, no x-y coupling):

$$\frac{1}{T_p} = A\left\langle \frac{\sigma_h^2}{\sigma_p^2} f(a,b,q) \right\rangle, \quad \frac{1}{T_x} = A\left\langle f\left(\frac{1}{a}, \frac{b}{a}, \frac{q}{a}\right) + \frac{\eta_x^2 \sigma_h^2}{\beta_x \epsilon_x} f(a,b,q) \right\rangle$$

$$\frac{1}{T_y} = A\left\langle f\left(\frac{1}{b}, \frac{a}{b}, \frac{q}{b}\right) + \frac{\eta_y^2 \sigma_h^2}{\beta_y \epsilon_y} f(a,b,q) \right\rangle$$

with

$$A = \frac{r_0^2 c N}{64\pi^2 \bar{\beta}^3 \gamma^4 \epsilon_x \epsilon_y \sigma_s \sigma_p}, \quad \frac{1}{\sigma_h^2} = \frac{1}{\sigma_p^2} + \frac{\eta_x^2}{\beta_x \epsilon_x} + \frac{\eta_y^2}{\beta_y \epsilon_y}$$

$$a = \frac{\sigma_h}{\gamma}\sqrt{\frac{\beta_x}{\epsilon_x}}, \quad b = \frac{\sigma_h}{\gamma}\sqrt{\frac{\beta_y}{\epsilon_y}}, \quad q = \sigma_h \bar{\beta}\sqrt{\frac{2d}{r_0}},$$

and

$$f(a,b,q) = 8\pi \int_0^1 \left\{ 2\ln\left[\frac{q}{2}\left(\frac{1}{P} + \frac{1}{Q}\right)\right] - 0.577\ldots \right\} \frac{1-3u^2}{PQ} du$$

$$P^2 = a^2 + (1-a^2)u^2, \quad Q^2 = b^2 + (1-b^2)u^2$$

d= min(σ_x, σ_y), N is bunch population, $\bar{\beta}$= v/c

In beam frame velocities are non-relativistic, with longitudinal velocities σ_p<< transverse velocities $\gamma\sigma_{x'}$, $\gamma\sigma_{y'}$

Piwinski shows that, because of symmetry of *f(a, b, q)*,

$$\epsilon_s\left(\frac{1}{\gamma^2}-\left\langle\frac{\eta_x^2}{\beta_x^2}\right\rangle-\left\langle\frac{\eta_y^2}{\beta_y^2}\right\rangle\right)+\left\langle\frac{\epsilon_x}{\beta_x}\right\rangle+\left\langle\frac{\epsilon_y}{\beta_y}\right\rangle=\text{const.}$$

$\Rightarrow$below transition the sum of three positive invariants is limited, and an equilibrium can exist. Above transition ε_x, ε_y, ε_s, can grow simultaneously and an equilibrium does not exist

Piwinski's solution finds IBS at locations where η_x', η_y'= 0. Martini generalized his solution by, most importantly, replacing η_x^2/β_x by dispersion invariant.

Bjorken-Mtingwa (BM) Formulation

IBS (amplitude) growth rates (i= x, y, or p):

$$\boxed{\begin{aligned}\frac{1}{T_i} &= 4\pi A(\log)\left\langle \int_0^\infty \frac{d\lambda\,\lambda^{1/2}}{[\det(L+\lambda I)]^{1/2}}\left\{ \right.\right.\\ &\left.\left. TrL^{(i)}Tr\left(\frac{1}{L+\lambda I}\right) - 3TrL^{(i)}\left(\frac{1}{L+\lambda I}\right)\right\}\right\rangle\end{aligned}}$$

with

$$A = \frac{r_0^2 cN}{64\pi^2\bar{\beta}^3\gamma^4\epsilon_x\epsilon_y\sigma_s\sigma_p}$$

$$L = L^{(p)} + L^{(x)} + L^{(y)} \quad ,$$

$$L^{(p)} = \frac{\gamma^2}{\sigma_p^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad ,$$

(log) is Coulomb log

$$L^{(x)} = \frac{\beta_x}{\epsilon_x}\begin{pmatrix} 1 & -\gamma\phi_x & 0 \\ -\gamma\phi_x & \gamma^2\mathcal{H}_x/\beta_x & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

r_0 is classical radius of electron

$\bar{\beta} = v/c$

$$L^{(y)} = \frac{\beta_y}{\epsilon_y}\begin{pmatrix} 0 & 0 & 0 \\ 0 & \gamma^2\mathcal{H}_y/\beta_y & -\gamma\phi_y \\ 0 & -\gamma\phi_y & 1 \end{pmatrix}$$

$H_x = [\eta_x^2 + (\beta_x\eta_x' - \beta_x'\eta_x/2)^2]/\beta_x$

$\phi_x = \eta_x' - \beta_x'\eta_x/2\beta_x$

Finding T_i^{-1} means performing a type of elliptical integral at all lattice positions

Nagaitsev algorithm for evaluating elliptical integrals speeds up the calculation a factor of ~25 (in Mathematica)

For finite result, Coulomb scattering calculation needs to be cut-off at large and small impact parameters, by (log)= log(b_{max}/b_{min}). Expected accuracy ~1/(log). For low emittance electron rings (log)~ 10 (including Raubenheimer tail cut).

Vertical emittance in a ring is usually due to either η_y, produced by orbit errors, and/or by *x*-*y* coupling. Formulas so far have been without coupling. IBS with coupling is described in A. Piwinski (1991), B. Nash et al (2002), V. Lebedev (2005).

Steady-State Emittances

In electron machines the IBS growth is counteracted by synchrotron radiation damping (with τ_i^{-1}>> T_i^{-1}), leading to increased stead-state emittances

Steady-state IBS emittance and energy spread (no *x*-*y* coupling):

$$\epsilon_x = \frac{\epsilon_{x0}}{1 - \tau_x / T_x}, \quad \epsilon_y = \frac{\epsilon_{y0}}{1 - \tau_y / T_y}, \quad \sigma_p^2 = \frac{\sigma_{p0}^2}{1 - \tau_p / T_p}$$

Solution involves (i) integration at every lattice element to obtain T_i^{-1}, (ii) averaging around the ring, (iii) solving the above three equations simultaneously (e.g. using Newton's method)

A heuristic approach often used for coupling dominated machine, with κ small, is to solve only ε_x, σ_p^2, equations and take $\varepsilon_y = \kappa\varepsilon_x$

Programs that solve IBS (mostly BM formulation) are ZAP, SAD, MAD-X, Elegant, …

SAD treats the three axes equally and includes coupling (e.g. *x*-*y*, *x*-*p*) in a general way by diagonalizing to normal modes

--The growth in the normal mode momentum is given by

$$\frac{\Delta\langle w_1^2\rangle}{\Delta t} = c_I[(g_2 - g_1) + (g_3 - g_1)],$$

$$\frac{\Delta\langle w_2^2\rangle}{\Delta t} = c_I[(g_1 - g_2) + (g_3 - g_2)],$$

$$\frac{\Delta\langle w_3^2\rangle}{\Delta t} = c_I[(g_1 - g_3) + (g_2 - g_3)],$$

From K. Kubo and K. Oide, PRST-AB 4, 124401 (2001)

where we see that the sum of the growth in three directions is zero

--SAD adds IBS as a momentum diffusion term

IBS calculation can be time consuming. Thus, simplified models have been developed by Parzen, Le Duff, Raubenheimer, Wei, …

A Simplified Model of IBS

(Coupling dominated version: $\varepsilon_y = \kappa\varepsilon_x$)

Longitudinal growth rate:

$$\frac{1}{T_p} \approx \frac{r_e^2 c N_b (\log)}{16\gamma^3 \epsilon_x^{3/4} \epsilon_y^{3/4} \sigma_z \sigma_p^3} \left\langle \sigma_H \, g(a/b) \; (\beta_x \beta_y)^{-1/4} \right\rangle$$

$$\frac{1}{\sigma_H^2} = \frac{1}{\sigma_p^2} + \frac{\mathcal{H}_x}{\epsilon_x} \,, \quad a = \frac{\sigma_H}{\gamma}\sqrt{\frac{\beta_x}{\epsilon_x}} \,, \quad b = \frac{\sigma_H}{\gamma}\sqrt{\frac{\beta_y}{\epsilon_y}}$$

$$g(\alpha) = \alpha^{(0.021 - 0.044 \ln \alpha)}$$

Transverse growth rate:

$$\frac{1}{T_x} = \frac{\sigma_p^2}{\epsilon_x} \langle \mathcal{H}_x \rangle \frac{1}{T_p} \quad \Longrightarrow \quad \frac{1}{T_x} = \frac{\sigma_p^2}{\epsilon_x} \langle \mathcal{H}_x \delta(1/T_p) \rangle$$

Valid for a, b<< 1, “High Energy approximation”

- Can be shown that, in this high energy regime, BM agrees with “modified P” (Piwinski with $\eta_x^2/\beta_x \rightarrow H_x$)

Recent IBS Calculations

Macroparticle tracking based on a binary collision model. For SuperB simulated 10^5 collisions, at each lattice element, until steady-state was reached—using many parallel processors. Will be useful for studying collision distributions (M. Biagini et al, IPAC 2011)

IBS can also be significant in the high brightness beams in the driver of a linac-based x-ray FEL (where $\sigma_p << \gamma\sigma_{x'}, \gamma\sigma_{y'}$):

--In LCLS, IBS is big enough to reduce microbunching instability (though not to interfere with lasing) (Z. Huang, LCLS-TN-02-8).

--Can limit Echo-Enabled Harmonic Generation (EEHG) to <~ 100 (G. Stupakov, FEL'11).

Measurements

IBS was measured in the ISR (1975), the SPS (1985), the AA (1984) at CERN, and deviations from theory was found to be <10—20%

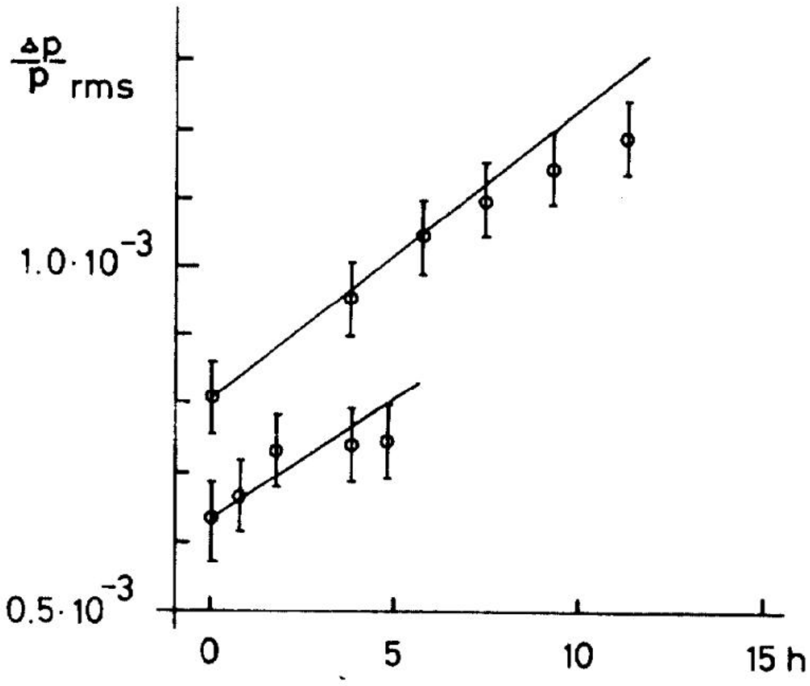


Fig. 3 – Rms momentum spread in an 1 A beam versus time at 26,6 GeV/c. Circles: measured points; full lines: computed growth.

K. Huebner, PAC75, p.1416

RHIC

Group at RHIC is maybe most active recently in IBS measurements and comparison with theory. Measurements and understanding of theory are improving. Latest reports shows near perfect agreement.

A. Fedotov et al, HB2006, p. 259

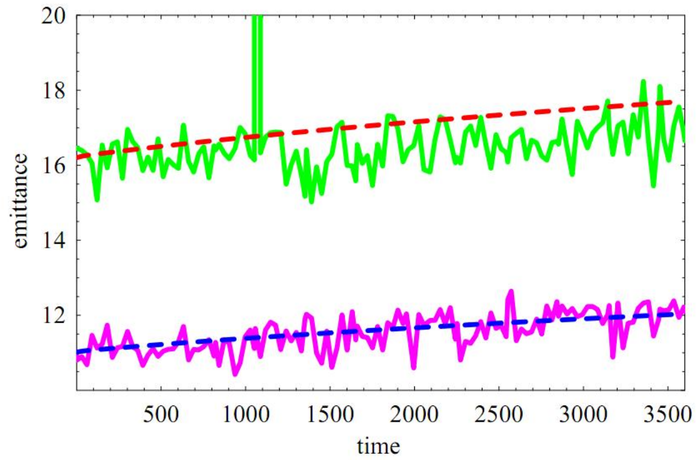


Figure 2: Horizontal and vertical 95% normalized emittance [μm] vs time [sec] for bunch intensity 2.9×10^9 Cu ions. Measured emittance: top green curve (horizontal), bottom pink curve (vertical). BETACOOL simulation using Martini's model: top red dash line (horizontal), blue dash line (vertical).

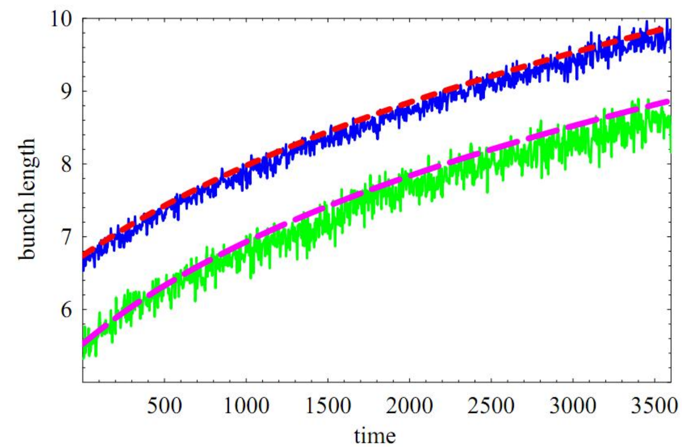


Figure 3: Growth of FWHM bunch length [ns] vs time [sec] for two bunch intensities: 2.9×10^9 (upper curve) and 1.4×10^9 (lower curve) Cu ions. Dash lines - simulations.

Tevatron

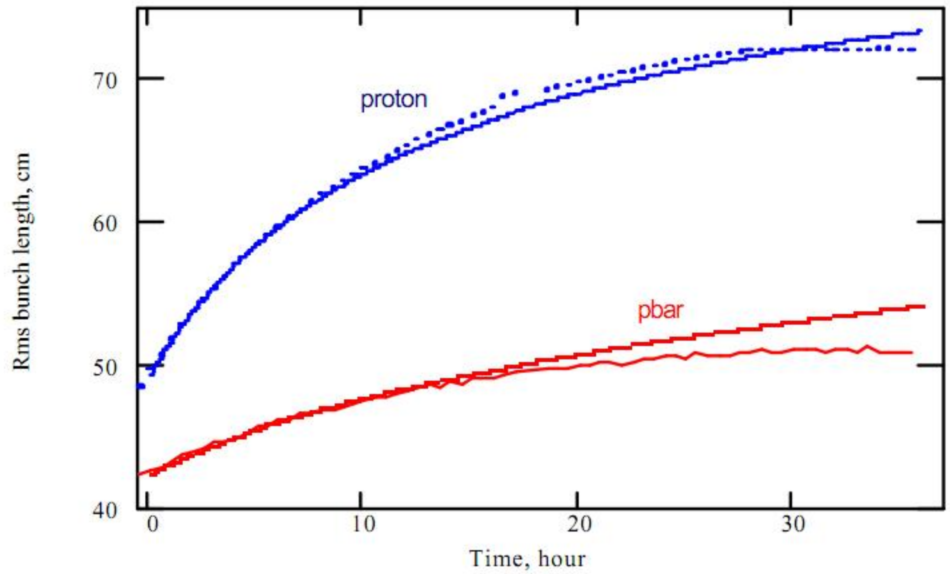


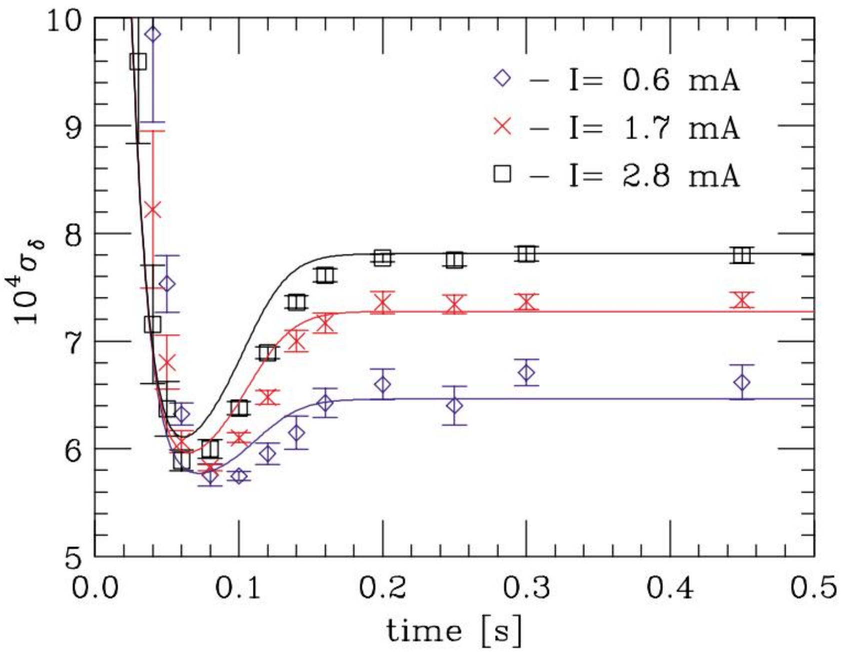
FIGURE 1. Measured (dotted line) and computed (solid line) dependence of rms bunch length on time for Store 3678.

V. Lebedev developed formalism to include x-y coupling, and non-linear and finite bucket size. In this comparison he included also effects: scattering on residual gas, particle loss due to luminosity, diffusion due to RF noise (V. Lebedev, AIP Conf. Proc. 773(1), 440 (2005))

KEK's Accelerator Test Facility (ATF)

ATF is an electron storage ring with C= 138 m, E= 1.28 GeV, ε_{x0}~ 1 nm, ε_{y0}~ 10 pm, maximum N~ 10^{10}. Complete IBS measurements performed in short time in April 2000

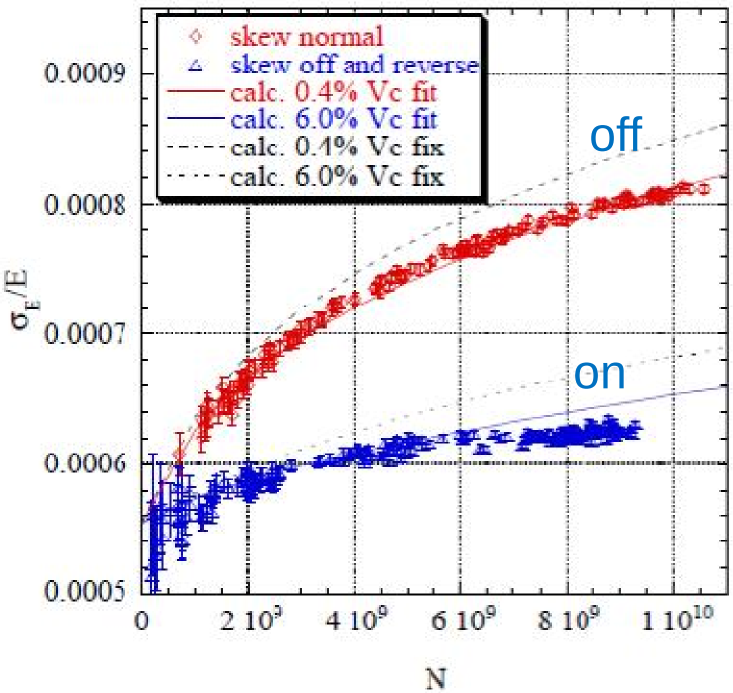
At ATF all bunch dimensions can be measured. Unique is that σ_p can be measured to a few percent (at high dispersion point after extraction). λ_z measured by streak camera.



Measured energy spread as function of time after injection, for three different currents (the plotting symbols). The curves give BM simulations assuming no potential well distortion.

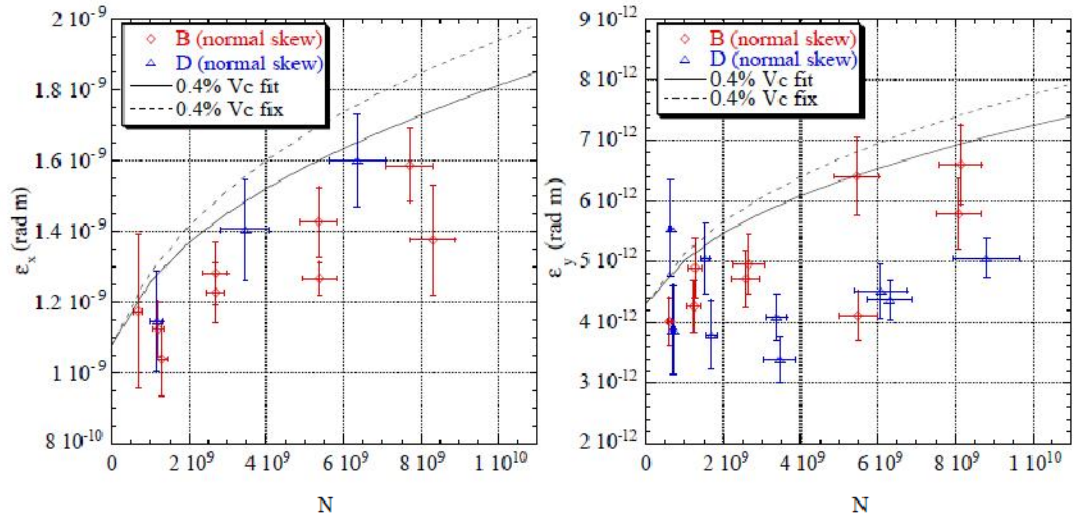
Evidence that we are seeing IBS: (i) on coupling resonance, σ_p becomes small, (ii) when reducing η_y, using dispersion correction, σ_p increases

ε_x, ε_y, measured by wire monitors (after extraction). Were also measured by laser wire in ring—at the time of measurement, very time consuming and large scatter in results.



σ_p vs N, on and off coupling resonance

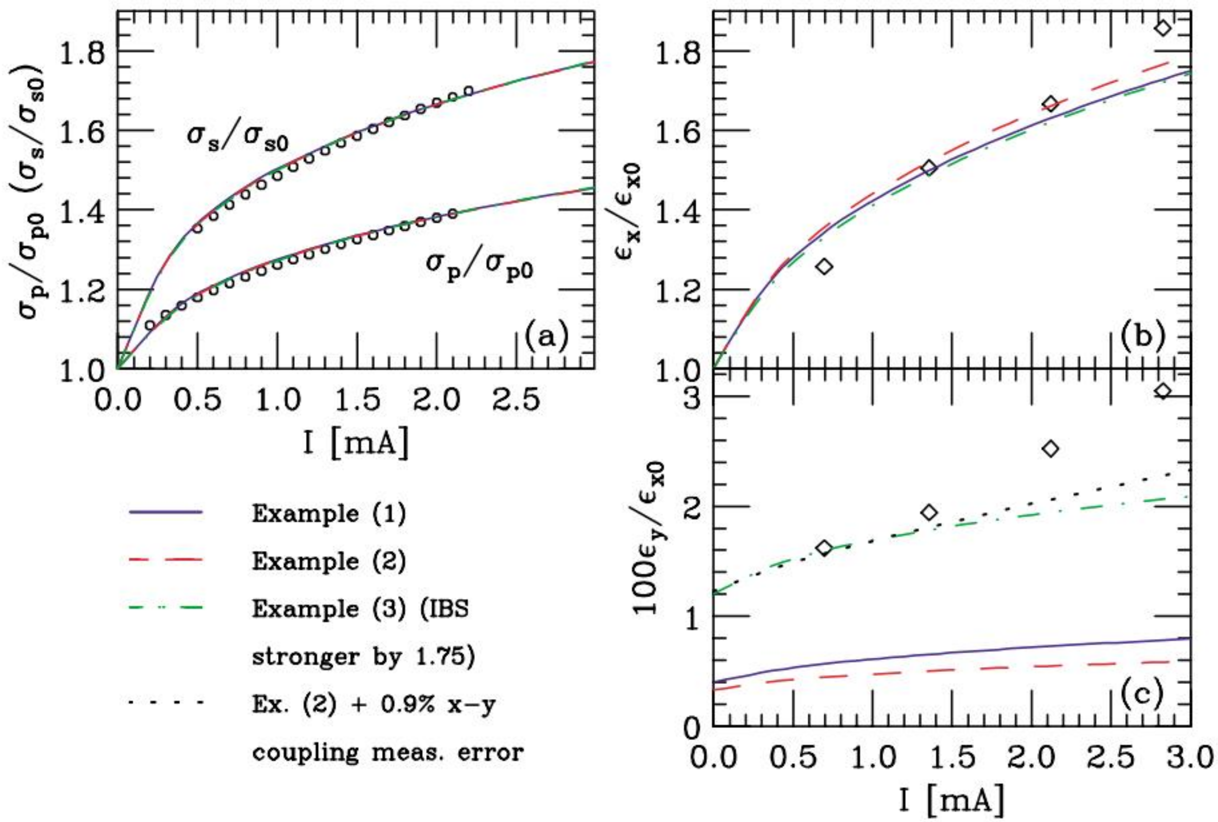
K. Kubo



Laser wire measurement of ε_x (left), ε_y (right) vs bunch population N

Potential well distortion is large in the ATF. We include this in the calculations by adding multiplicative factor $f_{pw}(I)$, obtained from measurement, to equation relating σ_s to σ_p

To compare with measurement, we set our one free parameter ε_{y0} so that σ_p agrees with measurement at large *I*



Examples:
(1) Vertical dispersion only, with $(\eta_y)_{rms}$= 5.6 mm

(2) Coupling dominated with κ= 0.33%

(3) Coupling dominated with κ= 1.2%, and IBS stronger by 1.75

(4) Same as (2) + 0.9% coupling meas. error

ATF measurement data (symbols) and IBS theory fits (the curves). The symbols in (a) give the smooth curve fits to measured data.

Note: after correction, typically $(\eta_y)_{rms} \approx 3$ mm

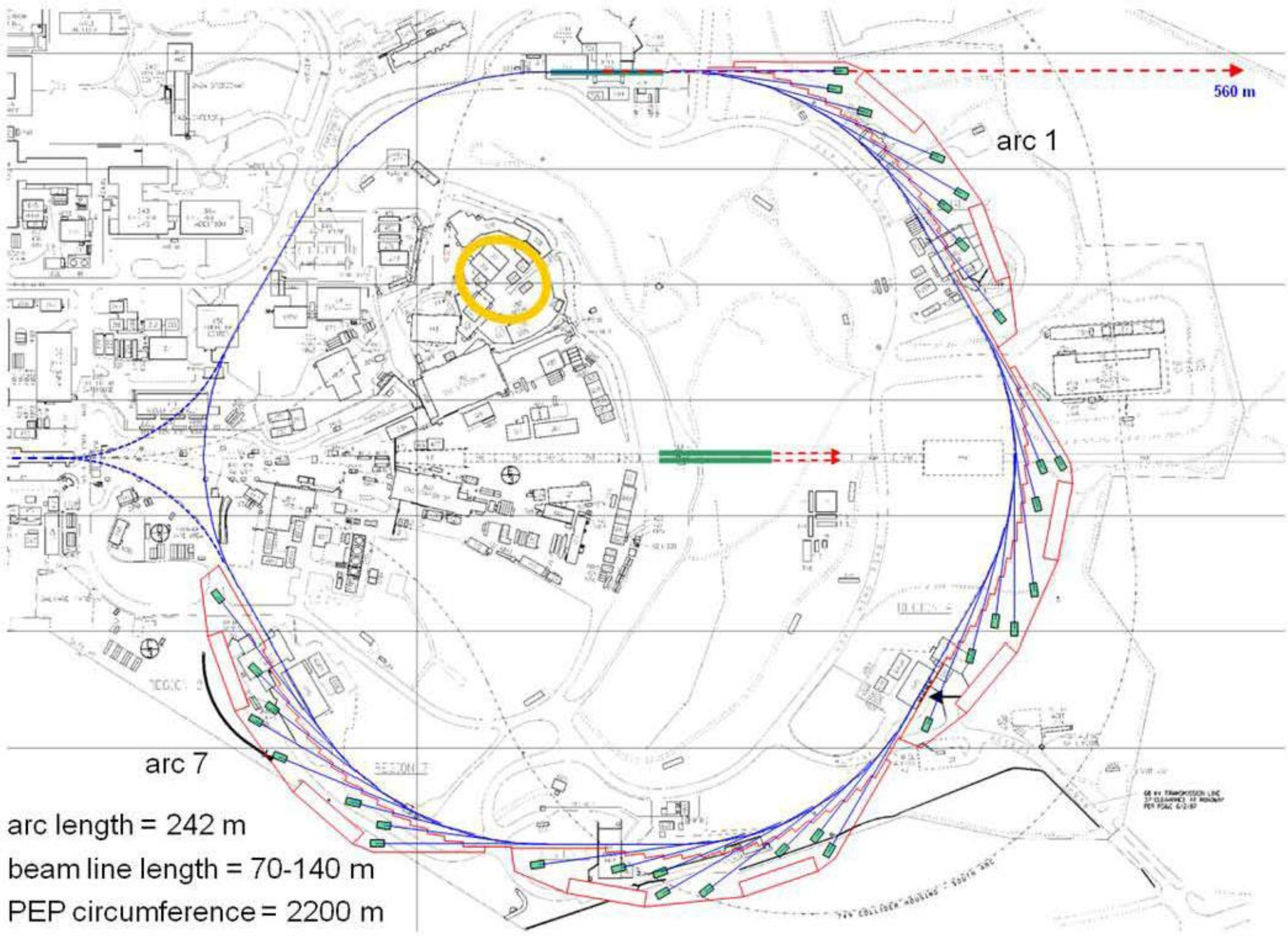
Detailed SAD simulations (including random rotations of quads and sextupoles, orbit errors, correction) find that when σ_p, σ_z, ε_x, agree with measurement the projected $\tilde{\varepsilon}_{y0}/\varepsilon_{x0} = (0.8 \pm 0.2)\%$; at I= 2.8 mA, $\tilde{\varepsilon}_y/\varepsilon_{x0} = (1.3 \pm 0.3)\%$, in better agreement—but still a factor 2 off at high current

ε_y measured in the extraction line was larger than that measured in the ring by laser wire. Now it is believed that field errors in the septum and a quad (in the extraction line) were responsible for a larger ε_y in the extraction line (K. Kubo)

The ATF has been improved in many ways over the last 10 years. It may be time to redo the series of IBS measurements

• CESR-TA has begun taking IBS measurements

PEP-X: An Ultimate Storage Ring



PEP-X Main Parameters

Lattice based on that of MAX IV, and optimized

Energy, GeV	4.5
Circumference, m	2199.32
Natural emittance, pm	11
Beam current, mA	200
Emittance at 200 mA, x/y, pm	12 / 12
Tunes, x/y/s	113.23 /65.14/0.007
Bunch length, mm	3.1
Energy spread	1.25×10^{-3}
Energy loss per turn, MeV	2.95
RF voltage, MV	8.3
RF harmonic number	3492
Length of ID straight, m	5.0
Wiggler length, m	90.0
Beta at ID center, x/y, m	4.92 / 0.80
Touschek lifetime, hour	3
Dynamic aperture, mm	10

IBS at PEP-X

(Thanks to A. Xiao, M. Borland, K. Kubo)

Run PEP-X near full coupling (κ= 1), so that ε_x= ε_y near diffraction limit for 1 angstrom light (8 pm)

To get approx. steady-state equation for ε_x, assume

$$\frac{\epsilon_{x0}}{\tau_x} + \frac{\epsilon_{y0}}{\tau_y} - \frac{\epsilon_x}{\tau_x} - \frac{\epsilon_y}{\tau_y} + \frac{\epsilon_x}{T_x} = 0$$

First two terms represent quantum excitation, next two radiation damping, last one IBS. If ε_x= $\varepsilon/(1+\kappa)$, ε_y= $\kappa\varepsilon_x$, ε_{x0}= $\varepsilon_0/(1+\kappa)$, ε_{y0}= $\kappa\varepsilon_{x0}$, =>

$$\epsilon_x = \frac{\epsilon_{x0}}{1 - \tau_x^*/T_x}$$

with τ_x^*= $\tau_x/(1+\kappa\tau_x/\tau_y)$, (and σ_p^2 equation as before).

κ	ε_{x0} [pm]	ε_x[pm]	ε_y [pm]	σ_p [10^{-3}]	σ_z [mm]	*T* [hrs]
1	5.5	11.8	11.8	1.15	3.15	3

Table. PEP-X IBS calculation results: coupling parameter, nominal (zero-current) emittance, and steady-state beam properties. The last column gives the Touschek lifetime (discussed below).

Note: almost no growth in p or z

At steady-state: T_x^{-1}= 52 s^{-1}, T_p^{-1}= 7.4 s^{-1} (high energy model gets T_x^{-1}= 53.7 s^{-1}, T_p^{-1}= 8.9 s^{-1})

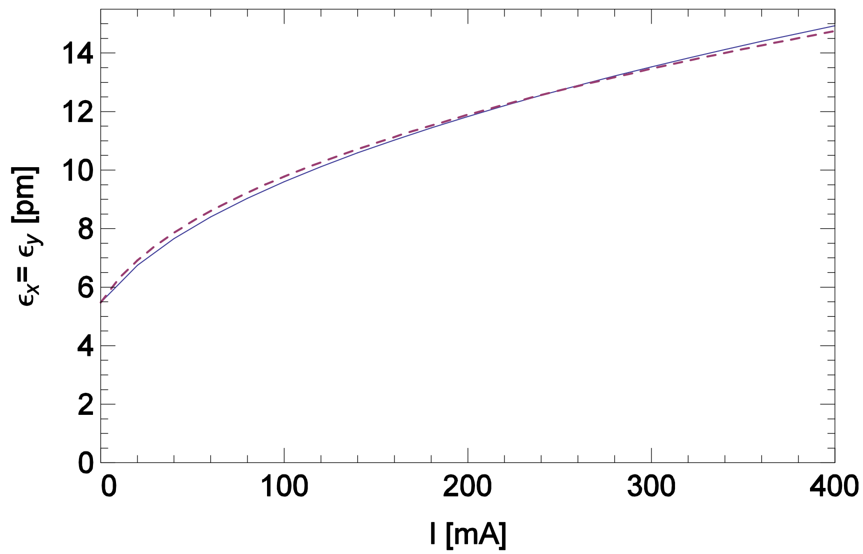
The result was checked by a more careful calculation with SAD (K. Kubo):
In dispersion-free regions: (i) quad strengths were adjusted to bring tunes close together, (ii) 800 quads were randomly rotated and then scaled to give $\varepsilon_{x0} \approx \varepsilon_{y0}$, (iii) IBS was calculated, and (iv) process was repeated for 10 seeds.

Result: $\varepsilon_x \approx \varepsilon_y \approx$ 11 pm, in reasonable agreement with our 11.8 pm. (Also agrees with Elegant.)

With little growth in σ_p, High Energy model gives:

$$\left(\frac{\epsilon_x}{\epsilon_{x0}}\right)^{5/2} - \left(\frac{\epsilon_x}{\epsilon_{x0}}\right)^{3/2} = \alpha\left(\frac{I}{I_A}\right)$$

with α a constant and I_A= 17 kA. Here the best fit is for α= 3.2×10^5.



Steady-state emittances as function of bunch current in PEP-X.
The dashed line represents the fit to the equation above.

- Touschek lifetime 3 hrs: based on momentum acceptance (with errors) +/–1.5%

Conclusions

Focusing on electron machines have

- Described the theory of IBS
- Discussed measurements, particularly at KEK’s ATF
- Given results for the latest iteration of PEP-X