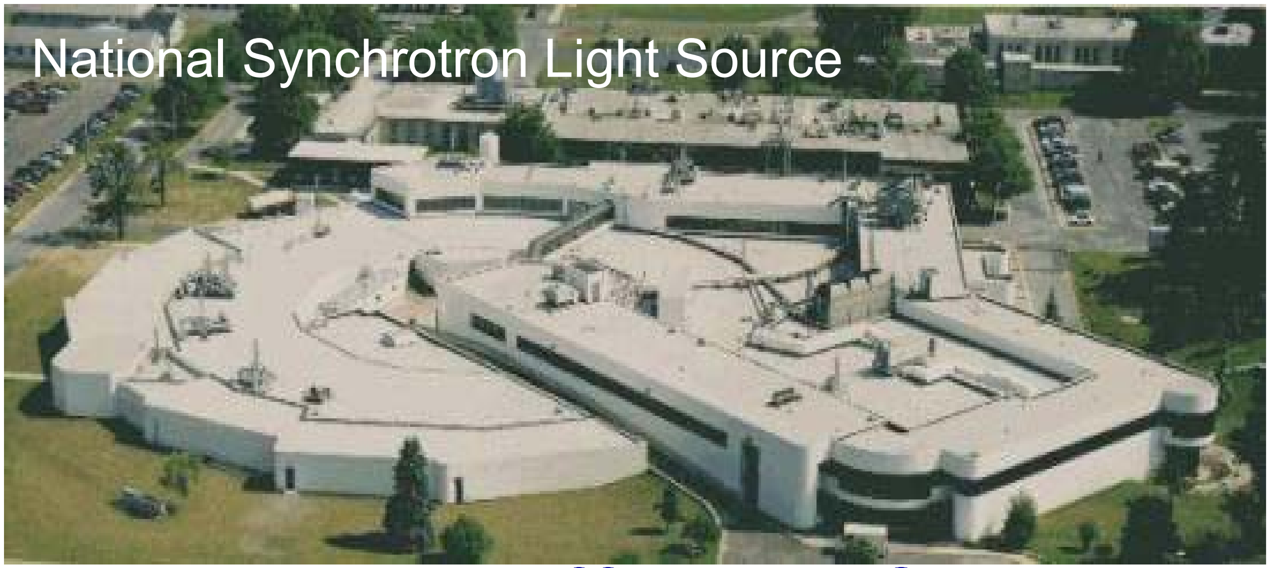
LER-2011, Heraklion, Greece



Physics and Efficient Calculation of Short Bunch Wakefields

Boris Podobedov (BNL) & Gennady Stupakov (SLAC)

boris@bnl.gov

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Outline

- **Motivation**
  - **General: why we care about short-bunch wake fields**
  - **How short is short**
  - **Why new methods/ideas are still needed**

- **New Method of Short-Bunch Wake Calculations: Basic Idea**
  - **The claim**
  - **Optical Model, Diffraction Model**
  - **Basic Idea**
  - **How to apply in practice**

- **New Method: Illustrative Examples**
  - **Simple stepped collimator**
  - **Simple cavity**
  - **Tapers & tapered collimators**
  - **Tesla 9-cell cavity**

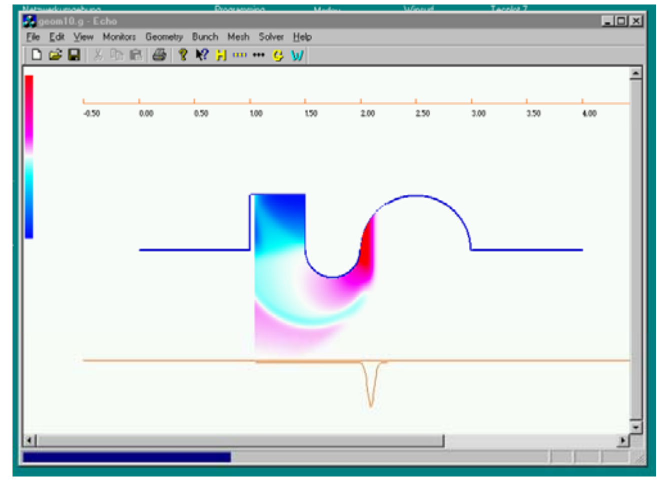
- **Summary**

References

1. *BP, GS, PAC-2011: basic idea of the method*
2. *BP, GS, PRST-AB (to be submitted): expanded version*
3. *GS, BP, PRST-AB (2010): short-bunch wake of a slowly tapered collimator from finite BW impedance calculations*

Motivation

- Knowledge of wakefields, incl. geometric ones, is critically important for accelerator beam dynamics.
- Detailed wakefield calculations for realistic vacuum chambers are done with time domain EM solvers, which calculate the fields due to finite length bunches.
- Extremely fine meshes are needed to compute wakes at small distances, where wake singularities dominate => calc's are slow and lots memory is req'd.

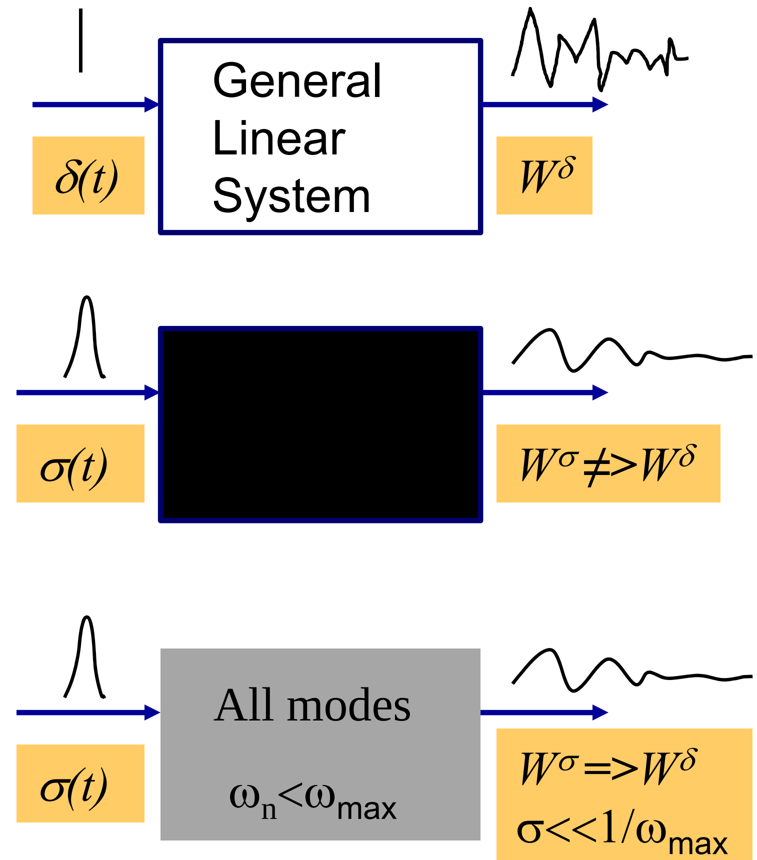


Electromagnetic
Code for
Handling
Of
Harmful
Collective
Effects

We suggest how to calculate short bunch wake-potentials, and even point-charge wakefields, from EM solver results for a long bunch. This saves greatly on calculation speed and provides physical insights.

We Are Not
Getting Something out of Nothing

- Can one get a δ-function impulse response, W^{δ}, using a finite duration Gaussian input, $\sigma(t) \sim \exp(-t^2/2\sigma^2)$?
- Or, equivalently, a frequency response, $Z(\omega)$, over infinite freq. range with finite BW excitation?
  - No, if the system is a black-box.
  - Yes, if the system is a gray-box, for example a set of harm. oscillators with (all normal modes) $\omega_n < \omega_{max}$



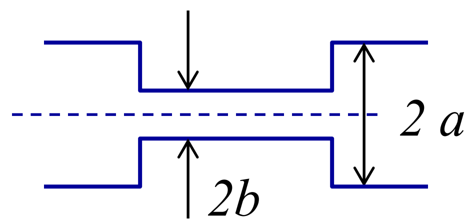
We claim that the problem of geometric impedance is similar to "gray box", since ω_{max}, as well as $Z(\omega \rightarrow \infty)$ asymptotic are known, or can be easily calculated. Similar is true in time-domain.

Preliminaries

- For simplicity we consider 2D geometry, longitudinal wakefields, and non-periodic structures.
- Extension to the transverse and/or to periodic structures is trivial.
- Extension to 3D case should be possible as well (work in progress)

Asymptotic Model for Short-Bunch Wakefields of Collimator-like Structures

- For collimator-like structures we use optical model:



$$W_{opt}^{\delta}(z) = k_{opt}\,\delta(z)$$

wake-function

$$W_{opt}^{\sigma}(z) = k_{opt}\,(2\pi)^{-1/2}\,\sigma^{-1}\,e^{-\frac{z^2}{2\sigma^2}}$$

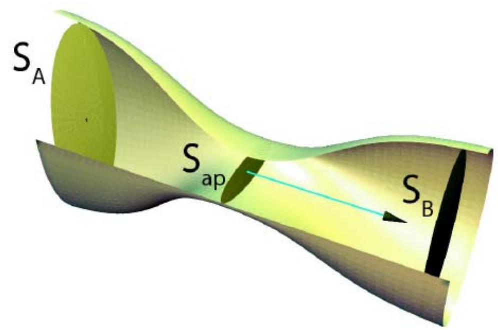
wake-potential

$$k_{opt} = -Z_0\, c\, Log(a/b)/\pi$$

$$W_{opt}^{\delta}(z \to 0) = \infty$$

$$W_{opt}^{\sigma\to 0}(z) = \infty$$

- Turns out this model describes all collimator-like structures, including 3D; A recipe to calculate geometry-dependent k_{opt} exists [see Stupakov, Bane, Zagorodnov, PRST-AB 10, 054401 (2007)]



Asymptotic Model for Short-Bunch Wakefields of Cavity-like Structures

- For cavity-like structures we use the diffraction model:

$$W_d^{\delta}(z) = k_d\, z^{-1/2}, \quad z > 0$$

wake-function

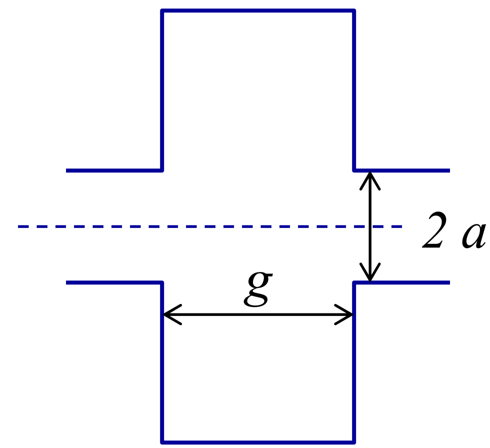
$$W_d^{\sigma}(z) = \frac{k_d}{\sqrt{\sigma}} f\left(z/\sigma\right)$$

wake-potential

$$k_d = -\frac{Z_0 c}{\pi^2 a}\sqrt{g/2}$$

$$f(s) = e^{-s^2/4}\sqrt{\tfrac{\pi}{8}|s|}\left(\mathrm{I}(-\tfrac{1}{4},\tfrac{s^2}{4}) + \mathrm{sign}(s)\,\mathrm{I}(\tfrac{1}{4},\tfrac{s^2}{4})\right)$$

I(…) are Bessel functions



- Wake-potentials for all cavity shapes (tapered or not, deep or shallow, etc.) converge to this model for short enough bunches and distances.
- Model is easily expandable to 3D geometries.

New Method: Idea

- Since short bunch asymptotic wake-potentials are singular at σ->0, and calculable analytically, we propose to subtract these singularities.

- The remaining, non-singular part of the wake-potential, $D^{\sigma}(z)$, does not change rapidly with σ. Furthermore, $D^{\sigma}(z)$ is almost σ-independent below certain bunch length $\underline{\sigma} << \lambda_g$, where λ_g is the shortest length-scale in the geometry of the structure.

- To find the wake of an arbitrary short bunch, it is thus sufficient to run the EM solver with drive-bunch length slightly below $\underline{\sigma}$. This allows for coarse mesh, relaxed memory requirements and faster calculations.

- This method allows one to calculate δ-function wakefields as well.

New Method: Details

- We represent the wake-potential as a sum of two parts, a short-bunch asymptotic model (that includes all singularities at $\sigma \to 0$), and function $D^{\sigma}(z)$ that remains finite for arbitrarily short bunches:

$$W^{\sigma}(z) = W^{\sigma}_{\mathrm{model}}(z) + D^{\sigma}(z).$$

- For bunches shorter than certain length $\underline{\sigma}$, the σ-dependence of $D^{\sigma}(z)$ is rather weak, so it can be dropped,

$$D^{\sigma}(z) \approx D(z), \quad 0 \le \sigma \le \underline{\sigma}.$$

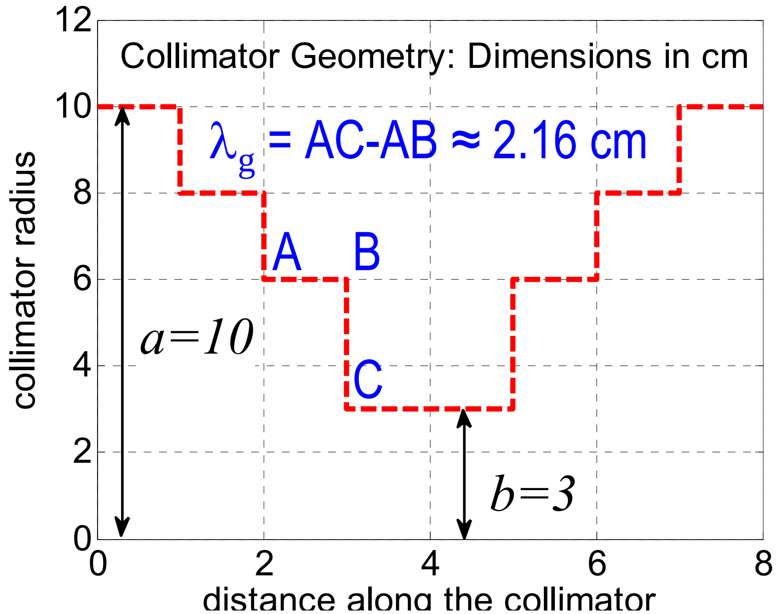
- We pick a bunch length $\sigma_0 \le \underline{\sigma}$, run EM solver to get $W^{\sigma_0}(z)$, and then find $D(z)$. A wake-potential of a shorter bunch is then*

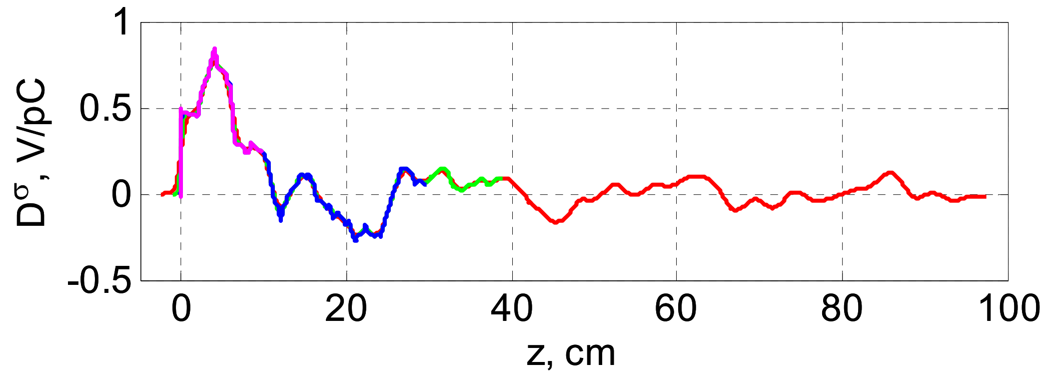
$$W^{\sigma}(z) \approx W^{\sigma}_{\mathrm{model}}(z) + D(z), \qquad 0 \le \sigma \le \sigma_0.$$

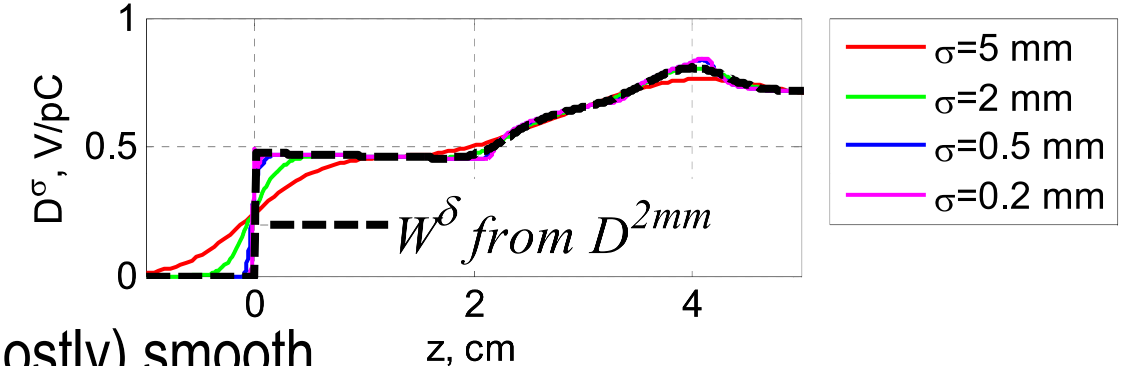
$$\boxed{W^{\delta}(z) \approx W^{\delta}_{\mathrm{model}}(z) + D(z), \quad \text{for point-charge, z>0}}$$

* *a better approximation at $z <$ few σ_0 to follow*

A Stepped Collimator Structure



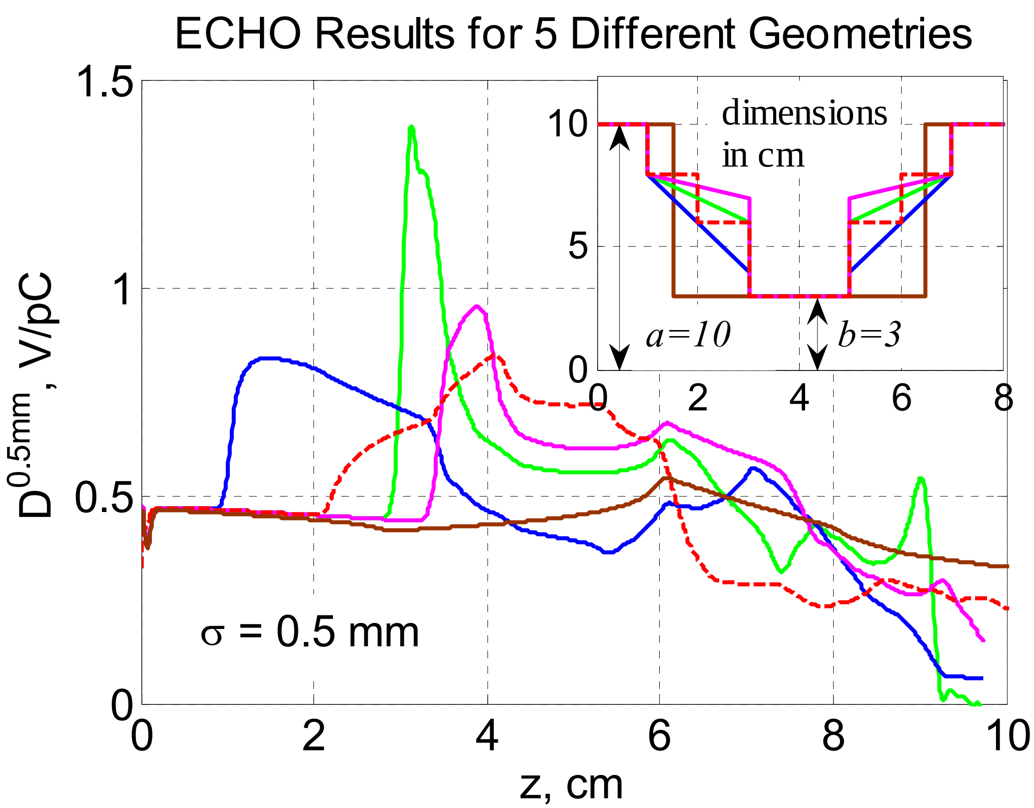




- $D^{\sigma}(z)$ is non-singular and (mostly) smooth
- $D^{\sigma}(z)$ weakly depends on σ, except near "corners" and near z=0
- Near z=0, extrapolation to σ->0 is trivial, i.e. $D^{\delta}(z) \approx 0.5\ V/pC$ plateau
- Point charge wake is well modeled by ($\sigma_0 << \lambda_g$ ≈2 cm)

$$W^{\delta}(z) = k_{opt}\delta(z) + D^{\sigma_0}(z)\times\theta(z-3\sigma_0) + 0.48\times\theta(z)\times\theta(3\sigma_0 - z)$$

A Family of Collimator Structures

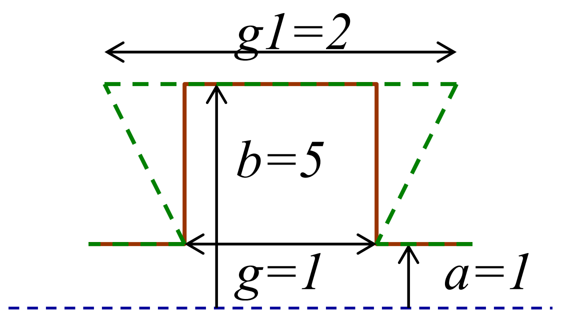


red dash - same as previous slide

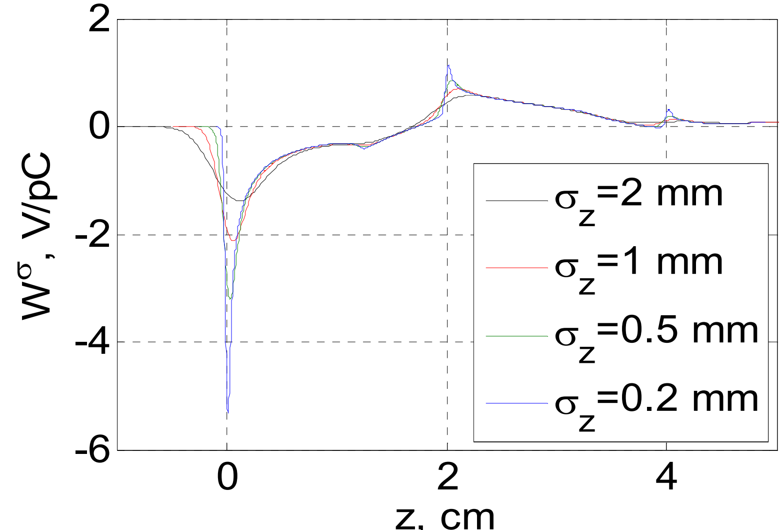
- Each structure has its own smallest geometric scale λ_g (related to the smallest radial or longitudinal step or their combination)
- Bunches much shorter than λ_g do not reveal any additional wake features
- For $z \ll \lambda_g$ and $\sigma \to 0$, all $D^{\sigma}(z)$ behave very simply (i.e. ~plateau)
- Point charge wake is easily modeled from $W^{\sigma_0}(z)$, when $\sigma_0 \ll \lambda_g$

A Simple Cavity Example

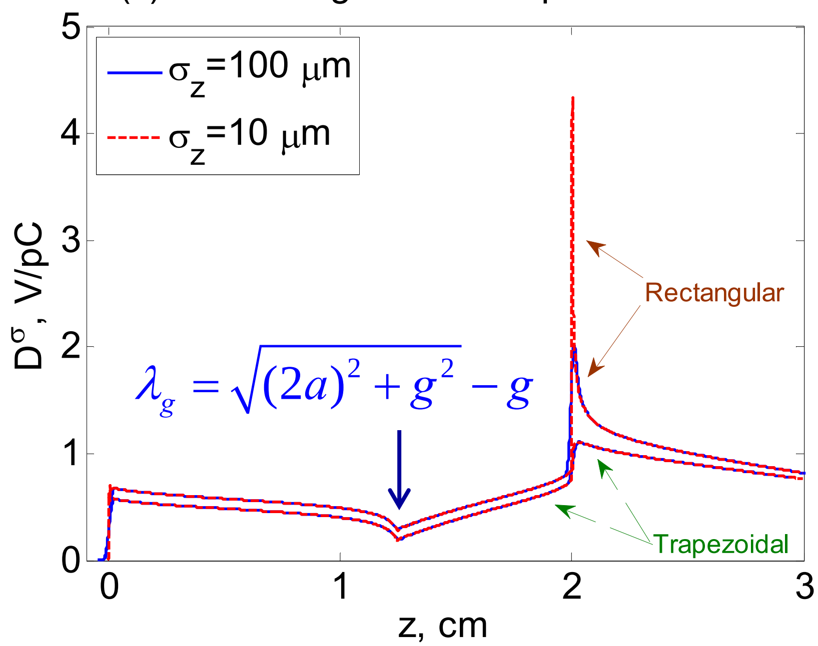
Geometry, dimensions in cm



Wake-potential for rectangular profile cavity

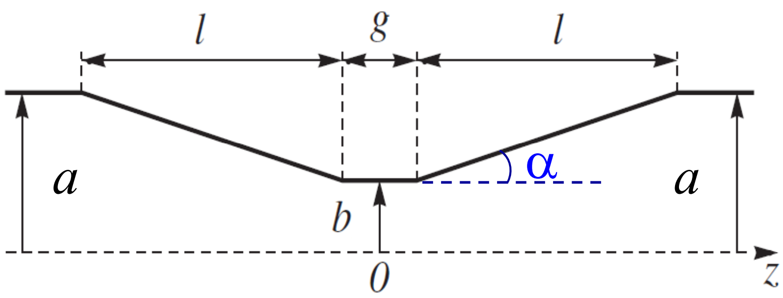


$D^{\sigma}(z)$ for rectangular and trapezoidal cavities



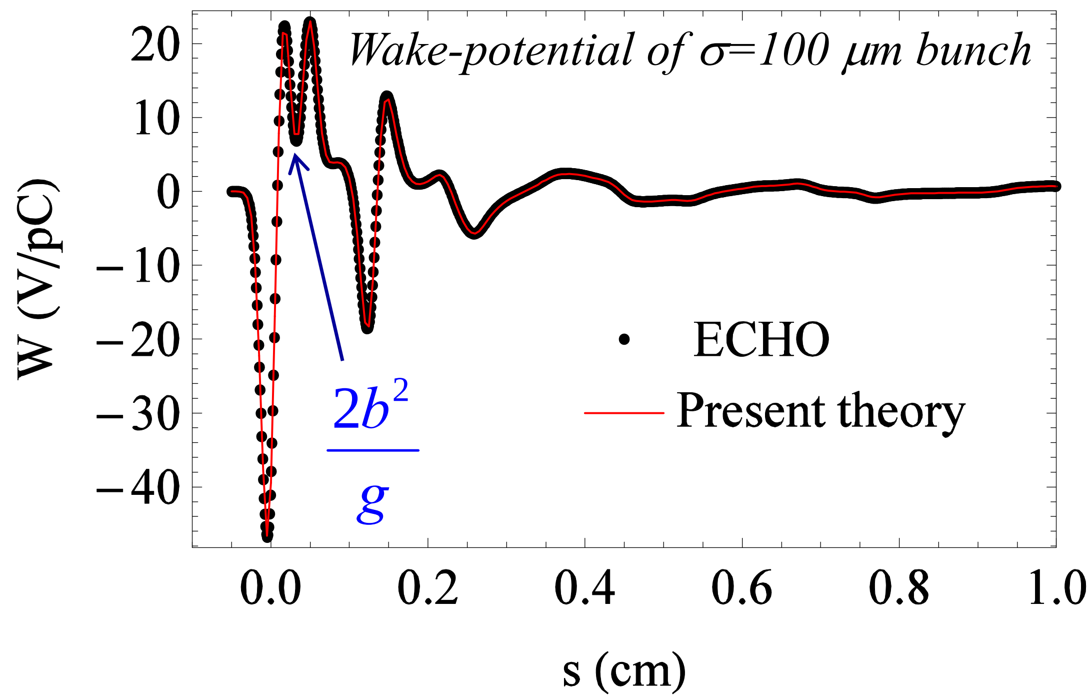
- Diffraction-model behaviour near z=0
- Singularities at z=2g, 4g, ... due to cavity wall reflections (rectangular); trapezoidal (or rounded-corner) cavities do not exhibit them.
- $D^{\sigma}(z)$ is σ-independent for $\sigma \ll \lambda_g$, $D^{\delta}(z)$ is obvious, $W^{\delta}(z)$ can be found, =>our method works well.

Reconstructing Short-Bunch Wake-potential for Small-Angle Tapered Collimator



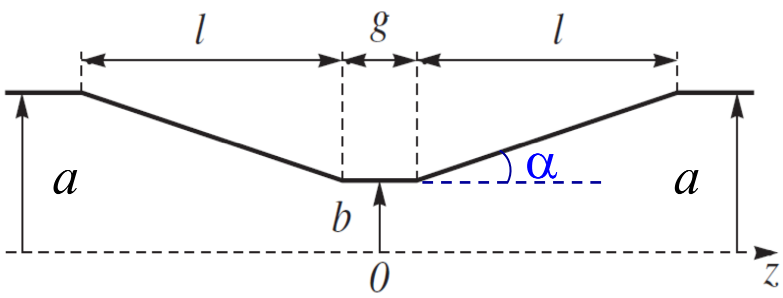
$l = g = 3$ cm; $a = 2b = 0.5$ cm
α=4.76 deg

G.Stupakov, B.Podobedov
PRST-AB 13, 104401 (2010)



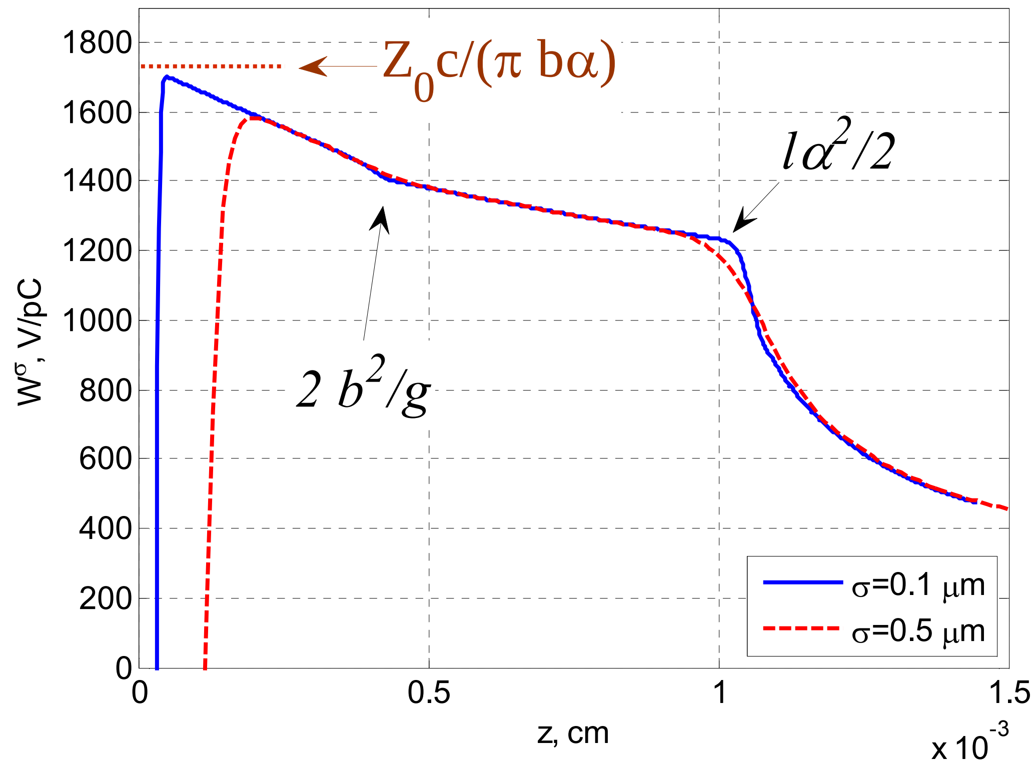
- Re $Z(\omega)$ calculated from mode-matching to f_{max}= 4THz (>> cutoff freq.)
- Optical model, $Z_{opt}(\omega)=Z_0/\pi\ Log(a/b)$, assumed above f_{max}
- Im $Z(\omega)$ calculated from Kramers-Kronig => $Z(\omega)$ is fully known
- Arbitrarily short bunch wake can be reconstructed
- Two length-scales show up in the wake: $l\alpha^2/2 \approx 100$ μm and $2b^2/g \approx 400$ μm

Small-Angle Tapered Collimator Con't



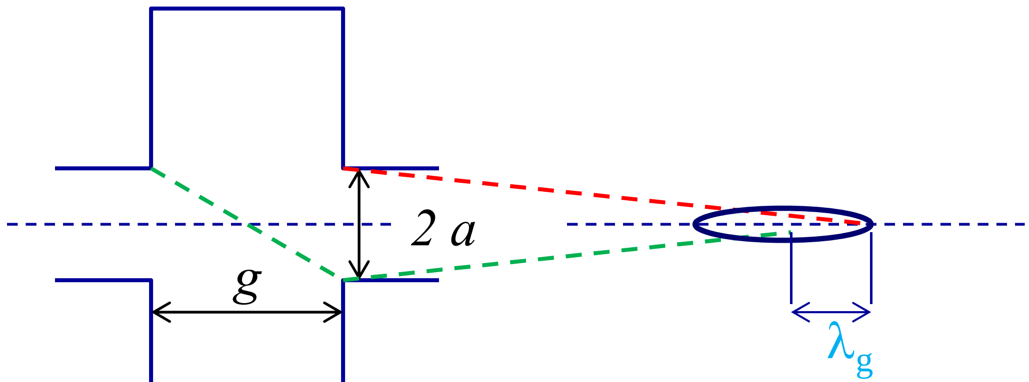
$l = g/10 = 3$ mm; $a = 2b = 0.5$ mm

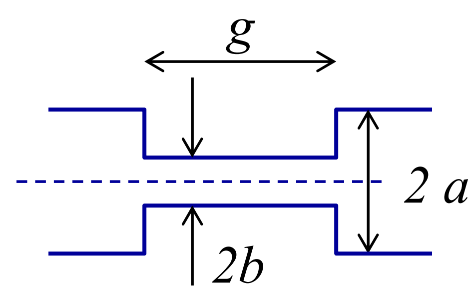
α=4.76 deg



- Let's scale down all dimensions, except for g, a factor of 10.
- Now the shortest length-scale is $\lambda_g = 2b^2/g < l\alpha^2/2$.
- Reconstruction of $W^{\sigma}(z)$ from $W^{\underline{\sigma}}(z)$ works for any $\sigma < \underline{\sigma} << \lambda_g$

Origin of the Smallest Length-Scale Parameter for Long Structures





$$\lambda_g = \sqrt{(2a)^2 + g^2} - g \approx 2a^2 / g, \quad (a << g)$$

$$\lambda_g = \sqrt{(2b)^2 + g^2} - g \approx 2b^2 / g, \quad (b << g)$$

- Red ray (spherical wave front) eventually catches up with ALL particles in the bunch, thus affecting the wakefield for all values of z.
- Green ray travels λ_g/c behind and it will never catch up with the front of the bunch, so λ_g emerges in the front portion of the wake.
- Depending on the ratio between a, b, and g, other combinations may define the smallest length-scale λ_g , i.e. $2(a-b)$, $2(a-b)^2/g$, etc.

Practical Application: TESLA 9-cell Cavity Wake Calculations

- As a more practical example we calculate a very short bunch wake for 9-cell TESLA structure.

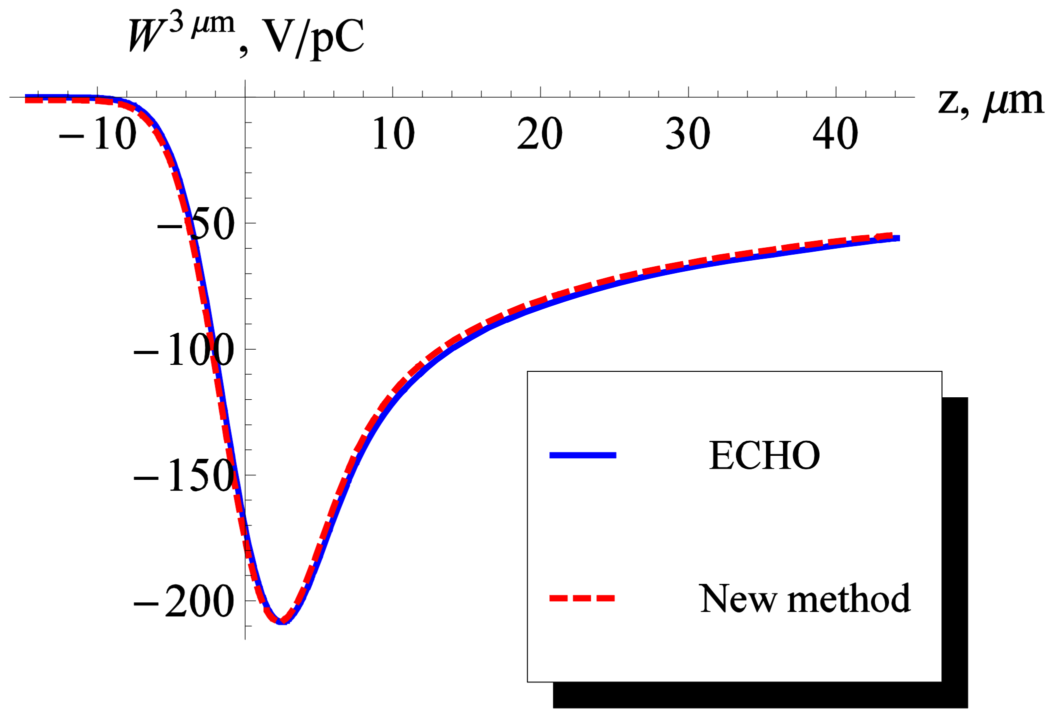
  iris $r_{\min}$=3.5 cm, period 11.5 cm, tot. length $g \approx 1$ m.

- Exact asymptotic wake model is unknown, although we expect a scaling similar to the diffraction model, i.e. $W^{\sigma}(z \approx 0) \sim \sigma^{-1/2}$
- We calculate $W^{\sigma 1}(z)$ & $W^{\sigma 2}(z)$ and "fit" for constant κ, assuming the equation:

$$W^{\sigma}(z) = \kappa W_d^{\sigma}(z) + D(z)$$

- Then we find $D(z)$, which allows us to calculate the wakefield of an arbitrary short bunch.

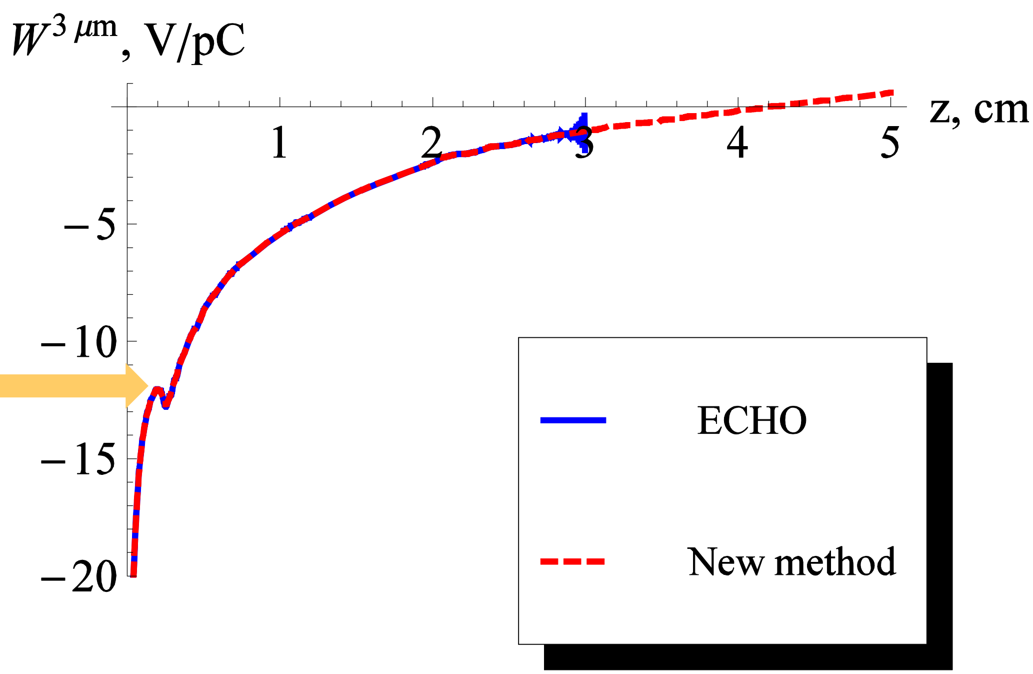
TESLA Cavity Wake: Results



- Wake for σ=3 μm bunch (red dash) was reconstructed from ECHO wakes for $\sigma 1$=100 μm & $\sigma 2$=30 μm bunches.
- Perfect agreement between the direct calculation and reconstructed wake.
- The smallest length scale is ~$r_{\min}^2/g \approx$ 1 mm => $\sigma 2 < \sigma 1 << 1$ mm

TESLA 9-cell Cavity Wake: Results Con't

Significant deviations from diffraction model start at
$z \approx 2r_{min}^2/g \approx 2.4$ mm



- Long range wake-potential is reconstructed successfully as well.
- Direct calculation by ECHO (blue) shows numerical instability at z~3 cm due to inadequately fine mesh ($mesh_z = \sigma/5$, $mesh_r = 200\,\sigma$)
- This calculation took ~200 hours on Intel Xeon 2.93 GHz CPU. Our method is many orders of magnitude faster.

Summary

- Wakefield calc's is important task for modern accelerators. However, in case of short bunches and large, smooth structures, direct EM solver calc's can be extremely resource-consuming, if not impossible.
- We describe a new method that allows us to accurately calculate geometric wakefields of arbitrary short bunches. So far we found this method to work quite well for a large number of 2D geometries.
- The method essentially consists of adding a (processed) long-bunch result from an EM solver to a singular analytical wakefield model.
- The bunch length to use in the EM solver must be shorter than a certain value, which can be found from simple geometric considerations.
- This bunch length often ends up much longer than the scale to which wakefield are needed. In such cases we save a lot of CPU time, and/or overcome computer memory limitations. We also gain analytical insight.
- In the future this work will be generalized to 3D geometries.