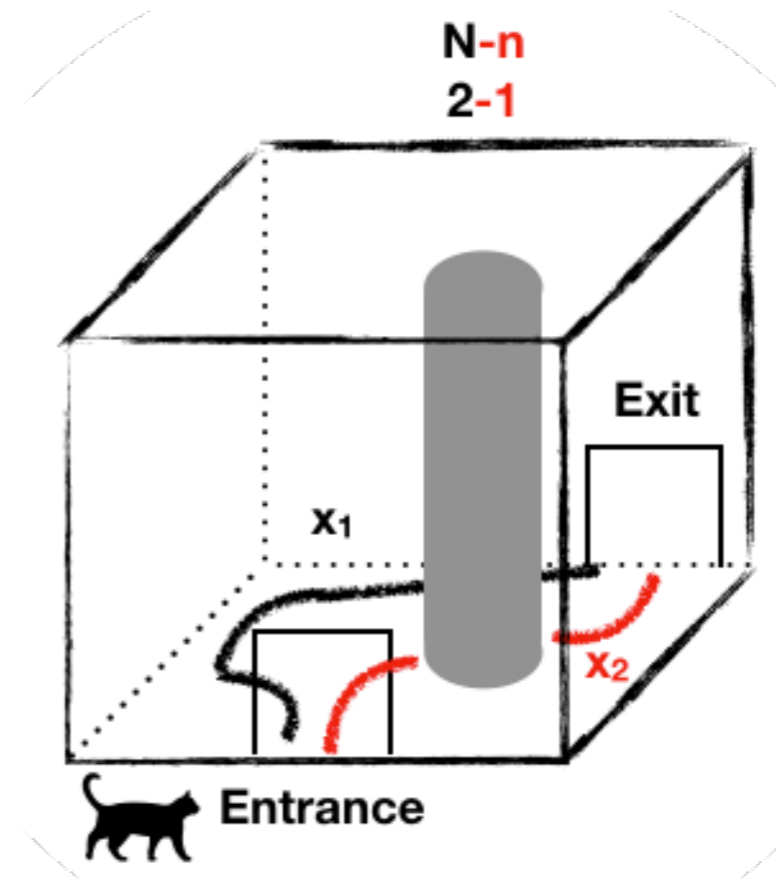
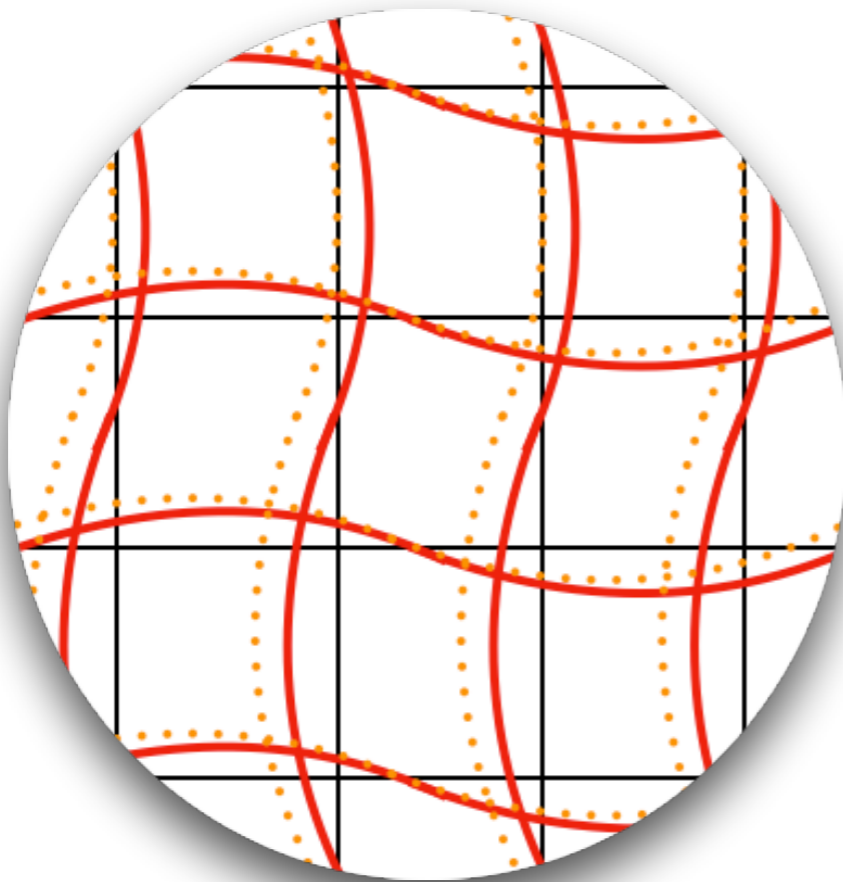


A probabilistic expanding universe under functors of actions theories



presented @



Extended session 12 Dec 2024

Pierros Ntelis

Postdoc researcher

Dec 2024

s. 1



Outline

Introduction

Functors of actions

Probabilistic universe

Dynamical Analysis

Conclusions and future

Cosmological Gravitology

Diagram remodified from
P.Ntelis & A.Morris '23
and novel theories

Functors
of
Actions.
 S_{FA}

$$S = \int_{\mathcal{V}} d^D x \sqrt{-g} \mathcal{L}$$

General Relativity
unique theory of
massless $g_{\mu\nu}$

Massive Gravity
 $m_g > 0$

Tensor
 $T_{\mu\nu}$

Additional Field

Vector V_μ

Scalar
 ϕ

Love-lock

Quint-essence

Brans-Dicke

Break Assumptions

Extra Dimensions

Probabilistic Dimensions

Hordenski

$f(R)$

Isotropy
homogeneity

locality

Causality

Non-Riemanniann

Curvature
-types
 $f(R^n)$

Torsion
 $f(T)$

non-
metricity
 $f(Q)$

Combined
 $f(R,T,Q)$

Beyond Hordenski
 $C(X)$ $D(X)$ $E(X)$

Gauss-Bonnet

Galileon
UEDM

Galileon

KGB

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Functor of actions (\mathcal{S}_{FA}) theories

Functor (F) is the generalisation concept of functionals

Functionals is the generalisation concept of functions

Action (S) in physics is a quantity which is the product of energy with time.

Action is a quantity which tell us the amount of possible ways a particle can travel from one point to another within a certain region

Functor of actions predict the existence of actionic fluctuations and field-particles which is an analogue of the energetic/topological fluctuations/field-particles in nature.

$$\mathcal{S}_{\text{FA}} \supset \int_{\Omega_S} dS' \supset \mathcal{S}_{\text{EFT}} \supset \int_{\Omega_S} dS' S' \leftarrow S = \int_{\mathcal{V}} d^D x \sqrt{-g} \mathcal{L}$$

Functors of actions

Prediction of Modification of Einstein Field Equations

$$S_{\text{FA}}^{\text{simple}} = \beta S_{\text{R}} + S_{\text{m}} + \boxed{S_3}$$

Prediction of Actionic fields
(similar to energy fields)

$$0 = \delta S_{\text{EFT}}^{\text{Simplified,2,GR,2}}$$

$$0 = \int d^4x \sqrt{-g} \delta g^{\mu\nu} \left(\beta^{(S_{\text{R}})} \frac{c^4}{16\pi G_{\text{N}}} \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} \right) - T_{\mu\nu}/2 \right) \boxed{+ \delta S'_3}$$

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{\beta^{(S_{\text{R}})}} \frac{8\pi G_{\text{N}}}{c^4} \left(T_{\mu\nu} + \boxed{\delta [\mathcal{L}_3]_{\mu\nu}} \right)$$

Functors of actions

Constraints and Prediction of Modification of EFE

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{1}{\beta(S_R)} \frac{8\pi G_N}{c^4} \left(T_{\mu\nu} + \delta [\mathcal{L}_3]_{\mu\nu} \right)$$

Prediction of Actionic fields,

by choosing a special Lagrangian fluctuation

$$\delta [\mathcal{L}_3]_{\mu\nu} \rightarrow \delta \mathcal{L}_3 \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Equation of state

$$w = - (1 + \delta \mathcal{L}_3)^{-1}$$

for $w \sim -1.1$

\Rightarrow

$$\delta \mathcal{L}_3 \sim -0.1$$

\Rightarrow

$$\delta S_3 \sim S d^D x (-g)^{1/2} \delta \mathcal{L}_3 \sim -0.1 V_D$$

$$T_{\mu\nu}^{(2)} = \begin{pmatrix} \rho(1 + \delta \mathcal{L}_3) & 0 & 0 & 0 \\ 0 & -P & 0 & 0 \\ 0 & 0 & -P & 0 \\ 0 & 0 & 0 & -P \end{pmatrix}$$

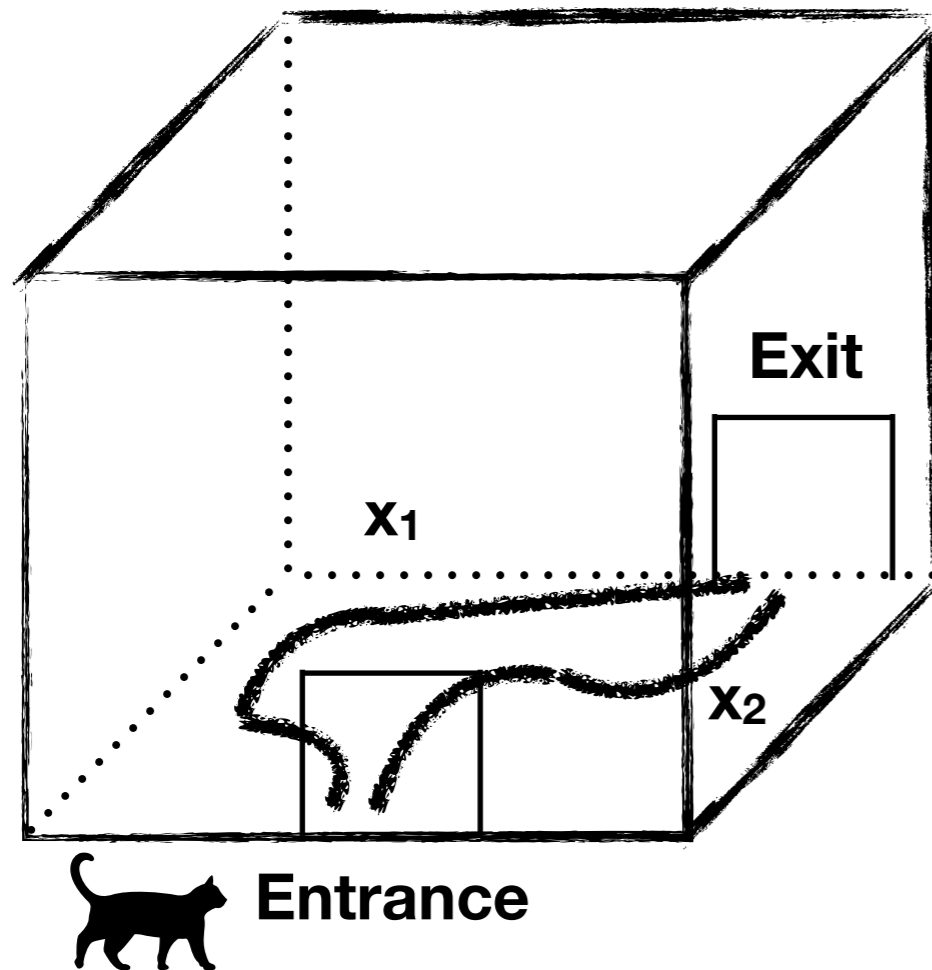
Under these assumptions simple actionions are 1/10th of the observed volume

Actionic field interpretation

Action answers to the question :

What is the number of all possible routes
a cat can use to pass through each room ?

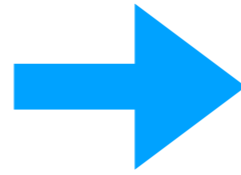
$N = 2$



x_1 is possible for both rooms, x_2 is not possible for the 2nd room
space, x



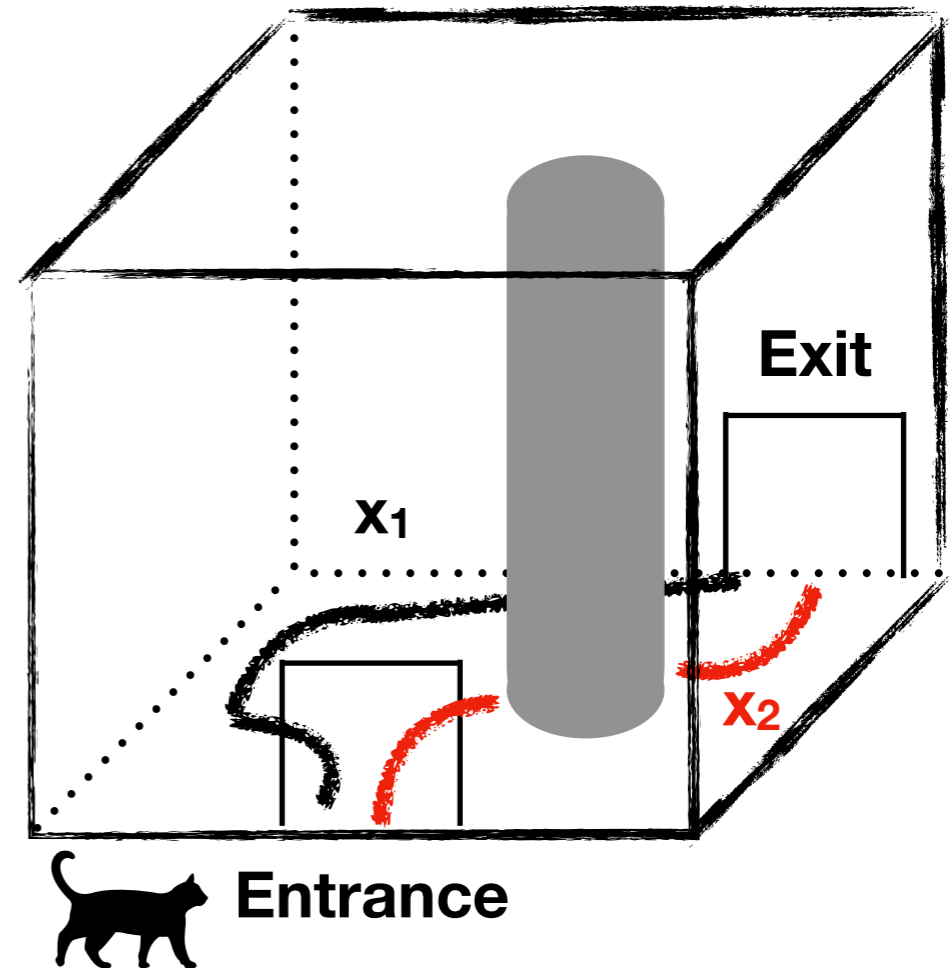
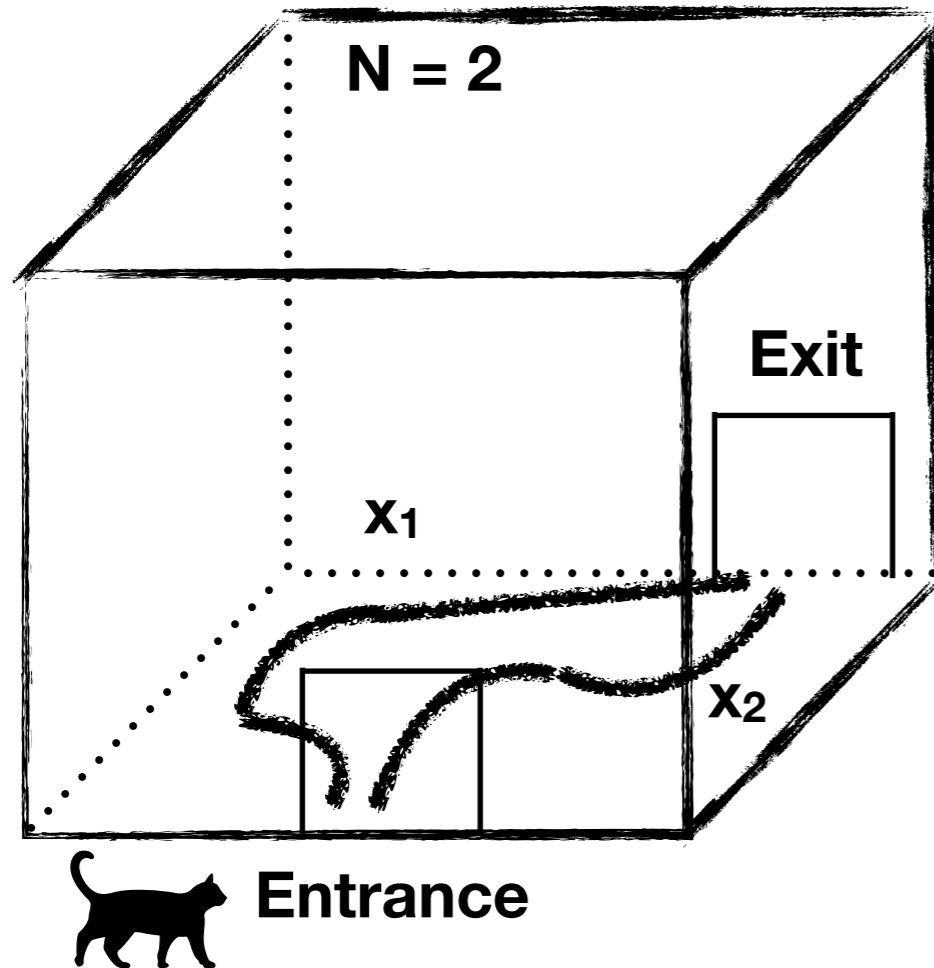
Actiononic field interpretation
Action answers to the question :



An actiononic field everywhere
QM \Leftrightarrow actionion field-particle

What is the number of all possible routes
a cat can use to pass through each room ?

$N-n$
 $2-1$



x_1 is possible for both rooms, x_2 is not possible for the 2nd room
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Functors of actions

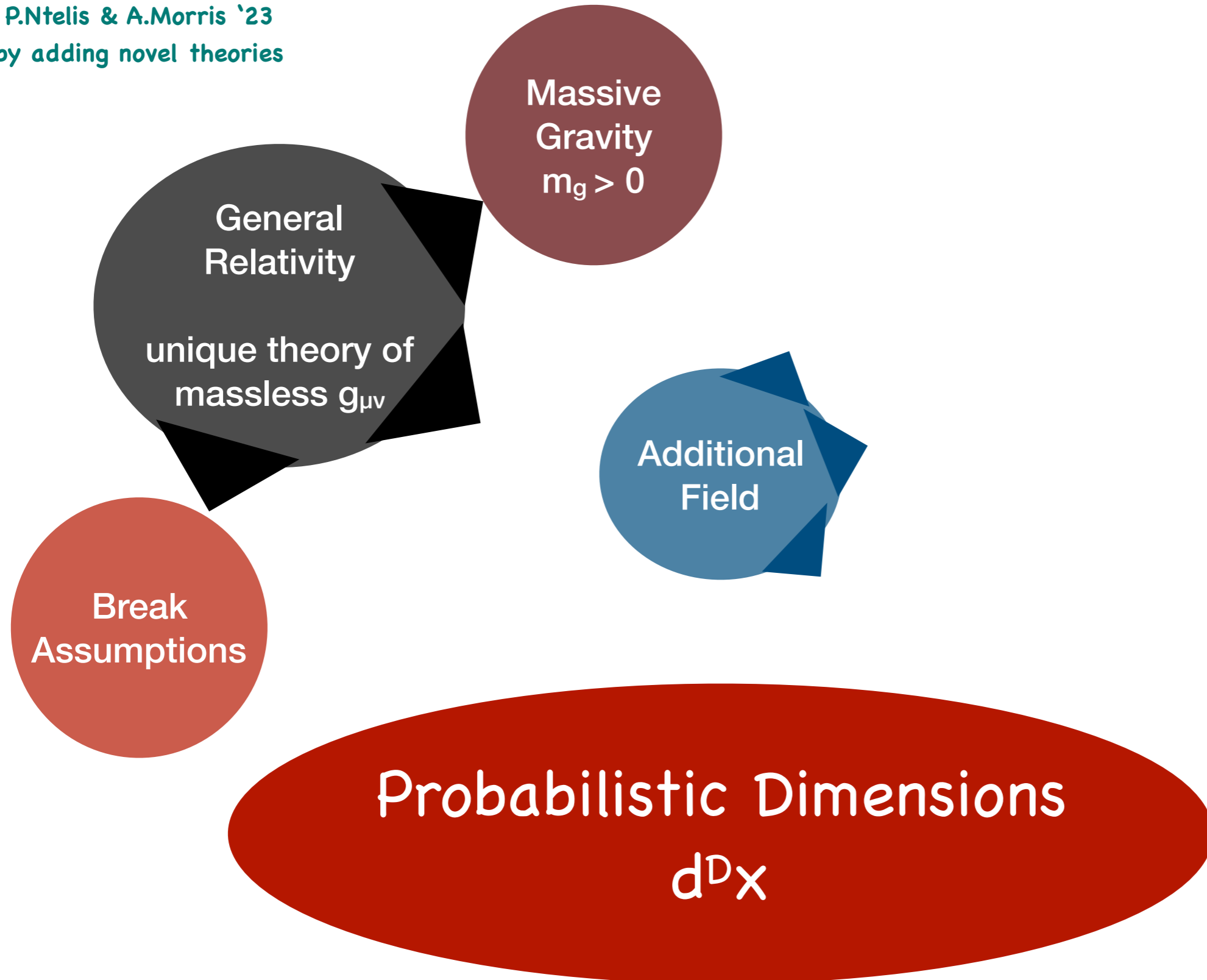
Probabilistic universe

Dynamical Analysis

Conclusions and future

Cosmological Gravitology

Diagram modified from
P.Ntelis & A.Morris '23
by adding novel theories



Cosmological Gravitology

Motivation 1

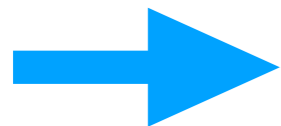
In 1942, Menger have introduced the concepts of statistical metrics [1].

In 1977, Drossos introduced the stochastic Menger spaces [2].

In the framework of modifications of gravity,

string theories predict extra dimensions [3],
while

Loop quantum gravity theories predict the non-existence of metric [4].



Natural to introduce an idea of a manifold in between
extra dimensional spacetime manifolds and non-metric manifolds

Motivation 2

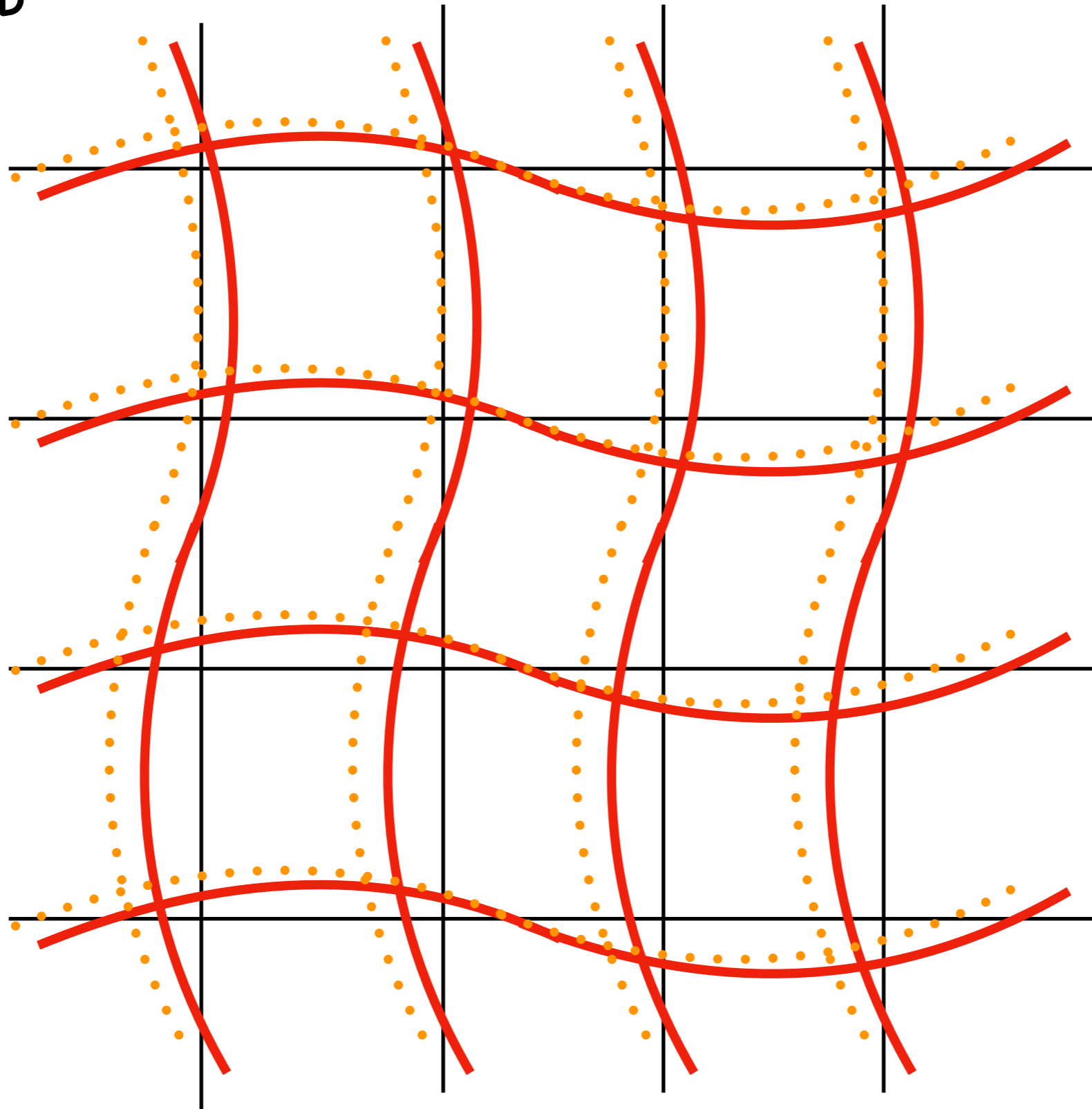
Evidence for a semi analytical model is found to be

Dimensions = 4 ± 0.1
from gravitational wave estimates
[Pardo, Fishbach, Holtz, 2018]

Probabilistic Dimensions
 $d^D x$

Space Time Continuum

2D



2D curved
probabilistic

2D curved

Easy
generalisation
in 3+1
dimensions

2D spatial curved probabilistic



Figure 2. A representation of a 2 dimensional spatially curved probabilistic expanding space. This spacetime is curved in the presence of some massive object, as well as it appears, disappears and expands in time. It is easy to generalise this concept in a (3+1)D spacetime continuum. [See section [3.14](#)]

Probabilistic dimensions turn up *probabilistic gravity*

$$D \rightarrow \bar{D} = \int_{\Omega_D} dX_D \text{ Gaussian}(X_D; D, \sigma_D)$$

modifying dimensions

means that we need to modify

- topology (metric, manifold, curvature tensor)

- matter content evolution

$$\mathcal{S}_{\text{Probabilistic gravity}} \propto \int d^{\bar{D}}x \sqrt{-g^{\bar{D}}} \left[\frac{R^{\bar{D}}}{16\pi G_N} + \mathcal{L}_m^{\bar{D}}(g_{\mu\nu}^{\bar{D}}, \psi_m, \dots) \right]$$

Sophisticated and **Simple modifications**

Simple probabilistic metric :

$$ds^2 = a^2(\tau) \left[-P^2(\tau) e^{2\Psi(\tau, \vec{x})} d\tau^2 + e^{-2\Phi(\tau, \vec{x})} dx^i dx^j \delta_{ij} \right]$$

Results

Einstein Field Equation results to modified Friedmann equations

$$\mathcal{H}^2(\tau) = \frac{8\pi G_N}{3c^4} a^2(\tau) P^2(\tau) \sum_{s \in \{m, r, \Lambda, k\}} \bar{\rho}_s(\tau)$$

$$2\mathcal{H}'(\tau) + \mathcal{H}^2(\tau) = -\frac{8\pi G_N}{3c^4} a^2(\tau) P^2(\tau) \sum_{s \in \{m, r, \Lambda, k\}} w_s(\tau) \bar{\rho}_s(\tau)$$

while continuity equation remains the same

$$\dot{\bar{\rho}}_s = -[1 + w_s(\tau)] \mathcal{H}(\tau) \bar{\rho}_s(\tau)$$

Currently working on solutions!

Results

This probabilistic perturbed expanding Manifold-metric

modifies

the Einstein-Boltzmann equations (find them in doc).

Results

A collection of Probabilistic Manifold-Metric pairs in the doc:

5. Simple toy models of probabilistic dimensions	11
5.1. 1D spatial probabilistic spacetime	12
5.2. Probabilistic timed perturbed FLRW metric	12
5.3. Probabilistic conformal timed perturbed FLRW metric	12
5.4. A probabilistic Schwarzschild metric	13
5.5. A spatially curved probabilistic and expanding metric around matter	14
5.6. Interpreting PFLRW with Gaussian probabilities	14
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5.9. Probabilistic Perturbed Einstein-Boltzmann Equations in conformal time	17
5.9.1. Main theorem	17

Qualitative results

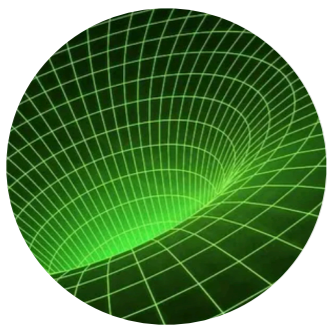
Connections of probabilities with information

$$\mathcal{I}(E) = -\log [\mathcal{P}(E)]$$

New kind of manifold-metric pairs
using probabilities and information

Correspondence with field-particles characterisation (spacions):

spaciallion,



timions,



probablons,



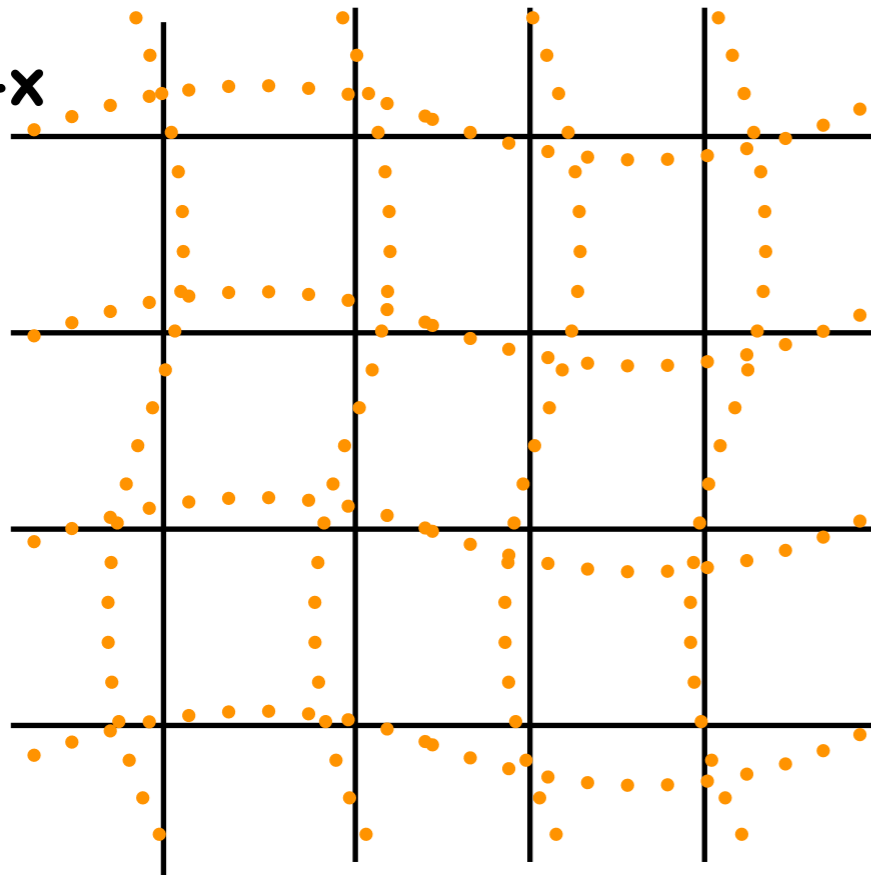
informatons.



A probabilistic expanding universe under functors of actions theories

$$ds^2 = a^2(\tau) \left[-P^2(\tau) e^{2\Psi(\tau, \vec{x})} d\tau^2 + e^{-2\Phi(\tau, \vec{x})} dx^i dx^j \delta_{ij} \right].$$

2D t-x



2D curved probabilistic

$$\mathcal{H}^2(\tau) = \frac{8\pi G_N}{3c^4} a^2(\tau) P^2(\tau) \bar{\rho}(\tau) + \frac{1}{3} a^2(\tau) P^2(\tau) \Lambda$$

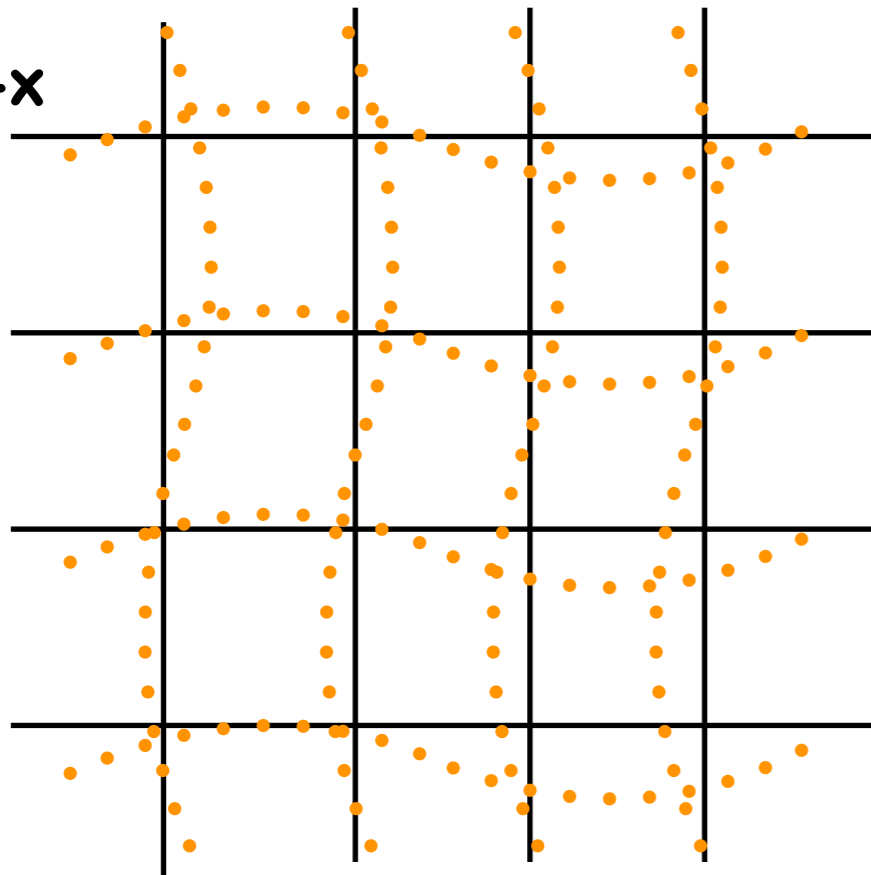
$$2\mathcal{H}'(\tau) + \mathcal{H}^2(\tau) = \frac{8\pi G_N}{3c^4} a^2(\tau) P^2(\tau) \bar{P}(\tau) + \Lambda a^2(\tau) P^2(\tau)$$

$$\mathcal{S}_{\text{FA}} \supset \int_{\Omega_S} dS' \supset$$

A probabilistic expanding universe under functors of actions theories

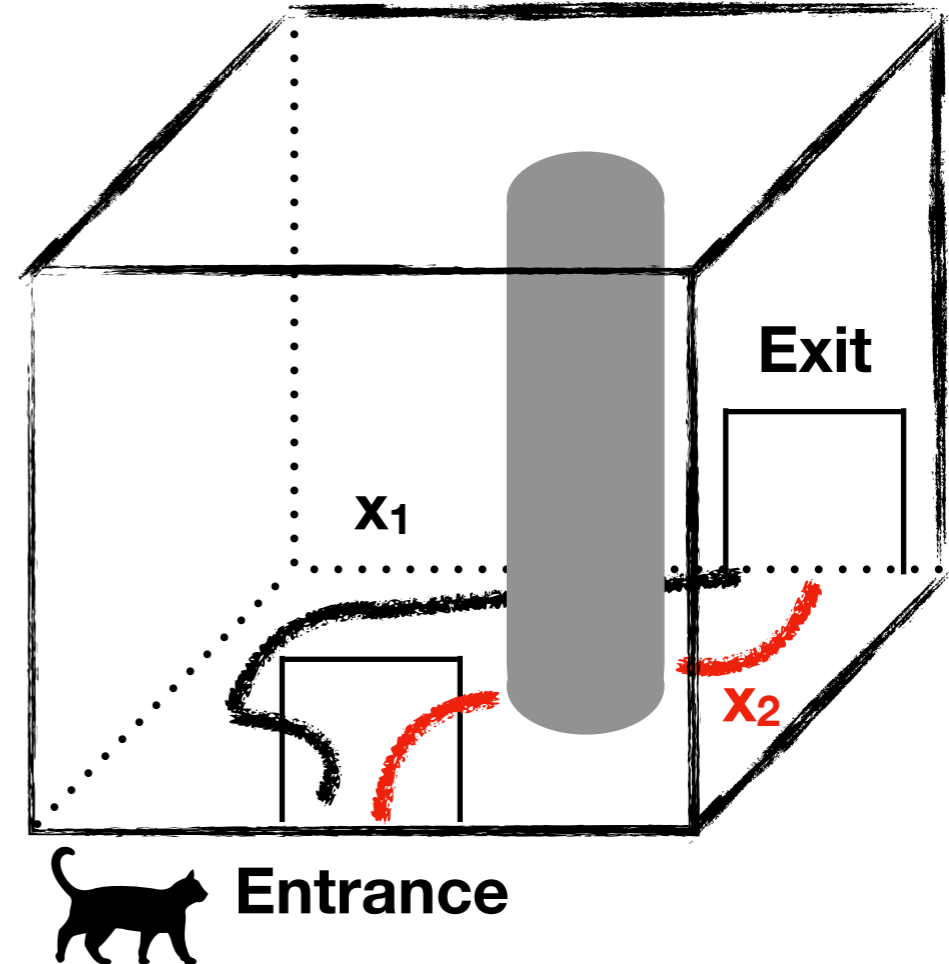
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2D t-x



2D curved probabilistic

N-n
2-1



$$\mathcal{H}^2(\tau) = \frac{8\pi G_N}{3c^4} a^2(\tau) P^2(\tau) \bar{\rho}(\tau) + \frac{1}{3} a^2(\tau) P^2(\tau) \Lambda$$

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$$\mathcal{S}_{FA} \supset \int_{\Omega_S} dS' \supset$$

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Conclusions

- Mathematical and Observational evidence for
 - Functors of actions theories predictions of actionions
 - Probabilistic dimensions, probabilistic manifold-metric pairs
- Dynamical analysis on standard Λ CDM cosmology
important tool for analysing other cosmic systems

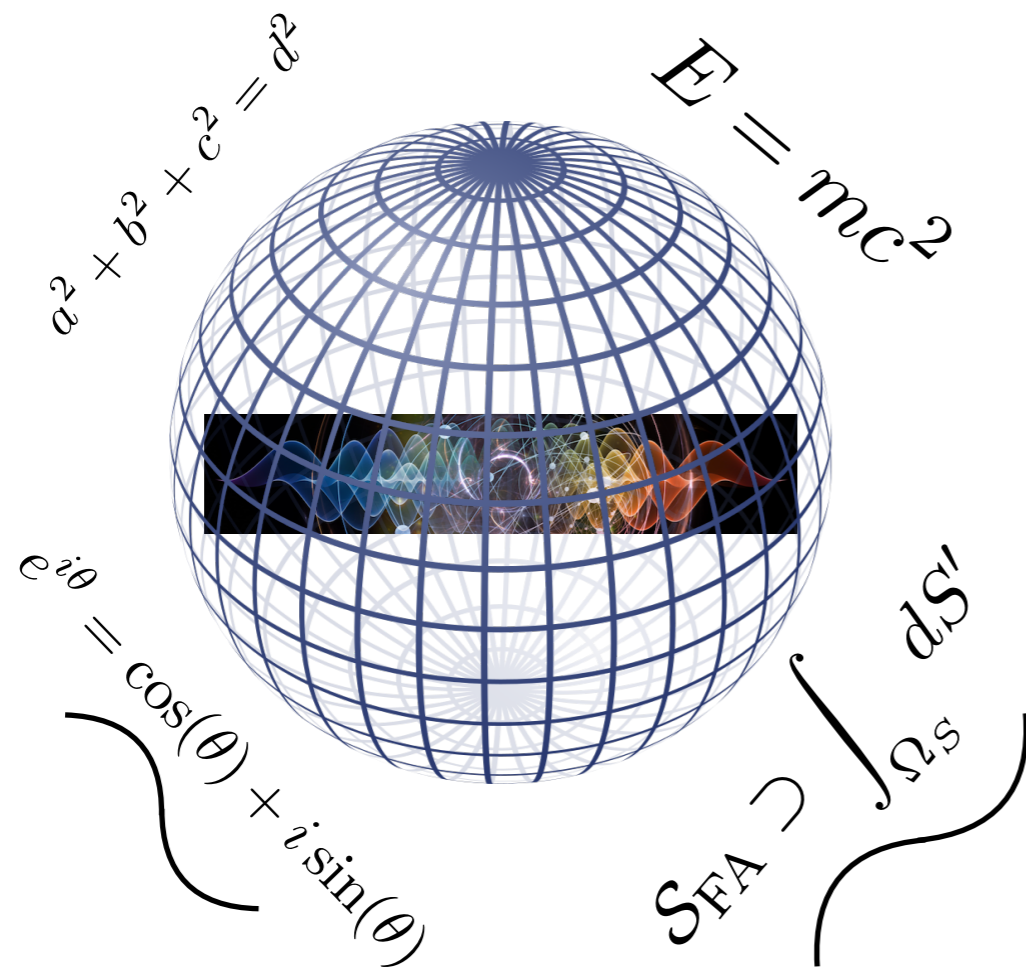
Future plan

Develop further 4 axis of research:

- Functors of actions through category theory
- Probabilistic and informatics model
- Dynamical analysis systems, numerical/analytical solutions
- Statistical tests of above theories (Early, Late Universe, GW)

Collaboration on models and tests with other cosmologists

Universions



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Thank you for your attention!

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