Thermodynamic free energy calculation under one loop correction at finite chemical potential

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ICNF, 12th December 2024,
Oslo, Norway

Outlines

- Partition Function and free Energy at a finite chemical potential
- QGP fireball in mean field potential
- One loop correction at finite chemical potential
- Result and conclusion

Partition Function and Free Energy at chemical potential

The general Partition function of the system defined as[1]:

$$Z(T,\mu,V) = Tr\{\exp[-\beta(\hat{H} - \mu\hat{N})]\}$$

$$F_i = TlnZ(T,\mu,V)$$

$$\beta = {}^{1}/_{T}$$

where, μ and $\hat{N}$ are the chemical potential and quark number.

QGP Fireball in mean field potential

Free energy is given by [2]

$$F_i = \mp T g_i \int dk \rho_i(k) \ln\left(1 \pm e^{-\left(\frac{\sqrt{m_i^2+k^2}}{T}\right)}\right)$$

Where, $\rho_i(k)$ is density of states of particular particle, i (quarks, gluons, pions etc.) being the number of states with momentum between k and k+dk in spherically symmetric situation. This density of state is calculated through Thomas-Fermi model

- sign corresponds to fermions
+ sign corresponds to bosons
T corresponds to temperature
$g_i =$ Color degeneracy, 6 for quarks and 8 for gluons

$m^2(T) = 2\gamma^2 g^2(k) T^2$

The Thomas Fermi model develops the density of states like the following

$$\int \rho_{q,g}(k)dk = \frac{\left(-V_{conf}(k)^3\right)\nu}{3\pi^2}$$

$$\rho_{q,g}(k) = \left(\frac{\nu}{\pi^2}\right)\left[-V_{Conf}(k)^2\left(\frac{dV_{Conf}(k)}{dk}\right)\right]_{q,g}$$

Where

ν = volume occupied by QGP

k = relativistic four momentum in natural units

V_{conf} = confining potential for quarks , gluons

$$V(k) = \frac{8\pi}{k}\gamma\alpha_s(k)T^2 - \frac{m_0^2}{2k}$$

Interface Free Energy, which is used to replace MIT Bag is [4]

$$F_{interface} = \frac{1}{4}\gamma R^2 T^3$$

$$\gamma = \sqrt{2} \times \sqrt{\left(\frac{1}{\gamma_g}\right)^2 + \left(\frac{1}{\gamma_q}\right)^2}$$

Where,

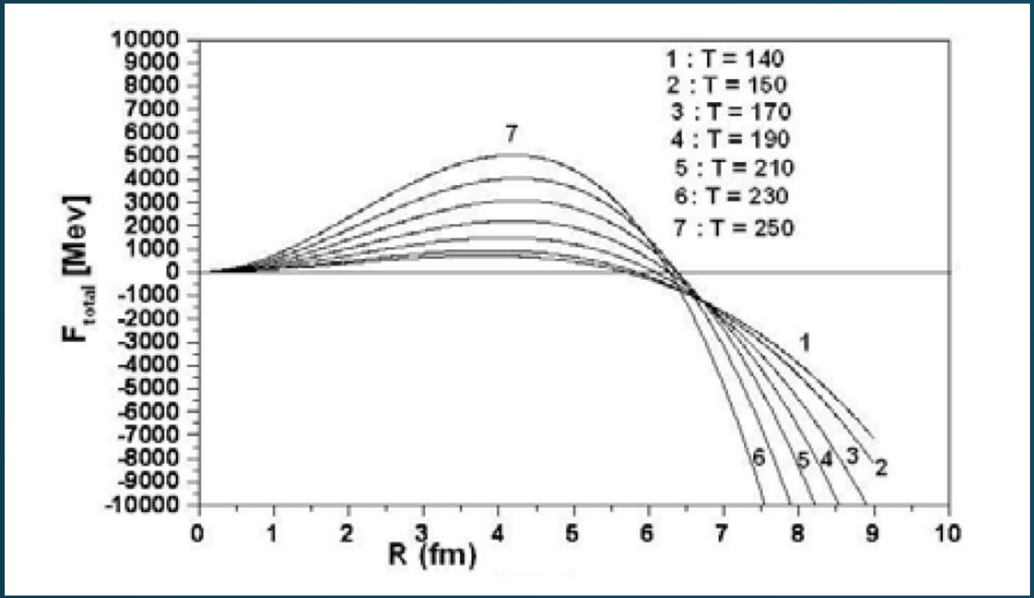
γ = modification introduced to take care of quark and gluons respectively

R = radius of droplet

γ_q, γ_g are the quark and gluons flow parameters

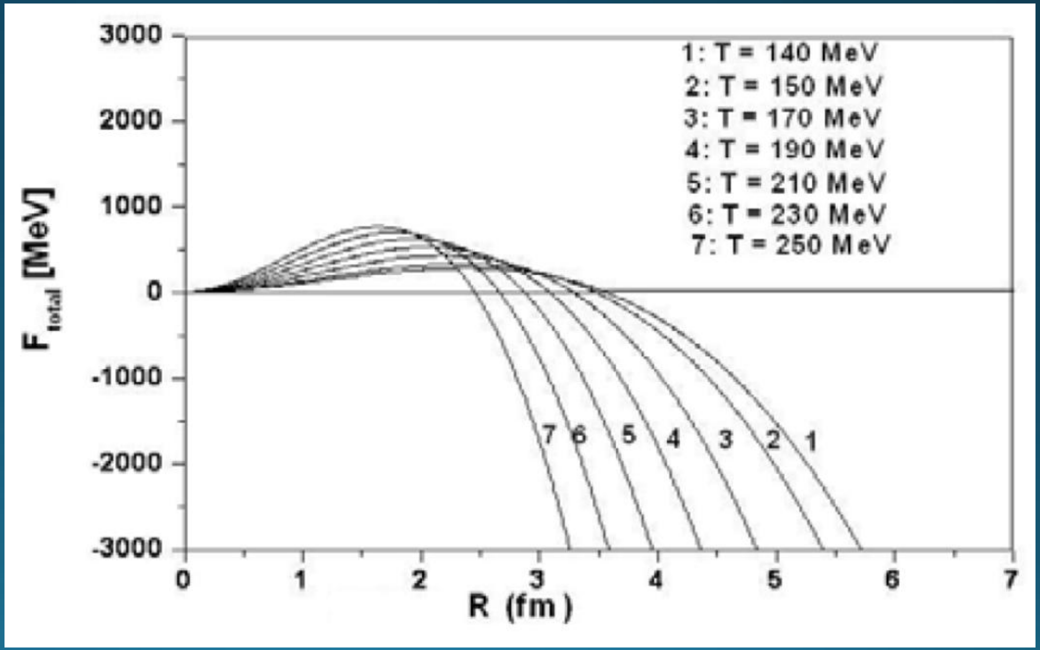
Total free energy of QGP through this model is

$$F_{total} = \sum F_{q=u,d,s} + F_{interface} + F_{mesons} + F_{gluons}$$



F_{total} at $\gamma_g = 6\gamma_q$, $\gamma_q = 1/6$
for various temperatures

F_{total} at $\gamma_g = 8\gamma_q$, $\gamma_q = 1/6$
for various temperatures



One loop correction in the mean field potential

$$F_i = \mp T g_i \int dk \rho_{q,g}(k) \ln\left(1 \pm e^{-\left(\frac{\sqrt{m_i^2+k^2}}{T}\right)}\right)$$

g_i = degeneracy factor for every particles

With the minimum energy cutoff in the free energy which is obtained through the potential w.r.t size

$$k_{min} = \left(\gamma_{g,q} N T^2 \Lambda^2/2\right)^{1/4}$$

Where, $N = (4/3)\left[12\pi/\left(33 - 2n_f\right)\right]$

$$m^2(T) = 2\gamma^2 g^2(k) T^2 (1 + g^2(k) a_1)$$

$$V_{conf}(k) = \frac{8\pi}{k}\gamma_{g,q}\alpha_s(k)T^2\left[1+\frac{\alpha_s(k)}{4\pi}\alpha_1\right]-\frac{m_0^2}{2k}$$

$Where,$

$\gamma_q = 1/8$

$\gamma_g = (8-10)\gamma_g$

$$\alpha_s(k) = \frac{1}{(33-2n_f)\ln\left(1+\frac{k^2}{\Lambda^2}\right)}$$

$\Lambda =$ QCD Parameter having taken as 1.5 GeV

Density of state in phase space with one loop correction [3]

$$\int \rho_{q,g} dk = \nu/\pi^2 \left(-V_{conf}(k)\right)^2 \frac{dV_{conf}}{dk}$$

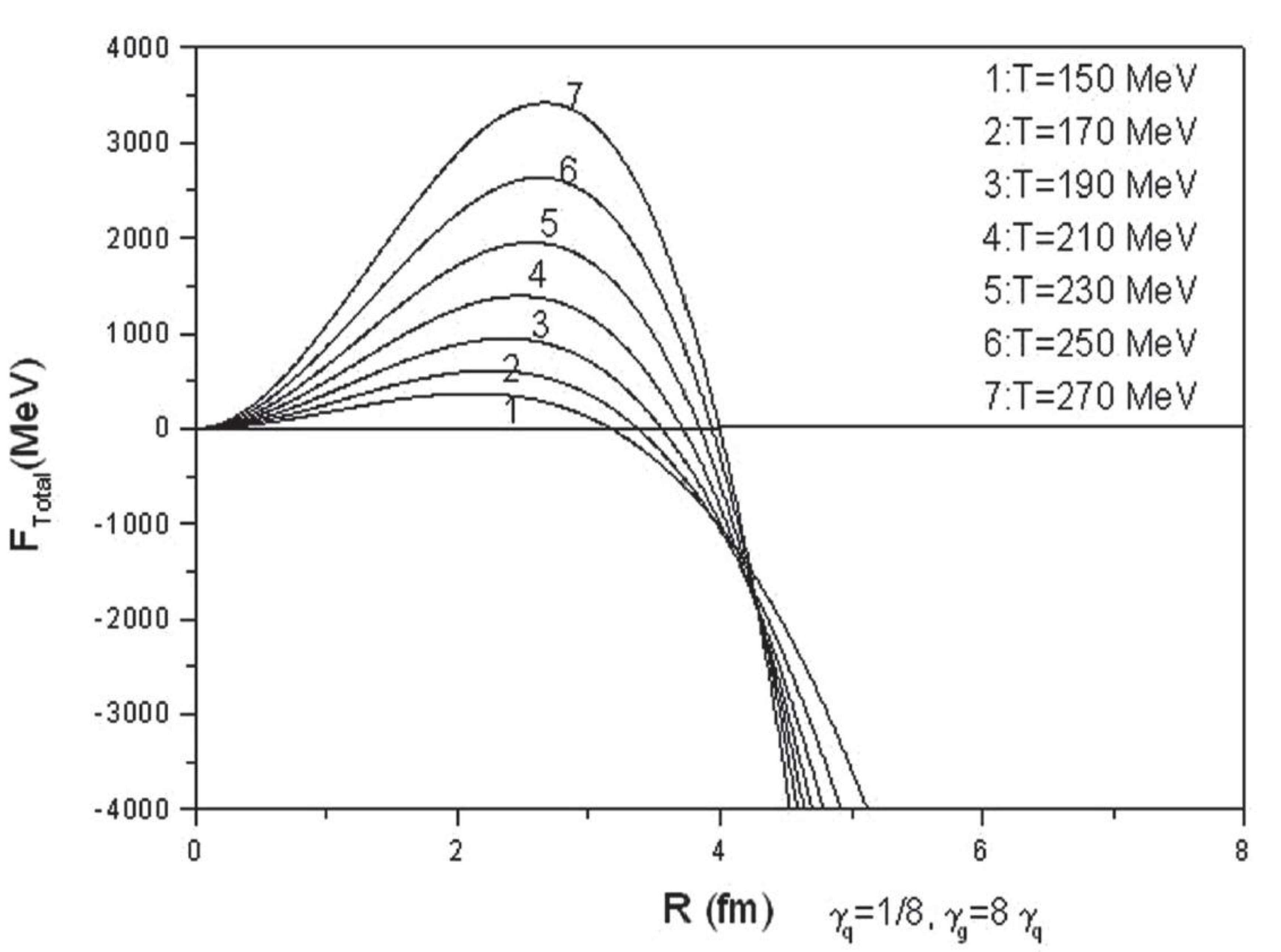
$$\rho_{q,q}(k) = \left(\frac{\nu}{\pi^2}\right)\left[\frac{\gamma_{g,q}^3 T^2}{2}\right]^3 g^6(k)A$$

Where

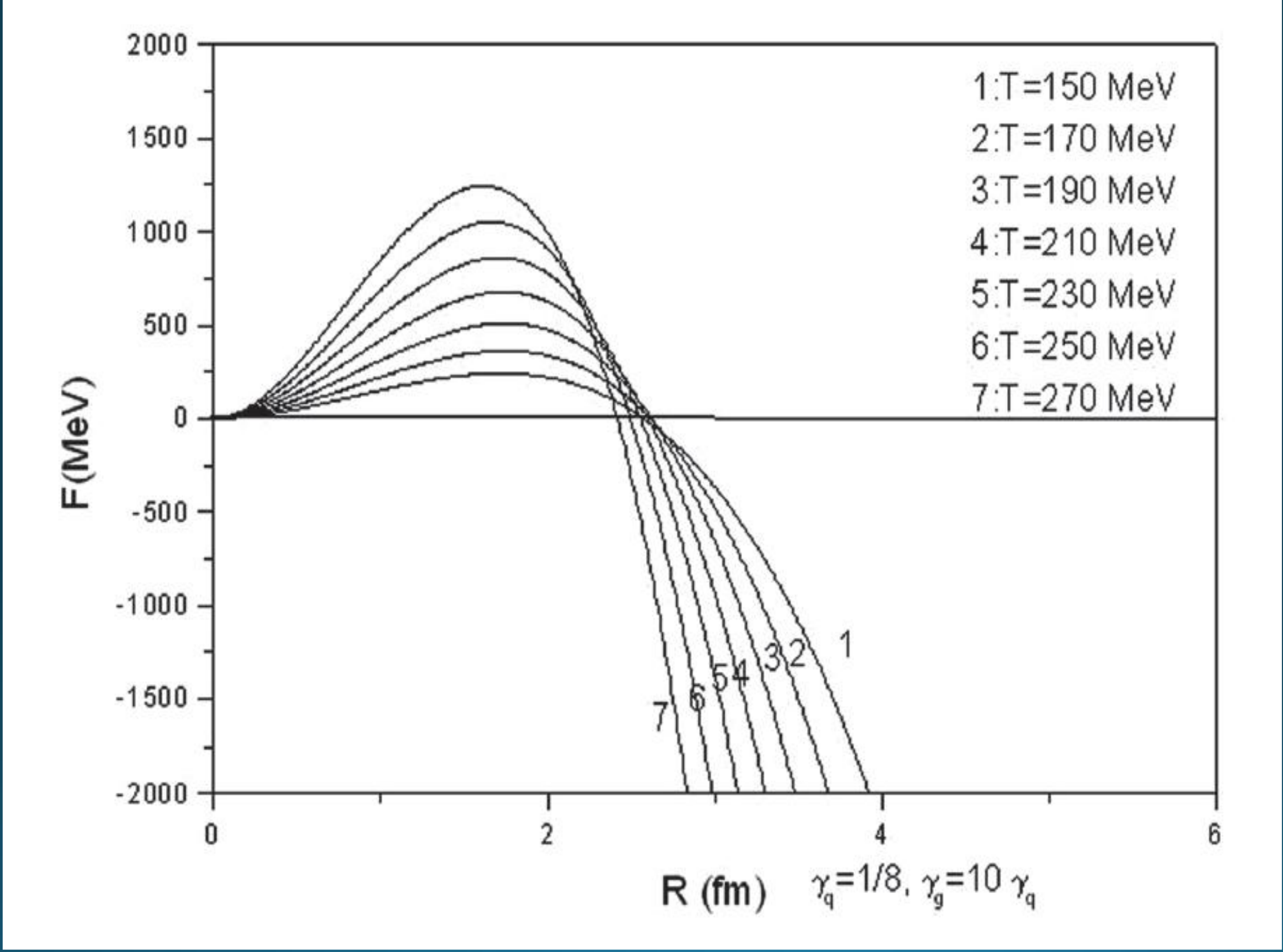
$$A = \left\{1 + \frac{(\alpha_s(k)\alpha_1)}{\pi}\right\}^2 \left[\frac{1 + \frac{(\alpha_s(k)\alpha_1)}{\pi}}{k^4} + \frac{2\left(1 + 2\frac{(\alpha_s(k)\alpha_1)}{\pi}\right)}{k^2(k^2 + \Lambda^2)\ln\left(1 + \frac{k^2}{\Lambda^2}\right)}\right]$$

$$g^2(k) = 4\pi\alpha_s$$

$a_1 = 2.5833 - 0.2778 n_1$
$\nu =$ Volume occupied by QGP
$n_1 =$ Number of light quark elements



4000
3000
2000
1000
0
-1000
-2000
-3000
-4000
0
2
4
6
8
F_{Total}(MeV)
R (fm)
γ_q=1/8, γ_g=8 γ_q
1:T=150 MeV
2:T=170 MeV
3:T=190 MeV
4:T=210 MeV
5:T=230 MeV
6:T=250 MeV
7:T=270 MeV
1
2
3
4
5
6
7


1:T=150 MeV
2:T=170 MeV
3:T=190 MeV
4:T=210 MeV
5:T=230 MeV
6:T=250 MeV
7:T=270 MeV
F(MeV)
2000
1500
1000
500
0
-500
-1000
-1500
-2000
0
2
4
6
1
2
3
4
5
6
7
R (fm)
γ_q=1/8, γ_g=10 γ_q

Density of state in phase space with one loop correction at chemical potential

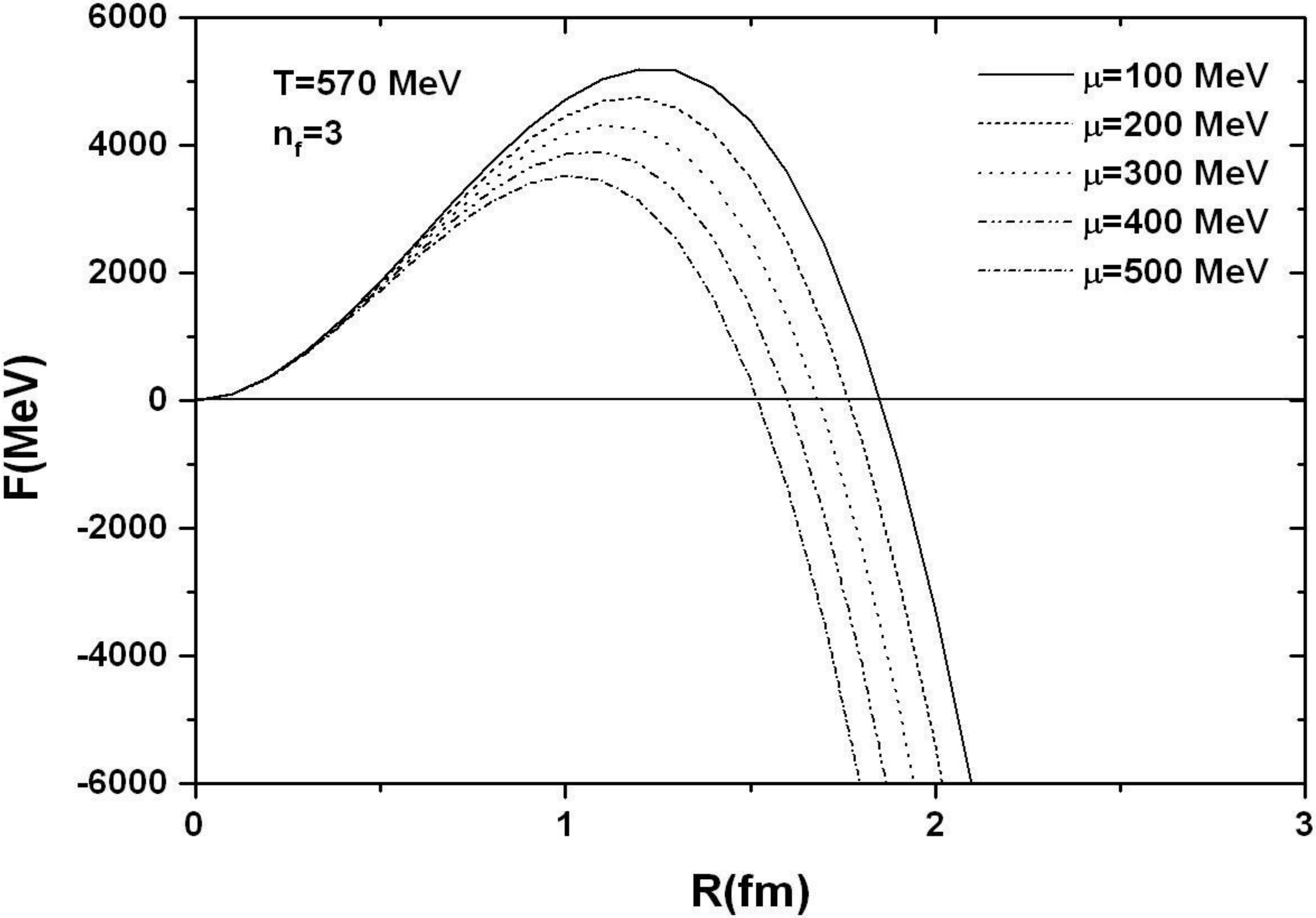
$$m^2(T,\mu) = 2\gamma^2 g^2(k)\left(T^2 + \frac{\mu^2}{\pi^2}\right)(1 + g^2(k)a_1)$$

$$V(k) = \frac{8\pi}{k}\gamma\alpha_s(k)\left(T^2 + \frac{\mu^2}{\pi^2}\right)\left[1 + \frac{\alpha_s(k)}{4\pi}a_1\right] - \frac{m_0^2}{2k}$$

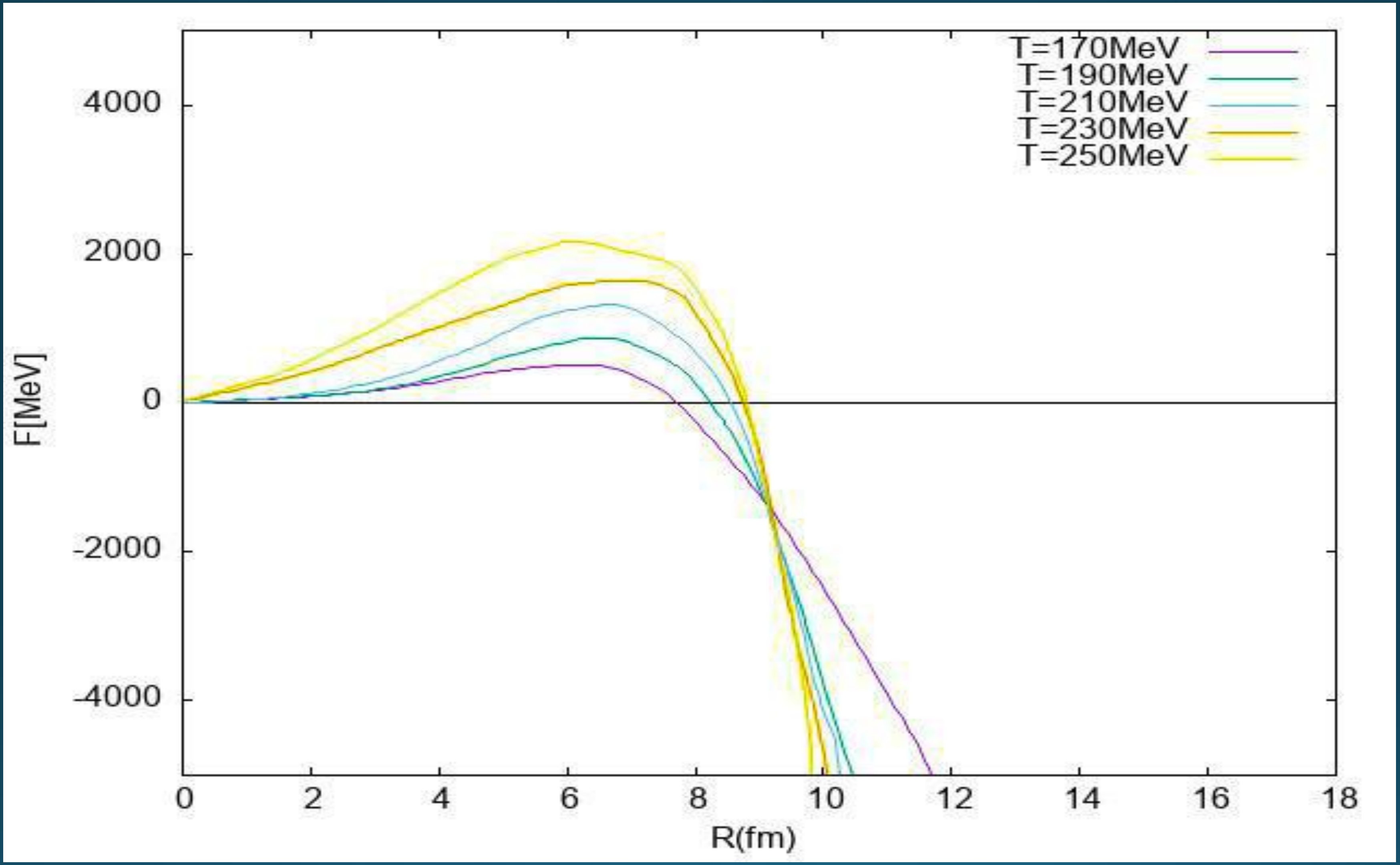
$$\rho_{q,q}(k,\mu) = \left(\frac{\nu}{\pi^2}\right)\left[\frac{\gamma^3(T^2+\mu^2/\pi^2)}{2}\right]^3 g^6(k)A$$

$$A = \left\{1 + \frac{(\alpha_s(k)\alpha_1)}{\pi}\right\}^2 \left[\frac{1 + \frac{(\alpha_s(k)\alpha_1)}{\pi}}{k^4} + \frac{2\left(1 + 2\frac{(\alpha_s(k)\alpha_1)}{\pi}\right)}{k^2(k^2+\Lambda^2)\ln\left(1 + \frac{k^2+\mu^2}{\Lambda^2}\right)}\right]$$

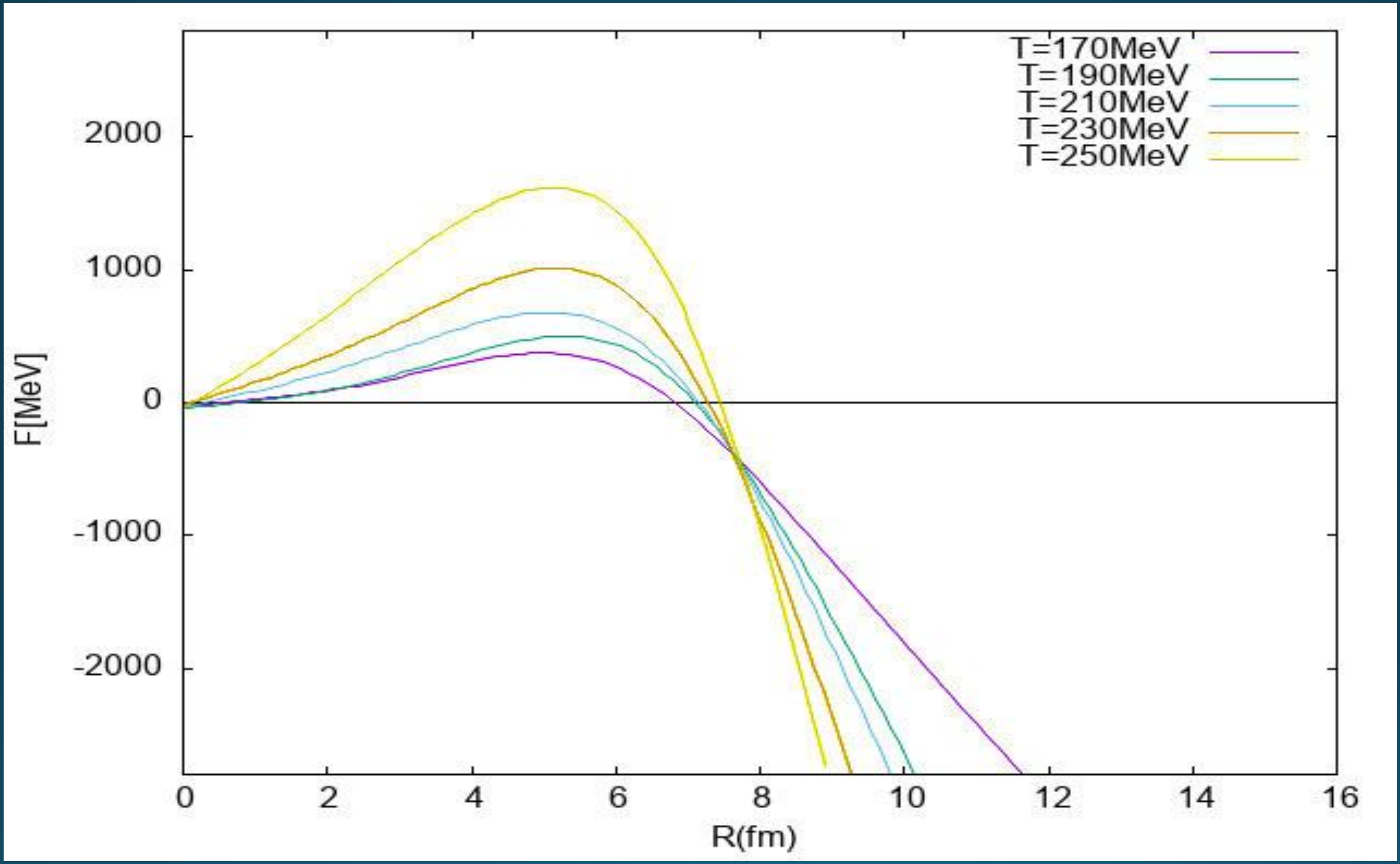
$$\alpha_s(k,\mu) = \frac{4\pi}{29\ln\left(1 + \frac{k^2+\mu^2}{\pi^2}\right)}$$



T=570 MeV
n_f=3
μ=100 MeV
μ=200 MeV
μ=300 MeV
μ=400 MeV
μ=500 MeV
6000
4000
2000
0
-2000
-4000
-6000
0
1
2
3
F(MeV)
R(fm)

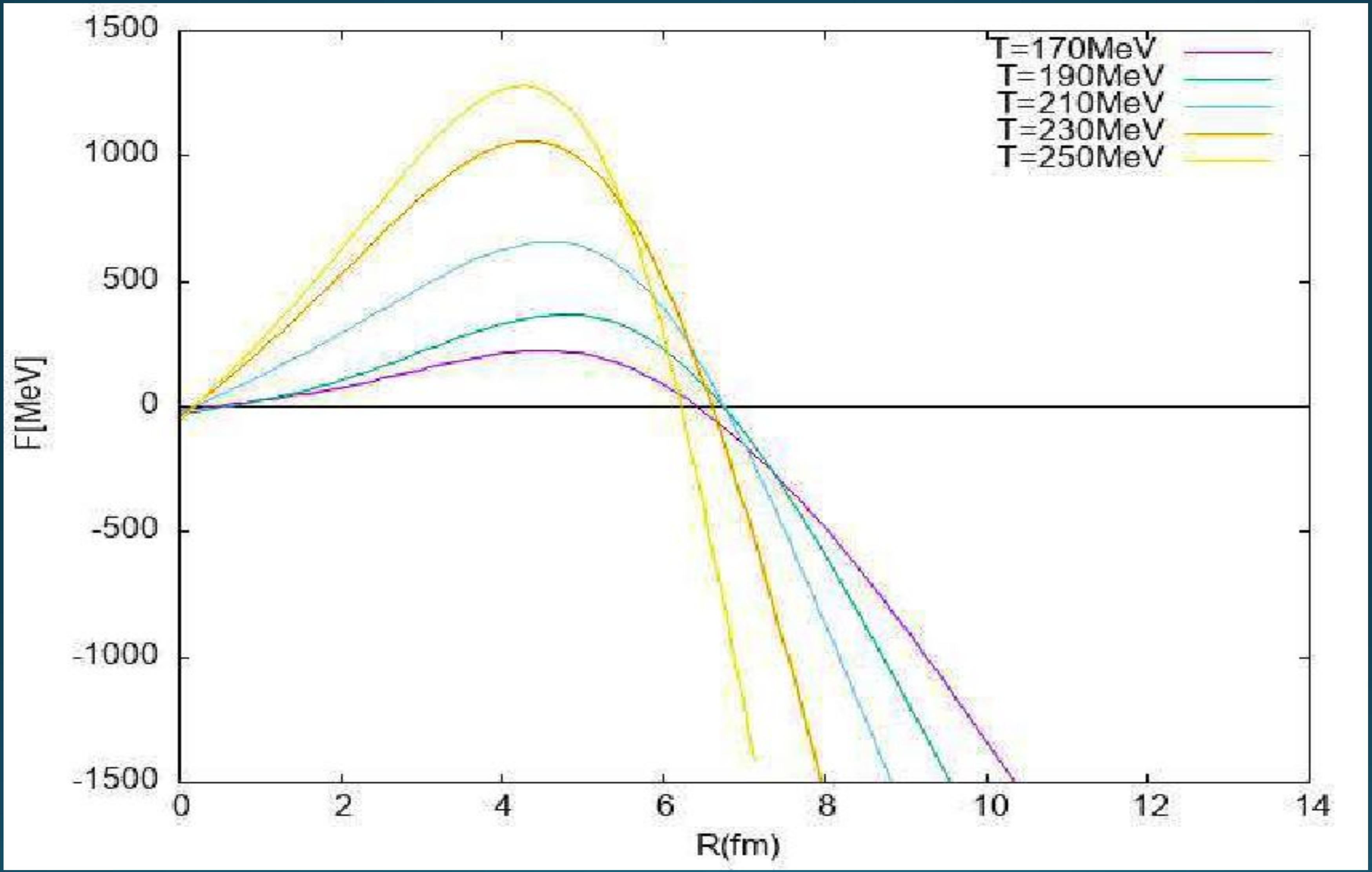


$$\gamma_q = \frac{1}{8}, \qquad \gamma_g = 12\gamma_q$$
$$at\ \mu = 300\ MeV$$



$$\gamma_q = \frac{1}{8}, \qquad \gamma_g = 14\gamma_q$$

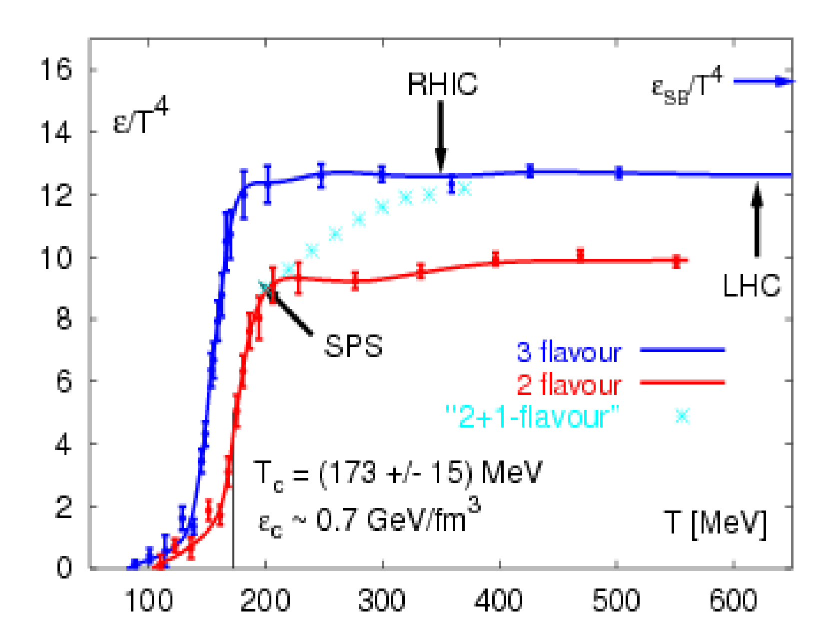
$at\ \mu = 300\ MeV$

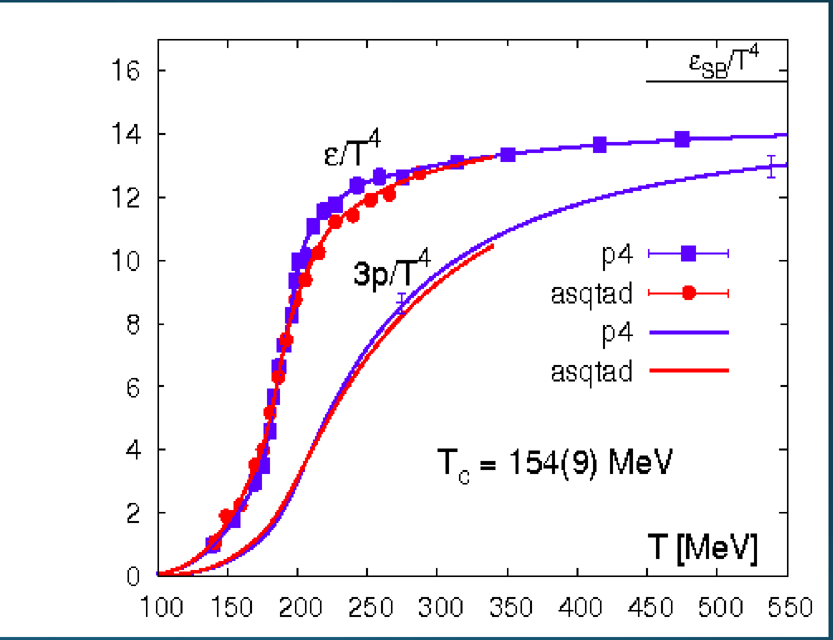


$$\gamma_q = \frac{1}{8}, \qquad \gamma_g = 16\gamma_q$$

$$at\ \mu = 300\ MeV$$

Equation of State





To look all these equations of state of QGP, We need to set up models and depending on the model, it can predict the QCD phase structure. There are many phenomenological models, we also create a very simple model to see all these physical parameters through one loop correction.

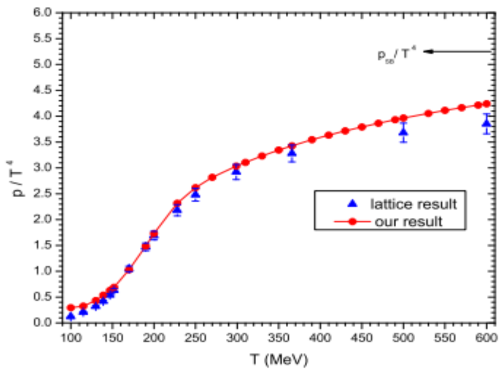


FIG. 4: Pressure vs. T at $\gamma_q = 1/14$, $\gamma_g = 68\gamma_q$.

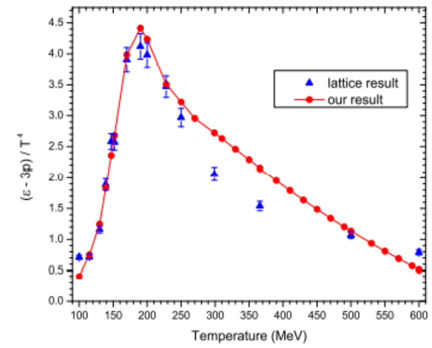
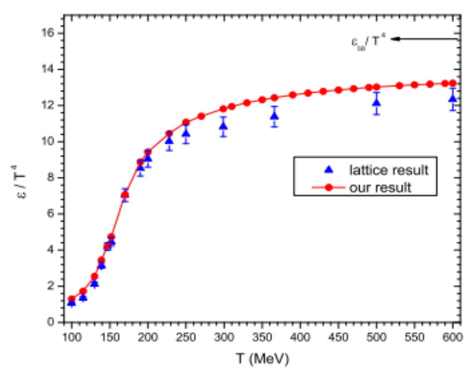


FIG. 6: Interaction measurement vs. T at $\gamma_q = 1/14$, $\gamma_g = 68\gamma_q$.



Energy density vs. T at $\gamma_q = 1/14$, $\gamma_g = 68\gamma_q$.

Conclusion

QGP Fireball under mean field potential and one loop at finite chemical potential

- It is also consistent with expectations of QGP- hadron phase transition. Due to presence of one loop correction in mean field potential, The following parameters are observed.
- The stability of droplets increases.
- Their size decreases in comparison with result with uncorrected potential.

$\pm$

- Energy evolution with effect of one loop correction in potential shows a higher transition temperature in range of T= 180 to 250 MeV.
- The transition of temperature is effected by dynamical flow parameter used in potential.
- It results in decreasing observable QGP droplets of stable radius 2.5-4.5 fm.

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Thank you