Far from equilibrium counterparts of near-equilibrium phenomena

Michal P. Heller

Funded by the European Union

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2307.07545 with Mazeliauskas and Preis [PRL], **2502.01622** with De Lescluze and **250x.xxxxx** with Berges, Denicol and Preis

Motivation

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Theory challenge: studying thermalization dynamics of closed quantum systems

2005.12299 with Berges, Mazeliauskas, Venugopalan

Highly relevant for → ultrarelativistic nuclear collisions at RHIC and LHC

Highly relevant for → ultracold atoms experiments

The theory targets specific parametric regimes (strong or weak coupling)

Connecting disparate languages offers novel opportunities →

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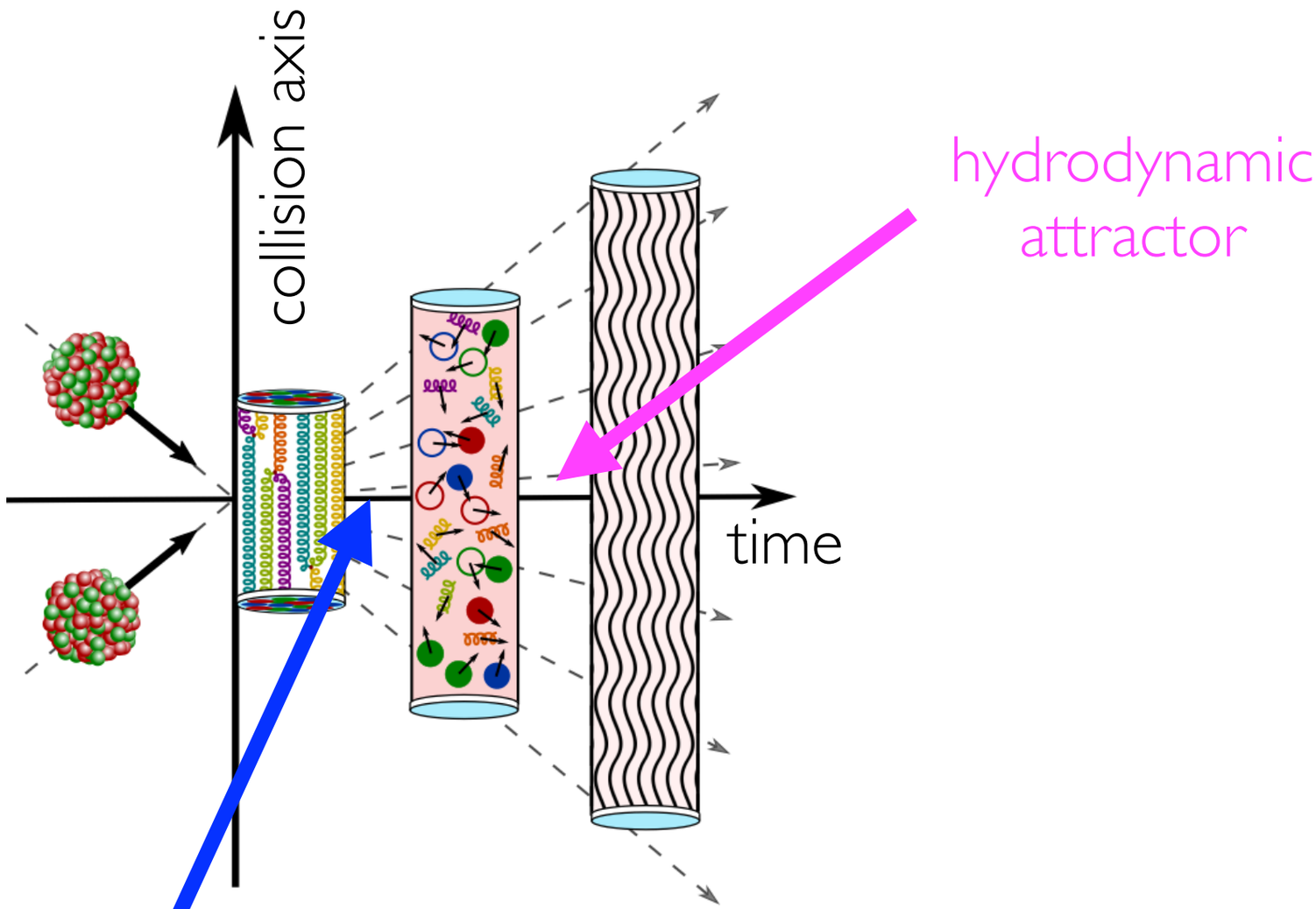
Attractors view on QCD thermalization

2005.12299 with Berges, Mazeliauskas and Venugopalan



Far from equilibrium ingredient: nonthermal fixed point(s)

Nonthermal attractor

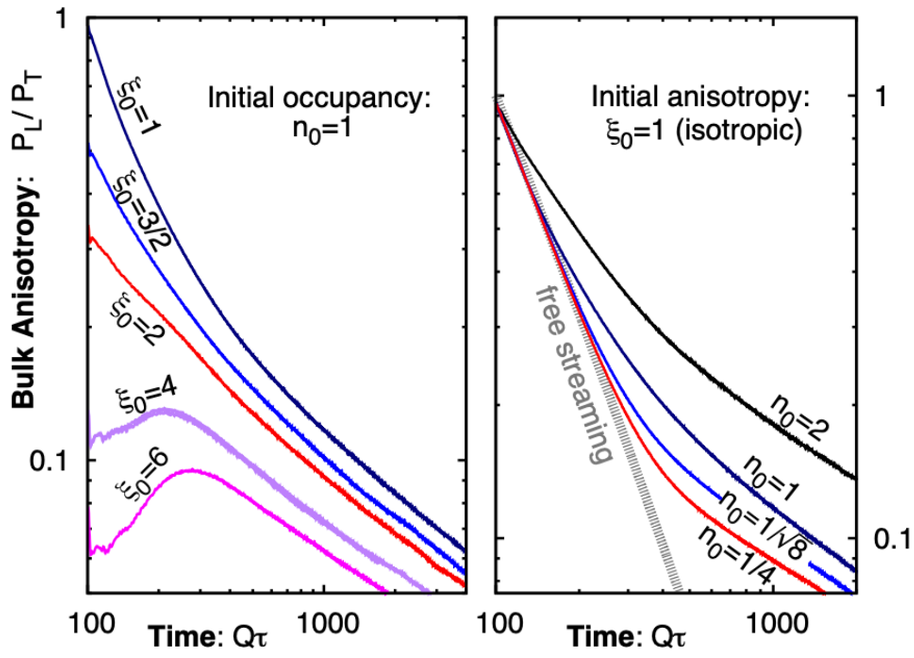
1303.5650 and **1311.3005** by Berges, Boguslavski, Schlichting and Venugopalan

After a collision at weak coupling: overoccupied gluons (strong classical fields):

$$f(p_T, p_z, \tau_0) = \frac{n_0}{8\pi\alpha_S}\,\Theta\left(Q - \sqrt{p_T^2 + (\xi_0 p_z)^2}\right) \quad \text{with} \quad \alpha_S = 10^{-5}$$

Ab initio simulations revealed significant insensitivity to plasma instabilities and free streaming, consistent with elastic scattering dominance among hard particles in the kinetic theory regime $Q\tau \sim log^2(1/\alpha_S) = O(10^2)$: stage I of the "bottom up" thermalization

hep-ph/0009237 by Baier, Mueller, Schiff and Son



Overoccupation is expected then to drive the system to a self similar regime

$$f(p_T, p_z, \tau) = (Q\tau)^{\alpha} f_S\left((Q\tau)^{\beta} p_T, (Q\tau)^{\gamma} p_z\right) \quad \text{with } \alpha = -\frac{2}{3}, \beta = 0, \gamma = \frac{1}{3}$$

Nonthermal attractor

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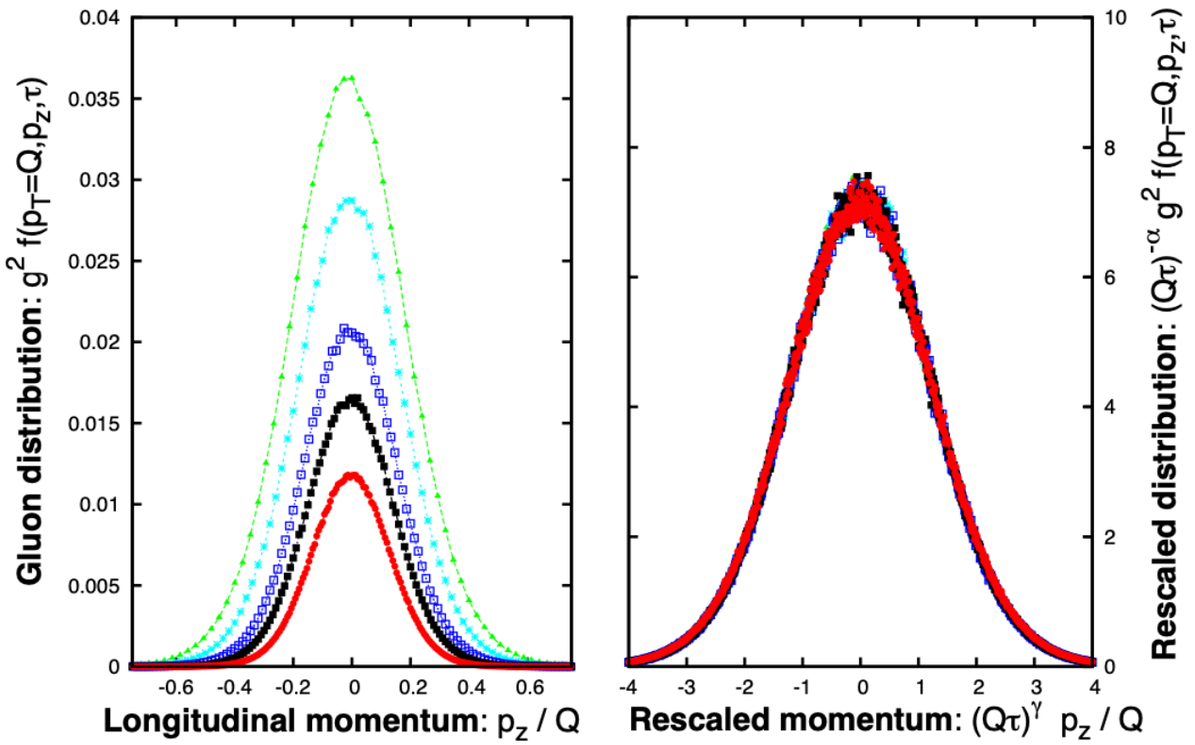
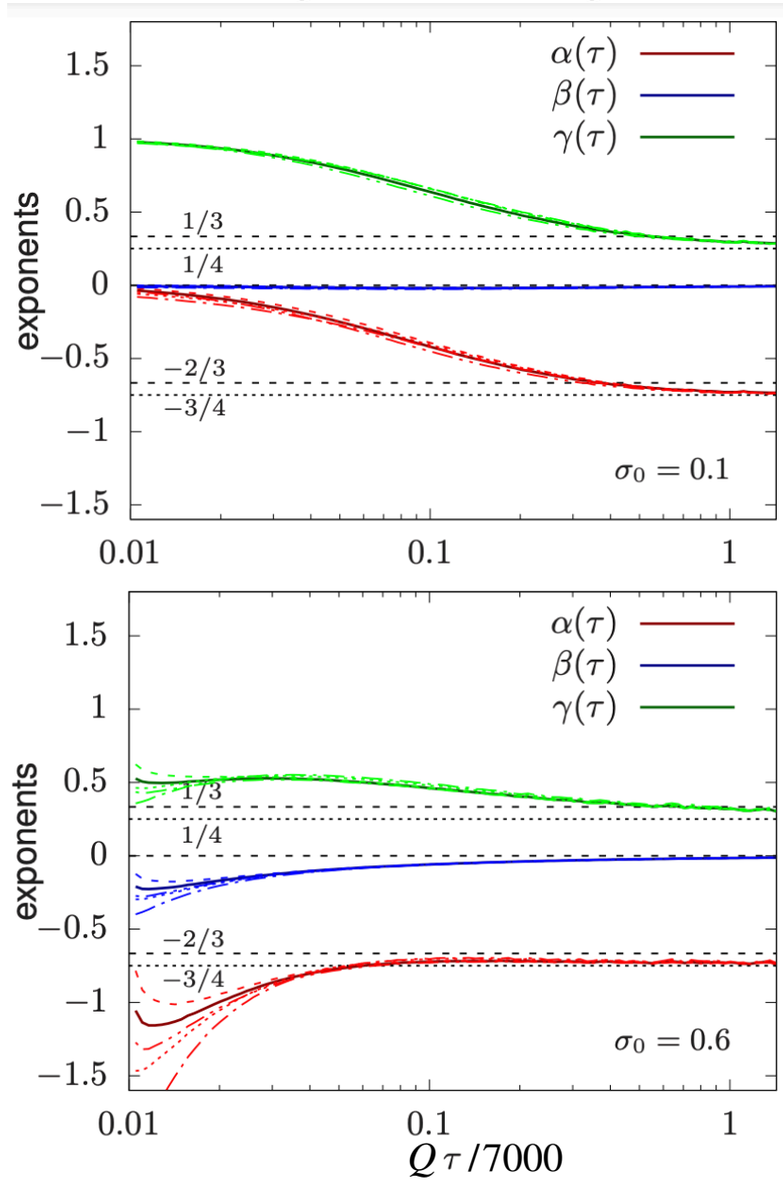


Classical statistical simulations of SU(2) gluons

1303.5650 and **1311.3005**

by Berges, Boguslavski, Schlichting and Venugopalan



g = 10^{-3} EKT with 2 massless quarks

1810.10554

by Berges and Mazeliauskas

Simplification in this talk: isotropy

No expansion: $ds^2 = -dt^2 + d\vec{x}^2$ and time dependence on t

Distribution function: $f(t, p \equiv |\vec{p}\,|)$

Nonthermal fixed point has only two scaling exponents: $t^{\alpha} f_s\left(t^{\beta} p\right)$

Prescaling: $A(t) f_s\left(B(t)\, p\right)$

Isotropic nonthermal fixed points

e.g. **1810.08143** by Schmied, Mikheev and Gasenzer
or **2005.12299** with Berges, Mazeliauskas, Venugopalan

Ingredients:

weak coupling + overoccupied initial states $f(t=0,p) \gg f_{eq}(p)\Big|_{\text{same energy density}}$

Outcome:

simulations + experiments show prolonged self-similar evolution in time

$$f(t,p) \approx A(t) \times f_s\left(B(t)p\right) \quad \text{with} \quad A(t) = \left(t/t_{\text{ref}}\right)^{\alpha} \quad \text{and} \quad B(t) = \left(t/t_{\text{ref}}\right)^{\beta}$$

Observation: the choice of the origin of time

$$f(t,p) \approx A(t) \times f_s\left(B(t)p\right) \quad \text{with} \quad A(t) = \left(t/t_{\text{ref}}\right)^{\alpha} \quad \text{and} \quad B(t) = \left(t/t_{\text{ref}}\right)^{\beta}$$

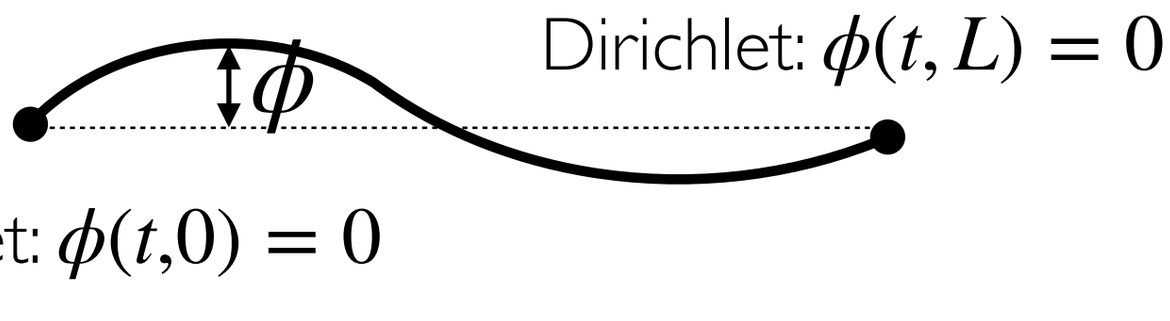
Special points for scaling phenomena $\begin{cases} \nearrow\ t = 0 \\ \searrow\ t \to \infty \end{cases}$

However, t used above has an origin chosen in a rather arbitrary way. Implication?

Near-equilibrium ingredient: quasinormal modes

Normal modes

Wave equation in a cavity $\Box_{\text{cavity}}\,\phi = 0$, $\phi\big|_{\text{bdries}} = 0$ ⟶ spectrum of normal modes e.g.



$$-\partial_t^2\phi + \partial_x^2\phi = 0$$

$$\phi_{\text{NM}} \sim e^{-i\omega_{\text{NM}}t}\sin(n\pi x/L) \quad \text{with} \quad \omega_{\text{NM}} = \pm\, n\pi/L$$

If excited, normal modes just oscillate (possibly interfering with each other)

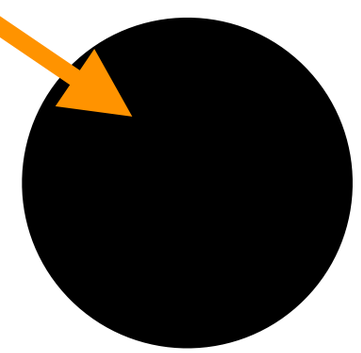
Quasinormal modes e.g. **0905.2975** by Berti, Cardoso and Starinets

Option I: the equation breaks time symmetry explicitly, e.g.

$\partial_t \phi - D\partial_x^2 \phi = 0$ on a line $\phi_{\mathrm{QNM}} \sim e^{-i\omega_{\mathrm{QNM}} t} e^{ikx}$ with $\omega_{\mathrm{QNM}} = -iDk^2 \in \mathbb{C}$

Option II: the boundary condition makes the problem non-Hermitian

black hole classically only absorbs : $\Box_{\text{black hole}}\, \phi = 0$ +



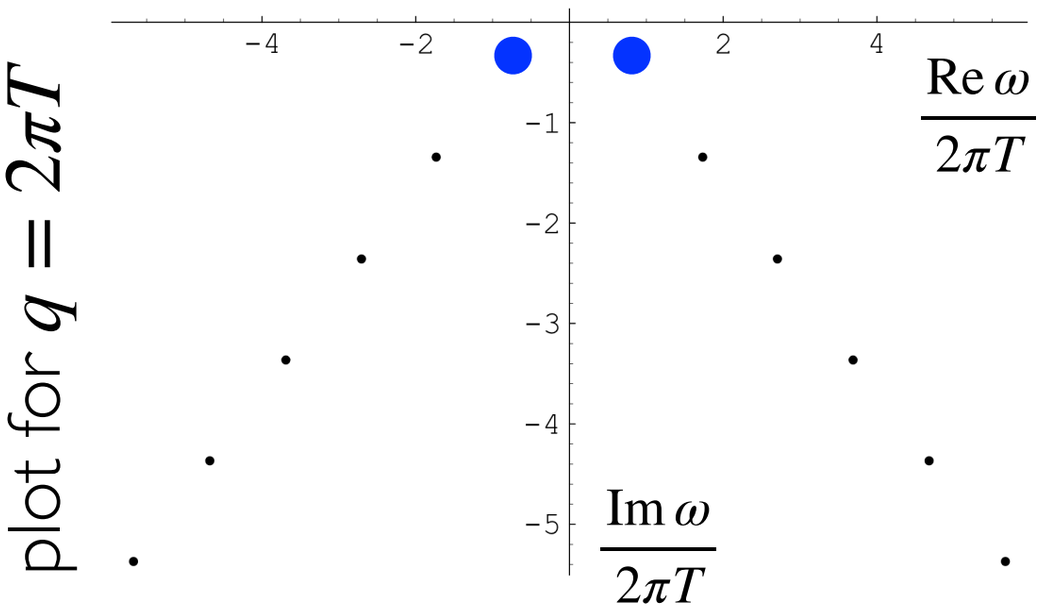
$\longrightarrow$ a tower of $\omega_{\mathrm{QNM}} \in \mathbb{C}$

If excited, quasinormal modes decay in time (and often also oscillate)

Holographic quasinormal modes (QNMs)

Horowitz and Hubeny hep-th/9909056; Kovtun and Starinets hep-th/0506184

Strongly-coupled QFTs relax via dual QNMs: $\delta g_{ab} \sim \delta\langle T_{\mu\nu}\rangle \sim e^{-i\omega t + i\vec{q}\cdot\vec{x}}$



Consequences for thermalization

- lots of short-lived excitations
- a few long-lived hydrodynamic modes

Ingredient III: (relativistic) hydrodynamics

Relativistic hydrodynamics e.g. Florkowski, Heller & Spaliński 1707.02282

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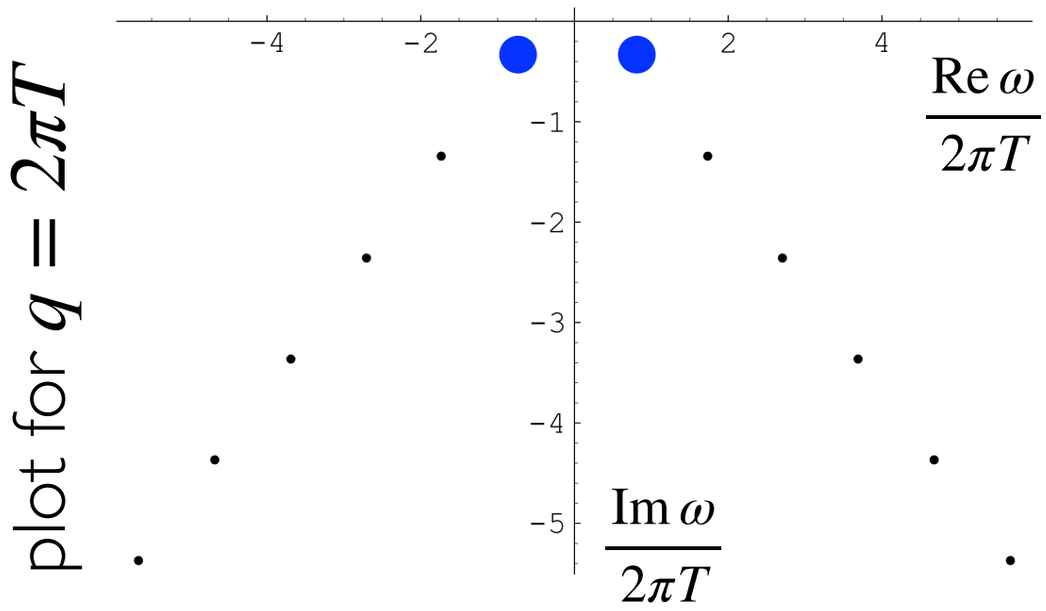


Relativistic Navier-Stokes equations dictate the properties of sound propagation:

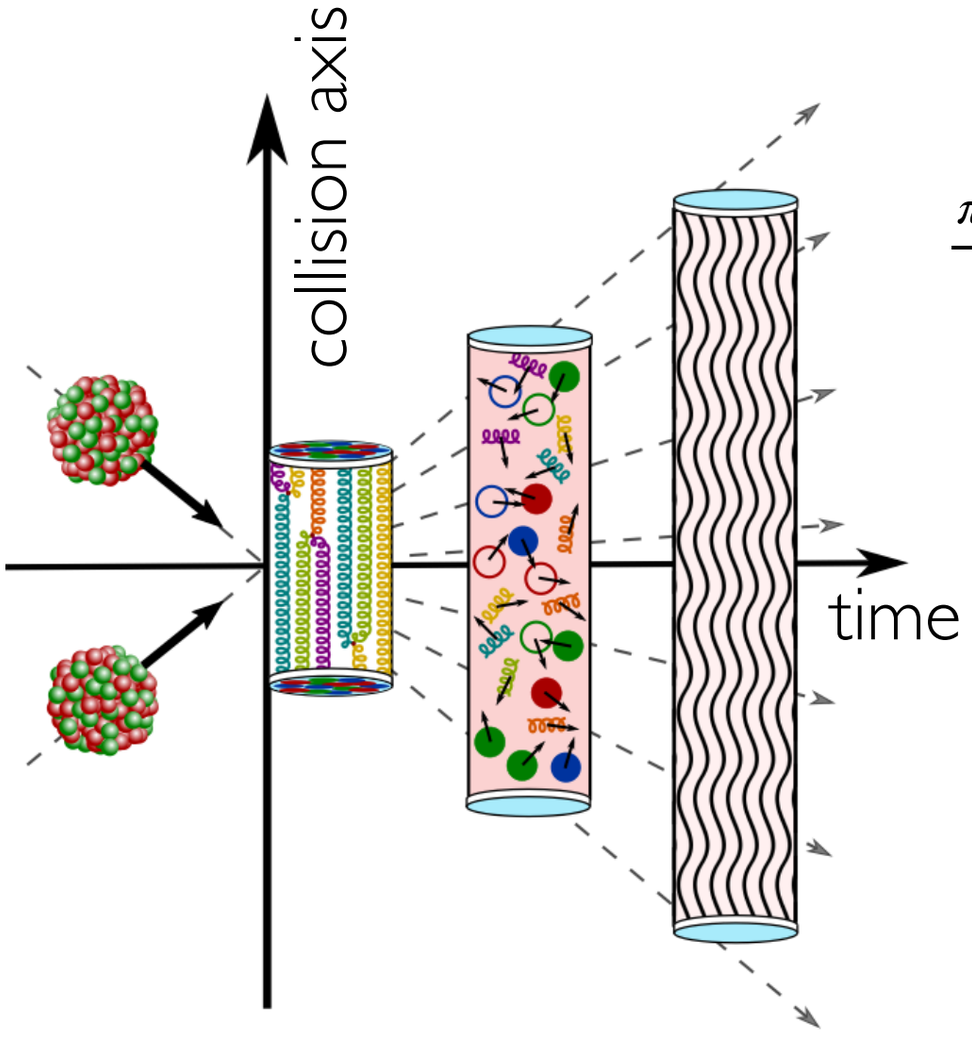
$$\omega = \pm\, c_s\, q - i\frac{4}{3}\frac{\eta}{s}q^2 + \ldots$$

(shear) viscosity describes the dominant dissipative effect

Are there sound waves propagating on top of nonthermal fixed points?

Hydro and transient QNMs in action

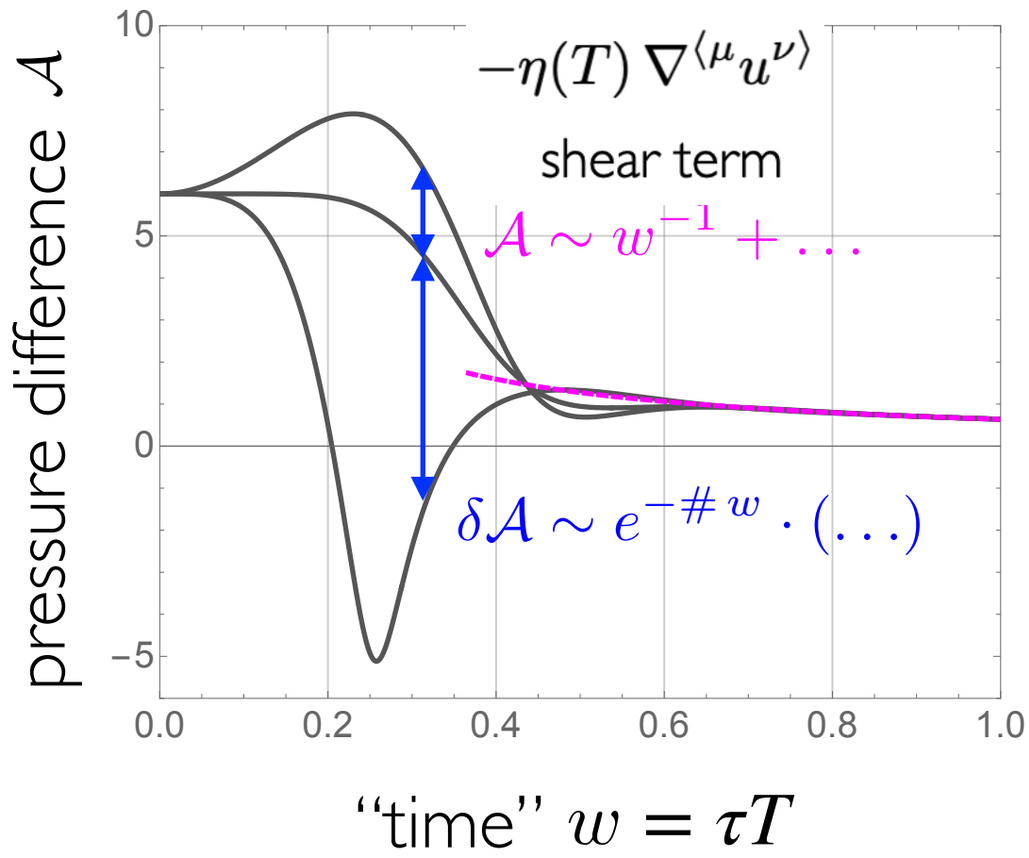
heavy-ion collisions
at RHIC and LHC



2005.12299
with Berges, Mazeliauskas & Venugopalan

behaviour in
of theoretical models
(here: holographic boost-invariant flow)

$$\frac{\pi^T_{\ T} - \pi^L_{\ L}}{\mathcal{P}(T)} = \mathcal{A}$$



1103.3452 with Janik & Witaszczyk

Setup

Kinetic theory setup

We will be using (relativistic) isotropic kinetic theory

$$\partial_t f(t,p) = C[f](t,p)$$

Example of $C[f]$ is the Fokker-Planck QCD collision kernel

$$C_{FP}[f](t,p) \sim \mathcal{L}\left[I_a \frac{1}{p^2}\partial_p(p^2\partial_p f) + I_b \frac{1}{p^2}\partial_p(p^2 f(1+f))\right]$$

$$\mathcal{L} = \int \frac{dq}{q} = \log\frac{\sqrt{\langle p^2\rangle}}{m_D}, \qquad \mathscr{L} = \text{const},$$

where $I_a = N_c \int \frac{d^3p}{(2\pi)^3} f(1+f),$

$$I_b = 2N_c \int \frac{d^3p}{(2\pi)^3}\frac{f}{p}.$$

This collision kernel leads to a direct cascade in the UV

Result 1:
extending nonthermal fixed points to earlier times

Prescaling

1807.07514 by Schmied, Mikheev and Gasenzer
1810.10554 by Berges and Mazeliauskas

Scaling with slowly varying in time exponents

$$f(t,p) \approx A(t) \times f_s\left(B(t)p\right) \quad \text{with} \quad A(t) = \left(t/t_{\text{ref}}\right)^{\alpha(t)} \text{and} \quad B(t) = \left(t/t_{\text{ref}}\right)^{\beta(t)}$$

might precede the exact scaling with α, β constant

Important for theory and experiment, since makes scaling visible much earlier

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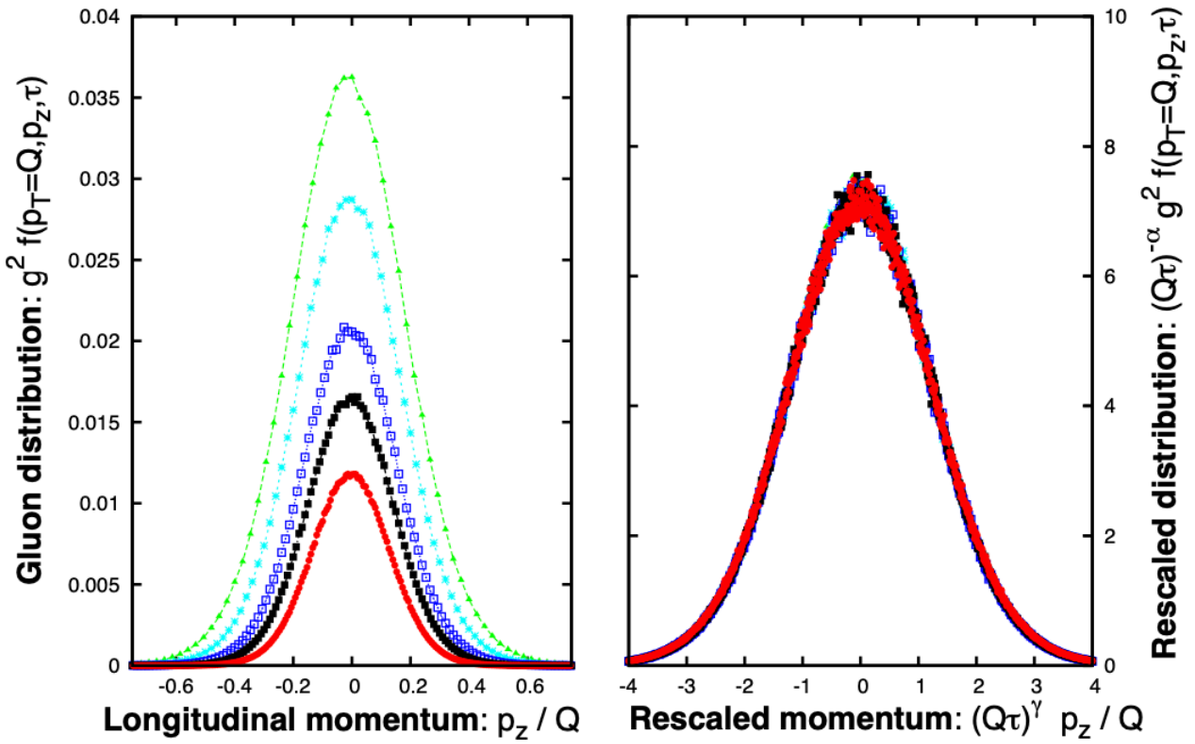

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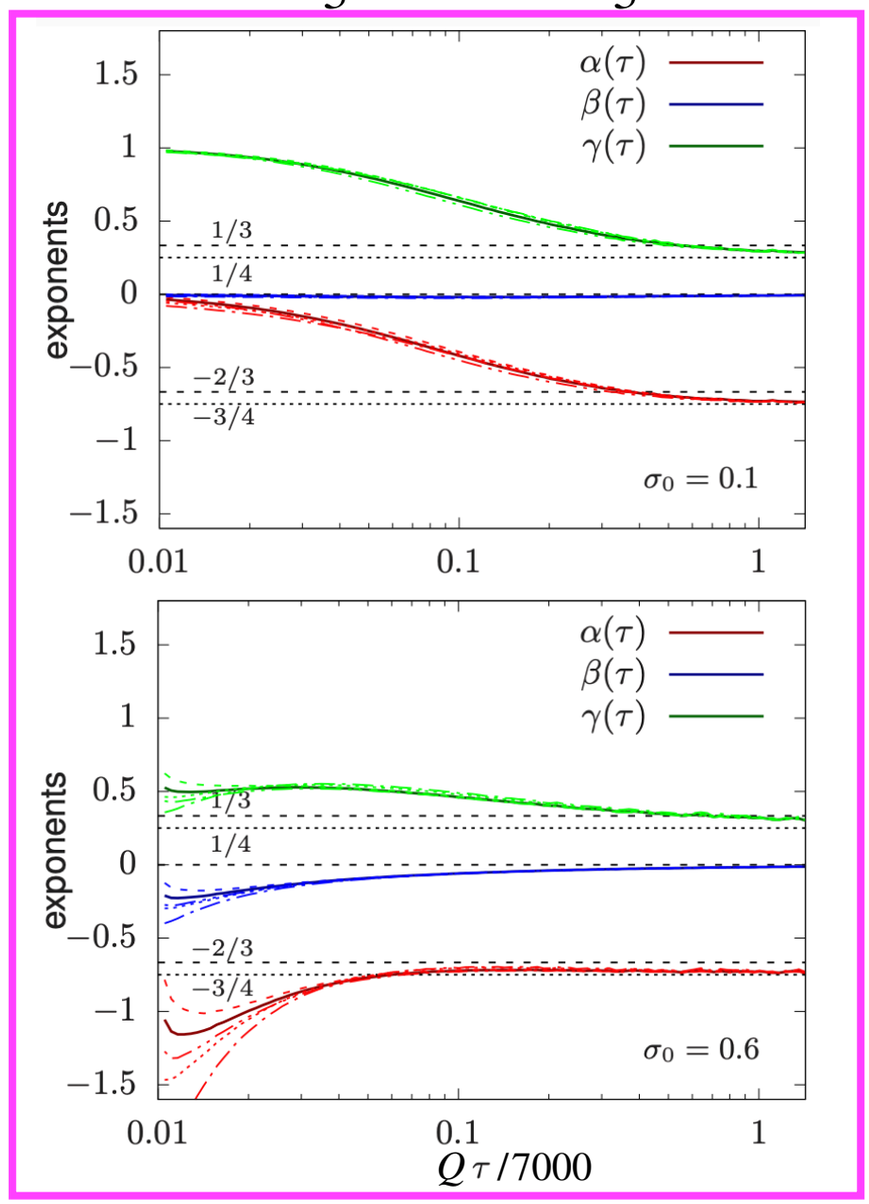

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The origin of (pre)scaling 2307.07545 with Mazeliauskas and Preis

Overoccupation simplifies $C[f(t,p)]$ as the highest power of f dominates

$$C[f(t,p)] \sim \int dP_1 \ldots \; (\ldots) f^{\kappa} \text{ with } \kappa_{2\leftrightarrow 2} = 3 \text{ and } \kappa_{1\leftrightarrow 2} = 2$$

This might make it possible to factor out t-dependence for $A(t) f_s\left(B(t)\, p\right)$:

$$C[f(t,p)] \equiv A(t)^{\mu_\alpha} B(t)^{\mu_\beta} C[f_s(\bar{p} \equiv B(t)p)]$$

Overoccupation leads to most of the energy (or particle number) in the relevant range of momenta, but the simplified kernel alters conservation properties

Imposing conservation of energy (or particle number) leads then to

$$A(t) \sim B(t)^{\sigma} \quad \text{with } \sigma = 4 \text{ (or } \sigma = 3)$$

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Prescaling as dimensionality reduction

2307.07545 with Mazeliauskas and Preis

All in all, overoccupation + conservation factorizes then the Boltzmann equation

$$\frac{B(t)^{1-1/\beta_\infty}}{\partial_t B(t)} = \frac{1}{D_1} = \frac{[\sigma + \bar{\mathbf{p}} \cdot \partial_{\bar{\mathbf{p}}}]\, f_S(\bar{\mathbf{p}})}{\mathcal{C}[f_S](\bar{\mathbf{p}})} \quad \text{where} \quad \frac{1}{\beta_\infty} \equiv (1-\mu_\alpha)\sigma - \mu_\beta$$

time dependence
(closed form eom for $B(t)$)

momentum dependence

Vast reduction of complexity in a sense of dimensionality reduction

Physics of prescaling 2307.07545 with Mazeliauskas and Preis

The solutions of

$$\frac{B(t)^{1-1/\beta_\infty}}{\partial_t B(t)} = \frac{1}{D_1} = \frac{[\sigma + \bar{\mathbf{p}} \cdot \partial_{\bar{\mathbf{p}}}]\, f_S(\bar{\mathbf{p}})}{\mathcal{C}[f_S](\bar{\mathbf{p}})} \quad \text{where} \quad \frac{1}{\beta_\infty} \equiv (1-\mu_\alpha)\sigma - \mu_\beta$$

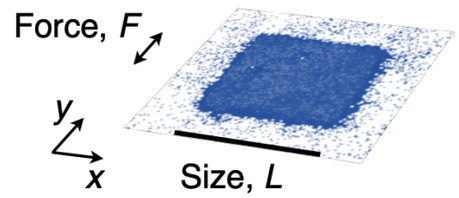
predict the prescaling as

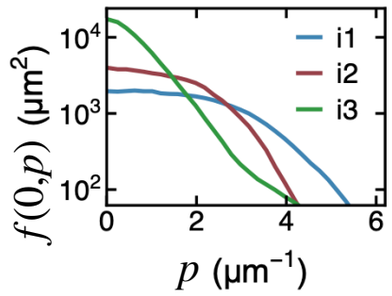
$$B(t) = \left(\frac{t-t_*}{t_{\rm ref}}\right)^{\beta_\infty} \approx \left(\frac{t}{t_{\rm ref}}\right)^{\beta_\infty} \left(1 - \beta_\infty \frac{t_*}{t} + \ldots\right)$$

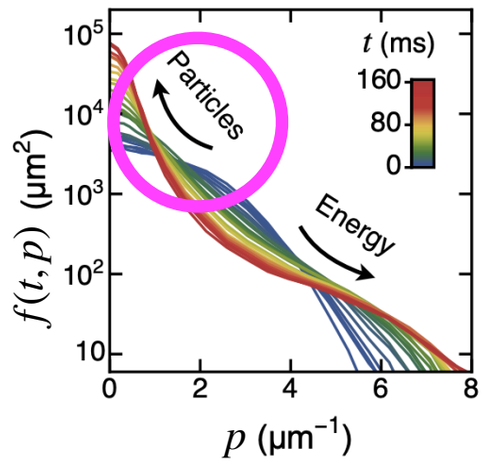
Prescaling is then just a scaling with an initial-condition dependent offset of time

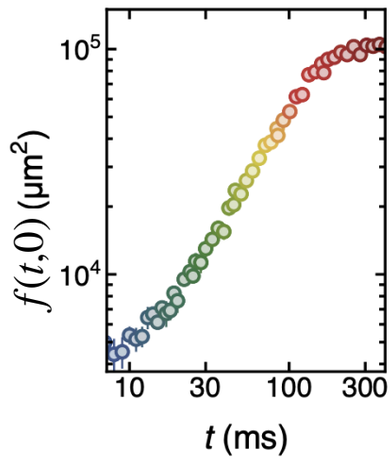
Experimental confirmation

2312.09248 by Gazo, Karailiev, Satoor, Eigen, Gałka and Hadzibabic

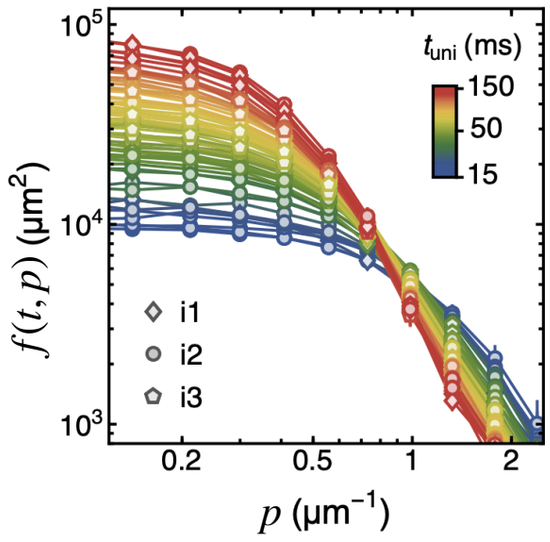




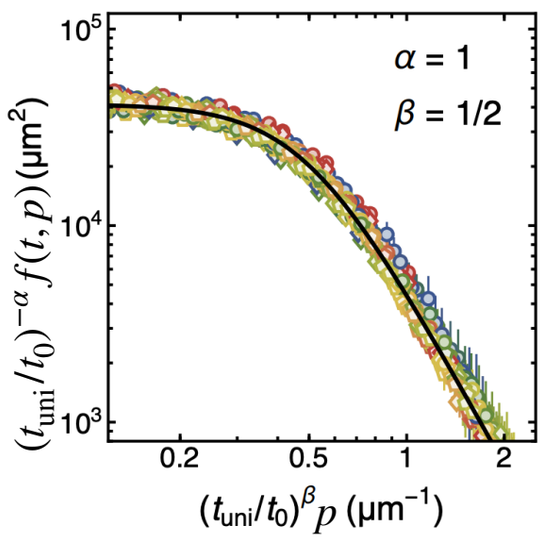




We start with a quasi-pure interacting 2D condensate of 7×10^4 atoms of ^{39}K in the lowest hyperfine state, confined in a square box trap of size $L = 50\,\mu\text{m}$ [40]. The interactions in the gas, characterized by the scattering length a, are tuneable via the magnetic Feshbach resonance at 402.7 G [41]. To prepare our far-from-equilibrium initial states, we temporarily turn off the interactions ($a \to 0$) and shake the gas with a spatially uniform oscillating force F (see Fig. 1A). This destroys the condensate and, as previously studied in 3D [42, 43], results in an isotropic highly nonthermal f distribution. After preparing one of the three different initial states i1–i3 shown in Fig. 1A, we stop the shaking, reinstate the interactions ($a \to 30\,a_0$, where a_0 is the Bohr radius), and let the gas relax. The states i1–i3 do not have a defined temperature, but $E = \int \varepsilon(k)\,\mathrm{d}k$, where $\varepsilon = 2\pi\hbar^2k^3n_k/(2m)$ and m is the atom mass, gives the total energy. We get $E/k_\text{B} = 4.1(3)$ mK, 2.2(3) mK, and 1.0(3) mK, for i1–i3 respectively; in all cases E is sufficiently low for a condensate to emerge during relaxation [44].



$$t_\text{uni} \equiv t - t_*$$



Some perspectives from prescaling

Nonthermal fixed points = overoccupation + dimensionality reduction

(Pre)scaling is then a consequence of the equations of motion

Prescaling exhausts the dimensionally reduced ansatz $f(t,p) \approx A(t) \times f_s\left(B(t)p\right)$

There is sense in which nonthermal fixed points are static: $f_s(\bar{p})$

As a result, the notion of quasinormal mode approach makes sense

Result II:
transient QNMs of nonthermal fixed points

Quasinormal modes of nonthermal fixed points

2502.01622 with De Lescluze

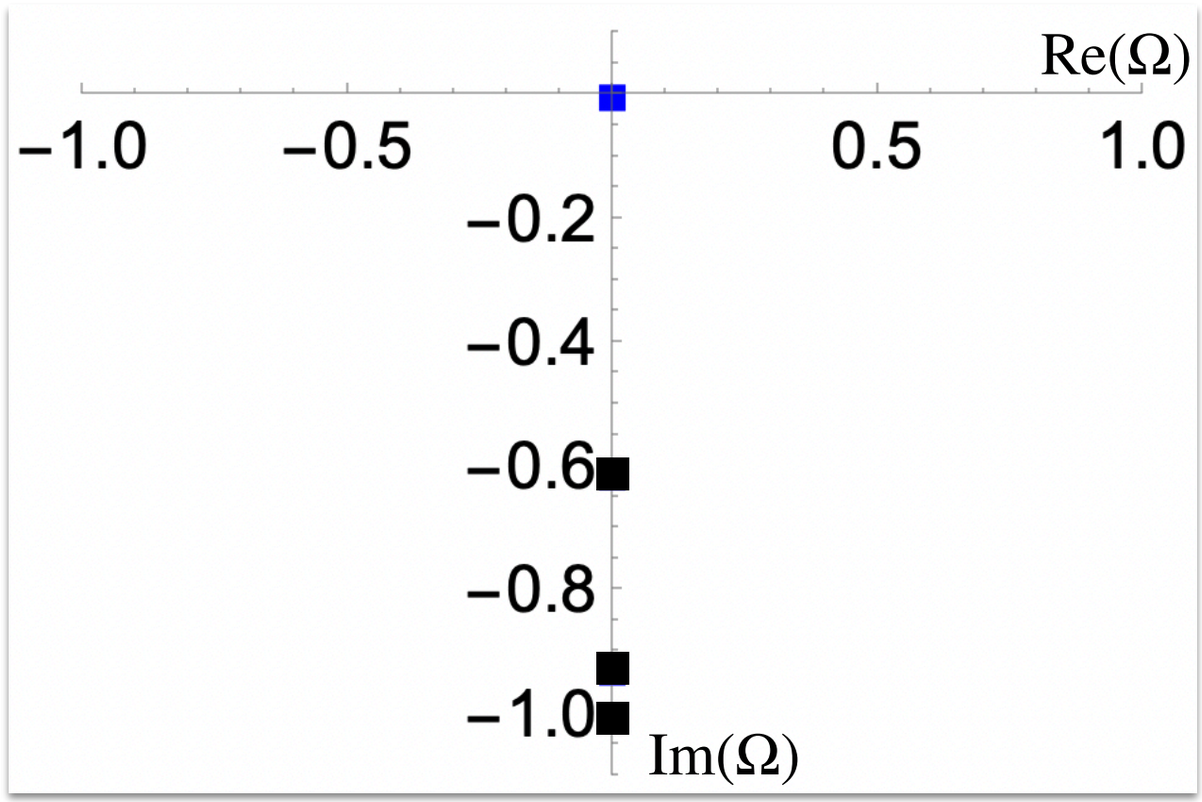
On a nonthermal fixed point:

$$f(t,p) \approx \left(t_{\rm uni}/t_0\right)^{\alpha} f_{\rm scaling}\left[\left(t_{\rm uni}/t_0\right)^{\beta} p\right]$$

with $t_{\rm uni} \equiv t - t_*$

directly before that:

$$\left(t_{\rm uni}/t_0\right)^{-\alpha} f(t,p) \approx f_{\rm scaling}\left[\left(t_{\rm uni}/t_0\right)^{\beta} p\right] + \sum_{\Omega} \left(t_{\rm uni}/t_0\right)^{-i\Omega} \delta f_{\Omega}\left[\left(t_{\rm uni}/t_0\right)^{\beta} p\right]$$



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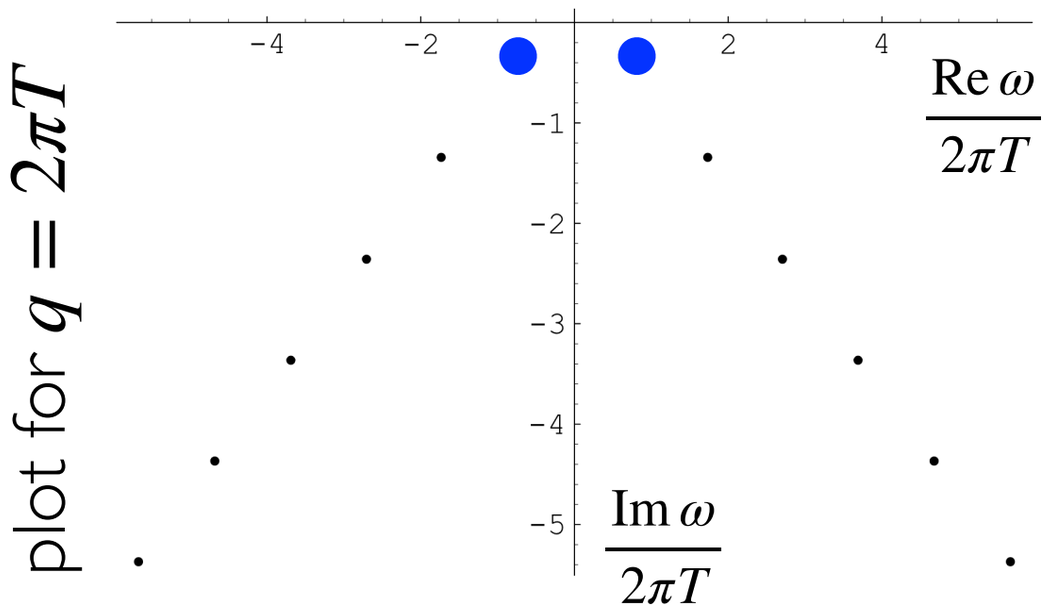

Consequences for thermalization

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How about nonthermal fixed points?

Quasinormal modes of nonthermal fixed points

2502.01622 with De Lescluze

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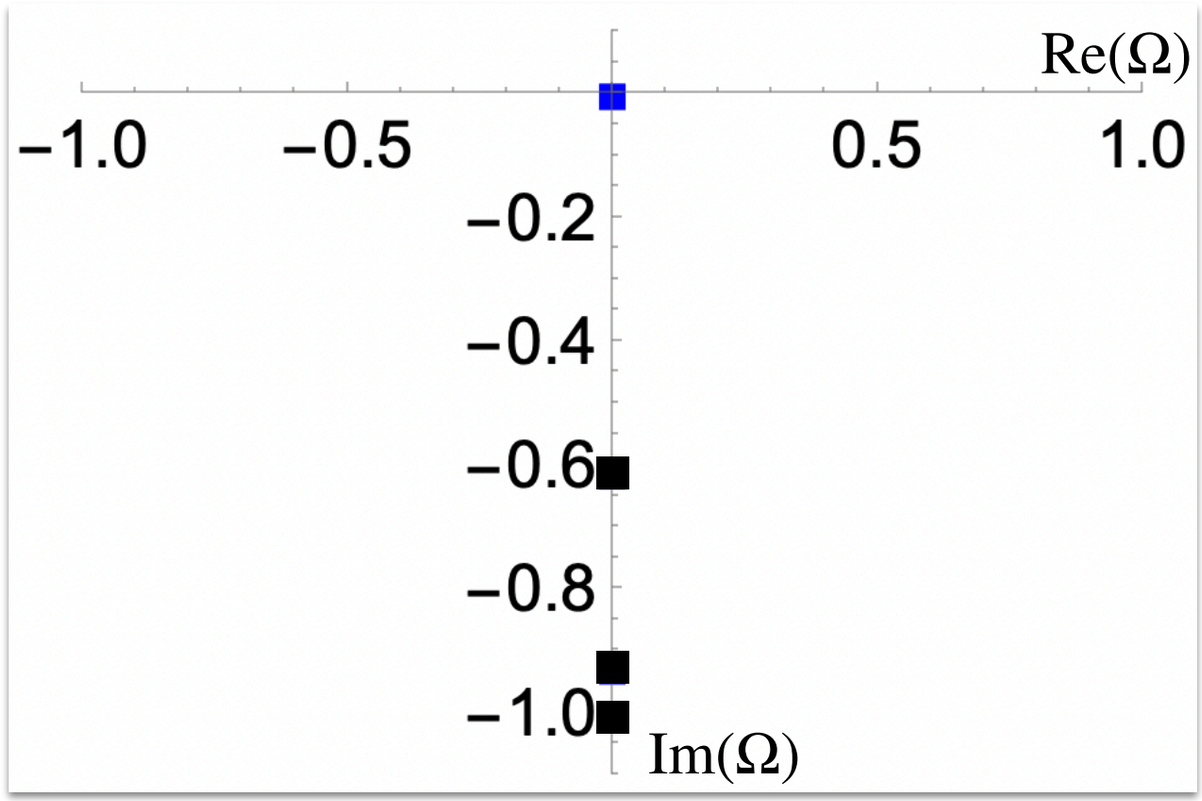
directly before that:

$$\left(t_{\rm uni}/t_0\right)^{-\alpha} f(t,p) \approx f_{\rm scaling}\left[\left(t_{\rm uni}/t_0\right)^{\beta} p\right] + \sum_{\Omega} \left(t_{\rm uni}/t_0\right)^{-i\Omega} \delta f_{\Omega}\left[\left(t_{\rm uni}/t_0\right)^{\beta} p\right]$$



Quasinormal modes of nonthermal fixed points

2502.01622 with De Lescluze

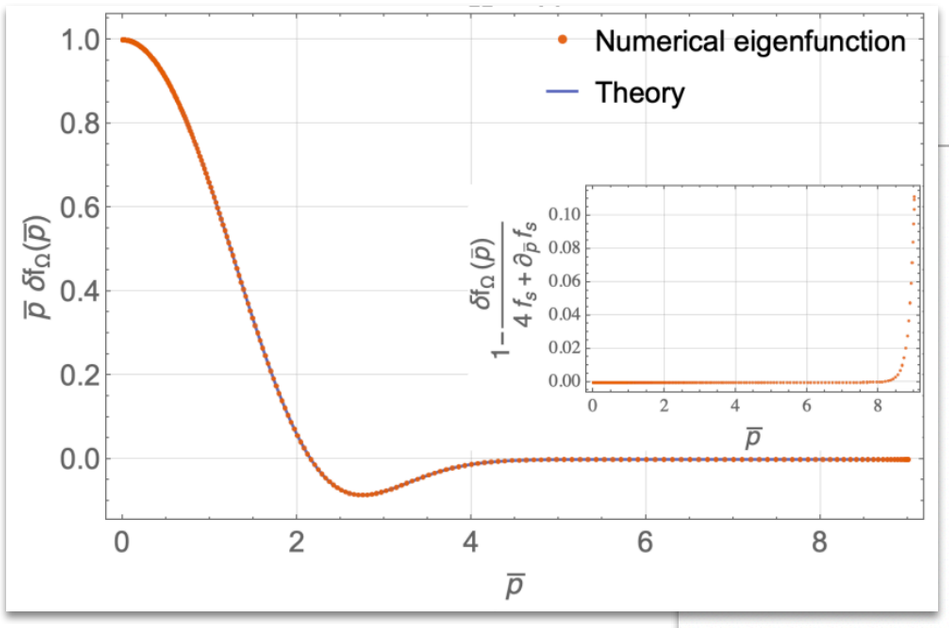
On a nonthermal fixed point:

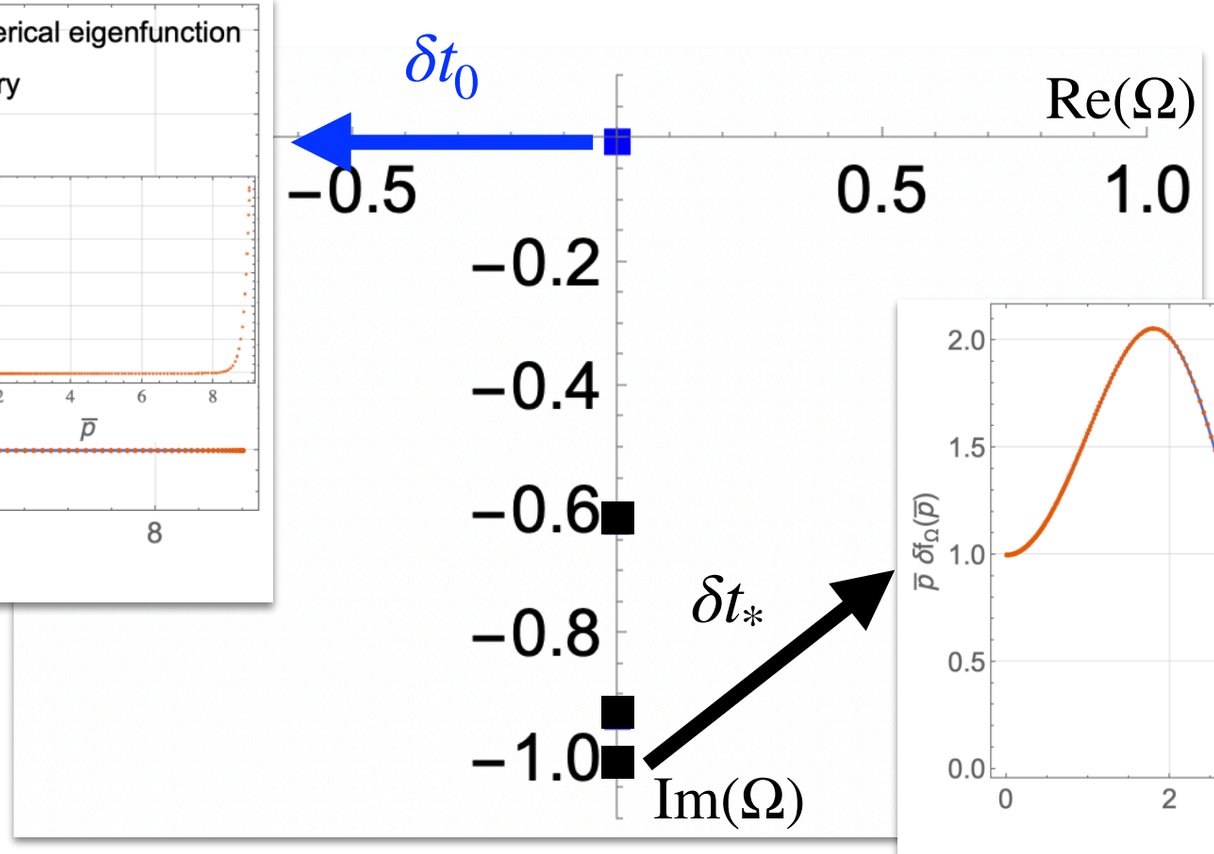
$$f(t,p) \approx \left(t_{\rm uni}/t_0\right)^{\alpha} f_{\rm scaling}\left[\left(t_{\rm uni}/t_0\right)^{\beta} p\right]$$

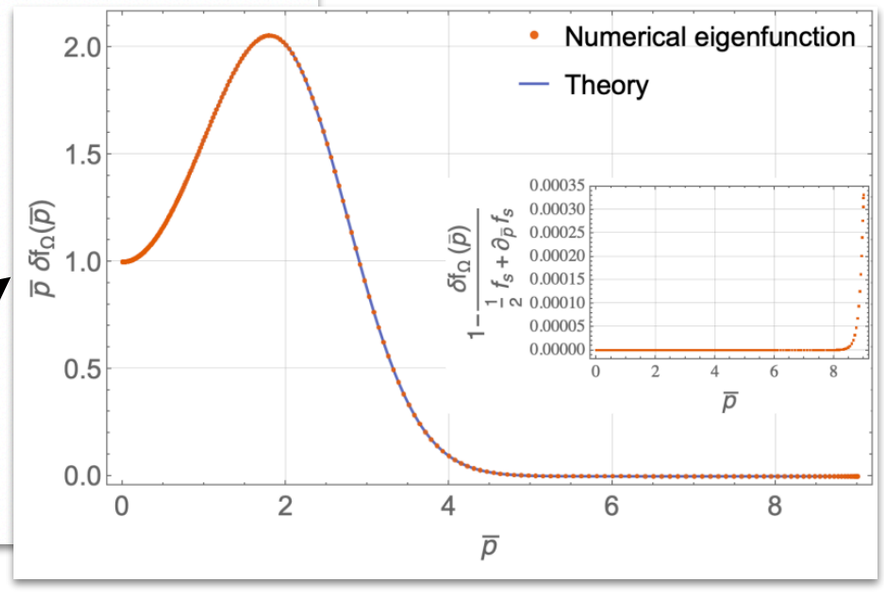
with $\quad t_{\rm uni} \equiv t - t_*$

directly before that:

$$\left(t_{\rm uni}/t_0\right)^{-\alpha} f(t,p) \approx f_{\rm scaling}\left[\left(t_{\rm uni}/t_0\right)^{\beta} p\right] + \sum_{\Omega} \left(t_{\rm uni}/t_0\right)^{-i\Omega} \delta f_{\Omega}\left[\left(t_{\rm uni}/t_0\right)^{\beta} p\right]$$





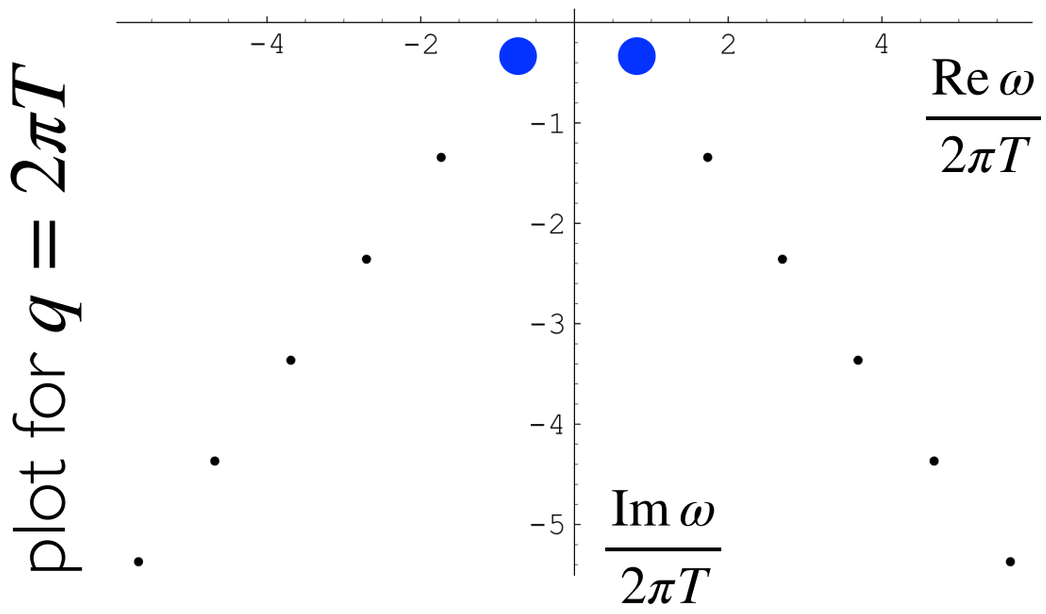


Result III:
hydrodynamics of nonthermal fixed points

Holographic quasinormal modes (QNMs)

Horowitz and Hubeny hep-th/9909056; Kovtun and Starinets hep-th/0506184

Strongly-coupled QFTs relax via dual QNMs: $\delta g_{ab} \sim \delta\langle T_{\mu\nu}\rangle \sim e^{-i\omega t + i\vec{q}\cdot\vec{x}}$



Consequences for thermalization

- lots of short-lived excitations
- a few long-lived hydrodynamic modes

How about nonthermal fixed points?

Hydrodynamics of nonthermal fixed points

250x.xxxxx with Berges, Denicol and Preis

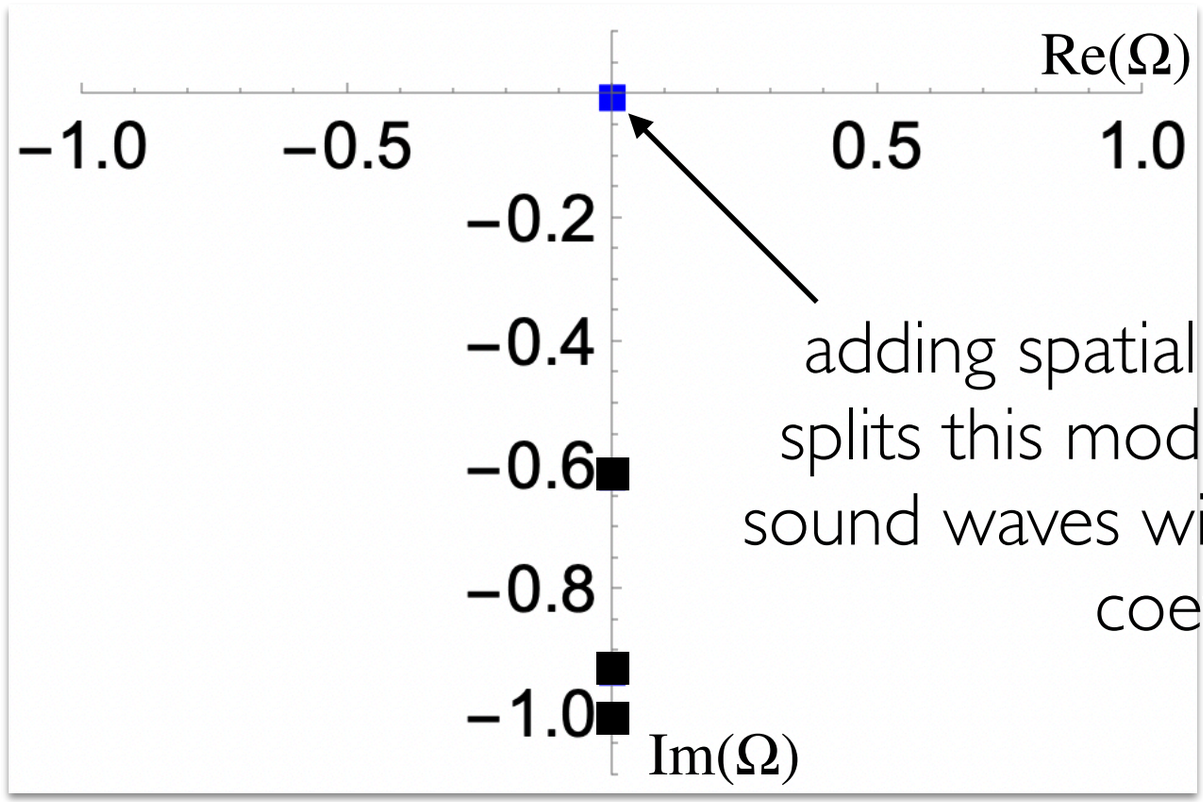
On a nonthermal fixed point:

$$f(t,p) \approx \left(t_{\rm uni}/t_0\right)^{\alpha} f_{\rm scaling}\left[\left(t_{\rm uni}/t_0\right)^{\beta} p\right]$$

with $t_{\rm uni} \equiv t - t_*$

directly before that:

$$\left(t_{\rm uni}/t_0\right)^{-\alpha} f(t,p) \approx f_{\rm scaling}\left[\left(t_{\rm uni}/t_0\right)^{\beta} p\right] + \sum_{\Omega} \left(t_{\rm uni}/t_0\right)^{-i\Omega} \delta f_{\Omega}\left[\left(t_{\rm uni}/t_0\right)^{\beta} p\right]$$



adding spatial momentum e^{iqx} splits this mode into propagating sound waves with scaling transport coefficients

Summary

Summary

Prescaling is the scaling with correctly accounted for the origin of time
Consequence: nonthermal fixed points extend to earlier times

2307.07545 with Mazeliauskas and Preis

The emergence of nonthermal fixed points can be thought of as originating from the decay of transient quasinormal modes around them

2502.01622 with De Lescluze

Adding spatial momentum reveals hydrodynamic modes and opens a window on studying inhomogeneous nonthermal fixed points

250x.xxxxx with Berges, Denicol and Preis

Open problems (cold atoms / nuclear collisions)

Prescaling is the scaling with correctly acc[illegible]e
Consequence: nonthermal fixed [illegible]ies

experimentally confirmed in the Hadzibabic Lab

prescaling in the expanding geometry?

2507.07545 with Mazeliauskas and Preis

The emergence of nonthermal fix[illegible] ca[illegible]ght of as originating from the decay of transient qua[illegible] around them

experiment?

relation to the adiabatic evolution?

2502.01622 with De Lescluze

Adding spatial momentum reveals hyd[illegible]mic m[illegible]s and opens a window on studying inhomogeneous non[illegible]

experiment?

nonthermal fixed point with initial state fluctuations?

xxxx with Berges, Denicol and Preis