



Signals of heavily irradiated particle detectors

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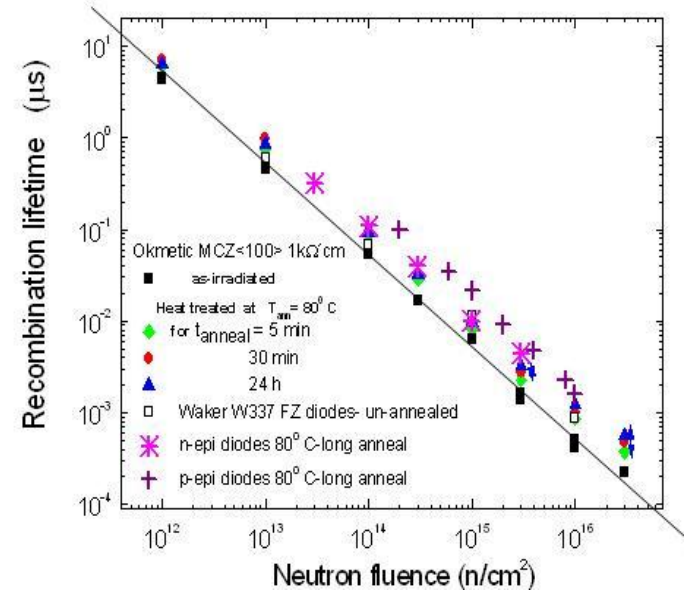
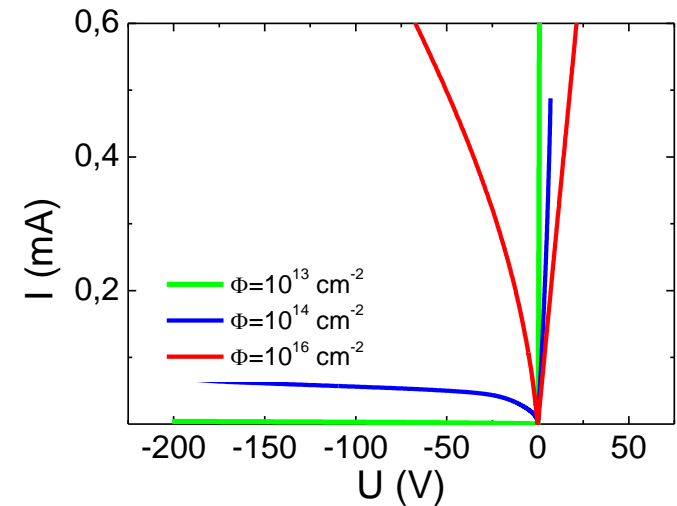
Outline

- Motivation (Why do we analyze the problem?)
- Short comment of views on Ramo's theorem
- Recombination and generation current in heavily irradiated detectors
- Current transients and charge collection efficiency
- Summary

The main aspects:

The radiation induced defects change the $I(V)$ dependence of detector:

- The reverse current increases a few orders of magnitude
- $I(V)$ becomes similar to the asymmetric resistor



The Charge Collection Efficiency (CCE) decreases with the fluence, but:

- It restores by an increase of bias
- It was observed CCE more than 100 % in highly irradiated samples

Then it follows (if the structure becomes as photoresistor, and if the electrostatic approximation is valid):

- The generation of deep levels in the midgap enhances the generation current in the structure, and it change an electric field distribution in it.
- The radiation creates a main free carrier capture centre, and its efficiency linearly increase with the fluence, **i.e., the capture probability depends on the distance between centers.**
- Therefore the CCE depends on the ratio of free carrier diffusion time to the capture center and its drift time through the base of detector.

$$\frac{1}{\tau} = \gamma P_R = aF$$

$$\tau_{\text{diffusion}} / t_{\text{transit}}$$

$$\tau_{\text{diffusion}} = d^2/4D,$$

d – distance between the centers,
D - diffusivity

The questions:

- Is any change of mechanism of the response on the excitation in a case of increase of generation current in the structure?
 - Does the structure become as a photoresistor?
 - What is the detector response time, i.e., RC?
 - Do the moving carriers induce the response in the circuit, or there is only the photoconductivity signal
- What is an origin of $CCE > 100\%$ in high fluence samples?
 - If it is caused by impact ionization, why it is absent in the lower irradiated samples?

The answers could show the ways to improve the properties of detector:

- If the detector operates as the photoresistor,
 - then the increase of CCE depends on the ratio of $\tau_{\text{lifetime}}/t_{\text{transit}}$,
 - then better results will be in the higher bias devicesand it could be recommended
 - to increase of τ_{lifetime} but not more than the collection time.
- If the detector is the relaxation type (the electrostatic approximation is not valid), and in any case, the understanding of conductivity mechanism (electric field distribution) and signal formation process is necessary to understand.



Motivation of the MORE GENERAL analysis

- To predict signal changes and to foresee possible modifications of the detector design and voltage regimes
- To evaluate the field redistribution effects
- To evaluate the optimal thickness and doping parameters
- To evaluate processes of the signal amplification

I.e., as a great role is given to the Ramo's theorem in analysis of the signal formation, it is actual to look on it in more details, because Ramo's model is an electrostatic approach **that is correct if the processes are slow.**

Si

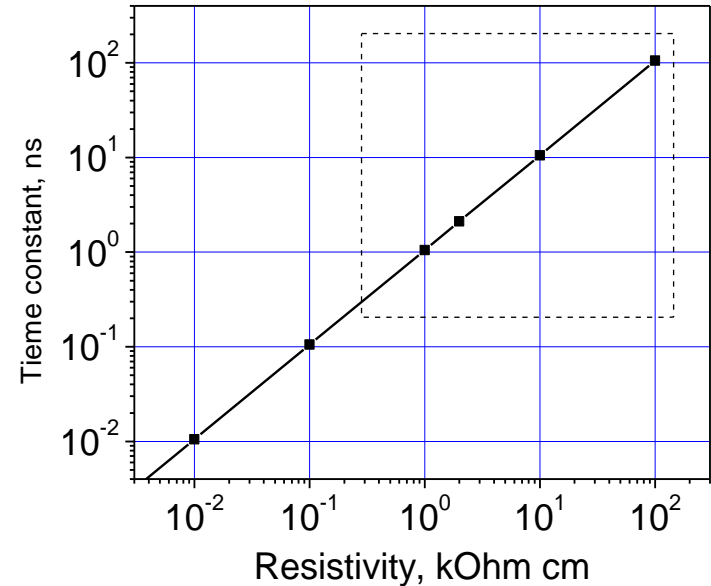
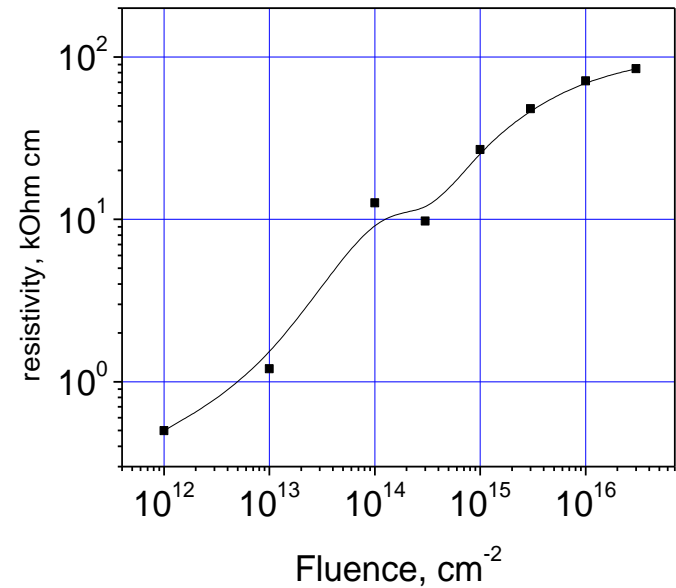
- Dielectric permittivity $\epsilon / \epsilon_0 = 11.9$
– $\epsilon_0 = 8.85 \cdot 10^{-14} \text{ F/cm}$
- Conductivity – $10^{-2} - 10^{-4} \Omega^{-1} \text{ cm}^{-1}$
- $\tau = RC = \epsilon \epsilon_0 / \sigma$

As the conductivity of detector material is $10^{-2} - 10^{-4} \Omega^{-1} \text{ cm}^{-1}$,

therefore it appears a question:

what influence on the Ramo approximation makes this non-static behaviour.

***i.e.*, electrostatic approach fits worse and worse if the compensation of semiconductor increases**



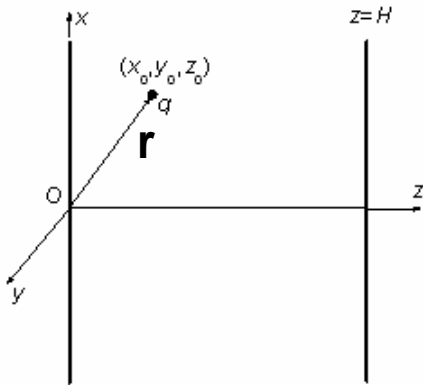


A validity of the Ramo's approximation

- It was analyzed in many works, because a straight transfer the vacuum conditions to the semiconductor is not possible.
- We concentrate on the results of:
 - I.V.Kotov, NIMA 539, 267-268 (2005)
 - L.A.Hamel, M.Julien. NIMA 597, 207-211 (2008)
- and we performed some additional analysis starting from more general principles.
- Our attempt to analyze the electrodynamic approach **will be in a future** because calculations prolonged more time than was proposed.



Ramo's theorem and it's validity



Ramo's theorem based on energy balance using Green's and Gauss' theorems with specific boundary conditions

The same technique was used by I.V.Kotov
L.A.Hamel, M.Julien approach - local field conception, (but the boundary conditions different from p-i-n diode)

- In this work:
- a) the solution of the Poisson equation (JB)
 - b) the boundary conditions different from p-i-n diode included (EG)
 - c) space charge effects neglected (JK)

Ramo's boundary conditions

ϕ is the potential of the electrostatic field,

in the region between conductors

$$\nabla^2 \phi = 0 \quad (R1)$$

ϕ_e is the potential of the tiny sphere and on the conductors

$$\phi_j = 0 \quad (R2)$$

Electron removed, conductor $j=A$ raised to unit potential

$$\phi_{j=A}' = 1 \text{ V}, \quad (R3)$$

and the other conductors grounded. Call the potential of the field in this case ϕ' , so that

$$\nabla^2 \phi' = 0 \quad (R4)$$

For capacitor:

$$\partial C_{eA} / \partial t = 0,$$

for diode:

$$\partial C_{eA} / \partial t \neq 0$$

$Q_A \phi_A' = -e \phi_e$ - Ramo classical equation. actually, balance of electrostatic energy with $\phi_A' = 1V$

The linear inter-electrode media without any space charge excepted elementary one $q=e$



Ramo's theorem and it's validity

Interpretation of Hamel-Julien based on analysis of electrostatic energy balance

The charge ΔQ induced in an electrode just behind the movement (single-dimensional, two-electrode approach) region $z=H$ (i.e. brought from the external circuit into the electrode) by the movement of a single carrier of charge q over a distance Δz is given by

$$\Delta Q = \pm q \Delta z/H = \pm q \Delta U/U \quad (1)$$

This is equivalent to calculating the charge ΔQ_j induced in electrode j when the test charge is moved from the ground, where $\phi_{el} = 0$, to the final position \mathbf{r} . In this case, Eq.(1) is replaced by

$$\sum_{j=1}^N \Delta Q_j U_j = -q \phi_{el}(\mathbf{r}) = q \sum_{j=1}^N \phi_j(\mathbf{r}) \text{ with } \phi_{el}(\mathbf{r}) = \sum_{j=1}^N \phi_j(\mathbf{r}) \quad (2)$$

If all U_j are 0 except one $\phi_{j=A}(\mathbf{r})$ which is at 1V, then the potential is simply $\phi_{el}(\mathbf{r}) = \phi_{j=A}(\mathbf{r})$, which is then equivalent to electrode j 's weighting potential $\phi_{jw}(\mathbf{r})$. **This is Ramo's classical result**, albeit $\phi_{jw}(\mathbf{r})$ now includes the induced surface ($Q \rightarrow U_j$) and space charge ($q \rightarrow \phi_{el}(\mathbf{r})$) potentials explicitly:

$$\Delta Q_j = -q \phi_{el}(\mathbf{r})/U_j = q \phi_{jw}(\mathbf{r}), \text{ i.e. } \phi_{jw}(\mathbf{r}) = \phi_{el}(\mathbf{r})/U_j \quad (3)$$

The second way (**Hamel-Julien approach**) to use Eq.(2) is to take the derivative with respect to U_j while keeping the other potentials constant at their operating biases:

$$\Delta Q_j|_{U_{bias}} = -q [\partial \phi_{el}(\mathbf{r}) / \partial U_j]|_{U_{bias}} = q \phi_{jw}^{HJ}(\mathbf{r}), \text{ with } \phi_{jw}^{HJ}(\mathbf{r}) = [\partial \phi_{el}(\mathbf{r}) / \partial U_j]|_{U_{bias}} \quad (4)$$

In a linear media, Eqs. (3) and (4) are fully equivalent.

The classical result Eq.(3) does not hold true for non-linear (e.g. diode) media ($L=H$):

$$\vec{E}_{wj}(\vec{x}) = \left. \frac{\partial \vec{E}_{el}(\vec{x})}{\partial V_j} \right|_{V_{op}} = -\nabla \phi_{wj}(\vec{x}) \quad \left. \begin{array}{l} \text{underdepleted : } E_w(x) = \frac{\partial E(x)}{\partial V} = \sqrt{\frac{Ne}{2V\epsilon}} = \frac{1}{W_d} \\ \text{depleted : } E_w(x) = \frac{\partial E(x)}{\partial V} = \frac{1}{L} \end{array} \right\}$$



Ramo's theorem and it's validity (JB)

Electro-static approach - direct Poisson equation solution

$$\nabla^2 \phi \neq 0$$

Invalid Ramo's boundary conditions R1, R3

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \Phi = -\frac{q}{\epsilon_0 \epsilon} \delta(x-x_0) \delta(y-y_0) \delta(z-z_0), \quad 0 \leq z \leq H, \quad 0 < z_0 < H$$

$$\Phi|_{z=0} = U, \quad \Phi|_{z=H} = 0$$

If the solutions will be used in the HJ approximation, the dependence on "z" is obtained.

$$\Phi(x, y, z) = \begin{cases} U \frac{H-z}{H} + \frac{q}{4\pi\epsilon_0\epsilon} \sum_{n=0}^{\infty} \left\{ \frac{1}{\sqrt{R^2 + (2nH + z_0 - z)^2}} - \frac{1}{\sqrt{R^2 + [2(n+1)H - z_0 - z]^2}} - \frac{1}{\sqrt{R^2 + (2nH + z_0 + z)^2}} + \frac{1}{\sqrt{R^2 + [2(n+1)H - z_0 + z]^2}} \right\}, & z \leq z_0 \\ U \frac{H-z}{H} + \frac{q}{4\pi\epsilon_0\epsilon} \sum_{n=0}^{\infty} \left\{ \frac{1}{\sqrt{R^2 + (2nH - z_0 + z)^2}} - \frac{1}{\sqrt{R^2 + [2(n+1)H - z_0 - z]^2}} - \frac{1}{\sqrt{R^2 + (2nH + z_0 + z)^2}} + \frac{1}{\sqrt{R^2 + [2(n+1)H + z_0 - z]^2}} \right\}, & z \geq z_0 \end{cases}$$

$$E_{wi-R}(z_0) = \frac{E(W)}{U=1V} = -\frac{1}{U=1V} \nabla \Phi(z_0)$$

$$Q_A = -e \phi_e / \phi_A' = C_{eA} \phi_e$$

Ramo's case (moving charge potential causes capacitance change)

$$E_{wi-HJ}(z_0) = \frac{\partial E(W)}{\partial U} = -\frac{\partial^2}{\partial z \partial U} \Phi(z)$$

Hamel-Julien

Hamel-Julien approach, when ϕ_e is not proportional to ϕ_A' like in non-linear media



Ramo's theorem and it's validity

According **P.Blood, J.W. Orton. Extended depletion approximation**
(non analytical case, numeric evaluation).

Invalid R1, R3, R4

Diode: W_d in approach more rigorous than depletion approximation (W_a)

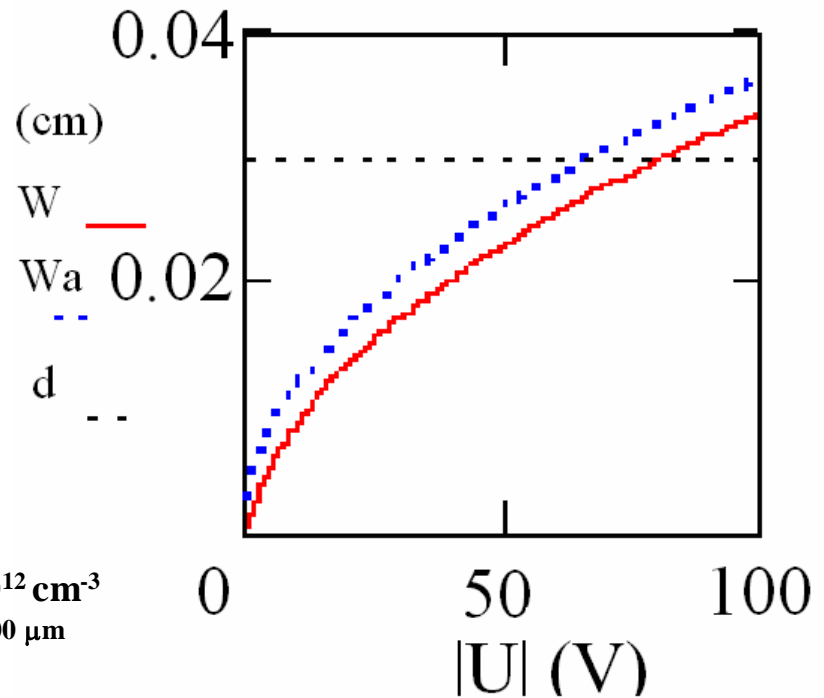
$$W = \int_0^{\varphi(x)} d\varphi / E(\varphi(x))$$

$$\frac{d\varphi(x)}{dx} = \sqrt{\frac{2eN_d}{\epsilon\epsilon_0} \left\{ \varphi(x) - \frac{kT}{e} \left(1 - \exp\left(\frac{e\varphi(x)}{kT}\right) \right) \right\}}$$

$$W_a = (2\epsilon_0\epsilon U / qN_D)^{1/2}$$

simple depletion approximation

$N_D \sim 10^{12} \text{ cm}^{-3}$
 $H=d= 300 \text{ }\mu\text{m}$





Ramo's theorem and it's validity

$$\nabla^2 \phi \neq 0$$

$$q_e = 8 \cdot 10^{-13} \text{ C/cm}^2$$

Approach (EG) of diode (at depletion approximation) with induced surface charge q_e at z_0

$$E_{wi-HJ+gv}(z_0) = \frac{1}{W_{qd}} \quad \text{HJ and EG}$$

$$W_{qd}(t) = \sqrt{\frac{2\epsilon\epsilon_0 U}{eN_{Deff}(t)} + \frac{q_e(t)}{eN_{Deff}(t)} z(t)}$$

Additional term related to the moving charge appears

$$E_{wi-R}(z_0) = \frac{E(W_{qd})}{U} = -\frac{1}{U} \nabla \phi(z_0) \neq \frac{1}{W_{qd}}$$

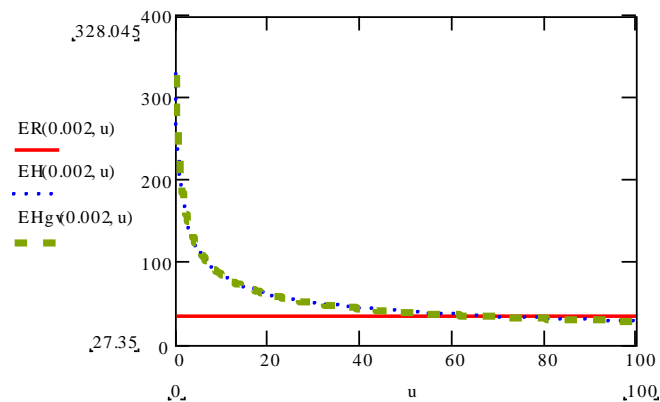
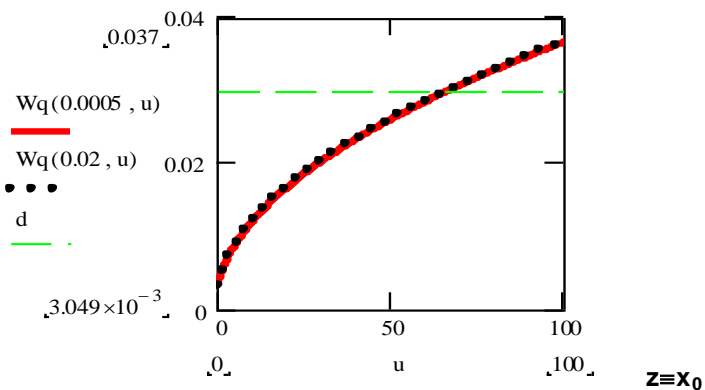
Ramo

$$E_{wi-R}(z_0) = \frac{1}{d}$$

$$E_{wi-HJ}(z_0) = \frac{\partial E(W_{qd})}{\partial U} = -\frac{\partial^2}{\partial z \partial U} \phi(z_0) = \frac{1}{W_{qd}}$$

Hamel-Julien

$$E_{wi-HJ}(z_0) = \frac{1}{W} = \sqrt{\frac{eN_D}{2\epsilon\epsilon_0(U_{bi} + U)}}$$





Ramo's theorem and it's validity

$$W_{qd}(t) = \sqrt{\frac{2\epsilon\epsilon_0 U}{eN_{Deff}(t)} + \frac{q_e(t)}{eN_{Deff}(t)}} z(t)$$

$$E_{wi-HJ}(z_0) = \frac{\partial E(W_{qd})}{\partial U} = -\frac{\partial^2}{\partial x \partial U} \phi(z_0) = \frac{1}{W_{qd}}$$

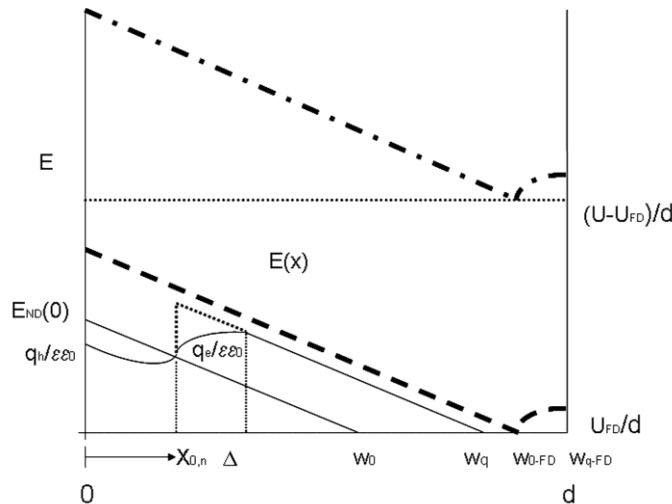
Hamel-Julien

Approach of diode (at depletion approximation) with induced inhomogeneously distributed charge domain $q_e = q_{e0}(z_0)\exp[-\alpha(z-z_0)]$

$\nabla^2 \phi' \neq 0$ invalid R4,

$\nabla^2 \phi' = -q_e(z_0)\exp[-\alpha(z-z_0)]|_0^\Delta$, Δ -charge domain width

$$\langle E_{qe} \rangle = en_{ex0} \alpha^{-1} [1 - \exp(-\alpha\Delta)] / 2\epsilon_0\epsilon; \quad q_{e0} = en_{ex0} \alpha^{-1}$$



EG approximation

Complementarily, broadening of the induced excess carrier domain

$$\Delta(t) = \Delta_0 + \delta(\mu E)t$$

leads to an increase of the transient current within ulterior phase of the induced current pulse

Diode (at depletion approximation) with induced inhomogeneously distributed charge

Ramo's invalid $\nabla^2 \phi \neq 0$ - invalid R1

Actually, Hamel-Julien approach of local Ramo's field helps only in evaluation of instant weighing field. Due to $W_{qd}(t)$, this field ($=1/W_{qd}(t)$) varies with time during charge drift (t_{dr}):

$$q(t) = eN_{Deff}(t) W_{qd}(q,t) - eN_{Deff}(t) z(t) / [1 - e^{-(t_{dr} + t_0)/\tau_M}]$$

t_0 - duration of excess carrier ($n_{ex} - p_{ex}$) pairs separation



Ramo's theorem and its validity including traps

Using simple Ramo's relation
but including trapping as $q_e(t)$

$$Q_A = -q_e(t) \phi_e / \phi_A' \quad \text{with } q_e(t) = q_{e0} \exp(-t/\tau_{\text{capt}})$$



Generally, Ramo's approximation is doubtful

$$\begin{aligned} I_R &= \partial Q_A / \partial t = q_e(t) \{ (1/\phi_A') [\partial \phi_q / \partial z] \} (\partial z / \partial t) - (q_e(t) / \tau_{\text{capt}}) (\phi_q / \phi_A') = \\ &= q_e(t) E_{\text{wj}} v - (q_e(t) / \tau_{\text{capt}}) (\phi_q / \phi_A') = q_e(t) [E_{\text{wj}} v - (1/\tau_{\text{capt}}) (\phi_q / \phi_A')] \end{aligned}$$

1) For $\tau_{\text{capt}} (\phi_A' / \phi_e) \gg \tau_{\text{drift}} \cong 1/E_{\text{wj}} v$, and scale of $t \cong \tau_{\text{drift}}$, $q_e(t) = q_{e0} = \text{const}$,
and at $\phi_q / \phi_A' \cong 1$, then, current leads to

Ramo's approximation is valid

$I_R(t) = q_{e0} E_{\text{wj}} v$, during $t < \tau_{\text{drift}}$, and

$I_R(t) = 0$, for $t \geq \tau_{\text{drift}}$, **charge drift is recorded**

$$\text{CCE} = (1/q_{\text{MIP}}) \int I_R(t) dt$$

2) For $\tau_{\text{capt}} (\phi_A' / \phi_e) \ll \tau_{\text{drift}}$ $q_e(t) = q_{e0} \exp(-t/\tau_{\text{capt}})$
current follows recombination decay

Ramo's approximation is invalid

(ϕ_A' / ϕ_e) should be known

$I_R(t) = (q_{e0} / \tau_{\text{capt}}) \exp(-t/\tau_{\text{capt}})$ for all the time (t) scale;
then, **charge drift cannot be resolved**

In the linear case $\tau_{\text{drift}} = d^2 / \mu U$, - then and only then Ramo's relations are valid.
Therefore, to keep condition (1), it is necessary to increase U and to decrease d.

Induced current within diode

EG approximation:

Approach of diode (at depletion approximation) with induced surface charge domain q and including generation $\{N_{Deff}(t)=N_D \pm (N_T - n_T(t))\}$, induced charge capture $q_e(t)=q_{e0} \exp(-t/\tau_C)$ currents and domain drift $v_q=dz_0/dt$

$$W_q(t) = \sqrt{\frac{2\epsilon\epsilon_0 U}{eN_{Deff}(t)} \pm \frac{q_e(t)}{eN_{Deff}(t)} z_0(t)}$$

$$C_q(t) = \frac{\epsilon\epsilon_0 S}{W_{qd}(t)}$$

$$i_C(t) = \frac{C_q(t)U}{2} \frac{W_q^2|_{U=0}(t)}{W_q^2(t)} \left[\frac{eN_{deff}(t)}{q(t)z_0(t)} \frac{1}{\tau_{gd}} \frac{n_T(t)}{N_{deff}(t)} + \frac{1}{\tau_{Cq}} \pm \frac{v_q(t)}{z_0(t)} \right]$$

Displacement current:
pseudo-Ramo's component

Conductivity (within transitional layer at W_q boundary) current due to q capture within SCR

Conductivity (within transitional layer at W_q boundary) current due to thermal emission from fast (shallow) filled traps

$$i_\Sigma(t) = i_C(t) + \frac{en_i S W_q(t)}{\tau_{g\Sigma,slow}} + i_{Rec,n^+p^+}(t)$$

If the electrostatic approximation is valid:

- Analysis shows that if the generated charge is very small then the signal is related with to the generation and capture components.
- If the induced charge is bigger then the signal start to depend on the redistribution of charge inside the detector (and the photoelectrical gain **has to be observed**).



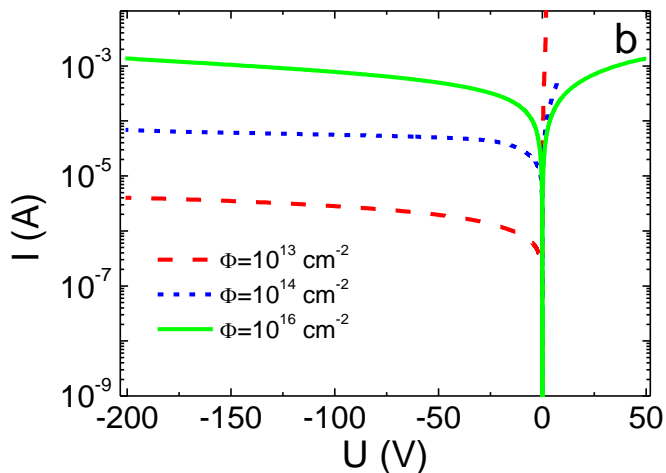
Additional experimental findings:

- Generation current evaluated from $I(V)$ and it destroys the $C(V)$ (and DLTS) analysis.
- Capacitance of diode reaches the geometrical capacitance value at fluence $\sim 10^{15} \text{ cm}^{-2}$ and more
- The lifetime decrease corresponds to the generation current increase.



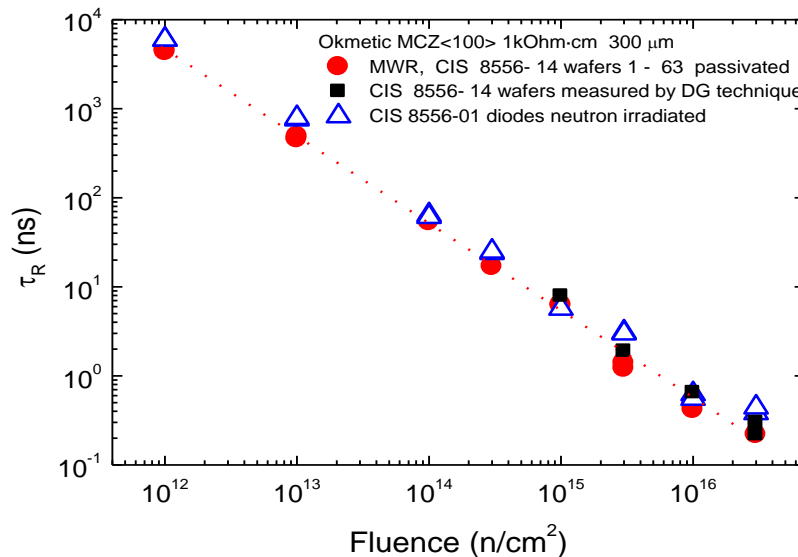
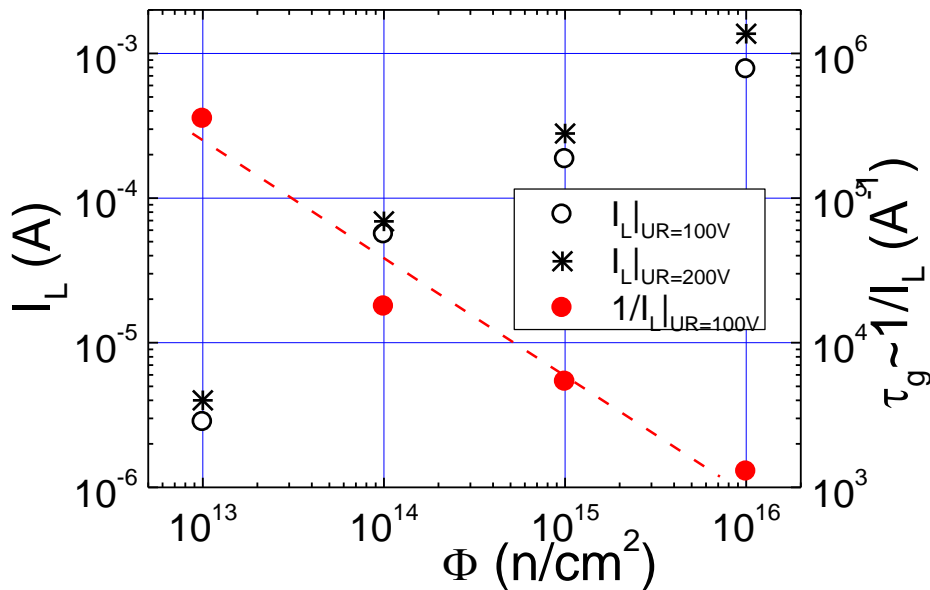
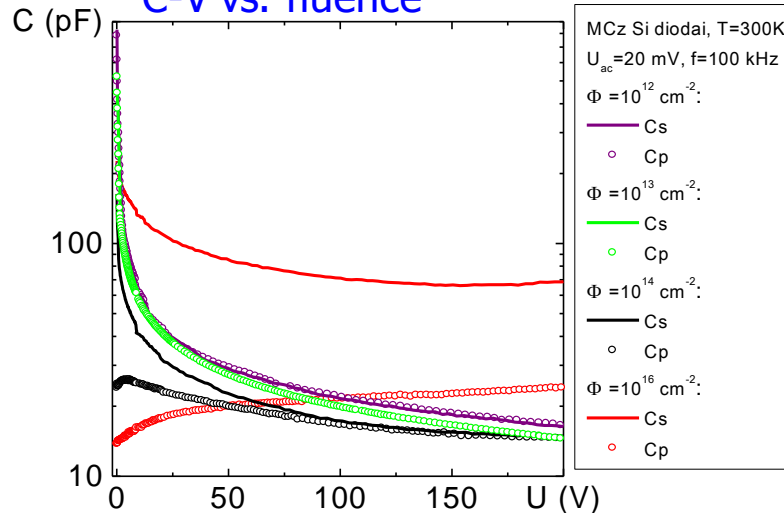
Carrier generation/emission lifetimes in Si detectors (for $M_{rad} \gg n_0$)

I-V vs. fluence



after E.Gaubas et al JAP, **110** (2011) 033719

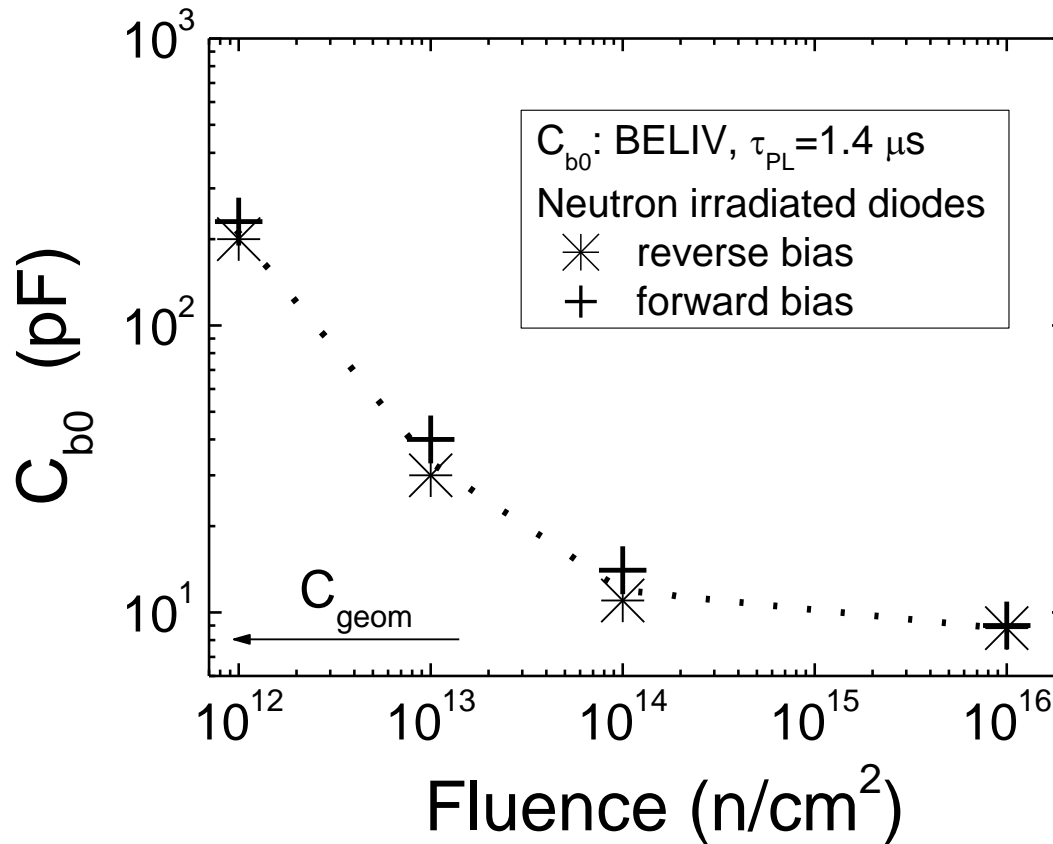
C-V vs. fluence



Nearly linear reduction of generation lifetime with enhancement of fluence is similar to that of recombination lifetime characteristic

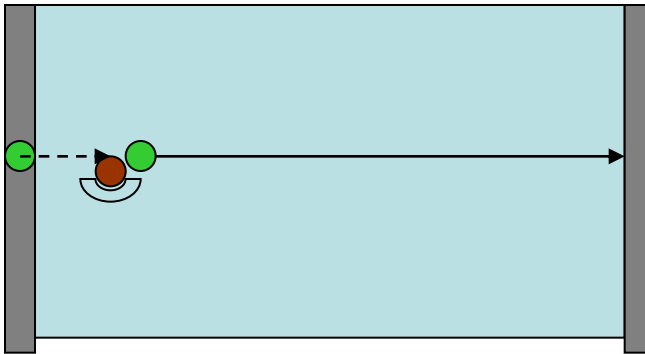
Qualitative emission lifetime dependence on fluence

Barrier capacitance of Si pad-detectors as a function of reactor neutrons fluence



after E.Gaubas et al JAP, **110** (2011) 033719

If the semiconductor works as photoresistor: Photo-conductivity gain (PCG)



The process will repeat until electron will recombine, or the hole will be extracted to the contact.

$$PCG = t_{dr,h} / t_{dr,e} \Rightarrow CCE > 1 \text{ if } t_{dr,e} < t_{dr,h} < \tau_{Rec}$$

$$CCE \Rightarrow 0 \text{ for } \tau_{Rec} < t_{dr}$$



Summary 1

- **Ramo's theorem with some corrections is "partly" working but the electrodynamic approach is necessary to solve for the investigation the processes in the high resistivity Si. Space charge, and trapping, leads to more complicated description of current transients.**
- **The high resistivity material based pin diodes become bad capacitors (with long relaxation – small operational frequency) under high irradiation fluences. (Only carriers emitted from shallow levels are able to be in equilibrium with band, which determine operational characteristics of detector. Deep levels actually do not govern the conductivity type).**



Summary 2

For high density of traps and small carrier density, **S-R-H recombination statistics is invalid**, and relaxation is determined co-existence of many different centres which contribution is controlled by the hole capture rate (A.Rose model).

Heavily irradiated diode behaves like a slow capacitor or un-stable resistor. There photo-electrical gain effect is probable. Also, the impact ionization avalanche caused gain is possible within electrode edge region when a virtual photo-electrode is formed by high density of photo-generated excess carriers.



Thank You for attention!