CORRELATION AND ANTICORRELATION BETWEEN STOCKS

ECONOPHYSICS & FINANCIAL MARKETS

Financial markets can be described using principles similar to critical phenomena in physics.

Critical Phenomena:

- Occur in space and time.

Focus of This Chapter:

- Moving beyond single asset analysis.
- Investigating multiple stock-price time series within a portfolio.

CROSS-CORRELATIONS IN FINANCIAL MARKETS

Importance of Cross-Correlations:

- Known to exist between pairs of stocks (and anticorrelations).
- Crucial in selecting the most efficient portfolio.

Study on Synchronization:

- Examines the synchronization in stock dynamics for a pair of stocks in a financial market.

Covariance Matrix Analysis:

- Extensive study on the covariance matrix of stock returns in a given portfolio.

Additional Studies:

- Detecting economic factors influencing stock prices.
- Evaluating deviations between market data and random matrix theory.

SIMULTANEOUS DYNAMICS OF PAIRS OF STOCKS

Simultaneous Trading of Stocks:

- Many stocks are traded simultaneously, making it essential to study their relationships.

Correlation Coefficient ($\boldsymbol{\rho}$ij):

- Measures similarities and differences in the synchronous time evolution of two stocks.
- Specifically, it analyzes the daily logarithmic price changes of stocks i and j.

Generalizing:

$$\boldsymbol{S(t) \equiv lnY(t+\Delta t) - lnY(t)}$$

Based on it we can define:

$$\boldsymbol{S_i \equiv lnY_i(t) - lnY_i(t-1)}$$

$$\boldsymbol{\rho_{ij}} = \frac{\langle S_i S_j \rangle - \langle S_i \rangle \langle S_j \rangle}{\sqrt{\langle S_i^2 - \langle S_i \rangle^2 \rangle - \langle S_j^2 - \langle S_j \rangle^2 \rangle}}$$

Yi & Si:

- Yi: Daily closure price of stock i at time t.
- Si: Daily change in the logarithm of the price of stock i.

Time Average:

- The angular brackets represent a time average over the trading days within the analysis period.

Correlation Coefficient (**ρ**ij):

- Ranges from -1 to 1:
  - 1: Completely correlated price changes.
  - 0: Uncorrelated price changes.
  - -1: Completely anticorrelated price changes.

DOW-JONES INDUSTRIAL AVERAGE PORTFOLIO

DJIA Stock Set:

30 stocks, resulting in 435 unique pairs of correlation coefficients (ρ_{ij}). (30*29/2=435)

Summary of Correlation Values:

- Typical Maximum (ρ_{ij}): Above 0.5, indicating positively correlated pairs.
- Typical Minimum (ρ_{ij}): Close to 0, showing that anticorrelations are minimal.

Time period	Minimum	Maximum
1990	0.02	0.73
1991	−0.01	0.63
1992	−0.10	0.63
1993	−0.16	0.63
1994	−0.06	0.51

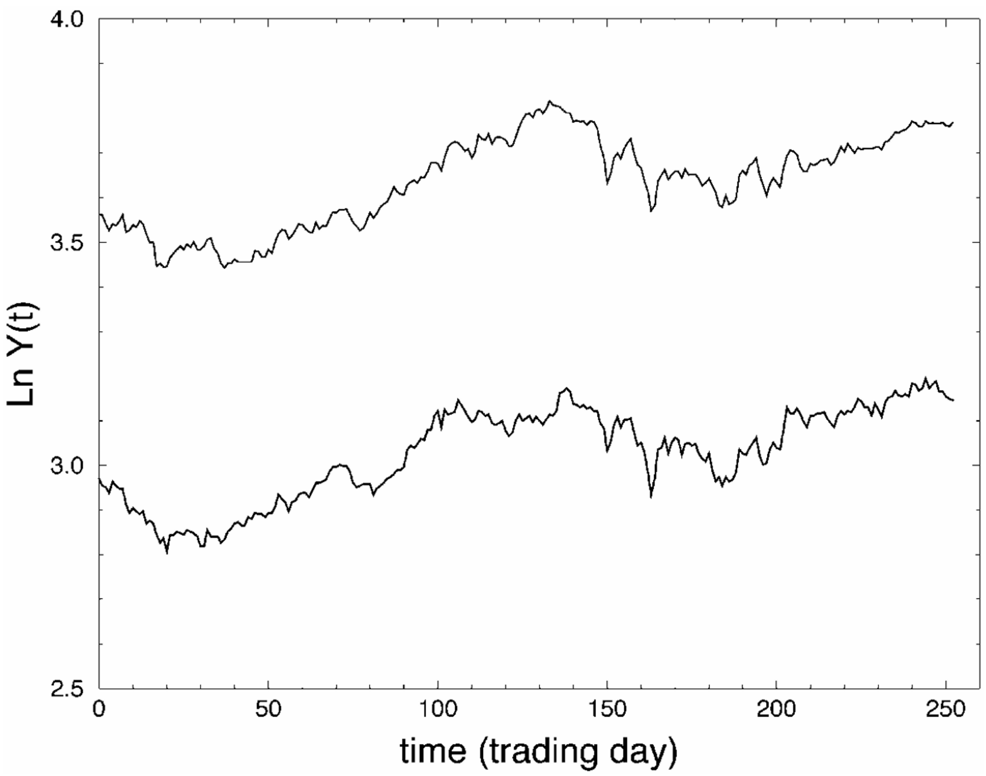

Fig. 12.1. Time evolution of $\ln Y(t)$ for Coca Cola (bottom curve) and Procter & Gamble (top curve) in the year 1990.

HIGH CORRELATION EXAMPLE: COCA COLA & PROCTER & GAMBLE (1990)

Maximum Correlation (**ρ**ij = 0.73):

- Observed in 1990 for the pair of stocks: Coca Cola and Procter & Gamble.

Time Evolution of Stock Prices:

- Figure 12.1 illustrates the synchronized price movements (lnY(t)) of both stocks.

Conclusion:

- The two stocks display remarkable synchronization in their price dynamics.

CORRELATION COEFFICIENT DISTRIBUTION (P(ρIJ))

Table 12.1 Overview:

- Lists only minimum and maximum values of ρ_{ij} for each time interval.

Full Set Analysis:

- $P(\rho_{ij})$: Probability density function of the 435 correlation coefficients.
- Shape: Bell-shaped curve.

Key Observations:

- The average value of ρ_{ij} is slowly time-dependent.
- The standard deviation (σ) remains almost constant.

CHARACTERIZING TIME-DEPENDENT CORRELATION ($\boldsymbol{\delta}_{IJ}$)

- Time Evolution of $\boldsymbol{\rho}$ij:
- $\boldsymbol{\rho}$ij changes with time for all 435 pairs of stocks.
- Example: Coca Cola and Procter & Gamble analyzed over 5 years (Figure 12.2).
- Deviation of $\boldsymbol{\rho}$ij ($\boldsymbol{\delta}$ij):
- Defined as
- $\boldsymbol{\delta_{ij}} \equiv \frac{\boldsymbol{\rho_{ij}} - \langle \boldsymbol{\rho_{ij}} \rangle}{\boldsymbol{\sigma}}$
- Where:
- $\boldsymbol{\sigma}$ - standard deviation
- $\langle \boldsymbol{\rho_{ij}} \rangle$ is the average of $\boldsymbol{\rho}$ij over all pairs of stocks ij in the portfolio analyzed
- Coca Cola & Procter & Gamble:
- For this pair, $\boldsymbol{\delta}$ij > 1 for all five years, the correlation coefficient varies

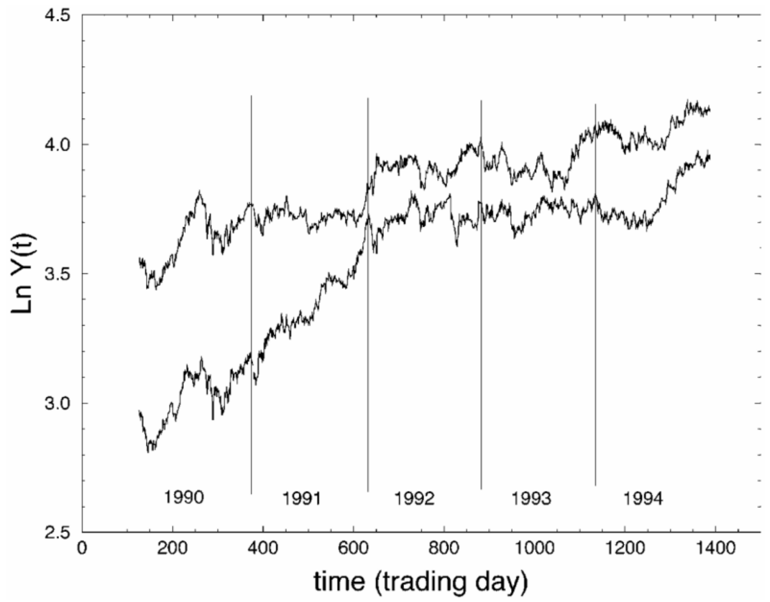

Fig. 12.2. Time evolution of $\ln Y(t)$ for Coca Cola and Procter & Gamble for the five calendar years investigated, 1990 to 1994. The value of ρ_{ij} is 0.73, 0.47, 0.28, 0.33, and 0.39 during the five years from 1990 through 1994, respectively, whereas δ_{ij} is 2.62, 1.73, 1.25, 2.44, and 2.27, respectively, during the same five years.

S&P 500 PORTFOLIO

S&P 500 Stock Set:

- 124,750 unique pairs of correlation coefficients (**ρ**ij) for the 500 stocks.

Correlation Results:

- High Synchronization: Similar to the DJIA, pairs of stocks show strong synchronization.
- Prominent Case (1994): **ρ**ij = 0.82 between Homestake Mining and Placer Dome, Inc..

Anticorrelation:

- Less common than correlation.
- Strongest Anticorrelation (1990): **ρ**ij = -0.30 between Barrick Gold and Nynex Corporation.

Time period	Minimum	Maximum
1990	—0.30	0.81
1991	—0.29	0.74
1992	—0.25	0.73
1993	—0.27	0.81
1994	—0.25	0.82

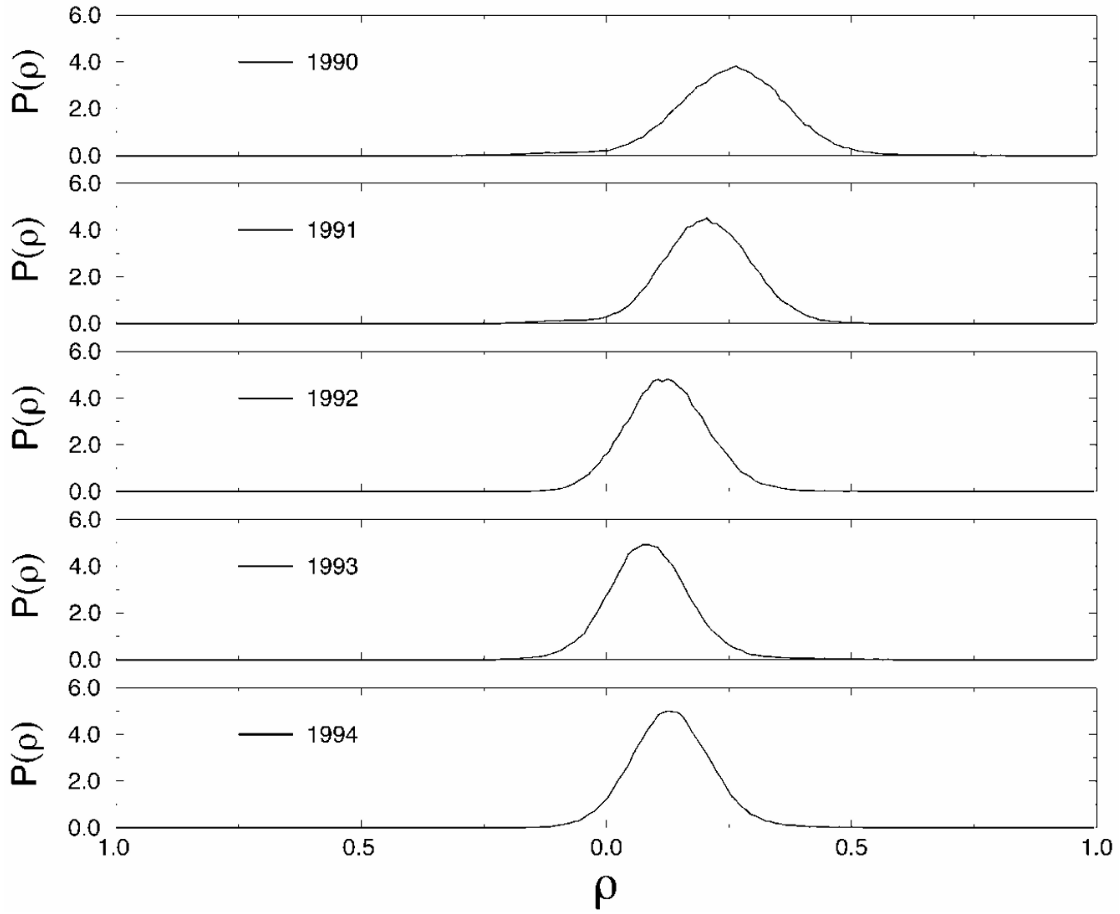

Fig. 12.3. Correlation coefficients for the S&P 500: $P(\rho_{ij})$ is shown for each of the five calendar years 1990 to 1994.

CORRELATION DISTRIBUTION IN THE S&P 500

Larger Sample Size:

- The number of correlation coefficients (**ρ**ij) is much larger than for the DJIA, providing greater statistical reliability.

P(**ρ**ij) PDF for S&P 500:

- Shown in Figure 12.3 for each of the five calendar years.
- Similar to the DJIA case, the center of the distribution moves slowly over time.
- The width of the distribution remains approximately constant.

STATISTICAL PROPERTIES OF CORRELATION MATRICES

Economic vs Physics Research:

- Both fields investigate the correlation matrix of returns, but with different goals.
- Economic Research Goal:
- Determine the number of k factors affecting a financial market.
- Arbitrage Pricing Theory (Ross, 1976) is commonly used.
- Arbitrage Pricing Model:
- Asset returns $\boldsymbol{R_n}$ are modeled as

$$\boldsymbol{R_n = R_{n0} + B\xi_k + \epsilon_n}$$

Where:

- Rn0 : Risk-free and factor-risk premia mean returns.
- B: Matrix of factor weights.
- $\boldsymbol{\xi_k}$: Time series of the k factors.
- n: Asset-specific risk

Assumptions:

Zero means for $\boldsymbol{\xi_k}$ and n.

No covariance between factors (cov($\boldsymbol{\xi k}$,$\boldsymbol{\xi}$)=0).

EIGENVALUES OF THE COVARIANCE MATRIX

Random Matrix Theory:

- Statistical properties of eigenvalues are well-documented.
- Empirical studies reveal deviations from random matrix predictions.

Arbitrage Pricing Theory:

- Dominant eigenvalues indicate a small number of economic k factors driving asset returns.
- Empirical evidence supports the presence of few k factors, including one prominent factor.

Physicists' Findings:

- Detect a prominent eigenvalue significantly larger than expected.
- Additional eigenvalues slightly exceed random matrix theory predictions.

Theoretical Insights:

- Anderson Localization Theory: Highlights importance of lowest eigenvalues.
- Eigenvectors linked to lowest eigenvalues are influenced by fewer independent elements.

DISCUSSION

Findings on Correlation and Market Dynamics

- Synchronization in Markets:
  - Significant cross-correlation exists between asset pairs.
  - Needs to be considered in financial market modeling.
- Economic Factors:
  - A small number of economic factors drive many assets.
  - Consistent with the Efficient Market Hypothesis (no direct predictability of future prices).
- Potential Market Inefficiencies:
  - Cross-correlations with time lags or precise factor dynamics could enable arbitrage opportunities.
  - Observed Deviation: Returns of large stocks tend to lead those of smaller stocks.

THANK YOU FOR YOUR ATTENTION!