How do we collide the beams in the future?

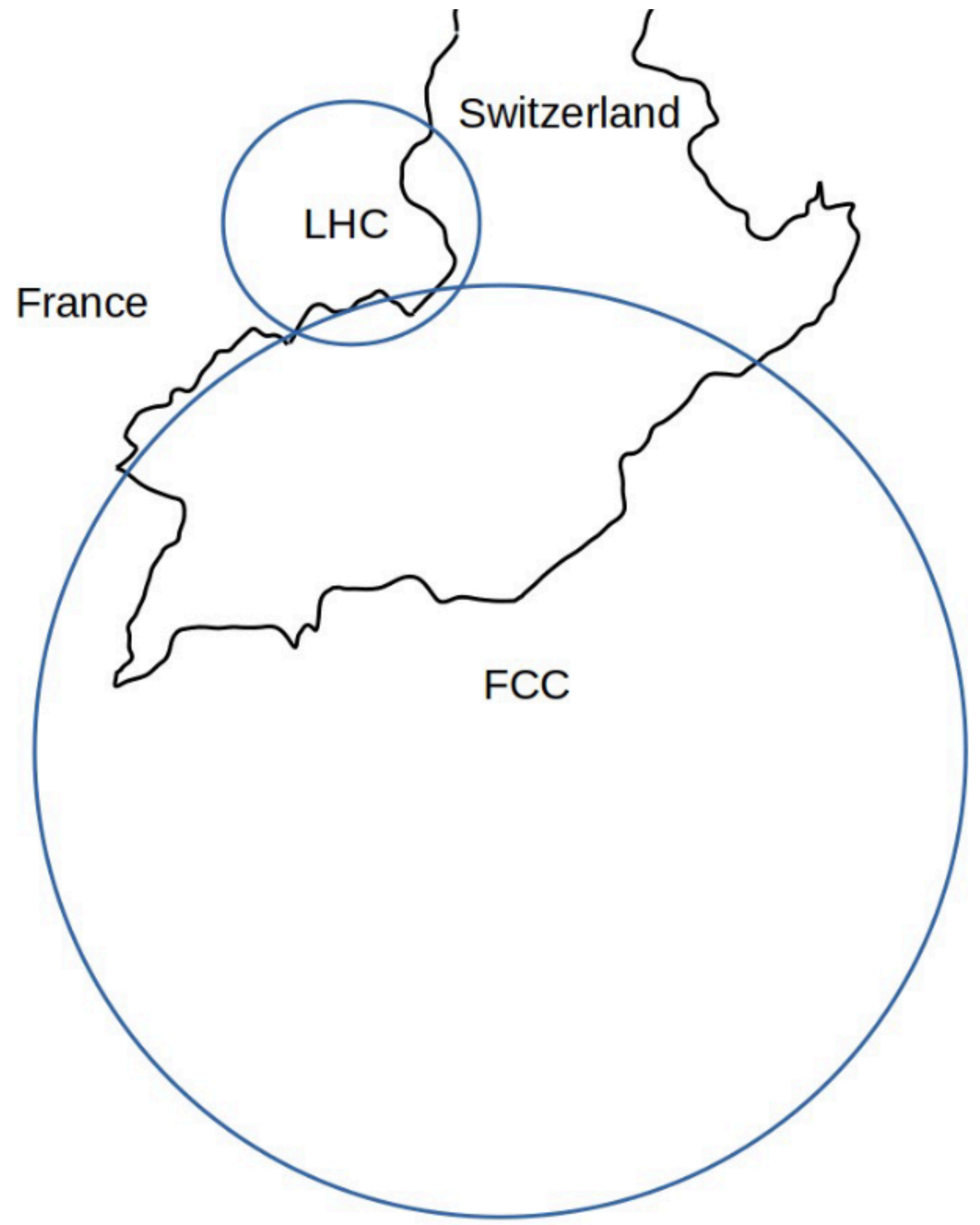
Kyriacos Skoufaris

FCC Early Career Forum - 13 Jan. 2025

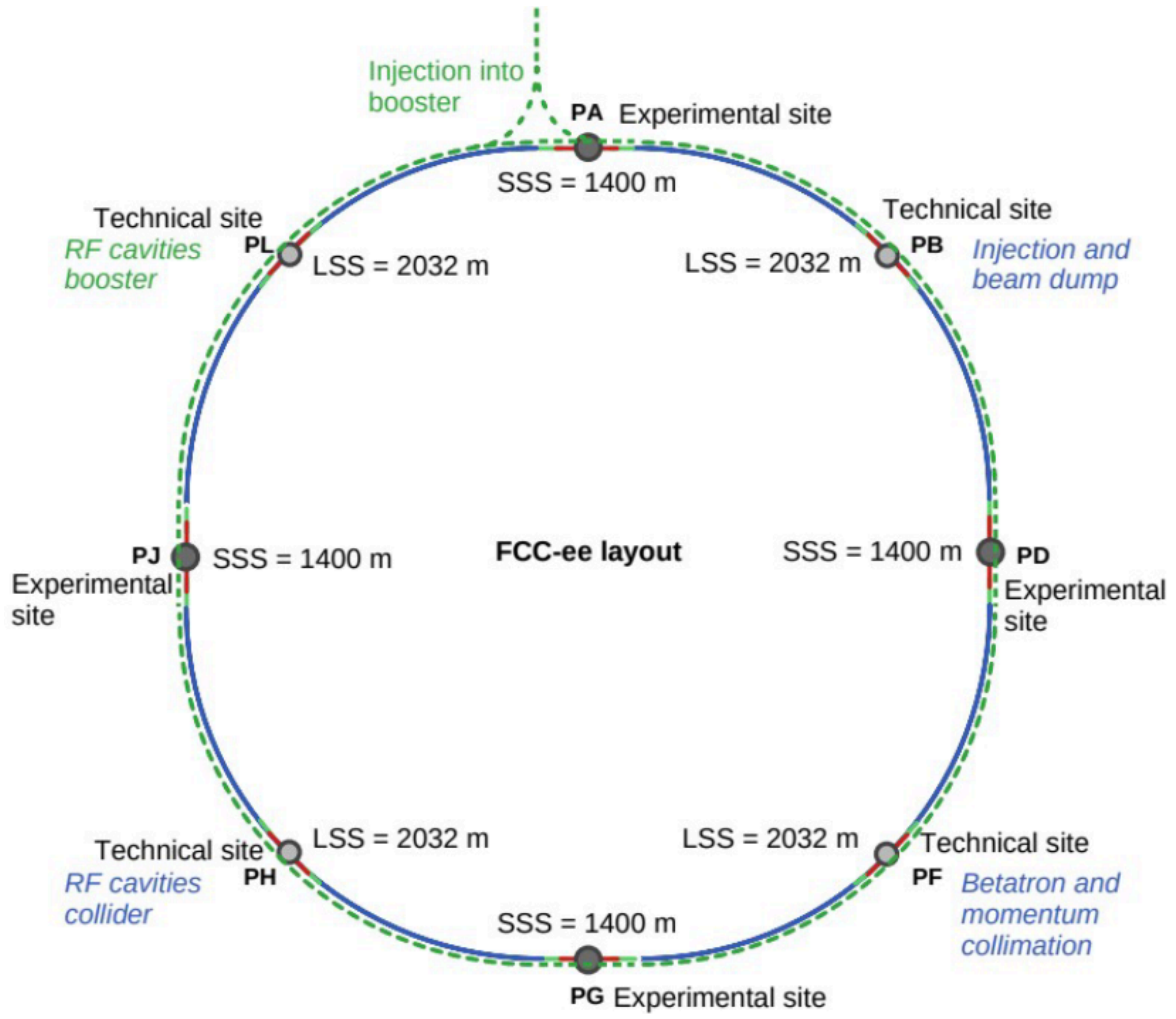
Special thanks to: CAS and FCC team for the material used

FCC-ee collider

Re-using CERN infrastructure



FCC-ee ~2045-2060
90.7 km with 8 surface points



FCC-hh ~2070-2090
Compatible lattice designs

Midterm Report
(February 2024)

Executive Summary of the
Future Circular Collider
Midterm Report

February 2024

Edited by:
B. Auchmann, W. Bartmann, M. Benedikt, J.P. Burnet, P. Charitos, P. Craievich, M. Giovannozzi, C. Grojean, J. Gutleber, K. Hanke, P. Janot, M. Mangano, J. Osborne, J. Poole, T. Raubenheimer, A. Unnervik, T. Watson, F. Zimmermann

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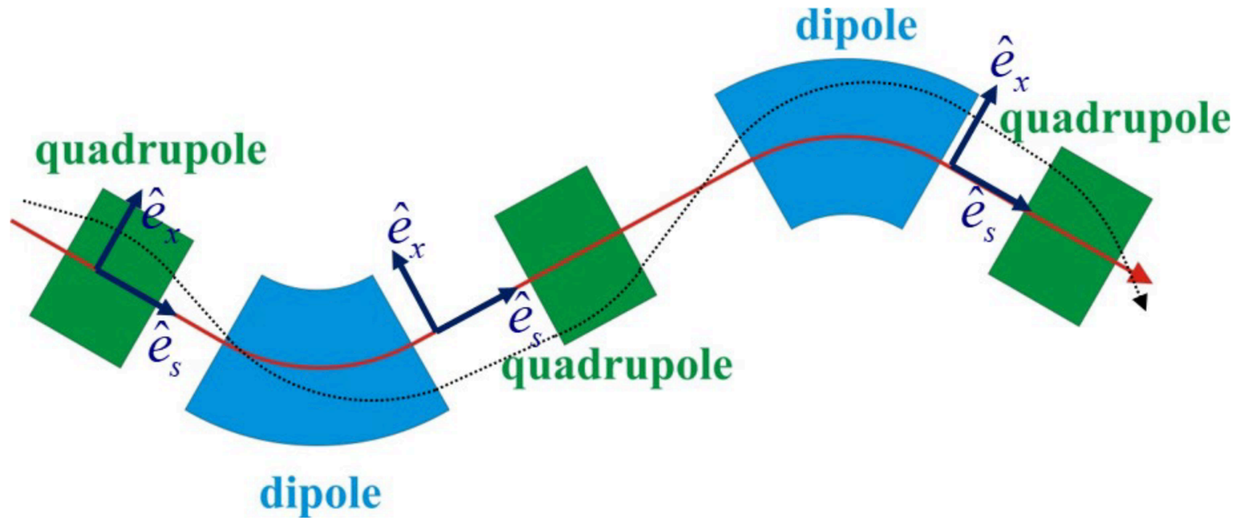
Beam control

Use of Electromagnetic fields $\overrightarrow{F} = q(\overrightarrow{E} + \vec{v} \times \overrightarrow{B})$

Steering



The magnetic field is more efficient

$$F_c = \gamma_r m_0 \frac{v^2}{\rho} = q(E_\perp + vB_\perp)$$

$$= \frac{pv}{q} = \rho(E_\perp + vB_\perp)$$

For ultra relativistic particles ($v \approx c$) and $\rho E_\perp \equiv c\rho B_\perp$, the required:

electric rigidity ($E_\perp \rho$) is: 300MV -> “**impossible”**

magnetic rigidity ($B_\perp \rho$) is: 1Tm -> “**easy”**

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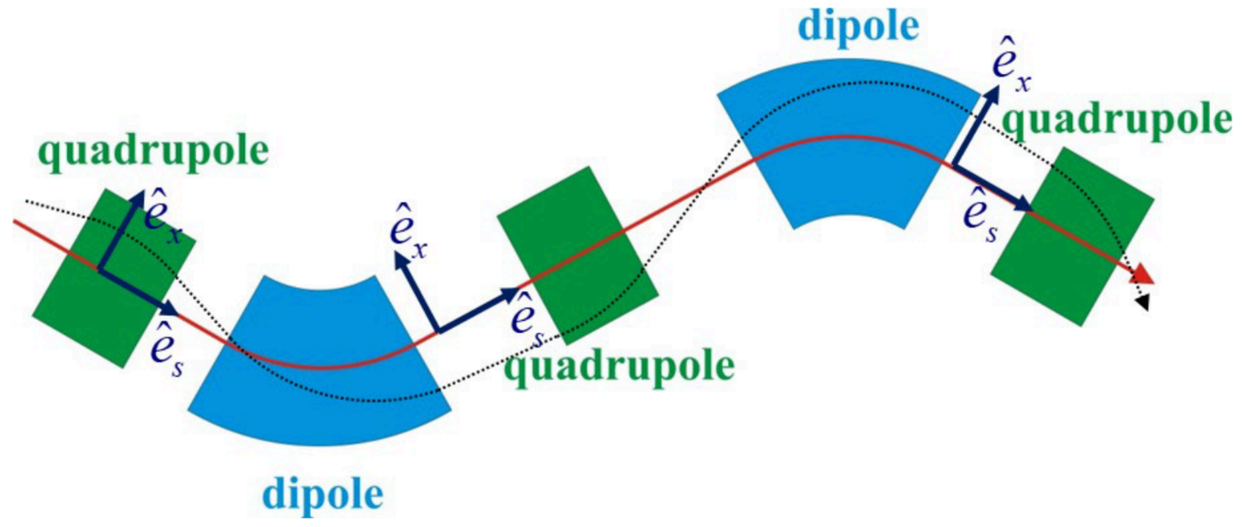

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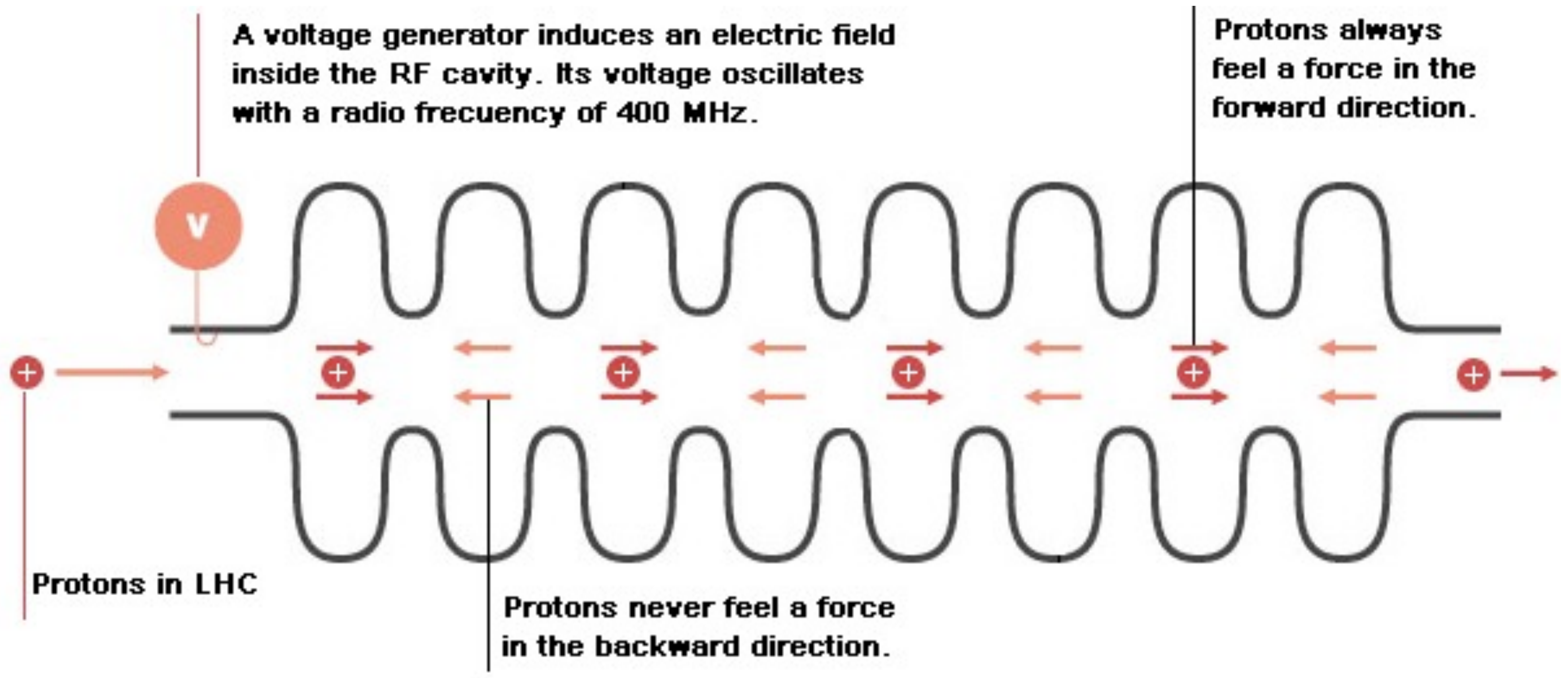
Acceleration



Time varying electric fields - RF cavities/resonators (metal container for electromagnetic field)

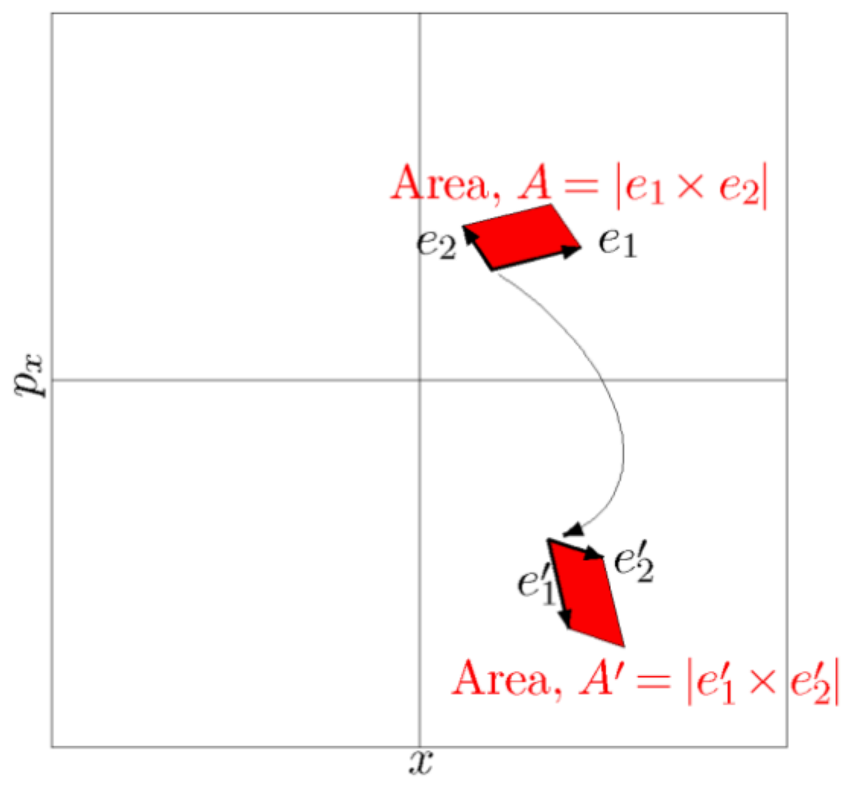
The magnetic field does not accelerate at all ($F = q\vec{v} \times \vec{B}$)

DC acceleration if not impossible ($\oint \vec{E} \cdot \vec{dl} = 0$) then is not effective (breakdown voltages)

Particle trajectory & Coordinate system

The calculation of single particle trajectories (in complex fields) is the backbone of beam dynamics

$$\vec{u} = \mathscr{M}\ \vec{u}_0 \quad \vec{u} = (x, p_x, y, p_y, l, \delta)^T$$



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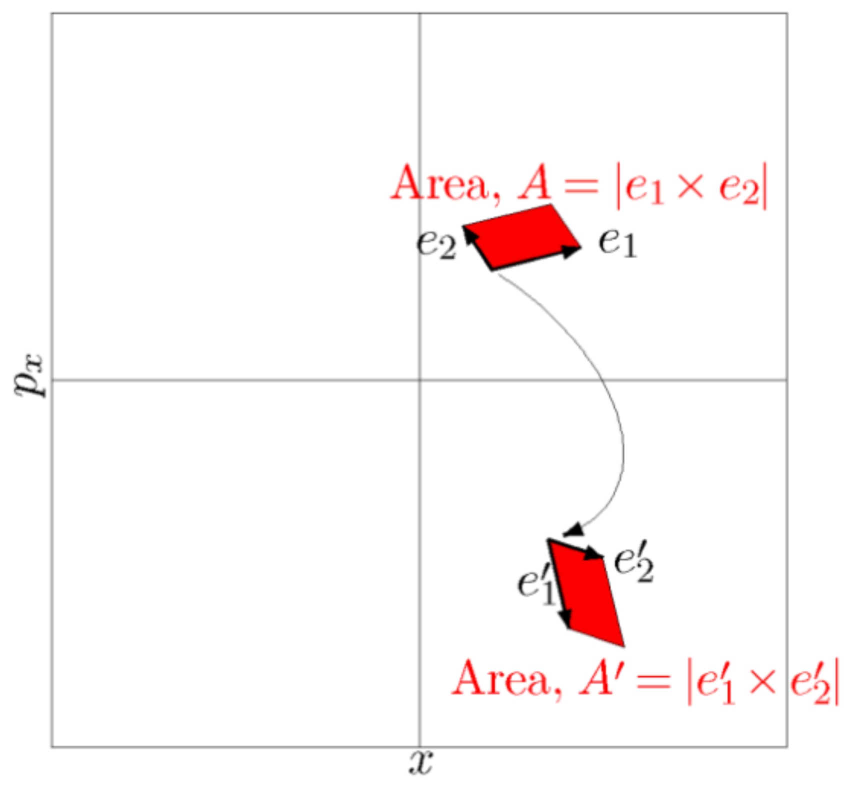

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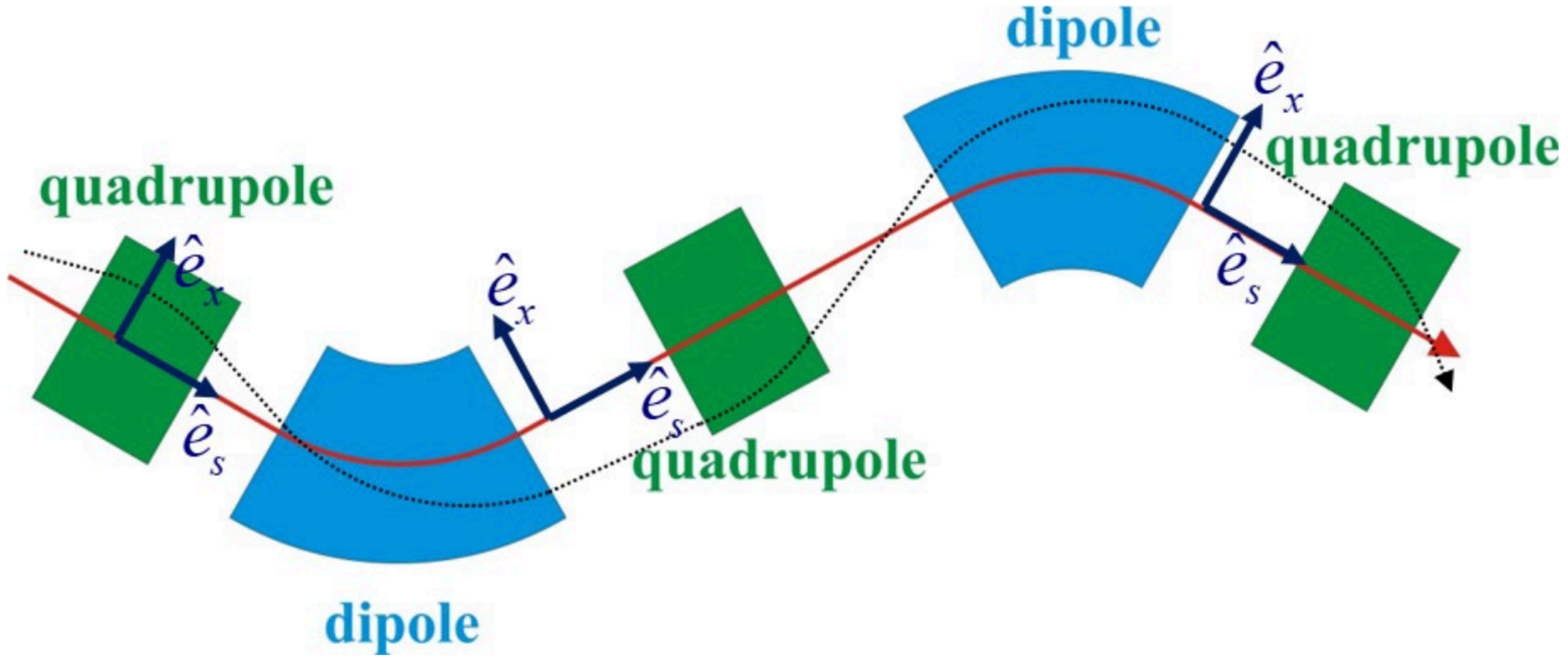
Coordinate system (curvilinear) is "following" the design orbit:

- is rotating in dipole magnets due to curved design orbit
- is not co-moving with reference particle
- s-position is measured along the reference orbit



Equations of motion in accelerator

Use of Hamiltonia formalism

Generalized equations of motion

$$\frac{dq}{ds} = \frac{\partial H}{\partial p_q} \qquad q : x, y, l \qquad\qquad \frac{dp_q}{ds} = -\frac{\partial H}{\partial q} \qquad p_q : p_x, p_y, \delta$$

$$H(x, p_x, y, p_y, l, \delta; s) = \frac{\delta}{\beta_r} - (1 + hx)\left[\sqrt{\left(\delta + \frac{1}{\beta_r} - \frac{q\phi}{cP_0}\right)^2 - (p_x - \alpha_x)^2 - (p_y - \alpha_y)^2 - \frac{1}{\beta_r^2\gamma_r^2}} + \alpha_s\right]$$

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Scalar potential for acceleration - RF cavities

$$\phi = \frac{V_{RF}}{\omega_{RF} C_0}\left(sin(\varphi_s)\omega_{RF} z - cos\left(\varphi_s - \frac{\omega_{RF} z}{c}\right) c\right) - \frac{cP_0}{2q}\eta_p \delta^2$$

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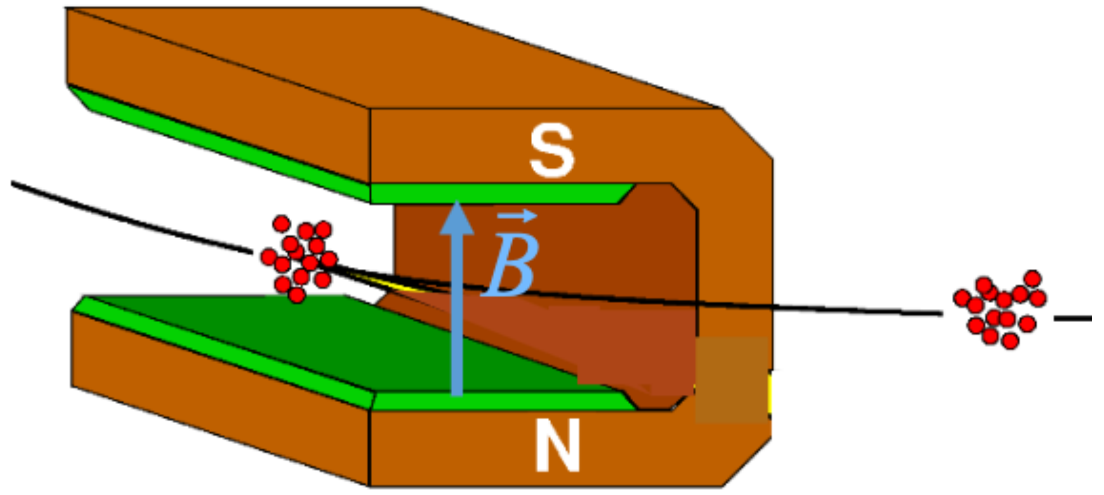
Vector potential for steering - most common multipoles (2n poles)

$$\alpha_x = \alpha_y = 0 \qquad \alpha_s = \frac{q}{P_0}A_s = -\frac{q}{P_0}B_{ref}\Re\left[\sum_{n=2}^{\infty}(b_n + ia_n)\frac{(x+iy)^n}{nR_{ref}^{n-1}}\right] = -\Re\left[\sum_{n=2}^{\infty}(k_{n-1} + iks_{n-1})\frac{(x+iy)^n}{n!}\right]$$

Deflection & Focusing

For **beam deflection** is need a homogenous (constant) magnetic field:



- **dipole magnet** $\rightarrow B = B_y = const \rightarrow a_s = -k_0 x + \dfrac{k_0 h x^2}{2(1+hx)}$

Iron dominated (warm):
field determined by
geometry of poles
(2 flat poles)

Superconducting:
field determined by
geometry of coils
$(j(\phi) \propto \cos\phi)$

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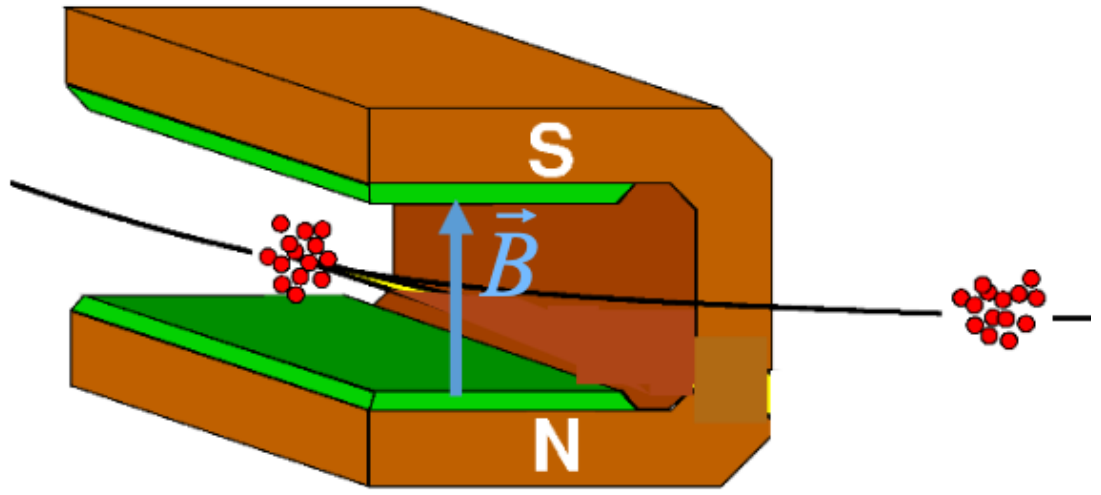

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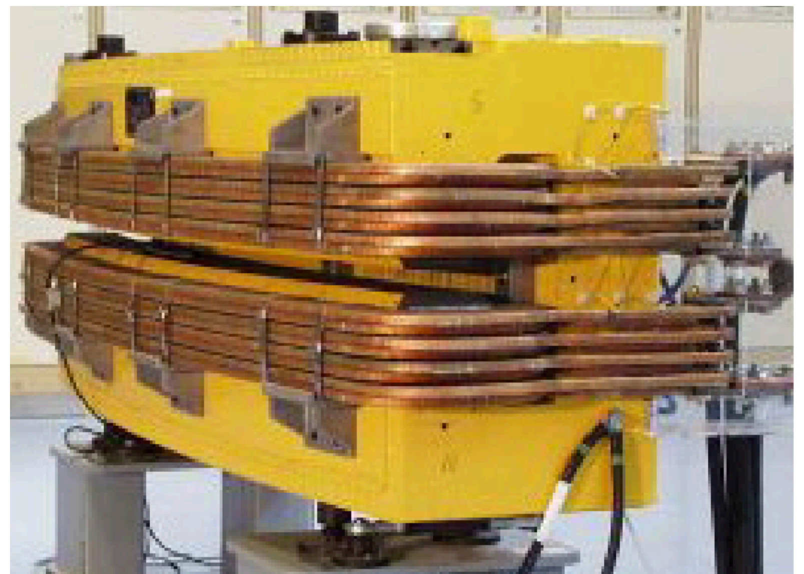
Picture taken from CERN Document Server

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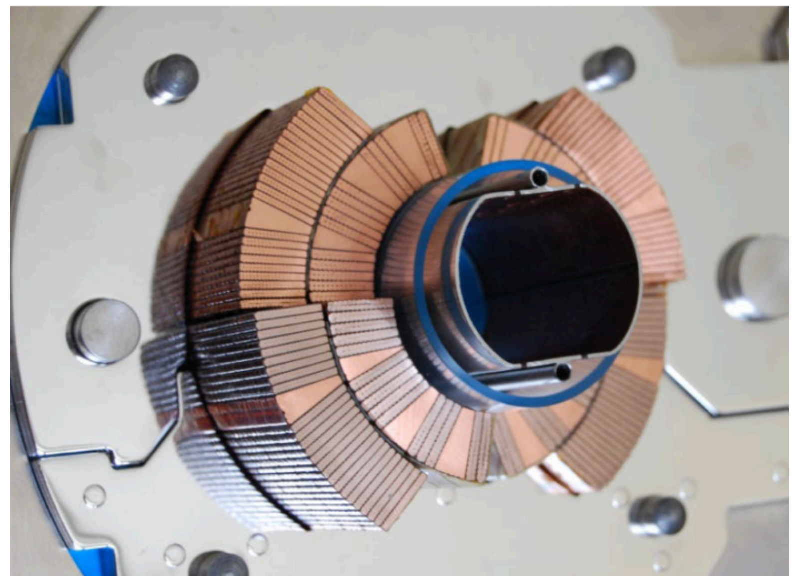
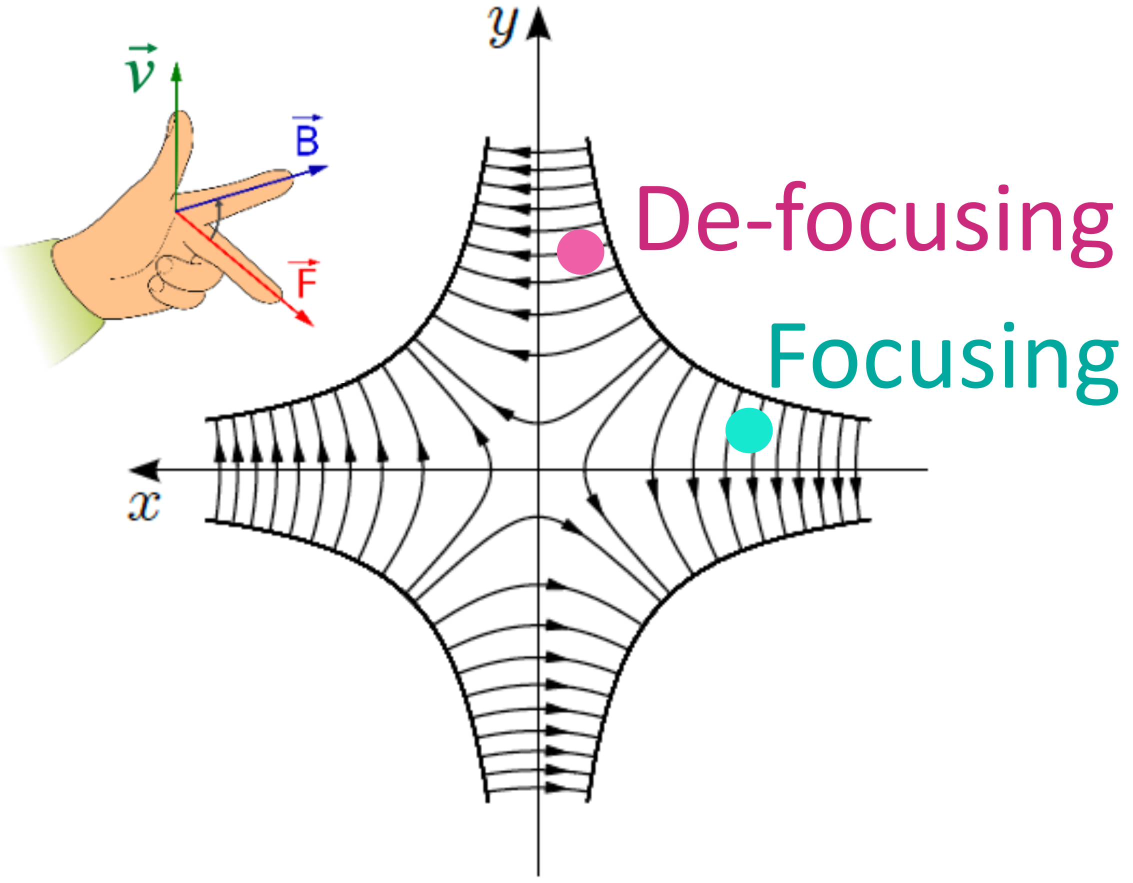
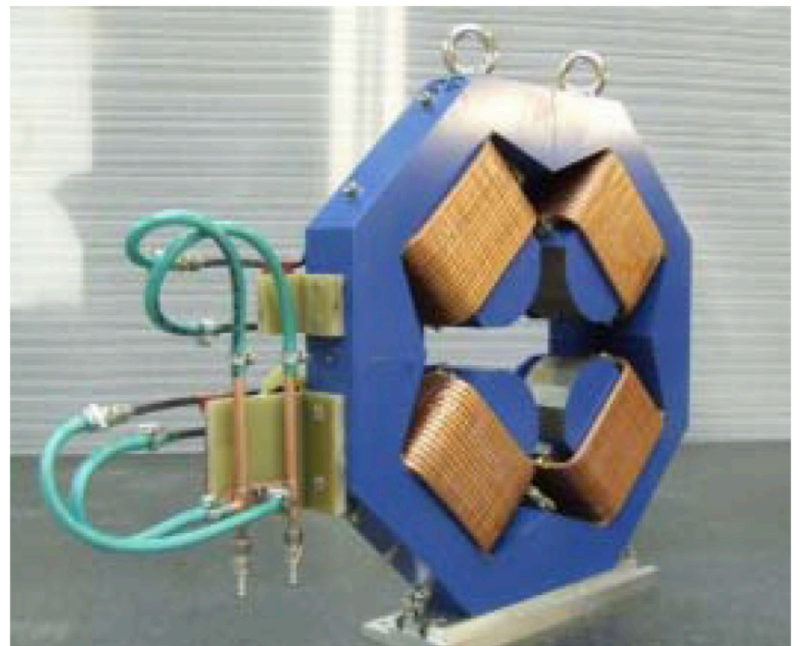

Picture taken from G. de Rijk, arXiv:2017.03177

For **beam focusing** a magnetic field that linearly increase from magnet center:

- **quadrupole magnet** $\rightarrow B_y + iB_x = k_1(x+iy) \rightarrow a_s = -\frac{k_1}{2}(x^2 - y^2)$



Iron dominated (warm):
field determined by
geometry of poles
(4 hyperbolic poles)


Picture taken from https://cds.cern.ch/record/1333874/plots

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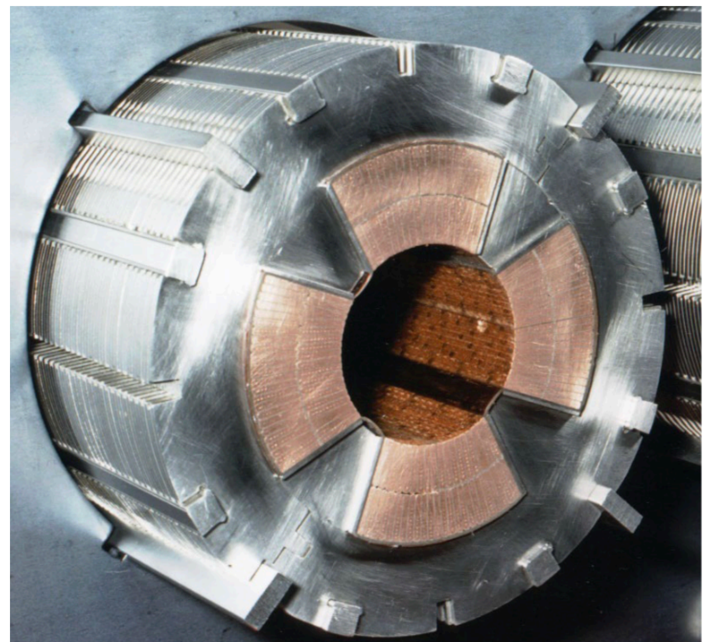
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Need for FODO cells
for focusing at x and y

Linear transfer maps

$$\vec{u} = \mathscr{M}\ \vec{u}_0 = M\ \vec{u}_0 + m$$

$$M = \begin{pmatrix} c_x & s_x & 0 & 0 & 0 & \bar{h}\frac{1-c_x}{\omega_x^2} \\ -\omega_x^2 s_x & c_x & 0 & 0 & 0 & \bar{h}s_x \\ 0 & 0 & c_y & s_y & 0 & 0 \\ 0 & 0 & \omega_y^2 s_y & c_y & 0 & 0 \\ -\bar{h}s_x & -\bar{h}\frac{1-c_x}{\omega_x^2} & 0 & 0 & 1 & \frac{L}{\beta_0^2\gamma_0^2} - \bar{h}^2\frac{L-s_x}{\omega_x^2} \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$m = \begin{pmatrix} (h-k_0)\frac{1-c_x}{\omega_x^2} \\ (h-k_0)s_x \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$s_x = \frac{\sin(\omega_x L)}{\omega_x}, \quad c_x = \cos(\omega_x L), \quad s_y = \frac{\sinh(\omega_y L)}{\omega_y}, \quad c_y = \cosh(\omega_y L), \quad \bar{h} = \frac{h}{\beta_0}, \quad \omega_x = \sqrt{hk_0 + k_1}, \quad \omega_y = \sqrt{k_1}.$$

For “ideal”:

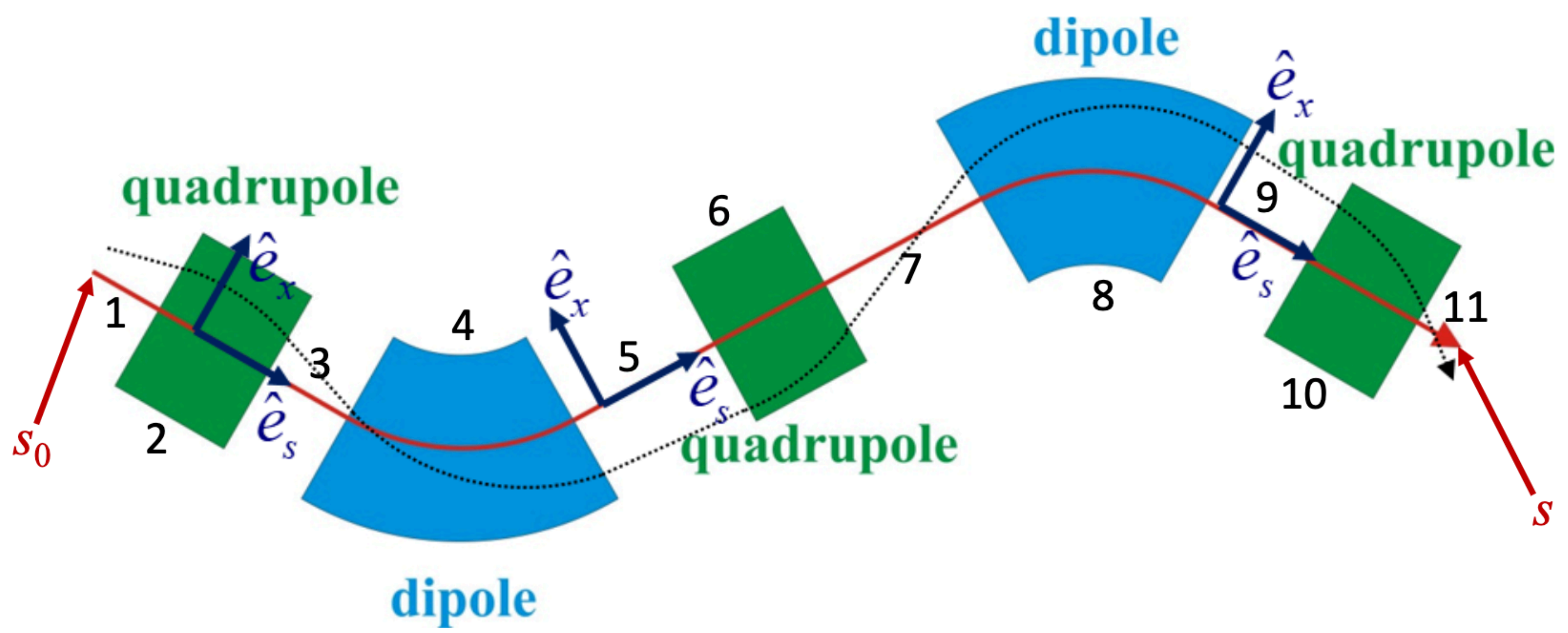
- **drifts** (no field) $\rightarrow h = k_0 = k_1 = 0$
- **dipole magnets** $\rightarrow h = k_0 \neq 0,\ k_1 = 0$
- **quadrupole magnets** $\rightarrow h = k_0 = 0,\ k_1 \neq 0$
- **combined dipole-quadrupole magnets** $\rightarrow h = k_0 \neq 0,\ k_1 \neq 0$

Linear transfer maps

Particle trajectory through linear lattice:

$$\vec{u} = \ldots \cdot M_{D8} \cdot M_{d7} \cdot M_{Q6} \cdot M_{d5} \cdot M_{D4} \cdot M_{d3} \cdot M_{Q2} \cdot M_{d1} \; \vec{u}_0$$

Transfer matrix M



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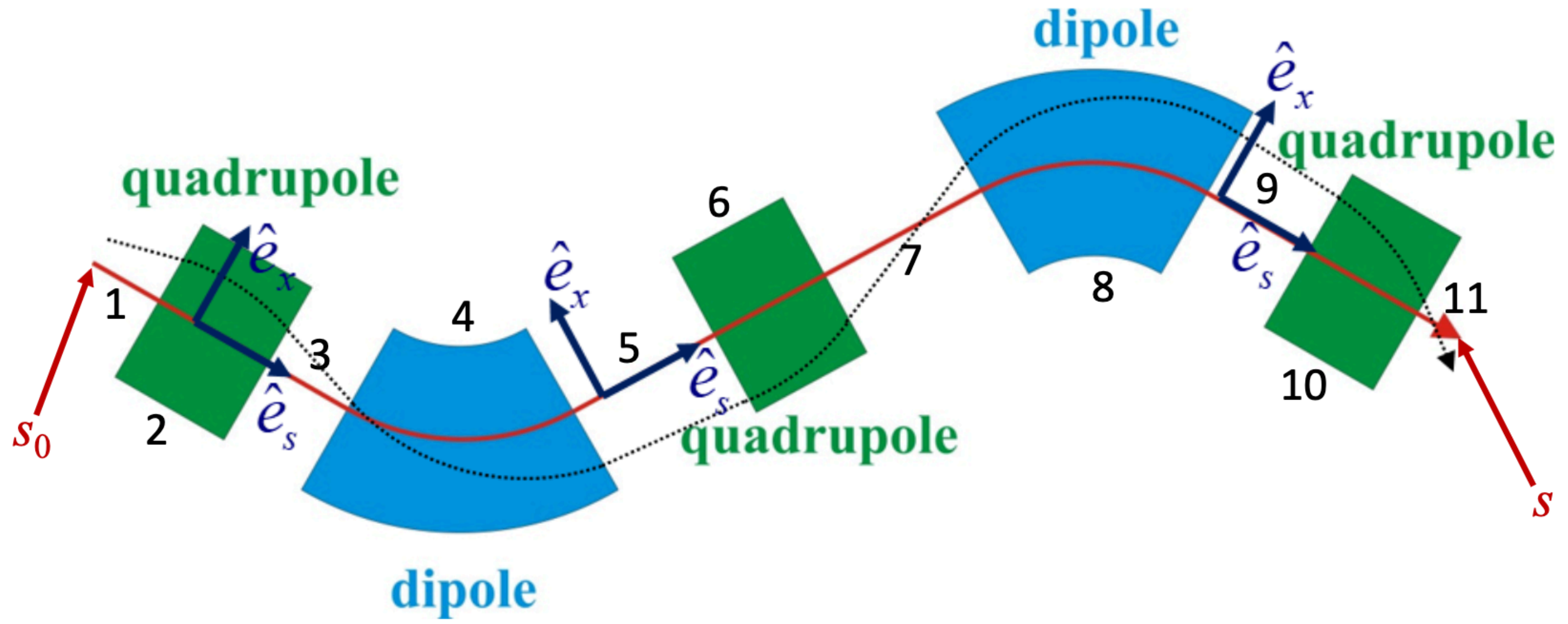
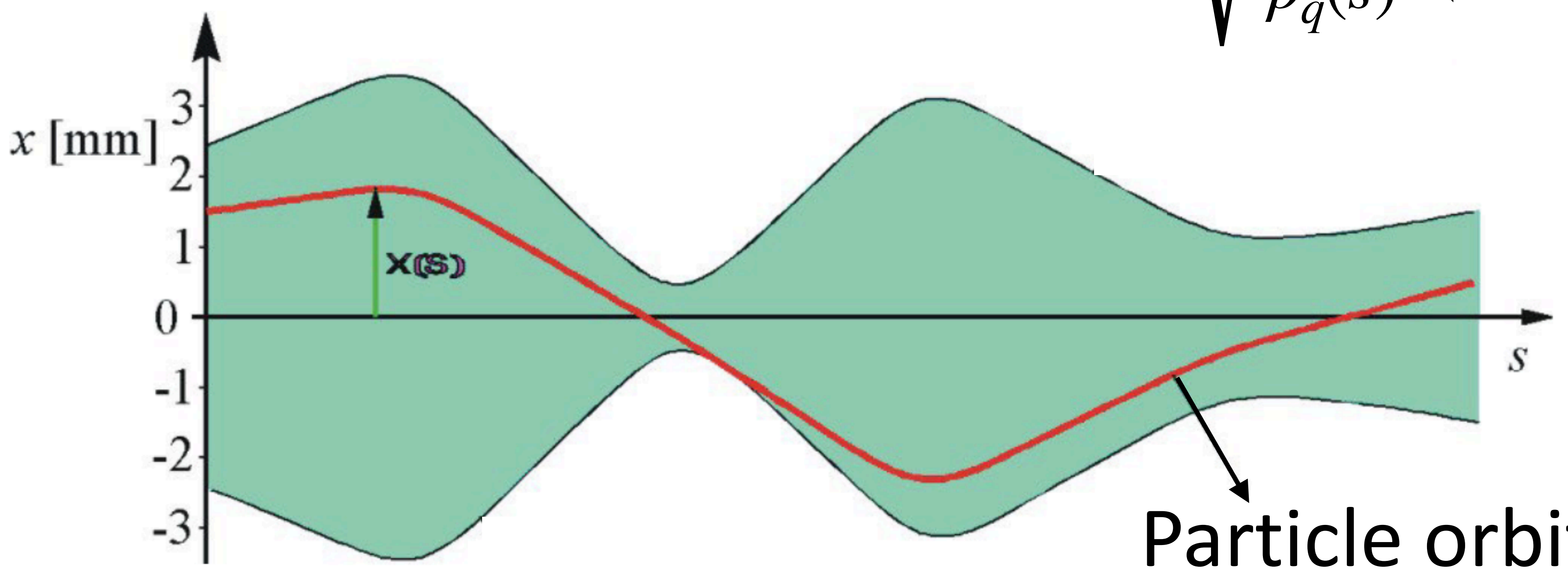


Periodic solution in (x,y):

$$q(s) = \sqrt{2\beta_q(s)J_q}\cos\mu_q(s)$$

$$p_q(s) = -\sqrt{\frac{2J_q}{\beta_q(s)}}\left(\sin\mu_q(s) + \alpha_q(s)\cos\mu_q(s)\right)$$

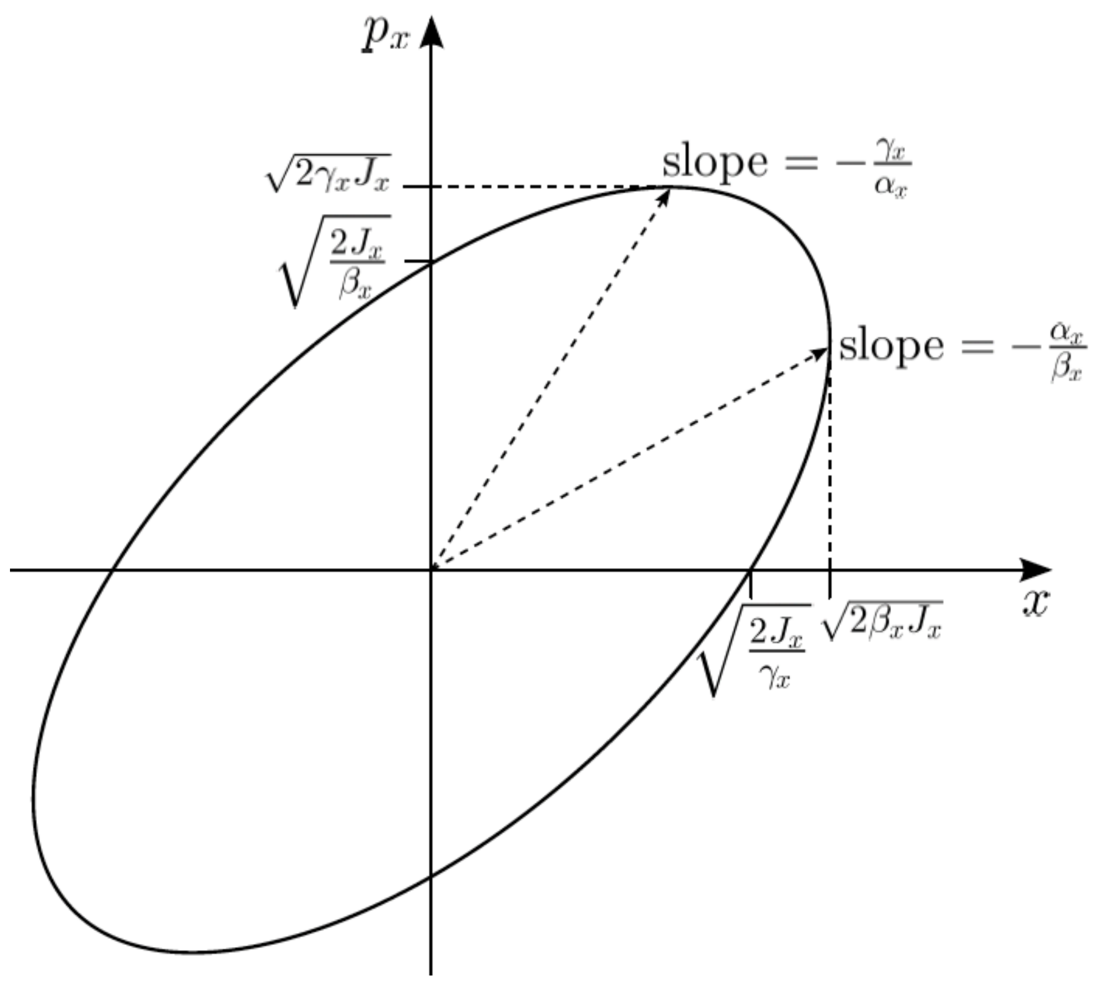


- $\beta_q(s)$ defines beam envelop shape

$$\alpha_q = -\frac{d\beta_q}{2ds},\ \gamma_q = \frac{1+\alpha_q^2}{\beta_q}$$

- J_q is characteristic for each particle and constant of motion in linear machines (const. ellipse area)

$$J_q = 0.5\left(\gamma_q q^2 + 2\alpha_q q p_q + \beta_q p_q^2\right)$$



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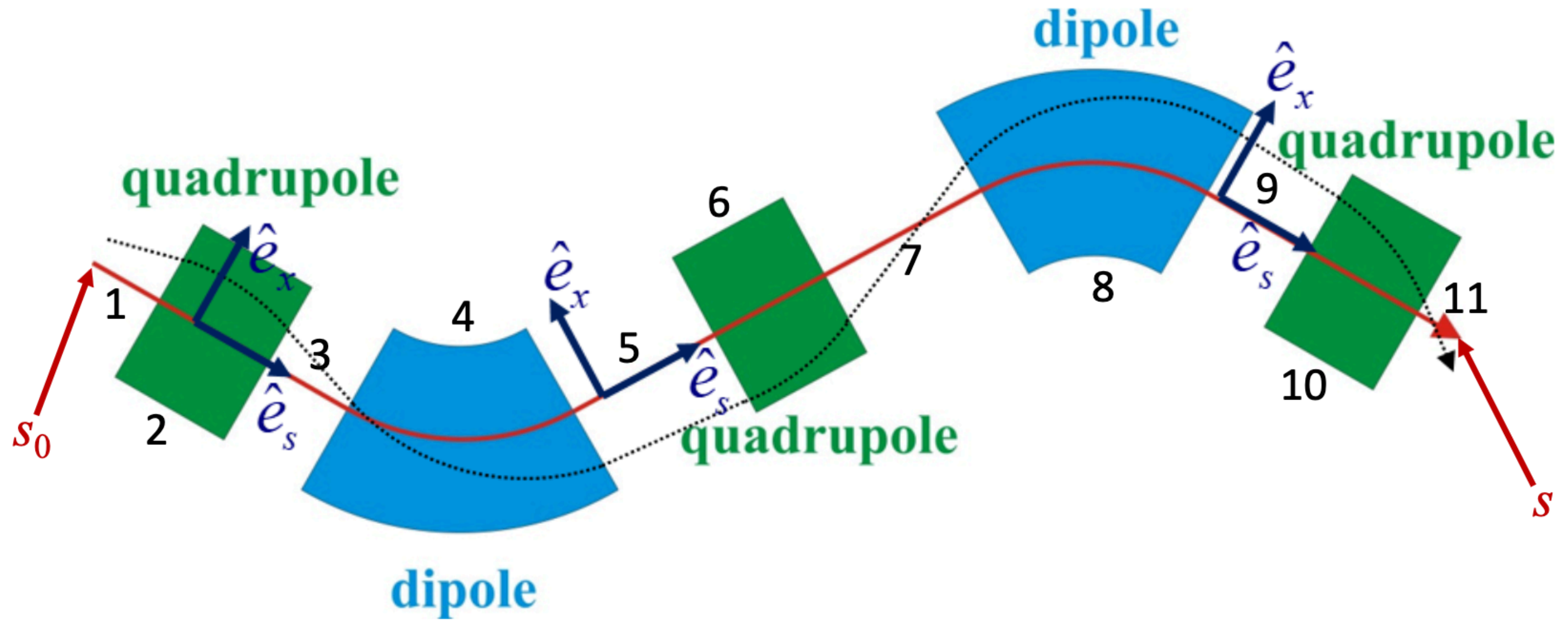

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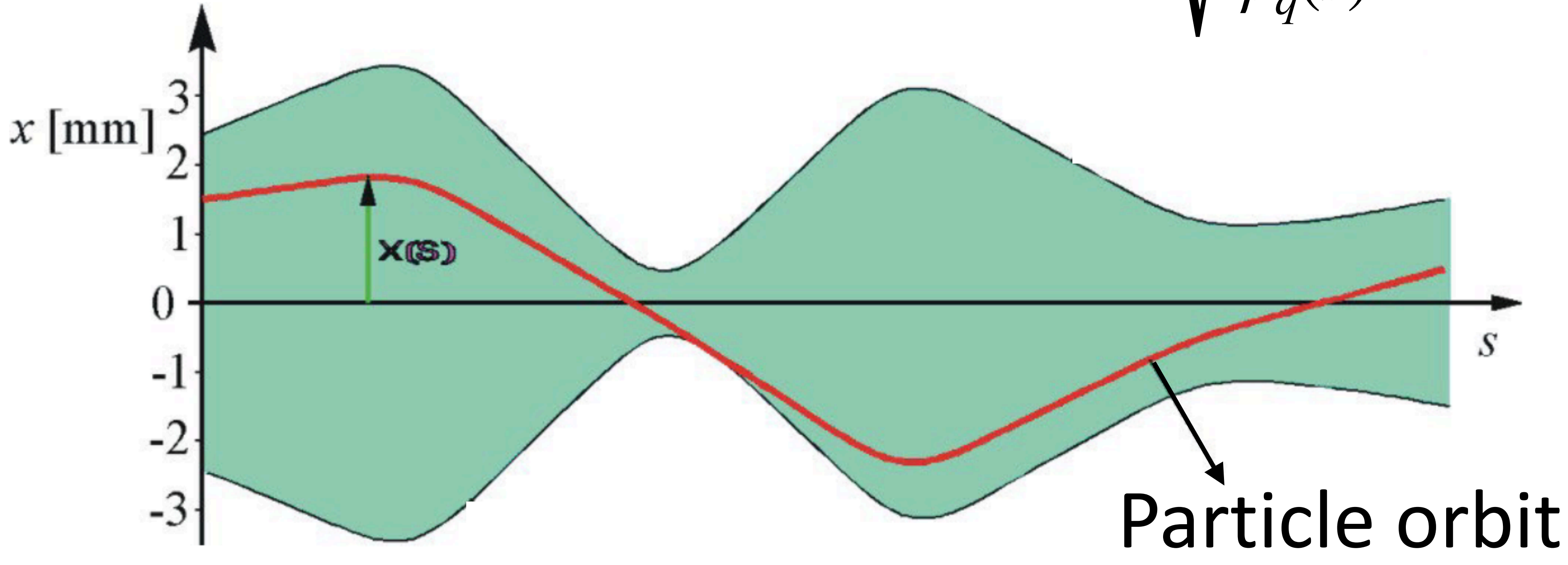

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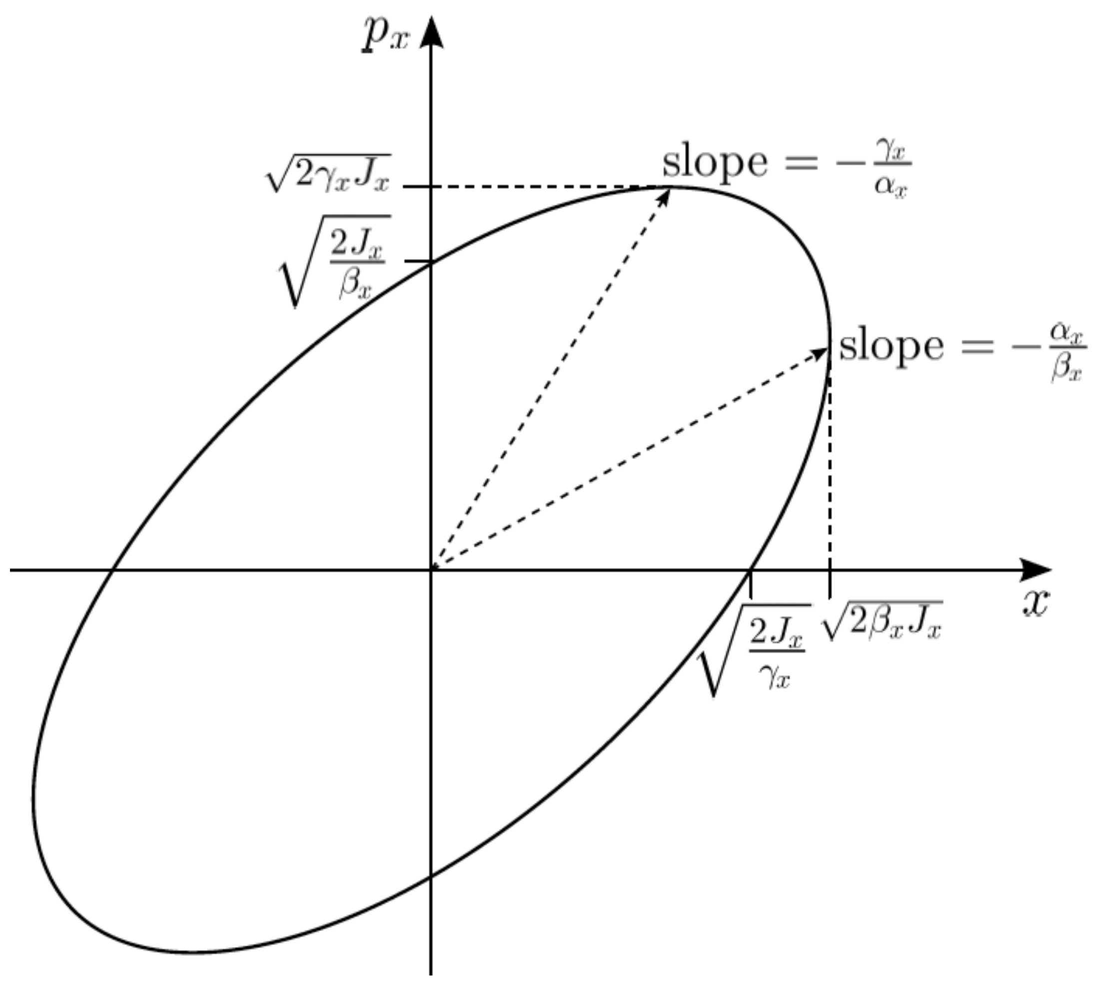
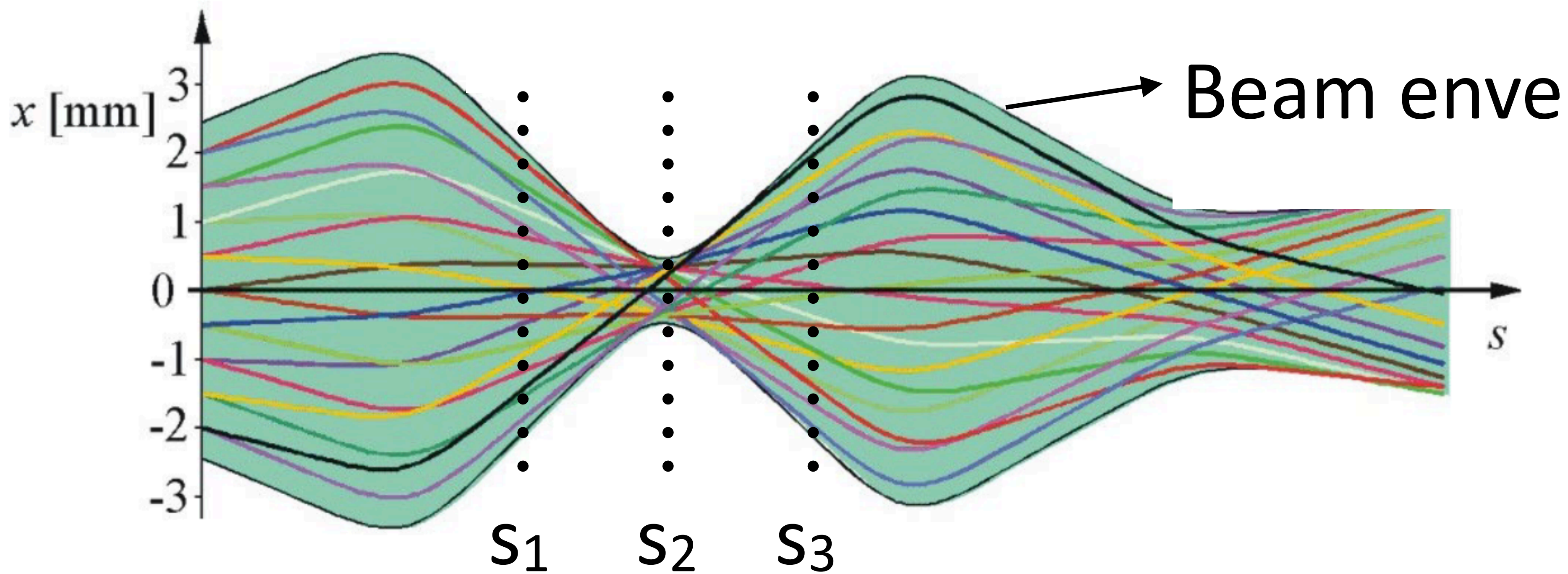
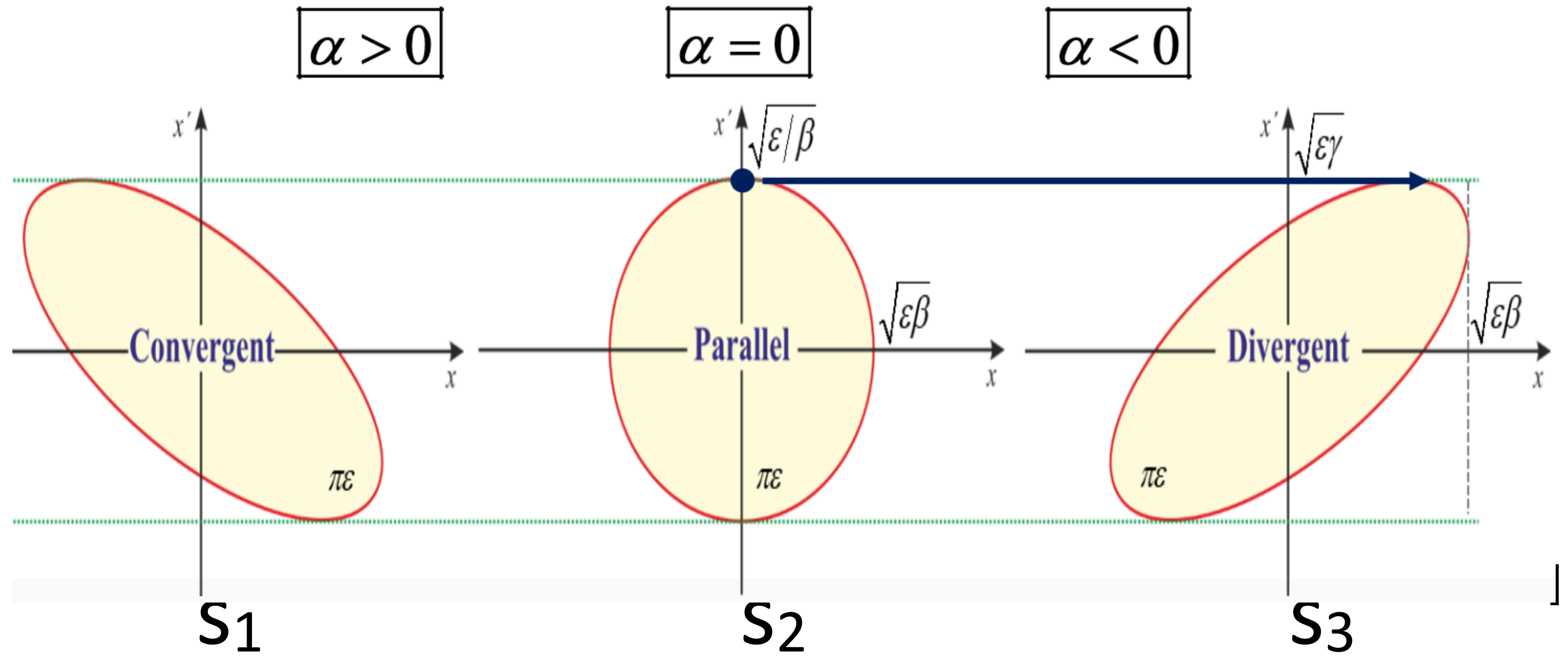




Beam envelop: $\sigma_q(s) = \sqrt{\beta_q(s)\varepsilon_q}$

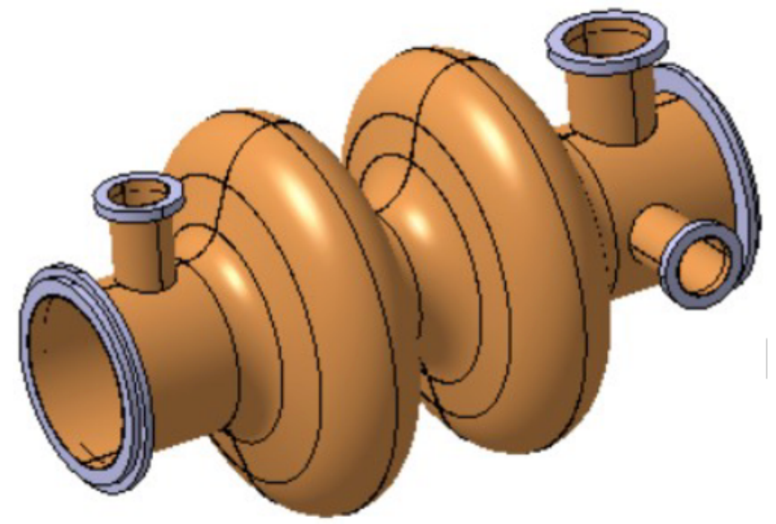
$$\varepsilon_q = \langle J_q \rangle$$

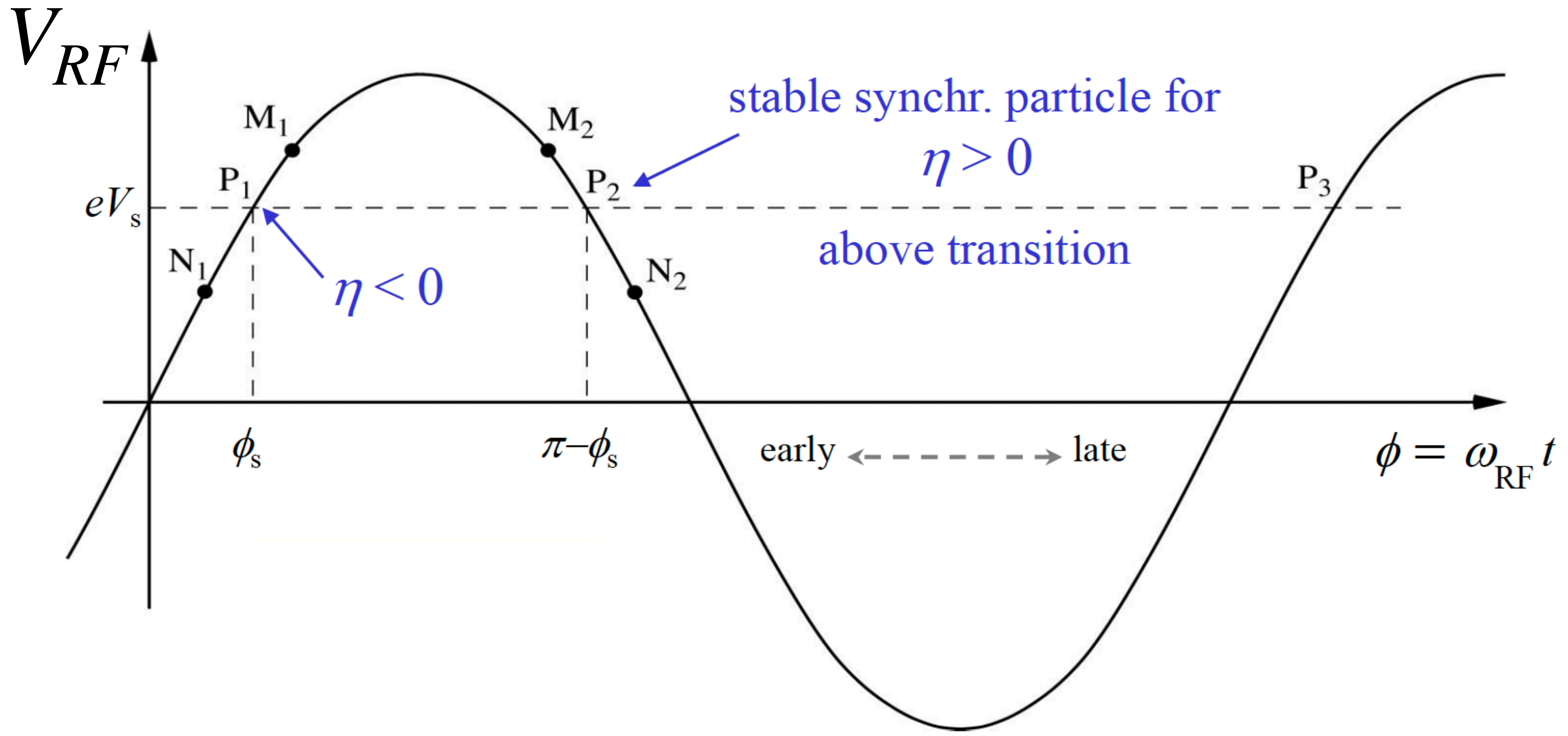


Longitudinal motion

In synchrotron during acceleration:

- the magnetic fields should increase with time (keep particles on the closed orbit)
- ω_{RF} should increase (match increase of revolution frequency)





- In machines with synchrotron radiation, like FCC-ee, there is energy-loss (that is energy dependant)
- The synchrotron motion is damped toward an equilibrium bunch length and energy spread

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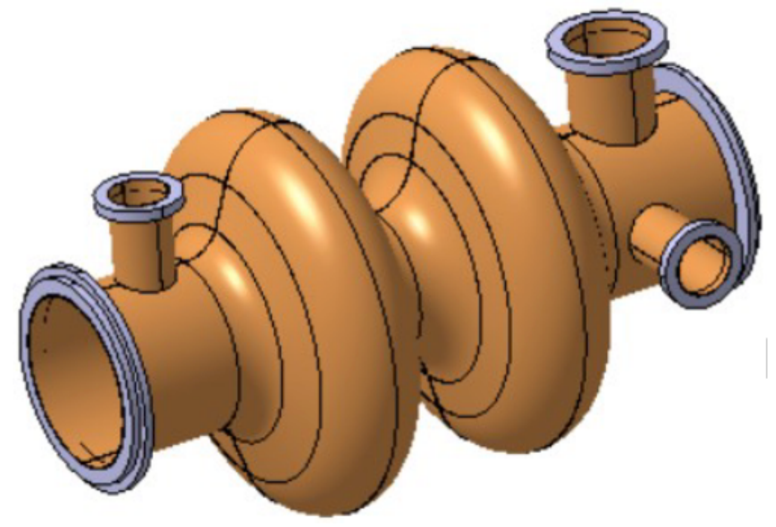

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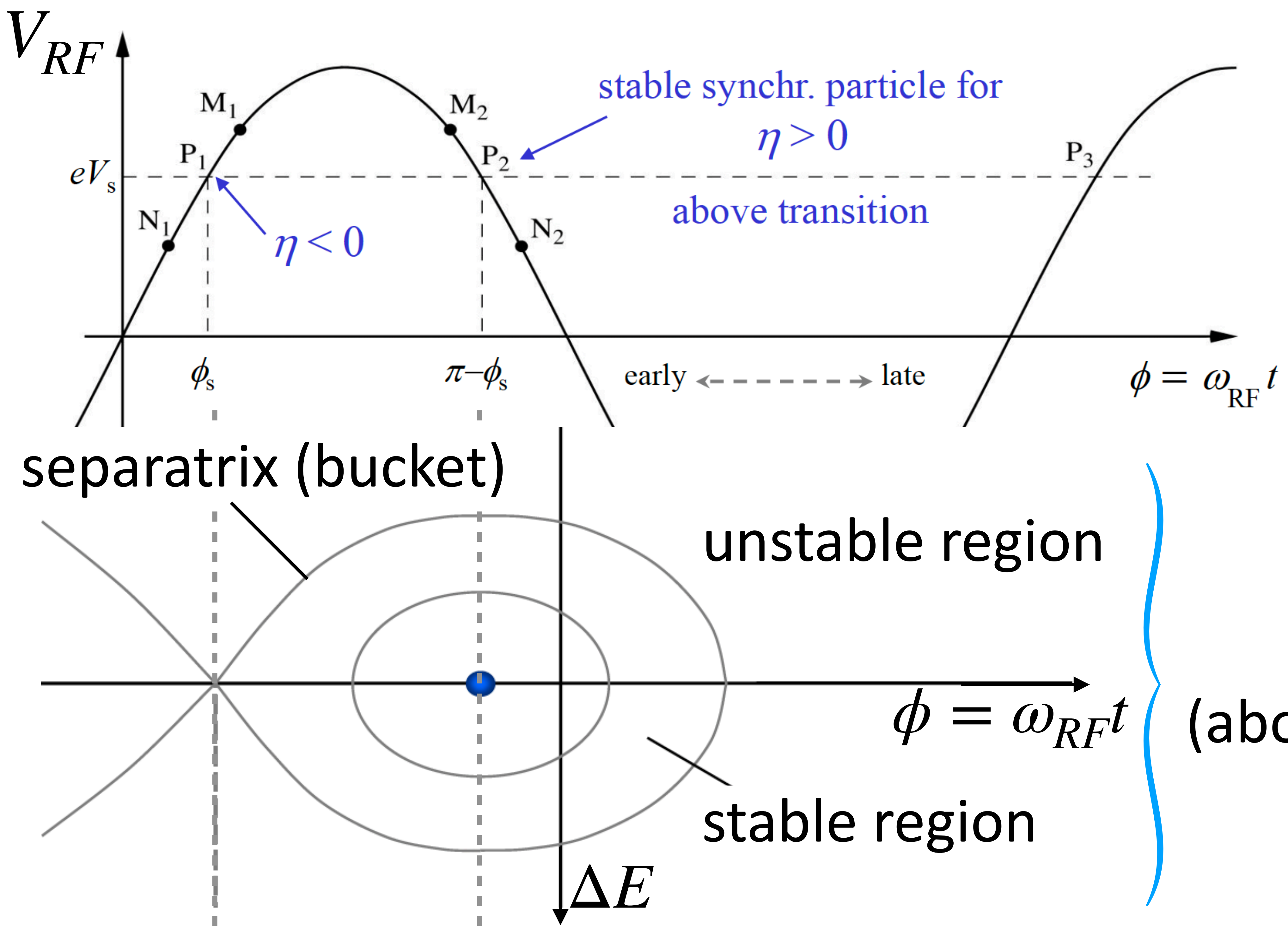
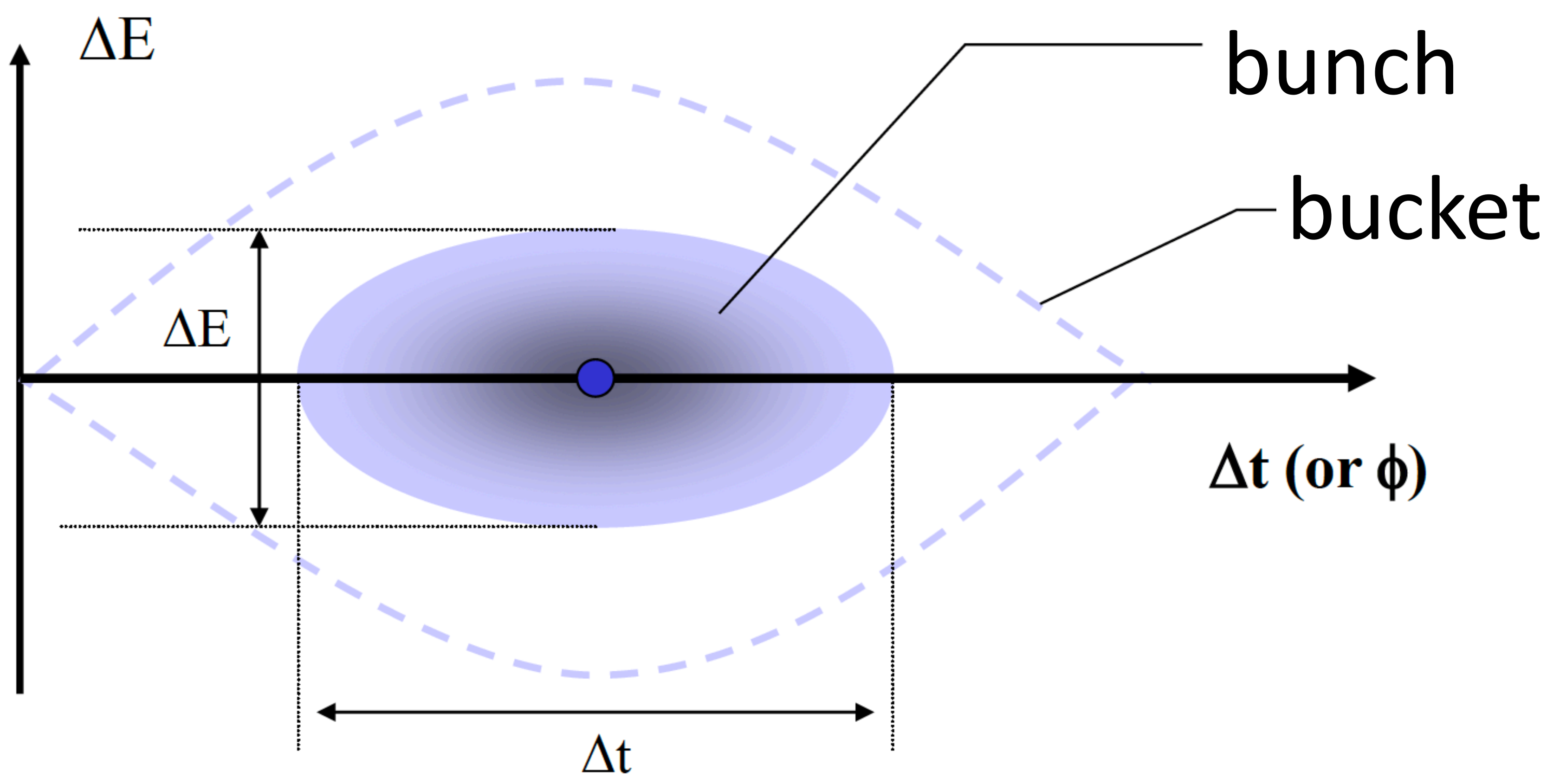


For $\eta > 0$
(above transition)



Bucket area: longitudinal acceptance

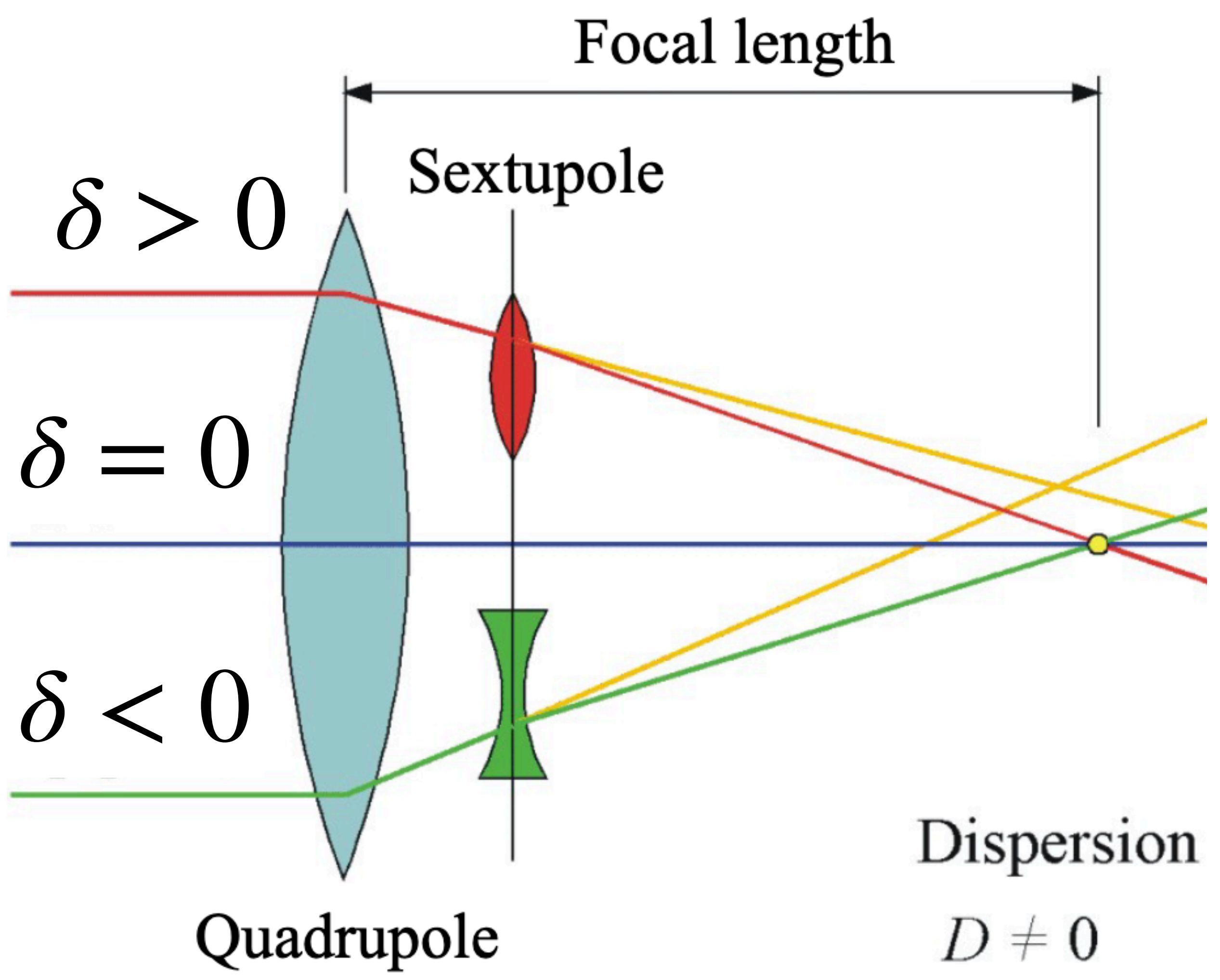
Bunch area: longitudinal beam emittance $= \pi\sigma_E\sigma_t$
(not unique definition)

- In machines with synchrotron radiation, like FCC-ee, there is energy-loss (that is energy dependant)
- The synchrotron motion is damped toward an equilibrium bunch length and energy spread

Non-linear elements & imperfections

To improve the performance of the collider, the use of non-linear magnets (sextuples, octuples, …) are needed

- As an example, the sextuples can correct the focussing issues for particles with not ideal energy ($\delta \neq 0$)



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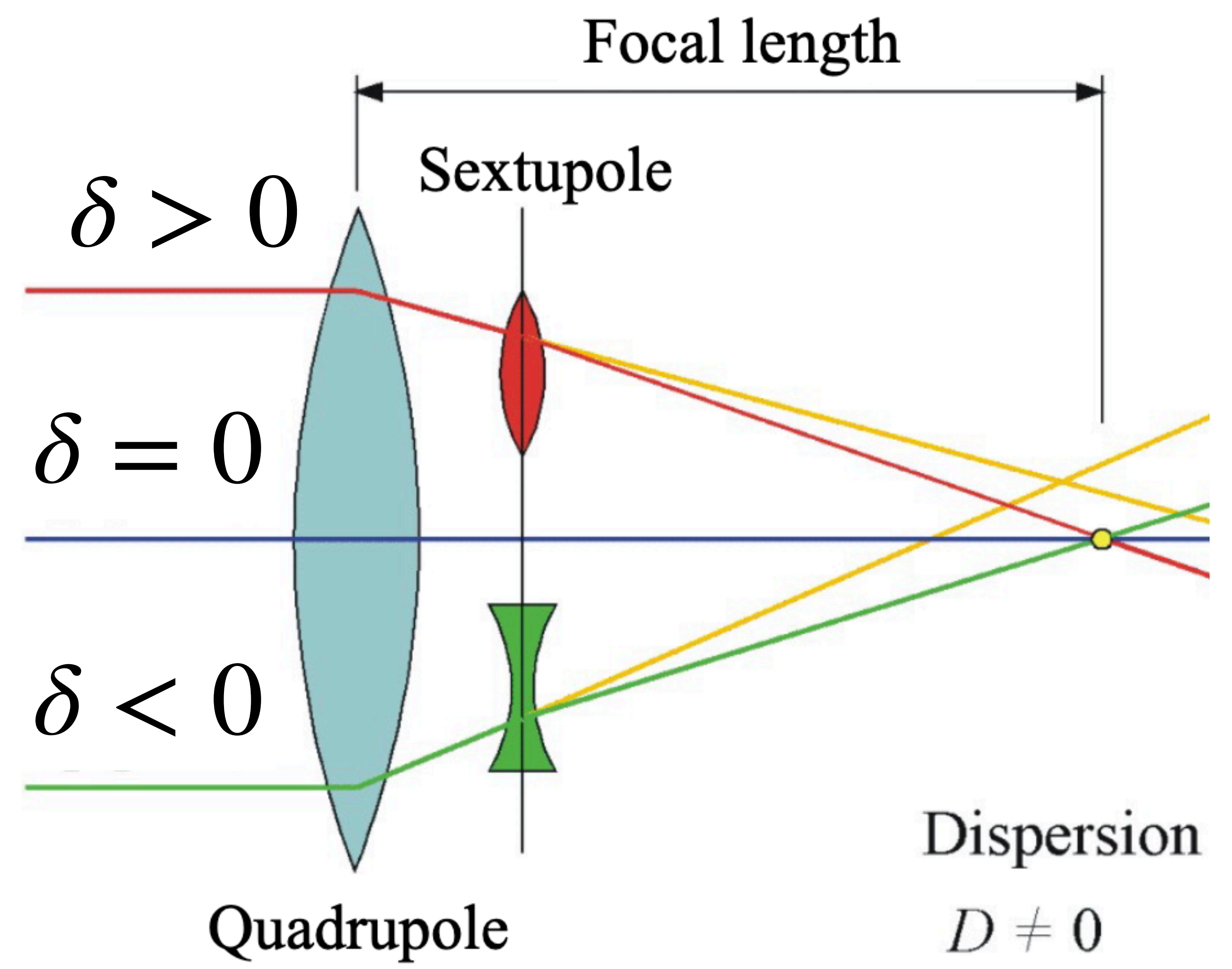
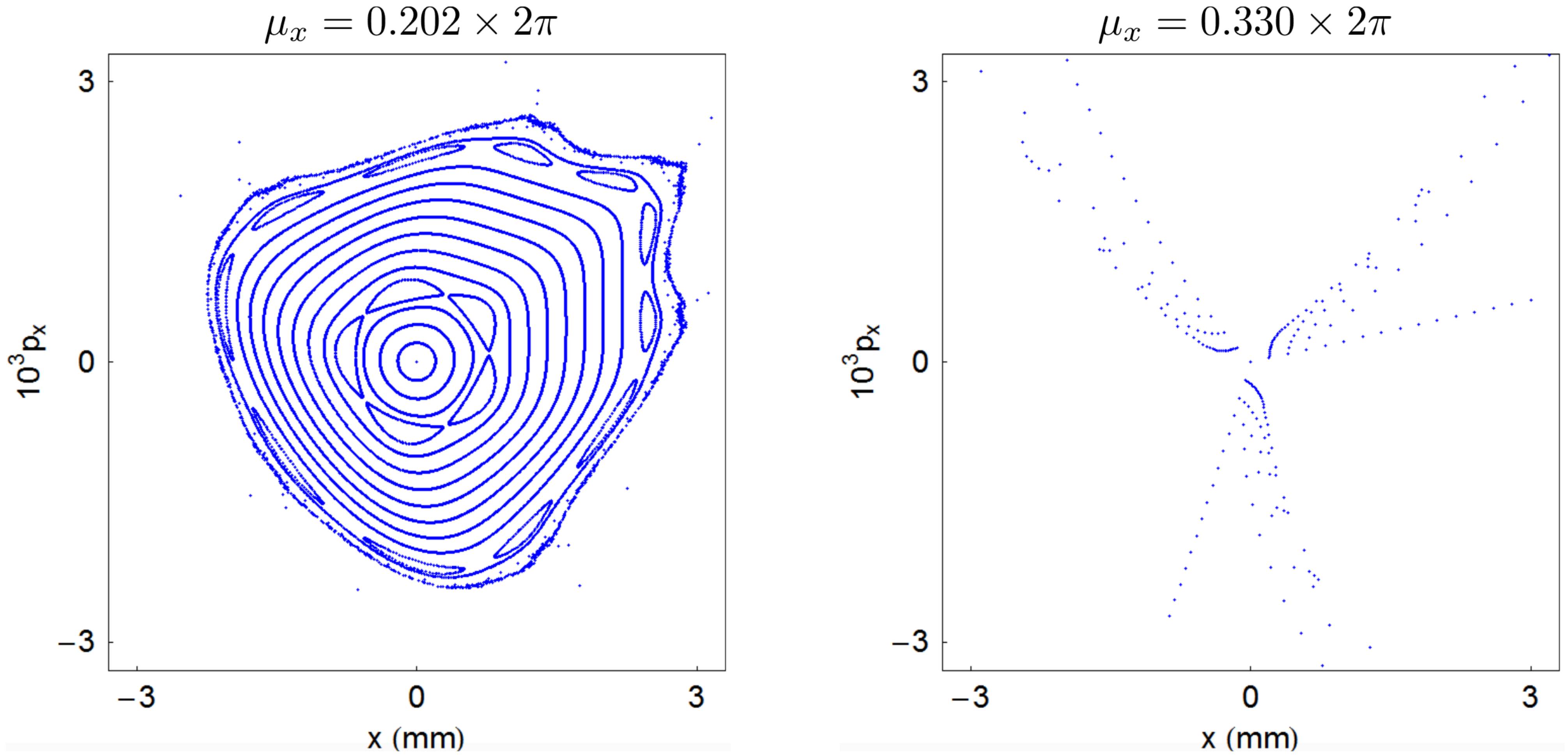


Any magnet imperfections experienced periodically (circular machines) can be detrimental for the beam quality if resonance conditions are satisfied

$$\text{Tune: } \nu_q = \frac{\mu_q(s_0 \,|\, s_0 + C_0)}{2\pi}$$

- As an example, the phase space for different tunes in the presence of a single sextupole



Need for non-linear magnets

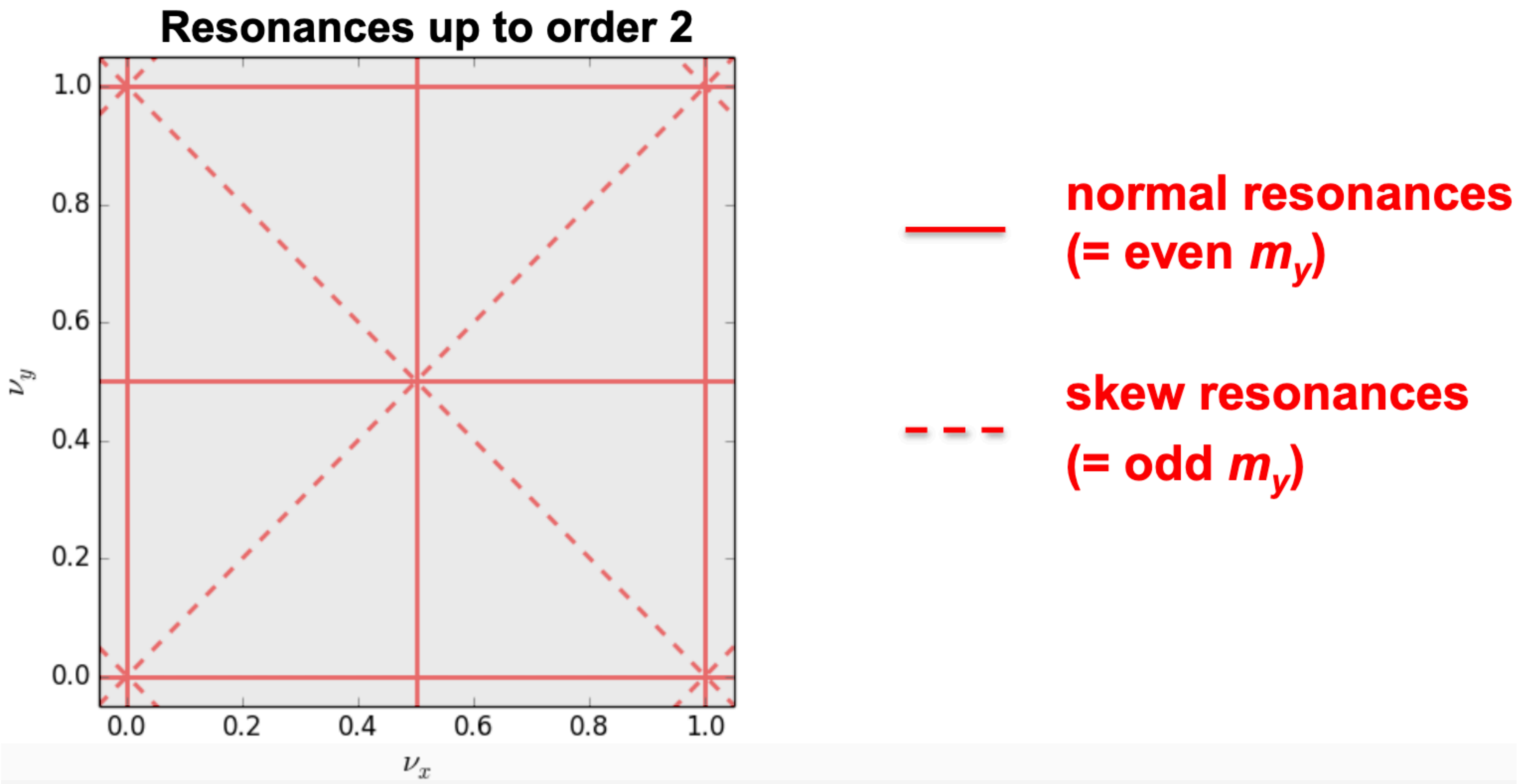
It is paramount to have some understanding of machine error and nonlinear dynamics for optimizing the design and operation of many accelerator systems

- Among the different strategies to mitigate the impact of the machine errors and improve the particle non-linear dynamic is to properly choose the working point (ν_x, ν_y, ν_s) so to avoid the resonance conditions

$$m_x\nu_x + m_y\nu_y = l$$

Where m_x, m_y *and* l are integers

Resonance is of order $|m_x| + |m_y|$



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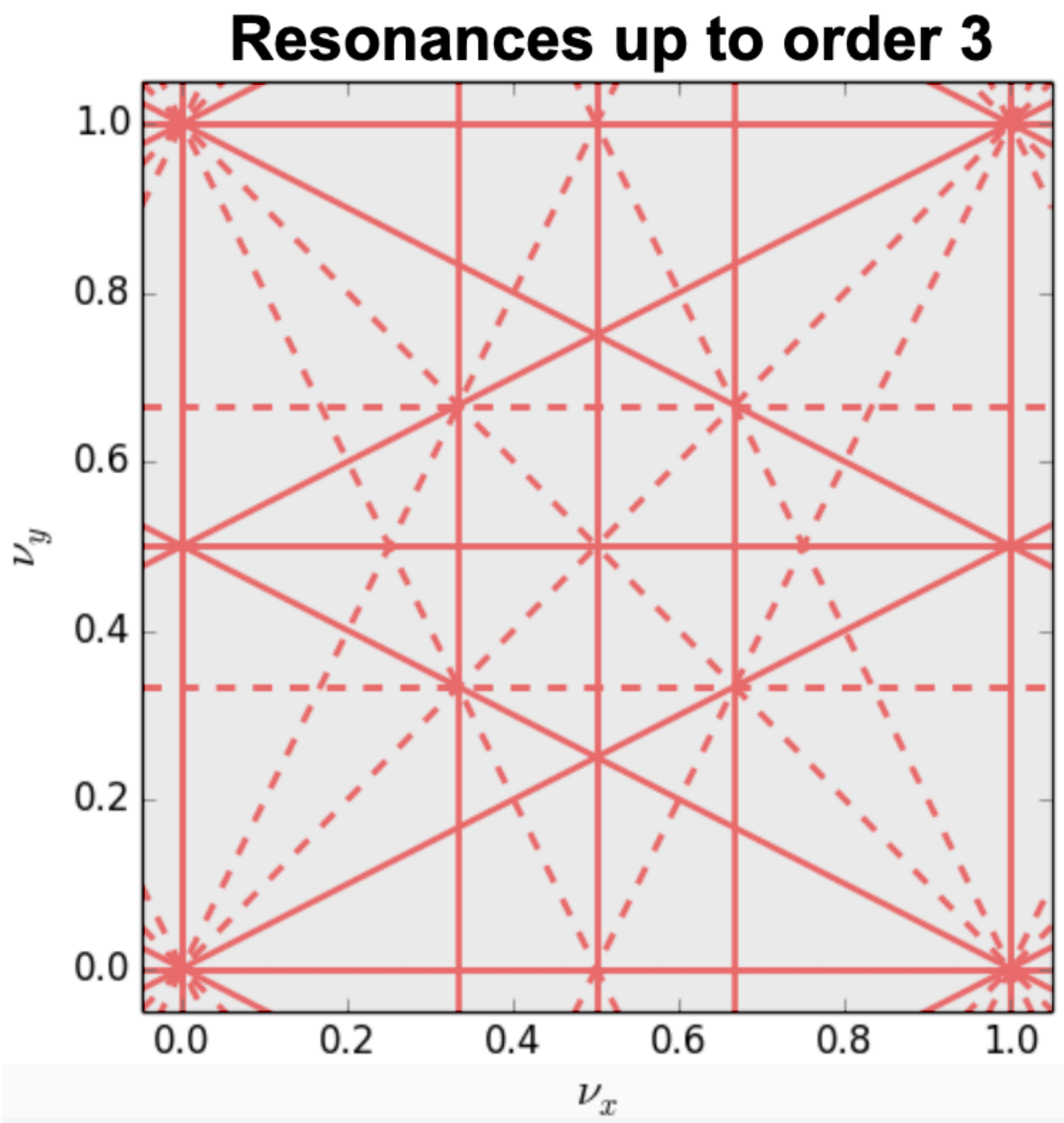

normal resonances (= even m_y)

skew resonances (= odd m_y)

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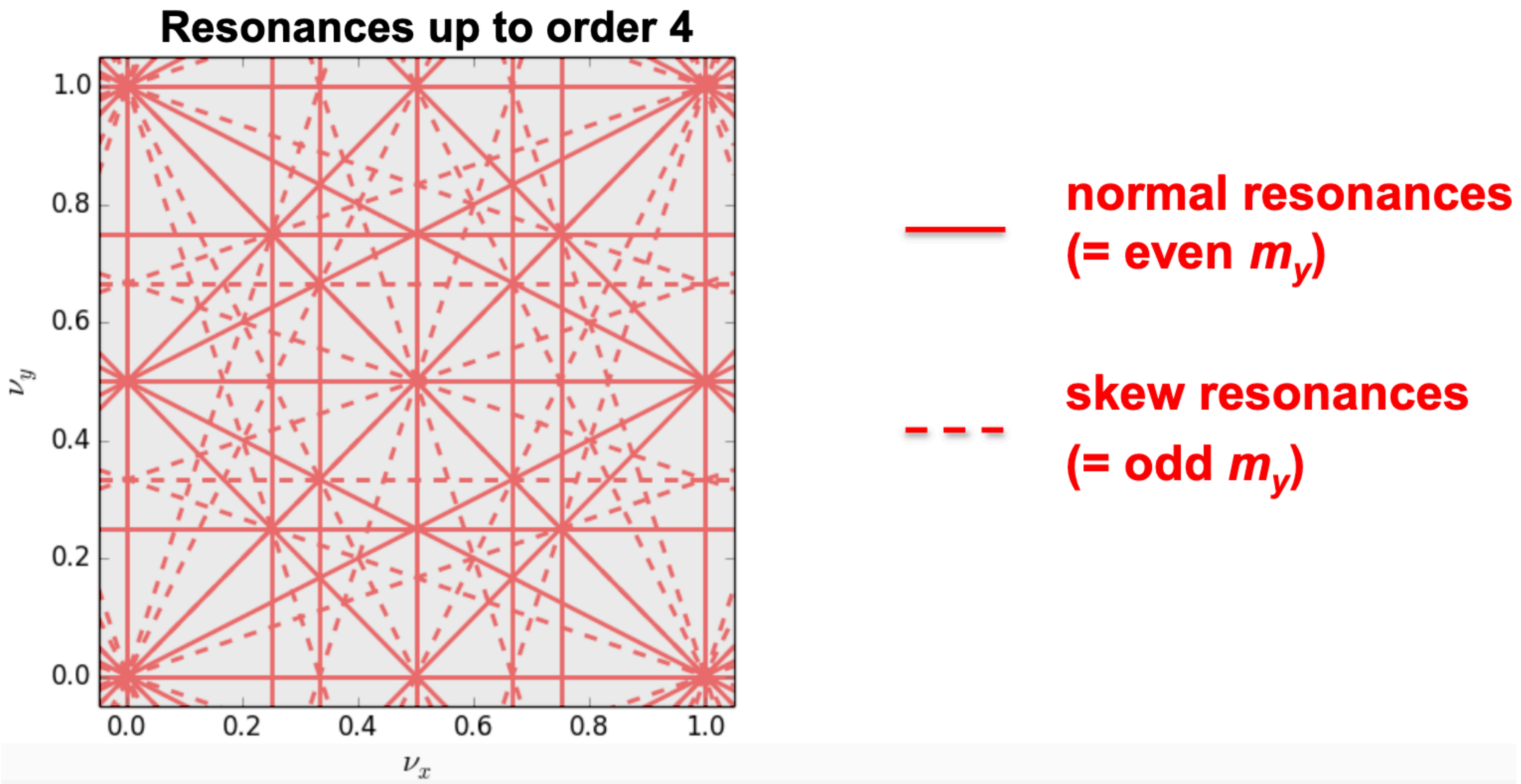
$$m_x \nu_x + m_y \nu_y = l$$

Where m_x, m_y *and* l are integers

Resonance is of order $|m_x| + |m_y|$



Need for non-linear magnets

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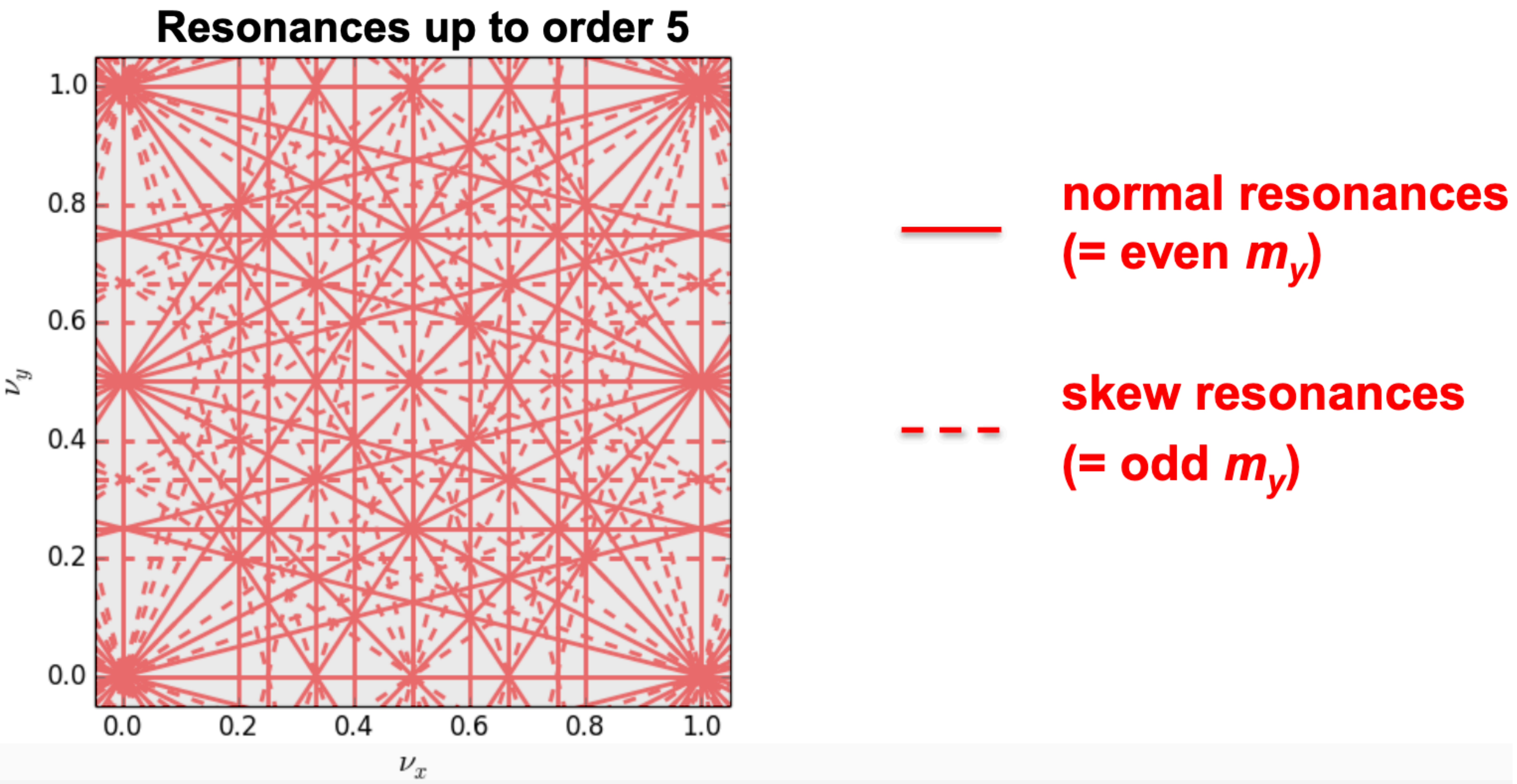

Need for non-linear magnets

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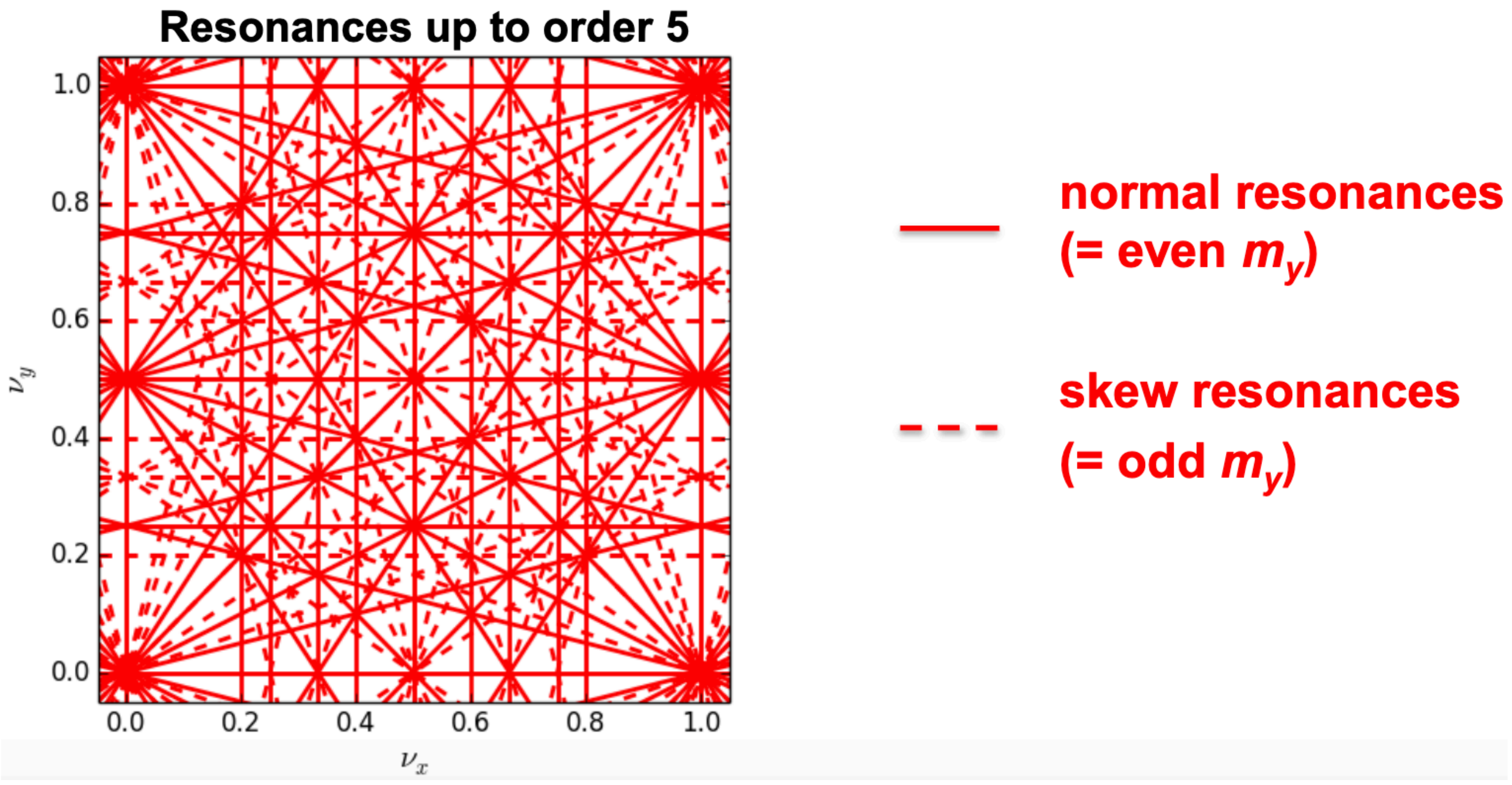
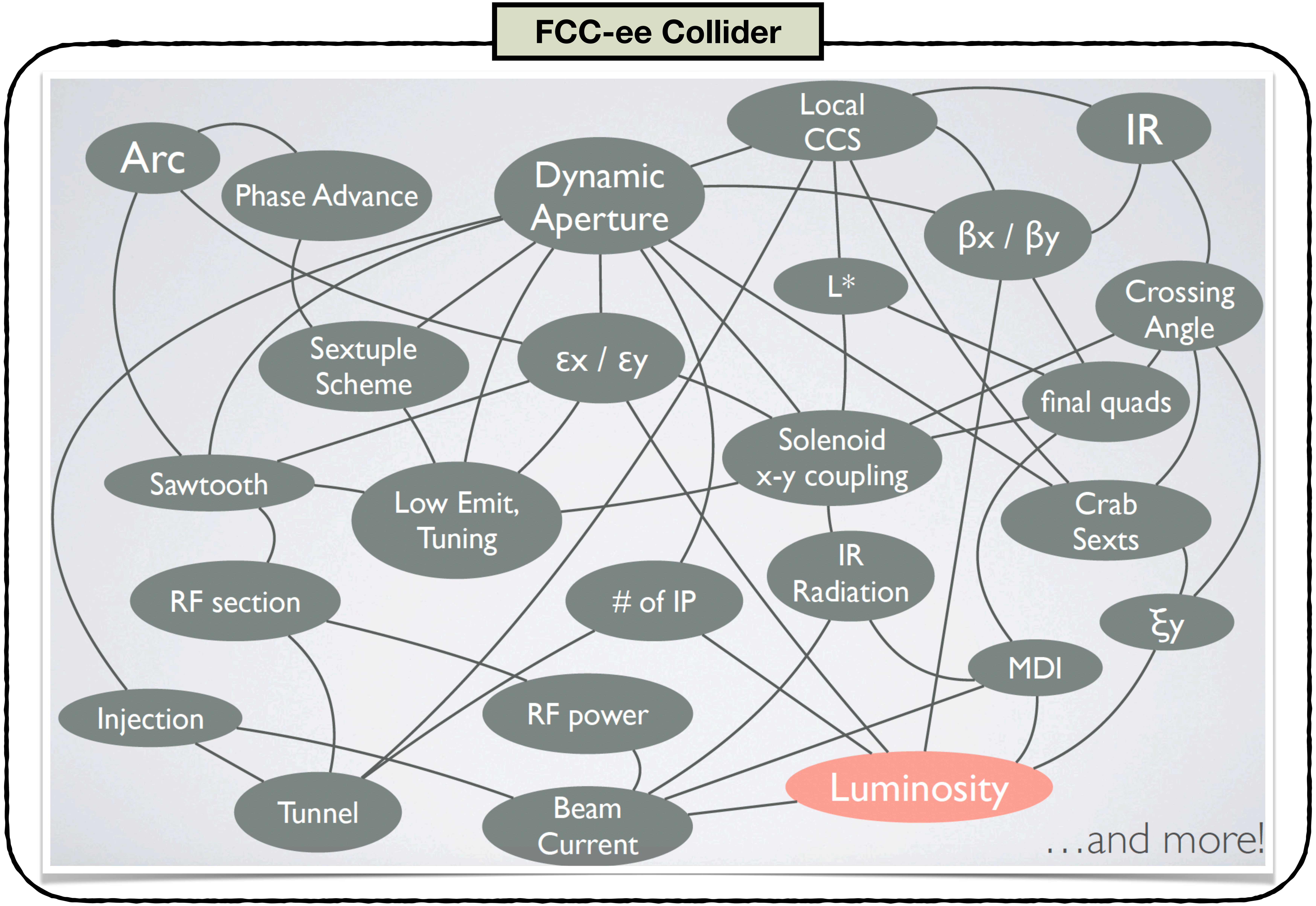
- **In high intensity/energy machines like the FCC:**
  - **Beam-Beam Effects** (due to beams cross each other, the electromagnetic fields generated by one beam can affect the other beam)
  - **Space Charge Effect** (mutual repulsion between particles in the beam)
  - **Impedance Effects** (interactions between the beam and the vacuum chamber or other structures)
  - **Electron Cloud** (high-energy particles hit the chamber walls/residual gas or the emitted SR, can release secondary electrons)
  - **Wakefields** (beam induce electromagnetic fields that affect subsequent particles)



Complexity, as usual



Current FCC-ee parameters

Tow lattice designs with different features:

- Global Hybrid Correction optics
- Local Chromaticity Correction optics
- Lattices can be found here

FCC-ee collider parameters for the GHC lattice at Z, Nov. 6, 2024.

Beam energy	[GeV]	45.6		
Layout		PA31-3.0		
# of IPs		4		
Circumference	[km]	90.658728		
Bend. radius of arc dipole	[km]	10.021		
Arc cell		Long 90/90		
Momentum compaction α_p	$[10^{-6}]$	28.67		
Arc sext families		75		
Energy loss / turn	[GeV]	0.0390		
SR power / beam	[MW]	50		
Beam current	[mA]	1283		
Harm. number for 400 MHz		121200		
RF frequency (400 MHz)	MHz	400.787129		
Long. damping time	[turns]	1171		
Beam crossing angle at IP θ_x	[mrad]	± 15		
Crab waist ratio	[%]	50		
RF voltage 400/800 MHz	[GV]	0.079 / 0	0.103 / 0	0.120 / 0
RF acceptance	[%]	1.06	1.41	1.62
Synchrotron tune Q_s		0.0289	0.0340	0.0371
Colliding bunches / beam		11200	11220	
Colliding bunch population	$[10^{11}]$	2.180	2.176	
Hor. emittance at collision ε_x	[nm]	0.70		
Ver. emittance at collision ε_y	[pm]	1.90	2.25	2.40
Lattice ver. emittance $\varepsilon_{y,\mathrm{lattice}}$	[pm]	0.76	1.06	1.09
$\beta^*_{x/y}$	[mm]	110 / 0.7	130 / 0.7	
Transverse tunes $Q_{x/y}$		218.158 / 222.200	218.144 / 222.220	218.158 / 222.220
Chromaticities $Q'_{x/y}$		+5 / +5		
Energy spread (SR/BS) σ_δ	[%]	0.039 / 0.110	0.039 / 0.121	0.039 / 0.123
Bunch length (SR/BS) σ_z	[mm]	5.53 / 15.7	4.70 / 14.6	4.31 / 13.7
Energy acceptance (DA)	[%]	± 1.0		
Beam-beam ξ_x/ξ_y [a]		0.0022 / 0.0985	0.0025 / 0.0981	0.0034 / 0.1008
X-Z threshold param. Q_s/ξ_x		13.1	13.6	10.9
Piwinski angle $(\theta_x \sigma_{z,\mathrm{BS}})/\sigma^*_x$		26.9	25.0	21.4
Lifetime (q + BS + lattice)	[sec]	13000	3100	2600
Lifetime (lum) [b]	[sec]	1320	1320	1320
Luminosity / IP	$[10^{34}/\mathrm{cm}^2\mathrm{s}]$	145.2	145.0	145.1

Design and parameters dominated by choice to allow for 50 MW synchrotron radiation power per beam

Thank you for your time