



Nuclear Matrix Elements for Neutrinoless Double-Beta Decay

Lotta Jokiniemi (she/her)
TRIUMF, Theory Department
TH Heavy Ion Coffee, CERN
8/1/2025



Arthur B. McDonald
Canadian Astroparticle Physics Research Institute



Discovery,
accelerated



D. Araujo Najera, M. Gennari, M. Drissi, P. Navrátil



UNIVERSITAT DE
BARCELONA

D. Castillo, P. Soriano, J. Menéndez



K. Kravvaris



THE UNIVERSITY
of NORTH CAROLINA
at CHAPEL HILL

B. Romeo



JYVÄSKYLÄN YLIOPISTO
UNIVERSITY OF JYVÄSKYLÄ

J. Kotila, J. Suhonen

Introduction

Corrections to $0\nu\beta\beta$ -Decay Nuclear Matrix Elements

Correlations with Other Observables to Constrain the Matrix Elements

Summary and Outlook

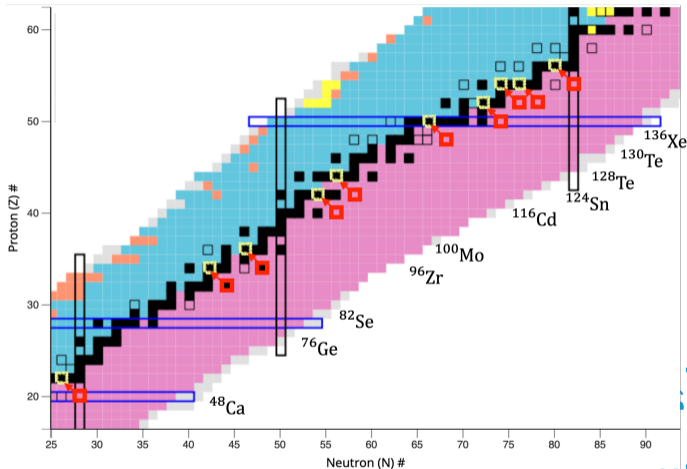
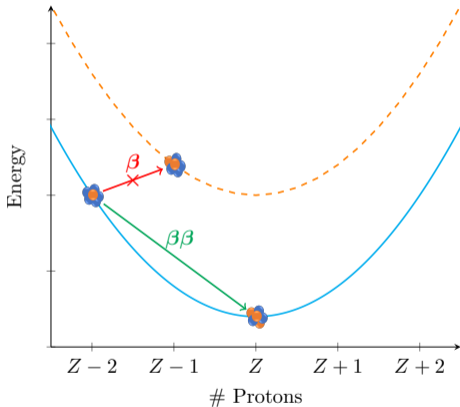
Introduction

Corrections to $0\nu\beta\beta$ -Decay Nuclear Matrix Elements

Correlations with Other Observables to Constrain the Matrix Elements

Summary and Outlook

Double-Beta Decay



nndc.bnl.gov

Neutrinoless Double-Beta ($0\nu\beta\beta$) Decay

- Violates lepton-number conservation

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\nu_e$$

Maria Goeppert-Mayer



$2\nu\beta\beta$

1935

Ettore Majorana



Majorana particles

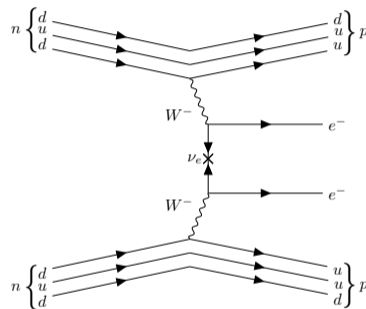
1937

Wendell H. Furry



$0\nu\beta\beta$

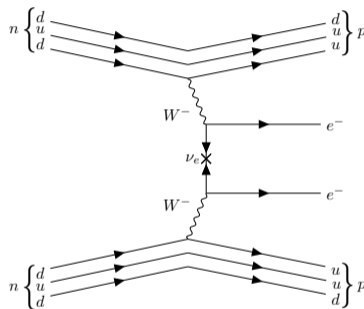
1939



Neutrinoless Double-Beta ($0\nu\beta\beta$) Decay

- Violates lepton-number conservation
- Requires that neutrinos are Majorana particles

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\nu_e$$



Maria Goeppert-Mayer



$2\nu\beta\beta$

1935

Ettore Majorana



Majorana particles

1937

Wendell H. Furry



$0\nu\beta\beta$

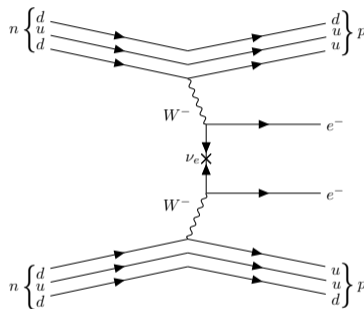
1939

...

Neutrinoless Double-Beta ($0\nu\beta\beta$) Decay

- Violates lepton-number conservation
- Requires that neutrinos are Majorana particles
- If observed, $t_{1/2}^{0\nu} \gtrsim 10^{25}$ years

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\nu_e$$



Maria Goeppert-Mayer



$2\nu\beta\beta$

1935

Ettore Majorana



Majorana particles

1937

Wendell H. Furry



$0\nu\beta\beta$

1939

...

Neutrinoless Double-Beta ($0\nu\beta\beta$) Decay

- Violates lepton-number conservation
- Requires that **neutrinos are Majorana particles**
- If observed, $t_{1/2}^{0\nu} \gtrsim 10^{25}$ years
 ($t_{1/2}^{2\nu} \approx 10^{20}$ years,
 age of the Universe $\approx 10^{10}$ years)

Maria Goeppert-Mayer

Ettore Majorana

Wendell H. Furry



$2\nu\beta\beta$

Majorana particles

$0\nu\beta\beta$

1935



1937

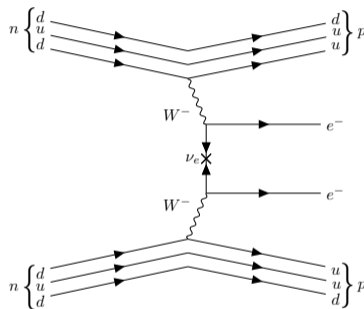


1939



...

$$(A, Z) \rightarrow (A, Z + 2) + 2e^- + 2\nu_e$$



$0\nu\beta\beta$ -Decay Experiments

SNOLAB (Canada):
SNO+ (^{130}Te)

SURF (USA):
MAJORANA (^{76}Ge)
LZ-nat (^{136}Xe)

WIPP (USA):
EXO-200 (^{136}Xe)

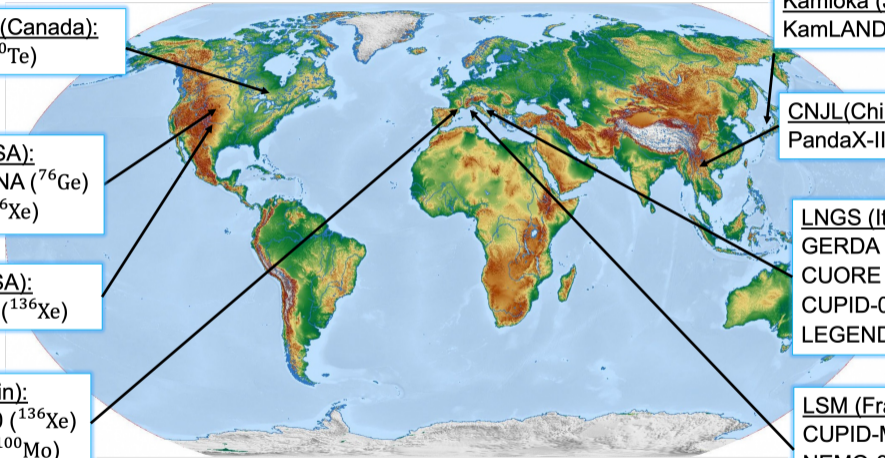
LSC (Spain):
NEXT-100 (^{136}Xe)
CROSS (^{100}Mo)

Kamioka (Japan):
KamLAND-Zen (^{136}Xe)

CNJL(China):
PandaX-III-200 (^{136}Xe)

LNGS (Italy):
GERDA (^{76}Ge)
CUORE (^{130}Te)
CUPID-0 (^{82}Se)
LEGEND-200 (^{76}Ge)

LSM (France):
CUPID-Mo (^{100}Mo)
NEMO-3 (^{100}Mo)
SuperNEMO-D (^{82}Se)



Current record: $t_{1/2}^{0\nu\beta\beta} (^{136}\text{Xe}) > 3.8 \times 10^{26}$ years

KamLAND-Zen, arXiv:2407:11438

accelerated

Next-Generation $0\nu\beta\beta$ -Decay Experiments

SNOLAB (Canada):
SNO+II (^{130}Te)

Kamioka (Japan):
KamLAND2-Zen (^{136}Xe)

Yemilab (Korea):
PandaX-III-200 (^{136}Xe)

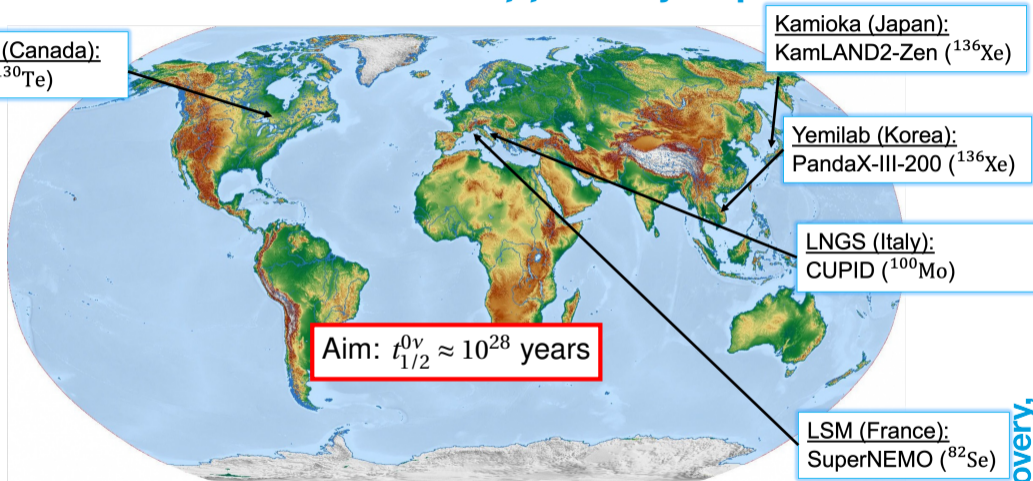
LNGS (Italy):
CUPID (^{100}Mo)

LSM (France):
SuperNEMO (^{82}Se)

+nEXO (^{136}Xe), LEGEND-1000 (^{76}Ge), NEXT-HD (^{136}Xe), Darwin (^{136}Xe), ...

M. Agostini et al., Rev. Mod. Phys. 95, 025002 (2023)

Next-Generation $0\nu\beta\beta$ -Decay Experiments



+nEXO (^{136}Xe), LEGEND-1000 (^{76}Ge), NEXT-HD (^{136}Xe), Darwin (^{136}Xe), ...

M. Agostini et al., Rev. Mod. Phys. 95, 025002 (2023)

*What would be
measured*

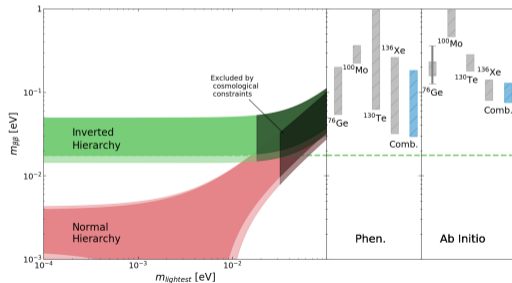
$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G_{0\nu} |M^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

$0\nu\beta\beta$ -Decay Half-Life

What would be measured

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G_{0\nu} |M^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

Majorana mass
 $m_{\beta\beta} = \sum_k (U_{ek})^2 m_k$



T. Shickele, LJ, A. Belley, J. D. Holt, in preparation

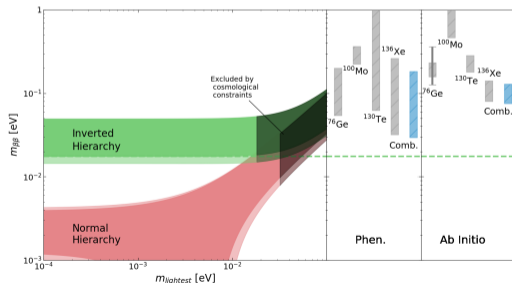
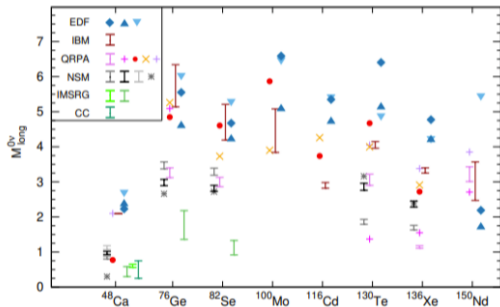
$0\nu\beta\beta$ -Decay Half-Life

What would be measured

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G_{0\nu} |M^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

Majorana mass
 $m_{\beta\beta} = \sum_k (U_{ek})^2 m_k$

Nuclear matrix element



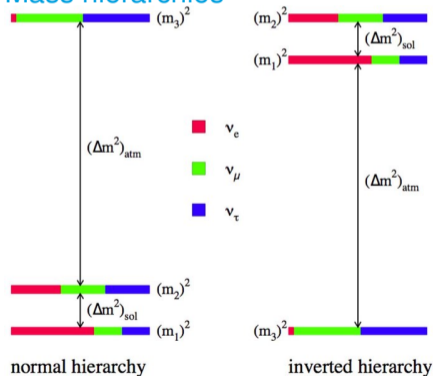
$$m_{\beta\beta} = \sum_k (U_{ek})^2 m_k$$

Neutrino mixing matrix:

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

Flavor eigenstates

Mass hierarchies



$$m_{\beta\beta} = \sum_k (U_{ek})^2 m_k$$

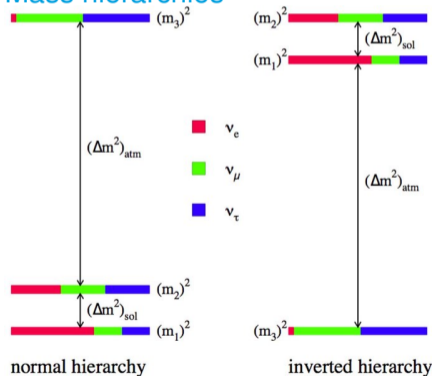
Neutrino mixing matrix:

$$\begin{bmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{bmatrix} = \begin{bmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu1} & U_{\mu2} & U_{\mu3} \\ U_{\tau1} & U_{\tau2} & U_{\tau3} \end{bmatrix} \begin{bmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{bmatrix}$$

Flavor eigenstates

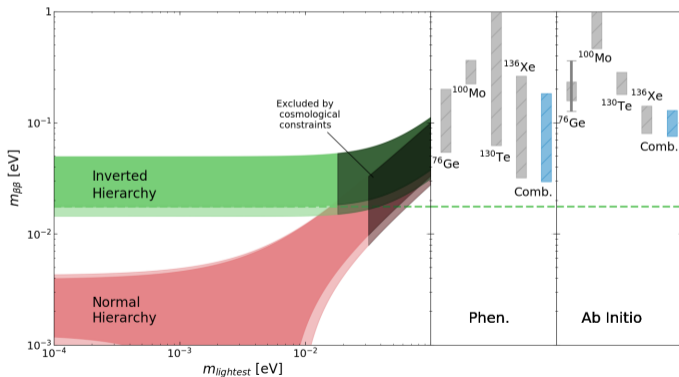
Mass eigenstates

Mass hierarchies



Effective Majorana Mass

$$m_{\beta\beta} = \frac{m_e}{g_A^2 |M^{0\nu}|} \frac{1}{\sqrt{G^{0\nu} t_{1/2}^{0\nu}}}$$



T. Shickele, L.J. A. Belley, J. D. Holt, in preparation

Nuclear Matrix Element

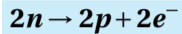
Operator

$2n \rightarrow 2p + 2e^-$

$$M^{0\nu} = \langle \Psi_f^{(A)} || \mathcal{O}^{0\nu\beta\beta} || \Psi_i^{(A)} \rangle$$

Nuclear Matrix Element

Operator



$$M^{0\nu} = \langle \Psi_f^{(A)} || \mathcal{O}^{0\nu\beta\beta} || \Psi_i^{(A)} \rangle$$

Initial state

Nuclear Matrix Element

Operator

$2n \rightarrow 2p + 2e^-$

$$M^{0\nu} = \langle \Psi_f^{(A)} || \mathcal{O}^{0\nu\beta\beta} || \Psi_i^{(A)} \rangle$$

Final state

Initial state

Nuclear Matrix Element

Operator

$2n \rightarrow 2p + 2e^-$

$$M^{0\nu} = \langle \Psi_f^{(A)} || \mathcal{O}^{0\nu\beta\beta} || \Psi_i^{(A)} \rangle$$

Final state

Initial state

$$H\Psi^{(A)} = E\Psi^{(A)}, \quad H = \sum_i \frac{p_i^2}{2m_i} + \sum_{i \neq j} V_{ij}^{2N} + \sum_{i \neq j \neq k} V_{ijk}^{3N}$$

Nuclear Matrix Element

Operator

$$2n \rightarrow 2p + 2e^-$$

$$M^{0\nu} = \langle \Psi_f^{(A)} || \mathcal{O}^{0\nu\beta\beta} || \Psi_i^{(A)} \rangle$$

Final state

Initial state

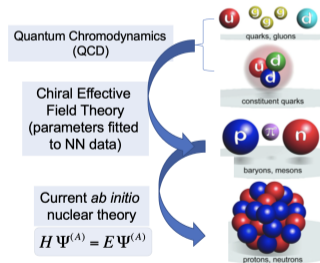
$$H\Psi^{(A)} = E\Psi^{(A)}, \quad H = \sum_i \frac{p_i^2}{2m_i} + \sum_{i \neq j} V_{ij}^{2N} + \sum_{i \neq j \neq k} V_{ijk}^{3N}$$

Schrödinger equation + Nuclear Hamiltonian

Nuclear Many-body Methods

$$H^{(A)}\Psi^{(A)} = E^{(A)}\Psi^{(A)}$$

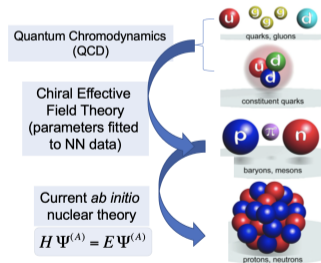
- *Ab initio* methods (IMSRG, NCSM,...)



Nuclear Many-body Methods

$$H^{(A)}\Psi^{(A)} = E^{(A)}\Psi^{(A)}$$

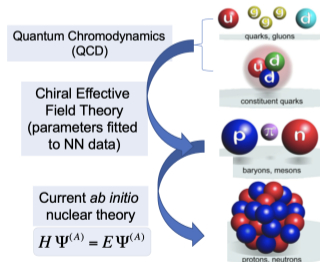
- *Ab initio* methods (IMSRG, NCSM,...)
 - + Aim to solve nuclear Schrödinger equation (SE) starting from interactions between nucleons



Nuclear Many-body Methods

$$H^{(A)}\Psi^{(A)} = E^{(A)}\Psi^{(A)}$$

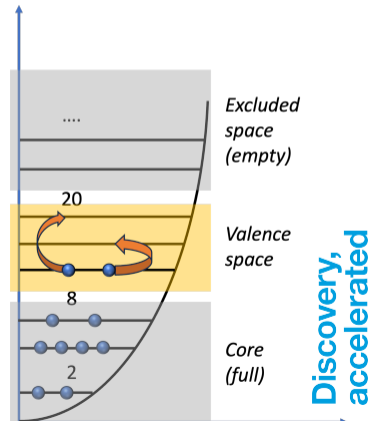
- *Ab initio* methods (IMSRG, NCSM,...)
 - + Aim to solve nuclear Schrödinger equation (SE) starting from interactions between nucleons
 - **VERY complex problem** → **computational limitations**



Nuclear Many-body Methods

$$H^{(A)}\Psi^{(A)} = E^{(A)}\Psi^{(A)}$$

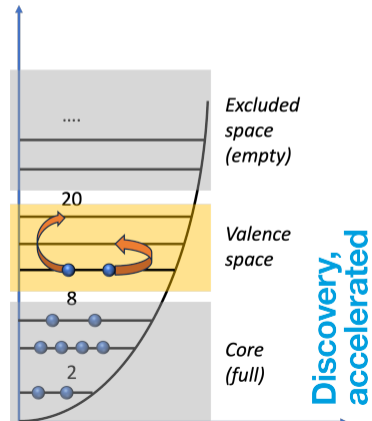
- *Ab initio* methods (IMSRG, NCSM,...)
 - + Aim to solve nuclear Schrödinger equation (SE) starting from interactions between nucleons
 - **VERY complex problem** → **computational limitations**
- Nuclear Shell Model (NSM)



Nuclear Many-body Methods

$$H^{(A)}\Psi^{(A)} = E^{(A)}\Psi^{(A)}$$

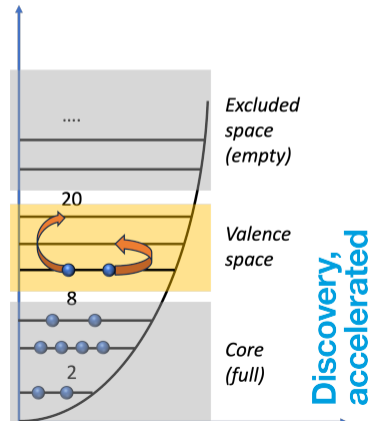
- *Ab initio* methods (IMSRG, NCSM,...)
 - + Aim to solve nuclear Schrödinger equation (SE) starting from interactions between nucleons
 - **VERY complex problem** → **computational limitations**
- Nuclear Shell Model (NSM)
 - ▶ Solves the SE in valence space



Nuclear Many-body Methods

$$H^{(A)}\Psi^{(A)} = E^{(A)}\Psi^{(A)}$$

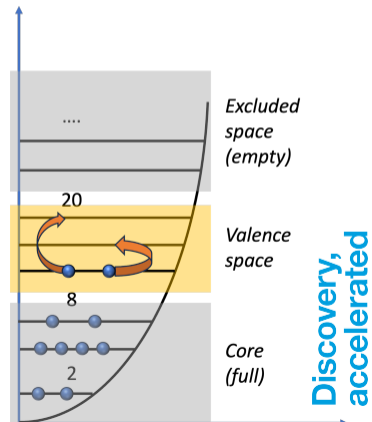
- *Ab initio* methods (IMSRG, NCSM,...)
 - + Aim to solve nuclear Schrödinger equation (SE) starting from interactions between nucleons
 - **VERY complex problem** → **computational limitations**
- Nuclear Shell Model (NSM)
 - ▶ Solves the SE in valence space
 - + **Less complex** → **wider reach**



Nuclear Many-body Methods

$$H^{(A)}\Psi^{(A)} = E^{(A)}\Psi^{(A)}$$

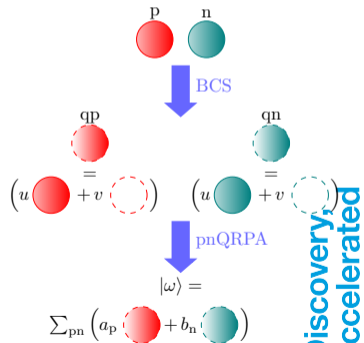
- *Ab initio* methods (IMSRG, NCSM,...)
 - + Aim to solve nuclear Schrödinger equation (SE) starting from interactions between nucleons
 - **VERY complex problem** → **computational limitations**
- Nuclear Shell Model (NSM)
 - ▶ Solves the SE in valence space
 - + **Less complex** → **wider reach**
 - **Effective Hamiltonian relies on experimental data**



Nuclear Many-body Methods

$$H^{(A)}\Psi^{(A)} = E^{(A)}\Psi^{(A)}$$

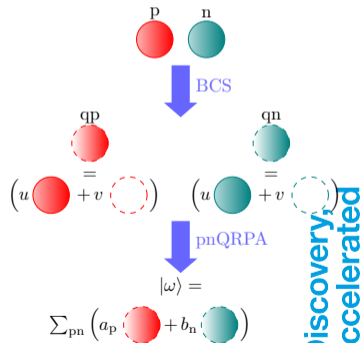
- *Ab initio methods* (IMSRG, NCSM,...)
 - + Aim to solve nuclear Schrödinger equation (SE) starting from interactions between nucleons
 - **VERY complex problem** → computational limitations
- Nuclear Shell Model (NSM)
 - ▶ Solves the SE in valence space
 - + **Less complex** → wider reach
 - **Effective Hamiltonian relies on experimental data**
- Quasiparticle Random-Phase Approximation (QRPA)



Nuclear Many-body Methods

$$H^{(A)}\Psi^{(A)} = E^{(A)}\Psi^{(A)}$$

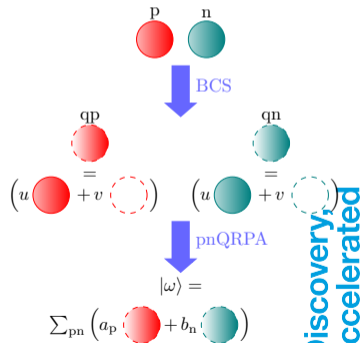
- *Ab initio methods* (IMSRG, NCSM,...)
 - + Aim to solve nuclear Schrödinger equation (SE) starting from interactions between nucleons
 - **VERY complex problem** → **computational limitations**
- Nuclear Shell Model (NSM)
 - ▶ Solves the SE in valence space
 - + **Less complex** → **wider reach**
 - **Effective Hamiltonian relies on experimental data**
- Quasiparticle Random-Phase Approximation (QRPA)
 - ▶ Describes nuclei as two-quasiparticle excitations



Nuclear Many-body Methods

$$H^{(A)}\Psi^{(A)} = E^{(A)}\Psi^{(A)}$$

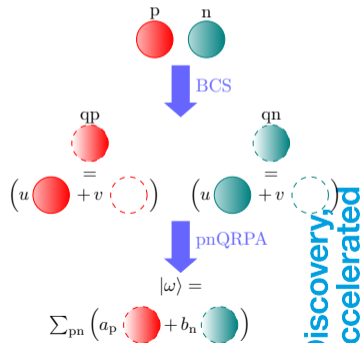
- *Ab initio methods* (IMSRG, NCSM,...)
 - + Aim to solve nuclear Schrödinger equation (SE) starting from interactions between nucleons
 - **VERY complex problem** → **computational limitations**
- Nuclear Shell Model (NSM)
 - ▶ Solves the SE in valence space
 - + **Less complex** → **wider reach**
 - **Effective Hamiltonian relies on experimental data**
- Quasiparticle Random-Phase Approximation (QRPA)
 - ▶ Describes nuclei as two-quasiparticle excitations
 - + **Large model spaces, wide reach**



Nuclear Many-body Methods

$$H^{(A)}\Psi^{(A)} = E^{(A)}\Psi^{(A)}$$

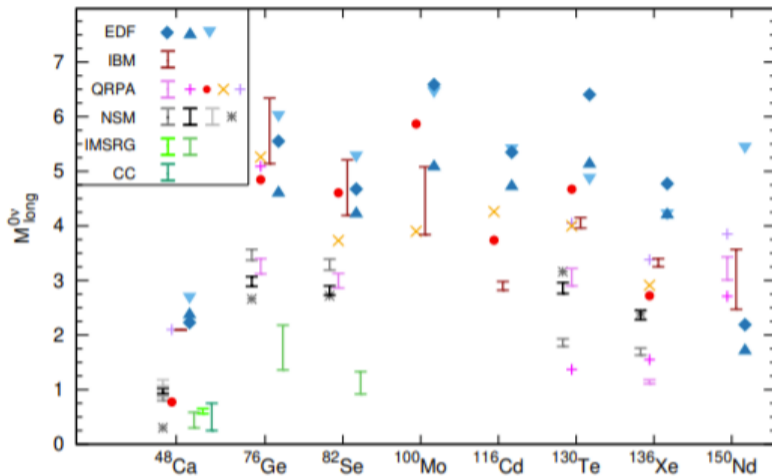
- *Ab initio methods* (IMSRG, NCSM,...)
 - + Aim to solve nuclear Schrödinger equation (SE) starting from interactions between nucleons
 - **VERY complex problem** → computational limitations
- Nuclear Shell Model (NSM)
 - ▶ Solves the SE in valence space
 - + **Less complex** → wider reach
 - **Effective Hamiltonian relies on experimental data**
- Quasiparticle Random-Phase Approximation (QRPA)
 - ▶ Describes nuclei as two-quasiparticle excitations
 - + **Large model spaces, wide reach**
 - **Missing correlations, adjustable parameters,...**



$$H^{(A)}\Psi^{(A)} = E^{(A)}\Psi^{(A)}$$

- *Ab initio methods* (IMSRG, NCSM,...)
 - + Aim to solve nuclear Schrödinger equation (SE) starting from interactions between nucleons
 - **VERY complex problem** → **computational limitations**
- Nuclear Shell Model (NSM)
 - ▶ Solves the SE in valence space
 - + **Less complex** → **wider reach**
 - **Effective Hamiltonian relies on experimental data**
- Quasiparticle Random-Phase Approximation (QRPA)
 - ▶ Describes nuclei as two-quasiparticle excitations
 - + **Large model spaces, wide reach**
 - **Missing correlations, adjustable parameters,...**
- ...

Different Many-body Methods Disagree



M. Agostini et al., Rev. Mod. Phys. 95, 025002 (2023)

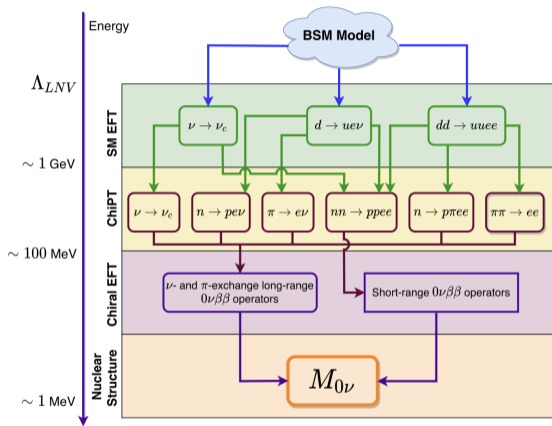
Introduction

Corrections to $0\nu\beta\beta$ -Decay Nuclear Matrix Elements

Correlations with Other Observables to Constrain the Matrix Elements

Summary and Outlook

Effective Field Theory For $0\nu\beta\beta$ Decay

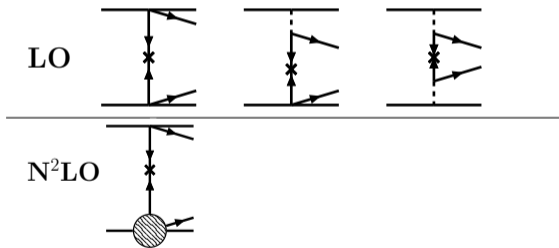


V. Cirigliano et al., *J. Phys. G: Nucl. Part. Phys.* 49, 120502 (2022)

Effective-Field-Theory Corrections to $0\nu\beta\beta$ Decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} |M_L^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e} \right)^2$$

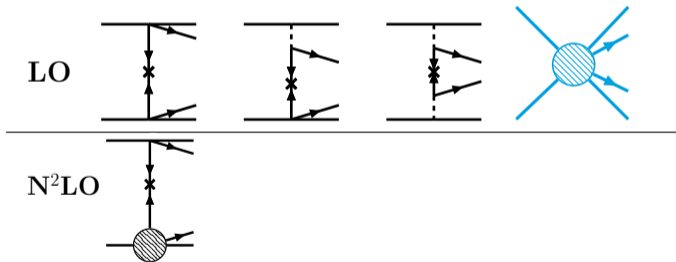
V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018), Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)



Effective-Field-Theory Corrections to $0\nu\beta\beta$ Decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} |M_L^{0\nu} + M_S^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

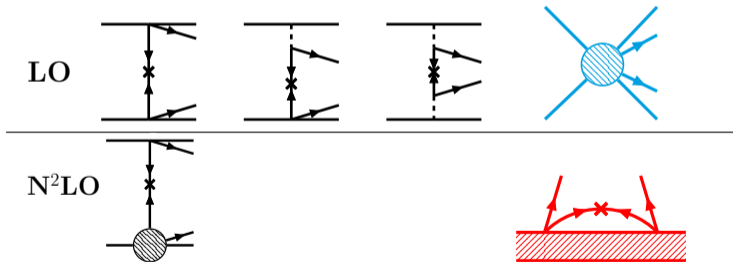
V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018), Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)



Effective-Field-Theory Corrections to $0\nu\beta\beta$ Decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} |M_L^{0\nu} + M_S^{0\nu} + M_{\text{usoft}}^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

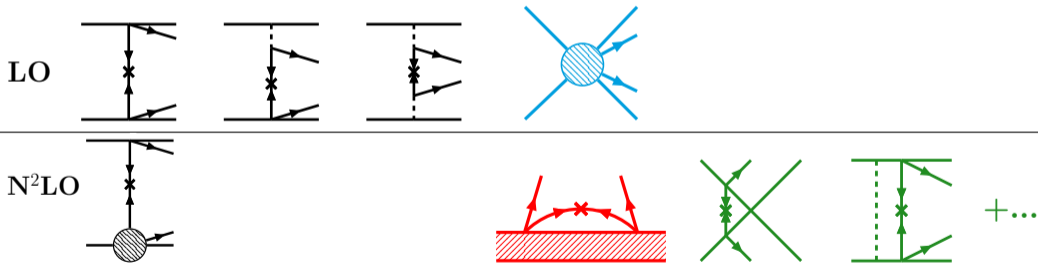
V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018), Phys. Rev. Lett. 120, 202001 (2018), Phys. Rev. C 100, 055504 (2019)



Effective-Field-Theory Corrections to $0\nu\beta\beta$ Decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} |M_L^{0\nu} + M_S^{0\nu} + M_{\text{usoft}}^{0\nu} + M_{N^2\text{LO}}^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

V. Cirigliano et al., *Phys. Rev. C* 97, 065501 (2018), *Phys. Rev. Lett.* 120, 202001 (2018), *Phys. Rev. C* 100, 055504 (2019)



Traditional $0\nu\beta\beta$ -Decay Operators

$$M^{0\nu} = \frac{4\pi R}{g_A^2} \int \mathbf{d}\mathbf{x} \int \mathbf{d}\mathbf{y} \int \frac{\mathbf{d}\mathbf{k}}{(2\pi)^3} \frac{e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}}{|\mathbf{k}|} \sum_n \frac{\langle f | J_\mu(\mathbf{x}) | n \rangle \langle n | J^\mu(\mathbf{y}) | i \rangle}{|\mathbf{k}| + E_n - \frac{1}{2}(E_i + E_f)}$$

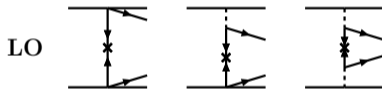
Traditional $0\nu\beta\beta$ -Decay Operators

$$M^{0\nu} = \frac{4\pi R}{g_A^2} \int d\mathbf{x} \int d\mathbf{y} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}}{|\mathbf{k}|} \sum_n \frac{\langle f | J_\mu(\mathbf{x}) | n \rangle \langle n | J^\mu(\mathbf{y}) | i \rangle}{|\mathbf{k}| + E_n - \frac{1}{2}(E_i + E_f)}$$

- Traditionally, the nuclear current includes the leading-order (LO) transition operators

$$\mathcal{J}^0 = \tau [g_V(0)]$$

$$\mathbf{J} = \tau [g_A(0)\boldsymbol{\sigma} - g_P(0)\mathbf{p}(\mathbf{p}\cdot\boldsymbol{\sigma})]$$



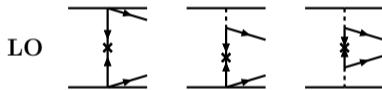
Traditional $0\nu\beta\beta$ -Decay Operators

$$M^{0\nu} = \frac{4\pi R}{g_A^2} \int d\mathbf{x} \int d\mathbf{y} \int \frac{d\mathbf{k}}{(2\pi)^3} \frac{e^{i\mathbf{k}\cdot(\mathbf{x}-\mathbf{y})}}{|\mathbf{k}|} \sum_n \frac{\langle f | J_\mu(\mathbf{x}) | n \rangle \langle n | J^\mu(\mathbf{y}) | i \rangle}{|\mathbf{k}| + E_n - \frac{1}{2}(E_i + E_f)}$$

- Traditionally, the nuclear current includes the leading-order (LO) transition operators

$$\mathcal{L}^0 = \tau [g_V(0)]$$

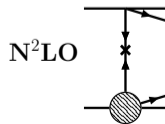
$$\mathbf{J} = \tau [g_A(0)\boldsymbol{\sigma} - g_P(0)\mathbf{p}(\mathbf{p}\cdot\boldsymbol{\sigma})]$$



- and next-to-next-to-leading-order (N²LO) corrections absorbed into **form factors** and **induced weak-magnetism terms**

$$\mathcal{L}^0 = \tau [g_V(\mathbf{p}^2)]$$

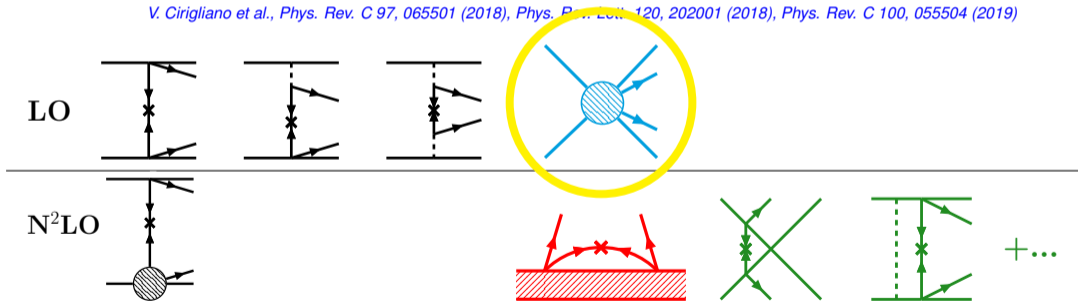
$$\mathbf{J} = \tau \left[g_A(\mathbf{p}^2)\boldsymbol{\sigma} - g_P(\mathbf{p}^2)\mathbf{p}(\mathbf{p}\cdot\boldsymbol{\sigma}) + ig_M(\mathbf{p}^2) \frac{\boldsymbol{\sigma} \times \mathbf{p}}{2m_N} \right]$$



Leading-order short-range contribution to $0\nu\beta\beta$ decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} |M_L^{0\nu} + M_S^{0\nu} + M_{\text{usoft}}^{0\nu} + M_{N^2\text{LO}}^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

V. Cirigliano et al., *Phys. Rev. C* 97, 065501 (2018), *Phys. Rev. Lett.* 120, 202001 (2018), *Phys. Rev. C* 100, 055504 (2019)



Contact Term in pnQRPA and NSM

- The contact term reads

$$M_S^{0\nu} = \frac{2R}{\pi g_A^2} \langle 0_f^+ | \sum_{m,n} \tau_m^- \tau_n^- \int j_0(qr) h_S(q^2) q^2 dq | 0_i^+ \rangle$$

with

$$h_S(q^2) = 2g_v^{NN} e^{-q^2/(2\Lambda^2)}.$$

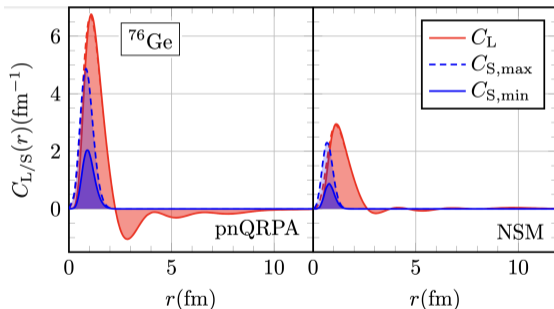
In pnQRPA:

$$M_S/M_L \approx 30\% - 80\%$$

In NSM:

$$M_S/M_L \approx 15\% - 50\%$$

$$\int C_{L/S}(r) dr = M_{L/S}^{0\nu}$$

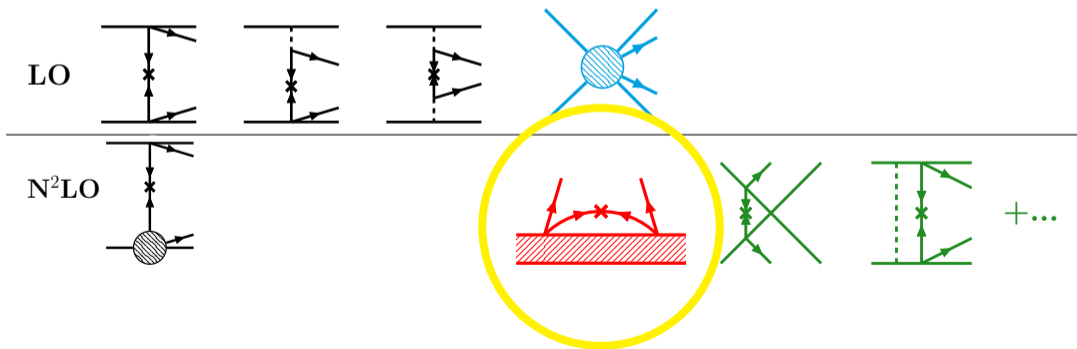


LJ, P. Soriano and J. Menéndez, Phys. Lett. B 823, 136720 (2021)

Ultrasoft-neutrino contribution to $0\nu\beta\beta$ decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} |M_L^{0\nu} + M_S^{0\nu} + M_{\text{usoft}}^{0\nu} + M_{N^2\text{LO}}^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

V. Cirigliano et al., *Phys. Rev. C* 97, 065501 (2018), *Phys. Rev. Lett.* 120, 202001 (2018), *Phys. Rev. C* 100, 055504 (2019)



Ultrasoft Neutrinos in pnQRPA and NSM

- Contribution of ultrasoft neutrinos ($|\mathbf{k}| \ll k_F \approx 100 \text{ MeV}$) to $0\nu\beta\beta$ decay:

V. Cirigliano et al., Phys. Rev. C 97, 065501 (2018)

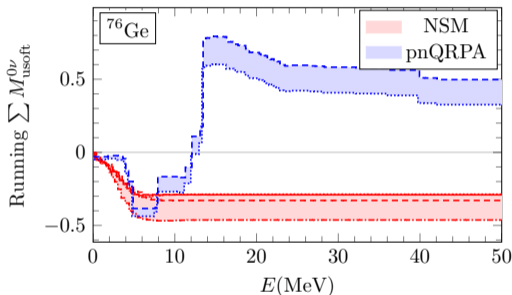
$$M_{\text{usoft}}^{0\nu} = -\frac{2R}{\pi} \sum_n \langle f || \sum_a \sigma_a \tau_a^+ || n \rangle \langle n || \sum_b \sigma_b \tau_b^+ || i \rangle \times (E_e + E_n - E_i) \left(\ln \frac{\mu_{\text{us}}}{2(E_e + E_n - E_i)} + 1 \right)$$

In pnQRPA:

$$|M_{\text{usoft}}^{0\nu} / M_L^{0\nu}| \leq 30\%$$

In NSM:

$$|M_{\text{usoft}}^{0\nu} / M_L^{0\nu}| \leq 10\%$$

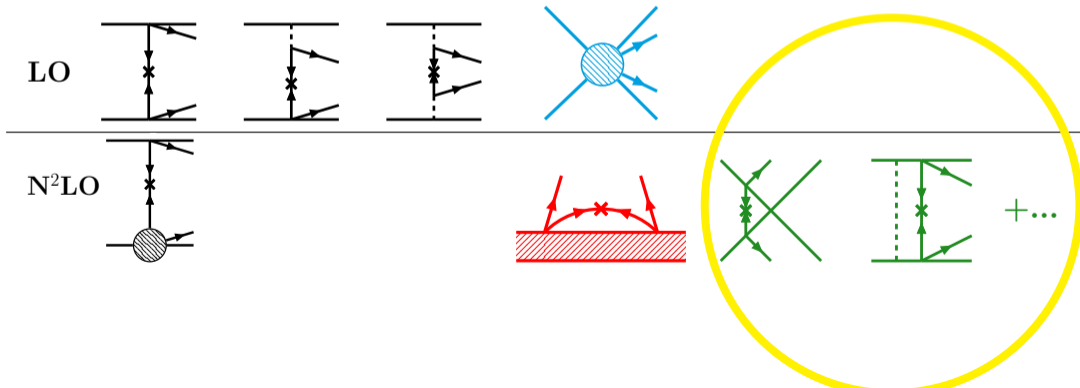


D. Castillo, L.J., P. Soriano, J. Menéndez, Phys. Lett. B 860, 139181 (2021)

N²LO Loop Corrections to $0\nu\beta\beta$ Decay

$$\frac{1}{t_{1/2}^{0\nu}} = g_A^4 G^{0\nu} |M_L^{0\nu} + M_S^{0\nu} + M_{\text{usoft}}^{0\nu} + M_{\text{N}^2\text{LO}}^{0\nu}|^2 \left(\frac{m_{\beta\beta}}{m_e}\right)^2$$

V. Cirigliano et al., *Phys. Rev. C* 97, 065501 (2018), *Phys. Rev. Lett.* 120, 202001 (2018), *Phys. Rev. C* 100, 055504 (2019)



- The N²LO loop corrections read as

$$M_{\text{loops}}^{0\nu} = \frac{4R}{\pi g_A^2} \langle 0_f^+ | \sum_{a,b} \tau_a^- \tau_b^- \int e^{-\frac{q^2}{2\Lambda^2}} j_u(qr) V_{\nu,2}^{(a,b)} q^2 dq | 0_i^+ \rangle$$

$$\int C_{\text{N}^2\text{LO}}^{0\nu}(r) dr = M_{\text{loops}}^{0\nu}$$

with

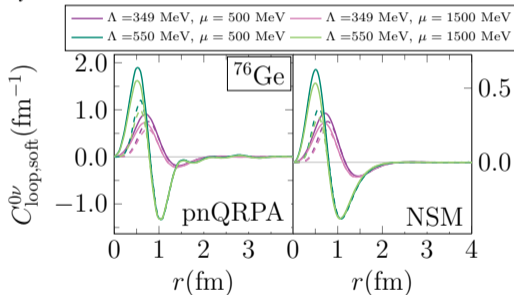
$$V_{\nu,2}^{(a,b)} = V_{\text{VV}}^{(a,b)} + V_{\text{AA}}^{(a,b)} + \ln \frac{m_\pi^2}{\mu_{\text{us}}^2} V_{\text{us}}^{(a,b)} + V_{\text{CT}}^{(a,b)}$$

In pnQRPA:

$$|M_{\text{N}^2\text{LO}}/M_{\text{L}}| \approx 2\% - 10\%$$

In NSM:

$$|M_{\text{N}^2\text{LO}}/M_{\text{L}}| \approx 4\% - 10\%$$



D. Castillo, L.J. P. Soriano, J. Menéndez, Phys. Lett. B 860, 139181 (2025)

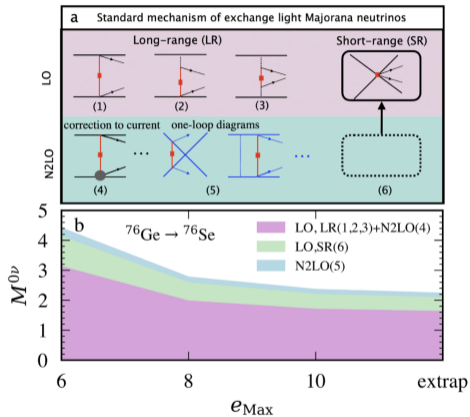
Similar effects found in *ab initio* studies

- In ^{76}Ge :

$$M_S^{0\nu} / M_L^{0\nu} \sim 40\%,$$

$$M_{\text{loop}}^{0\nu} / M_L^{0\nu} \sim 5\% \text{ }^a$$

A. Belley et al. arXiv:2308.15634 (2023)



^aI found some errors in the expressions

A. Belley et al. arXiv:2308.15634 (2023)

Similar effects found in *ab initio* studies

- In ^{76}Ge :

$$M_S^{0\nu} / M_L^{0\nu} \sim 40\%,$$

$$M_{\text{loop}}^{0\nu} / M_L^{0\nu} \sim 5\% \text{ }^a$$

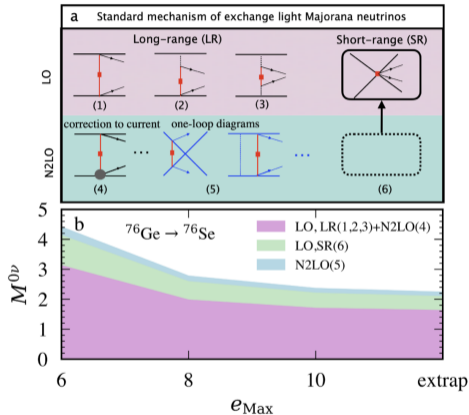
A. Belley et al. arXiv:2308.15634 (2023)

- In ^{130}Te and ^{136}Xe :

$$M_S^{0\nu} / M_L^{0\nu} \sim 20\% - 120\%$$

A. Belley et al. arXiv:2307.15156 (2023)

^aI found some errors in the expressions



A. Belley et al. arXiv:2308.15634 (2023)

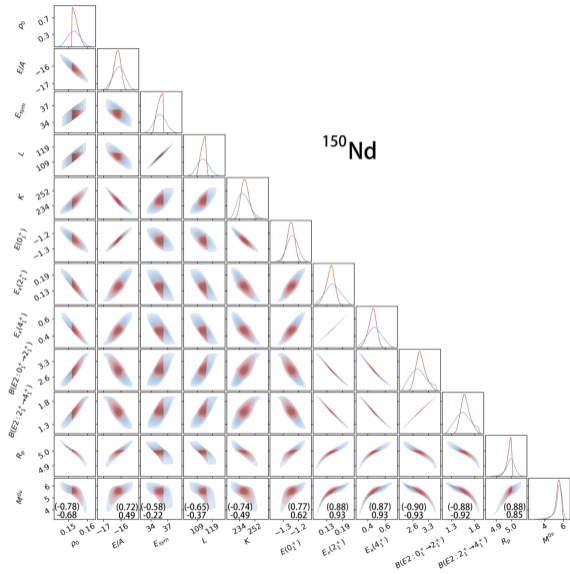
Introduction

Corrections to $0\nu\beta\beta$ -Decay Nuclear Matrix Elements

Correlations with Other Observables to Constrain the Matrix Elements

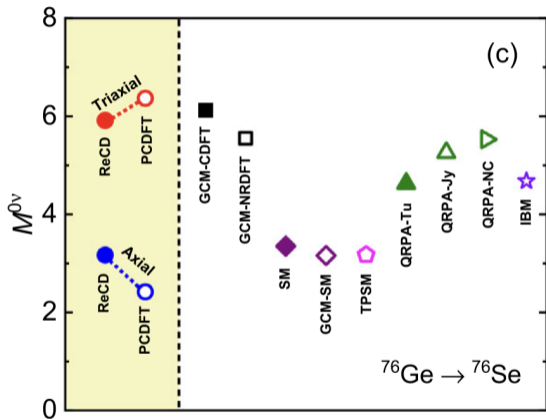
Summary and Outlook

Correlations with Structure Observables



X. Zhang, C. C. Wang, C. R. Ding, and J. M. Yao, arXiv:2408:13209[nucl-th]

Effect of Triaxial Deformation on $M^{0\nu}$ of ^{76}Ge



Y. Wang et al., Science Bulletin 69, 2017–2020 (2024)

$0\nu\beta\beta$ Decay vs Double-Charge-Exchange Reactions

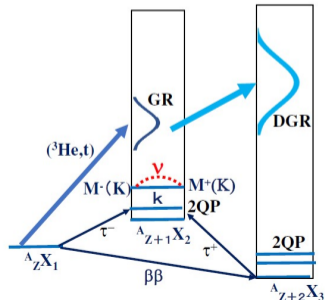
$$M^{0\nu} = M_{\text{GT}}^{0\nu} - \left(\frac{g_V}{g_A}\right)^2 M_{\text{F}}^{0\nu} + M_{\text{T}}^{0\nu} + M_{\text{S}}^{0\nu} + M_{\text{N}^2\text{LO}}^{0\nu}$$

Leading contribution

$$M_{\text{GT}}^{0\nu} = \langle f || \sum_{jk} \tau_j^- \tau_k^- \sigma_j^- \sigma_k^- V_{\text{GT}}(r_{jk}) || i \rangle$$

- Double-Gamow-Teller (DGT) strength function

$$B(\text{DGT}; \lambda) = \frac{1}{2J_i + 1} |\langle f || [\sum_{jk} \sigma_j \tau_j^- \times \sigma_k \tau_k^-]^{(\lambda)} || i \rangle|^2$$



$0\nu\beta\beta$ Decay vs Double-Charge-Exchange Reactions

$$M^{0\nu} = M_{\text{GT}}^{0\nu} - \left(\frac{g_V}{g_A}\right)^2 M_{\text{F}}^{0\nu} + M_{\text{T}}^{0\nu} + M_{\text{S}}^{0\nu} + M_{\text{N}^2\text{LO}}^{0\nu}$$

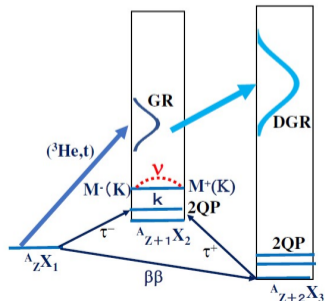
Leading contribution

$$M_{\text{GT}}^{0\nu} = \langle f || \sum_{jk} \tau_j^- \tau_k^- \sigma_j^- \sigma_k^- V_{\text{GT}}(r_{jk}) || i \rangle$$

- Double-Gamow-Teller (DGT) strength function

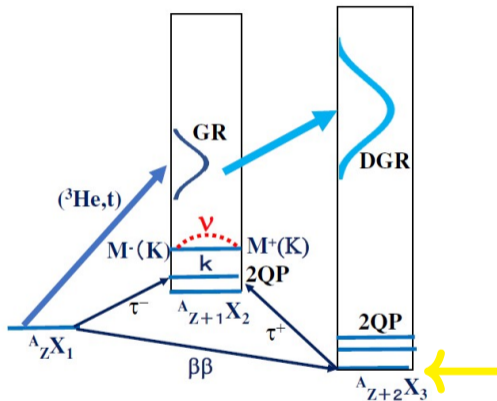
$$B(\text{DGT}; \lambda) = \frac{1}{2J_i + 1} |\langle f || [\sum_{jk} \sigma_j \tau_j^- \times \sigma_k \tau_k^-]^{(\lambda)} || i \rangle|^2$$

- Could we probe $0\nu\beta\beta$ decay by DGT reactions?



Correlations Between DGT and $0\nu\beta\beta$ Decay

$$M_{\text{DGT}} = -\langle 0_{\text{gs},f}^+ | [\sum_{jk} \sigma_j \tau_j^- \times \sigma_k \tau_k^-]^{(0)} | 0_{\text{gs},i}^+ \rangle$$

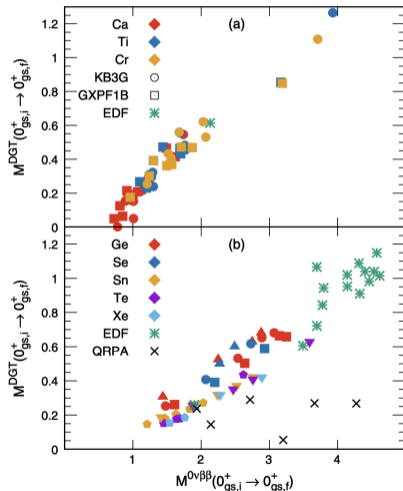


H. Ejiri, L.J. J. Suhonen, Phys. Rev. C 105, L022501 (2022)

Correlations Between DGT and $0\nu\beta\beta$ Decay

$$M_{\text{DGT}} = -\langle 0_{\text{gs},f}^+ || [\sum_{jk} \sigma_j \tau_j^- \times \sigma_k \tau_k^-]^{(0)} || 0_{\text{gs},i}^+ \rangle$$

- Correlation between $M^{0\nu}$ and M_{DGT} found in **nuclear shell model** and **EFT**

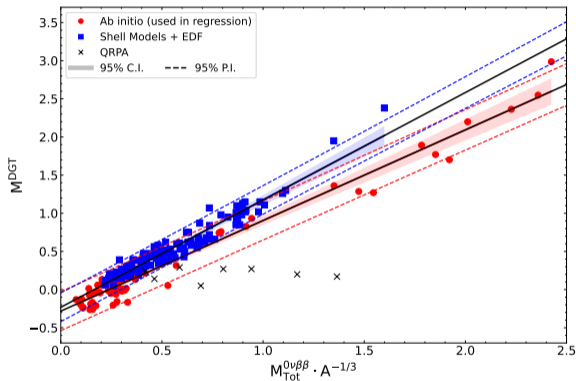


N. Shimizu, J. Menéndez, K. Yako, *Phys. Rev. Lett.* 120, 142502 (2018)

Correlations Between DGT and $0\nu\beta\beta$ Decay

$$M_{\text{DGT}} = -\langle 0_{\text{gs},f}^+ | [\sum_{jk} \sigma_j \tau_j^- \times \sigma_k \tau_k^-]^{(0)} | 0_{\text{gs},i}^+ \rangle$$

- Correlation between $M^{0\nu}$ and M_{DGT} found in **nuclear shell model** and **EFT**
- Correlation also holds in *ab initio* **VS-IMSRG**

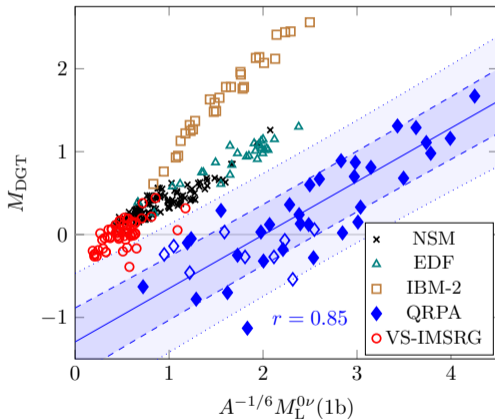


J. M. Yao, I. Ginnett, A. Belley et al., Phys. Rev. C 106, 014315 (2022)

Correlations Between DGT and $0\nu\beta\beta$ Decay

$$M_{\text{DGT}} = -\langle 0_{\text{gs},f}^+ | [\sum_{jk} \sigma_j \tau_j^- \times \sigma_k \tau_k^-]^{(0)} | 0_{\text{gs},i}^+ \rangle$$

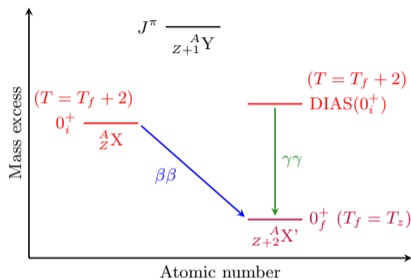
- Correlation between $M^{0\nu}$ and M_{DGT} found in **nuclear shell model** and **EFT**
- Correlation also holds in *ab initio* **VS-IMSRG**
- ...and **QRPA**, when proton-neutron pairing varied
 - ▶ **Observation of M_{DGT} → constraints for $M^{0\nu}$**



LJ, J. Menéndez, *Phys. Rev. C* 107, 044316 (2023)

Probing $0\nu\beta\beta$ Decay by Gamma Decays

- Double magnetic dipole (M1) decay (**electromagnetic interaction**) can be related to $0\nu\beta\beta$ decay (**weak interaction**)



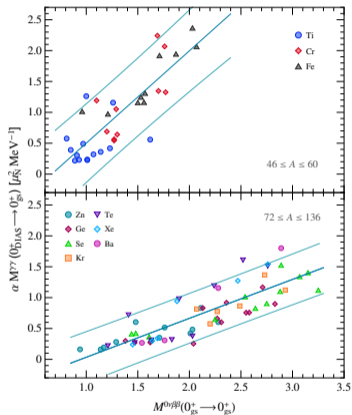
$$M^{\gamma\gamma}(M1M1) = \sum_n \frac{(0_f^+ || \mathbf{M}_1 || 1_n^+)(1_n^+ || \mathbf{M}_1 || 0_i^+)}{E_n - (E_i + E_f)/2},$$

$$\mathbf{M}_1 = \mu_N \sqrt{\frac{3}{4\pi}} \sum_{i=1}^A (g_i^l \boldsymbol{\ell}_i + g_i^s \mathbf{s}_i)$$

Probing $0\nu\beta\beta$ Decay by Gamma Decays

- Double magnetic dipole (M1) decay (**electromagnetic interaction**) can be related to $0\nu\beta\beta$ decay (**weak interaction**)
- Correlation between these processes observed in NSM

B. Romeo, J. Menéndez, C. Peña-Garay, Phys. Lett. B **827**, 136965 (2022)



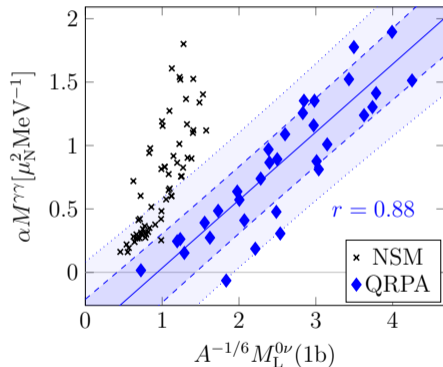
B. Romeo, J. Menéndez, C. Peña-Garay, Phys. Lett. B **827**, 136965 (2022)

Probing $0\nu\beta\beta$ Decay by Gamma Decays

- Double magnetic dipole (M1) decay (**electromagnetic interaction**) can be related to $0\nu\beta\beta$ decay (**weak interaction**)
- Correlation between these processes observed in NSM

B. Romeo, J. Menéndez, C. Peña-Garay, Phys. Lett. B **827**, 136965 (2022)

- Correlation also found in QRPA



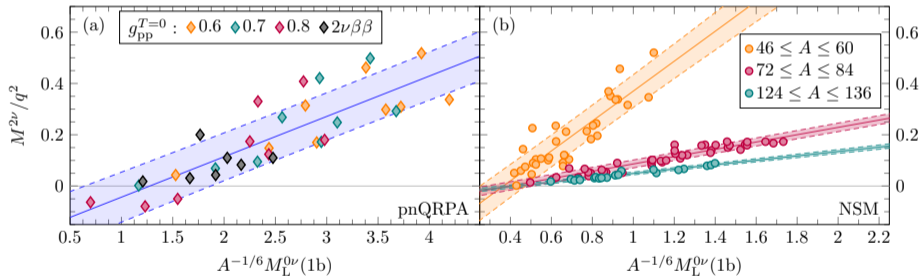
LJ, J. Menéndez, Phys. Rev. C **107**, 044316 (2023)

Probing $0\nu\beta\beta$ Decay by $2\nu\beta\beta$ Decay

- *How about $2\nu\beta\beta$ decay?*

Probing $0\nu\beta\beta$ Decay by $2\nu\beta\beta$ Decay

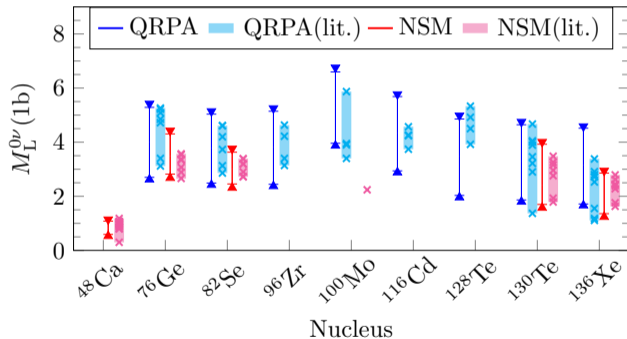
- *How about $2\nu\beta\beta$ decay?*
- $2\nu\beta\beta$ -decay also correlated with $0\nu\beta\beta$ -decay!



LJ, B. Romeo, P. Soriano and J. Menéndez, *Phys. Rev. C* **107**, 044305 (2023)

Probing $0\nu\beta\beta$ Decay by $2\nu\beta\beta$ Decay

- **How about $2\nu\beta\beta$ decay?**
- $2\nu\beta\beta$ -decay also correlated with $0\nu\beta\beta$ -decay!
- We can use the existing data to estimate $0\nu\beta\beta$ -decay NMEs!



LJ, B. Romeo, P. Soriano and J. Menéndez, *Phys. Rev. C* **107**, 044305 (2023)

Introduction

Corrections to $0\nu\beta\beta$ -Decay Nuclear Matrix Elements

Correlations with Other Observables to Constrain the Matrix Elements

Summary and Outlook

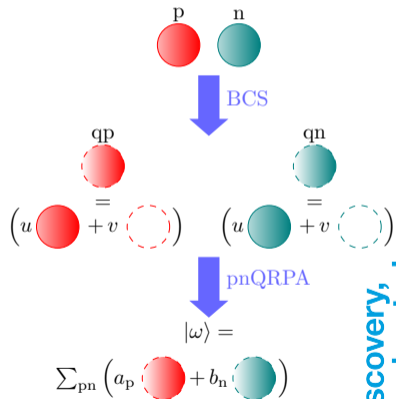
- The nuclear matrix elements of $0\nu\beta\beta$ decay are sensitive to nuclear structure
- χ EFT corrections to $0\nu\beta\beta$ -decay seem to respect the power counting, but N²LO corrections still significant
- Correlation between $0\nu\beta\beta$ and $2\nu\beta\beta$ decays helped us predict $0\nu\beta\beta$ -decay NMEs with uncertainties
- Correlations with DGT and M1M1 transitions with future data can help us further constrain the NMEs

Thank you
Merci



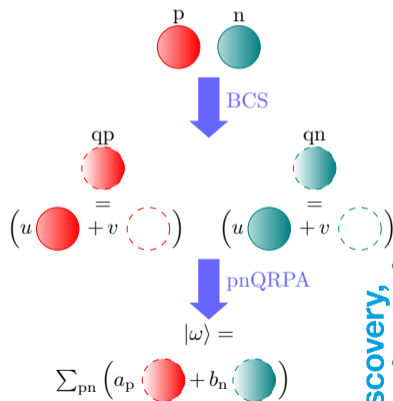
Spherical proton-neutron quasiparticle random-phase approximation (pnQRPA)

- Single-particle bases from Woods-Saxon potential



Spherical proton-neutron quasiparticle random-phase approximation (pnQRPA)

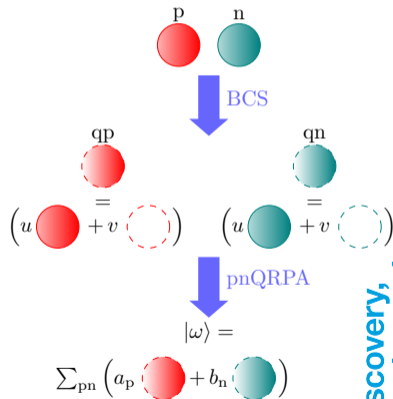
- Single-particle bases from Woods-Saxon potential
- Quasiparticle bases from BCS equations with Bonn-A two-body G -matrix



Spherical proton-neutron quasiparticle random-phase approximation (pnQRPA)

- Single-particle bases from Woods-Saxon potential
- Quasiparticle bases from BCS equations with Bonn-A two-body G -matrix
- Intermediate states \equiv two-quasiparticle excitations

$$|J_k^\pi\rangle = \sum_{pn} \left(X_{pn}^{J_k^\pi} [a_p^\dagger a_n^\dagger]_J - Y_{pn}^{J_k^\pi} [a_p^\dagger a_n^\dagger]^\dagger \right) |\text{QRPA}\rangle$$



Spherical proton-neutron quasiparticle random-phase approximation (pnQRPA)

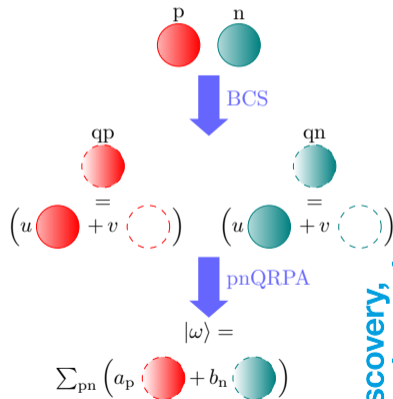
- Single-particle bases from Woods-Saxon potential
- Quasiparticle bases from BCS equations with Bonn-A two-body G -matrix
- Intermediate states \equiv two-quasiparticle excitations

$$|J_k^\pi\rangle = \sum_{pn} \left(X_{pn}^{J_k^\pi} [a_p^\dagger a_n^\dagger]_J - Y_{pn}^{J_k^\pi} [a_p^\dagger a_n^\dagger]^\dagger \right) |\text{QRPA}\rangle$$

- Adjustable parameters:

$$g_{\text{ph}} \langle p' n'^{-1}, J | V | p n^{-1}, J \rangle$$

$$g_{\text{pp}} \langle p' n', J | V | p n, J \rangle$$

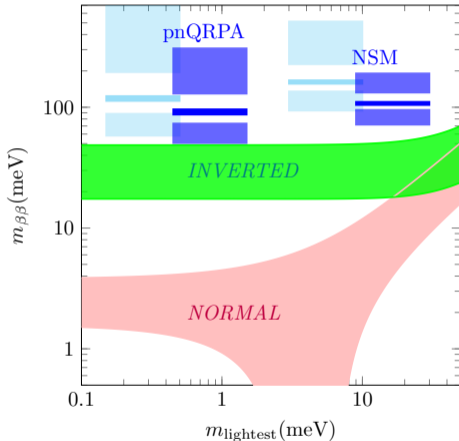


Effective Neutrino Masses

- Effective neutrino masses combining the likelihood functions of GERDA (^{76}Ge), CUORE (^{130}Te), EXO-200 (^{136}Xe) and KamLAND-Zen (^{136}Xe)

S. D. Biller, Phys. Rev. D **104**, 012002 (2021)

- Middle bands: $M_L^{(0\nu)}$
 Lower bands: $M_L^{(0\nu)} + M_S^{(0\nu)}$
 Upper bands: $M_L^{(0\nu)} - M_S^{(0\nu)}$



LJ, P. Soriano and J. Menéndez, Phys. Lett. B **823**, 136720 (2021)

Traditional nuclear matrix elements of neutrinoless double-beta decay

$$M^{0\nu} = \frac{R}{g_A^2} \int \frac{d\mathbf{k}}{2\pi^2} \frac{e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})}}{E_\nu} \sum_n \frac{\langle f | J_\mu(\mathbf{x}) | n \rangle \langle n | J^\mu(\mathbf{y}) | i \rangle}{E_\nu + E_n - \frac{1}{2}(E_i + E_f) - \frac{1}{2}(E_1 - E_2)}$$

- Energy of the virtual neutrino typically $E_\nu = \sqrt{m_\nu^2 + \mathbf{k}^2} \sim |\mathbf{k}| \sim k_F \sim 100 \text{ MeV}$ ("soft neutrinos")

Traditional nuclear matrix elements of neutrinoless double-beta decay

$$M^{0\nu} = \frac{R}{g_A^2} \int \frac{d\mathbf{k}}{2\pi^2} \frac{e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})}}{E_\nu} \sum_n \frac{\langle f | J_\mu(\mathbf{x}) | n \rangle \langle n | J^\mu(\mathbf{y}) | i \rangle}{E_\nu + E_n - \frac{1}{2}(E_i + E_f) - \frac{1}{2}(E_1 - E_2)}$$

- Energy of the virtual neutrino typically $E_\nu = \sqrt{m_\nu^2 + \mathbf{k}^2} \sim |\mathbf{k}| \sim k_F \sim 100 \text{ MeV}$ ("soft neutrinos")
- Electrons carry away roughly the same amount of energy: $E_1 - E_2 \sim 0 \text{ MeV}$

Traditional nuclear matrix elements of neutrinoless double-beta decay

$$M^{0\nu} = \frac{R}{g_A^2} \int \frac{d\mathbf{k}}{2\pi^2} \frac{e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})}}{E_\nu} \sum_n \frac{\langle f | J_\mu(\mathbf{x}) | n \rangle \langle n | J^\mu(\mathbf{y}) | i \rangle}{E_\nu + E_n - \frac{1}{2}(E_i + E_f) - \frac{1}{2}(E_1 - E_2)}$$

- Energy of the virtual neutrino typically $E_\nu = \sqrt{m_\nu^2 + \mathbf{k}^2} \sim |\mathbf{k}| \sim k_F \sim 100 \text{ MeV}$ ("soft neutrinos")
- Electrons carry away roughly the same amount of energy: $E_1 - E_2 \sim 0 \text{ MeV}$

$$\rightarrow M^{0\nu} = \frac{R}{g_A^2} \int \frac{d\mathbf{k}}{2\pi^2} \frac{e^{i\mathbf{k}(\mathbf{x}-\mathbf{y})}}{|\mathbf{k}|} \sum_n \frac{\langle f | J_\mu(\mathbf{x}) | n \rangle \langle n | J^\mu(\mathbf{y}) | i \rangle}{|\mathbf{k}| + E_n - \frac{1}{2}(E_i + E_f)}$$

Closure approximation

Without closure approximation:

$$M^{0\nu} \propto \sum_n \frac{\langle f | J_\mu(\mathbf{x}) | n \rangle \langle n | J^\mu(\mathbf{y}) | i \rangle}{|\mathbf{k}| + E_n - \frac{1}{2}(E_i + E_f)}$$

With closure approximation:

Closure approximation

Without closure approximation:

$$M^{0\nu} \propto \sum_n \frac{\langle f | J_\mu(\mathbf{x}) | n \rangle \langle n | J^\mu(\mathbf{y}) | i \rangle}{|\mathbf{k}| + E_n - \frac{1}{2}(E_i + E_f)}$$

- Intermediate states $|n\rangle$ **with all spin-parities J^π up to high energies**

With closure approximation:

Closure approximation

Without closure approximation:

$$M^{0\nu} \propto \sum_n \frac{\langle f | J_\mu(\mathbf{x}) | n \rangle \langle n | J^\mu(\mathbf{y}) | i \rangle}{|\mathbf{k}| + E_n - \frac{1}{2}(E_i + E_f)}$$

- Intermediate states $|n\rangle$ **with all spin-parities J^π up to high energies**
 - ▶ Typically used in **pnQRPA**

With closure approximation:

Closure approximation

Without closure approximation:

$$M^{0\nu} \propto \sum_n \frac{\langle f | J_\mu(\mathbf{x}) | n \rangle \langle n | J^\mu(\mathbf{y}) | i \rangle}{|\mathbf{k}| + E_n - \frac{1}{2}(E_i + E_f)}$$

- Intermediate states $|n\rangle$ **with all spin-parities J^π up to high energies**
 - ▶ Typically used in **pnQRPA**

With closure approximation:

- Assuming that $|\mathbf{k}| \gg E_n - \frac{1}{2}(E_i + E_f)$:
 $E_n - \frac{1}{2}(E_i + E_f) \rightarrow \langle E \rangle$

Closure approximation

Without closure approximation:

$$M^{0\nu} \propto \sum_n \frac{\langle f | J_\mu(\mathbf{x}) | n \rangle \langle n | J^\mu(\mathbf{y}) | i \rangle}{|\mathbf{k}| + E_n - \frac{1}{2}(E_i + E_f)}$$

- Intermediate states $|n\rangle$ **with all spin-parities J^π up to high energies**
 - ▶ Typically used in **pnQRPA**

With closure approximation:

- Assuming that $|\mathbf{k}| \gg E_n - \frac{1}{2}(E_i + E_f)$:
 $E_n - \frac{1}{2}(E_i + E_f) \rightarrow \langle E \rangle$
- Use the relation $\sum_n |n\rangle \langle n| = \mathbf{1}$

Closure approximation

Without closure approximation:

$$M^{0\nu} \propto \sum_n \frac{\langle f | J_\mu(\mathbf{x}) | n \rangle \langle n | J^\mu(\mathbf{y}) | i \rangle}{|\mathbf{k}| + E_n - \frac{1}{2}(E_i + E_f)}$$

- Intermediate states $|n\rangle$ with all spin-parities J^π up to high energies
 - ▶ Typically used in pnQRPA

With closure approximation:

- Assuming that $|\mathbf{k}| \gg E_n - \frac{1}{2}(E_i + E_f)$:
 $E_n - \frac{1}{2}(E_i + E_f) \rightarrow \langle E \rangle$
- Use the relation $\sum_n |n\rangle \langle n| = \mathbf{1}$

$$\rightarrow M^{0\nu} \propto \frac{\langle f | J_\mu(\mathbf{x}) J^\mu(\mathbf{y}) | i \rangle}{|\mathbf{k}| + \langle E \rangle}$$

Closure approximation

Without closure approximation:

$$M^{0\nu} \propto \sum_n \frac{\langle f | J_\mu(\mathbf{x}) | n \rangle \langle n | J^\mu(\mathbf{y}) | i \rangle}{|\mathbf{k}| + E_n - \frac{1}{2}(E_i + E_f)}$$

- Intermediate states $|n\rangle$ **with all spin-parities J^π up to high energies**
 - ▶ Typically used in **pnQRPA**

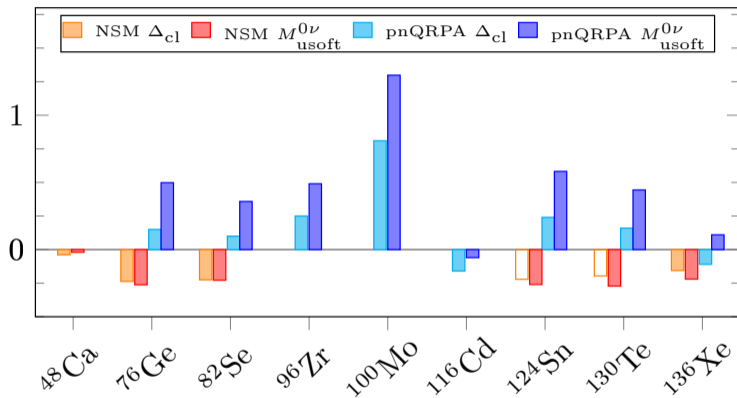
With closure approximation:

- Assuming that $|\mathbf{k}| \gg E_n - \frac{1}{2}(E_i + E_f)$:
 $E_n - \frac{1}{2}(E_i + E_f) \rightarrow \langle E \rangle$
- Use the relation $\sum_n |n\rangle \langle n| = \mathbf{1}$

$$\rightarrow M^{0\nu} \propto \frac{\langle f | J_\mu(\mathbf{x}) J^\mu(\mathbf{y}) | i \rangle}{|\mathbf{k}| + \langle E \rangle}$$

- ▶ Typically used **with other nuclear methods**

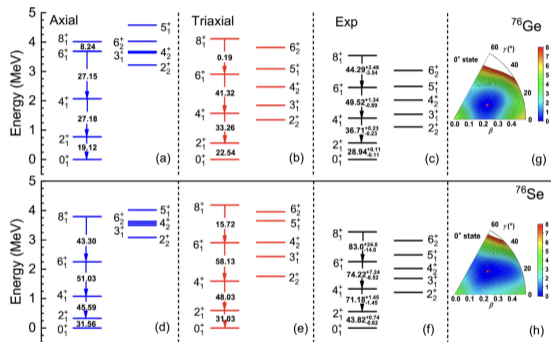
Ultrasoft Neutrinos as Closure Correction



$$\Delta_{cl} = M_{\text{non-cl}}^{0\nu} - M_{cl}^{0\nu}$$

D. Castillo, L.J. P. Soriano, J. Menéndez, Phys. Lett. B 860, 139181 (2025)

A = 76 Energies in ReCD



Y. Wang et al., Science Bulletin 69, 2017–2020 (2024)