# Status of calculations of $M^{0\nu}$

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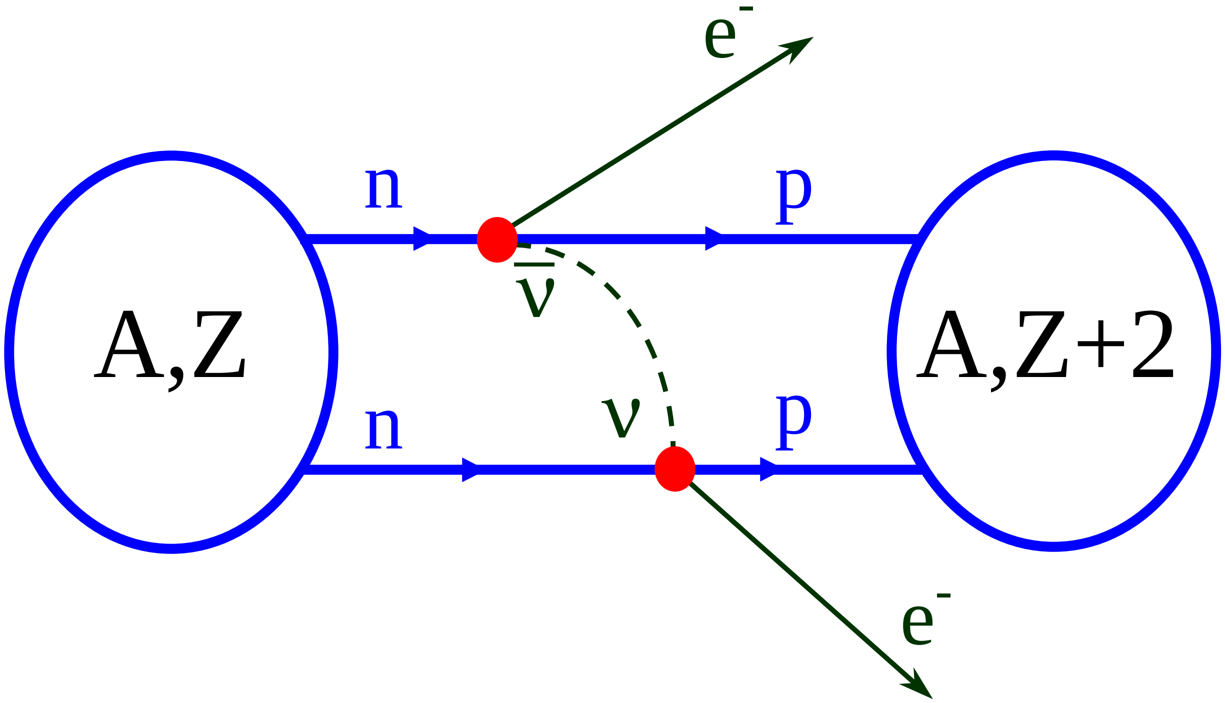
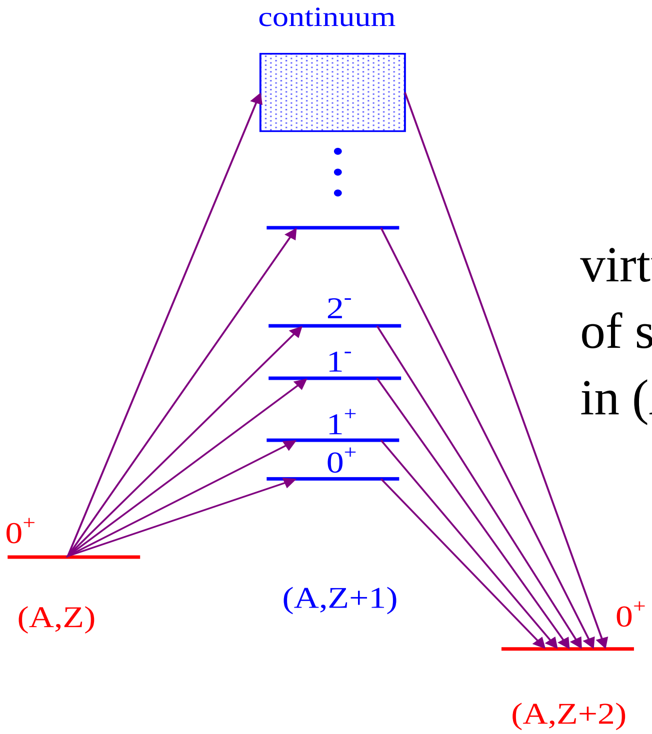
# Introduction

## Nuclear $0\nu\beta\beta$-decay ($\overline{\nu} = \nu$)

strong in-medium modification of the basic process $dd \rightarrow uue^-e^-(\overline{\nu}_e\overline{\nu}_e)$

Light neutrino
exchange mechanism





virtual excitation
of states of all multipolarities
in (A,Z+1) nucleus

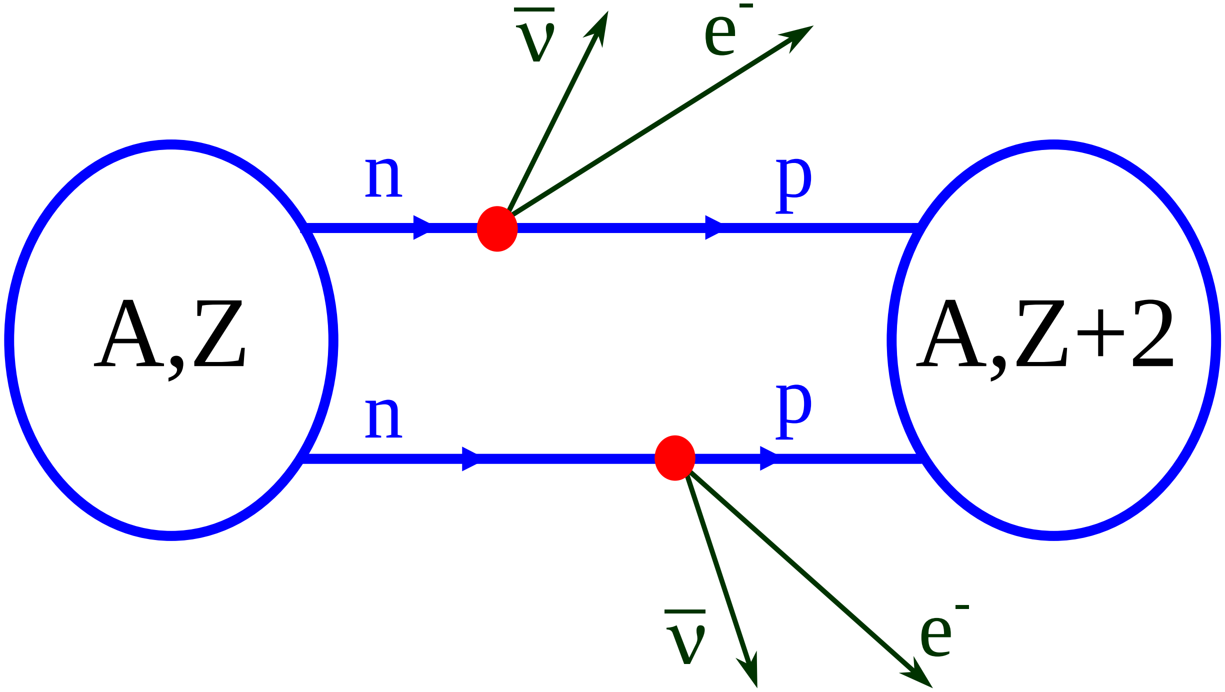
GT amplitudes to $1^+$ states

— from charge-exchange reactions

(H. Ejiri, D. Frekers, H. Sakai, R. Zegers, et al.)

# Nuclear 2$\nu\beta\beta$-decay

second order weak process
within SM



# Introduction

## measured $T^{2\nu}_{1/2}$ (compilation of A. Barabash, PRC **81** 2010)

| Isotope | $T^{2\nu}_{1/2}$, in $10^{19}$ y |
|---|---|
| $^{48}$Ca | $4.4^{+0.6}_{-0.5}$ |
| $^{76}$Ge | $150 \pm 10$ |
| $^{82}$Se | $9.2 \pm 0.7$ |
| $^{96}$Zr | $2.3 \pm 0.2$ |
| $^{100}$Mo | $0.71 \pm 0.04$ |
| $^{116}$Cd | $2.8 \pm 0.2$ |
| $^{128}$Te | $(1.9 \pm 0.4) \times 10^5$ |
| $^{130}$Te | $68^{+12}_{-11}$ |
| $^{136}$Xe | $211 \pm 25$ ← EXO-200, 1108.4193 |
| $^{150}$Nd | $0.82 \pm 0.09$ |
| $^{238}$U | $200 \pm 60$ |

$2\nu\beta\beta$ $0\nu\beta\beta$

# Inverse Half-Lives $[T_{1/2}(0^+ \to 0^+)]^{-1}$

$$G^{2\nu}(Q,Z)\,|M^{2\nu}_{GT}|^2$$

$$m^2_{\beta\beta}\,G^{0\nu}(Q,Z)\,\left|M^{0\nu}_{GT} - \frac{g_V^2}{g_A^2}M^{0\nu}_F\right|^2$$

Eff. neutrino mass $m_{\beta\beta} = \sum_j m_j U^2_{ej}$

$U_{ej}$ — first raw of the neutrino mixing matrix

$2\nu\beta\beta$

$0\nu\beta\beta$

# Nuclear Matrix Elements

$$M_{GT}^{2\nu} =$$

$$\sum_s \frac{\langle 0_f \| \hat{\beta}^- \| s \rangle \langle s \| \hat{\beta}^- \| 0_i \rangle}{E_s - (M_i + M_f)/2}$$

$$\hat{\beta}^- = \sum_k \boldsymbol{\sigma}_k \tau_k^-$$

$$M_{GT}^{0\nu} =$$

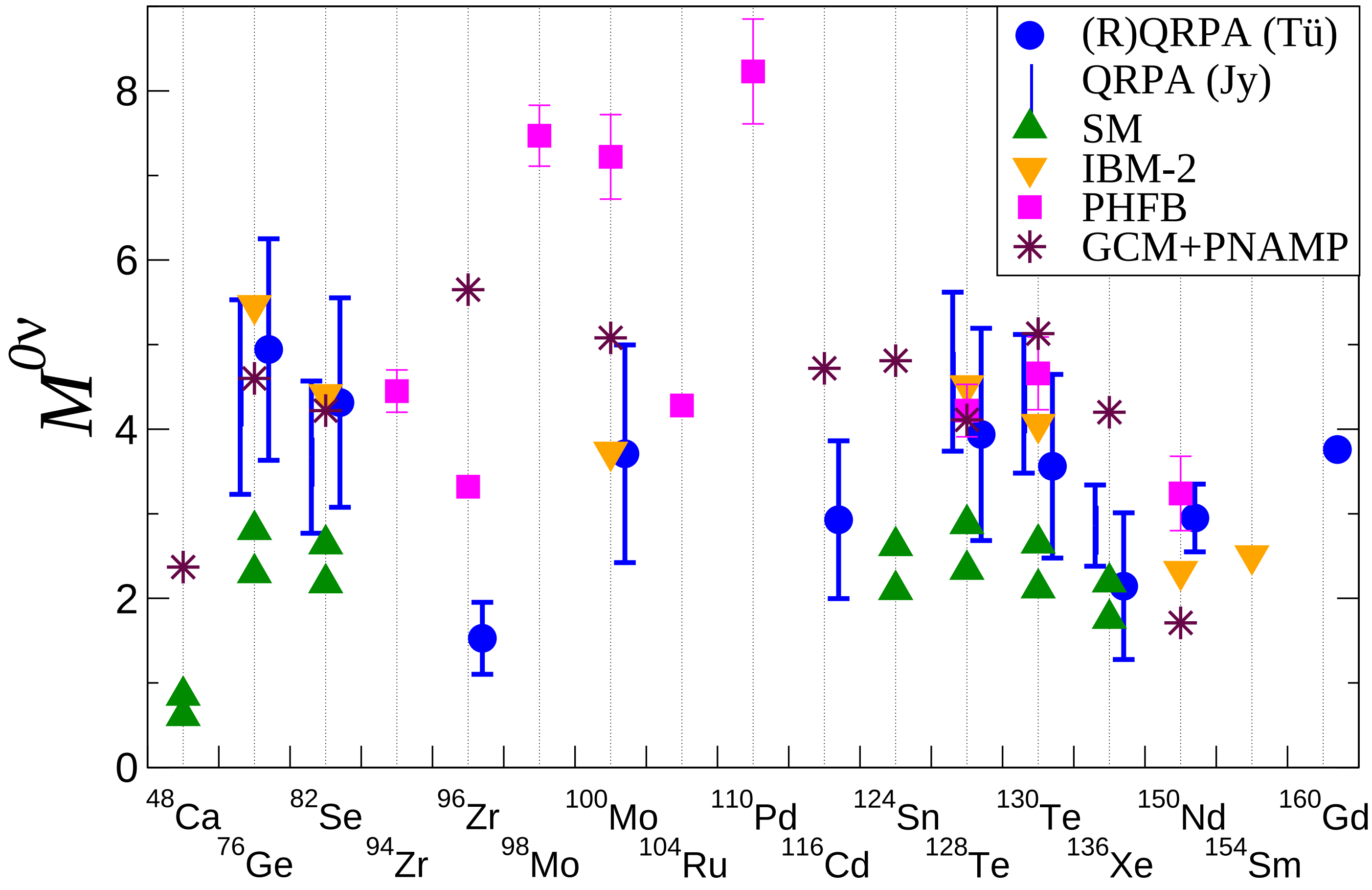
$$\langle 0_f | \sum_{ik} P_\nu(r_{ik}, \overline{\omega}) \tau_i^- \tau_k^- \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_k | 0_i \rangle$$

Neutrino potential : $P_\nu(r, \overline{\omega}) =$

$$\frac{2R}{\pi r} \int_0^\infty dq \frac{q \sin(qr)}{\omega(\omega + \overline{\omega})}$$

$$\approx \frac{R}{r} \; \phi(\overline{\omega} r)$$

# World status of $M^{0\nu}$, light neutrino mass mechanism



**QRPA: (Tü)** F. Šimkovic, A. Faessler, V.R., P. Vogel and J. Engel, PRC **77** (2008);

$^{150}$Nd,$^{160}$Gd with deformation: D. Fang, A. Faessler, V.R., F. Šimkovic, PRC **82** (2010); PRC **83**(2011)

**(Jy)** J. Suhonen, O. Civitarese, NPA **847** (2010)

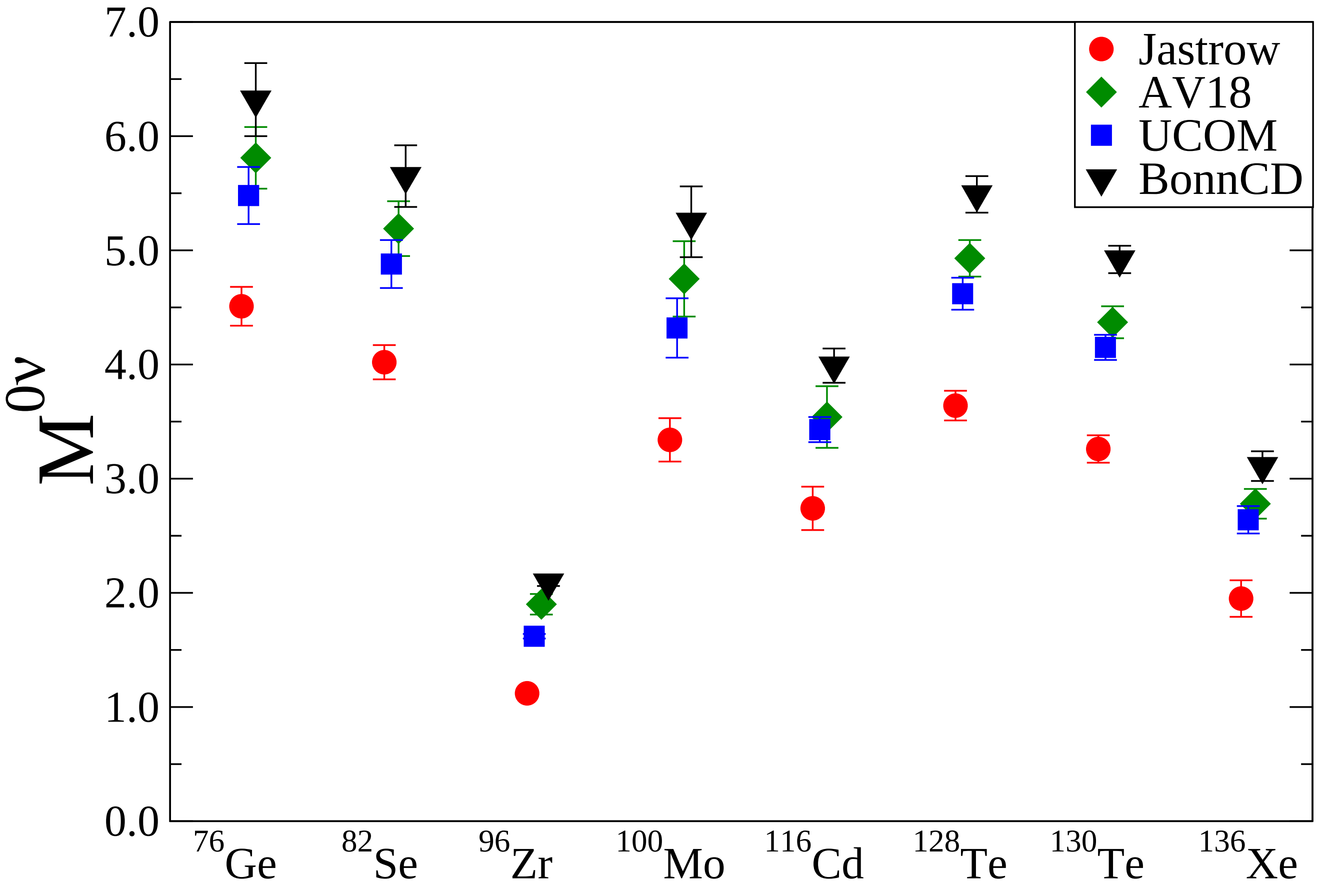
**SM** E. Caurier, J. Menendez, F. Nowacki, A. Poves, PRL **100** (2008) & NPA **818** (2009) **IBM-2** J. Barea and F. Iachello, PRC **79** (2009);

PHFB P.K. Rath *et al.*, PRC **82** (2010); GCM+PNAMP T. R. Rodriguez and G. Martinez-Pinedo, PRL **105** (2010)

# Introduction

Effect of short range correlation

F. Šimkovic, A. Faessler, H. Muether, V.R., M. Stauf, PRC **79** (2009)



# Nuclear models to calculate $\beta\beta$-amplitudes

Mean field ⟶ s.p. states ⟶ configuration space ⟶ diagonalization

Structure of intermediate states and $M^{2\nu}$:

QRPA,SM **"+"**    IBM-2,PHFB,GCM "–"

| | QRPA | NSM |
|---|---|---|
| s.p. bases | $N\hbar\omega$ | $0\hbar\omega$ |
| configurations | limited | all |

# QRPA vs. SM

NSM seems to be appealing *ab initio* approach:

neglects nothing and treats all configurations on the same footing

describes well spectroscopy of low-energy nuclear levels

# QRPA vs. SM

NSM seems to be appealing *ab initio* approach:

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describes well spectroscopy of low-energy nuclear levels

**BUT:**

- The Wall: factorial ($N! \sim N^N$) growth of configuration-space dimension

  ⇒ Severe s.p. basis truncation in medium and heavy nuclei

- Different phenomenological quenching factors and effective charges

  ⇒ effective operators are needed instead of the “bare" ones

Extensive recent work on effective $0\nu\beta\beta$ operators:

– J. Engel and G. Hagen, PRC **79**, 064317 (2009) [arXiv:0904.1709 [nucl-th]].

– J. Engel, J. Carlson and R. B. Wiringa, PRC **83**, 034317 (2011) [arXiv:1101.0554 [nucl-th]].

– D. Shukla, J. Engel and P. Navratil, arXiv:1108.3069 [nucl-th].

and also presentation by J. Engel at MEDEX’11

# QRPA vs. SM

## Shell Model

**Model Space:** $^{48}$Ca — $fp$; $^{76}$Ge, $^{82}$Se — $p, f_{5/2}, g_{9/2}$

$^{96}$Zr, $^{100}$Mo — $s, d, g$; $^{128,130}$Te, $^{136}$Xe — $s, d, g_{7/2}, h_{11/2}$

- several spin-orbit partners are missing even in $0\hbar\omega$ model space

  A lot of GT strength is missing (Ikeda Sum Rule is violated up to 40%)

  Inclusion of the spin-orbit partners is crucial for the quality of calculated $M^{0\nu}$

  A. Escuderos, A. Faessler, V. R., F. Šimkovic, J. Phys. G **37**, (2010)

  J. Suhonen, O. Civitarese, NPA **847** (2010)

- Many $0\nu\beta\beta$-transitions via negative parity intermediate states (dipole, spin-dipole etc.) are missing. They contribute a lot to $M^{0\nu}$ (shown by QRPA)

# QRPA vs. SM

## QRPA

- Works quite well when applied to description of collective states

- Fulfills exactly various model-independent sum rules

$M^{0\nu}$ and $M^{2\nu}$ are integral quantities (sums over all intermediate states)
challenge for experimental verification, but favors QRPA description

Notorious $g_{pp}$-problem
But $2\nu\beta\beta$ is sensitive to degree of violation of the Wigner SU(4) symmetry ⇒ $g_{pp}$-sensitivity is unavoidable!
V. R.,M. Urin, A. Faessler, NPA **747**, 297 (2005);
V. R., A. Faessler, PRC **84**, 014322 (2011).

# Partial contributions to $M^{0\nu}$

- $M^{0\nu} = \sum_{J^\pi} M^{0\nu}_{J^\pi}$ — particle-hole channel

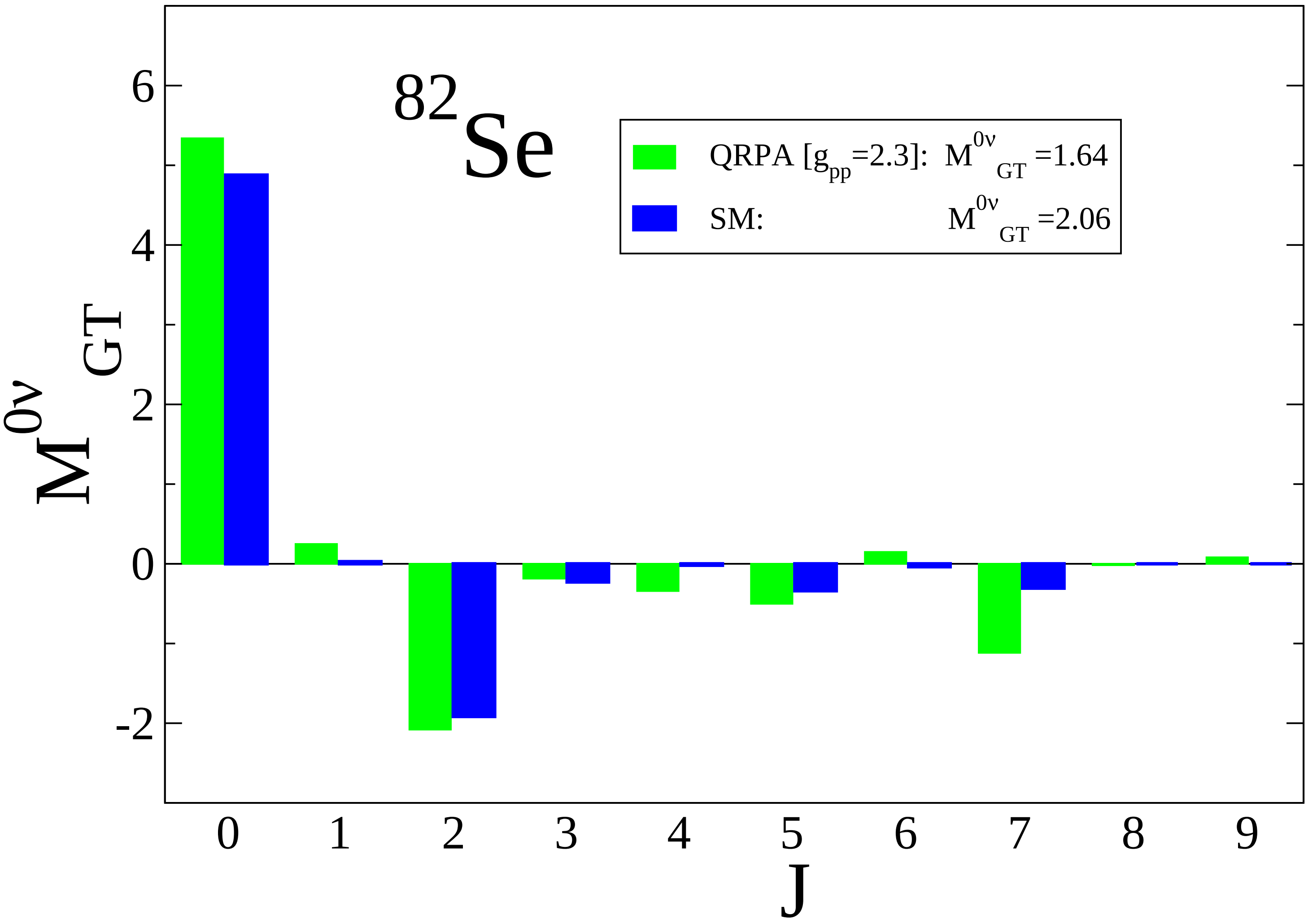
  $$M^{0\nu}_{J^\pi} = \sum_{pnp'n'} a_{pnp'n'} \langle 0_f | \left[ c^\dagger_p \tilde{c}_n \right]_J \left[ c^\dagger_{p'} \tilde{c}_{n'} \right]_J | 0_i \rangle$$

- $M^{0\nu} = \sum_{\mathcal{J}^\pi} M^{0\nu}_{\mathcal{J}^\pi}$ —particle-particle channel

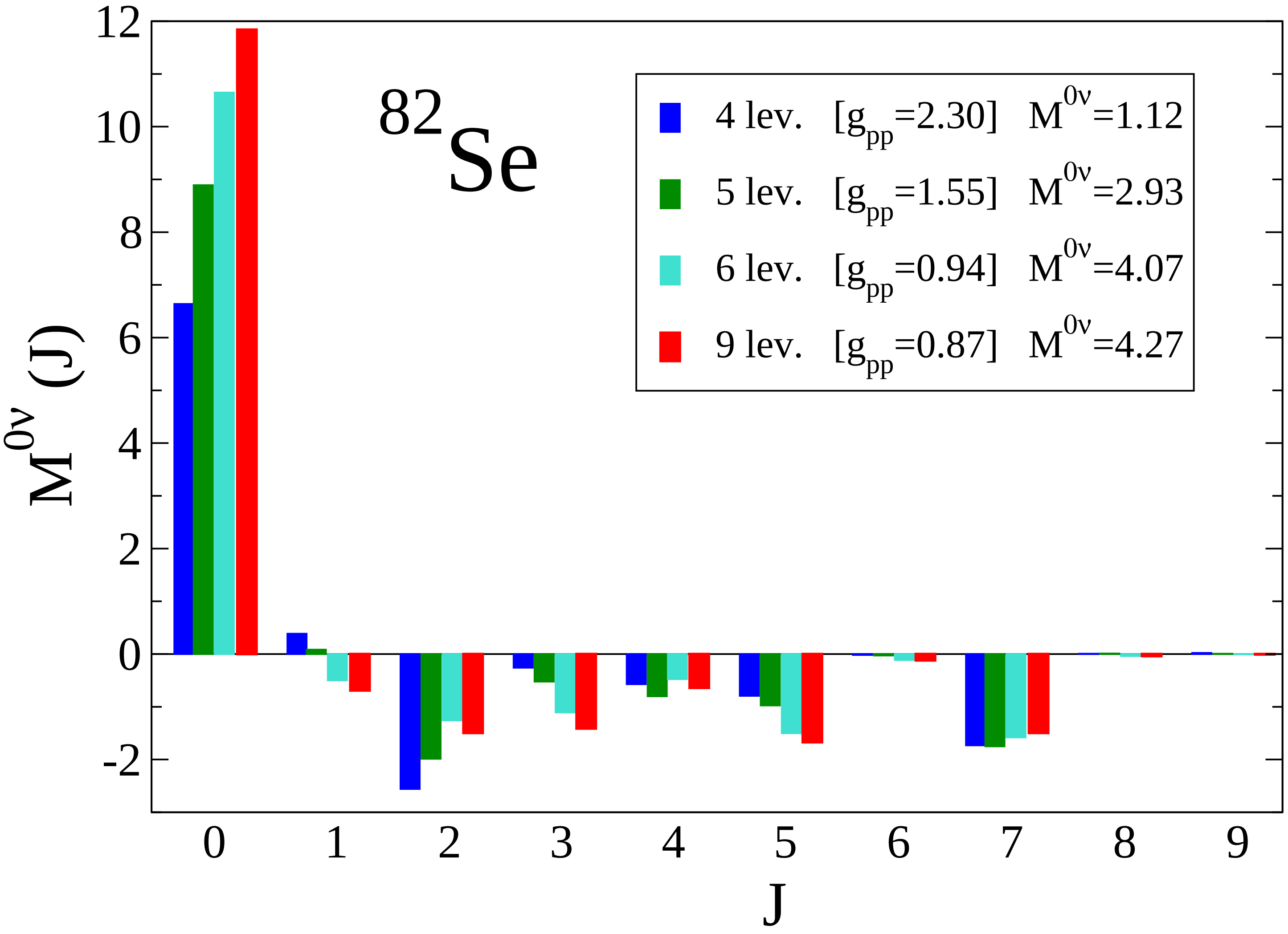
  $$M^{0\nu}_{\mathcal{J}^\pi} = \sum_{pnp'n'} b_{pnp'n'} \langle 0_f | \left[ c^\dagger_p c^\dagger_{p'} \right]_{\mathcal{J}} \left[ c_n c_{n'} \right]_{\mathcal{J}} | 0_i \rangle$$

# QRPA vs. SM

A. Escuderos, A. Faessler, V. R., F. Šimkovic, J.Phys.G37 (2010) arXiv:1001.3519 [nucl-th]



# QRPA vs. SM



$^{82}$Se
4 lev. [$g_{pp}$=2.30] $M^{0\nu}$=1.12
5 lev. [$g_{pp}$=1.55] $M^{0\nu}$=2.93
6 lev. [$g_{pp}$=0.94] $M^{0\nu}$=4.07
9 lev. [$g_{pp}$=0.87] $M^{0\nu}$=4.27
$M^{0\nu}$ (J)
12
10
8
6
4
2
0
-2
0
1
2
3
4
5
6
7
8
9
J

# QRPA vs. SM

## IBM

PHYSICAL REVIEW C **79**, 044301 (2009)

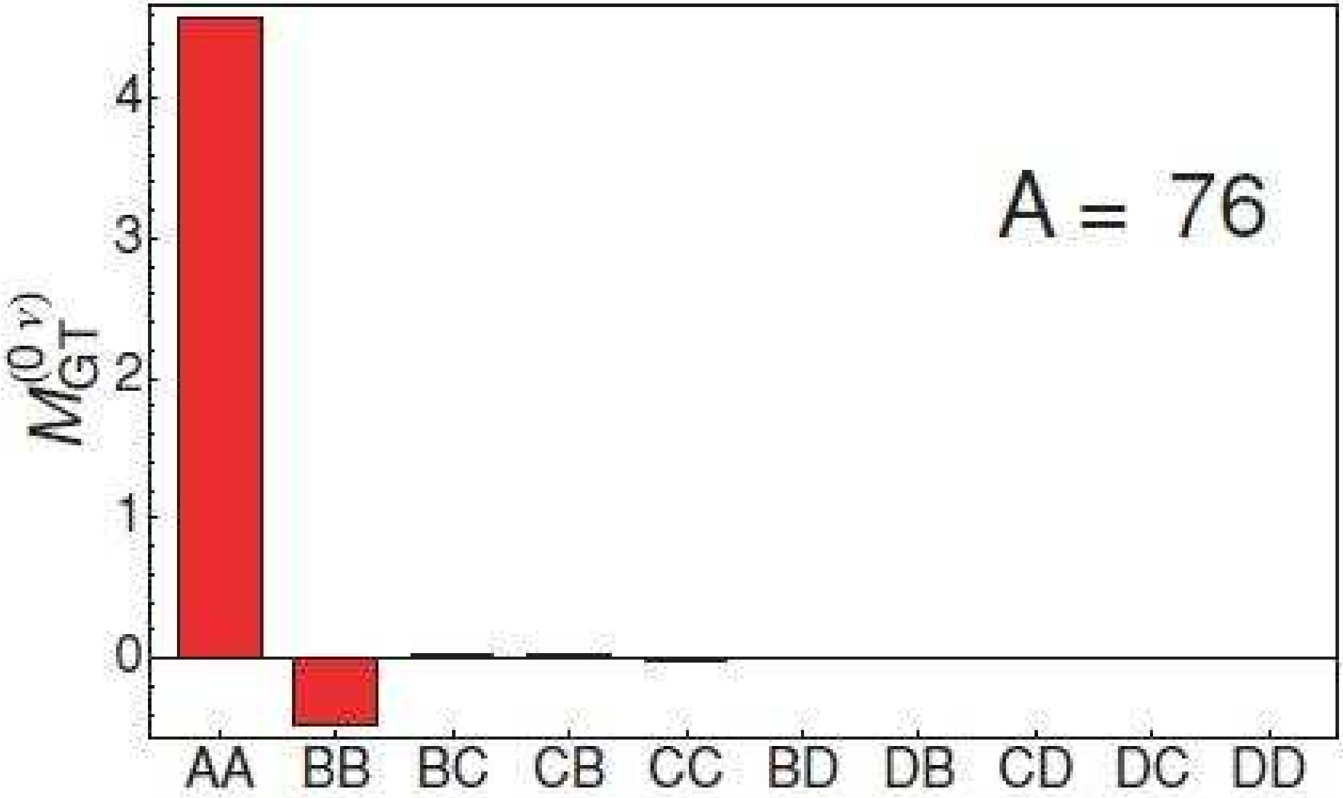


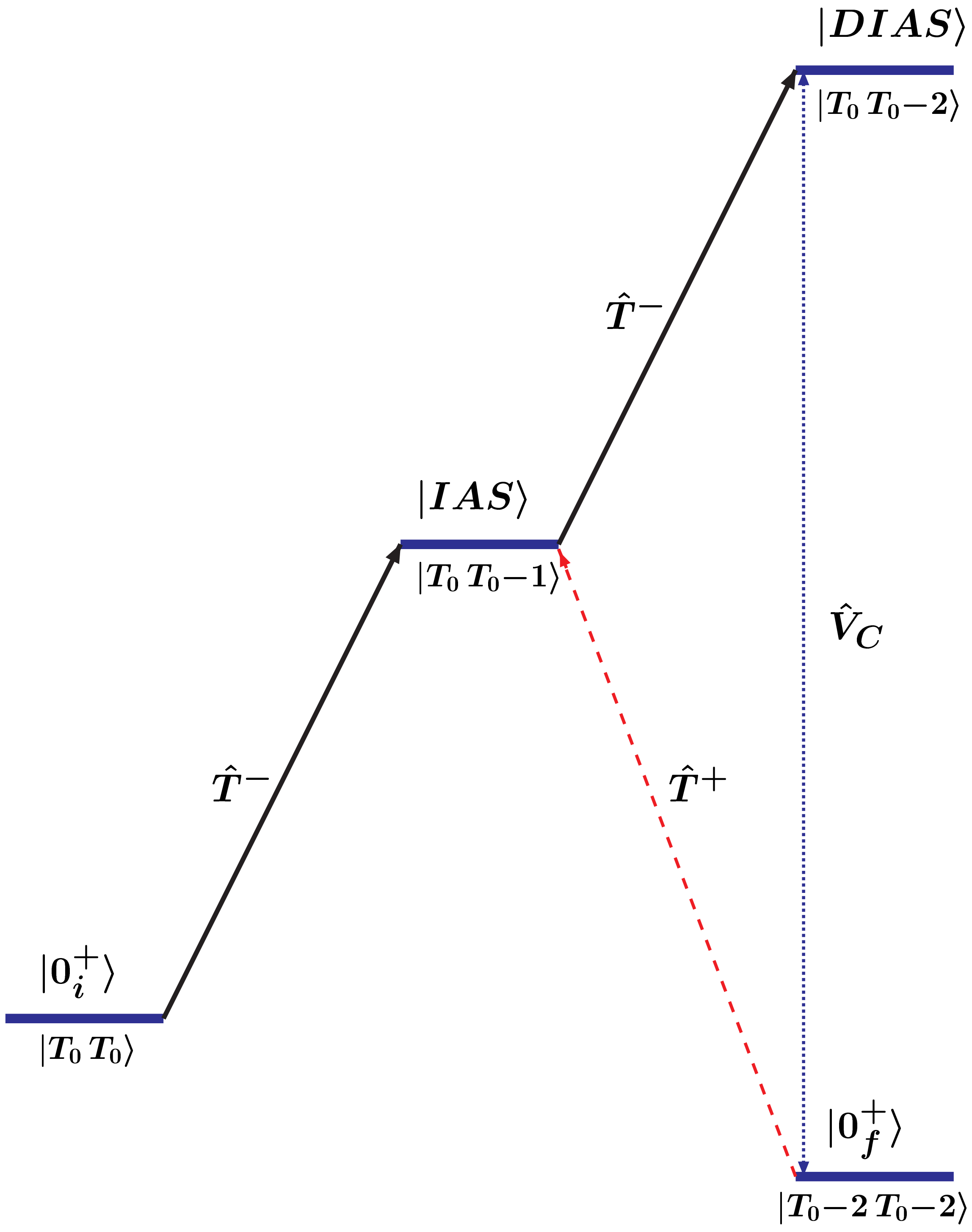
FIG. 1. (Color online) Contributions to the Gamow-Teller matrix elements of the $^{76}$Ge → $^{76}$Se decay in the boson expansion of Eq. (18).

# Measuring $M_F^{0\nu}$

## Can one measure nuclear matrix elements of neutrinoless double beta decay?

V.R., A. Faessler, PRC **80** , 041302(R) (2009) [arXiv:0906.1759 [nucl-th]]

PPNP **66**, 441 (2011); arXiv:1012.5176 [nucl-th]


$|DIAS\rangle$
$|T_0\,T_0-2\rangle$
$\hat{T}^-$
$|IAS\rangle$
$|T_0\,T_0-1\rangle$
$\hat{V}_C$
$\hat{T}^-$
$\hat{T}^+$
$|0_i^+\rangle$
$|T_0\,T_0\rangle$
$|0_f^+\rangle$
$|T_0-2\,T_0-2\rangle$

# Measuring $M_F^{0\nu}$

$$M_F^{0\nu} \approx \frac{-2}{e^2}\,\bar{\omega}_{IAS}\langle 0_f^+|\hat{T}^-|IAS\rangle\langle IAS|\hat{T}^-|0_i^+\rangle$$

$$\approx \frac{1}{e^2}\langle 0_f^+|\hat{V}_C|DIAS\rangle\langle DIAS|\left(\hat{T}^-\right)^2|0_i^+\rangle$$

Measure the $\Delta T = 2$ isospin-forbidden matrix element $\langle 0_f^+|\hat{T}^-|IAS\rangle$ [charge-exchange (*n*, *p*)-type reaction]

Challenge: $\langle IAS|\hat{T}^+|0_f^+\rangle \sim 0.001\langle IAS|\hat{T}^-|0_i^+\rangle$

$$\frac{M_F^{0\nu}(QRPA)}{M_F^{0\nu}(SM)} \approx 3 \div 5 \quad \text{and} \quad \frac{M_{GT}^{0\nu}}{M_F^{0\nu}} \approx -2.5$$

$\boxed{^{48}\mathrm{Ca}\rightarrow{}^{48}\mathrm{Ti}}$

# IAS of $^{48}$Ca ($T = 4, T_z = 3$) in $^{48}$Sc

1. is located at $E_x$ =6.678 MeV ($\bar{\omega}_{IAS}$ ≈8.5 MeV) under threshold of particle emission

2. 100% $\gamma$-decay to $1^+$ state at $E_x$ =2.517 MeV ($E_\gamma$=4.160 MeV)

3. a single state - no fragmentation

Reaction: $^{48}$Ti(n,p)$^{48}$Sc(IAS)

V.R., arXiv:1108.5108 [nucl-th]

$$\boxed{{}^{48}\mathrm{Ca}\rightarrow{}^{48}\mathrm{Ti}}$$

$$\frac{\langle IAS|\hat{T}^{+}|0_f\rangle}{\langle IAS|\hat{T}^{-}|0_i\rangle} = -\frac{e^2 M_F^{0\nu}}{2\bar{\omega}_{IAS}R}\cdot\frac{1}{N-Z}$$

$$\text{QRPA:}\qquad M_F^{0\nu} = 0.6 \Rightarrow \left|\frac{\langle IAS|\hat{T}^{+}|0_f\rangle}{\langle IAS|\hat{T}^{-}|0_i\rangle}\right|^2 \approx 2\cdot 10^{-6}$$

$\frac{d^2\sigma_{pn}}{d\Omega dE} \approx 10$ mb/(sr MeV), $E_p = 134$ MeV (B.D.Anderson et al., PRC 31 (1985))

$$\Rightarrow \frac{d^2\sigma_{np}}{d\Omega dE} \approx 20 \text{ nb/(sr MeV)}$$

# $M_F^{2\nu}$

byproduct of the calculation:

$$M_F^{2\nu}({}^{48}Ca) \approx -\frac{e^2 M_F^{0\nu}}{2R\overline{\omega}_{IAS}^2} = -1.4 \cdot 10^{-3}\ \mathrm{MeV}^{-1}$$

If $M_F^{2\nu}$ is measured $\Rightarrow M_F^{0\nu} \approx -\frac{2R\overline{\omega}_{IAS}^2}{e^2} M_F^{2\nu}$

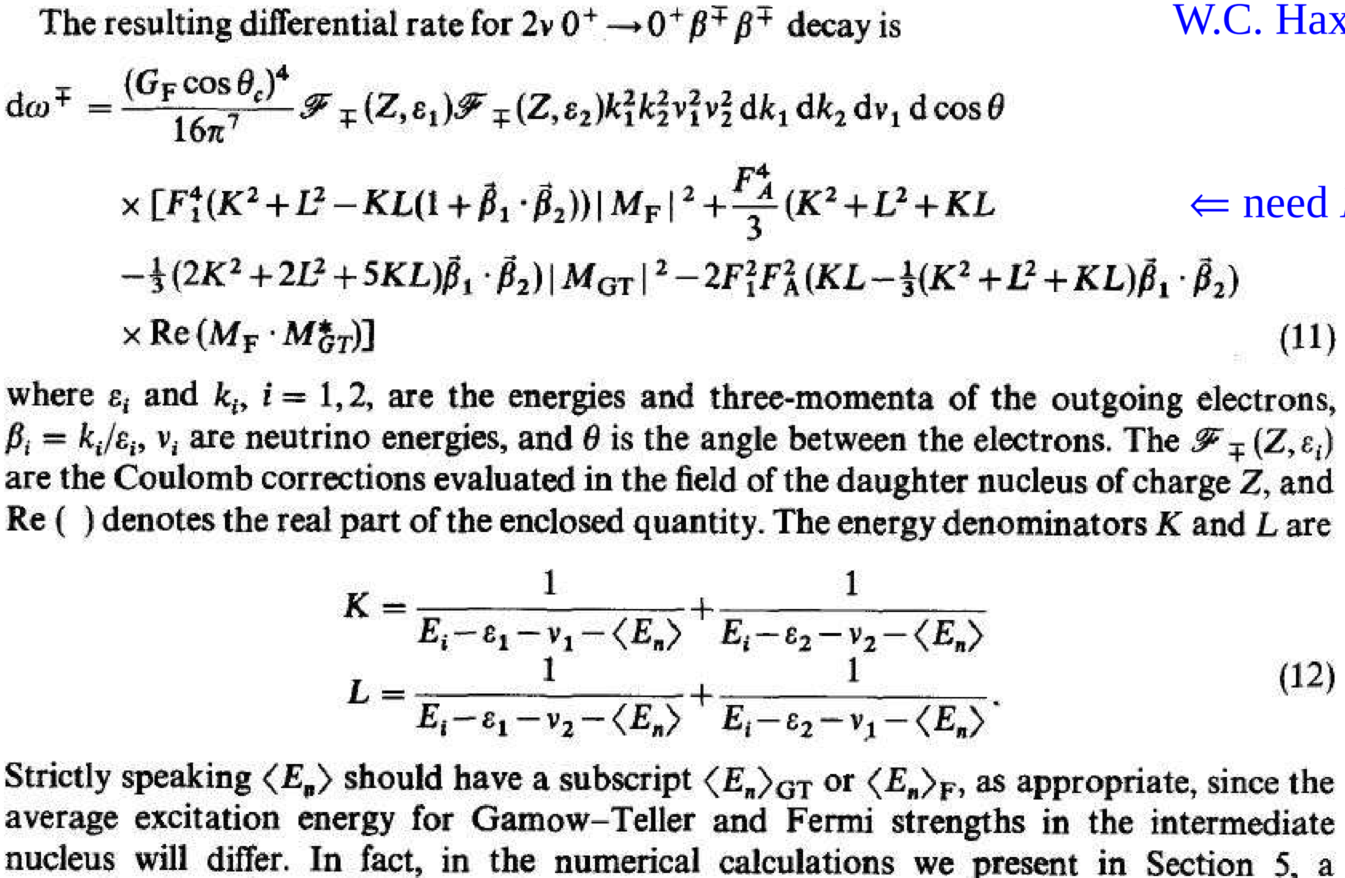
# $\boxed{M_F^{2\nu}}$

$$M^{2\nu} = M_{GT}^{2\nu} - \frac{g_V^2}{g_A^2} M_F^{2\nu}$$

- SSD for $M_{GT}^{2\nu}$: compare $M^{2\nu}$ vs. $M_{GT}^{2\nu}$ (from charge-exchange reactions)

- $e^-$-angular distribution

W.C. Haxton, G.J. Stephenson PPNP (1984)



The resulting differential rate for $2\nu\ 0^+ \rightarrow 0^+ \beta^\mp \beta^\mp$ decay is

$$\begin{aligned} \mathrm{d}\omega^\mp = {} & \frac{(G_\mathrm{F} \cos\theta_c)^4}{16\pi^7} \mathscr{F}_\mp(Z,\varepsilon_1) \mathscr{F}_\mp(Z,\varepsilon_2) k_1^2 k_2^2 \nu_1^2 \nu_2^2 \,\mathrm{d}k_1 \,\mathrm{d}k_2 \,\mathrm{d}\nu_1 \,\mathrm{d}\cos\theta \\ & \times [F_1^4(K^2+L^2-KL(1+\vec{\beta}_1\cdot\vec{\beta}_2))|M_\mathrm{F}|^2 + \frac{F_A^4}{3}(K^2+L^2+KL \\ & -\tfrac{1}{3}(2K^2+2L^2+5KL)\vec{\beta}_1\cdot\vec{\beta}_2)|M_\mathrm{GT}|^2 - 2F_1^2F_\mathrm{A}^2(KL-\tfrac{1}{3}(K^2+L^2+KL)\vec{\beta}_1\cdot\vec{\beta}_2) \\ & \times \mathrm{Re}\,(M_\mathrm{F}\cdot M_{GT}^*)] \end{aligned} \qquad (11)$$

$\Leftarrow$ need $K_{GT} \neq L_{GT}$, since $K_F = L_F = \overline{K}_F$

where $\varepsilon_i$ and $k_i$, $i = 1, 2$, are the energies and three-momenta of the outgoing electrons, $\beta_i = k_i/\varepsilon_i$, $\nu_i$ are neutrino energies, and $\theta$ is the angle between the electrons. The $\mathscr{F}_\mp(Z,\varepsilon_i)$ are the Coulomb corrections evaluated in the field of the daughter nucleus of charge $Z$, and Re ( ) denotes the real part of the enclosed quantity. The energy denominators $K$ and $L$ are

$$\begin{aligned} K &= \frac{1}{E_i-\varepsilon_1-\nu_1-\langle E_n\rangle} + \frac{1}{E_i-\varepsilon_2-\nu_2-\langle E_n\rangle} \\ L &= \frac{1}{E_i-\varepsilon_1-\nu_2-\langle E_n\rangle} + \frac{1}{E_i-\varepsilon_2-\nu_1-\langle E_n\rangle}. \end{aligned} \qquad (12)$$

Strictly speaking $\langle E_n\rangle$ should have a subscript $\langle E_n\rangle_\mathrm{GT}$ or $\langle E_n\rangle_\mathrm{F}$, as appropriate, since the average excitation energy for Gamow–Teller and Fermi strengths in the intermediate nucleus will differ. In fact, in the numerical calculations we present in Section 5, a

# Conclusions

- $0\nu\beta\beta$-decay is an *experimentum crucis* for revealing the Majorana nature of neutrinos and a feasible way to determine the absolute neutrino mass scale down to 10 meV's.
- Uncertainties in the QRPA calculations of $M^{0\nu}$ could be greatly reduced by using the experimental data on $2\nu\beta\beta$.
- The $M^{0\nu}$ of the SM are substantially smaller than the QRPA and other $M^{0\nu}$. Reason for such a deviation is under active study now (too small basis of the SM?).

# Conclusions

- The way to reliable $M^{0\nu}$:
  Understanding essential nuclear physics of $0\nu\beta\beta$-decay
  Devising nuclear model capable of catching this physics
  Trying to separate out less model-dependent contributions to $M^{0\nu}$
  Comparison with relevant experimental data is indispensible

- $M_F^{0\nu}$ can be extracted from measured Fermi m.e. $\langle IAS|\hat{T}^{+}|0_f\rangle$ (e.g. in $(n, p)$-type charge-exchange reactions)

- Non-accelerator methods playing more and more important role in deciphering new physics.
  $M^{0\nu}$ — very probably not the last input needed from nuclear physics.

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