



**Imperial College** 

London

## Laser-hybrid Accelerator for Radiobiological Applications (LhARA)

## John Adams Institute Accelerator Design Project 2024

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#### Motivation



[1] Comparison of radiation dose as a function of depth.



[2] Flash timescales compared to conventional RT

- Development of more accessible, cheaper alternatives for RT (radiation therapy)
- Study of ion beam radiobiology
- Exploration of novel treatment modalities



#### **Beam Parameters**

- Energy
- Ion species
- Dose, dose spatial distribution, dose rate
- Biological end point



[3] Facility comparison showing where the planned LhARA S1 & S2 are in energy & dose rate.

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## Target Normal Sheath Acceleration (TNSA): Exploration of Hybrid Acceleration



[4] Solid target interaction using TNSA to produce proton beams

Parameter	Value or	Unit
	range	
Laser power	100	TW
Laser energy	2.5	J
Laser pulse length	25	$\mathbf{fs}$
Laser rep. rate	10	Hz
Proton energy	15	MeV



[5] Ion beam spectra characteristics



## **Gabor Lenses**

#### Advantages

- More efficient focusing compared to high-field solenoid
- Reduces costs
- Focus in both planes simultaneously
- Variable focusing strength proportional to plasma density



[6] Schematic of Gabor Lens to be used in LhARA



### LhARA Design Overview



[7] Proposed LhARA facility.





High energy (in vivo and in vitro)

- Variable Injection energy using stage 1 beam line focusing strengths allows variable proton energies
- 2. 15 MeV -127 MeV



## Lattice Design

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#### **Lattice Constraints**



## **Beam Size Optimisation – 3.0cm Configuration**

#### Methodical Accelerator Design (MAD-X)

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- General purpose accelerator design tool with a focus on beam dynamics and optics optimisation
- 1. Beam focus after Lens 3 (S = 5.5m)
- 2. Twiss alpha -> 0 between Gabor lenses 2-3
- 3. High dispersion and low Twiss beta in the arc
- 4. Twiss alpha and dispersion -> 0 at end station

To keep constraint 1 satisfied for all configurations, only lenses 4-6 were varied to achieve smaller spot sizes.





## **Beam Size Optimisation – MAD-X Solenoid Matching**



MATCH module used to vary solenoid strengths and apply lattice constraints to find lower spot size configurations

	Solenoid Strength, K <sub>s</sub>			
2σ Spot Size (cm)	Lens 4	Lens 5	Lens 6	Lens 7
3.0	1.80	1.61	1.24	1.91
2.0	1.94	1.48	1.82	0.65
1.0	1.93	1.33	2.49	0.88

Beyond 1.0 cm, MAD-X is unable to accurately reach the intended beam size AND sufficiently satisfy lattice constraints in Dispersion and Twiss Alpha





### Arc Optimisation - Quadrupole Strength



Alpha and dispersion for a variety of beta values

Required strengths: -21.9, 31.2, -32, -31.1, 31.5, -23.3 [1/m]



## **BDSIM Lattice Model**

#### **Beam Delivery Simulation (BDSIM)**

 Program utilising the Geant4 physics libraries to simulate the transport of a particle beam through a 3D model of the accelerator with realistic physics processes.

#### **Studies on the BDSIM Lattice**

- Energy loss and deposition along the beamline
- Dose rate calculations
- Beam uniformity through the octupole
- Gabor lens performance study (vs solenoids)
- Tracking through a 3D field map of the student designed RF cavity.

Studies on the BDSIM lattice use a 3.0 cm beam size to account for:

- BDSIM not including the effects of space charge
- The largest beam size being most effective for studying losses



[8]



#### **Beam and Energy Loss**

Solenoid run with 10000 protons excluding collimators

A Global aperture radius of **3.65cm** was found to minimize total beam loss across the lattice.

The "g4QGSP\_BIC\_EMZ" Geant4 physics list was used for simulation. Chosen as it is most common for handling physics for radiobiology/medical applications.

10

 $10^{-1}$ 

Fractional Beam Loss  $10^{-2}$  fractional Beam Loss  $10^{-3}$  for  $10^{-4}$   $10^{-5}$ 

 $10^{-6}$ 

 $10^{-7}$ 

Primary Hit

Primary Loss



15

10

S (m)

Energy and Deposition plot under the same conditions with 10 million protons.

#### Lattice

20



#### **Beam and Energy Loss**

**Collimator 1** – After GL3

- Energy Cleaning
- Positioned where the beam is at its smallest
- Circular aperture
  - $\circ~$  Radius of 1.8mm (~2 $\sigma$ )

Collimator 2 – Middle of Vertical Arc

- Momentum Cleaning
- At the point of maximum Dispersion
- Elliptical aperture
  - $\circ$  Y-width of 1.2cm (~2 $\sigma$ )
  - $\circ$  X-width of 2.0cm
- Particles lost in dispersive y-axis, minimal losses in x.

10<sup>0</sup>

 $10^{-1}$ 

ss 10<sup>-2</sup>

Bean 10<sup>-3</sup>

Fractional B 10<sup>-4</sup>

10<sup>-6</sup>

40



S (m)

17



To enable **Dose Calculation**, a model end station target is placed at the end of the stage 1 lattice.



the water comparable to a Markus ion chamber

Dose Rate for 1cm beam directly into end station (no losses) =  $122.63 \pm 1.41$  Gy/s Close to LhARA's theoretical maximum dose rate in literature (~120 Gy/s) [7]



#### **Dose Rate Calculation:**

- Dose per proton extracted from the scorer and scaled by a factor of 10<sup>10</sup> to represent the expected 10<sup>9</sup> particles per shot and the 10 Hz repetition rate of the laser
- Large errors due to small sample size compared with the real number of expected particles per shot

	Dose Rate (Gy/s)	Change w.r.t Reference (Gy/s)	Within Error of Reference?
Reference	$16.92 \pm 0.61$	n/a	n/a
3.65cm Aperture	17.23 ± 0.61	+ 0.31	Yes
w/ Collimator 1	17.51 ± 0.62	+ 0.59	Yes
w/ Collimator 1+2	14.78 ± 0.57	-2.14	No

Significant impact of the second, **Momentum Cleaning**, collimator on the dose rate validates the motivation for LhARA's smaller beam sizes.

Smaller beams will experience less loss in that second collimator and therefore correlate to a higher dose at the end station.



## **Beam Uniformity - Octupole Tracking**

#### Why?

We desire a **uniform** beam at the end station to provide a spatially consistent dose to the entire target.

#### How?

<u>Octupoles!</u> Spatial flattening of the distribution is captured via **kurtosis**.

#### So ... how can we measure success?



#### Definition:

$$\mu_4 = \mathbb{E}\left[\frac{(x - \mu_x)^4}{\sigma_x^4}\right]$$

using scipy stats

Gaussian:  $\mu = 3$ 

Perfectly Flat:  $\mu = 1$ 

#### Let's define a flatness threshold!



#### 10,000 particles through the beamline ...



#### **Selection Arc**







*This is useful, but it's not entirely clear how we can measure the kurtosis of the beam distribution ....* 

> We can **rotate** the beam post-mortem in code and get an accurate figure for the uniform width.

#### Kurtosis metric of 1.73 (Non-Fisher)

We can expect a largely uniform coverage of the end station target.





μ = 1.73

## **Octupole Tracking – Comparison with Gaussian Bunch**

How does this compare to the case with no octupole? What are the real-term gains?Uniform , Fewer Losses

SUCCESS



Clearly an improvement in uniformity in the region of interest. Tests used k-value = 30,000 mm<sup>-4</sup>. Can be scaled up with more current to shape the beam further.





A single kurtosis measurement ignores many other features of the data like skewness ...

A quantile-quantile (QQ) plot directly compares the quantiles of two distributions to check for similarity. When one of those distributions is theoretical, we have what is known as a probability plot.





Applying this to the octupole data ...





#### **Gabor Lens Comparison**

Performance study between solenoid and Gabor lens models in BDSIM

Confinement field neglected (~0.03T solenoid)

Plasma magnetic field negligible at proposed densities (~5e15 m<sup>-3</sup>)

Modelled as drift elements with field maps and scaling applied

Requires sufficient plasma density/uniformity to neutralise beam space charge and avoid instabilities

Lattice



#### Field map equivalent to 1T



### **Gabor Lens Comparison**

COBYLA optimisation for fine tuning optics [9]

Constraints remain satisfied

Solenoid strengths: 1.40, 0.57, 0.80, 1.04, 0.80, 1.40, 0.28

Gabor lens strengths: 1.38, 0.56, 0.81, 1.04, 0.80, 1.38, 0.32

Comparable strengths, same optics but much less power required





# **RF Cavity Design**

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## **RF Introduction and Requirements**



## **Frequency Choice**



**RF** Cavities

- RF frequency tuned for bunch length alignment on rising RF wave edge.
- Late particles (closer to the bunch end) gain energy, while early ones (closer to the beginning) lose it, ensuring **phase stability**.
- All particles receive a positive acceleration at the same time, and the beam remains grouped throughout its trajectory.

Frequency	Bunch Length "Stability"
201 MHz	2.5 ns
352 MHz	1.4 ns

• Energy spread > 2% ⇒ Frequency < 201MHz







#### 201 MHz sine wave with dots showing the start and end of a 2.5 ns bunch



## SuperFish CCL Geometry

8 Free Parameters:

- Length
- Gap Length
- Outer Corner Radius
- Inner Corner Radius
- Outer Nose Radius
- Inner Nose Radius
- Flat Length
- Cone Angle

Optimising for:

- Shunt Impedance
- Transit Time
- Bunching Capability

Automatically adjusts diameter to fit frequency

Automatically adjusts E field to reach Kilpatrick factor of 1.5





#### **Cavity Optimisation**





Need a way to measure bunching ability for each cavity design.

- Write a simple particle-tracking simulation code
- Generate N particles representing bunch distribution
- Treat most of accelerator as drift
- On passing through the RF cavity, change the energy of the particle based on the field profile calculated in SuperFish
- Record the bunch length and energy spread at exit















































**RF** Cavities

#### **Longitudinal Phase Space Simulation – Results**

**15 MeV Protons**  Entering Section
 Exiting Section 15.75 - 15.75 15.50 15.50 € 15.25 15.25 15.00 - 15.00 OFF Bunch Length **Energy Spread** te 14.75 14.75 14.50 - 14.50 4.00 ns 1.92% 14.25 14.25 -2 ò ż -4 4 -2 0 2 Time delay relative to golden particle (ns) Entering Section - 15.75 15.75 Exiting Section - 15.50 15.50 -1.18 ns 0.31% a 15.25 - 15.25 NO 15.00 - 15.00 TR 14.75 - 14.75 14.50 - 14.50 14.25 -- 14.25 -4 -2 0 2 4 -2 0 2 Time delay relative to golden particle (ns) -4



44



## **Final Cavity Geometry**



Parameter	Value
Frequency [MHz]	201
Shunt Impedance - Z [MOhm/m]	25.41
Transit time factor -T []	0.33
ZTT [MOhm/m]	2.91
Maximum E field [MV/m] @ kilpatrick – 1.5	22.15
Maximum E field on axis [MV/m] @ kilpatrick – 1.5	8.08



## **3D Simulation with CST**





#### SuperFish vs CST



- Scaled by the stored energy in the cavity.
- Size of the electric field is somewhat arbitrary. We are looking a relative differences here.



- Simulation in BDSIM using the 3D field map from CST.
- Validated the cavity design and shown good control of the longitudinal phase space
- Final energy spread 0.68%





- Designed & optimised a 3D cavity design for LhARA stage 1.
- Validated the design with 6D tracking in BDSIM.

## Future work:

- Investigate schemes to better control the carbon beam.
- Continue design is CST, waveguides and waveguide ports.
- Continue optimisation using BDSIM.
- Additional RF infrastructure, cavity phasing.





# Magnet Design

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#### **Magnet Design**

Beam Parameters	
Energy	15 MeV
Momentum	0.168 GeV/c
Rigidity	0.561 Tm
Diameter	7.5 mm (1σ radius)

#### Magnet materials:

- Pure *iron* yokes
  - *Copper* coils
- Vacuum/Air gaps



#### **Required Magnets:**

Vertical Arc Dipole: Bending/beam *selection* into vertical arc

Vertical Arc Quadrupole: *Twiss* manipulation and *focusing* in vertical arc

**Extraction Octupole:** Flat beam profile

Nozzle Quadrupole: Permanent magnet capturing beam after laser source

**Good Field Region:** 35mm (5σ radius) **Beam pipe radius:** 50mm





$$B_y = \sum_{n=1}^{\infty} C_n x^{n-1}$$

*Fit a curve* to the absolute B-field value on a *radial contour* from the beamline to the edge of the beampipe.

#### Less accurate

- Monomial functions are *not orthogonal* 
  - Fit depends on chosen monomials
    - Easy to *overfit* data

#### **Fourier Analysis**

$$C_n = \frac{1}{Mr_0^{n-1}} \sum_{m=0}^{M-1} B_m e^{-2\pi i n m/M}$$

Fourier transform of the B-field vector along an azimuthal
contour around the Good Field Region of the beampipe.

#### More accurate

- Fourier coefficients are *orthogonal* 
  - Fit is always the same
- Compare *normal & skew* components



## **Switching Dipole**



FEMM 4.2 output of ½-dipole magnet B-field, mesh size 0.03mm. C-shape for easy beam switching.

### **Dipole GFR Field**





#### Simple fit takes **average B** across the **GFR** to assign a value to the field.

Standard dipole equations: • w<sub>pole</sub>=w<sub>GFR</sub>+2.5h • B<sub>leg</sub>=B<sub>gap</sub>\*(w<sub>pole</sub>+1.2h)/w<sub>leg</sub>



## **Dipole Field Analysis**



Harmonic	K-value (normal)	K-value (skew)	B@R=R <sub>GFR</sub>
Dipole	0.0 m <sup>-1</sup>	0.551 m <sup>-1</sup>	0.551 T
Quadrupole	1.0x10 <sup>-5</sup> m <sup>-2</sup>	0.0 m <sup>-2</sup>	0.160 μΤ
Sextupole	1.2x10 <sup>-4</sup> m <sup>-3</sup>	1.0x10 <sup>-5</sup> m <sup>-3</sup>	0.041 μT
Octupole	4.5x10 <sup>-3</sup> m <sup>-4</sup>	2.9x10 <sup>-4</sup> m <sup>-4</sup>	0.018 μΤ
Decapole	0.291 m <sup>-5</sup>	0.0146 m <sup>-5</sup>	0.010 μΤ
Dodecapole	26.582 m <sup>-6</sup>	1.065 m⁻ <sup>6</sup>	0.007 μΤ
14-pole (k6)	3165.3 m <sup>-7</sup>	107.93 m <sup>-7</sup>	0.005 μΤ
16-pole (k7)	4.7x10 <sup>5</sup> m <sup>-8</sup>	1.3x10 <sup>4</sup> m <sup>-8</sup>	0.003 μT

N.B. all values given are positive, no distinction is given to ±k Main k in red bold, allowed harmonics in red italics.



## Vertical Arc Quadrupole



Initial requirements:

- $K_1 = 32.0 \text{ m}^{-2}$
- B<sub>max</sub> ≤ 2.0 T
- GFR field purity  $\geq$  99.9%

Coil Parameters	
Coil Area	4,070 mm <sup>2</sup>
<b>Current Density</b>	5.31 Amm <sup>-2</sup>
Turns	18
Cooling Method	Water cooled



FEMM 4.2 output of ¼-quadrupole magnet B-field, mesh size 0.03mm

## Quadrupole GFR Field





Quadrupole fitted with calculated  $k_1 = 36.96m^{-2}$ 

Residuals <0.1% for most of the GFR. Central part dominated by mesh error due to small fields





Harmonic	K-value (normal)	K-value (skew)	B@R=R <sub>GFR</sub>
Dipole	0.0 m <sup>-1</sup>	0.0 m <sup>-1</sup>	0.0 T
Quadrupole	36.958 m <sup>-2</sup>	0.058 m <sup>-2</sup>	0.726 T
Sextupole	0.0 m <sup>-3</sup>	0.0 m <sup>-3</sup>	0.0 T
Octupole	0.0 m <sup>-4</sup>	0.0 m <sup>-4</sup>	0.0 T
Decapole	0.0 m <sup>-5</sup>	0.0 m <sup>-5</sup>	0.0 T
Dodecapole	3.7x10 <sup>6</sup> m⁻ <sup>6</sup>	15,460 m <sup>-6</sup>	0.908 mT
20-pole (k9)	1.4x10 <sup>15</sup> m <sup>-10</sup>	1.7x10 <sup>13</sup> m <sup>-10</sup>	0.174 mT
28-pole (k13)	3.7x10 <sup>24</sup> m <sup>-14</sup>	5.1x10 <sup>22</sup> m <sup>-14</sup>	0.039 mT

N.B. all values given are positive, no distinction is given to ±k. Main k in red bold, allowed harmonics in red italics.



### **Extraction Octupole**



Initial requirements:

- K<sub>3</sub>≥60,000 m<sup>-4</sup>
- B<sub>max</sub> ≤ 2.0 T
- GFR field purity  $\geq$  99.9%

Coil Parameters	
Coil Area	3431 mm <sup>2</sup>
<b>Current Density</b>	2.48 Amm <sup>-2</sup>
Turns	10
Cooling Method	Water cooled



FEMM 4.2 output of ¼-octupole magnet B-field, mesh size 0.03mm

#### **Octupole GFR Field**





Octupole fitted with calculated  $k_3 = 60273m^{-4}$ 

Relative **residuals <0.1%** up until edge of GFR (maximum 0.2%)



## **Octupole Field Analysis**



Harmonic	K-value (normal)	K-value (skew)	B@R=R <sub>GFR</sub>
Dipole	0.0 m <sup>-1</sup>	0.0 m <sup>-1</sup>	0.0 T
Quadrupole	0.0 m <sup>-2</sup>	0.0 m <sup>-2</sup>	0.0 T
Sextupole	0.0 m <sup>-3</sup>	0.0 m <sup>-3</sup>	0.0 T
Octupole	60,273 m <sup>-4</sup>	<b>0.1667 m</b> <sup>-4</sup>	0.242 T
Decapole	0.0 m <sup>-5</sup>	0.0 m <sup>-5</sup>	0.0 T
Dodecapole	0.0 m⁻ <sup>6</sup>	0.0 m <sup>-6</sup>	0.0 T
24-pole (k11)	3.9x10 <sup>20</sup> m <sup>-12</sup>	2.4x10 <sup>17</sup> m <sup>-12</sup>	0.527 mT
40-pole (k19)	2.4x10 <sup>40</sup> m <sup>-20</sup>	7.9x10 <sup>38</sup> m <sup>-20</sup>	0.024 mT

N.B. all values given are positive, no distinction is given to ±k Main k in red bold, allowed harmonics in red italics.



- Immediately after the laser ion/proton source the beam is extremely small leading to significant space charge effects.
- A focusing element at very near to the source could allow for more beam to be captured.
- The element must be small, high gradient and radiation hard....





## Permanent Magnet Quadrupole Design



• Samarium–cobalt magnet material.





## Summary



#### Lattice Design:

- Optimised configurations for spot sizes 3.0-1.0 cm
- Performance comparison between Gabor lenses and solenoids
- Quantified beam losses and end station dose rate
- Demonstrated the effect of the octupole on beam uniformity



#### **Cavity Design:**

- 2D cavity geometry optimisation
- Particle tracking for bunching measurements
- 3D modelling using CST
- Phase-space comparison to BDSim

- 2D magnet design for dipoles, quadrupoles, octupole and PMQ
- Fourier harmonic analysis



## Bibliography

- 1. "Figure 1. Comparison of the Radiation Dose as Function of the Depth In..." ResearchGate, ResearchGate, 2020, <u>www.researchgate.net/figure/Comparison-of-the-radiation-dose-as-function-of-the-depth-in-tissue-for-X-rays</u> fig5 230912423. Accessed 8 Mar. 2024.
- 2. Hageman, Eline, et al. "Radiobiological Aspects of FLASH Radiotherapy." *Biomolecules*, vol. 12, no. 10, 26 Sept. 2022, pp. 1376–1376, www.mdpi.com/2218-273X/12/10/1376, https://doi.org/10.3390/biom12101376. Accessed 14 Mar. 2024.
- 3. Aymar, Galen, et al. "LhARA: The Laser-Hybrid Accelerator for Radiobiological Applications." Frontiers in Physics, vol. 8, 29 Sept. 2020, www.frontiersin.org/articles/10.3389/fphy.2020.567738/full, https://doi.org/10.3389/fphy.2020.567738. Accessed 8 Mar. 2024.
- 4. "Figure 2.4: Target Normal Sheath Acceleration-TNSA. A Thin Target Foil..." ResearchGate, ResearchGate, 2016, <u>www.researchgate.net/figure/Target-Normal-Sheath-Acceleration-TNSA-A-thin-target-foil-with-thickness-d-5-50 fig8 308718768</u>. Accessed 8 Mar. 2024.
- 5. Gizzi, Leonida A, et al. "Enhanced Laser-Driven Proton Acceleration via Improved Fast Electron Heating in a Controlled Pre-Plasma." *Scientific Reports*, vol. 11, no. 1, 2 July 2021, www.nature.com/articles/s41598-021-93011-3, https://doi.org/10.1038/s41598-021-93011-3. Accessed 14 Mar. 2024.
- 6. Palkovic, J.A., Mills, F.E., Schmidt, C., & Young, D.E. (1989). Gabor lens focusing of a negative ion beam (FNAL/C--89/115). United States
- 7. Kurup, A. & Long, K (2020) "LhARA: world-leading radiobiology and novel technology development" <u>https://www.imperial.ac.uk/news/198093/lhara-world-leading-radiobiology-novel-technology-development/</u> Accessed 13 Mar. 2024.
- 8. L.J. Nevay et al., BDSIM: An Accelerator Tracking Code with Particle-Matter Interactions, Computer Physics Communications 252 107200 (2020).
- Powell, M.J.D. (1994). A Direct Search Optimization Method That Models the Objective and Constraint Functions by Linear Interpolation. In: Gomez, S., Hennart, JP. (eds) Advances in Optimization and Numerical Analysis. Mathematics and Its Applications, vol 275. Springer, Dordrecht. <u>https://doi.org/10.1007/978-94-015-8330-54</u>
- 10. D. C. Meeker, Finite Element Method Magnetics, Version 4.2 (28Feb2018 Build), https://ww.femm.info



## Varying Gabor Lens and Quadrupole strength to achieve smallest possible beam size

MATCH, SEQUENCE=lhara, betx=init\_betx, bety=init\_bety, alfx=init\_alfx, alfy=init\_alfy; vary, name = LHA\_LEL\_MAG\_QUAD\_01->k1, step=1, lower=-33, upper=-15; // Vary k in gabor lens 4 vary, name = LHA\_LEL\_MAG\_QUAD\_02->k1, step=1, lower=10, upper=30; // Vary k in gabor lens 5 vary, name = LHA\_LEL\_MAG\_QUAD\_03->k1, step=1, lower=-33, upper=-15; // Vary k in gabor lens 6 vary, name = LHA\_LEL\_MAG\_QUAD\_04->k1, step=1, lower=-33, upper=-10; // Vary k in gabor lens 7 vary, name = LHA\_LEL\_MAG\_QUAD\_05->k1, step=1, lower=10, upper=33; // Vary k in gabor lens 7 vary, name = LHA\_LEL\_MAG\_QUAD\_06->k1, step=1, lower=-33, upper=-15; // Vary k in gabor lens 7 constraint, sequence=lhara, range = LHA\_LEL\_DIA\_COL\_04, dy>3.3; // Dispersion high in collimator constraint, sequence=lhara, range = LHA\_LEL\_DIA\_COL\_04, bety<60; // Dispersion = 0 before arc //constraint, sequence=lhara, range = LHA\_LEL\_DIA\_COL\_04, bety<60; // Dispersion = 0 before arc constraint, sequence=lhara, range = LHA\_LEL\_VAC\_DRI\_30, bety<br/>betaY; // Reduce Size at end station constraint, sequence=lhara, range = LHA\_LEL\_VAC\_DRI\_30, betx<betaX; //^^^^^^^ constraint, sequence=lhara, range = LHA\_LEL\_VAC\_DRI\_30, alfy=0; //Alfa 0 at end station constraint, sequence=lhara, range = LHA\_LEL\_VAC\_DRI\_30, alfx=0;//^^^^^^ constraint, sequence=lhara, range = LHA\_LEL\_VAC\_DRI\_30, dy=0;// Dispersion = 0 at end station constraint, sequence=lhara, range = LHA\_LEL\_VAC\_DRI\_30, dx=0;//^^^^^^

#### DOF

#### Constraints

#### Uses sum of squares of constraint functions