Beam-beam effects

JAI lectures - Hilary Term 2025

Hector Garcia-Morales

Physics PhD and Science Communicator

hector.garcia.morales@cern.ch

Table of contents

References

- W. Herr, Lectures on Beam-beam interaction, CERN accelerator school (2016)[1].
- D. Schulte, Beam-beam effects in linear colliders, CERN accelerator school (2017) [2].

[1] https://cds.cern.ch/record/1982430/files/431-459%20Herr.pdf
[2] https://indico.cern.ch/event/457349/attachments/1175828/1699810/Beam-beam2.pdf

Goals of this course

- Introduction to beam-beam interaction.
- This is a complex topic and we will cover a small part.
- Mostly related to induced tune shift.
- Introduce some concepts to compensate beam-beam effects.

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Beam-beam effects

When two beams collide, protons may collide or not:

- Wanted Physics
- Un-wanted Physics

In real colliders:

- Only a small fraction of the particles contained in the bunch collide.
- But the rest feel the EM interaction of the opposite beam.

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Luminosity and crossing angle

The interaction will depend on the beam parameters and the geometry of the collision:

- Beam size.
- Collision angle.

This will affect luminosity:

$$\mathcal{L} = \frac{N_1 N_2 f_{\text{rep}} n_b}{4\pi\sigma_x\sigma_y} R(\theta/2) \tag{1}$$

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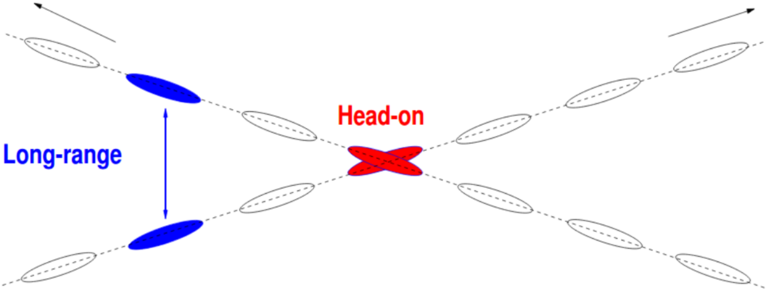
Crossing angle

In *pp* colliders, to avoid parasitic collisions, we need to introduce a crossing angle.



Now, the overlapping between bunches is not optimal. There are methods to mitigate this effect.

Beam-Beam force

The electrostatic field are obtained by integrating over the charge distribution.

Gaussian distribution

$$\rho_u(u) = \frac{1}{\sigma_u\sqrt{2\pi}} \exp\left(-\frac{u^2}{2\sigma_u^2}\right) \tag{2}$$

Electrostatic potential

$$U(x, y, \sigma_x, \sigma_y) = \frac{ne}{4\pi\epsilon_0} \int_0^\infty \frac{\exp\left(-\frac{x^2}{2\sigma_x^2+q} - \frac{y^2}{2\sigma_y^2+q}\right)}{\sqrt{(2\sigma_x^2+q)(2\sigma_y^2+q)}} dq \tag{3}$$

where n is the density of particles in the beam, e the elementary charge and ϵ_0 the permitivity of empty space.

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Beam-Beam force and tune shift

The field $\vec{E}$ is obtained by taking the gradient of the potential:

$$\vec{E} = -\nabla U(x, y, \sigma_x, \sigma_y) \tag{4}$$

Assuming round beams ($\sigma_x = \sigma_y = \sigma$) the Lorentz force $\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$ becomes,

$$\vec{F} = q(E_r + \beta c B_\Phi) \times \vec{r} \tag{5}$$

From the electrostatic potential in Eq. (3), we can write the fields, as,

$$E_r = -\frac{ne}{4\pi\epsilon_0} \frac{\delta}{\delta r} \int_0^\infty \frac{\exp\left(-\frac{r^2}{2\sigma^2+q}\right)}{2\sigma^2 + q} dq \tag{6}$$

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Beam-Beam force

From Eq. (6) and Eq. (7) we can finally obtain the radial force,

$$F_r(r) = -\frac{ne^2(1+\beta^2)}{2\pi\epsilon_0}\frac{1}{r}\left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right] \tag{8}$$

where, in cartesian coordinates, takes the form,

$$F_x(r) = -\frac{ne^2(1+\beta^2)}{2\pi\epsilon_0}\frac{x}{r^2}\left[1 - \exp\left(-\frac{r^2}{2\sigma^2}\right)\right] \tag{9}$$

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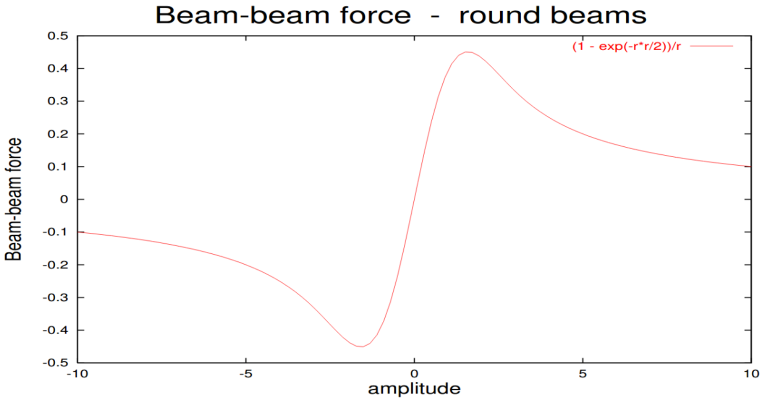


Beam-Beam parameter

When small amplitudes are considered, we can derive the linear tune shift produced by beam-beam interaction.

Kick received from the opposite beam:

$$\Delta r' = \frac{1}{mc\beta\gamma} \int_{-\infty}^{\infty} F_r(r, s, t) dt \tag{11}$$

$$\Delta r' = -\frac{2Nr_0}{\gamma} \frac{1}{r} \left[1 - \exp\left(-\frac{r^2}{2\sigma^2} \right) \right] \tag{12}$$

where $r_0 = e^2/4\pi\epsilon_0 mc^2$.
for small amplitudes, the asymptotic limit:

$$\Delta r'|_{r\to 0} = \frac{Nr_0 r}{4\pi\gamma\sigma^2} = -rf \tag{13}$$

Beam-Beam parameter

We already know how the focal length relates to a tune change.
Linear tune shift ξ:

$$\xi_{x,y} = \frac{N r_0 \beta^*_{x,y}}{2\pi\gamma\sigma_{x,y}(\sigma_x + \sigma_y)} \tag{14}$$

This expression is often used to quantify the strength of the interaction. However, it does not include the non-linear part of the interaction.

Tune shift

For small values of ξ and a tune far away from resonances:

$$\xi \approx \Delta Q \tag{15}$$

Non-linear effects

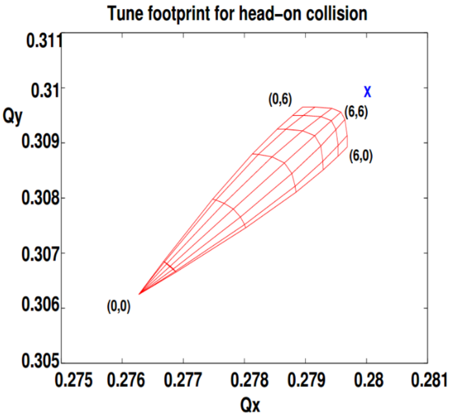
When we take the non-linear part of the beam-beam interaction:

- Amplitude-dependent tune shift.
- Tune spread.

Detuning with amplitude

$$\Delta Q(J) = \xi \cdot \frac{2}{J} \cdot (1 - I_0(J/2) \cdot e^{-J/2}) \quad (16)$$

where $I_0(x)$ is the modified Bessel function and $J = \epsilon\beta/2\sigma^2$.



Beam stability

When the beam-beam interaction becomes too strong, the beam can become unstable or the dynamics is strongly affected.

- Dynamic aperture reduction, particle loss and lifetime reduction.
- Beam optics distortion.
- Vertical blow-up.

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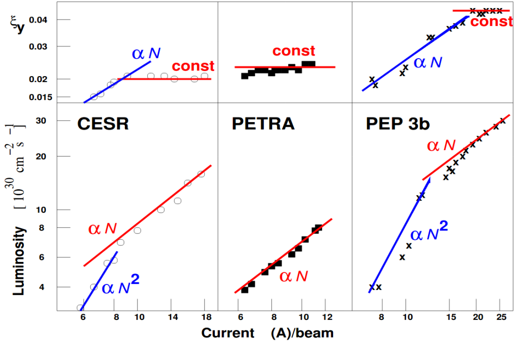
Beam-beam limit

Regular operation

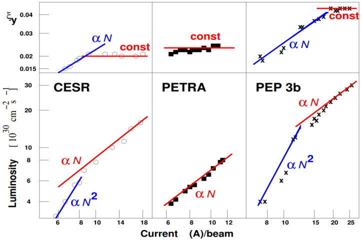
- Luminosity: $\mathcal{L} \sim N^2$.
- Beam-beam: $\xi \sim N$



High-current operation

- Luminostiy: $\mathcal{L} \sim N$.
- Beam-beam: $\xi \sim$ constant

$$\mathcal{L} = \frac{N^2 n_b f_{rep}}{4\pi\sigma_x\sigma_y} = \frac{N n_b f_{rep}}{4\pi\sigma_x}\frac{N}{\sigma_y} \qquad (17)$$



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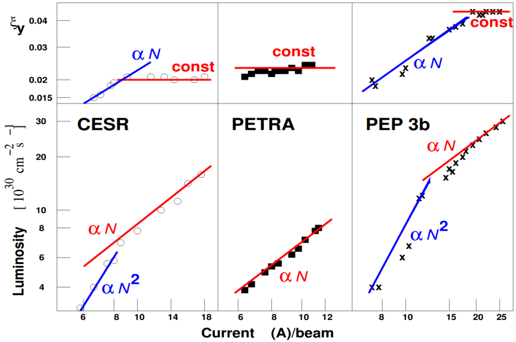

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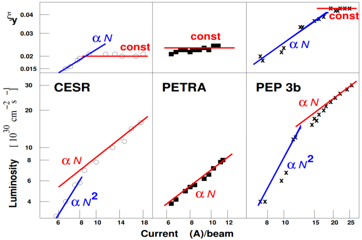

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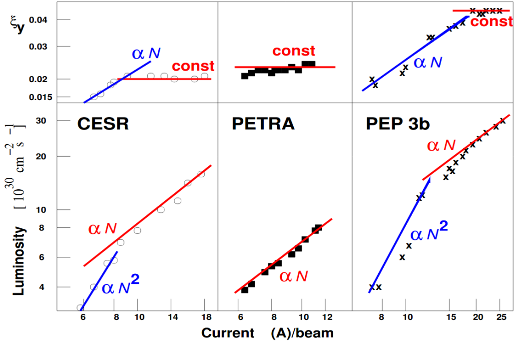

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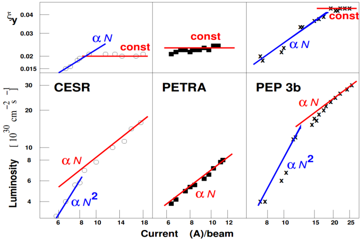

Weak-Strong and Strong-Strong interaction

Sometimes beam-beam effects are classified into different categories depending on the nature of the two colliding beams.

- Strong-Strong: both high-intensity beams are equally affected.
  - LHC, LEP, RHIC.
- Weak-Strong: Asymmetric beams. Only one of the beams is really affected.
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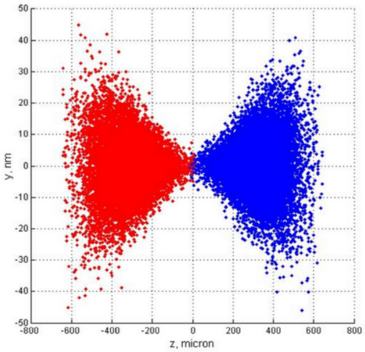
Pinch effect in e^+e^- colliders

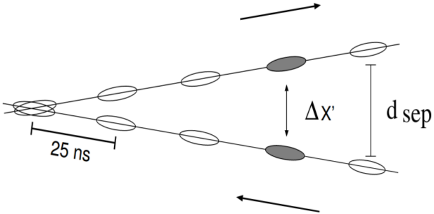
Due to the opposite charge of the beams, there exists an extra focusing (pinch effect).



This may increase luminosity up to a factor 2 (ILC, CLIC).

Long range interactions

- Symmetry breaking between planes.
- Mostly affect large-amplitude particles.
- Tune shift has opposite sign in the plane of separation compared to head-on tune shift.
- They cause changes in the closed orbit.



Strength of LR interactions

Assuming a separation d in the horizontal plane:

$$\Delta x' = -\frac{2Nr_0}{\gamma}\frac{x+d}{r^2}\left[1-\exp\left(-\frac{r^2}{2\sigma^2}\right)\right] \tag{18}$$

$$\Delta y' = -\frac{2Nr_0}{\gamma}\frac{y}{r^2}\left[1-\exp\left(-\frac{r^2}{2\sigma^2}\right)\right] \tag{19}$$

Tune spread:

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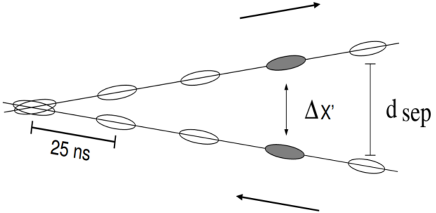

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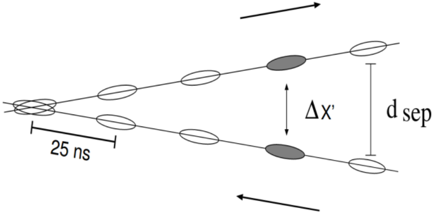

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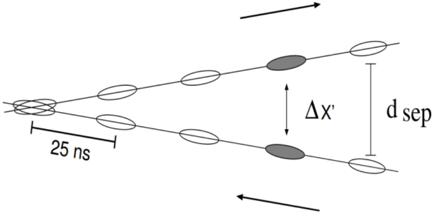

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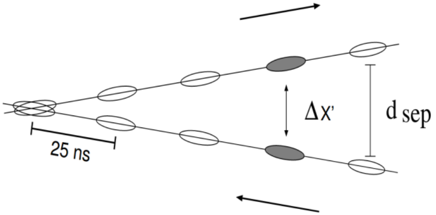

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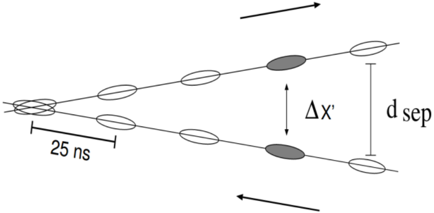

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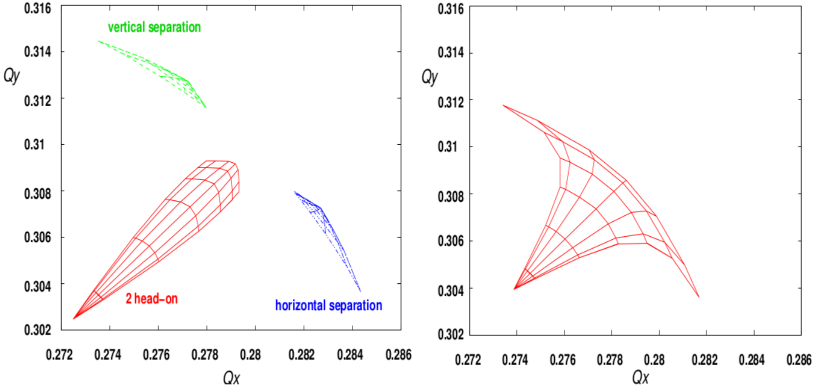


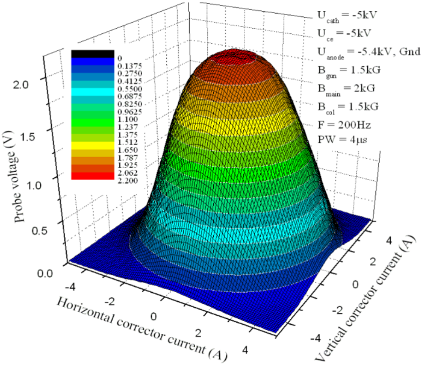
Figure: Tune footprint for two head-on interactions, LR in the H and V planes (left). Combined head-on and long-range interactions (right).

Beam-beam compensation

When the beam-beam effects limit the performance of the collider, several schemes are proposed to compensate the detrimental effects.

Build a non-linear lens to counteract the distortion from the non-linear beam-beam lens

- Head-on effects:
  - Electron lenses.
  - Linear lens to shift tunes.
  - Non-linear lens to decrease tune spread.
- Long-range effects:
  - At large distances, beam-beam force $\sim 1/r$.
  - Same force as a wire.
  - Crab cavities.



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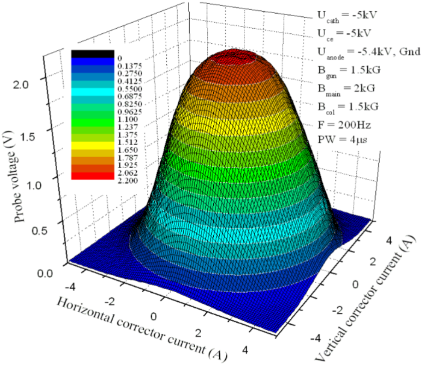

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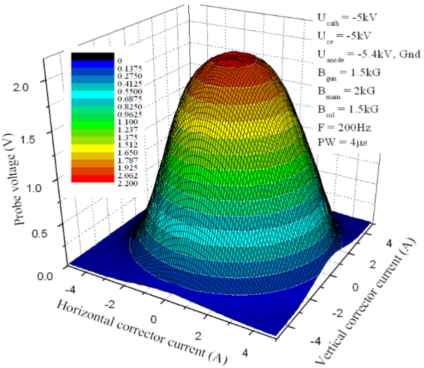

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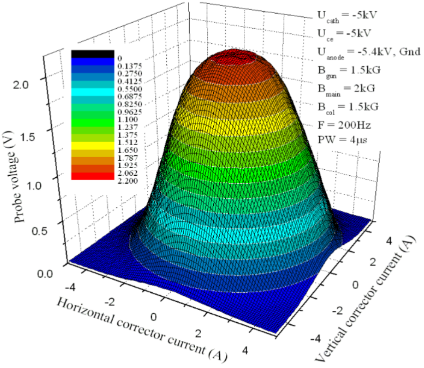
- Long-range effects:
  - At large distances, beam-beam force $\sim 1/r$.
  - Same force as a wire.
  - Crab cavities.



Electron lens

A proton beam travels through a counter-rotating high-current electron beam. The negative space charge reduces the effect from beam-beam interaction.

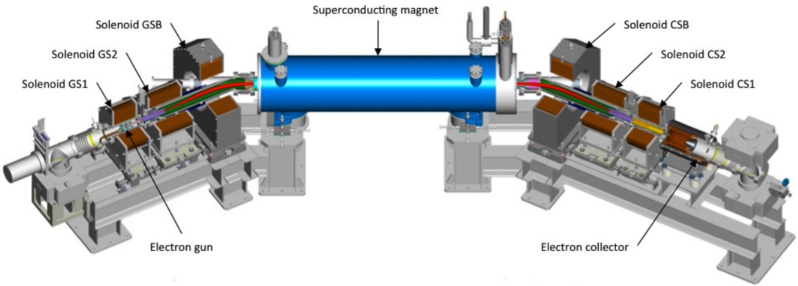
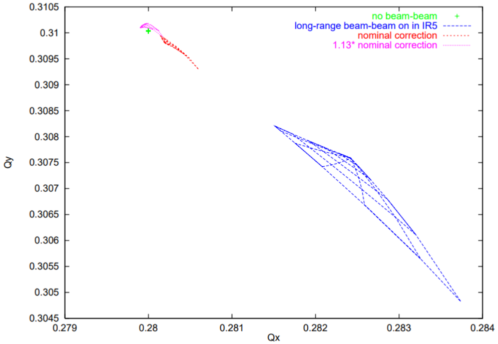


Figure: RHIC electron lens for beam-beam compensation.

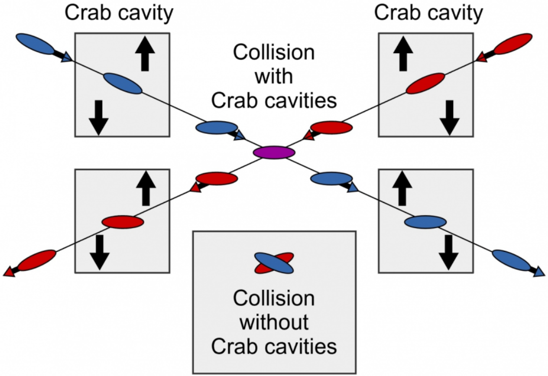
Electrostatic Wire

To compensate the tune spread from long-range interactions a non-linear lens is required. Since, for large amplitude, the beam-beam force goes like $1/r$ an electrostatic wire located parallel to the beam.



Crab cavities

We can increase the crossing angle so long-range interaction becomes larger.



Crab cavities does not compensate beam-beam interaction but help reducing its effects.

Summary

- Beam-beam interaction limits the performance of particle colliders.
- The linear effect is expressed in terms of the beam-beam parameters, ξ.
- There are some techniques to compensate its effects.

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Thank you!