An introduction to Magnets for Accelerators

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John Adams Institute Accelerator Course

23 Jan. 2025

If you want to know more...

- 1. Special CAS edition on magnets, St. Pölten, Nov.-Dec. 2023
- 2. Special CAS edition on magnets, Bruges, Jun. 2009
- 3. N. Marks, Magnets for Accelerators, JAI (John Adams Institute) course, Jan. 2015
- 4. D. Tommasini, Practical Definitions & Formulae for Normal Conducting Magnets
- 5. Lectures about magnets in CAS (CERN Accelerator School) and JUAS (Join Universities Accelerator School)
- 6. Superconducting magnets for particle accelerators in USPAS (U.S. Particle Accelerator Schools)
- 7. J. Tanabe, Iron Dominated Electromagnets
- 8. P. Campbell, Permanent Magnet Materials and their Application
- 9. K.-H. Mess, P. Schmüser, S. Wolff, Superconducting Accelerator Magnets
- 10. M. N. Wilson, Superconducting Magnets
- 11. A. Devred, Practical Low-Temperature Superconductors for Electromagnets
- 12. L. Rossi and E. Todesco, Electromagnetic design of superconducting dipoles based on sector coils

This is an introduction to magnets as building blocks of synchrotrons / transfer lines

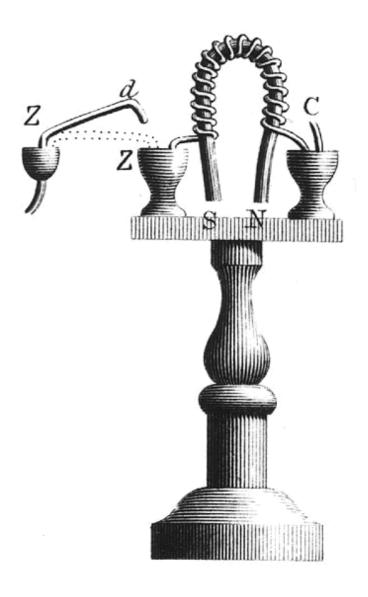
```
11
// MADX Example 2: FODO cell with dipoles
// Author: V. Ziemann, Uppsala University
// Date: 060911
TITLE, 'Example 2: FODO2.MADX';
BEAM, PARTICLE=ELECTRON, PC=3.0;
DEGREE:=PI/180.0;
OF: OUADRUPOLE, L=0.5, K1=0.2;
OD: QUADRUPOLE, L=1.0, K1=-0.2;
B: SBEND, L=1.0, ANGLE=15.0*DEGREE;
FODO: SEQUENCE, REFER=ENTRY, L=12.0;
 QF1: QF, AT=0.0;
 B1: B, AT=2.5;
```

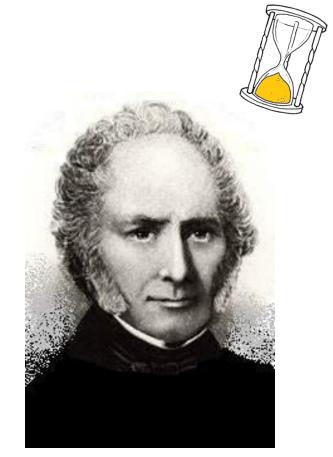
QD1: QD, AT=5.5;

- B2: B, AT=8.5;
- QF2: QF, AT=11.5;

ENDSEQUENCE ;

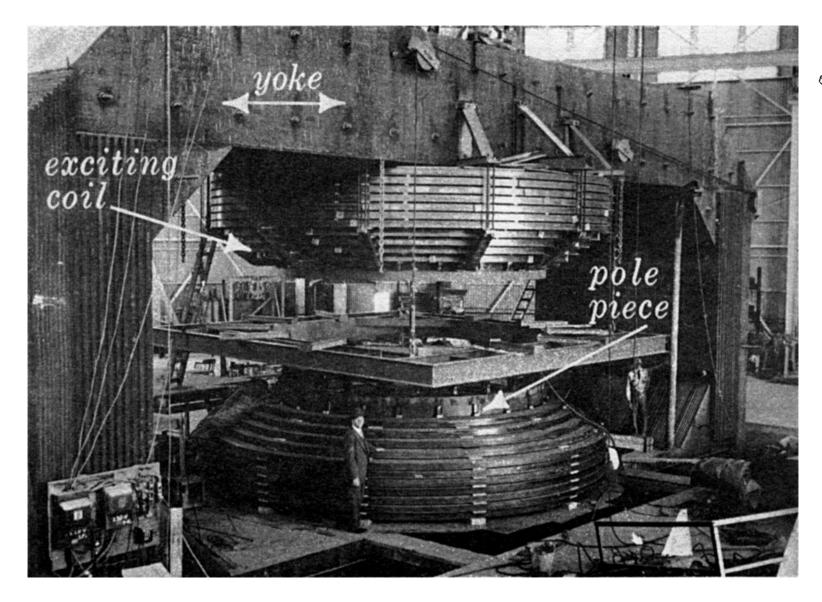
According to history, the first electromagnet (not for an accelerator) was built in England in 1824 by William Sturgeon





William Sturgeon

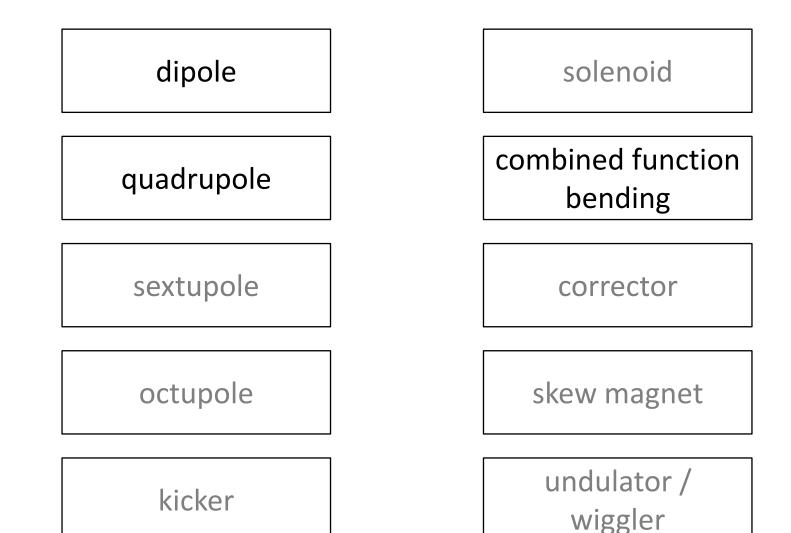
The working principle is the same as this large magnet, of the 184" (4.7 m) cyclotron at Berkeley (picture taken in 1942)



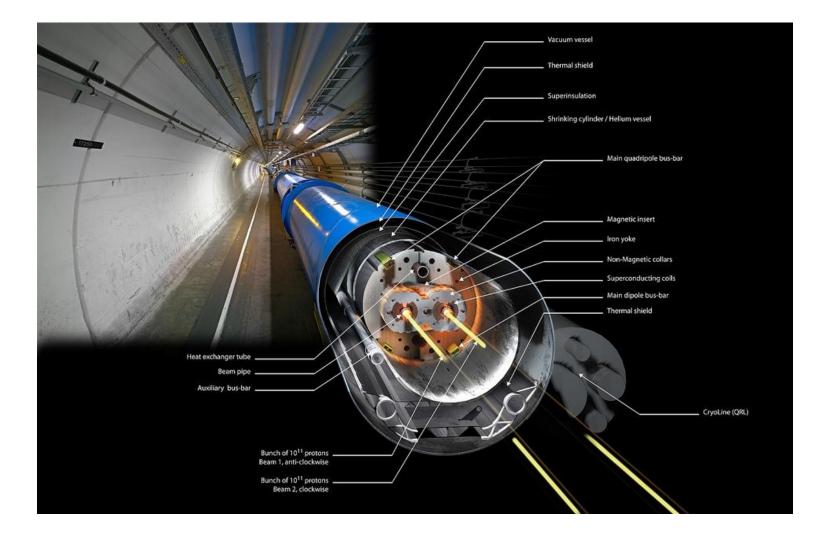
This short course is organized in several blocks

- 1. Introduction, jargon, general concepts and formulae
- 2. Resistive magnets
- 3. Superconducting magnets
- 4. Tutorial with FEMM

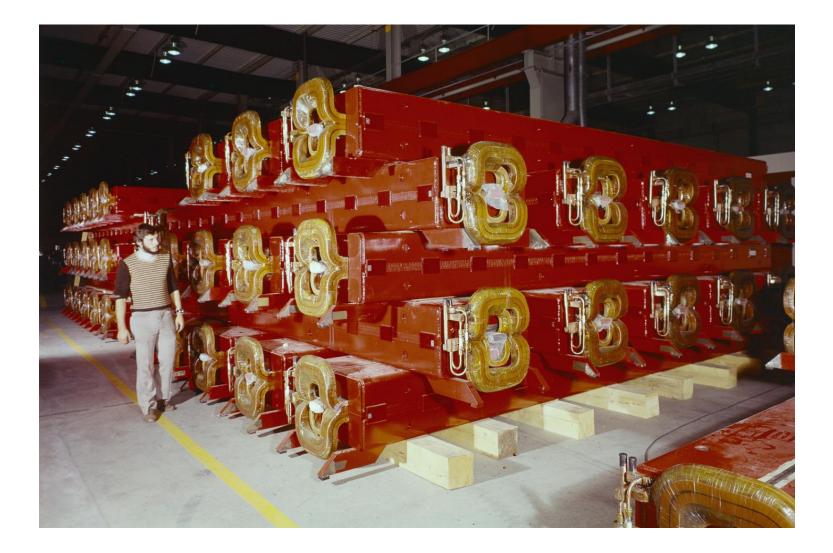
Magnets can be classified based on their geometry / what they do to the beam



This is a main dipole of the LHC at CERN: 8.3 T × 14.3 m



These are main dipoles of the SPS at CERN: 2.0 T × 6.3 m



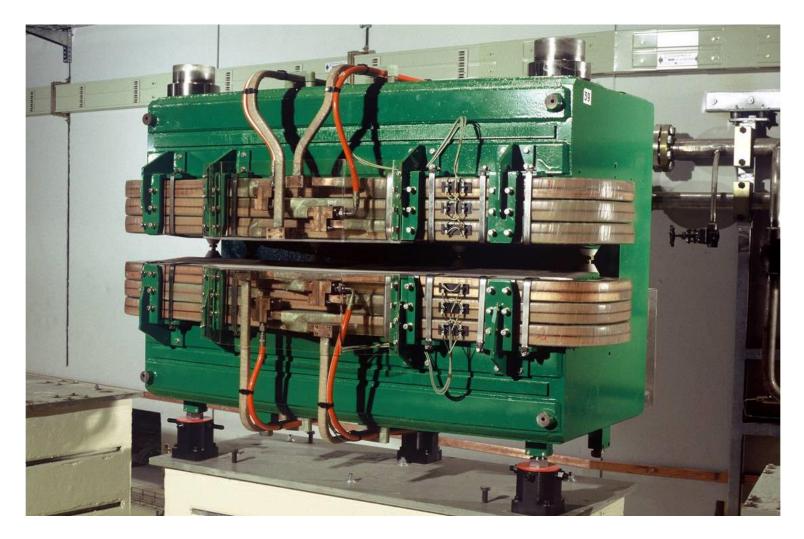
This is a cross section of a main quadrupole of the LHC at CERN: 223 T/m \times 3.2 m



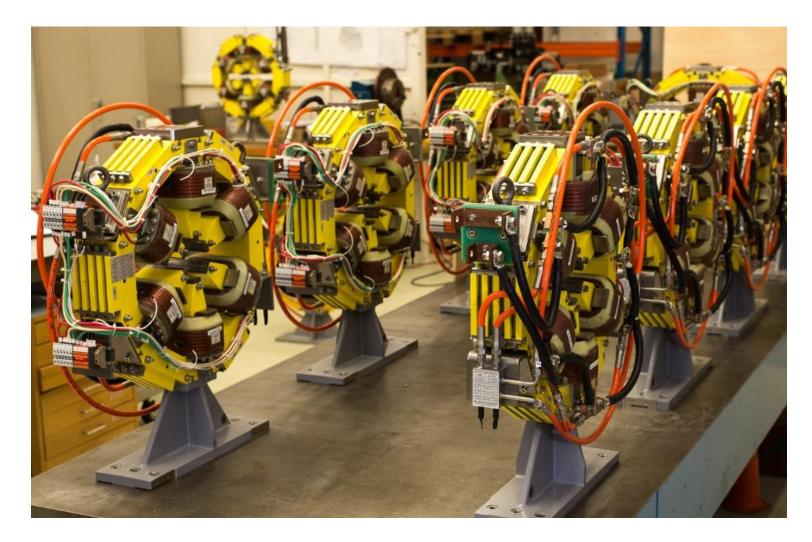
These are main quadrupoles of the SPS at CERN: $22 \text{ T/m} \times 3.2 \text{ m}$



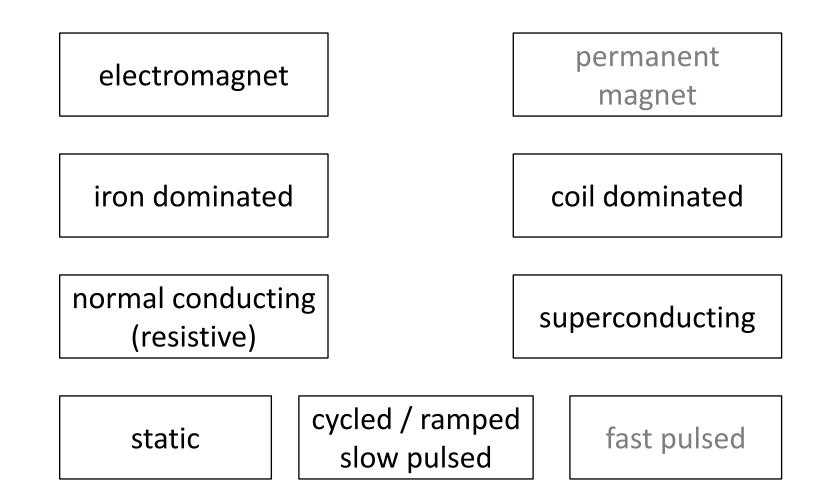
This is a combined function bending magnet of the ELETTRA light source



These are sextupoles (with embedded correctors) of the main ring of the SESAME light source



Magnets can be classified also differently, looking for example at their technology



Nomenclature

B

Η

magnetic field

B field magnetic flux density magnetic induction

T (Tesla)

H field magnetic field strength magnetic field

 μ_0 vacuum permeability

A/m (Ampere/m)

1.25663706212(19)·10⁻⁶ H/m 4π·10⁻⁷ H/m (Henry/m)

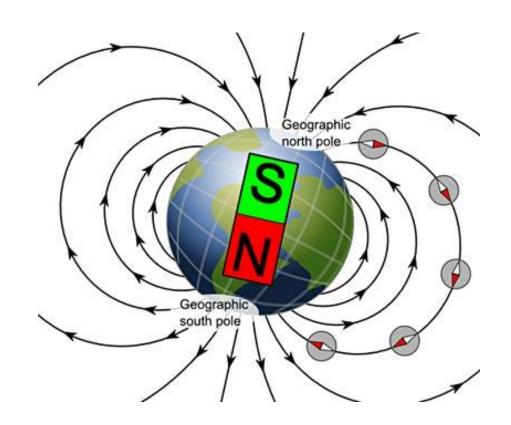
 μ_r relative permeability

dimensionless

 μ permeability, $\mu = \mu_0 \mu_r$

H/m

The polarity comes from the direction of the flux lines, that go from a North to a South pole (outside the magnetic material)

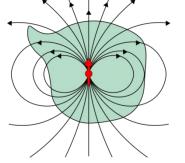




in Oxford, on 22/01/2025 |B| = 49099 nT = 0.0 49099 mT = 0.0000 49099 T Magnetostatic fields are described by Maxwell's equations, coupled with a law describing the material

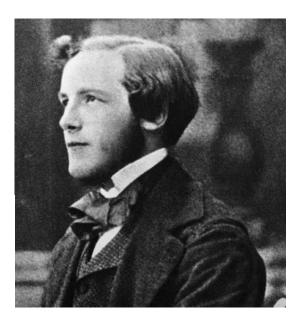
div
$$\vec{B} = 0$$

$$\oint_{S} \vec{B} \cdot \vec{dS} = 0$$



$$\operatorname{rot} \vec{H} = \vec{J}$$

$$\oint_{C} \vec{H} \cdot \vec{dl} = \int_{S} \vec{j} \cdot \vec{dS} = NI$$



James Clerk Maxwell

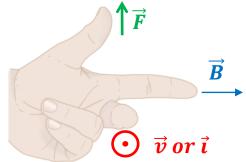
 $\vec{B} = \mu_0 \mu_r \vec{H}$

The Lorentz force is the link between electromagnetism and mechanics

 $\vec{F} = q \left[\vec{E} + \left(\vec{v} \times \vec{B} \right) \right]$ for charged particles

$$\vec{F} = \vec{B} \ x \vec{l} l$$

for conductors



My mnemonic: "F.B.I." \vec{v} and \vec{I} are a flow (current) of positive charges



Oliver Heaviside





Hendrik Lorentz

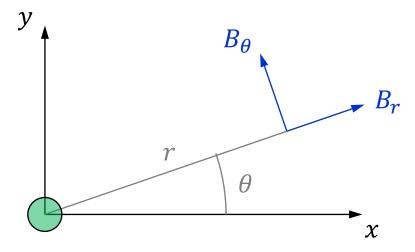
Pierre-Simon, marquis de Laplace

In synchrotrons / transfer lines magnets, the B field seen from the beam is often expressed as a series of multipoles

$$B_r = \sum_{n=1}^{\infty} \left(\frac{r}{R}\right)^{n-1} \left[B_n \sin(n\theta) + A_n \cos(n\theta)\right]$$

$$B_{\theta} = \sum_{n=1}^{\infty} \left(\frac{r}{R}\right)^{n-1} \left[B_n \cos(n\theta) - A_n \sin(n\theta)\right]$$

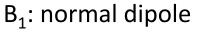
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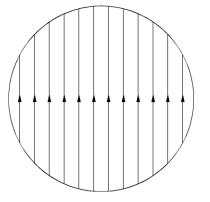


direction of the beam (orthogonal to plane)

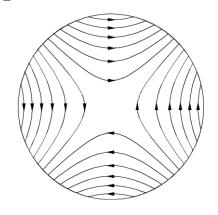
$$B_y(z) + iB_x(z) = \sum_{n=1}^{\infty} (B_n + iA_n) \left(\frac{z}{R}\right)^{n-1} \qquad z = x + iy = re^{i\theta}$$

Each multipole term corresponds to a field distribution; they can be added up (exploiting linear superposition)

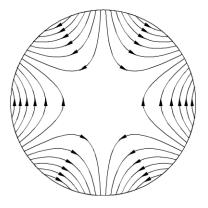




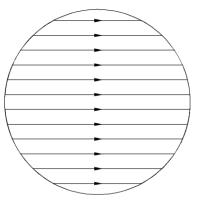
B₂: normal quadrupole



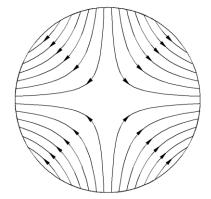
B₃: normal sextupole



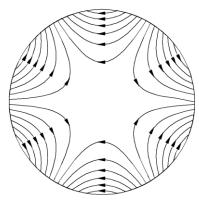
A₁: skew dipole



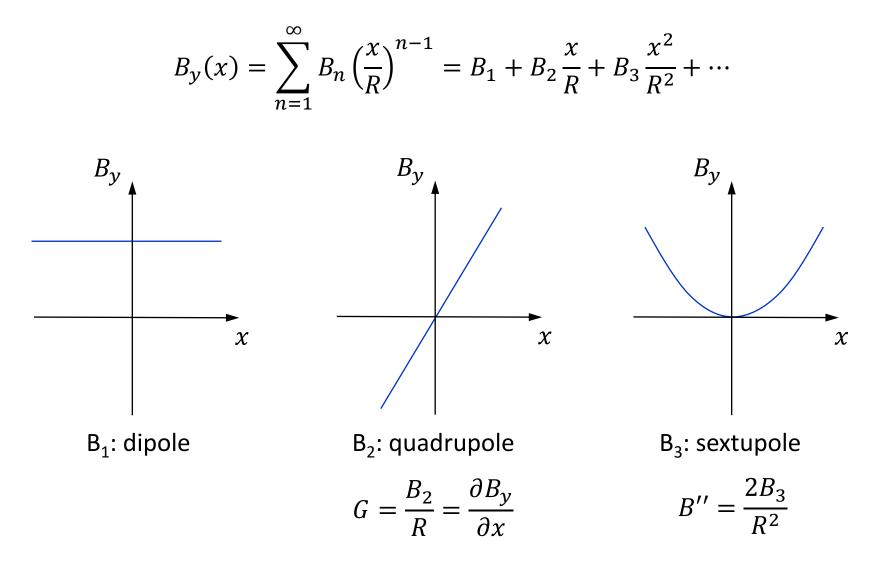
A₂: skew quadrupole



A₃: skew sextupole



The field profile in the horizontal plane follows a polynomial expansion



For the optics, usually the field or multipole component is given, together with the (magnetic) length: ex. from MAD-X

Dipolebend angle a [rad] & length L [m] k_0 [1/m] & length L [m] $k_0 = B / (B\rho)$ $B = B_1$



<u>Sextupole</u>

sextupole coefficient k₂ [1/m³] × length L [m] k₂ = (d²B_y/dx²) / (B ρ) (d²B_y/dx²)/2! = B₃/R² Here is how to compute magnetic quantities from MAD-X

entries, and vice versa

BEAM, PARTICLE=ELECTRON, PC=3.0; DEGREE:=PI/180.0; QF: QUADRUPOLE,L=0.5,K1=0.2; QD: QUADRUPOLE,L=1.0,K1=-0.2; B: SBEND,L=1.0,ANGLE=15.0*DEGREE;



(Bρ) = 10⁹/c*PC = 10^9/299792458*3.0 = 10.01 Tm

<u>dipole</u> (SBEND) B = |ANGLE|/L*(Bρ) = (15*pi/180)/1.0*10.01 = 2.62 T

<u>quadrupole</u> G = |K1|*(Bρ) = 0.2*10.01 = 2.00 T/m

$$B\rho = \frac{1}{c} \frac{A}{q} \sqrt{E_k^2 + 2E_k E_0}$$

 $B.Lmag = \theta.B\rho$

The harmonic decomposition is used also to describe the field quality (or field homogeneity), that is, the deviations of the actual B with respect to the ideal one

(normal) dipole

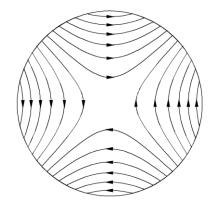
 $\vec{B}_{id}(x,y) = B_1 \vec{j}$

 $B_y(z) + iB_x(z) =$ $= B_1 + \frac{B_1}{10000} \left[ia_1 + (b_2 + ia_2) \left(\frac{z}{R}\right) + (b_3 + ia_3) \left(\frac{z}{R}\right)^2 + (b_4 + ia_4) \left(\frac{z}{R}\right)^3 + \cdots \right]$

$$b_2 = 10000 \frac{B_2}{B_1}$$
 $b_3 = 10000 \frac{B_3}{B_1}$ $a_1 = 10000 \frac{A_1}{B_1}$ $a_2 = 10000 \frac{A_2}{B_1}$...

The same expression can be written for a quadrupole

(normal) quadrupole





$$\vec{B}_{id}(x,y) = B_2[x\vec{j} + y\vec{i}]\frac{1}{R}$$

$$B_{y}(z) + iB_{x}(z) =$$

$$= B_{2}\frac{z}{R} + \frac{B_{2}}{10000} \left[ia_{2}\left(\frac{z}{R}\right) + (b_{3} + ia_{3})\left(\frac{z}{R}\right)^{2} + (b_{4} + ia_{4})\left(\frac{z}{R}\right)^{3} + \cdots \right]$$

$$b_3 = 10000 \frac{B_3}{B_2}$$
 $b_4 = 10000 \frac{B_4}{B_2}$ $a_2 = 10000 \frac{A_2}{B_2}$...

The *allowed / not-allowed* harmonics refer to some terms that shall / shall not cancel out thanks to design symmetries

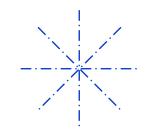
<u>fully symmetric dipoles</u> allowed: B_1 , b_3 , b_5 , b_7 , b_9 , etc. not-allowed: all the others

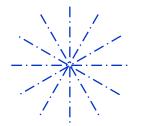




<u>half symmetric dipoles</u> allowed: B_1 , b_2 , b_3 , b_4 , b_5 , etc. not-allowed: all the others

<u>fully symmetric quadrupoles</u> allowed: B_2 , b_6 , b_{10} , b_{14} , b_{18} , etc. not-allowed: all the others





<u>fully symmetric sextupoles</u> allowed: B_3 , b_9 , b_{15} , b_{21} , etc. not-allowed: all the others

The field quality is often also shown with a $\Delta B/B$ plot



$$\frac{\Delta B}{B} = \frac{B(x, y) - B_{id}(x, y)}{B_{id}(x, y)}$$

done on one component, usually B_y for a dipole

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 Δ B/B can (at least locally) be expressed from the harmonics: this is the expansion for a dipole



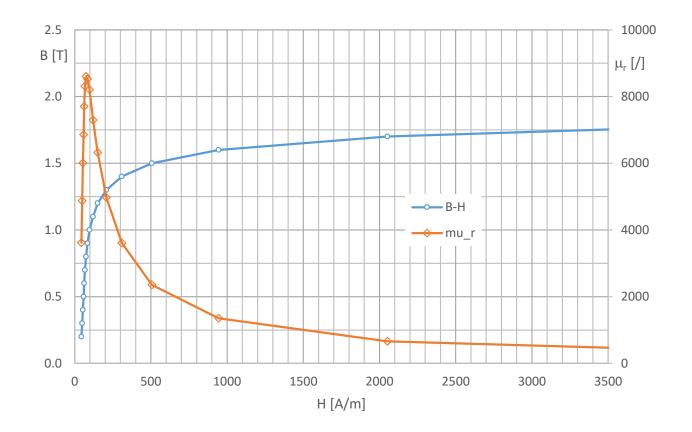
$$B_{y,id}(x) = B_1$$

$$B_{y}(x) = B_{1} + \frac{B_{1}}{10000} \left[b_{2} \left(\frac{x}{R} \right) + b_{3} \left(\frac{x}{R} \right)^{2} + b_{4} \left(\frac{x}{R} \right)^{3} + \cdots \right]$$

$$\frac{\Delta B}{B}(x) = \frac{1}{10000} \left[b_2 \left(\frac{x}{R}\right) + b_3 \left(\frac{x}{R}\right)^2 + b_4 \left(\frac{x}{R}\right)^3 + \cdots \right]$$

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- 2. Resistive magnets
- 3. Superconducting magnets
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Resistive magnets are in most cases "iron-dominated": the BH response of the yoke material is important



curves for typical M1200-100 A electrical steel

These are typical fields for resistive dipoles and quadrupoles, taken from machines at CERN

<u>PS @ 26 GeV</u> combined function bending B = 1.5 T(G = 6.2 T/m)

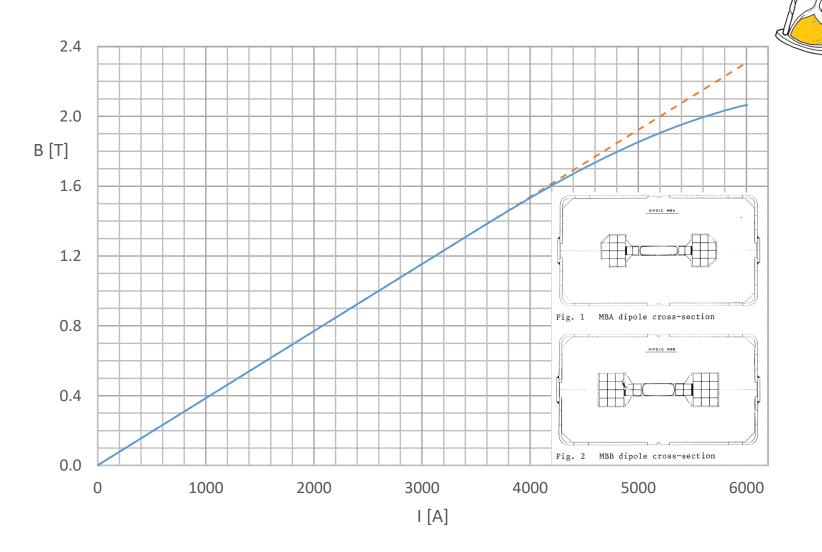


<u>SPS @ 450 GeV</u> bending quadrupole

B = 2.0 T B_{pole} = 21.7*0.044 = 0.95 T

TI2 / TI8 (transfer lines SPS to LHC, @ 450 GeV)bendingB = 1.8 Tquadrupole $B_{pole} = 53.5*0.016 = 0.86 T$

This is the (average) transfer function field B vs. current I for the SPS main dipoles



If the magnet is not dc, then an rms power / current is taken, considering the duty cycle



$$P_{rms} = RI_{rms}^2 = \frac{1}{T} \int_{0}^{T} R[I(t)]^2 dt$$

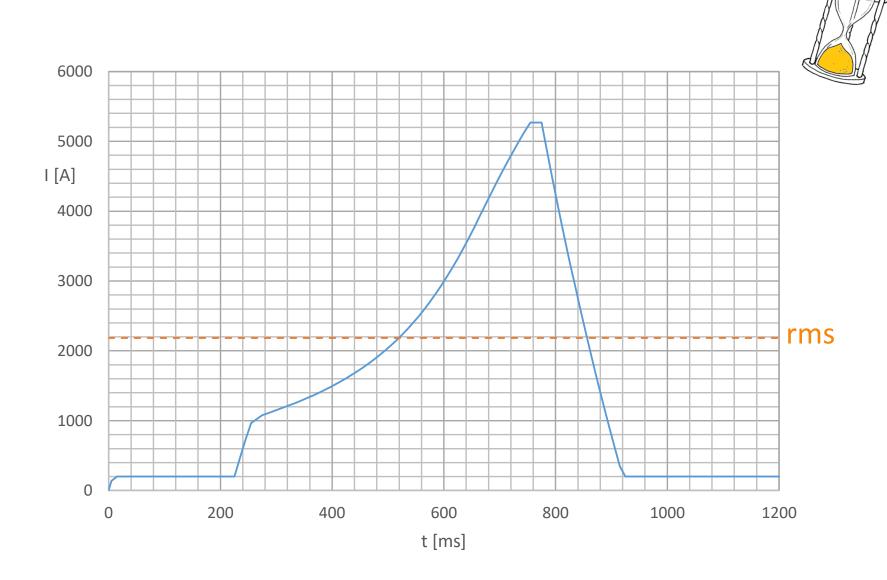
for a pure sine wave

$$I_{rms} = \frac{I_{peak}}{\sqrt{2}}$$

for a linear ramp from 0

$$I_{rms} = \frac{I_{peak}}{\sqrt{3}}$$

This is a cycle to 2.0 GeV of the PSB at CERN



For resistive coils, the material is most often copper, sometimes aluminum

Cu

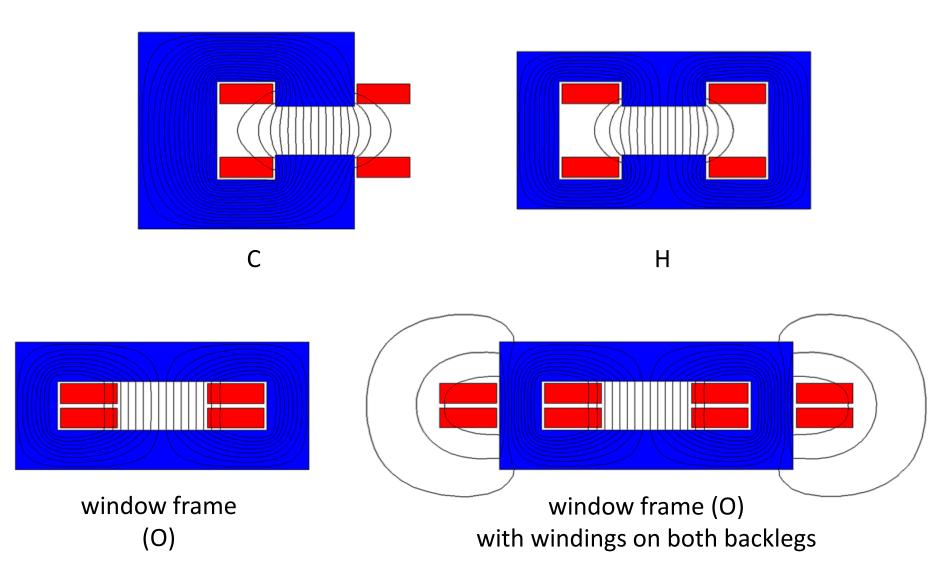
≈ 9300 \$/ton raw metal price electrical resistivity $1.72 \cdot 10^{-8} \Omega m$ 8.9 kg/dm³ density

Al ≈ 2650 \$/ton 2.65·10⁻⁸ Ωm 2.7 kg/dm^3

LHCb detector dipole Al coils coil mass 2 × 25 t power 2×2.1 MW



These are the most common types of resistive dipoles

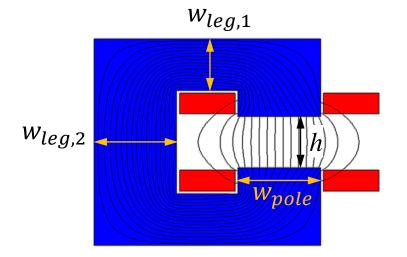


The table describes the field quality – in terms of allowed multipoles – for the different layouts of these examples



	C-shaped	H-shaped	O-shaped
b ₂	1.4	0	0
b ₃	-88.2	-87.0	0.2
b ₄	0.7	0	0
b ₅	-31.6	-31.4	-0.1
b ₆	0.1	0	0
b ₇	-3.8	-3.8	-0.1
b ₈	0.0	0	0
b ₉	0.0	0.0	0.0

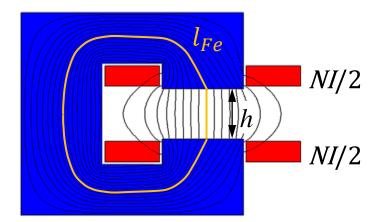
 b_n multipoles in units of 10⁻⁴ at R = 17 mm NI = 20 kA, h = 50 mm, w_{pole} = 80 mm The magnetic circuit is dimensioned so that the pole is wide enough for field quality, and there is enough room for the flux in the return legs



$$w_{pole} \cong w_{GFR} + 2.5h$$

$$B_{leg} \cong B_{gap} \frac{w_{pole} + 1.2h}{w_{leg}}$$

The Ampere-turns are a linear function of the gap and of the B field (at least up to saturation)



$$NI = \oint \vec{H} \cdot \vec{dl} = \frac{B_{Fe}}{\mu_0 \mu_r} \cdot l_{Fe} + \frac{B_{gap}}{\mu_0} \cdot h \cong \frac{B_{gap}h}{\mu_0}$$
$$NI = \frac{Bh}{\eta \mu_0} \quad \eta = \frac{1}{1 + \frac{1}{\mu_r} \frac{l_{Fe}}{h}}$$

Example of computation of Ampere-turns and current

central fieldB = 1.5 Ttotal gap80 mm

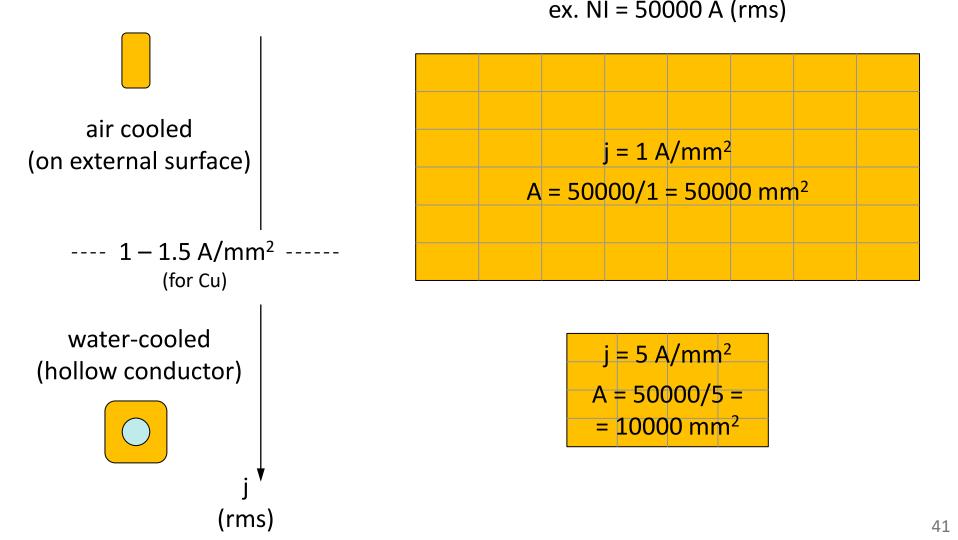
$$NI = \frac{Bh}{\eta\mu_0}$$

 $\eta \cong 0.97$

NI = (1.5*0.080)/(0.97*4*pi*10^-7) = 98446 A total

<u>low inductance option</u> 64 turns, I \cong 98500/64 = 1540 A L = 62.9 mH, R = 15.0 m Ω

<u>low current option</u> 204 turns, I \cong 98500/204 = 483 A L = 639 mH, R = 160 m Ω Besides the number of turns, the overall size of the coil depends on the current density, which drives the resistive power consumption (linearly)



These are common formulae for the main electric parameters of a resistive dipole (1/2)

Ampere-turns (total)
$$NI = \frac{Bh}{\eta\mu_0}$$

current $I = \frac{(NI)}{N}$

resistance (total) $R = \frac{\rho N L_{turn}}{A_{cond}}$

$$L \cong \eta \mu_0 N^2 A / h$$

 $A \cong (w_{pole} + 1.2h)(l_{Fe} + h)$

These are common formulae for the main electric parameters of a resistive dipole (2/2)



voltage
$$V = RI + L \frac{dI}{dt}$$

resistive power (rms)
$$P_{rms} = RI_{rms}^2$$

= $\rho j_{rms}^2 V_{cond}$

$$=\frac{\rho L_{turn} B_{rms} h}{\eta \mu_0} j_{rms}$$

magnetic stored energy
$$E_m = \frac{1}{2}LI^2$$

These are useful formulae for the main cooling parameters of a water-cooled resistive magnet

cooling flow
$$Q_{tot} \cong 14.3 \frac{P}{\Delta T}$$

$$Q_{tot} \cong N_{hydr}Q$$

water velocity

$$v = \frac{1000}{15\pi d^2}Q$$

1000

Reynolds number

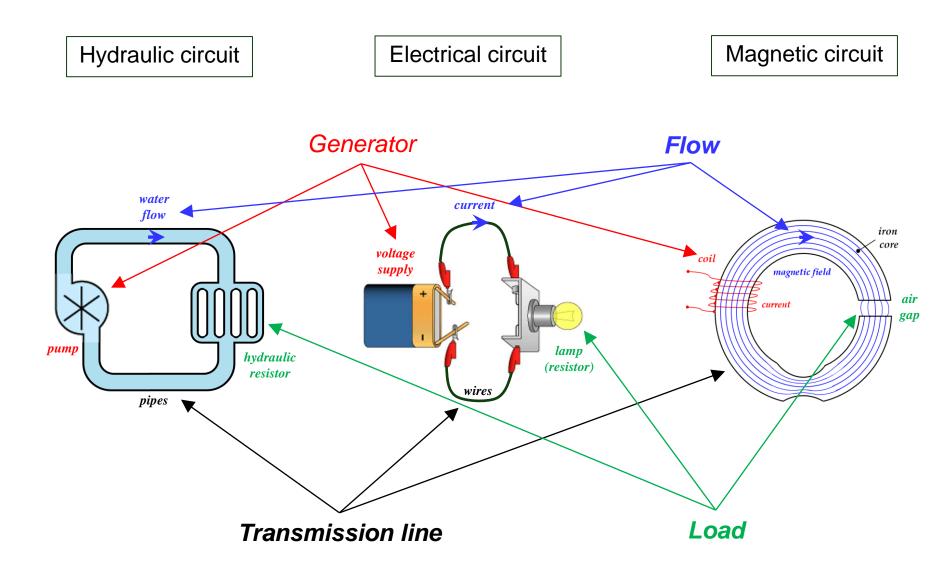
 $Re \cong 1400 dv$

pressure drop

$$\Delta p = 60L_{hydr} \frac{Q^{1.75}}{d^{4.75}}$$



Analogy of electromagnetism with other domains

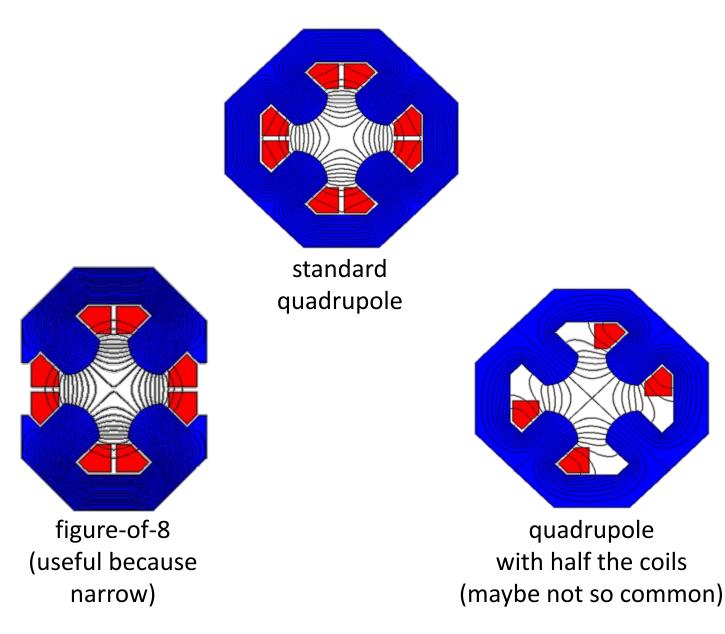


Analogy of electromagnetism with other domains

Fluid dynamics	Electricity	Electromagnetism		
ΔP = 🕿 x Q	ΔV = R x I	NI =		
Q = V x A	I = J x A	$\phi = B \times A$		
🕿 = L / ψ A	R = L / σ A	R = L / μ Α		
P: pressure : hydraulic resistance 	V: voltage R: resistance	NI: ampere-turns ℝ: reluctance		
Q: fluid volumic flow	I: current	φ: magnetic flux		
V: fluid velocity	J: current density	B: magnetic flux density		
A: pipe area	A: conductor area	A: magnetic circuit area		
L: pipe length	L: conductor length	L: magnetic circuit length		
ψ: hydraulic conductance	σ: electrical conductivity	μ: magnetic permeability		

Similar analogies exist in mechanics, electrostatics...

These are the most common types of resistive quadrupoles



These are useful formulae for standard resistive quadrupoles



pole tip field
$$B_{pole} = Gr$$

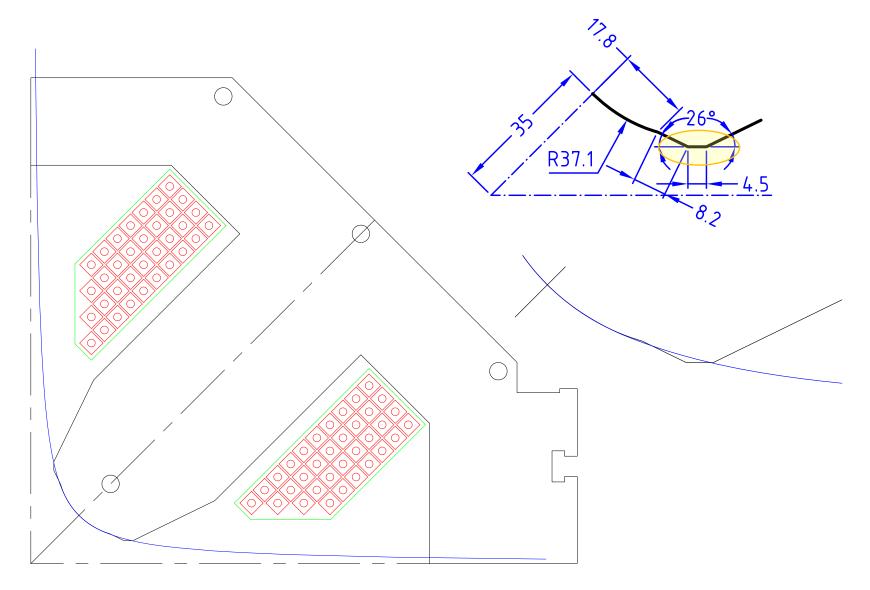
Ampere-turns (per pole)
$$NI = \frac{Gr^2}{2\eta\mu_0}$$

current $I = \frac{(NI)}{N}$

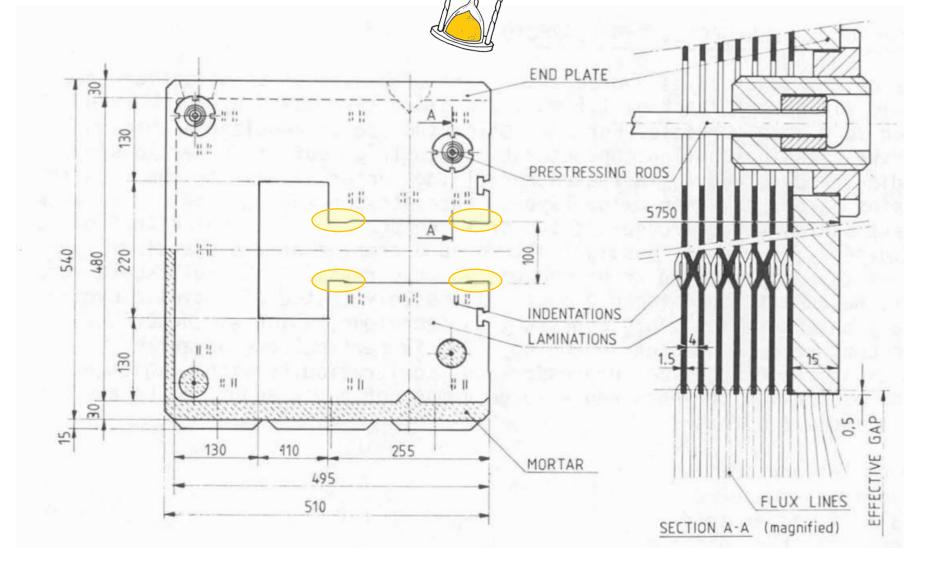
$$R = 4 \frac{\rho N L_{turn}}{A_{cond}}$$

The *ideal* poles for dipole, quadrupole, sextupole, etc. are lines of constant scalar potential

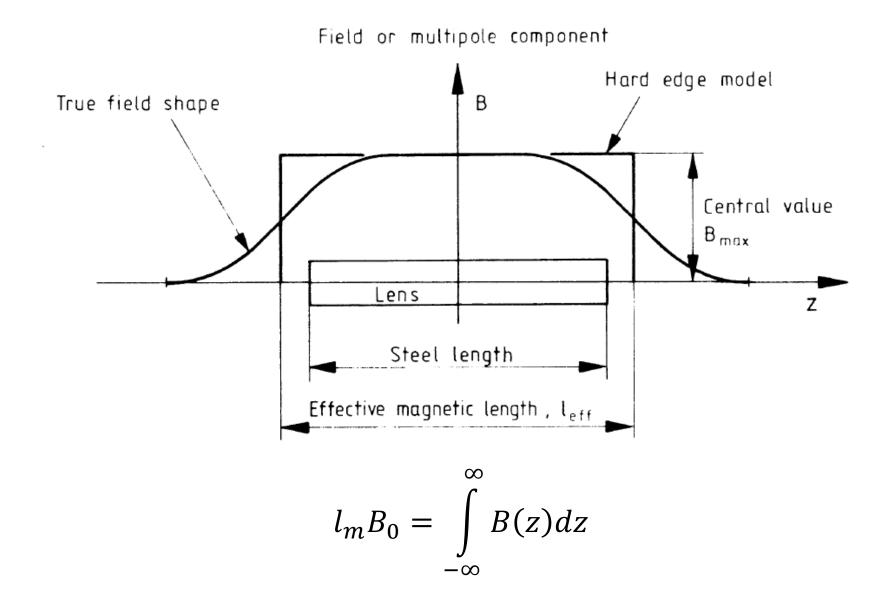
dipole $\rho \sin(\theta) = \pm h/2$ $y = \pm h/2$ (straight line) quadrupole $\rho^2 \sin(2\theta) = \pm r^2$ $2xy = \pm r^2$ (hyperbola) sextupole $\rho^3 \sin(3\theta) = \pm r^3$ $3x^2y - y^3 = \pm r^3$ As an example, this is the pole tip used in the SESAME quadrupoles vs. the theoretical hyperbola



This is the lamination of the LEP main bending magnets, with the pole shims well visible



In 3D, the longitudinal dimension of the magnet is described by a magnetic length



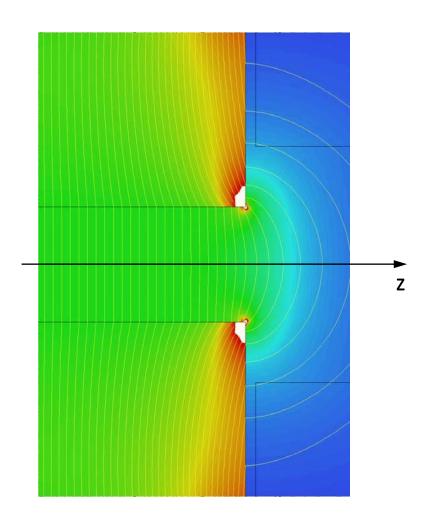
The magnetic length can be estimated at first order with simple formulae

 $l_m > l_{Fe}$



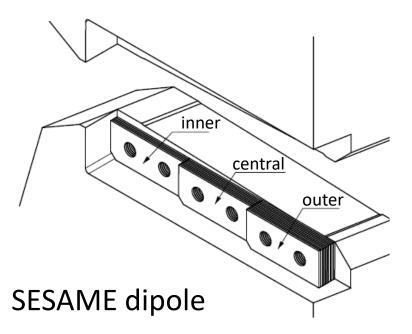
dipole $l_m \cong l_{Fe} + h$ quadrupole $l_m \cong l_{Fe} + 0.80r$ There are many different options to terminate the pole ends, depending on the type of magnet, its field level, etc.





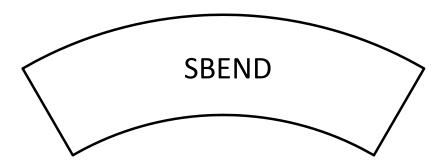


DIAMOND dipole

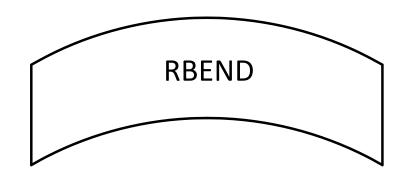


abrupt

Usually two dipole elements are found in lattice codes: the sector dipole (SBEND) and the parallel faces dipole (RBEND)

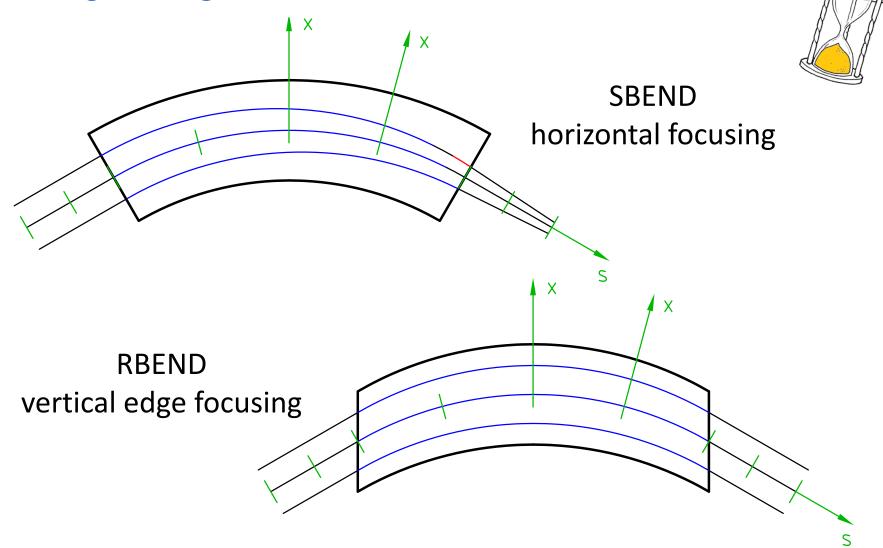






top views

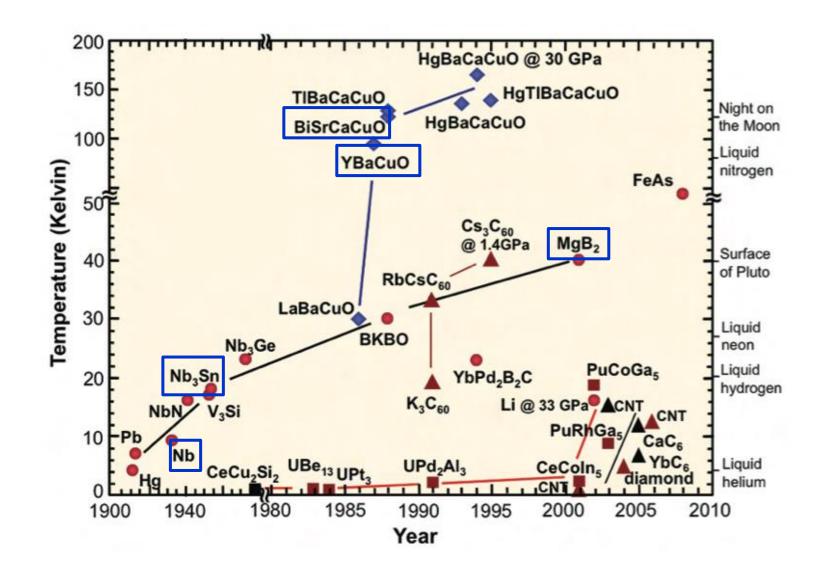
The two types of dipoles are slightly different in terms of focusing, for a geometric effect



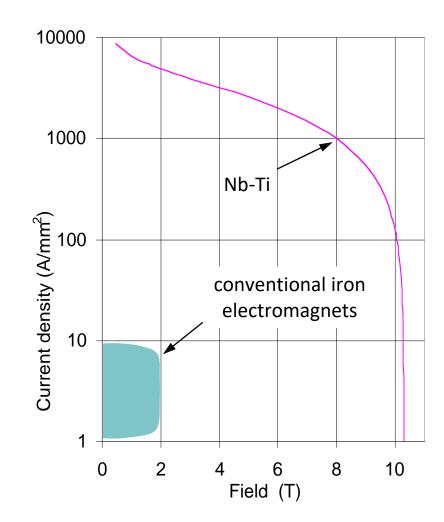
- and anything in between (playing with the edge angles) -

- 1. Introduction, jargon, general concepts and formulae
- 2. Resistive magnets
- 3. Superconducting magnets (thanks to Luca Bottura for the material of many slides)
- 4. Tutorial with FEMM

This is a history chart of superconductors, starting with Hg all the way to HTS (High Temperature Superconductors)



Superconductivity makes possible large accelerators with fields well above 2 T

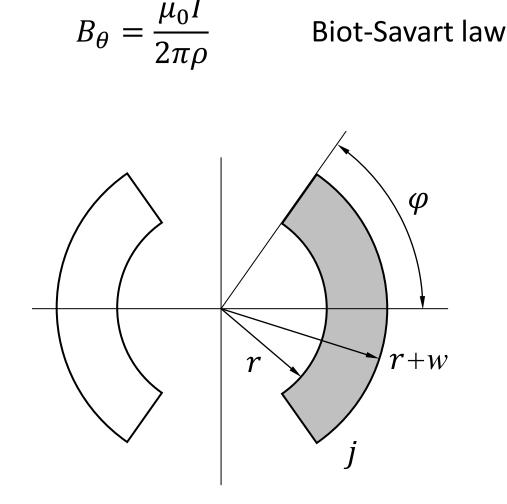


This is a summary of (somehow) practical superconductors

	LTS		HTS			
material	Nb-Ti	Nb₃Sn	MgB ₂	REBCO	BSCCO	Fe based
year of discovery	1961	1954	2001	1987	1988	2008
Т _с [К]	9.2	18.2	39	≈93	95 / 108	up to 58
B _{c2} [T]	≈14.5	≈30	>30	120250	≈200	>100

The field in the aperture of a superconducting dipole can be derived using Biot-Savart law (in 2D)

Biot-Savart law for an infinite wire

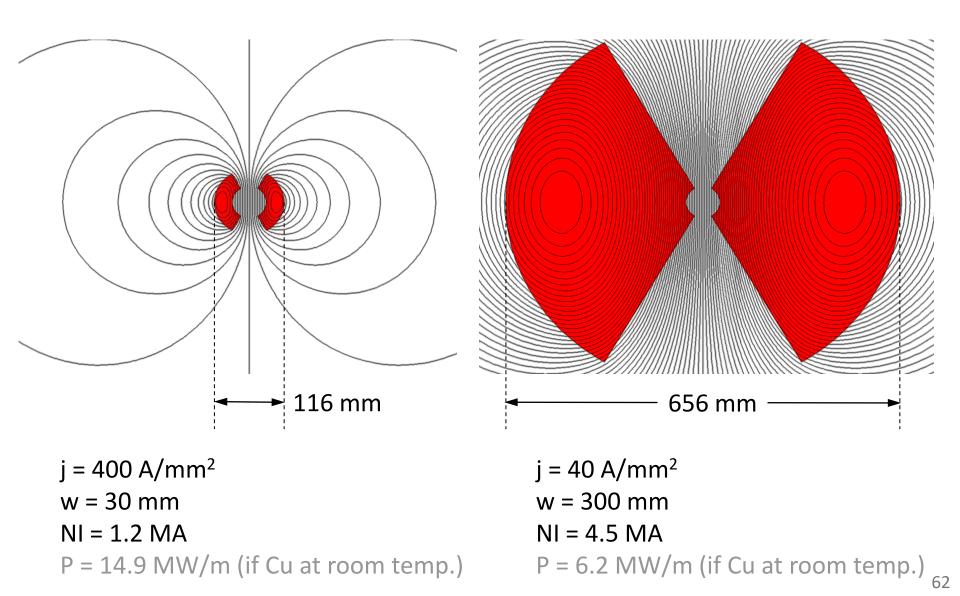


 $B = \frac{2\mu_0 \sin \varphi}{\pi} jw$
for a sector coil

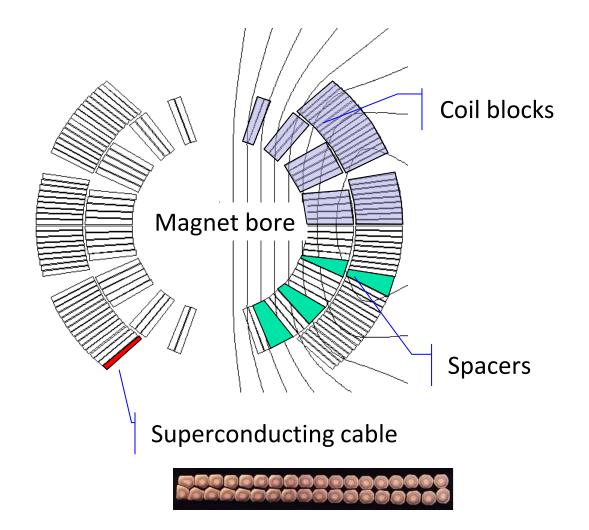
$$B = \frac{\sqrt{3\mu_0}}{\pi} jw$$

for a 60 deg sector coil

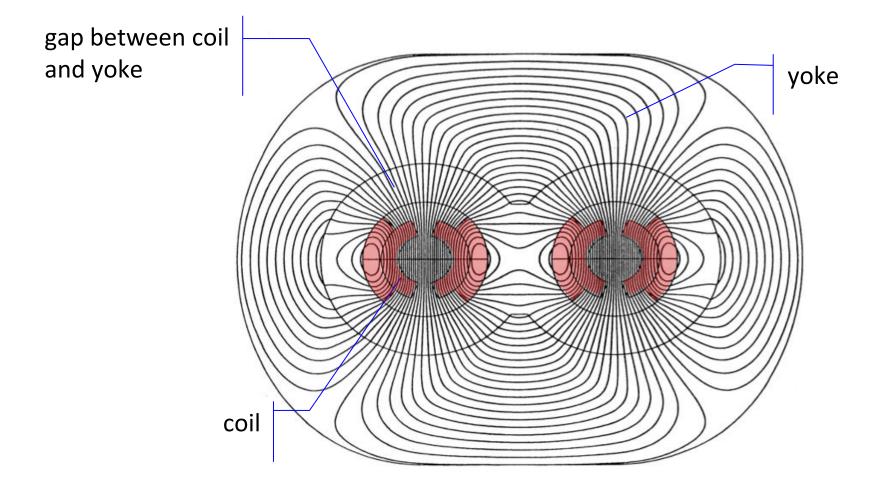
This is how it would look like one aperture of the LHC dipoles at 8.3 T, with two different current densities (without iron)



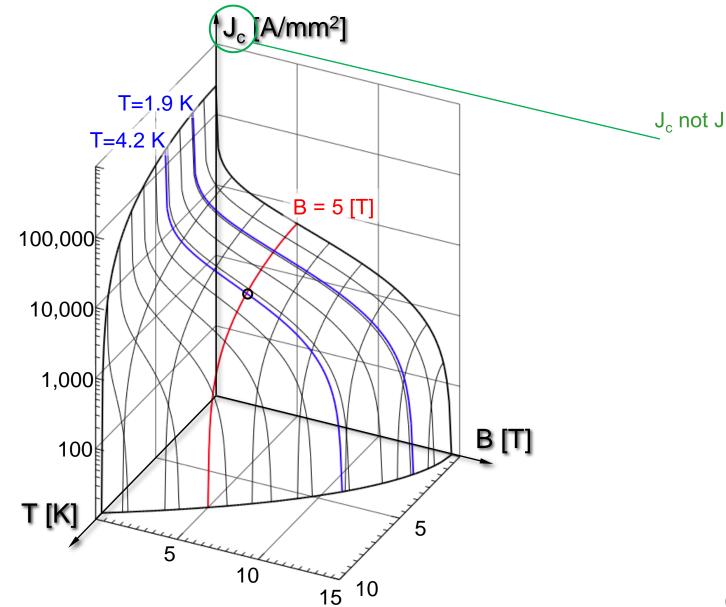
This is the actual coil of the LHC main dipoles (one aperture), showing the position of the superconducting cables



Around the coils, iron is used to close the magnetic circuit



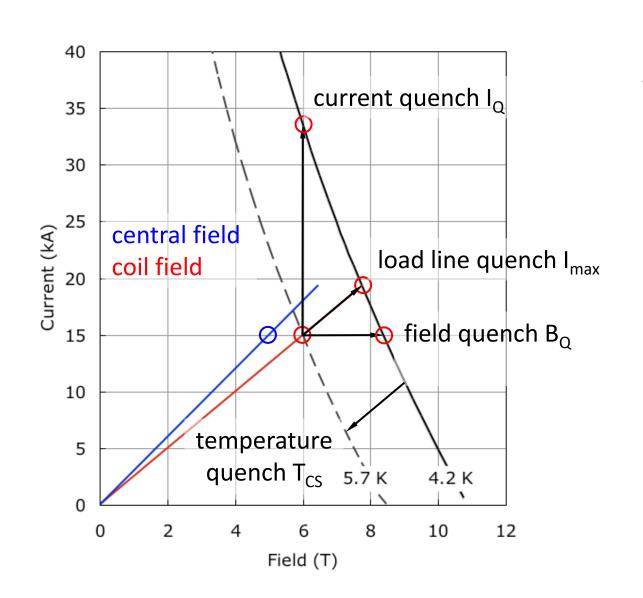
The allowable current density is high – though finite – and it depends on the temperature and the field



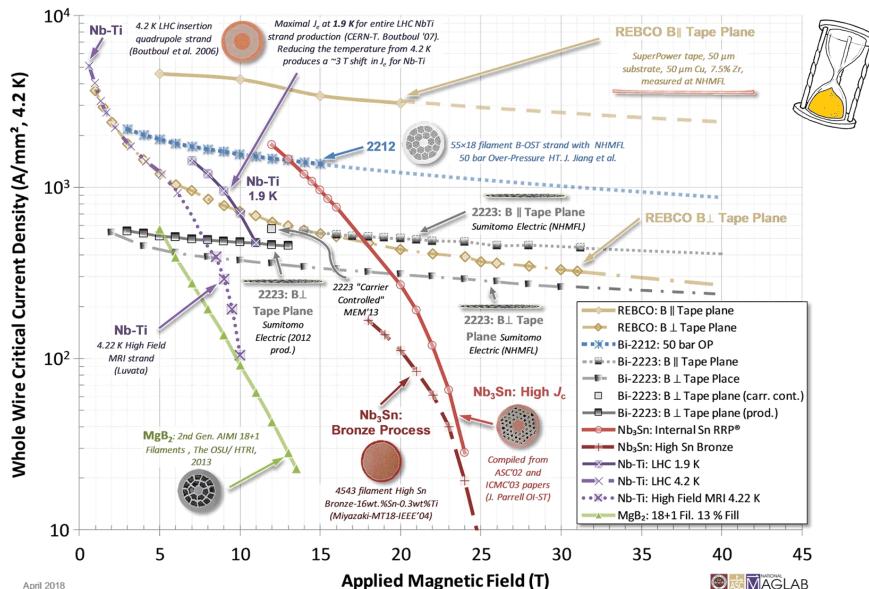
The maximum achievable field (on paper) depends on the amount of conductor and on the superconductor's critical line

Nb-Ti critical surface --- I_c = J_c x A_{sc} --- Nb-Ti critical current I_c (B) 40 35 6 Current/density [kA/mm² 30 25 25 20 20 15 quench! 5 T bore field 15 Temperaturett 10 conductor peak field 5 4.2 K Field [T] 100 10 2 0 4 6 8 10 12 14 Field (T)

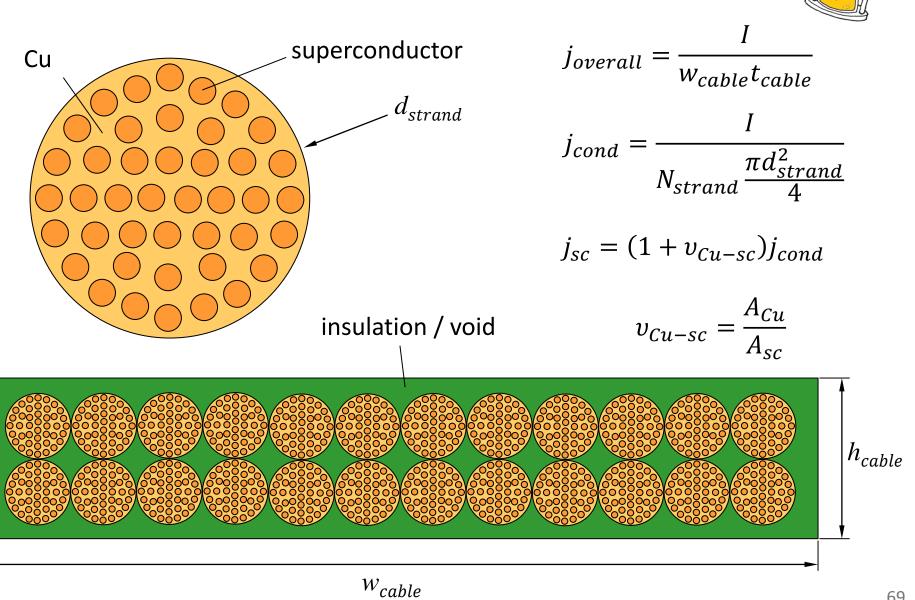
In practical operation, margins are needed with respect to this short sample limit



This is the best (Apr. 2018) critical current for several superconductors Applied Superconductivity Center at NHMFL



The overall current density is lower than the current density on the superconductor



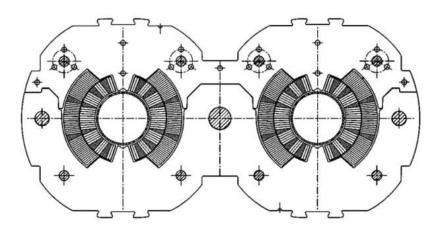
The forces can be very large, so the mechanical design is important



Nb-Ti LHC MB @ 8.3 T

 $F_x \approx 350$ t per meter

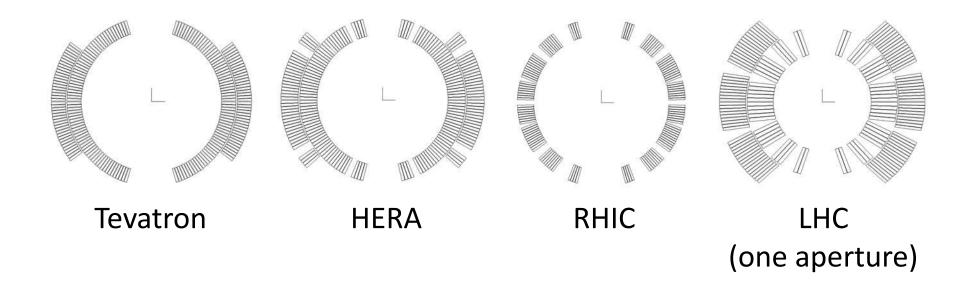
precision of coil positioning: 20-50 μm



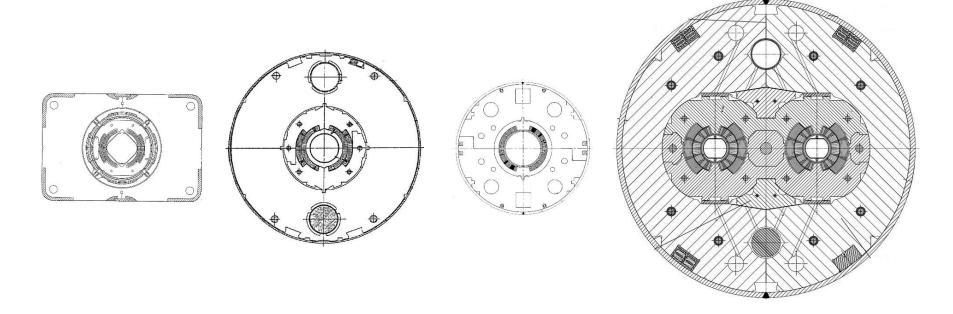
 $F_z \approx 40 \text{ t}$



The coil cross sections of several superconducting dipoles show a certain evolution; all were (are) based on Nb-Ti



Also the iron, the mechanical structure and the operating temperature can be quite diverse



Tevatron HERA	RHIC
76 mm bore 75 mm bore 8	80 mm bore
B = 4.3 T B = 5.0 T E	B = 3.5 T
T = 4.2 K T = 4.5 K T	Г = 4.3-4.6 К
first beam 1983 first beam 1991 f	irst beam 2000

56 mm bore B = 8.3 T T = 1.9 K first beam 2008

LHC

This is how they look in their machines









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There are different programs used for magnetic simulations

- 1. OPERA-2D and OPERA-3D, by Dassault Systèmes
- 2. ROXIE, by CERN
- 3. POISSON, by Los Alamos
- 4. FEMM
- 5. RADIA, by ESRF
- 6. ANSYS
- 7. Mermaid, by BINP
- 8. COMSOL

