Tomography of pp and $p\bar{p}$ Scattering

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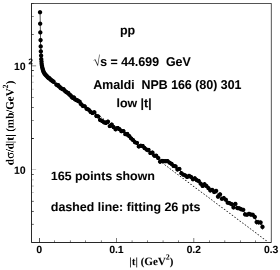
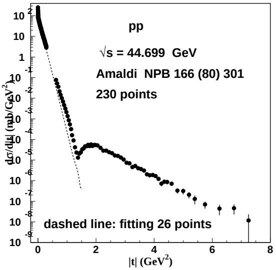
- Phenomenology of High Energy Scattering
- Use of Dispersion Relations
- Conclusions

Cern and Fermilab (1960 to 1990)

Data from 19 to 1800 GeV
Example: $\sqrt{s} = 44.699$ GeV





Description of scattering for small $|t|$

Nuclear plus Coulomb amplitudes

$$F^{C+N}(s,t) = F^{C}(s,t)e^{i\alpha\Phi(s,t)} + F^{N}(s,t)\ ,$$

Coulomb part :

$$F^{C} = (-/+)\ \frac{2\alpha}{|t|}\ F^{2}_{\rm proton}$$

with the proton electromagnetic form factor

$$F_{\rm proton} = (0.71/(0.71+|t|))^{2}\ .$$

$\Phi\ (s,t)$ is the relative phase and $F^{N}(s,t)$ is the complex nuclear amplitude.

Description of scattering for small $|t|$

Differential cross section

$$\frac{d\sigma}{dt} = \pi |F^{C+N}(s,t)|^2 = \pi |F^C(s,t) e^{i\alpha\Phi(s,t)} + F_R^N(s,t) + iF_I^N(s,t)|^2 \,,$$

For small angles : exponential forms

$$F^N(s,t) \approx F_R^N(s,0) e^{B_R t/2} + iF_I^N(s,0) e^{B_I t/2}$$

Usually B_R and B_I are treated as equal . However

$$B_R \neq B_I$$

(Need to generalize the Coulomb phase)

Differential cross section parameters

$$\rho = \frac{F_R^N(s,0)}{F_I^N(s,0)} \; ,$$

Optical theorem

$$\sigma = \frac{\mathrm{Im}\; F^N(s,0)}{s} \; ,$$

Four parameters ρ, σ,B_R and B_I describe $d\sigma/dt$:

$$\frac{d\sigma}{d|t|} = 0.389\pi \Bigg[\Big[\frac{\rho\sigma e^{B_R|t|/2}}{0.389 \times 4\pi} + F^C e^{\alpha\Phi_I} \cos{(\alpha\Phi_R)} \Big]^2 + \Big[\frac{\sigma e^{B_I|t|/2}}{0.389 \times 4\pi} + F^C e^{\alpha\Phi_I} \sin{(\alpha\Phi_R)} \Big]^2 \Bigg]$$

Actually, values of ρ and B_R are guided by dispersion relations.

The directly observed slope in

$$\frac{d\sigma}{dt} = \left(\frac{d\sigma}{dt}\right)_0 e^{-B|t|}$$

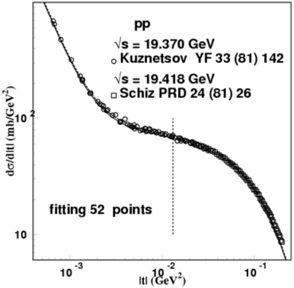
is

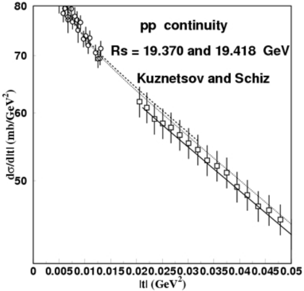
$$B(s) = \frac{B_R \rho^2 + B_I}{\rho^2 + 1}$$

Treatment of the data

Need to prepare data basis : compilation, selection, renormalization of original data.
Examples : 19.4 GeV for pp and 541 GeV for $\mathrm{p\bar{p}}$

Figure: pp forward scattering at 19.4 GeV





$p\bar{p}$ forward scattering at 541 GeV

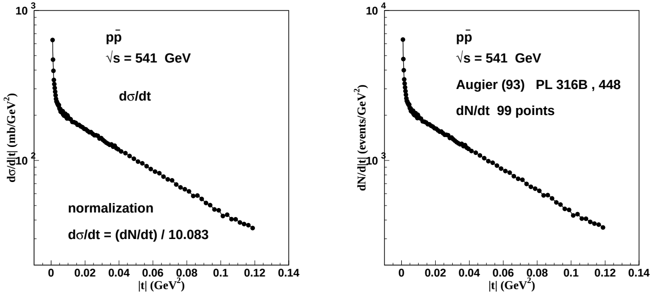


Figure: Event rate and differential cross section at 541 GeV .

Use of Dispersion Relations

Even and odd dispersion relations

$$\Re F_+(E,0) = K + \frac{2}{\pi}(E/m)^2 P \int_1^\infty \frac{\Im F_+(E')}{(E'/m)((E'/m)^2 - (E/m)^2)}\, d(E'/m),$$

$$\Re F_-(E,0) = \frac{2}{\pi}(E/m) P \int_1^\infty \frac{\Im F_-(E')}{(E'/m)^2 - (E/m)^2}\, d(E'/m)$$

Needed input : Use optical theorem for $\Im F_+(E',0)$ and $\Im F_-(E',0)$.

At high energies $\sqrt{s} = \sqrt{2mE}$ where $\sqrt{s}$ is center of mass energy.
Notation : Even and odd functional forms

$$P_+[F(E/m)] \equiv \frac{2}{\pi}(E/m)^2 P\int_1^\infty \frac{[F(E'/m)]}{(E'/m)((E'/m)^2-(E/m)^2)} d(E'/m)$$

and

$$P_-[F(E/m)] \equiv \frac{(2E/m)}{\pi} P\int_1^\infty \frac{[F(E'/m)]}{(E'/m)^2-(E/m)^2} d(E'/m)$$

Amplitudes in pp and $\mathrm{p\bar{p}}$ channels

$$F_+(E,0) \equiv \frac{F^{pp}(E)+F^{p\bar{p}}(E)}{2} \qquad F_-(E,0) \equiv \frac{F^{pp}(E)-F^{p\bar{p}}(E)}{2}$$

In terms of observables quantities

$$\frac{\Re F_{+}(E,0)}{2mE}=\frac{\sigma_{\mathrm{pp}}\rho_{\mathrm{pp}}+\sigma_{\mathrm{p\bar{p}}}\rho_{\mathrm{p\bar{p}}}}{2},$$

and

$$\frac{\Re F_{-}(E,0)}{2mE}=\frac{\sigma_{\mathrm{pp}}\rho_{\mathrm{pp}}-\sigma_{\mathrm{p\bar{p}}}\rho_{\mathrm{p\bar{p}}}}{2},$$

Need to determine de subtraction constant K.

Dispersion Relations for Slopes

With exponential $|t|$ dependence in the amplitudes :

$$\Re F_{pp}(E) e^{\frac{B_{pp}^R}{2}|t|} = \frac{K}{2mE} + \frac{1}{2mE}\left[P_+ \left[\Im F_+(E') e^{\frac{B_+^I}{2}|t|} \right] - P_- \left[\Im F_-(E') e^{\frac{B_-^I}{2}|t|} \right] \right]$$

Expand the exponentials to first order in $|t|$ and subtract the dispersion relations to obtain

$$\Re F_{pp}(E) B^R_{pp} = \frac{1}{2mE}\left[P_+\left[\Im F_+(E')B^I_+\right] - P_-\left[\Im F_-(E')B^I_-\right]\right]$$

or

$$\sigma_{\rm pp}\rho_{\rm pp} B^R_{\rm pp} = \frac{1}{(2mE)^2}\left[P_+\left[\Im F_+(E')B^I_+\right] - P_-\left[\Im F_-(E')B^I_-\right]\right]$$

and for $p\bar{p}$

$$\sigma_{p\bar{p}}\rho_{p\bar{p}} B^R_{\rm p\bar{p}} = \frac{1}{(2mE)^2}\left[P_+\left[\Im F_+(E')B^I_+\right] + P_-\left[\Im F_-(E')B^I_-\right]\right]$$

With explicit input for

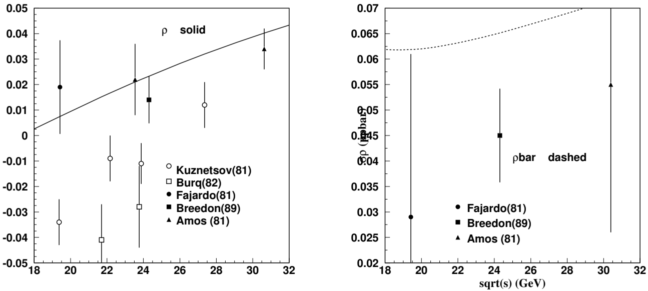
$$B_I(s) = c_1 + c_2 \ln(s)$$

we obtain predictions for B_R which can be used in the description of $d\sigma/dt$.

We have analysed the forward data using this information.

Note: the triple product does not depend on K.

ρ_{pp} and $\rho_{p\bar{p}}$



Long t Description

The exponential forms for the imaginary and real amplitudes have limited scope. We wish to have a description of $d\sigma/dt$ for all t. Parameterization of the imaginary and real parts of the amplitude

$$\mathcal{I}(t) \equiv 2\ \alpha \mathrm{e}^{-\beta|t|} + \lambda 2\rho \mathrm{e}^{\rho\gamma} A_\gamma(t) \tag{1}$$

and

$$\mathcal{R}(t) \equiv \alpha' \mathrm{e}^{-\beta'|t|} + \lambda' 2\rho \mathrm{e}^{\rho\gamma'} A_{\gamma'}(t)\ , \tag{2}$$

where we have a SHAPE FUNCTION with one parameter

$$A_\gamma(t) \equiv \frac{\mathrm{e}^{-\gamma\sqrt{\rho^2+a^2|t|}}}{\sqrt{\rho^2+a^2|t|}} - \mathrm{e}^{\rho\gamma} \frac{e^{-\gamma\sqrt{4\rho^2+a^2|t|}}}{\sqrt{4\rho^2+a^2|t|}}\ , \tag{3}$$

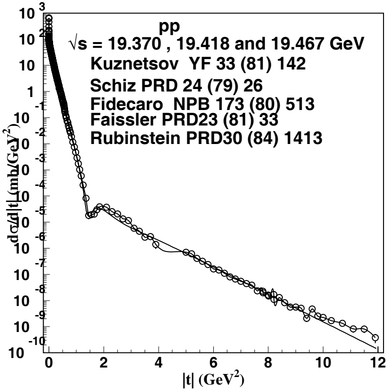
with $\rho = 3\pi/8$, and we have grouped the factors $2\rho e^{\rho\gamma} A_\gamma(t)$ in order to have $2\rho e^{\rho\gamma} A_\gamma(0) = 1$.

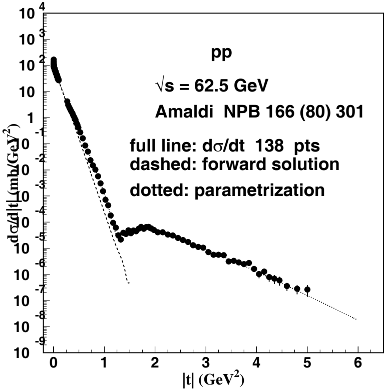
The dimensionless scattering amplitude and the elastic differential cross section are given respectively by

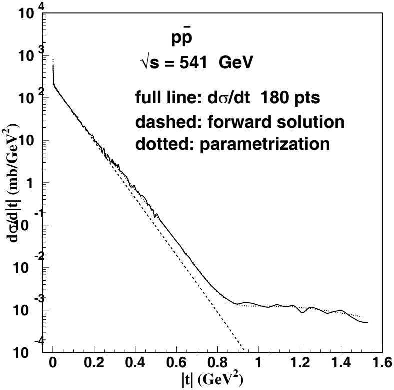
$$T(s,t) = 4\sqrt{\pi} s [i\mathcal{I}(t) + \mathcal{R}(t)] \ , \tag{4}$$

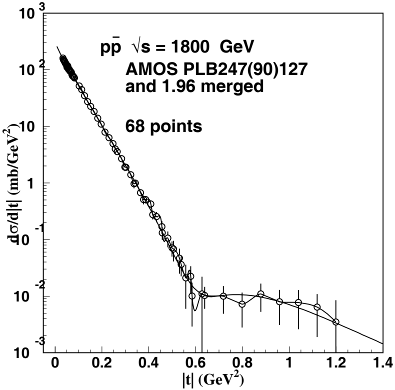
and

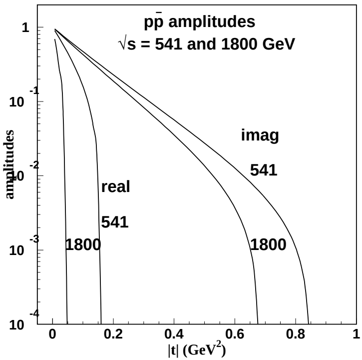
$$\frac{d\sigma^{e\ell}}{dt} = \frac{1}{16\pi s^2} |T(s,t)|^2 \ . \tag{5}$$

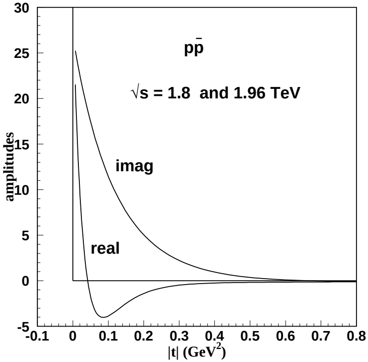

pp
√s = 19.370 , 19.418 and 19.467 GeV
Kuznetsov YF 33 (81) 142
Schiz PRD 24 (79) 26
Fidecaro NPB 173 (80) 513
Faissler PRD23 (81) 33
Rubinstein PRD30 (84) 1413
dσ/d|t| (mb/GeV²)
|t| (GeV²)

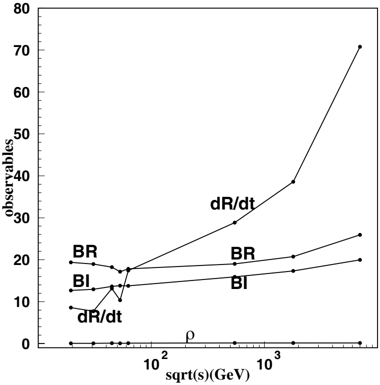

pp
√s = 62.5 GeV
Amaldi NPB 166 (80) 301
full line: dσ/dt 138 pts
dashed: forward solution
dotted: parametrization
dσ/d|t| (mb/GeV²)
|t| (GeV²)

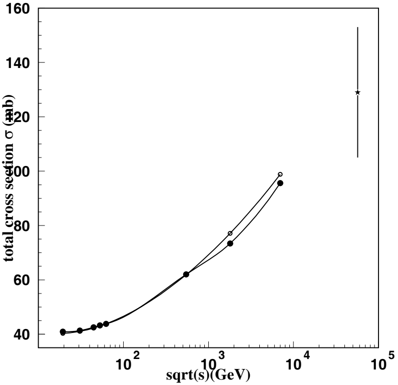

$p\bar{p}$
$\sqrt{s}$ = 541 GeV
full line: dσ/dt 180 pts
dashed: forward solution
dotted: parametrization
dσ/d|t| (mb/GeV^2)
|t| (GeV^2)

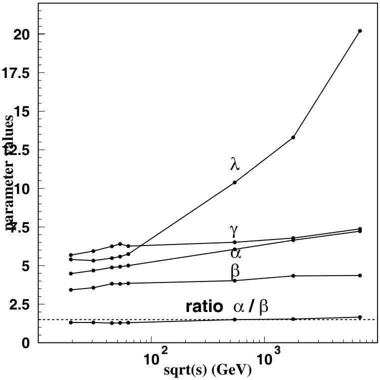

$p\bar{p}$ $\sqrt{s}$ = 1800 GeV
AMOS PLB247(90)127
and 1.96 merged
68 points
dσ/d|t| (mb/GeV^2)
|t| (GeV^2)

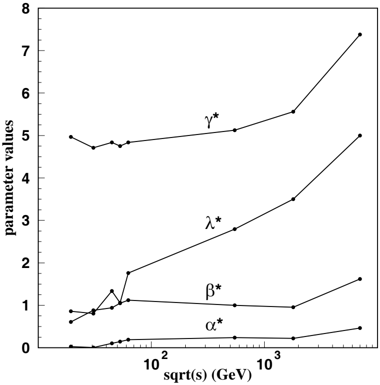

pp̄ amplitudes
√s = 541 and 1800 GeV
imag
541
real
541
1800
1800
amplitudes
|t| (GeV²)


pp̄
√s = 1.8 and 1.96 TeV
imag
real
amplitudes
|t| (GeV²)

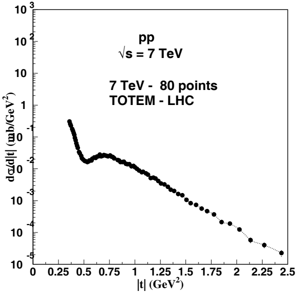

80
70
60
50
40
30
20
10
0
observables
dR/dt
BR
BR
BI
BI
dR/dt
ρ
10 2
10 3
sqrt(s)(GeV)

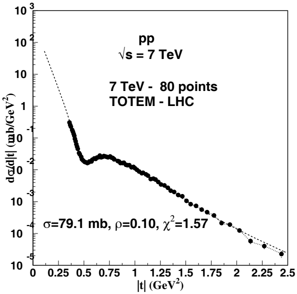

160
140
120
100
80
60
40
total cross section σ (mb)
10 2
10 3
10 4
10 5
sqrt(s)(GeV)

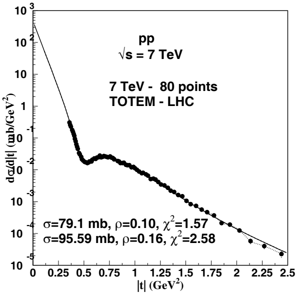

20
17.5
15
12.5
10
7.5
5
2.5
0
parameter values
λ
γ
α
β
ratio α / β
10^2
10^3
sqrt(s) (GeV)

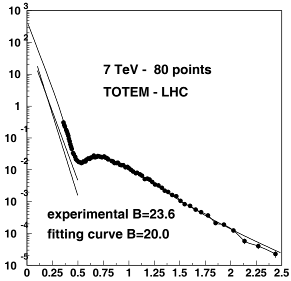

8
7
6
5
4
3
2
1
0
parameter values
γ*
λ*
β*
α*
10^2
10^3
sqrt(s) (GeV)

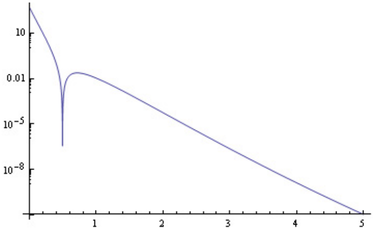
E AGORA ?

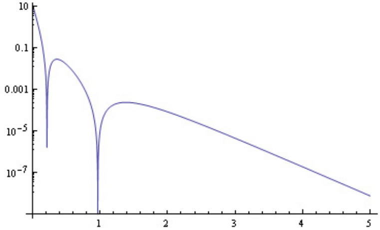

pp
√s = 7 TeV
7 TeV - 80 points
TOTEM - LHC
dσ/d|t| (mb/GeV²)
|t| (GeV²)
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5
10³
10²
10
1
10⁻¹
10⁻²
10⁻³
10⁻⁴
10⁻⁵

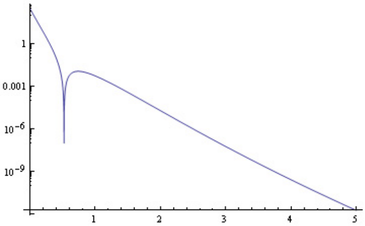

pp
√s = 7 TeV
7 TeV - 80 points
TOTEM - LHC
σ=79.1 mb, ρ=0.10, χ²=1.57
dσ/d|t| (mb/GeV²)
|t| (GeV²)
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5

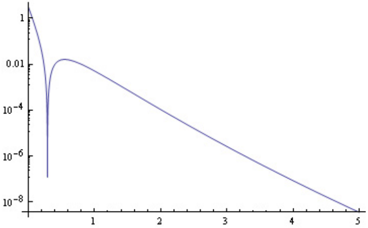

pp
√s = 7 TeV
7 TeV - 80 points
TOTEM - LHC
σ=79.1 mb, ρ=0.10, χ²=1.57
σ=95.59 mb, ρ=0.16, χ²=2.58
dσ/d|t| (mb/GeV²)
|t| (GeV²)
0 0.25 0.5 0.75 1 1.25 1.5 1.75 2 2.25 2.5

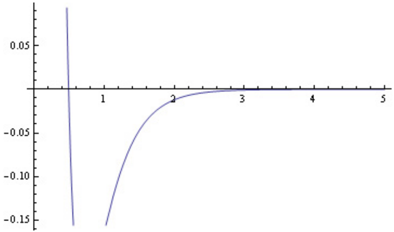

7 TeV - 80 points
TOTEM - LHC
experimental B=23.6
fitting curve B=20.0
10^3
10^2
10
1
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-5}
0
0.25
0.5
0.75
1
1.25
1.5
1.75
2
2.25
2.5

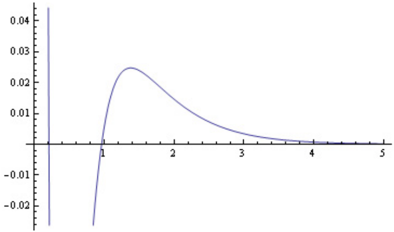

10
0.01
10^{-5}
10^{-8}
1
2
3
4
5

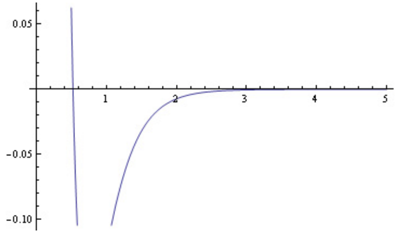

10
0.1
0.001
10^{-5}
10^{-7}
1
2
3
4
5

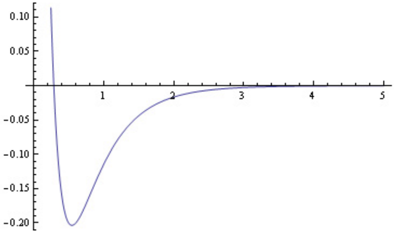

1
0.001
10^{-6}
10^{-9}
1
2
3
4
5


1
0.01
10^{-4}
10^{-6}
10^{-8}
1
2
3
4
5


0.05
-0.05
-0.10
-0.15
1
2
3
4
5


0.04
0.03
0.02
0.01
-0.01
-0.02
1
2
3
4
5


0.05
-0.05
-0.10
1
2
3
4
5


0.10
0.05
-0.05
-0.10
-0.15
-0.20
1
2
3
4
5