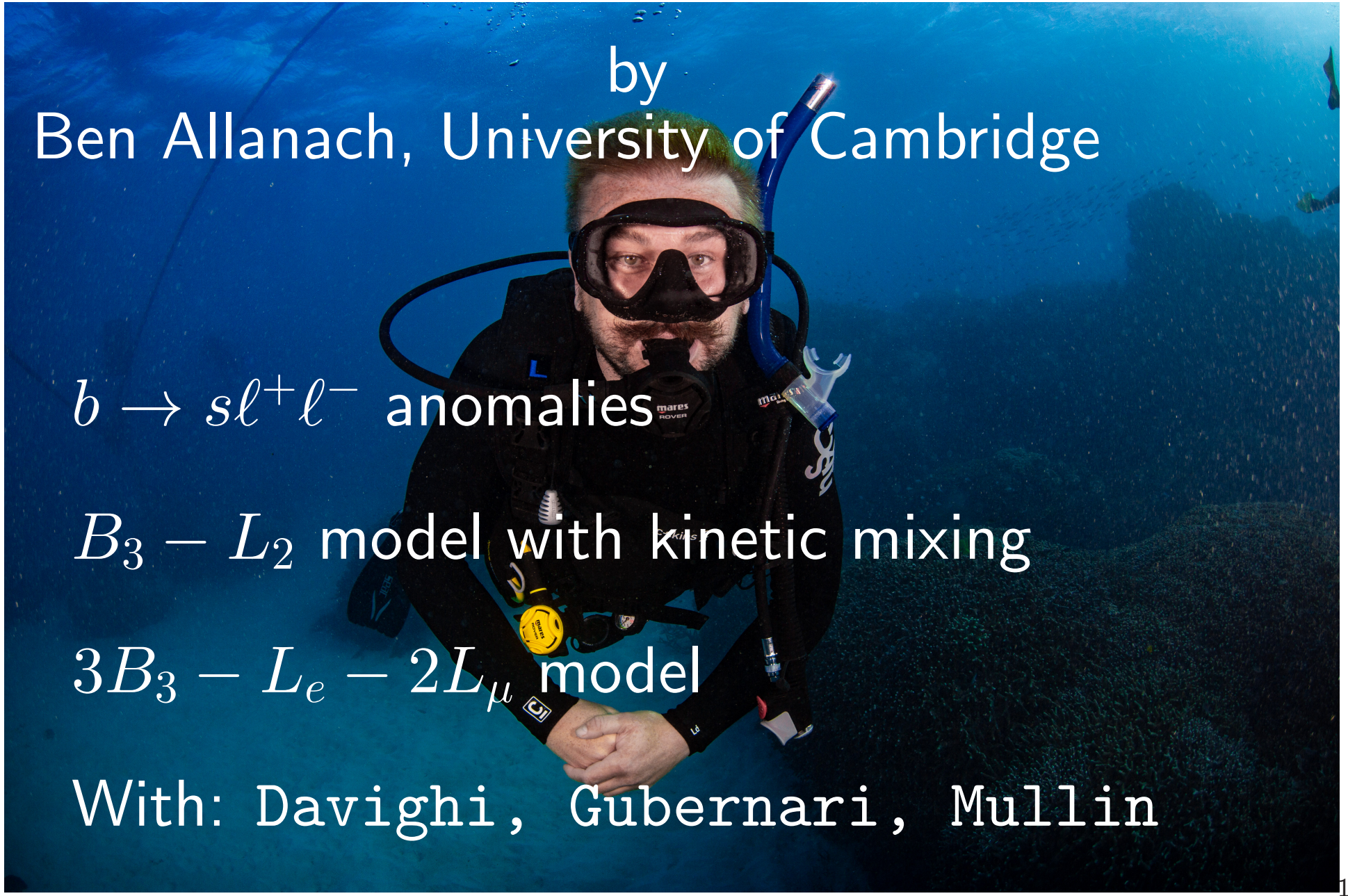
Z' models in light of LFU/anomalies



by
Ben Allanach, University of Cambridge

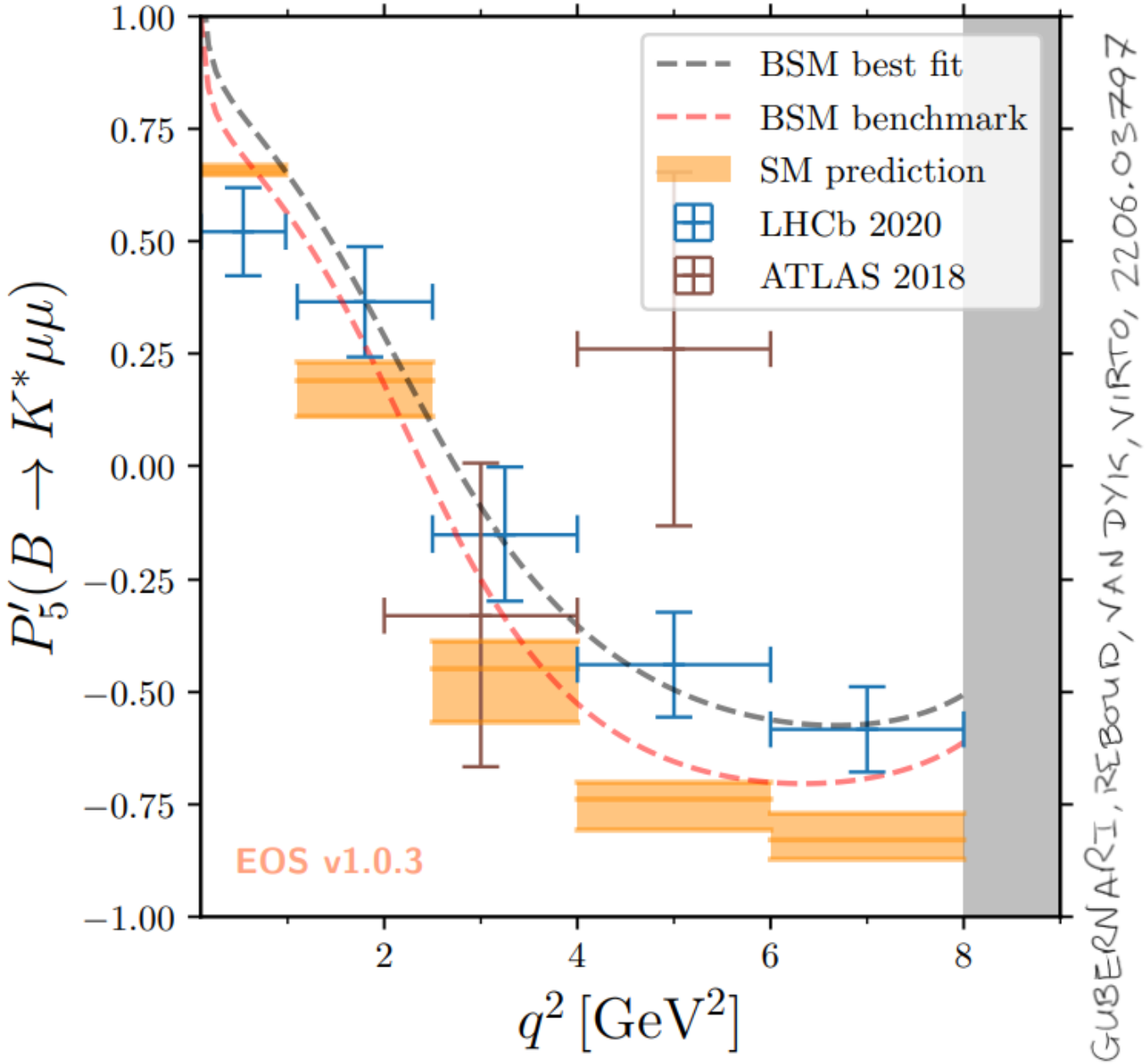
$b \to s\ell^+\ell^-$ anomalies

$B_3 - L_2$ model with kinetic mixing

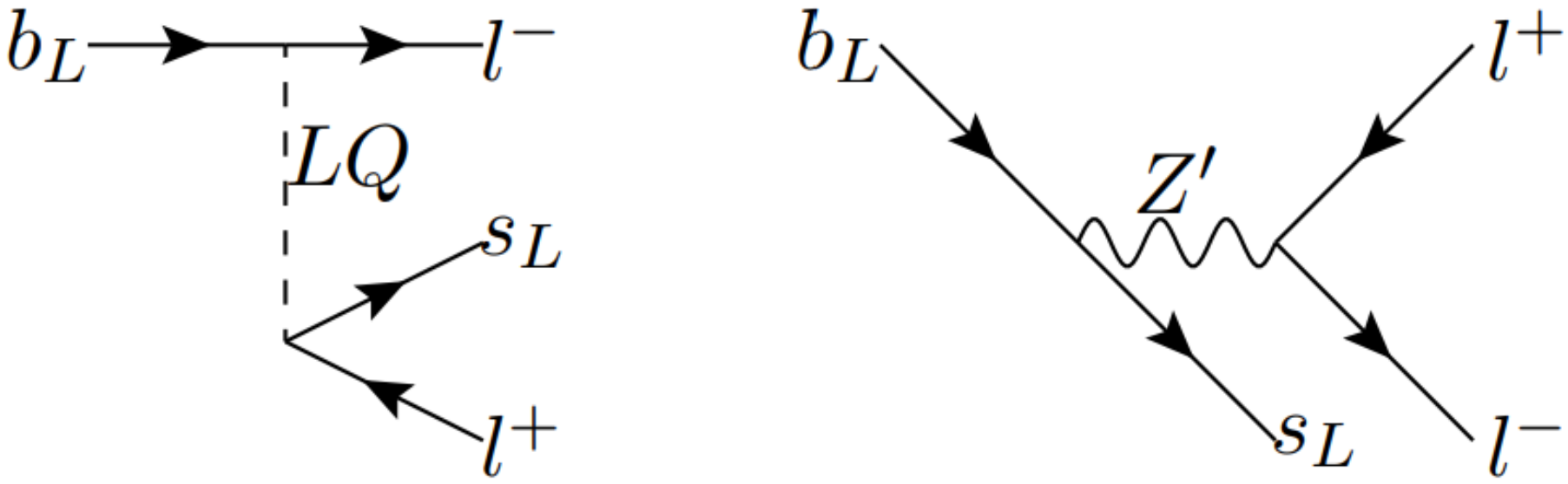
$3B_3 - L_e - 2L_\mu$ model

With: Davighi, Gubernari, Mullin

$$P_5' = S_5 / \sqrt{F_L(1 - F_L)}$$



Tree-level Explanations

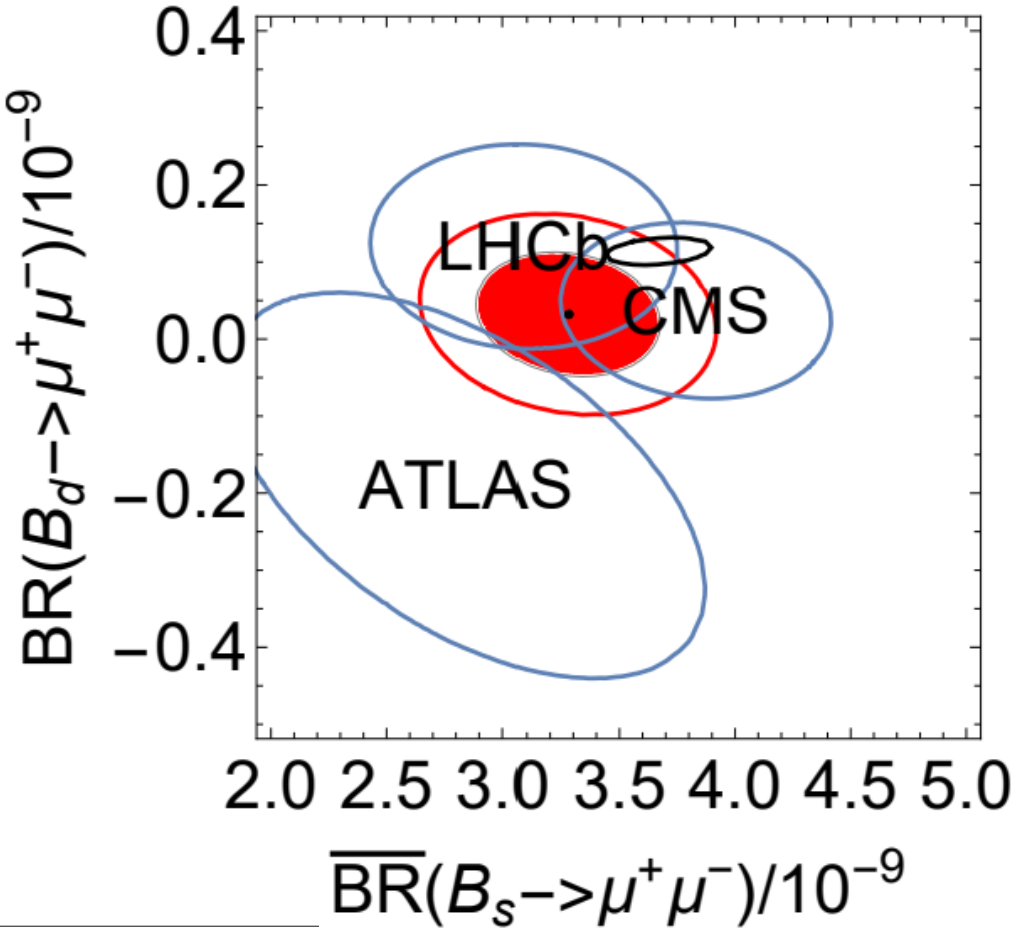


Interferes with Standard Model process

Vector-like Z' option leaves $BR(B_s \to \mu^+\mu^-)$ unchanged

$BR(B_s \to \mu^+\mu^-)$[1]

$B_s = (\bar{b}s)$, $B_d = (\bar{b}d)$



[1]SM: Feldmann, Gubernari, Huber, Seitz, 2211.04209; Combination: BCA, Davighi, 2211.11766

$B_3 - L_2$ model

SM-singlet scalar 'flavon' θ

Additional $U(1)_X$ gauge symmetry broken by $\langle\theta\rangle \sim$TeV$\Rightarrow M_X = g_X X_\theta \langle\theta\rangle \sim \mathcal{O}$(TeV)

SM+$3\nu_R$ fermion content

Zero charges for first two generations of quark

$X = B_3 - L_2$ postdicts some small CKM[2]; $X_\mu \leftrightarrow$ propagating Z'

[2] Bonilla *et al*, 1705.00915; Alonso *et al* 1705.03858, BCA 2009.02197 (*simplified EFT*)

Some Family Structure

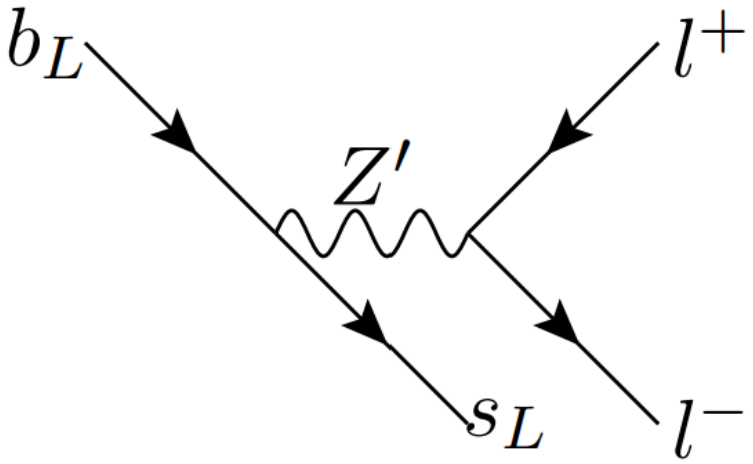
$$Y_u \sim \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix}, \qquad Y_d \sim \begin{pmatrix} \times & \times & 0 \\ \times & \times & 0 \\ 0 & 0 & \times \end{pmatrix},$$

Postdicts CKM angles $|V_{cb}|$, $|V_{ub}|$, $|V_{ts}|$, $|V_{td}|$ to be small

Important X Couplings

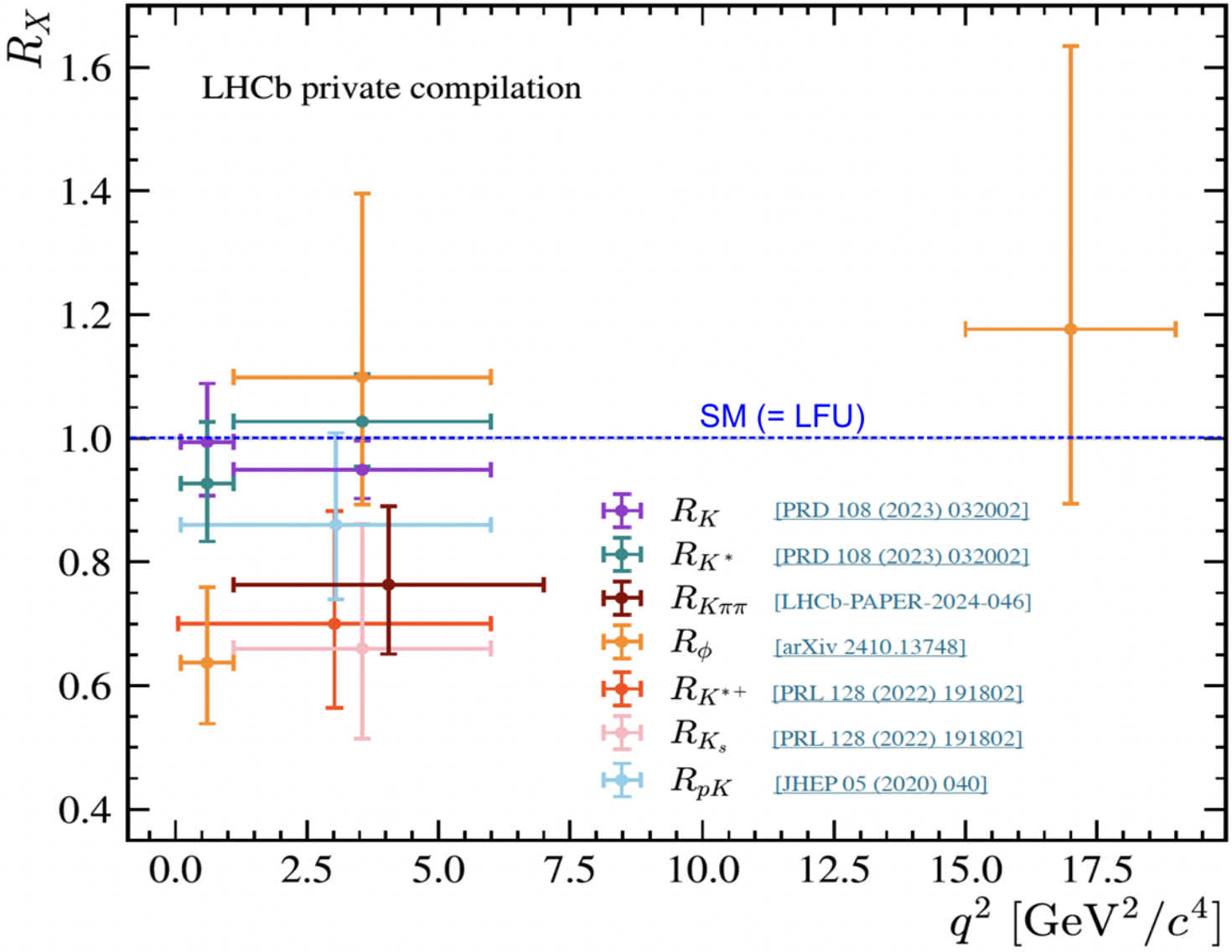
$$g_{Z'}\left[(\overline{d_L}\ \overline{s_L}\ \overline{b_L})\begin{pmatrix}0 & 0 & 0\\ 0 & \sin^2\theta_{sb} & \frac{1}{2}\sin 2\theta_{sb}\\ 0 & \frac{1}{2}\sin 2\theta_{sb} & \cos^2\theta_{sb}\end{pmatrix}\not{X}\begin{pmatrix}d_L\\ s_L\\ b_L\end{pmatrix}\right.$$

$$\left.-(\overline{e}\ \overline{\mu}\ \overline{\tau})\begin{pmatrix}0 & 0 & 0\\ 0 & 3 & 0\\ 0 & 0 & 0\end{pmatrix}\not{X}\begin{pmatrix}e\\ \mu\\ \tau\end{pmatrix}\right]$$

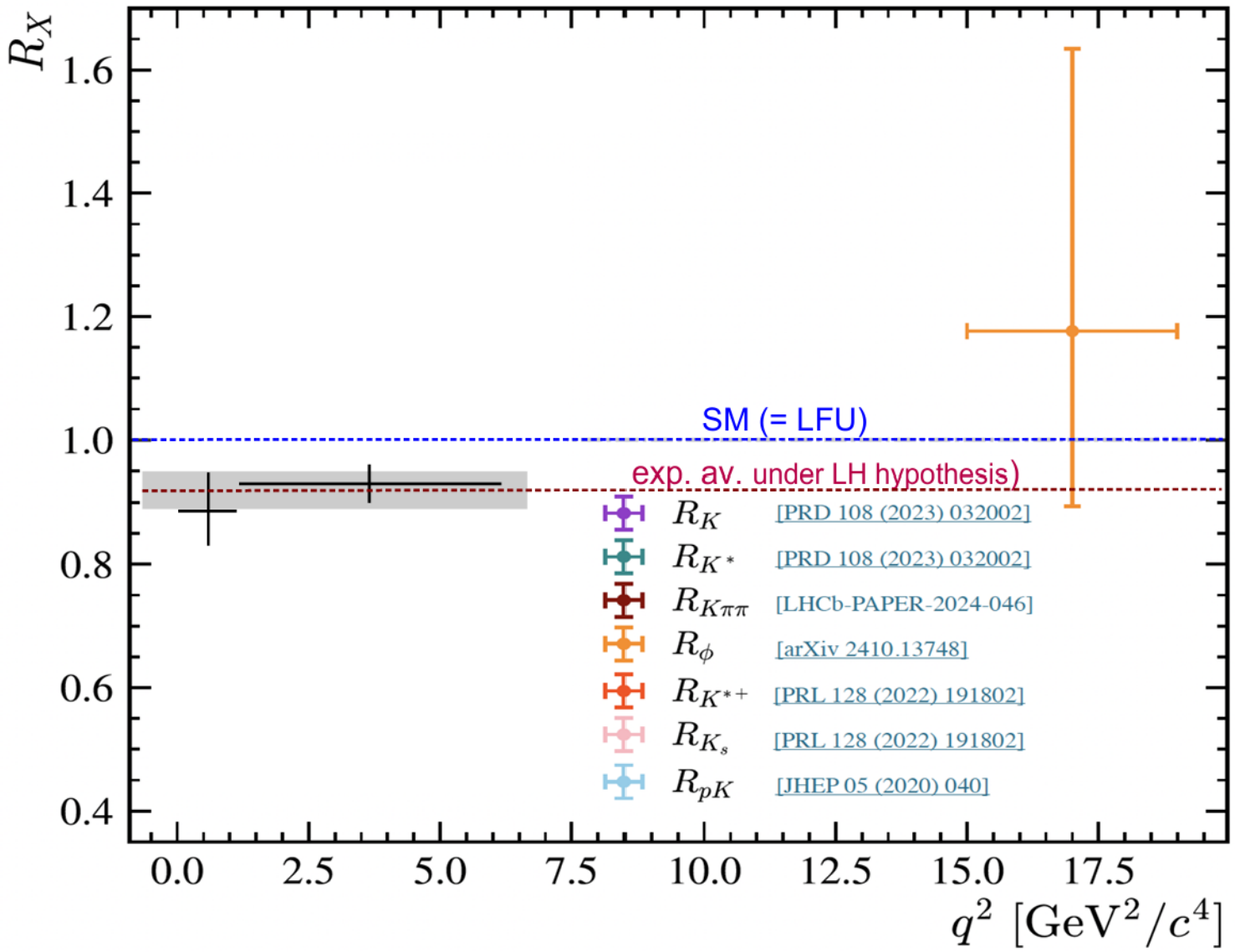


– LFU Violating? $C_9 \neq 0$

$$R_X = BR(B \to X\mu^+\mu^-)/BR(B \to Xe^+e^-)$$



Thanks to G Isidori



Kinetic Mixing BCA, Gubernari 2409.06804

This will induce a **family independent** component to the Z' couplings

$$J^\mu = g_X \sum_{\psi'} X_{\psi'} \overline{\psi'} \gamma^\mu \psi',$$

$$j_\mu = i g' Y_H \left[H^\dagger D_\mu H - (D_\mu H)^\dagger H \right] + g' \sum_{\psi'} Y_\psi \overline{\psi'} \gamma_\mu \psi'.$$

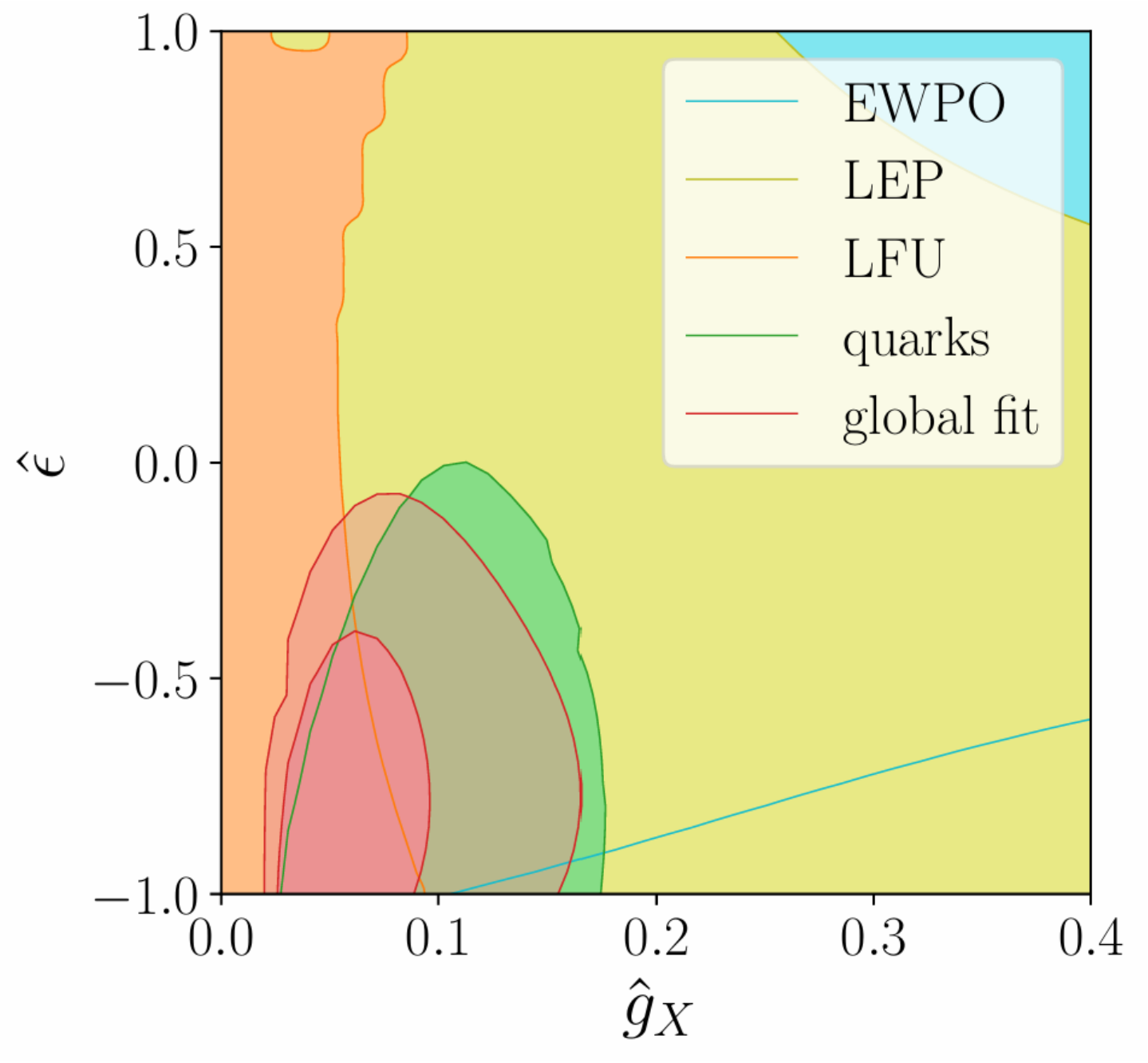
$$\mathcal{L}_{XB} = -\frac{\epsilon}{2} B_{\mu\nu} X^{\mu\nu} - X_\mu J^\mu - B_\mu j^\mu.$$

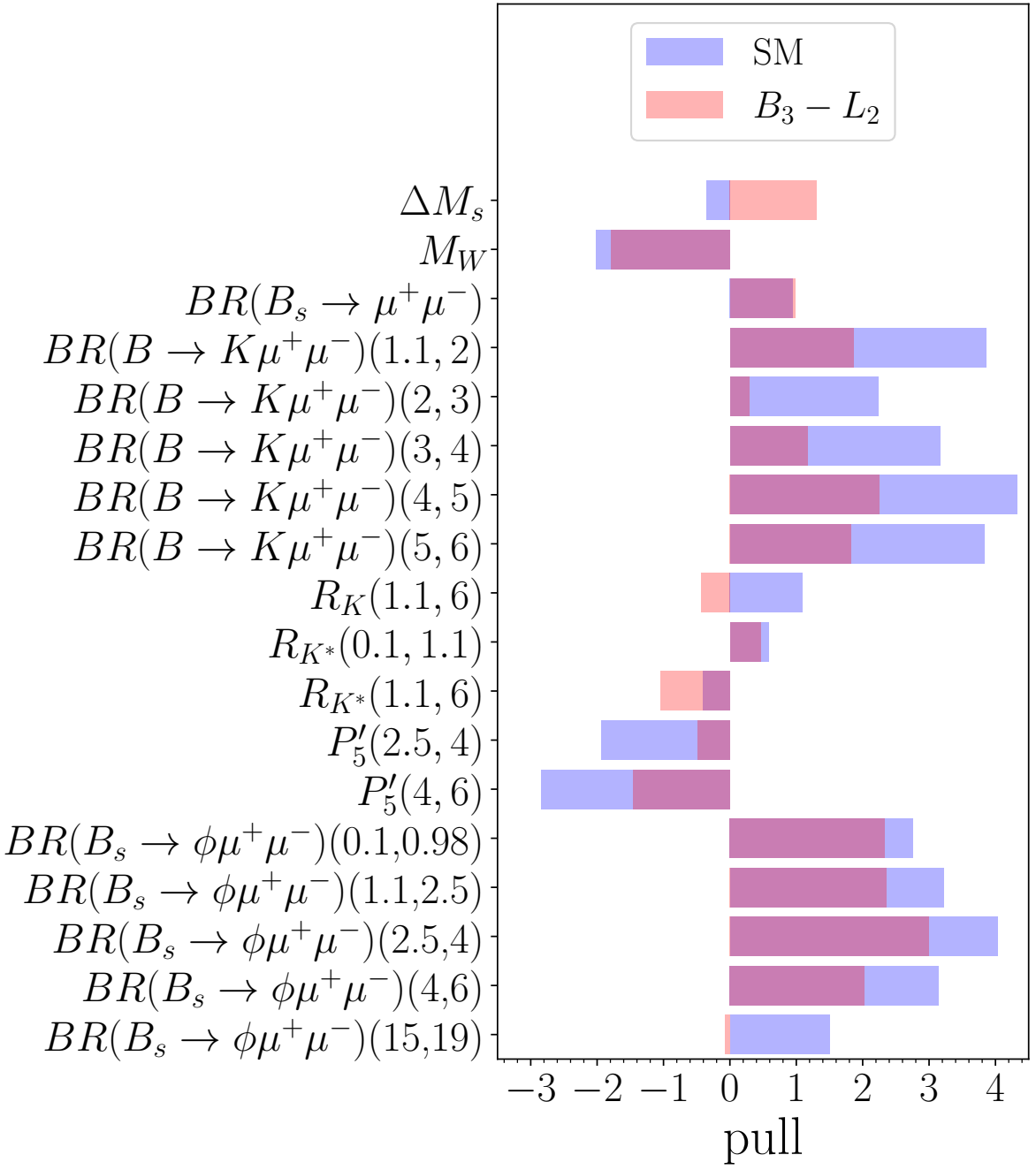
Global Fit

Observables: 306 quarks, 31 EWPOs, 148 LEP2, 24 LFU

$$\hat{\epsilon} := \epsilon \frac{3 \text{ TeV}}{M_X}, \qquad \hat{g}_X := g_X \frac{3 \text{ TeV}}{M_X}.$$

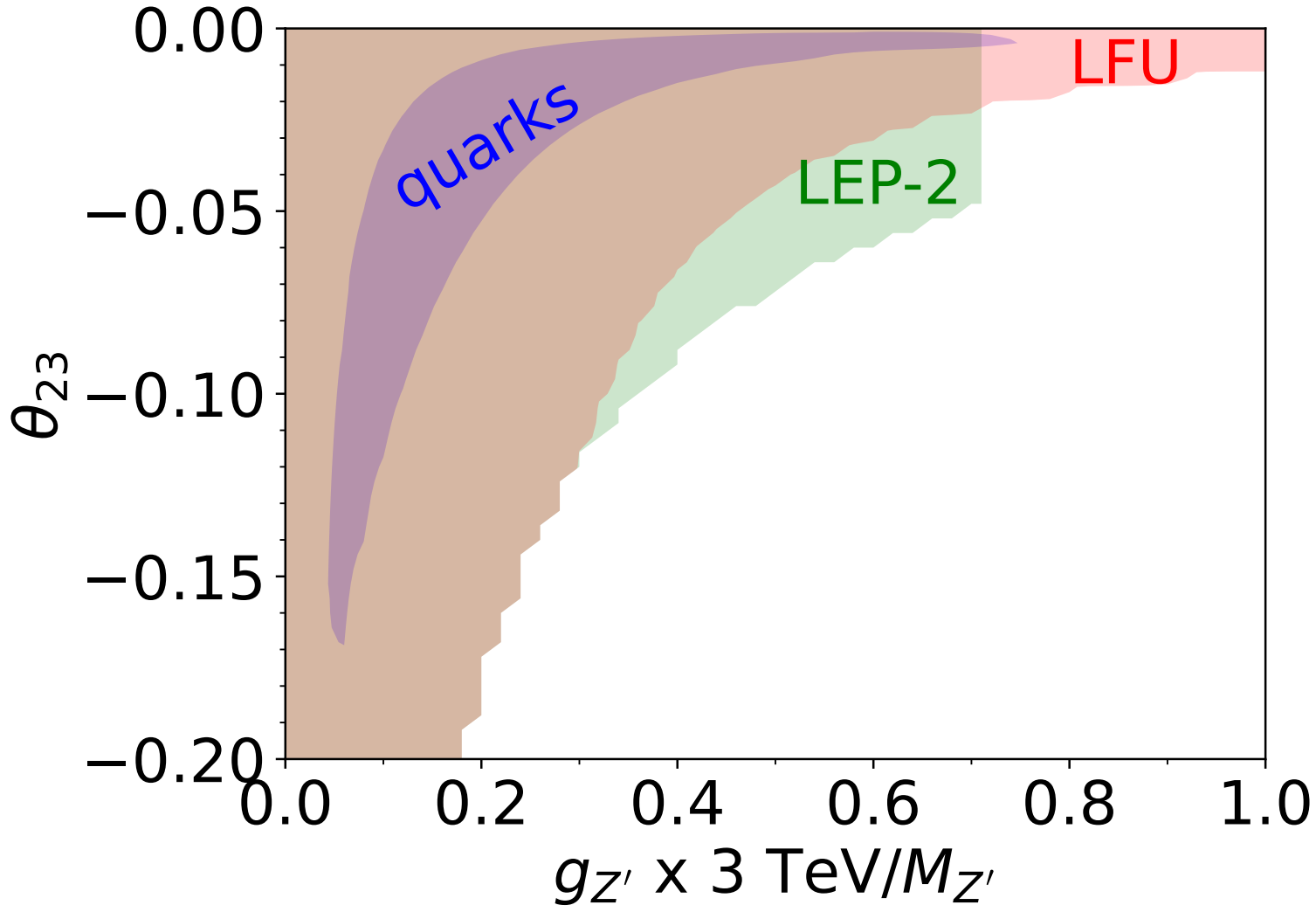
$\hat{\epsilon}$	$\hat{g}_X$	θ_{sb}	$\Delta\chi^2_{\text{quarks}}$	$\Delta\chi^2_{\text{EWPO}}$	$\Delta\chi^2_{\text{LEP2}}$	$\Delta\chi^2_{\text{LFU}}$	$\Delta\chi^2_{\text{global}}$
0	0.082	-0.11	36.2	0.0	0.00	-3.8	**32.8**
-0.86	0.048	-0.19	40.1	-0.4	-0.02	0.8	**40.1**


EWPO
LEP
LFU
quarks
global fit
1.0
0.5
0.0
−0.5
−1.0
0.0
0.1
0.2
0.3
0.4
ϵ
$\hat{g}_X$

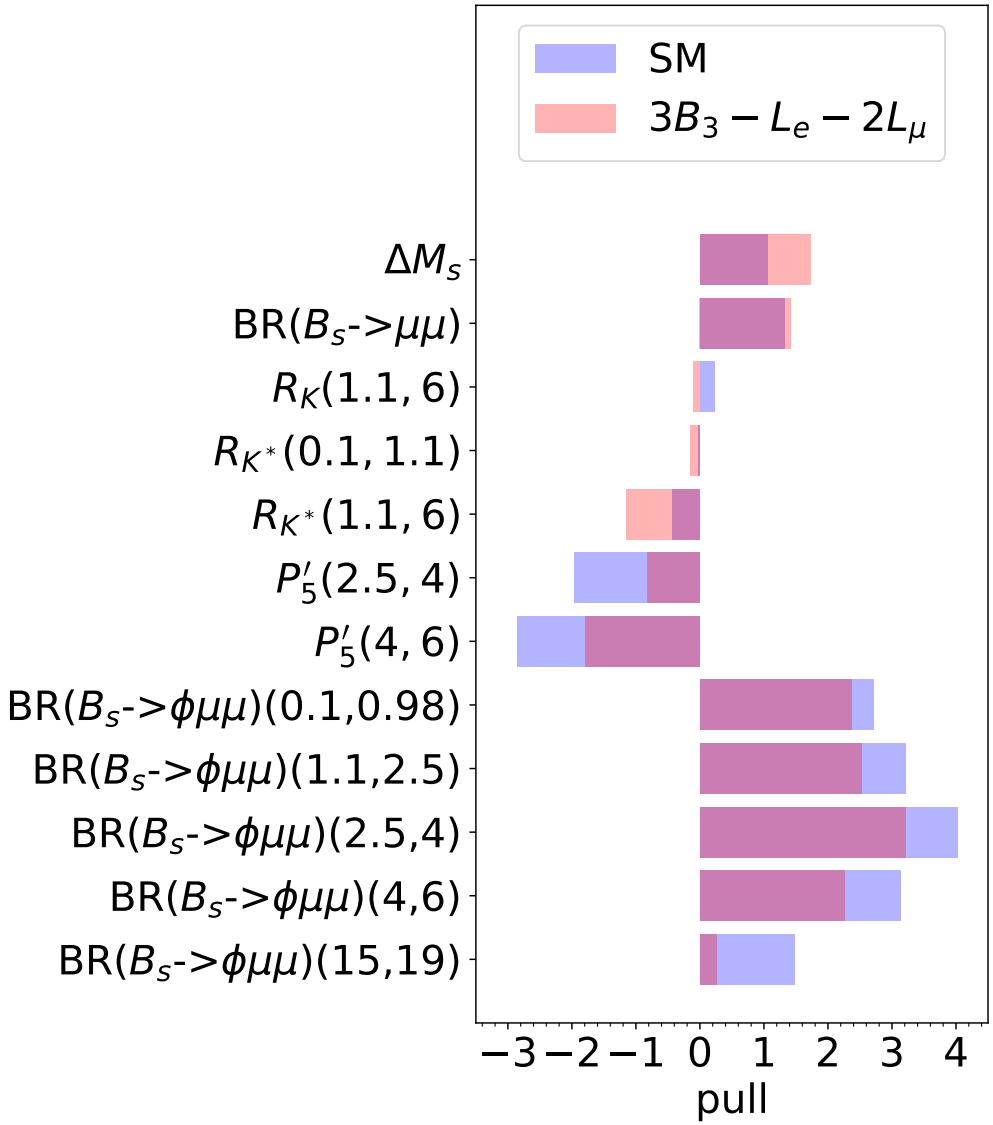


3

[3] pull=(theory-expt cntrl value)/error at best-fit point



$3B_3 - L_e - 2L_\mu$ (no kin mixing): BCA, Mullin, 2306.08669

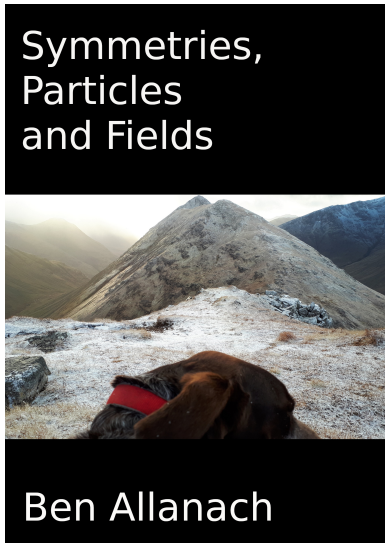

SM
$3B_3 - L_e - 2L_\mu$
ΔM_s
BR(B_s->$\mu\mu$)
$R_K(1.1, 6)$
$R_{K^*}(0.1, 1.1)$
$R_{K^*}(1.1, 6)$
$P_5'(2.5, 4)$
$P_5'(4, 6)$
BR(B_s->$\phi\mu\mu$)(0.1,0.98)
BR(B_s->$\phi\mu\mu$)(1.1,2.5)
BR(B_s->$\phi\mu\mu$)(2.5,4)
BR(B_s->$\phi\mu\mu$)(4,6)
BR(B_s->$\phi\mu\mu$)(15,19)
−3 −2 −1 0 1 2 3 4
pull

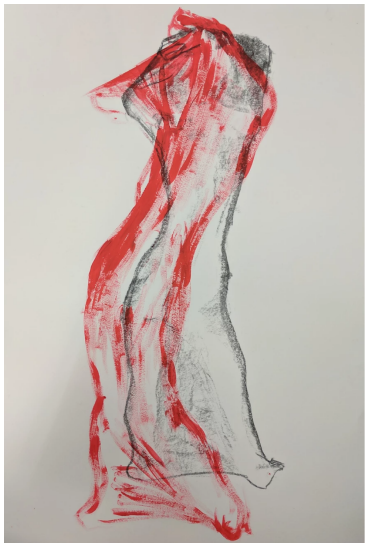
Epilogue

Family universal component of kinetic mixing is equivalent to $X := B_3 - L_2 + \alpha Y, \quad \alpha \in \mathbb{Q}$

X:=$3B_3 - L_e - 2L_\mu$ also works without KM

Links to my music, book (18€) and Quantum Selves art:





smelli

Aebischer, Kumar, Stangl, Straub, 1810.07698:

Input: SMEFT coefficients C_i/Λ^2.

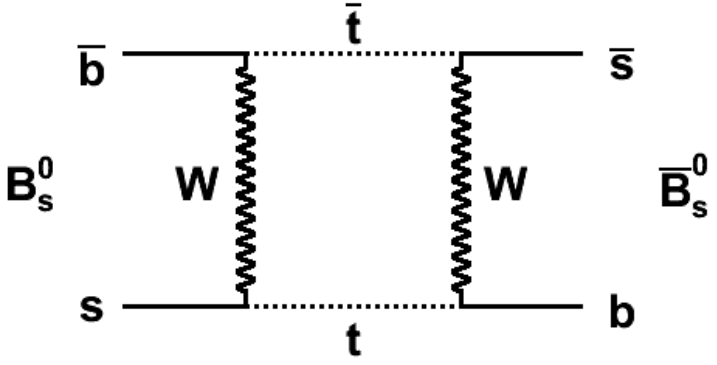
Output: χ^2

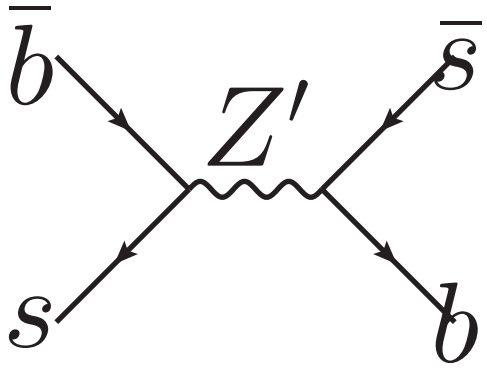
Hundreds of $B-$observables

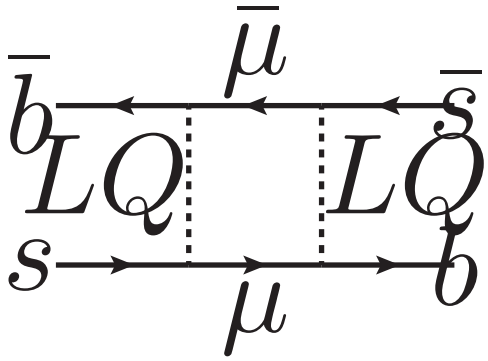
31 EWPOs

$B_s - \bar{B}_s$ Mixing

Measurement agrees with SM.



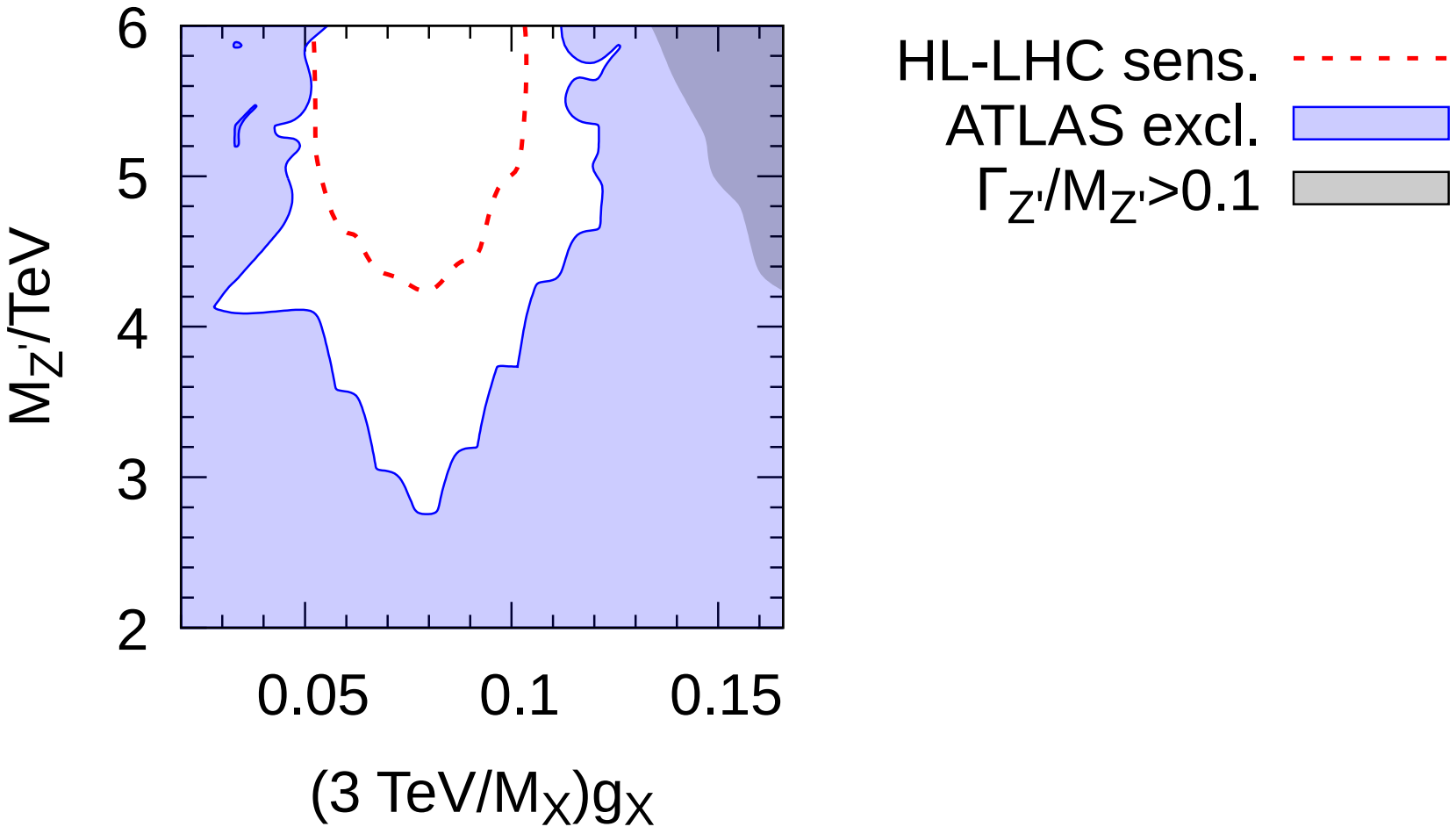




$$g_{sb} = \frac{g_X}{2} \sin 2\theta_{sb} \lesssim \frac{M_{Z'}}{194 \text{ TeV}}$$ but uncertain from QCD sum rules and lattice[4].

[4] King, Lenz, Rauh, arXiv:1904.00940

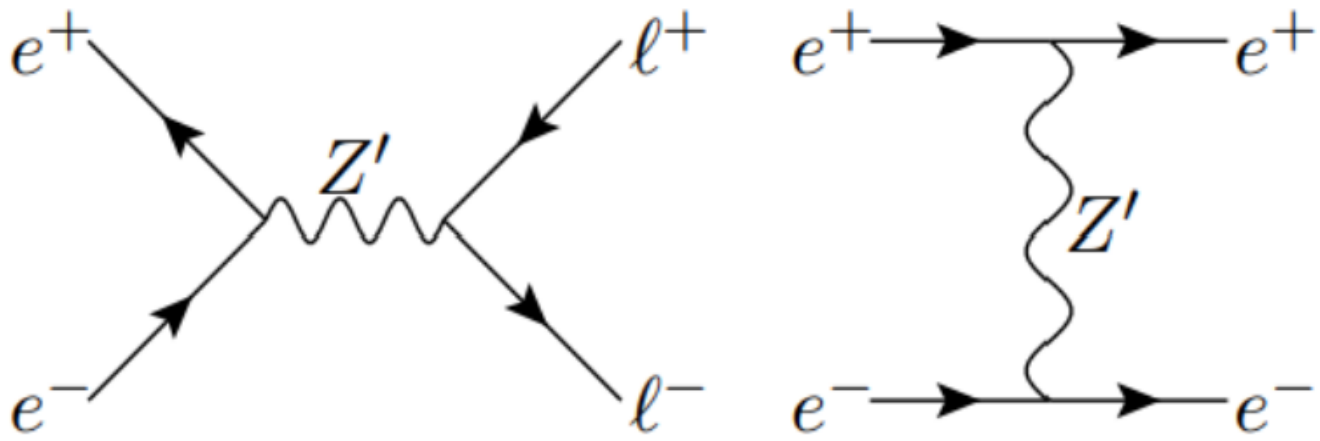
BCA, 2412.01956



Integrate out heavy X_μ

$$\begin{aligned}
\mathcal{L}_6 &= -\frac{1}{2M_X^2} J_\mu J^\mu - \frac{\epsilon}{M_X^2}(\partial_\nu B^{\mu\nu}) J_\mu \\
&\quad -\frac{\epsilon^2}{2M_X^2}(\partial_\nu B^{\mu\nu})(\partial^\rho B_{\mu\rho}) \\
&= -\frac{1}{2M_X^2}\left(J_\mu - \epsilon j_\mu\right)\left(J^\mu - \epsilon j^\mu\right)
\end{aligned}$$

LEP constraints BCA, Mullin, 2306.08669



SMEFT contributions: C_{ee}^{11ii}, C_{ll}^{11ii}, C_{le}^{1ii1}

Code into `flavio` (cf `Falkowski, Mimouni 1511.07434`): 148 LEP2 bins, σ, A_{FB}

UV Completion

The $B_3 - L_2$ model with kinetic mixing is approximately physically equivalent to an unmixed model with

$$X := B_3 - L_2 + \alpha Y$$

where $\alpha \in \mathbb{Q}$ is chosen appropriately.

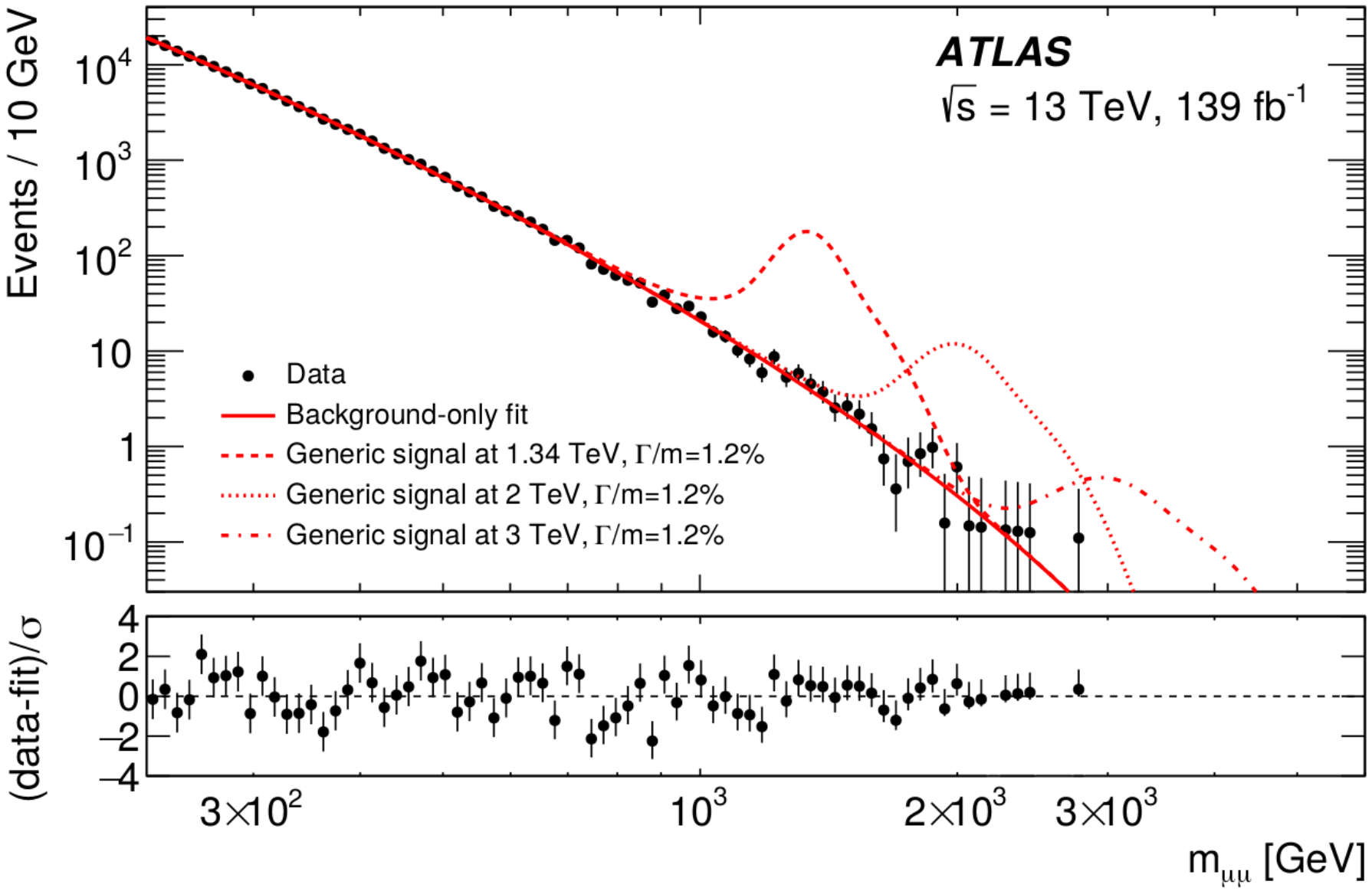
l^+l^- ATLAS 13 TeV 139 fb^{-1}

e.g. 2 track-based isolated μ, $p_T > 30$ GeV with reconstructed vertex.[5] Only keep pair with highest $(|p_{T_1}| + |p_{T_2}|)$.

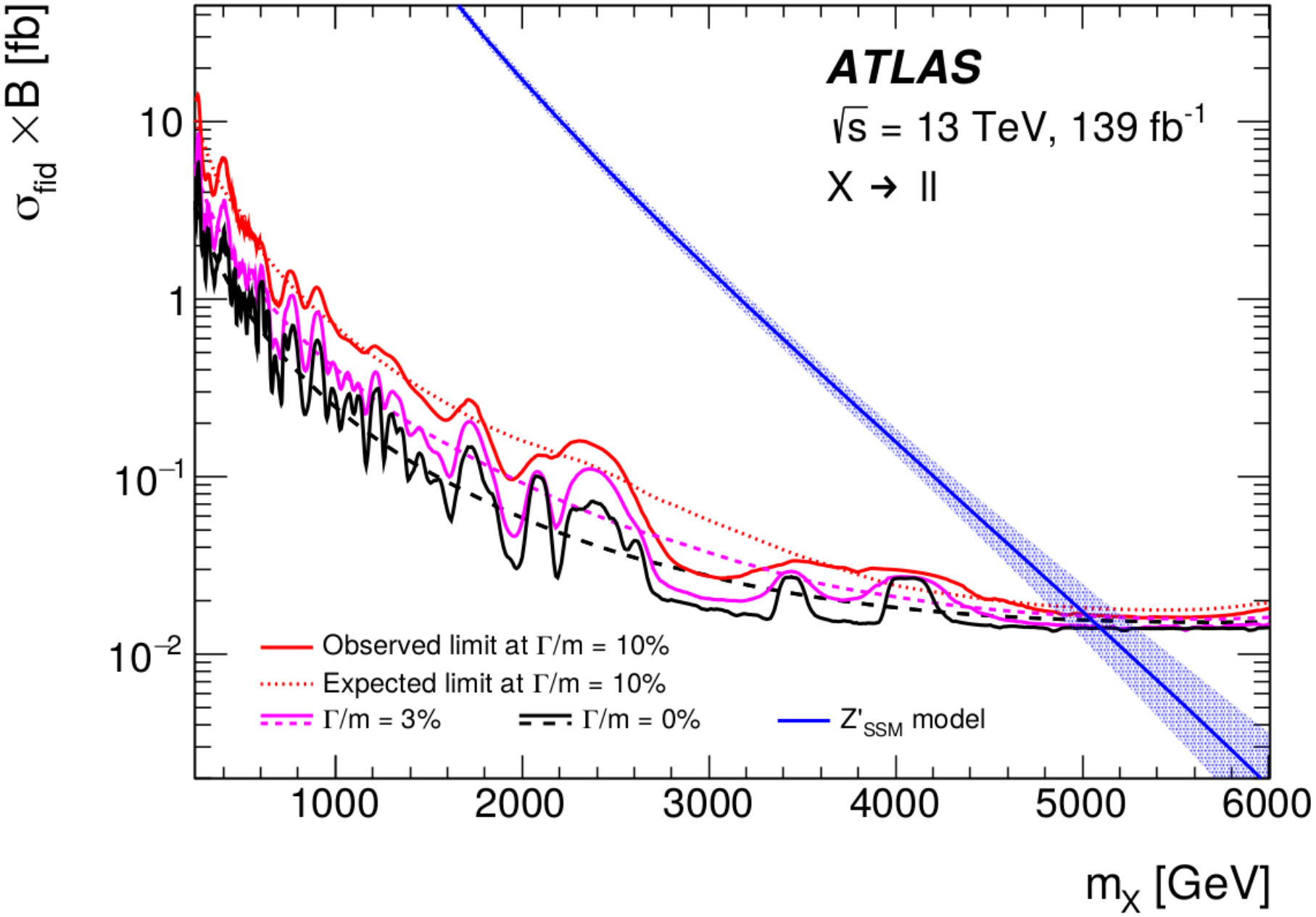
$$m^2_{\mu_1\mu_2} = (p_1^\mu + p_2^\mu)\left(p_{1\mu} + p_{2\mu}\right)$$

CMS also has a similar analysis[6]

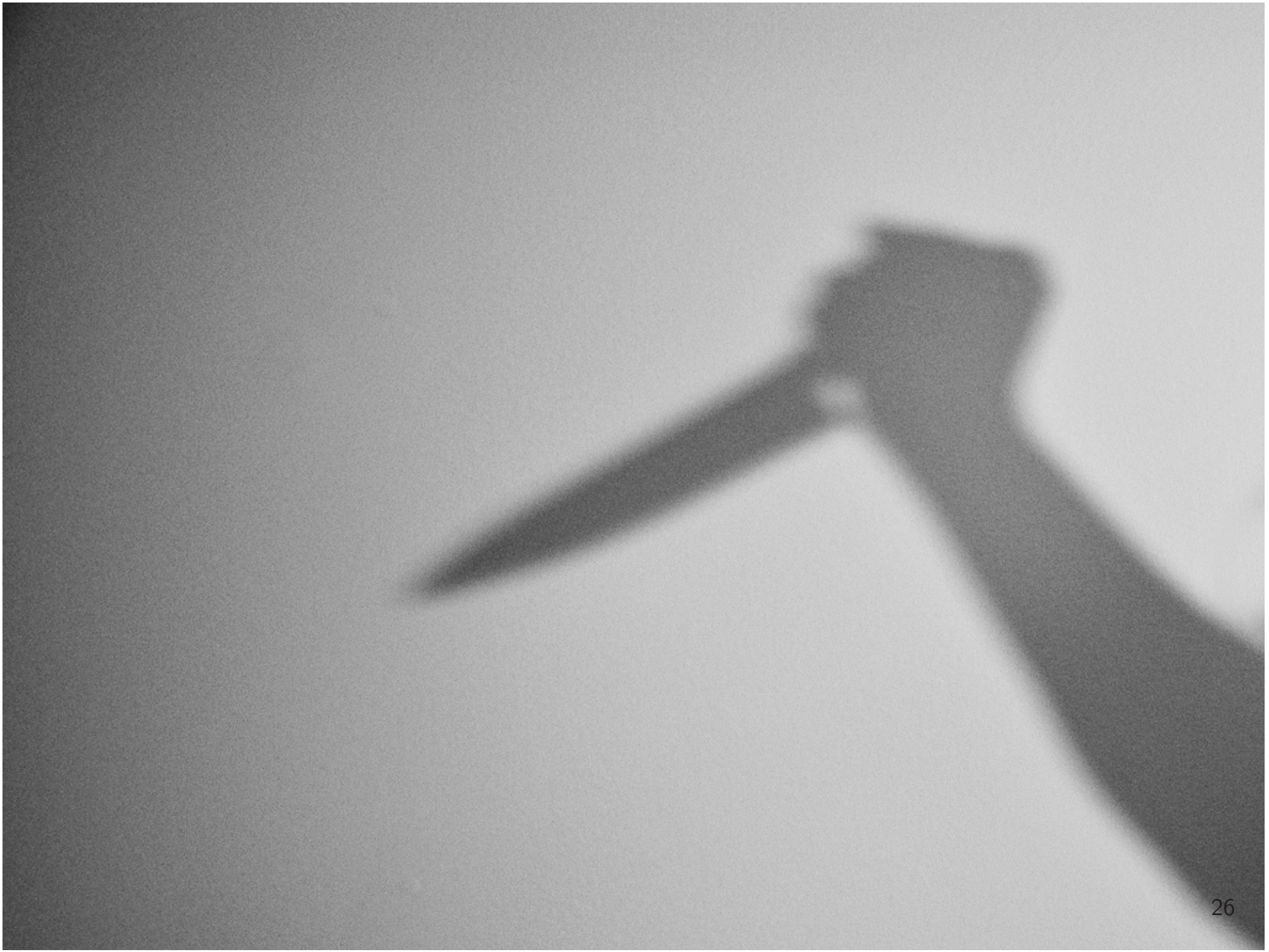
[5] ATLAS, 1903.06248
[6] CMS, 2103.02708


ATLAS
√s = 13 TeV, 139 fb-1
Events / 10 GeV
Data
Background-only fit
Generic signal at 1.34 TeV, Γ/m=1.2%
Generic signal at 2 TeV, Γ/m=1.2%
Generic signal at 3 TeV, Γ/m=1.2%
(data-fit)/σ
mμμ [GeV]

ATLAS l^+l^- limits



ATLAS, 1903.06248



smelli

Aebischer, Kumar, Stangl, Straub, 1810.07698:

Input: SMEFT coefficients C_i/Λ^2.

Output: χ^2

Hundreds of $B-$observables

31 EWPOs

SMEFT

Parameterises heavy new physics effects

$$\mathcal{L} = \mathcal{L}_{4D} + \sum_{d=5} \sum_{i} \frac{C_i}{\Lambda^{d-4}} \mathcal{O}.$$

Assumptions:

All BSM fields have mass scale $\Lambda \gg$ scale of observables.

Higgs doublet linearly realises EWSB

Important term

2499 $d = 6$ terms

$$\mathcal{L} = \ldots + \frac{(C_{lq}^{(1)})^{2223}}{\Lambda^2} \left(\overline{L_2}\gamma_\alpha L_2)\right) \left(\overline{Q_2}\gamma^\alpha Q_3\right)$$

mediates $b \to s\mu^+\mu^-$ transitions.

Here, from integrating Z' out:

$$\frac{(C_{lq}^{(1)})^{2223}}{\Lambda^2} = \frac{-3 \sin 2\theta_{sb} g_X^2}{M_X}$$

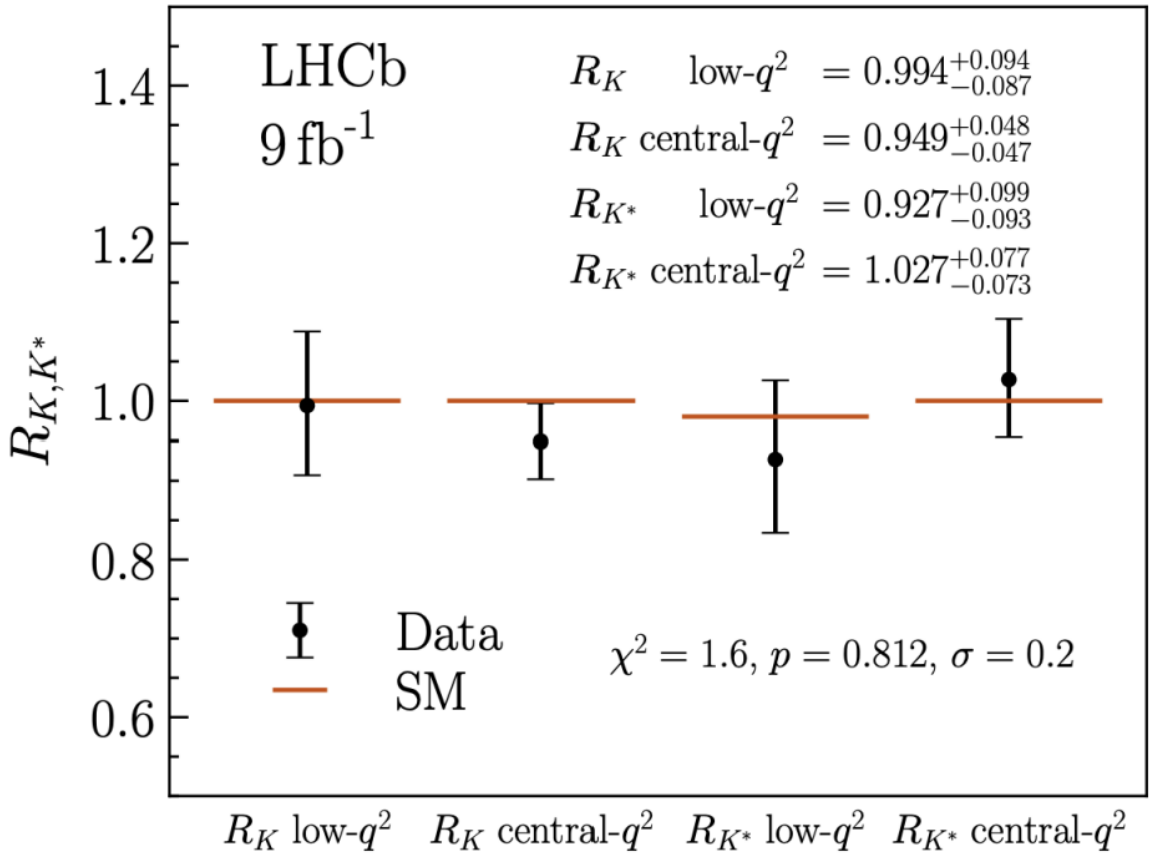
SMEFT WCs$/(g_X^2/M_X^2)$

BCA, Davighi, 2211.11766

WC	value	WC	value
C_{ll}^{2222}	$-\frac{9}{2}$	$(C_{lq}^{(1)})^{22ij}$	$3\Lambda_{\Xi\ ij}^{(d_L)}$
$(C_{qq}^{(1)})^{ijkl}$	$\Lambda_{\Xi\ ij}^{(d_L)}\Lambda_{\Xi\ kl}^{(d_L)}\frac{\delta_{ik}\delta_{jl}-2}{2}$	C_{ee}^{2222}	$-\frac{9}{2}$
C_{uu}^{3333}	$-\frac{1}{2}$	C_{dd}^{3333}	$-\frac{1}{2}$
C_{eu}^{2233}	3	C_{ed}^{2233}	3
$(C_{ud}^{(1)})^{3333}$	-1	C_{le}^{2222}	-9
C_{lu}^{2233}	3	C_{ld}^{2233}	3
C_{qe}^{ij22}	$3\Lambda_{\Xi\ ij}^{(d_L)}$	$(C_{qu}^{(1)})^{ij33}$	$-\Lambda_{\Xi\ ij}^{(d_L)}$
$(C_{qd}^{(1)})^{ij33}$	$-\Lambda_{\Xi\ ij}^{(d_L)}$		

| wilson | flavio | smelli > output

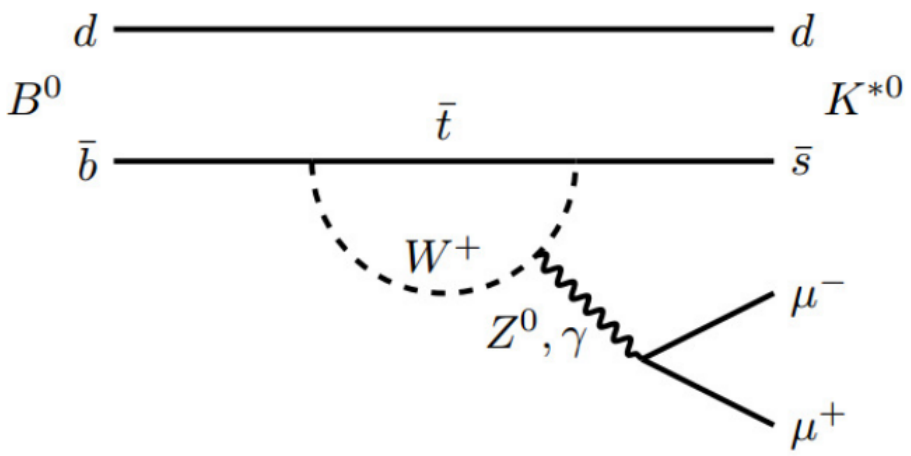
LHCb 2212.09152

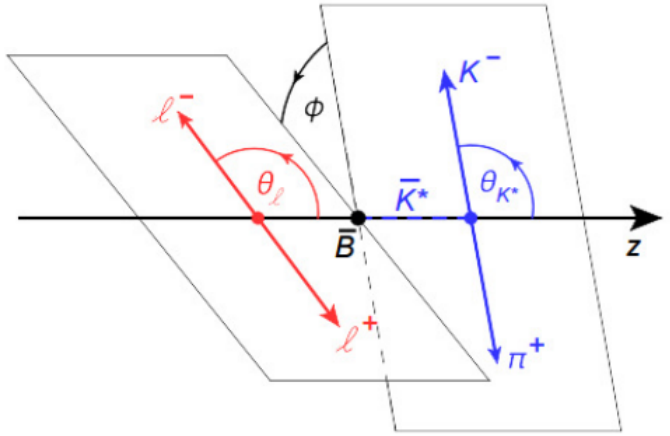


$$R_X(q^2) = \frac{BR(B \to X\mu^+\mu^-)}{BR(B \to Xe^+e^-)}(q^2)$$


MUSTER STATION 1
FIRE HOSE
NITROX
CLOSE VENTS DURING ADVERSE WEATHER

$B^0 \to K^{*0}(\to K^+\pi^-)\mu^+\mu^-$

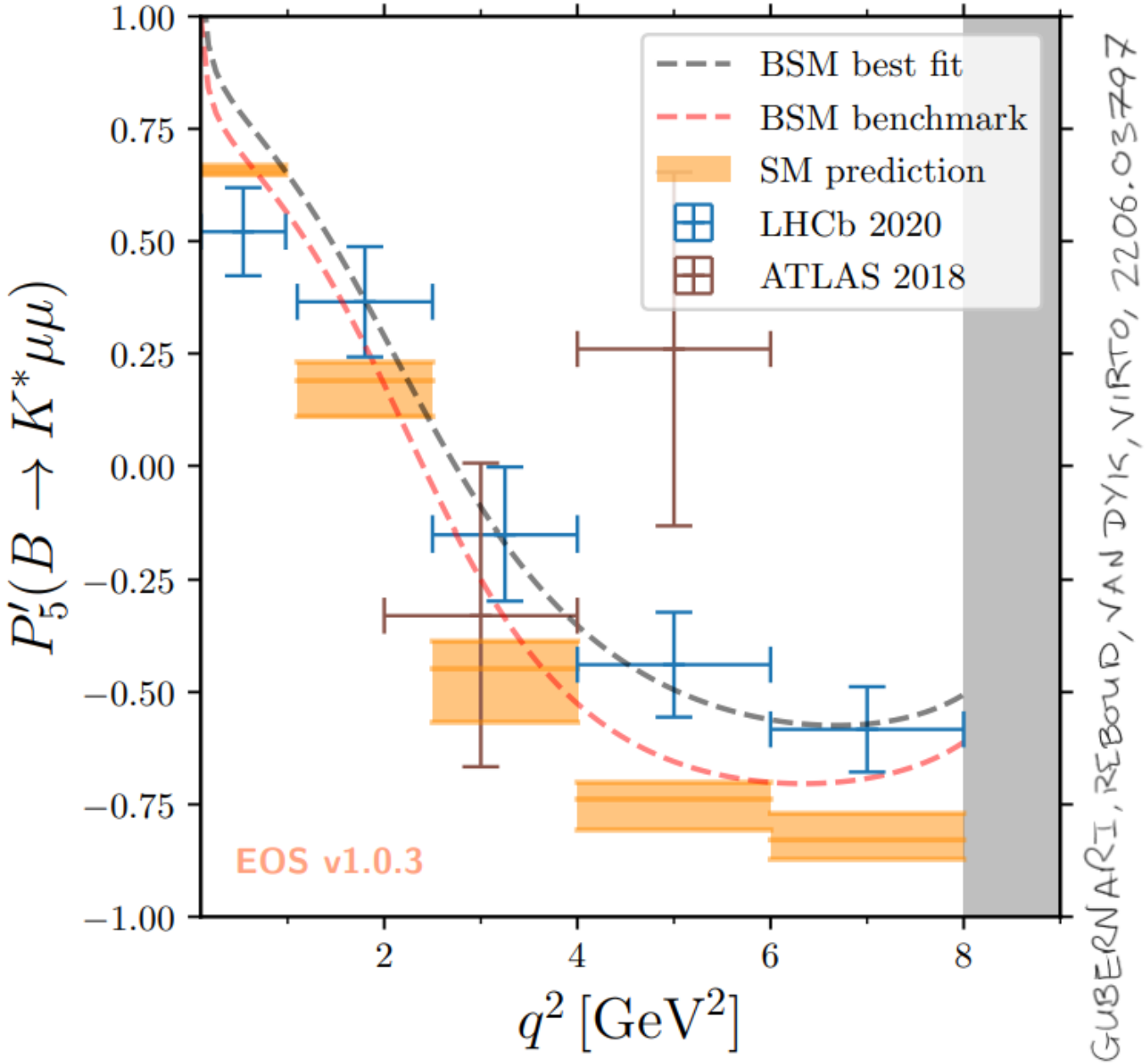




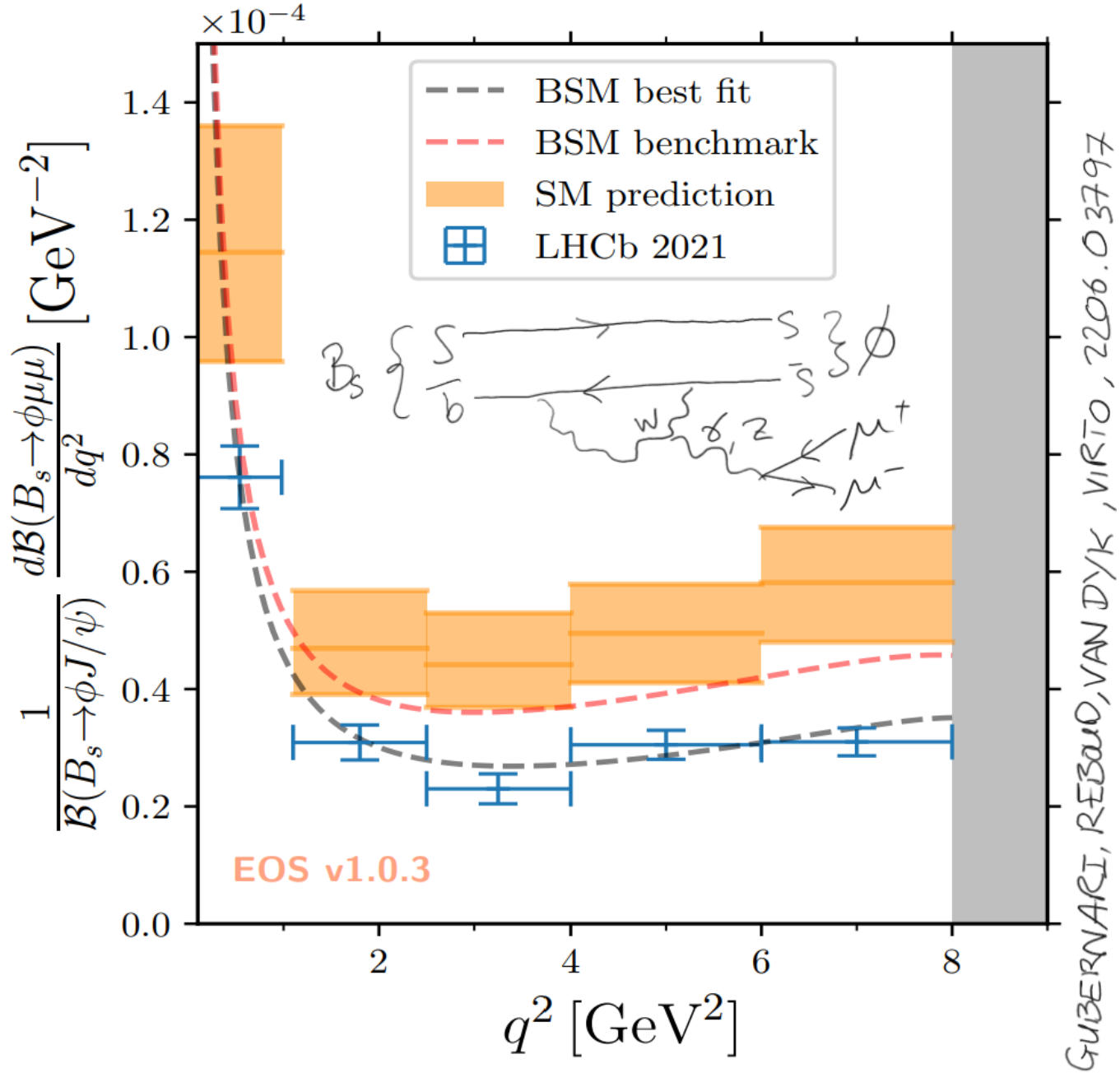
Decay fully described by three helicity angles $\vec{\Omega} = (\theta_\ell, \theta_K, \phi)$ and $q^2 = m^2_{\mu\mu}$

$$\frac{1}{\mathrm{d}(\Gamma+\bar{\Gamma})/\mathrm{d}q^2}\frac{\mathrm{d}^3(\Gamma+\bar{\Gamma})}{\mathrm{d}\vec{\Omega}} = \frac{9}{32\pi}\Big[\tfrac{3}{4}(1-F_\mathrm{L})\sin^2\theta_K + F_\mathrm{L}\cos^2\theta_K + \tfrac{1}{4}(1-F_\mathrm{L})\sin^2\theta_K\cos 2\theta_\ell$$
$$- F_\mathrm{L}\cos^2\theta_K\cos 2\theta_\ell + S_3\sin^2\theta_K\sin^2\theta_\ell\cos 2\phi$$
$$+ S_4\sin 2\theta_K\sin 2\theta_\ell\cos\phi + S_5\sin 2\theta_K\sin\theta_\ell\cos\phi$$
$$+ \tfrac{4}{3}A_\mathrm{FB}\sin^2\theta_K\cos\theta_\ell + S_7\sin 2\theta_K\sin\theta_\ell\sin\phi$$
$$+ S_8\sin 2\theta_K\sin 2\theta_\ell\sin\phi + S_9\sin^2\theta_K\sin^2\theta_\ell\sin 2\phi\Big]$$

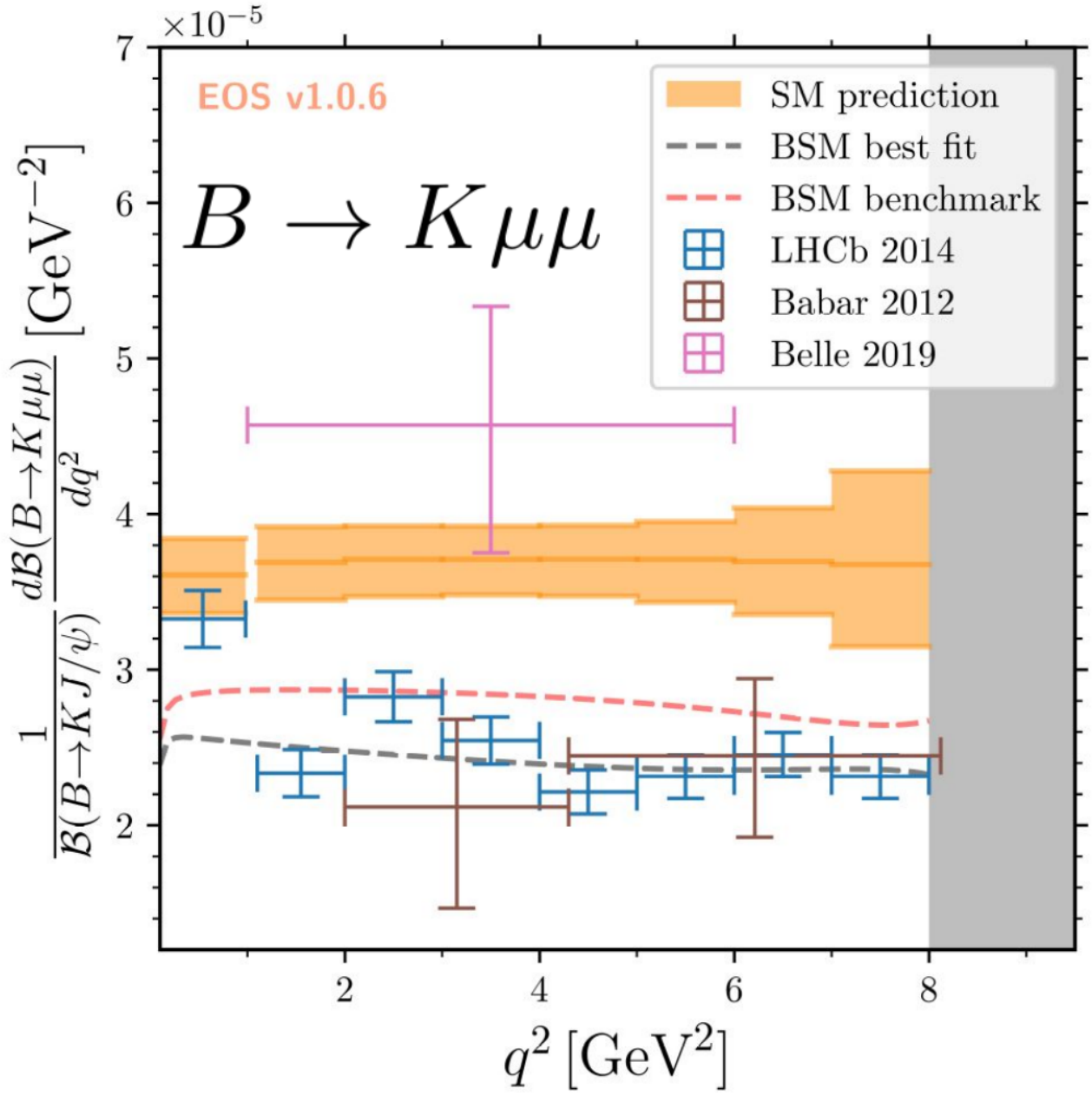
$$P_5' = S_5 / \sqrt{F_L(1 - F_L)}$$



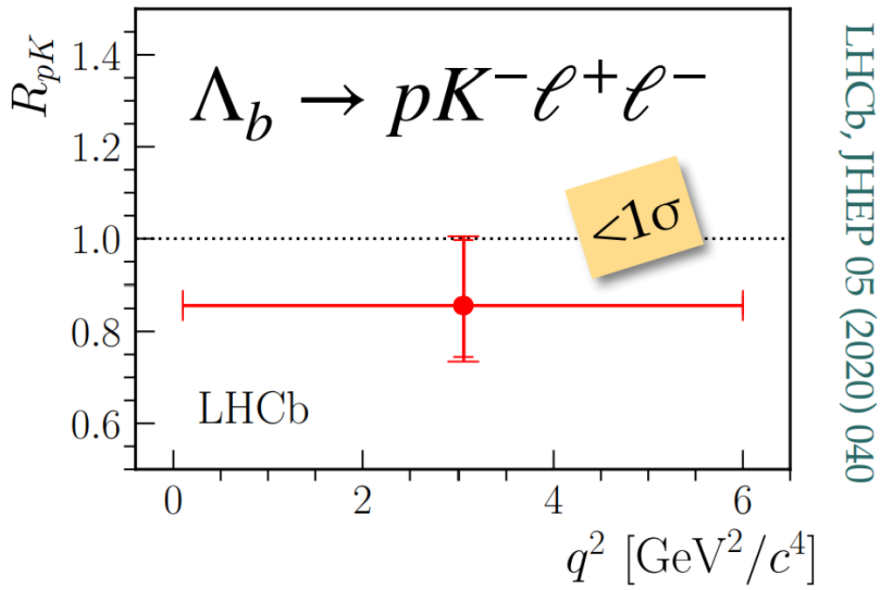
$B_s \to \phi\mu^+\mu^-$: $\phi = (s\bar{s})$



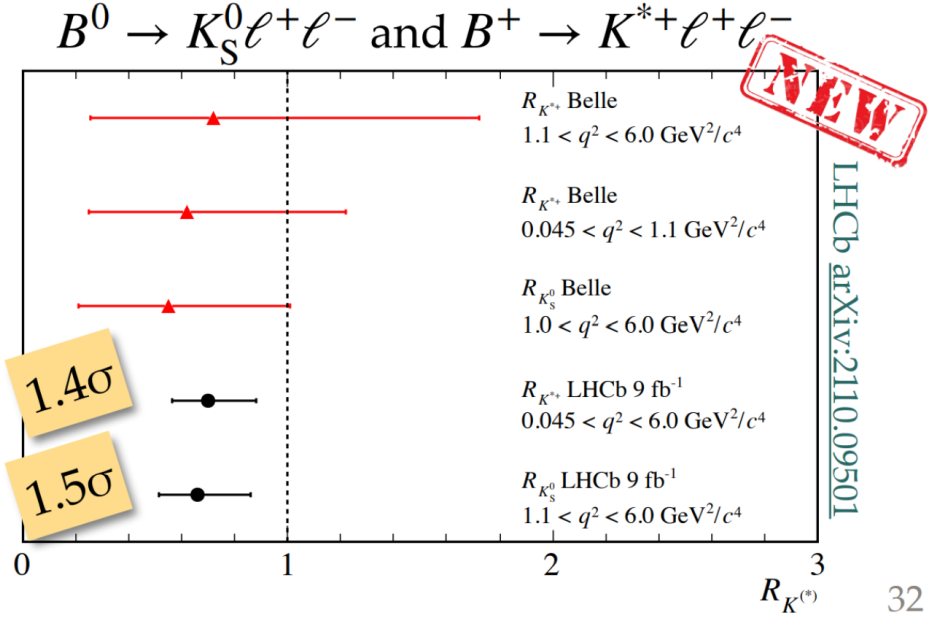
$BR(B \to K\mu^+\mu^-)$


EOS v1.0.6
$B \to K\mu\mu$
SM prediction
BSM best fit
BSM benchmark
LHCb 2014
Babar 2012
Belle 2019
$\times 10^{-5}$
$\frac{1}{\mathcal{B}(B\to KJ/\psi)} \frac{d\mathcal{B}(B\to K\mu\mu)}{dq^2}$ [GeV^{-2}]
q^2 [GeV^2]

Other LFU



LHCb, JHEP 05 (2020) 040



LHCb arXiv:2110.09501

32

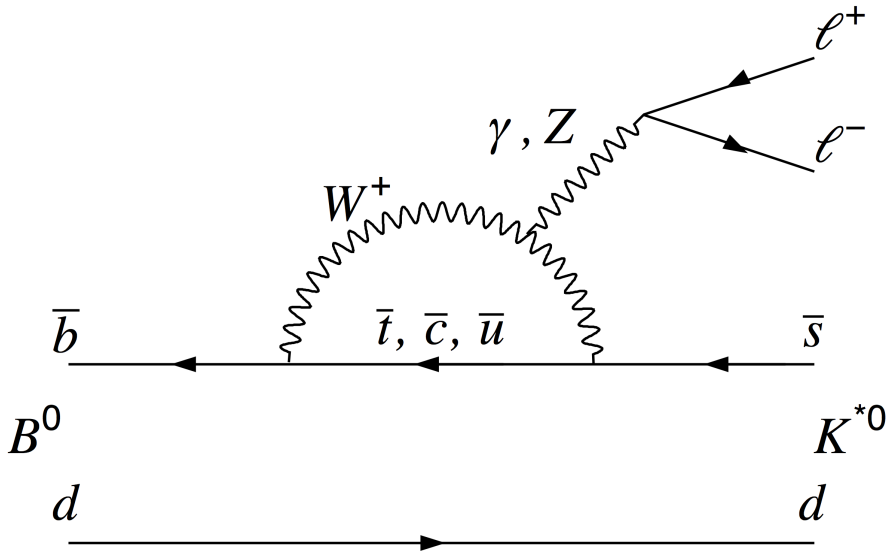
$$B_s \to \phi\ell^+\ell^-,$$

$$B \to \pi\ell^+\ell,$$

$$B \to K\pi^+\pi^-\ell^+\ell^-, \ldots \text{ to come}$$

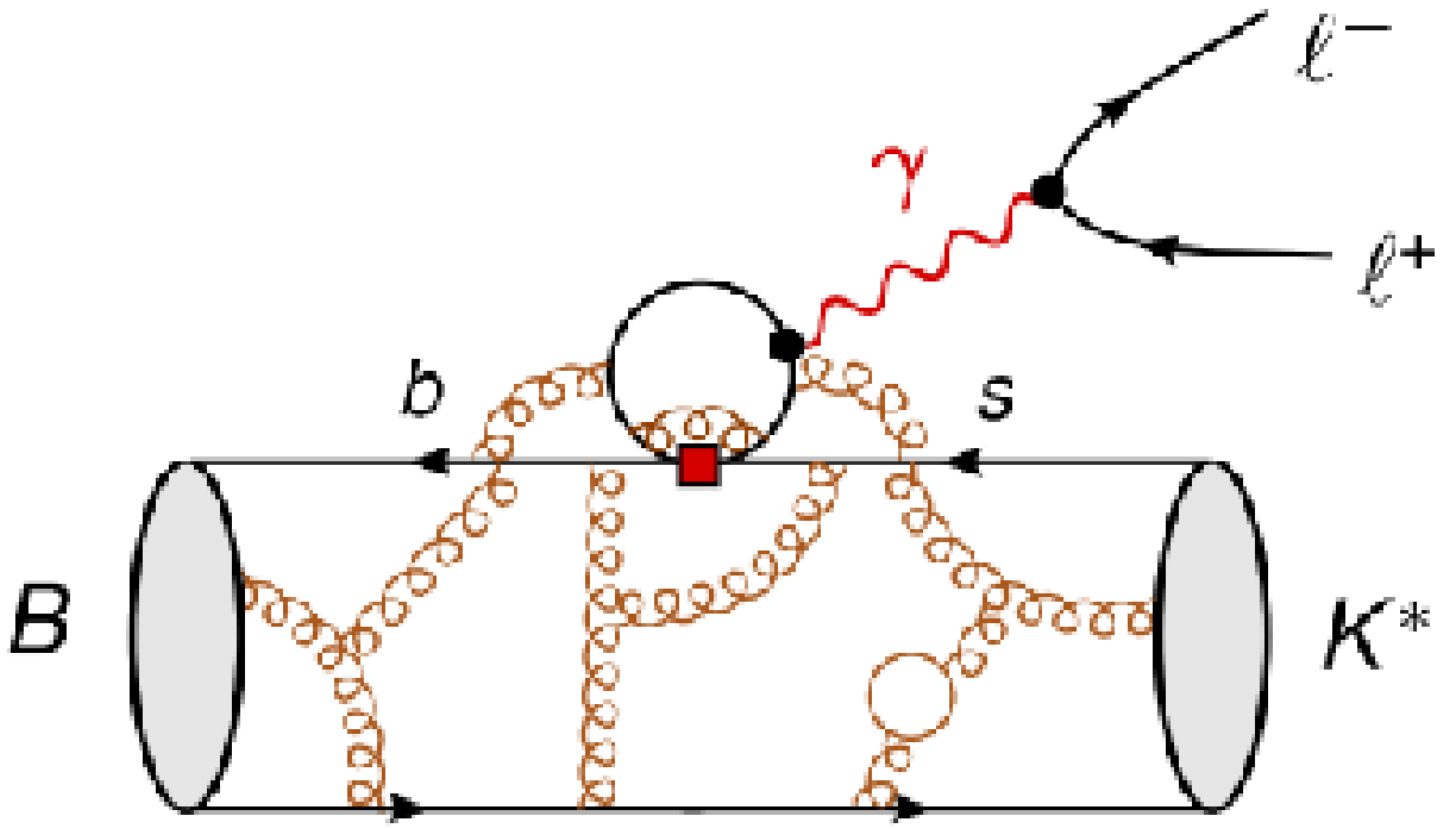
$b \to s l^+ l^-$ in Standard Model

BR$\sim \mathcal{O}(10^{-6})$: loop+EW+CKM



$$R_{K^*} = \frac{BR(B \to K^* \mu^+ \mu^-)}{BR(B \to K^* e^+ e^-)} = 1.00$$

Form Factors



Predicting $B \to M\ell^+\ell^-$: FFs

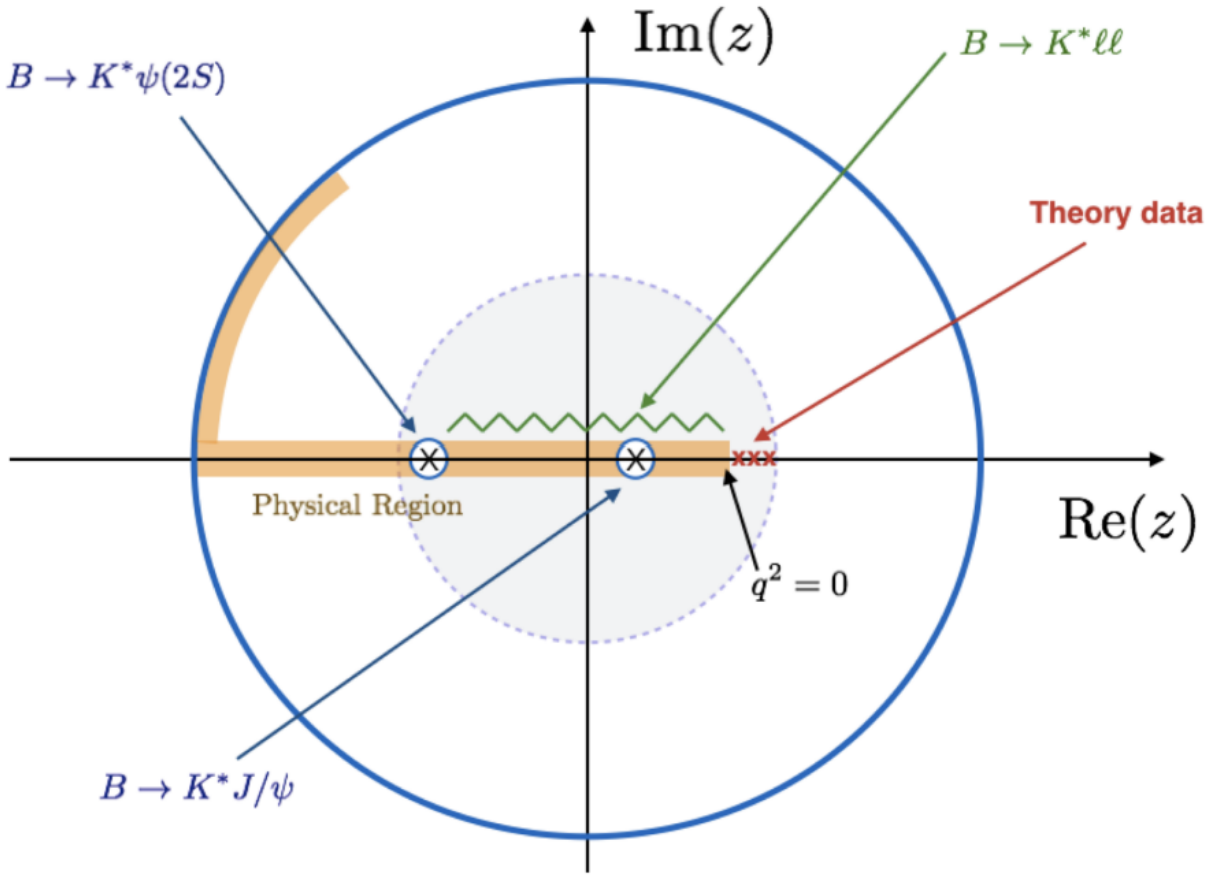
$$\mathcal{A} = \texttt{local} + \texttt{non-local}$$

`local`: interpolate lattice at high $q^2 = m_{ll}^2$ and LCSR at low q^2.

`non-local`: no lattice. Most use QCD factorisation: perturbative charm loop+ad-hoc

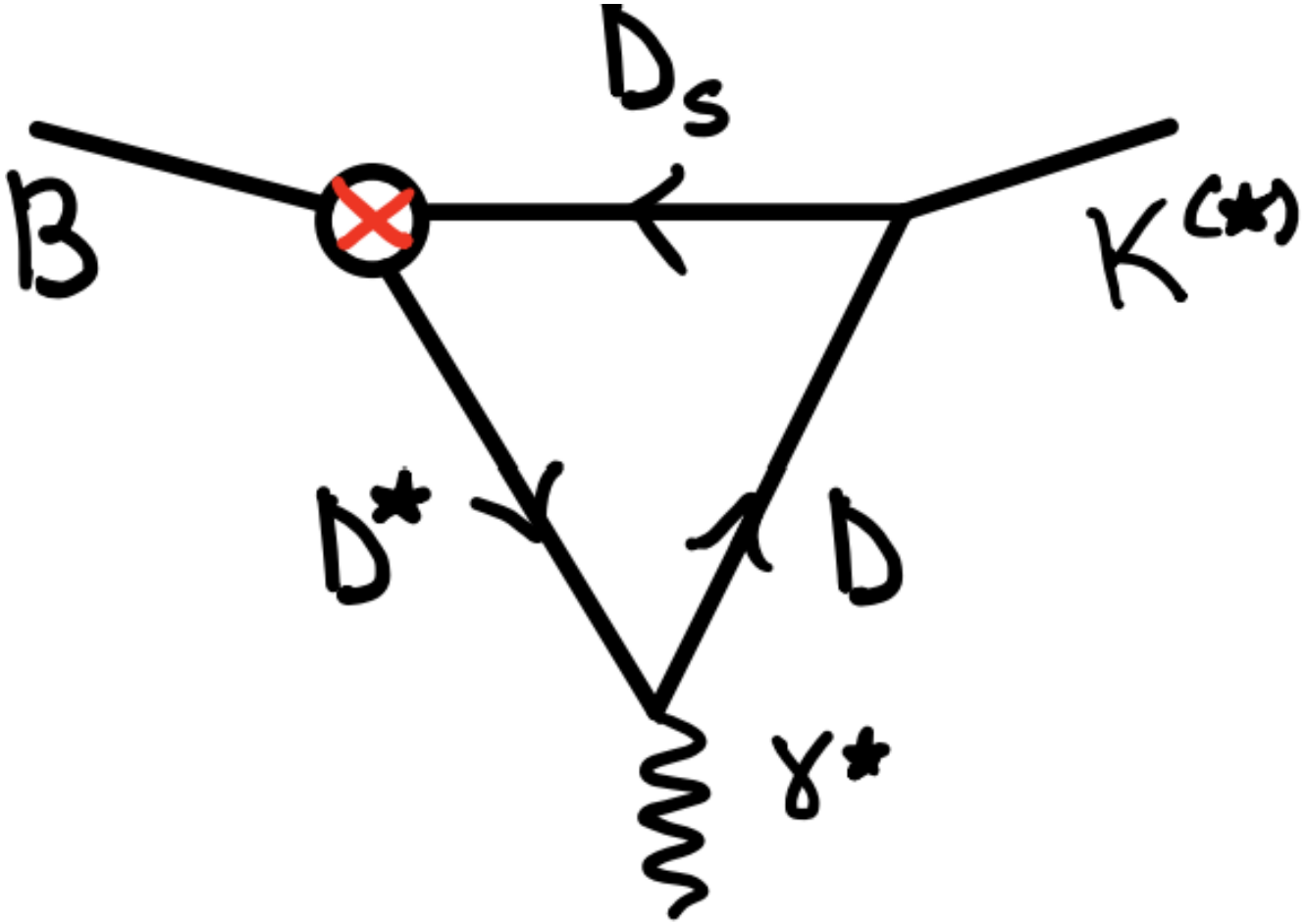
`EOS` approach: interpolate $q^2 < 0$ LCOPE and measurements of BRs/angular dists at $q^2 = M_{J/\psi}^2$.

$$q^2 \to z(q^2),\ |z| < 1$$



$C_9^{LD} \propto \sum_n a_n z^n$ 1707.07305 *truncation* 2205.03797

Caveat Emptor



2212.10516

Backup

Ultra-violet completion?

This model is *equivalent* to

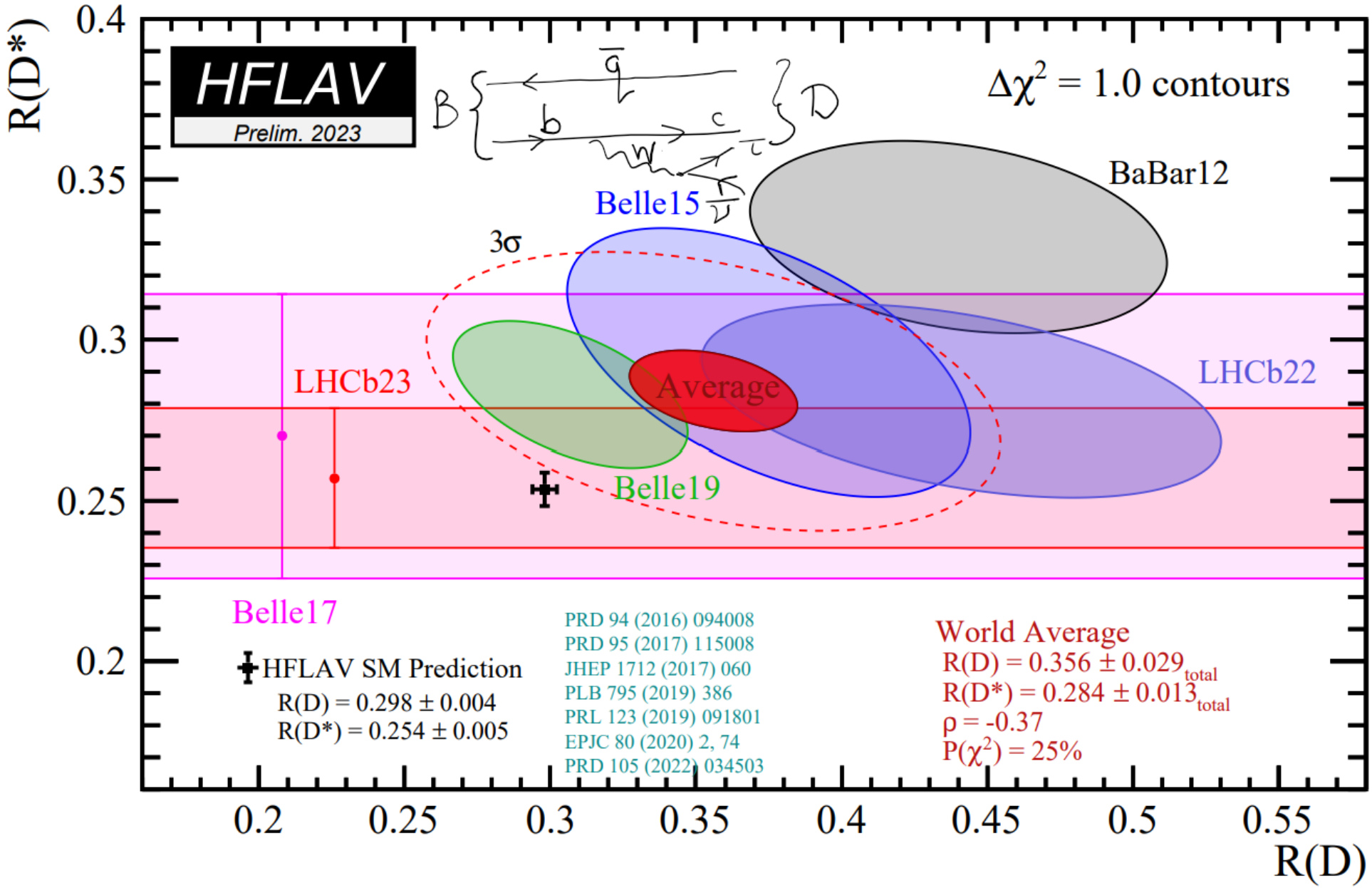
$$SU(3) \times SU(2) \times U(1)_Y \times U(1)_{X_1}$$

without kinetic mixing and

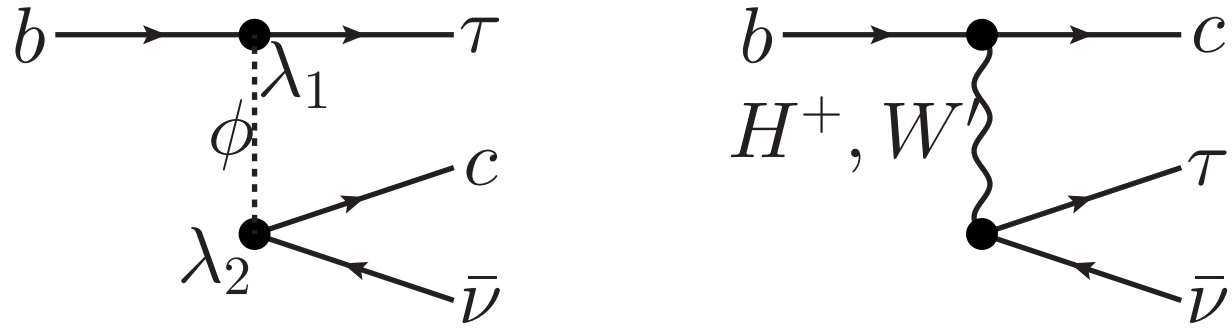
$$X_1 := B_3 - L_2 + \alpha Y,$$

where $\alpha \in \mathbb{Q}$.

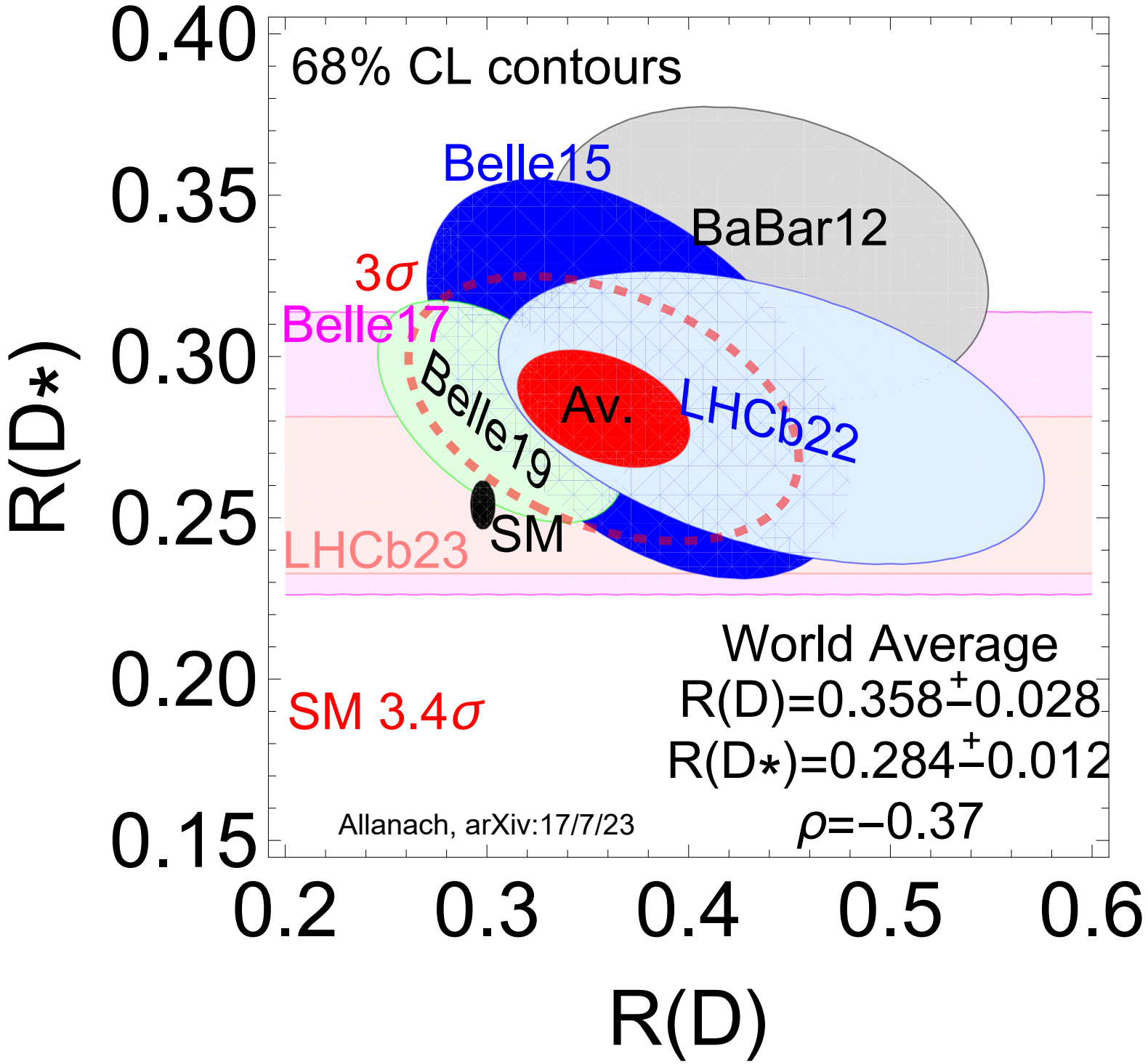
$$R_{D^{(*)}} = BR(B \to D^{(*)}\tau\nu)/BR(B \to D^{(*)}\ell\nu_\ell)$$



$R_{D(*)}$: BSM Explanations



$$\mathcal{L}_{WET} = -\frac{2\lambda_1\lambda_2}{M^2}\left(\bar{c}\gamma^\mu P_L \nu\right)\left(\bar{\tau}\gamma_\mu P_L b\right) + H.c.$$


68% CL contours
Belle15
BaBar12
3σ
Belle17
Belle19
Av.
LHCb22
SM
LHCb23
SM 3.4σ
World Average
R(D)=0.358±0.028
R(D∗)=0.284±0.012
ρ=−0.37
Allanach, arXiv:17/7/23
R(D∗)
R(D)
0.40
0.35
0.30
0.25
0.20
0.15
0.2
0.3
0.4
0.5
0.6

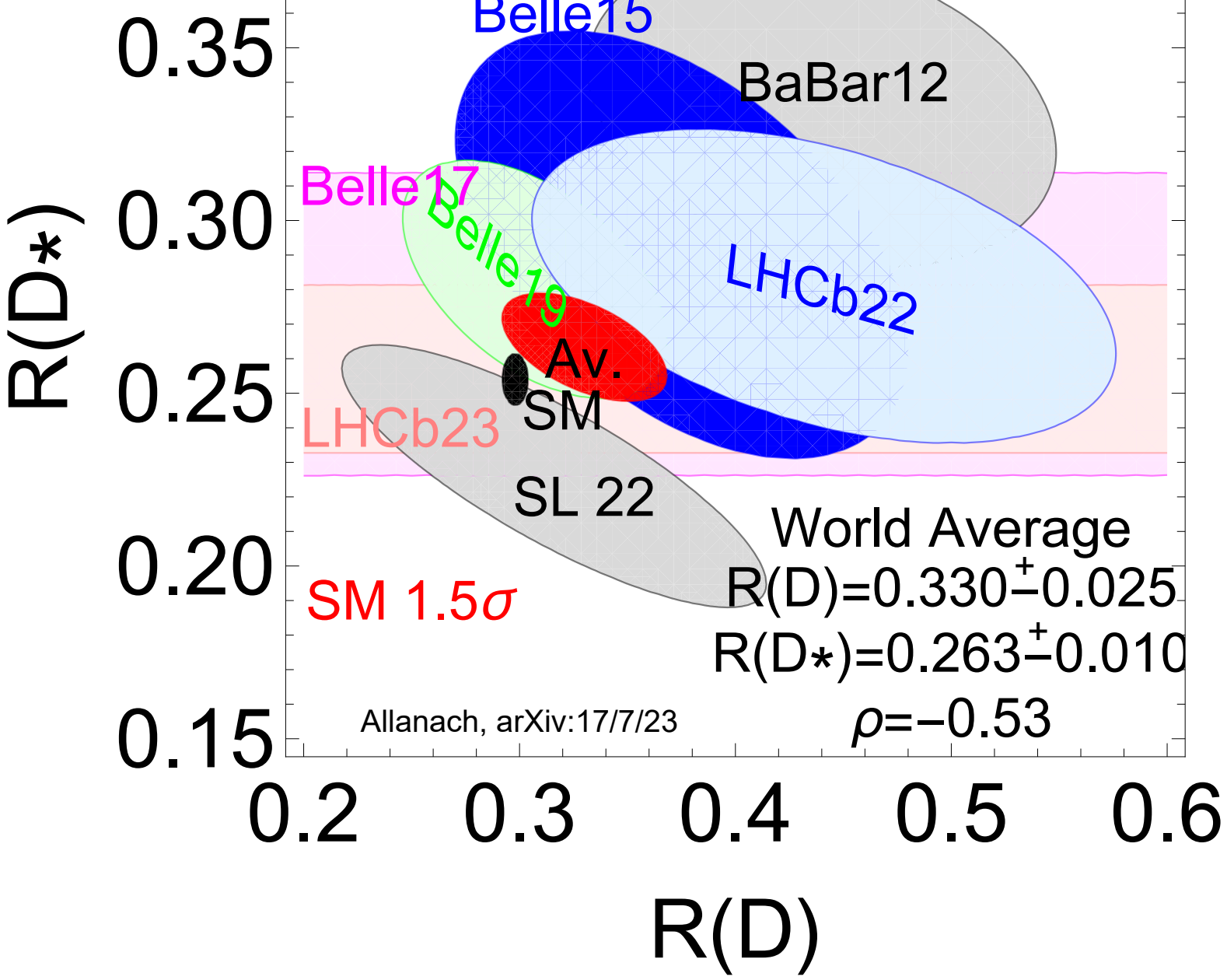
2022 Measurement

Using BaBar data (not official BaBar analysis) and *semi-leptonic* tag: (2012 used *hadronic*)

$$\begin{aligned} R(D) &= 0.316 \pm 0.062 \pm 0.019 \\ R(D*) &= 0.226 \pm 0.022 \pm 0.012 \\ \rho &= -0.82 \end{aligned}$$

Yunxuan Li, *Search for Beyond Standard Model Physics at BaBar*, (2022), Caltech Ph.D. thesis

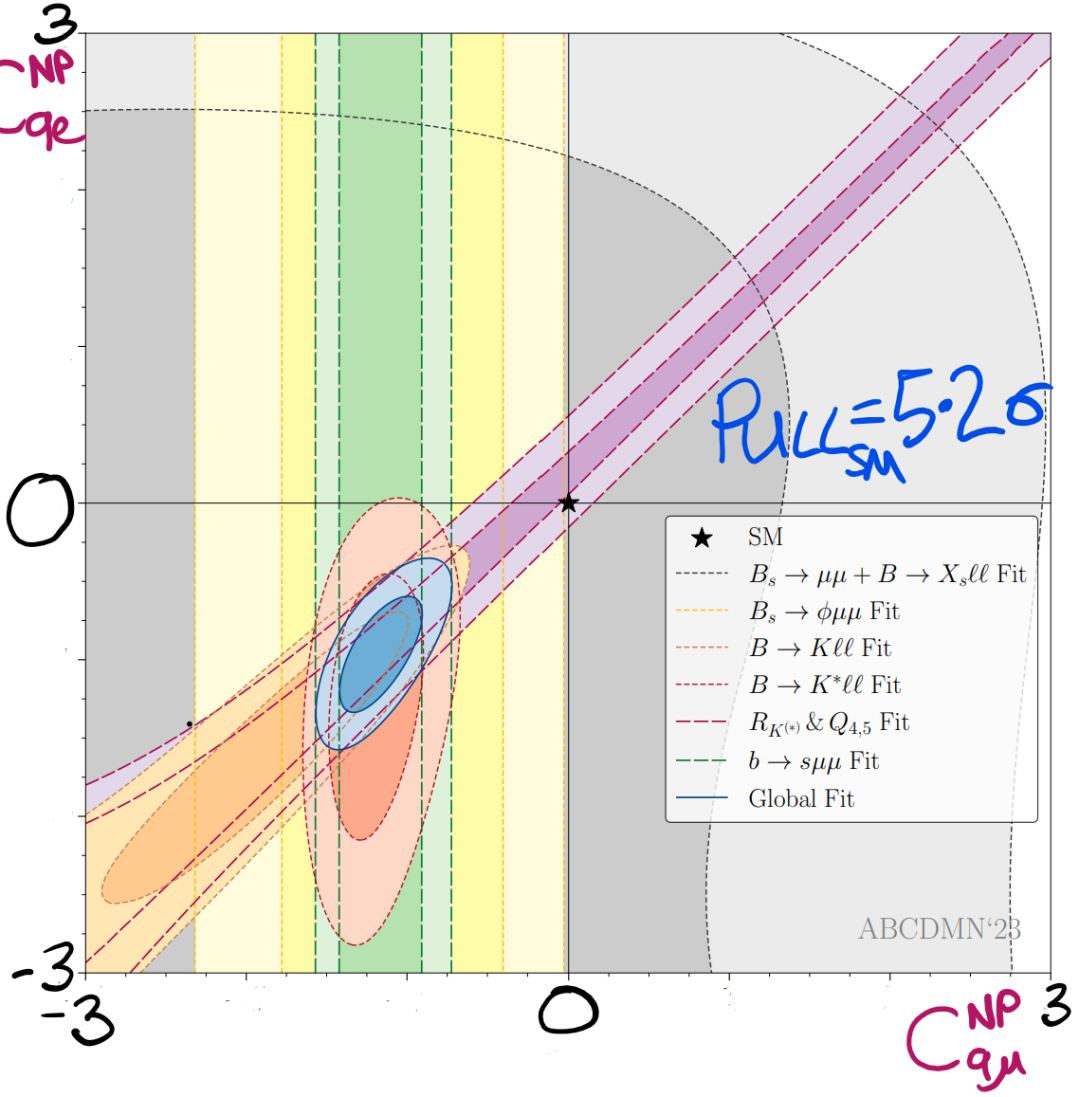
https://resolver.caltech.edu/CaltechTHESIS:05232022-144829107


Belle15
BaBar12
Belle17
Belle19
LHCb22
Av.
SM
LHCb23
SL 22
SM 1.5σ
World Average
R(D)=0.330±0.025
R(D∗)=0.263±0.010
ρ=−0.53
Allanach, arXiv:17/7/23
R(D∗)
R(D)
0.35
0.30
0.25
0.20
0.15
0.2
0.3
0.4
0.5
0.6

μ/e Neutral Current Fits

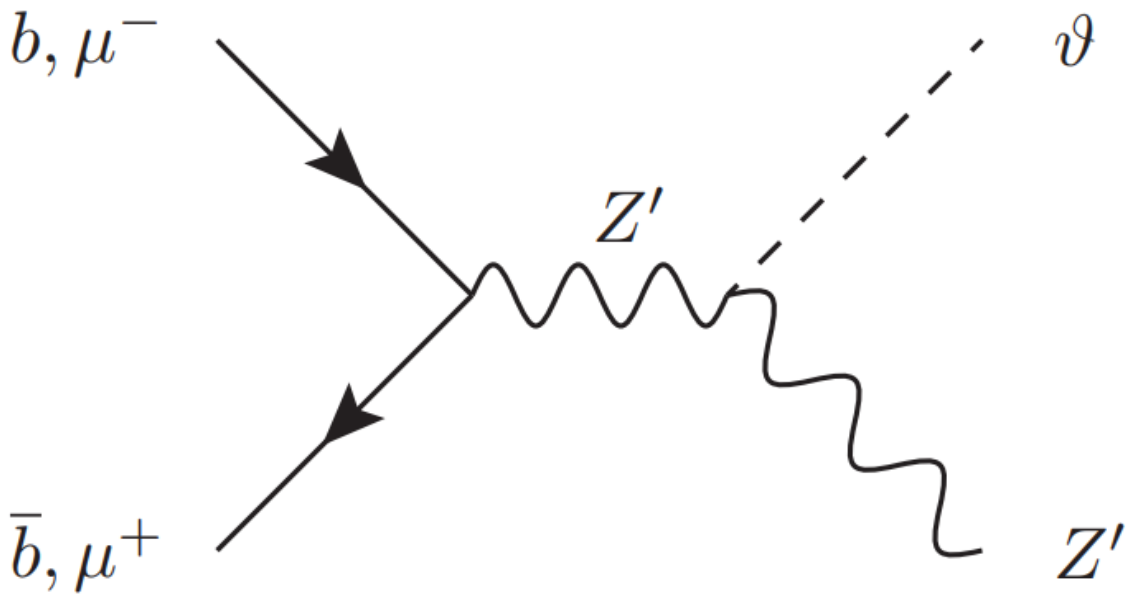
Alguero *et al*, 2304.07330

$$\mathcal{L} = N[C_{9\mu}^{NP}(\bar{b}_L\gamma^\alpha s_L)(\bar{\mu}\gamma_\alpha\mu) + C_{9e}^{NP}(\bar{b}_L\gamma^\alpha s_L)(\bar{e}\gamma_\alpha e)] + H.c.$$



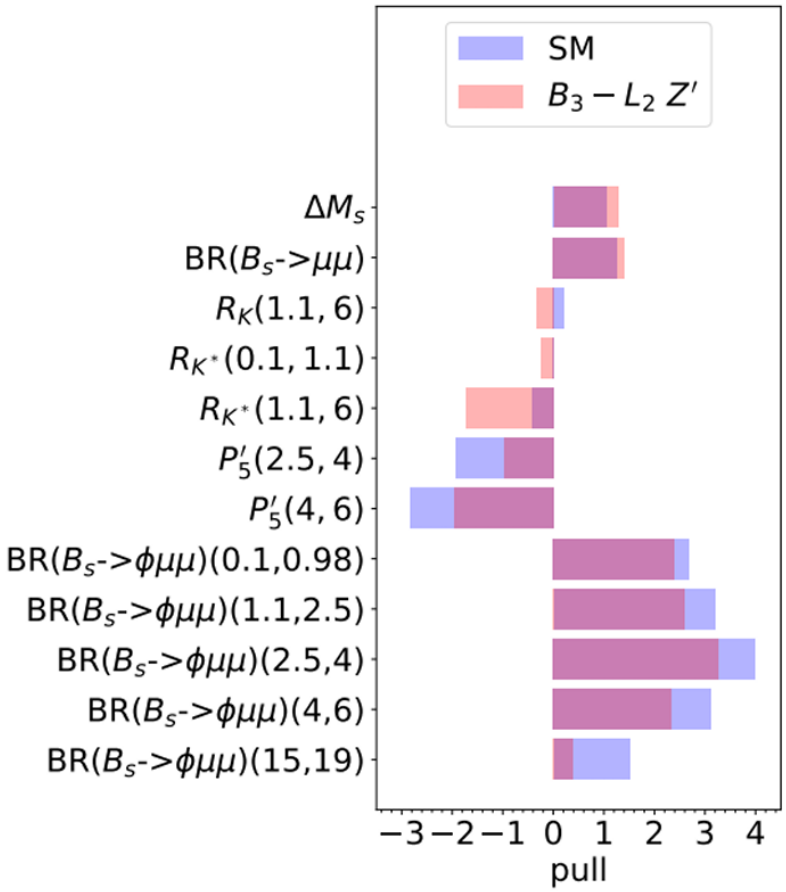
Flavonstrahlung

Models of Z' ilk possess $\mathcal{L} = \lambda H H^\dagger \theta \theta^\dagger \Rightarrow$ a *flavonstrahlung* signature:



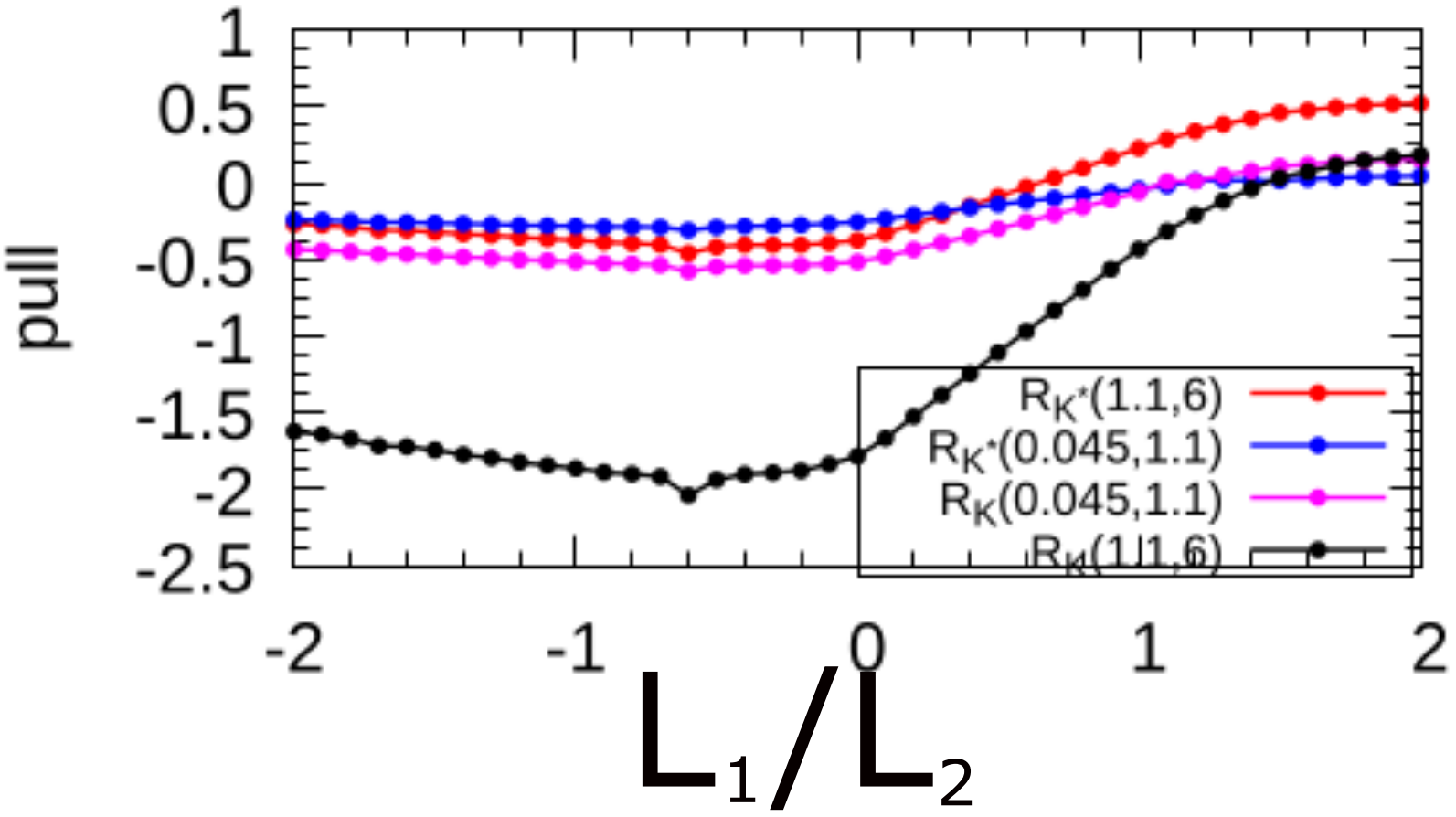
BCA, 2009.02197; BCA, Loisa, 2212.07440

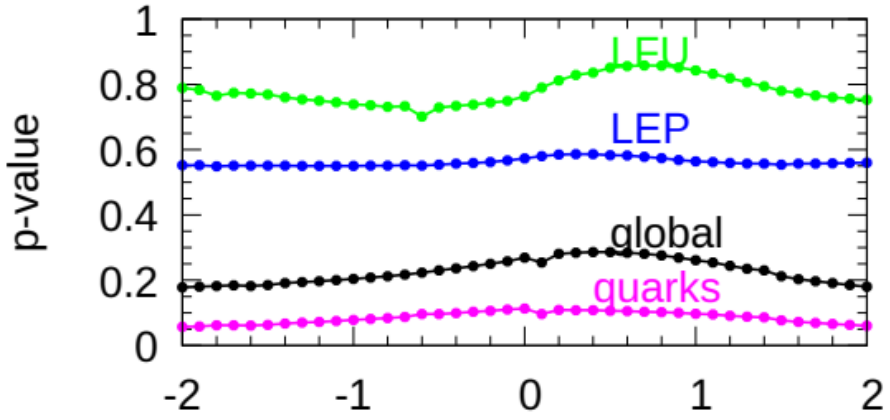
Pull=(theory-exp)/error

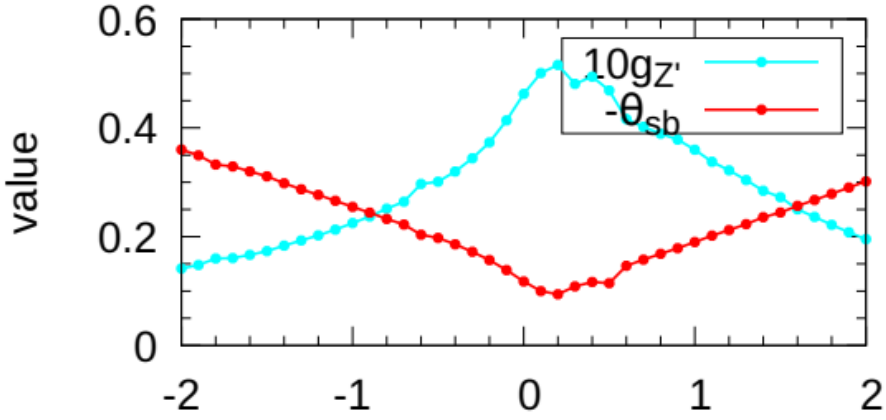


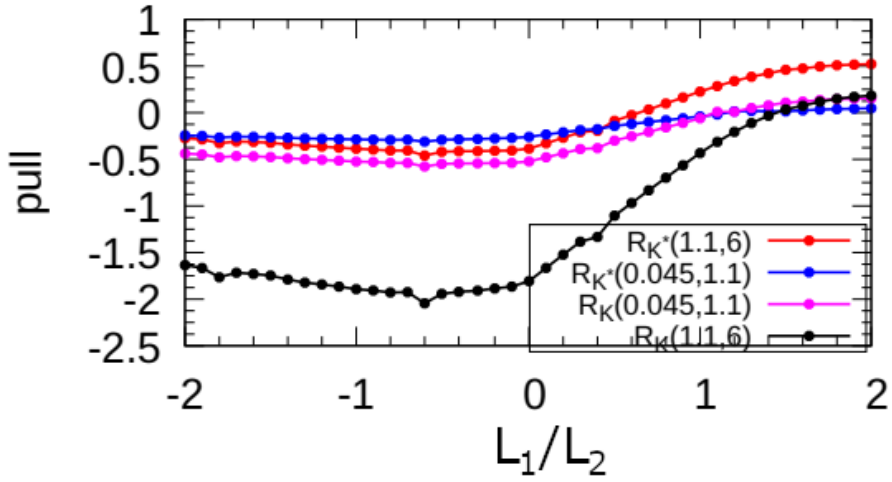
BCA, Davighi, 2211.11766

BCA, Mullin, 2306.08669




p-value
LFU
LEP
global
quarks

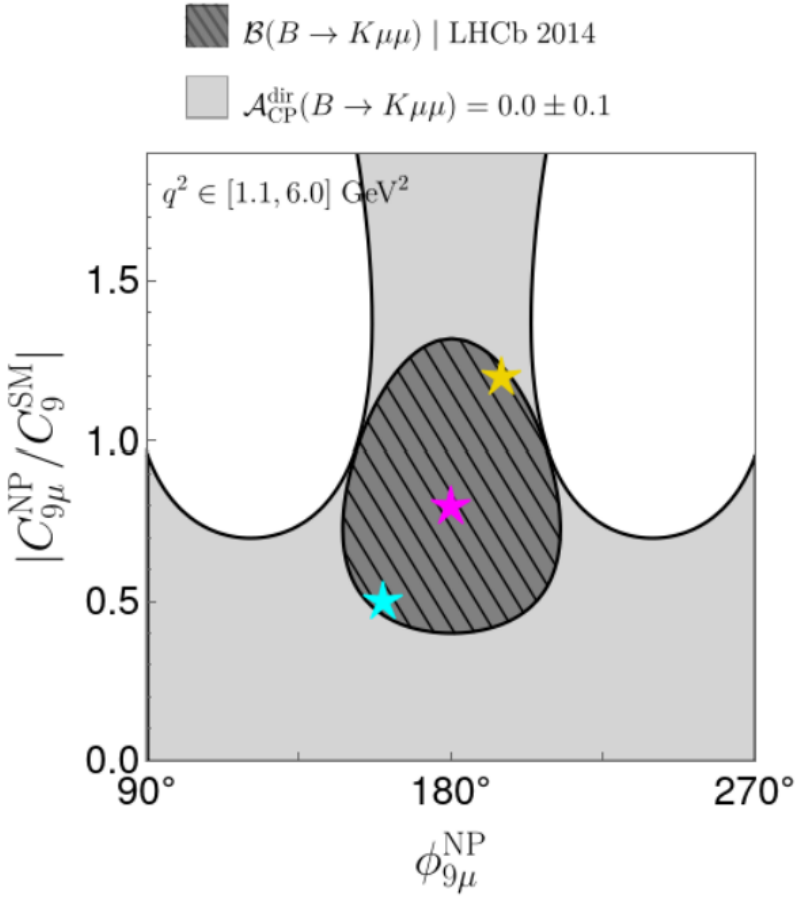

value
10g_Z'
-θ_sb

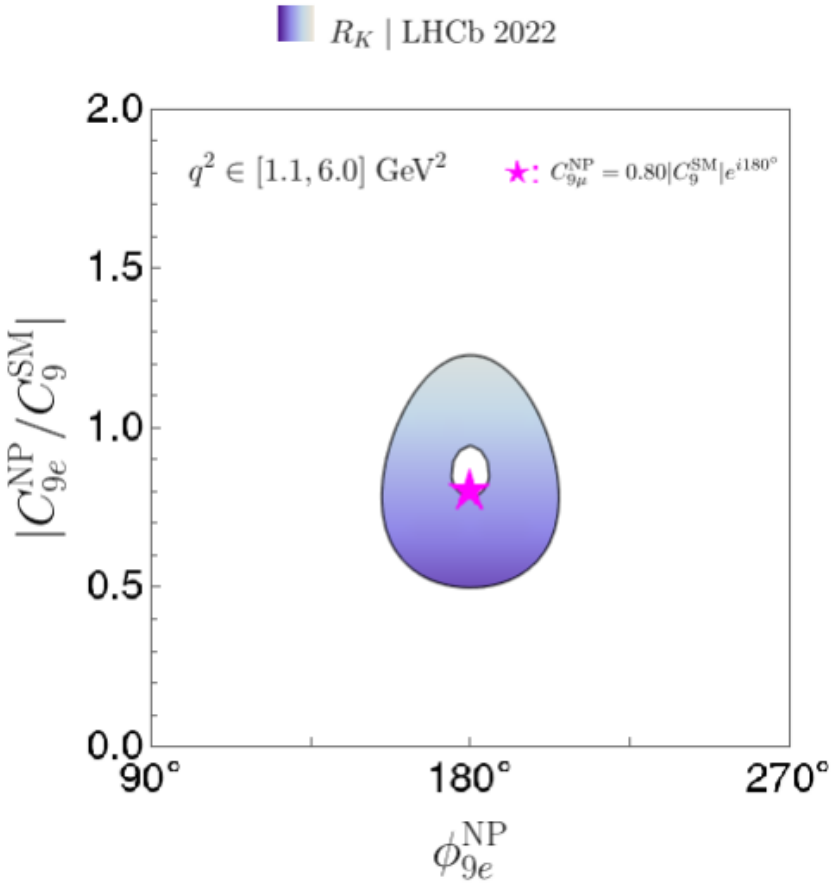

pull
L1/L2
R_K*(1.1,6)
R_K*(0.045,1.1)
R_K(0.045,1.1)
R_K(1.1,6)

$e \neq \mu$ allowed

Fleischer, Malami, Rehult, Keri Vos, 2303.08764; $C_{9\ell}^{NP} = |C_{9\ell}^{NP}| e^{i\phi_{9\ell}^{NP}}$

$$\mathcal{L} = N(\bar{b}_L \gamma^\alpha s_L)[C_{9\mu}^{NP}(\bar{\mu}\gamma_\alpha \mu) + C_{9e}^{NP}(\bar{e}\gamma_\alpha e)] + H.c.$$





Anomaly cancellation

Need to pick X charges for fermions consistent with QFT anomaly cancellation.

$$X = 3B_3 - (X_e L_e + X_\mu L_\mu + [3 - X_e - X_\mu] L_\tau)$$

works (proof in 2306.08669).

Trident Neutrino Process

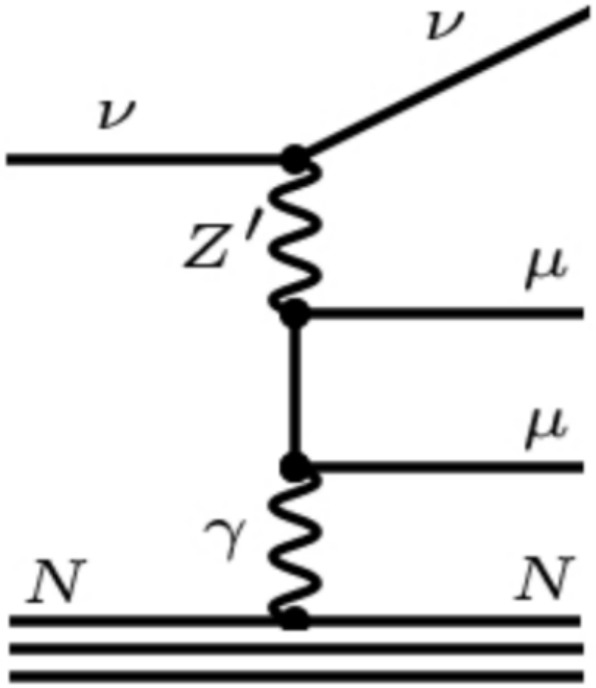
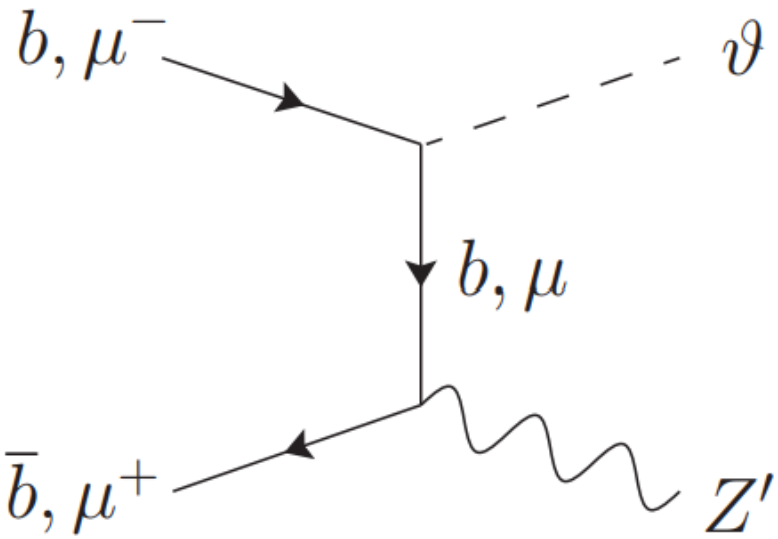
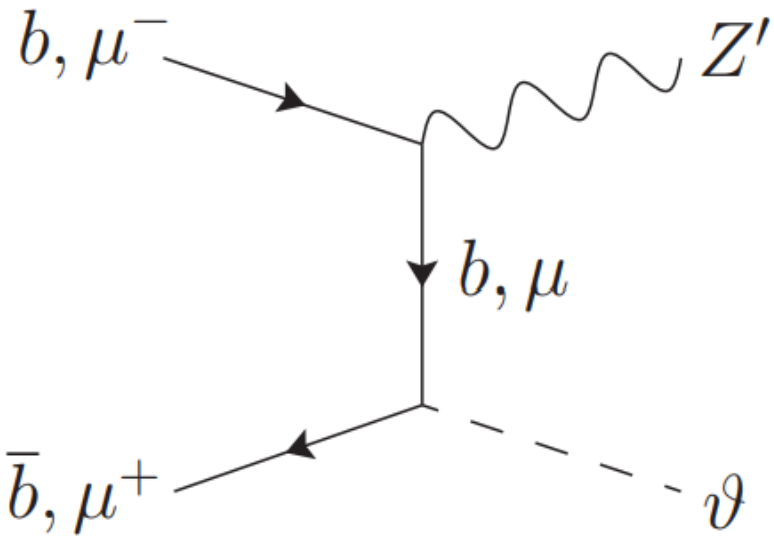


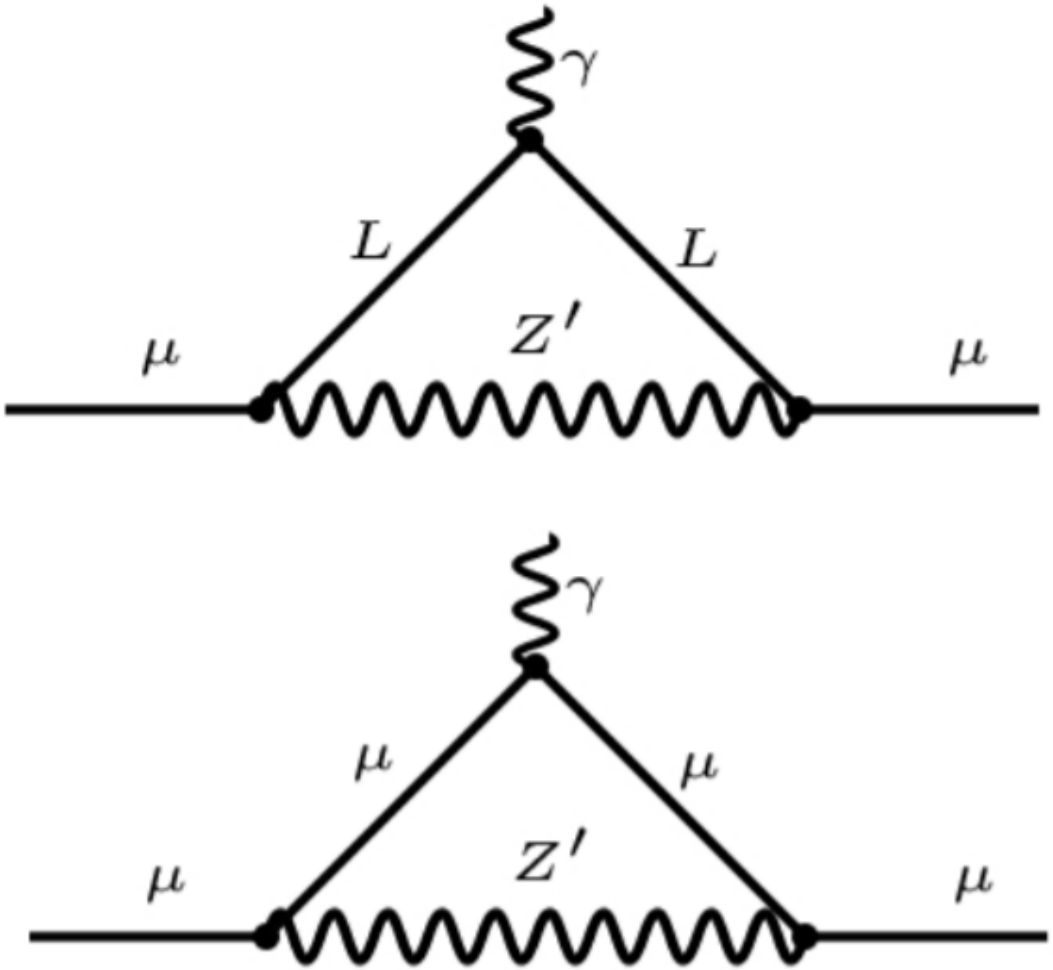
FIG. 10. Neutrino trident process that leads to constraints on the Z^μ coupling strength to neutrinos-muons, namely $M_{Z'}/g_{\nu\mu} \gtrsim 750$ GeV.

$t-$channel





$(g-2)_\mu$



$H\vartheta$ potential

$$
\begin{aligned}
V &= -\mu^2 H^\dagger H + \lambda_H (H^\dagger H)^2 - \mu_\theta^2 \theta^* \theta + \\
&\quad \lambda_\theta (\theta^* \theta)^2 + \lambda_{\theta H} \theta^* \theta H^\dagger H \\
&= -\frac{1}{2} \begin{pmatrix} h' & \vartheta' \end{pmatrix} M^2 \begin{pmatrix} h' \\ \vartheta' \end{pmatrix} + \dots \\
M^2 &= \begin{pmatrix} 2\lambda_H v_H^2 & \lambda_{\theta H} v_H v_\theta \\ \lambda_{\theta H} v_H v_\theta & 2\lambda_\theta v_\theta^2 \end{pmatrix}
\end{aligned}
$$

$H\vartheta$ mixing

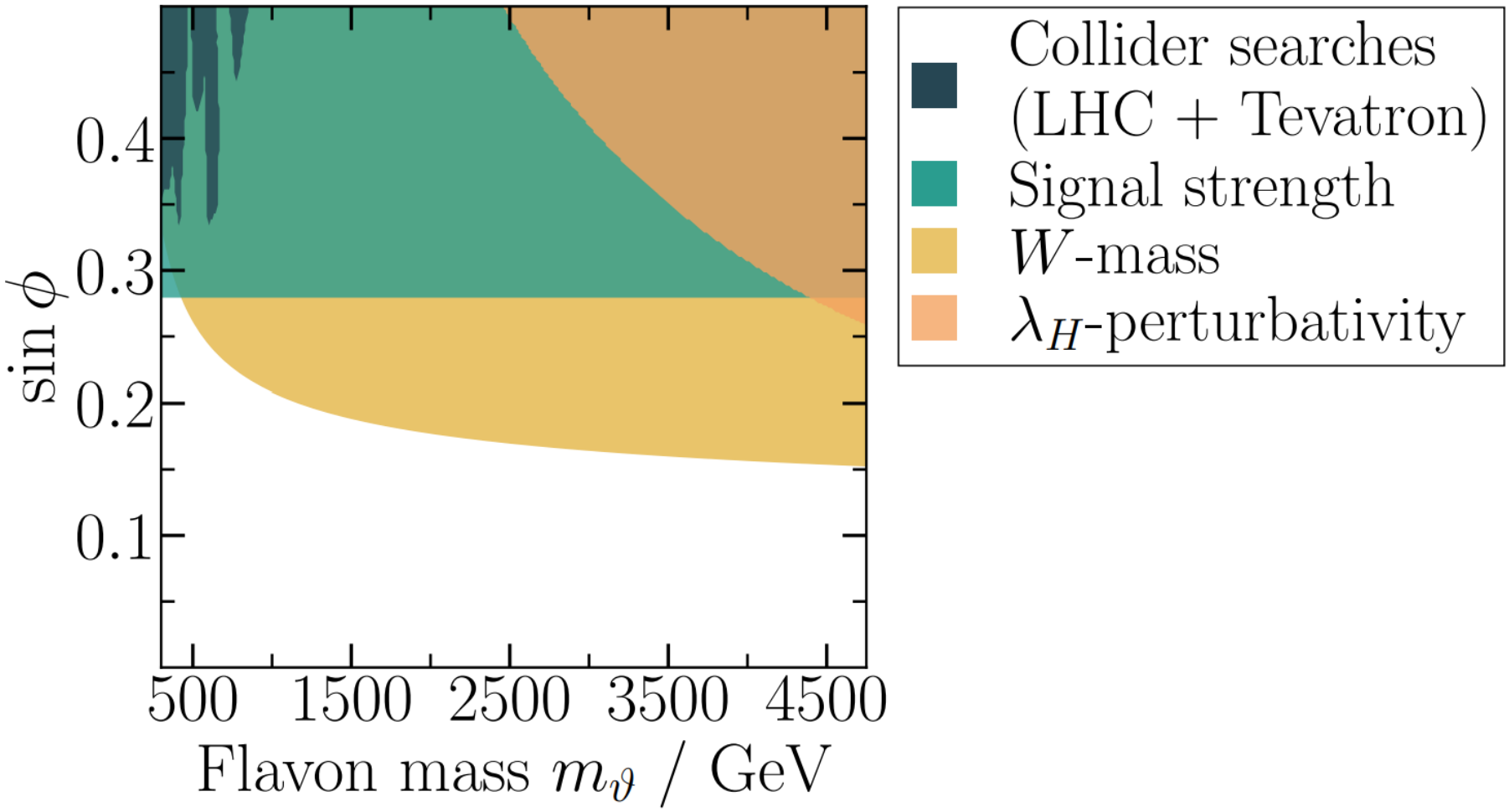
$$\begin{pmatrix} h \\ \vartheta \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} h' \\ \vartheta' \end{pmatrix}$$

$$\sin 2\phi = \frac{2\lambda_{\theta H} v_h v_\theta}{m_\vartheta^2 - m_h^2}. \qquad \text{(-9)}$$

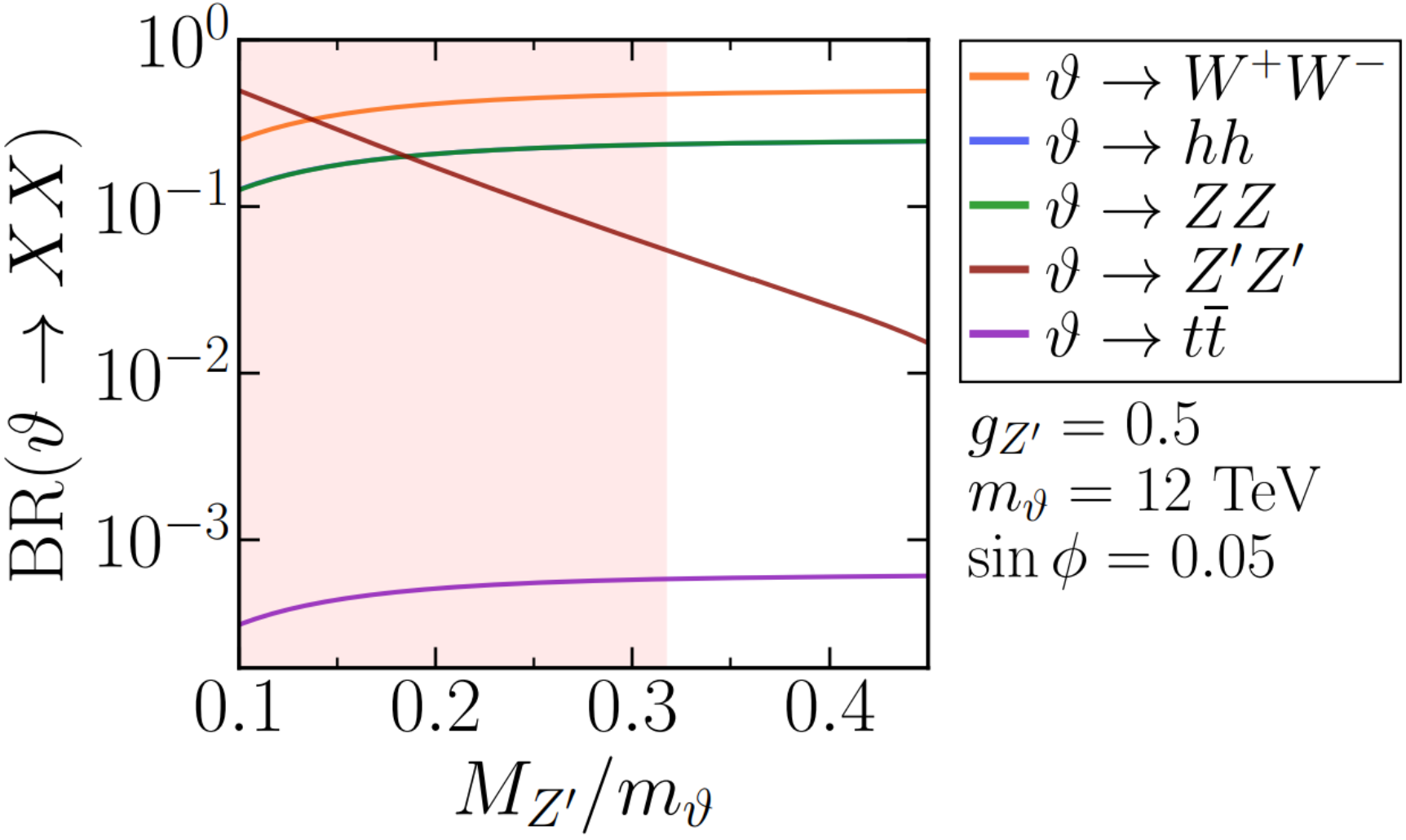
Three parameters: $v_\theta = M_{Z'}/g_{Z'}$, m_ϑ and ϕ.

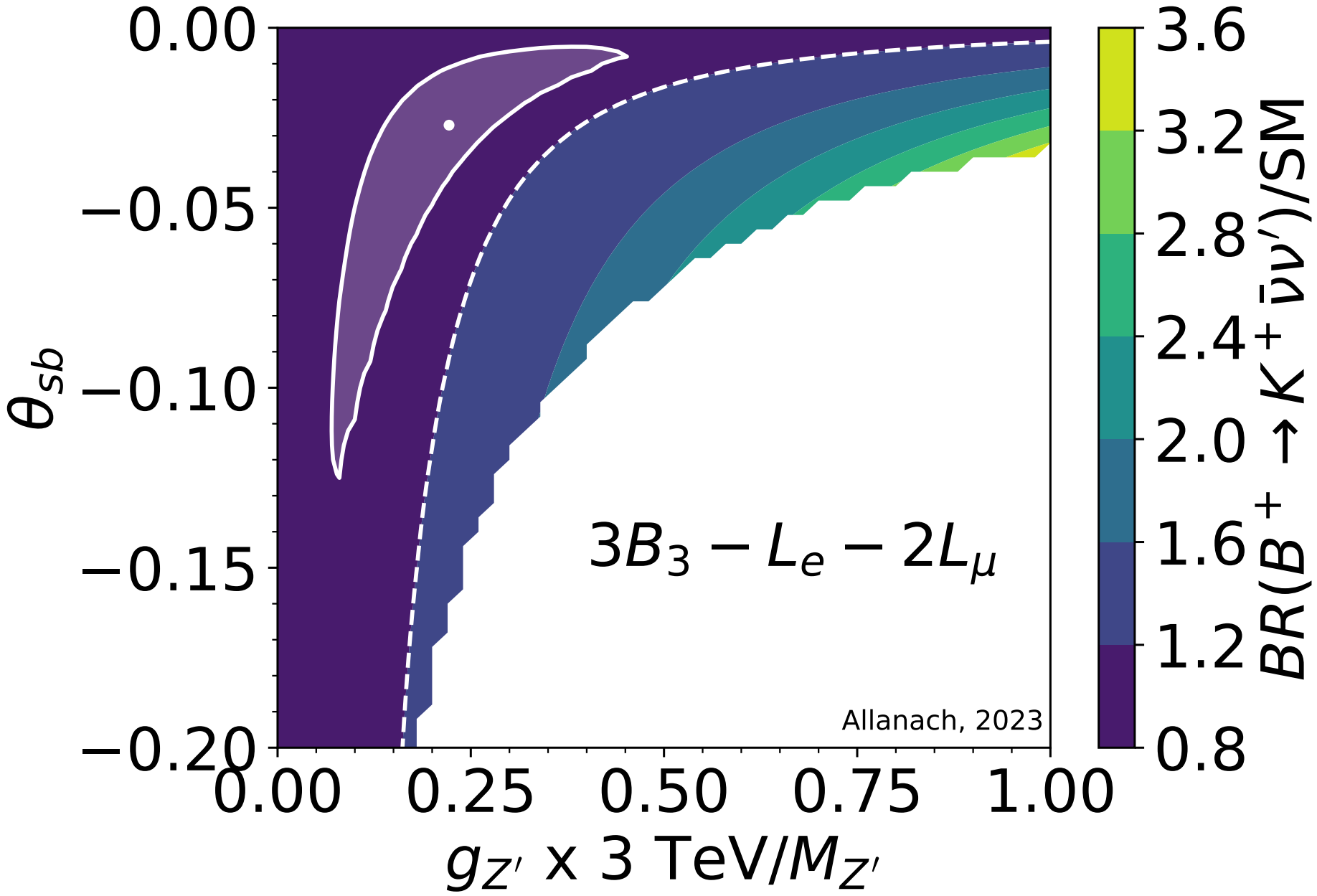
Higgs Signal Strength

BCA, Loisa, 2212.07440

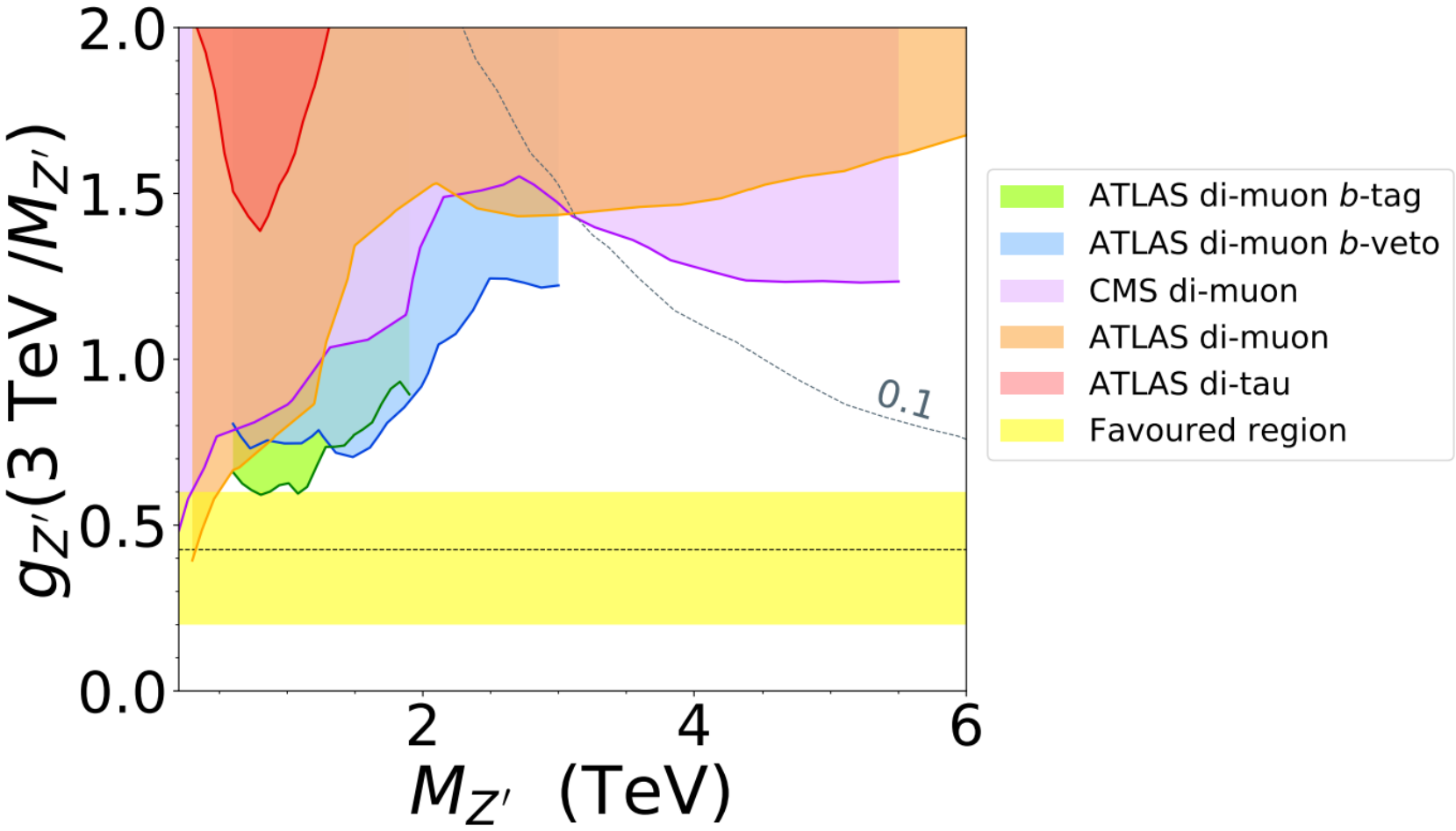


ϑ BRs

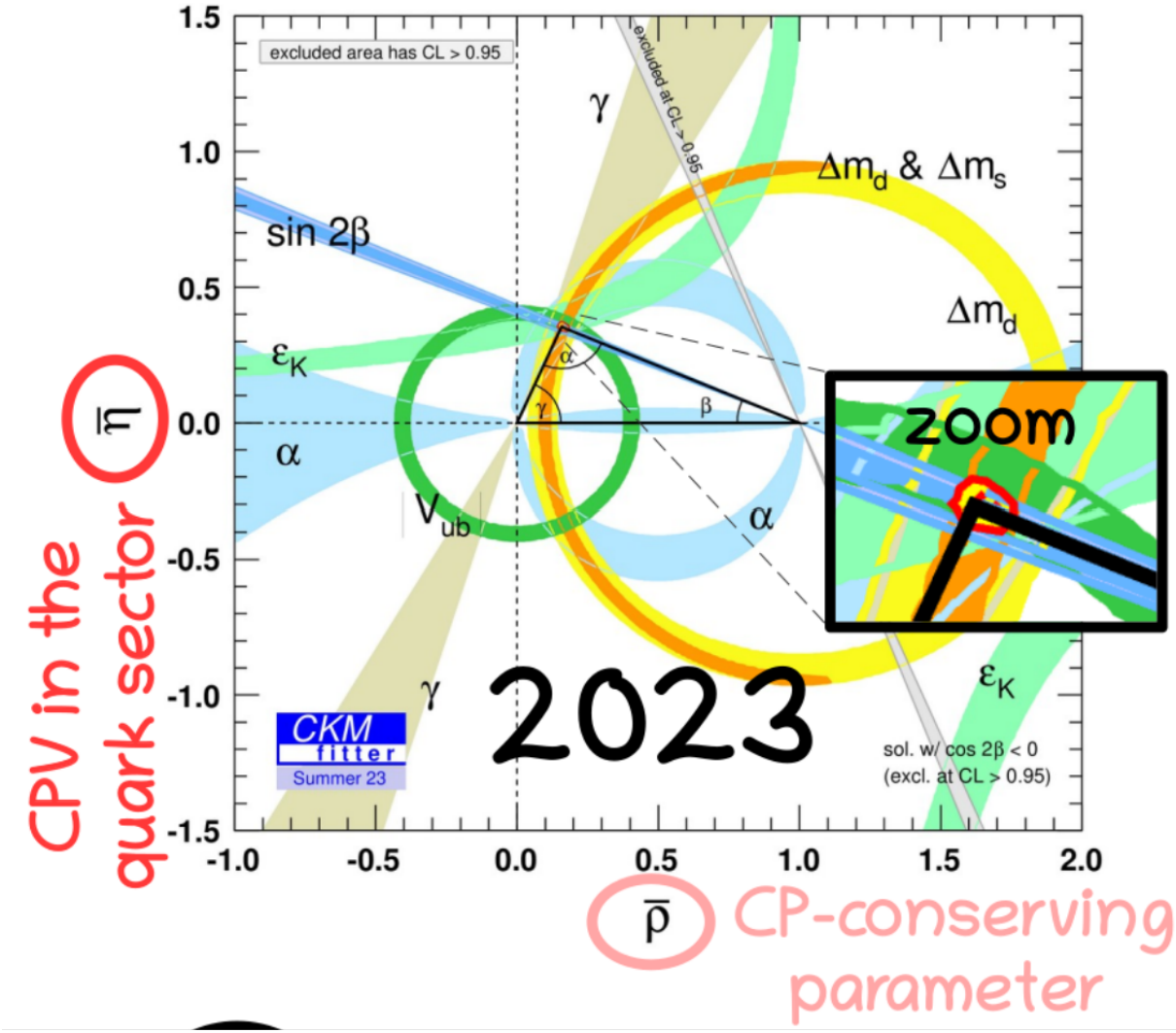


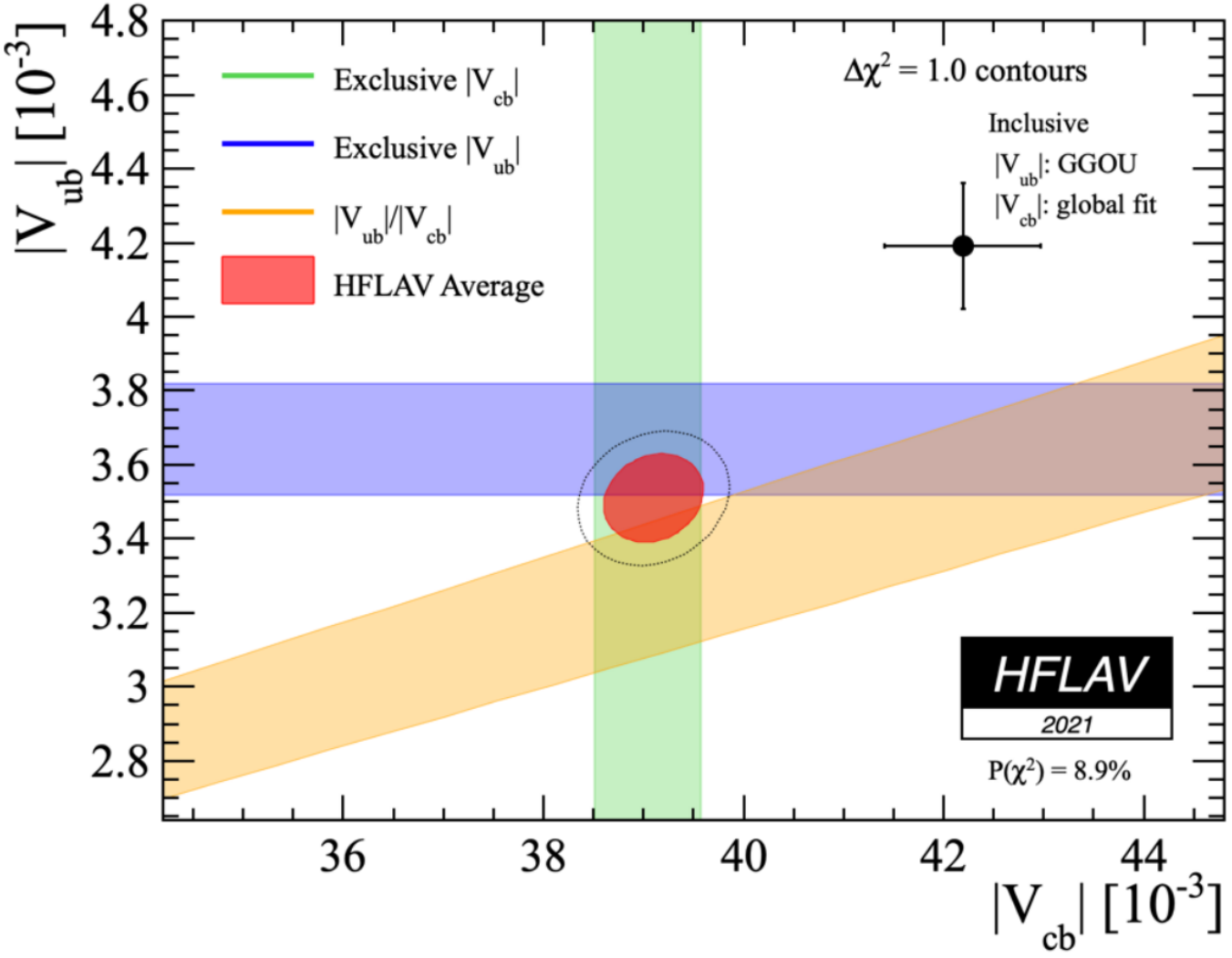

0.00
−0.05
−0.10
−0.15
−0.20
θ_{sb}
0.00
0.25
0.50
0.75
1.00
$g_{Z'}$ x 3 TeV/$M_{Z'}$
$3B_3 - L_e - 2L_\mu$
Allanach, 2023
3.6
3.2
2.8
2.4
2.0
1.6
1.2
0.8
$BR(B^+ \to K^+ \bar{\nu}\nu')/SM$

Z' Searches[7]



[7] BCA, Banks, 2111.06691

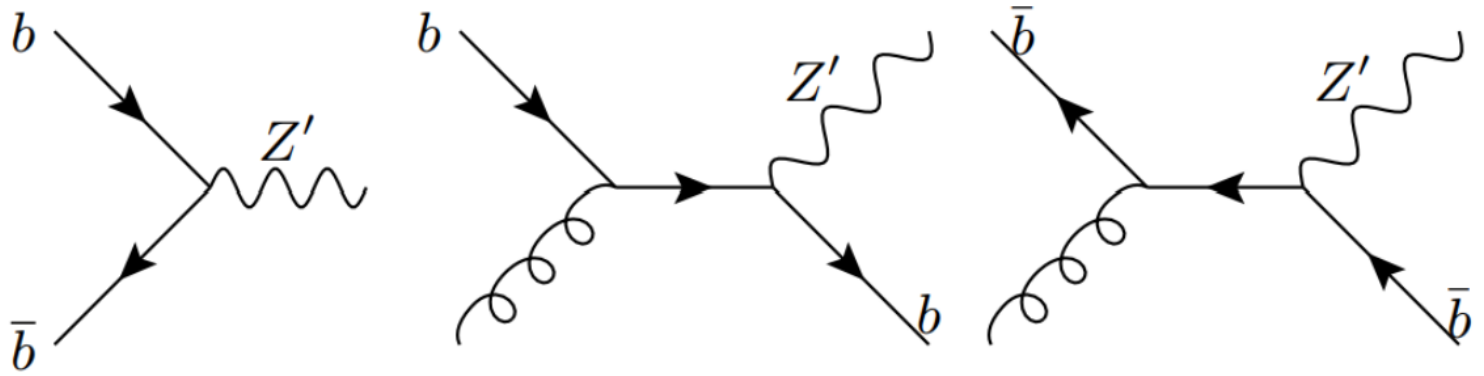

excluded area has CL > 0.95
excluded at CL > 0.95
γ
Δm_d & Δm_s
sin 2β
Δm_d
ε_K
α
V_ub
α
zoom
γ
2023
ε_K
CKM fitter
Summer 23
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
CPV in the quark sector
η̄
ρ̄
CP-conserving parameter


Exclusive |V_cb|
Exclusive |V_ub|
|V_ub|/|V_cb|
HFLAV Average
Δχ² = 1.0 contours
Inclusive
|V_ub|: GGOU
|V_cb|: global fit
HFLAV
2021
P(χ²) = 8.9%
|V_ub| [10^-3]
|V_cb| [10^-3]
4.8
4.6
4.4
4.2
4
3.8
3.6
3.4
3.2
3
2.8
36
38
40
42
44

Z' Decay Modes

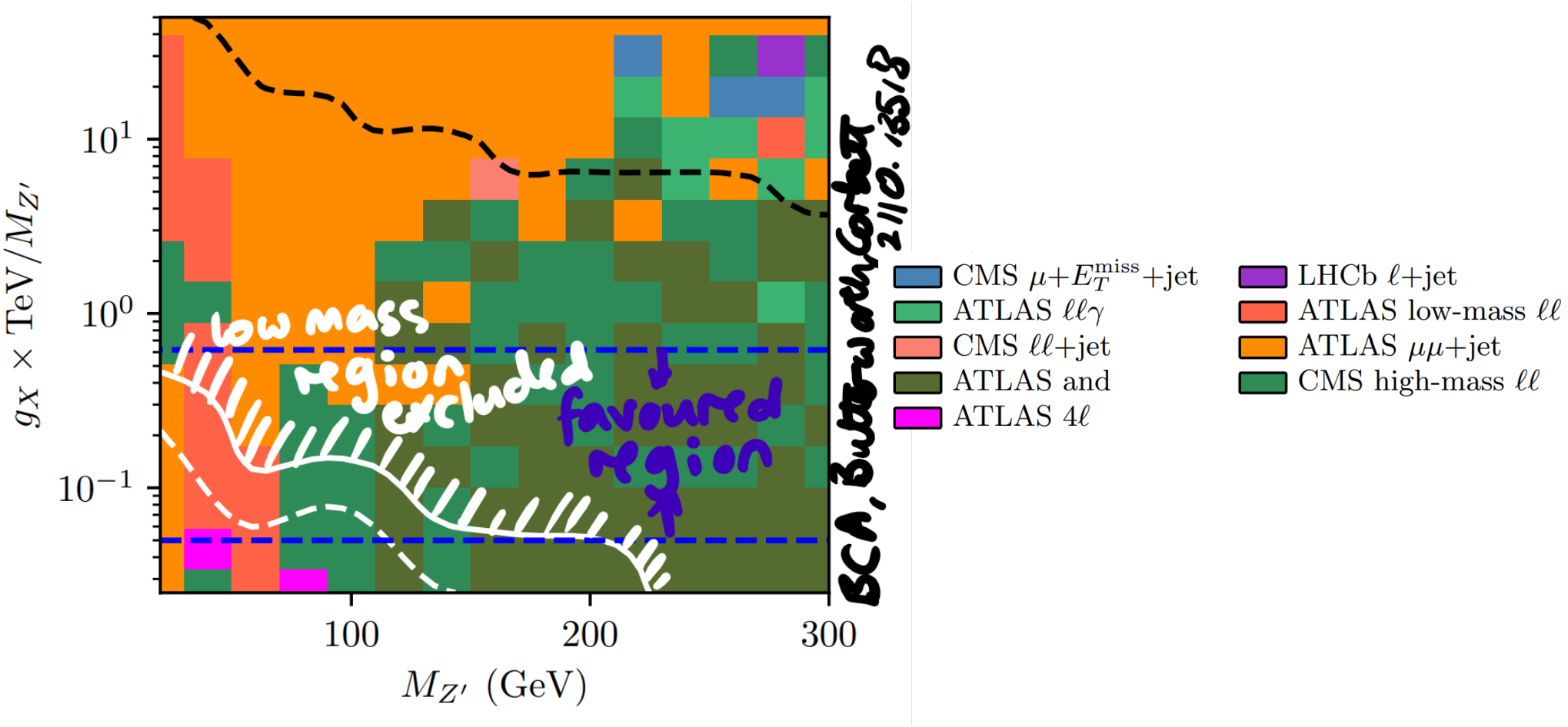
Mode	BR	Mode	BR	Mode	BR
$t\bar{t}$	0.15	$b\bar{b}$	0.15	$\nu\bar{\nu}'$	0.23
$\mu^+\mu^-$	0.37	e^+e^-	0.09		

pp Z' Production:



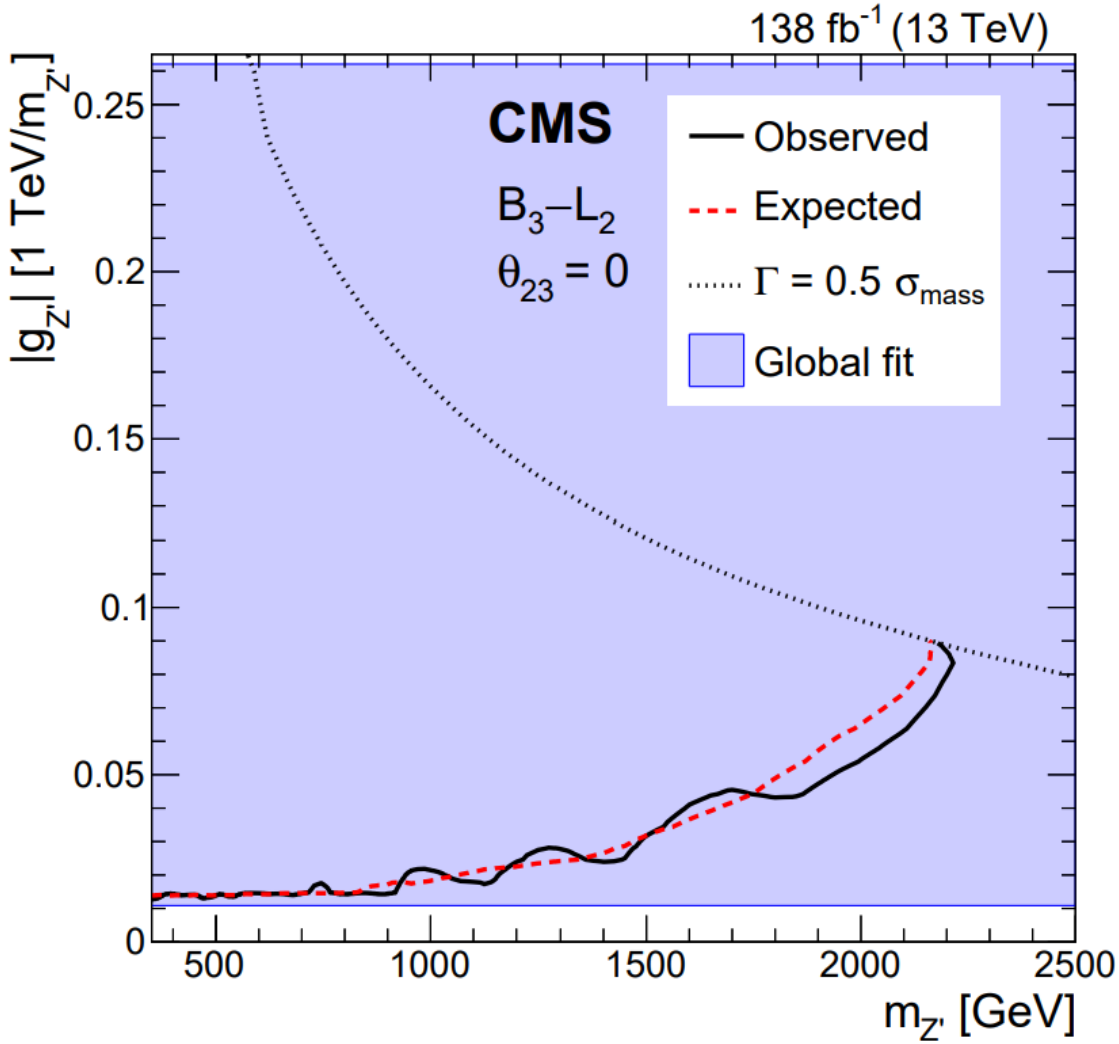
$$\sigma_{prod} \propto g_{Z'}^2 \cos^4\theta_{sb} = g_{Z'}^2 \left(1 - 2\theta_{sb}^2 + \mathcal{O}(\theta_{sb}^4)\right)$$

$B_3 - L_2$ model's[8] Z'

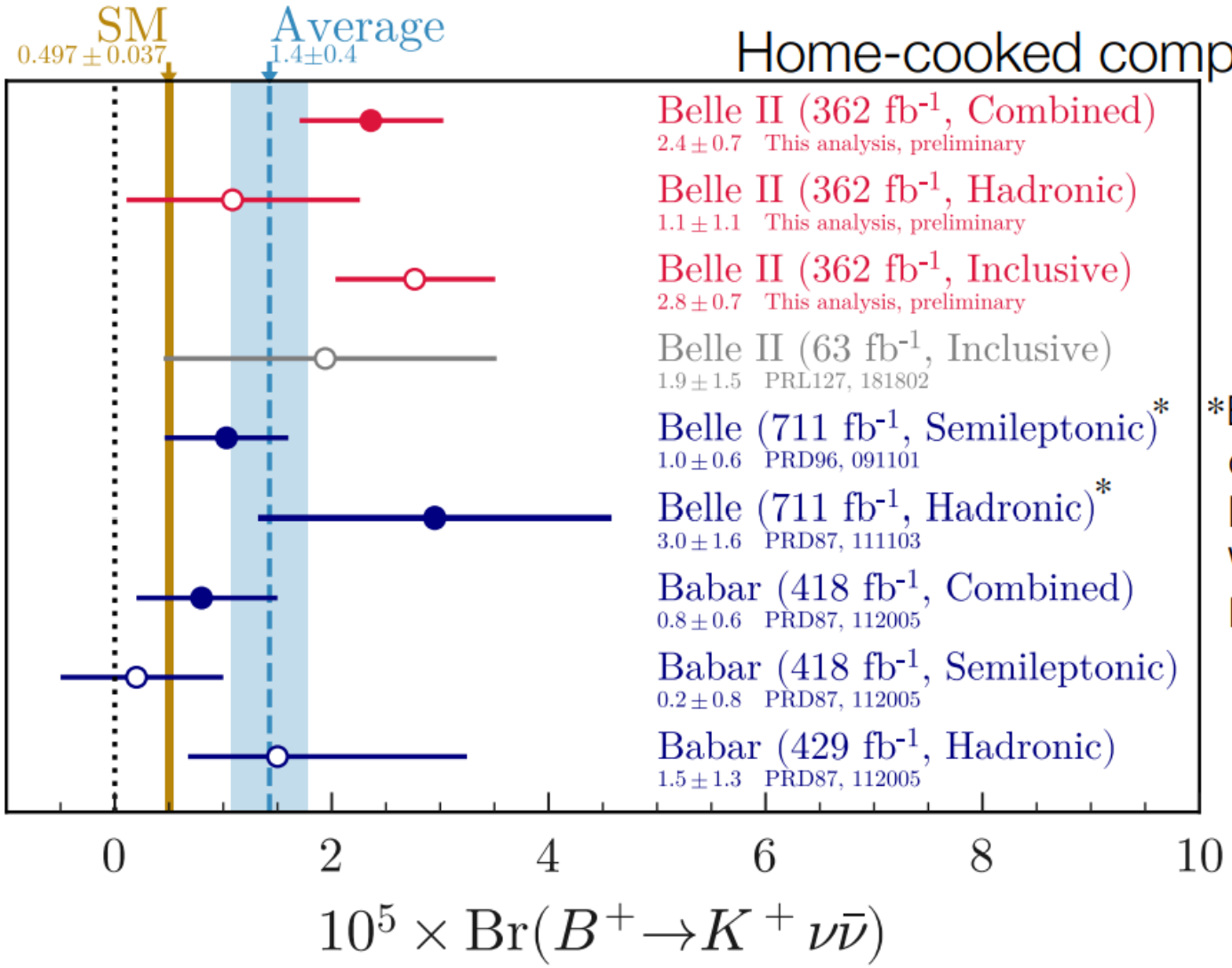


[8] Bonilla, Modak, Srivastava, Valle, 1705.00915; Alonso, Cox, Han, Yanagida 1705.03858

CMS $\mu^+\mu^- b$ 2307.08708

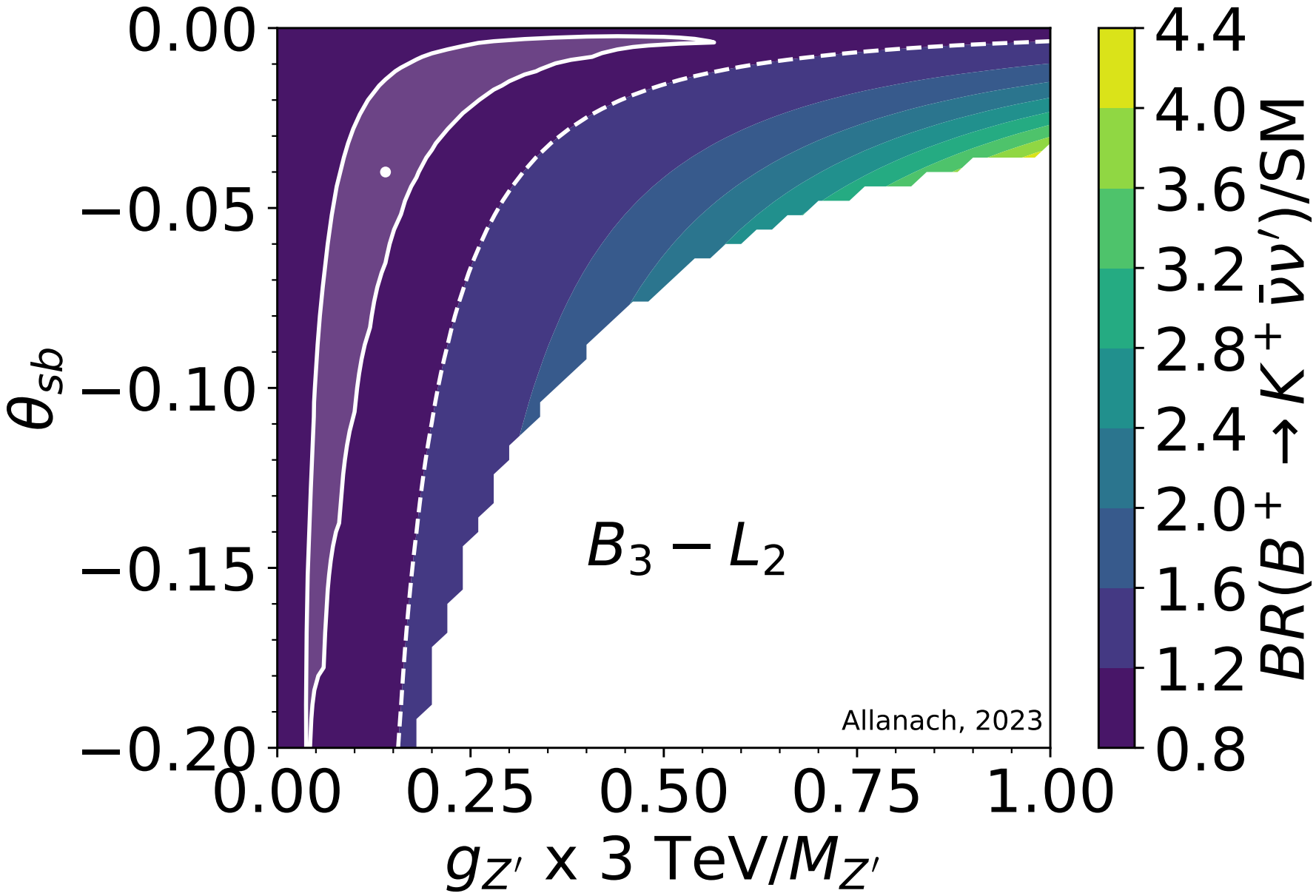


Home-cooked comparison

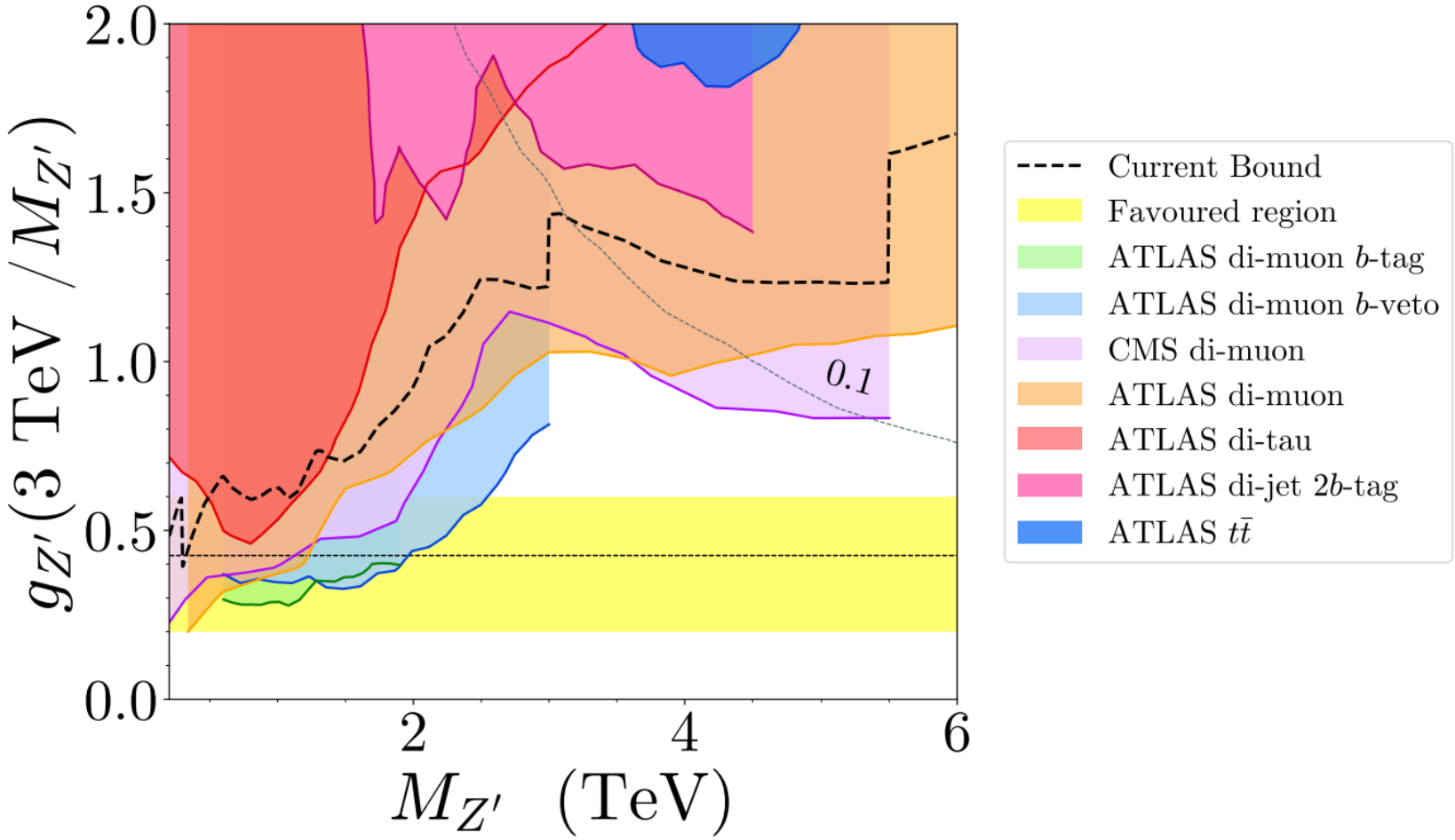


*Belle reports only upper limits. We calculate BF ourselves

Overall compatibility is good χ^2/ndf = 4.3/4

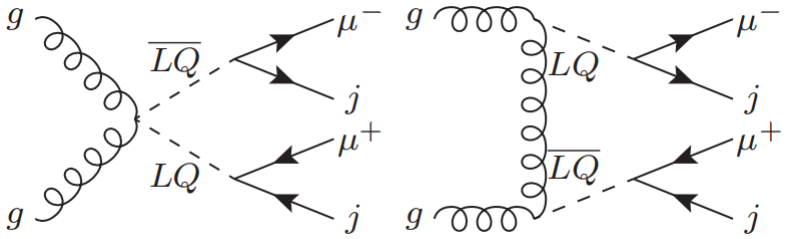

0.00
−0.05
−0.10
−0.15
−0.20
θ_{sb}
0.00
0.25
0.50
0.75
1.00
$g_{Z'}$ x 3 TeV/$M_{Z'}$
$B_3 - L_2$
Allanach, 2023
4.4
4.0
3.6
3.2
2.8
2.4
2.0
1.6
1.2
0.8
$BR(B^+ \to K^+ \bar{\nu}\nu')$/SM

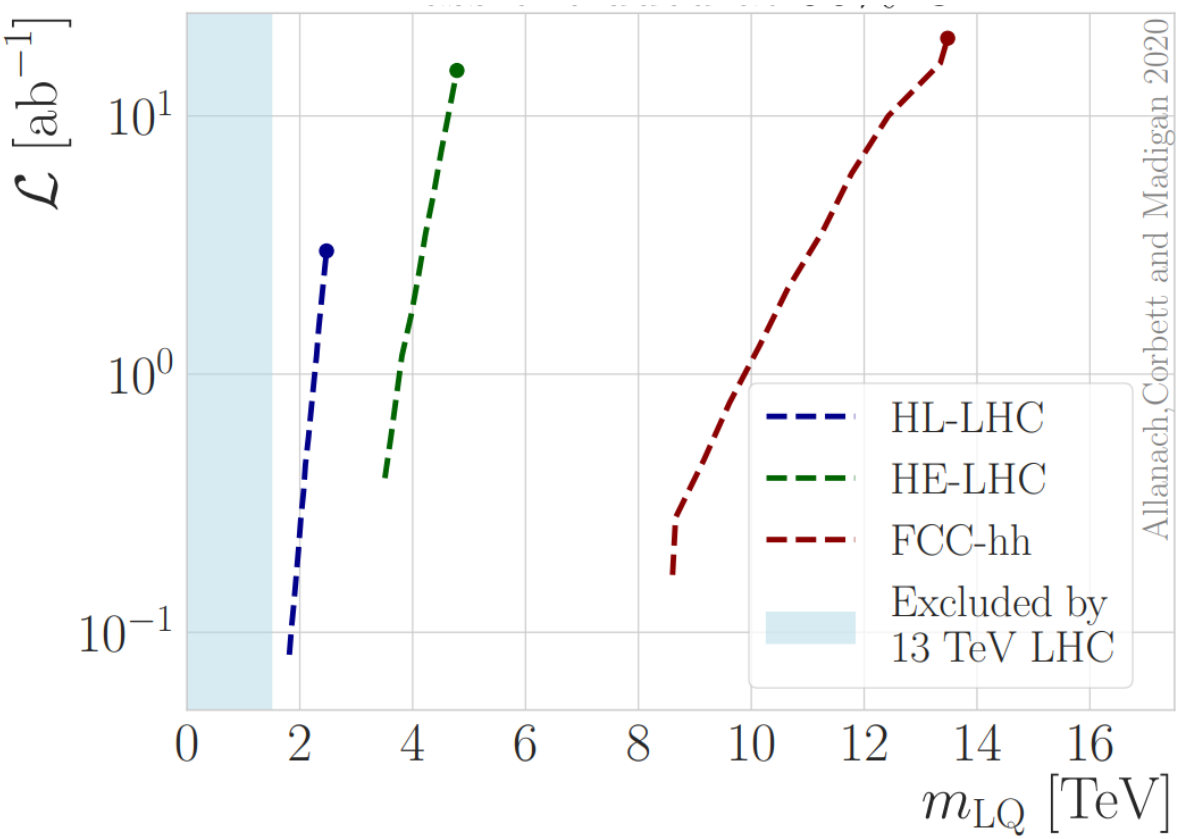
Y_3 HL-LHC sensitivity[9]



[9] BCA, Banks, 2111.06691

Scalar LQ[10]: eg $S_3 \sim (\bar{\mathbf{3}}, \mathbf{3}, 1/3)$





[10]BCA, Corbett, Madigan, 1911.0445