Charmonium resonances from lattice QCD

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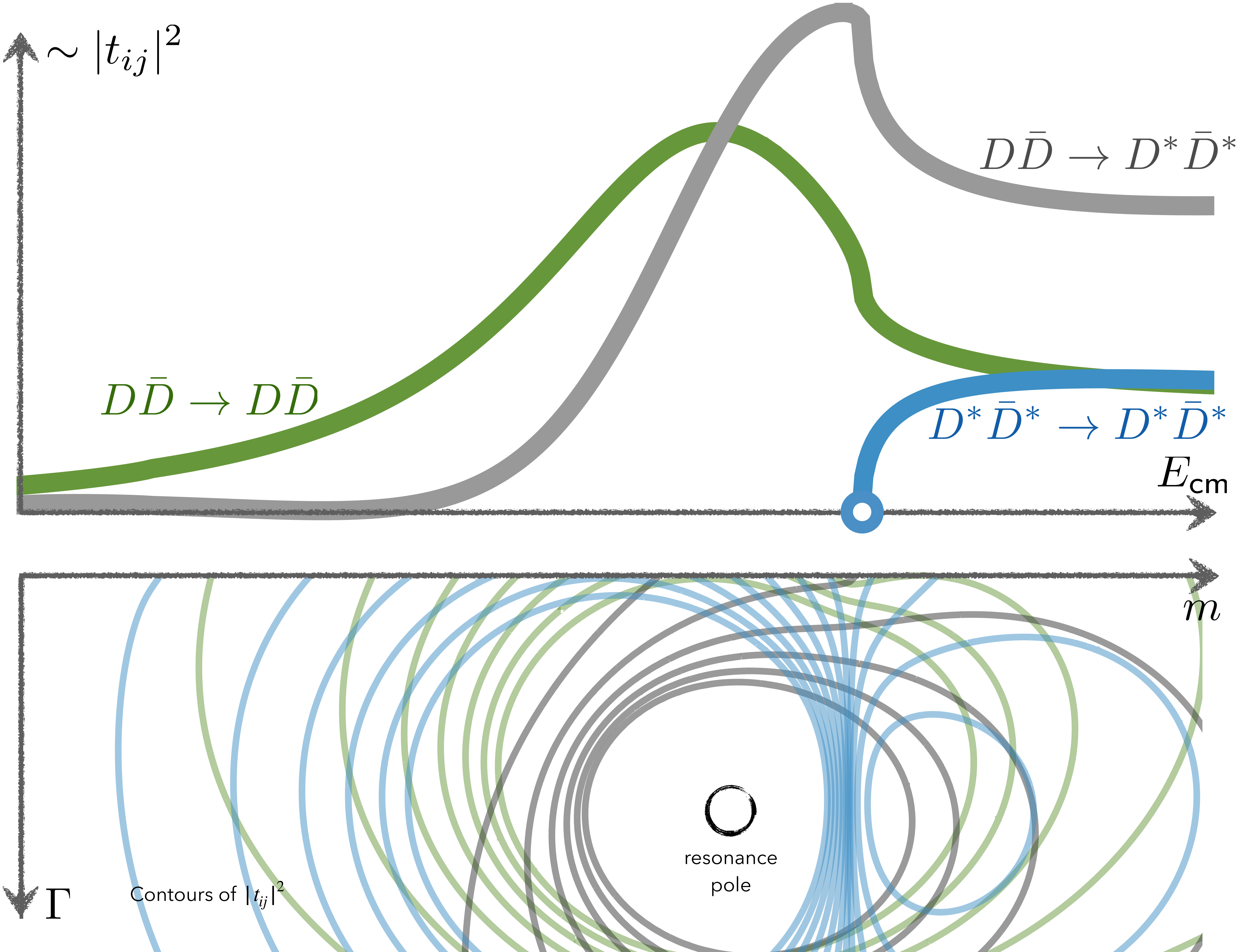
LHCb UK meeting
7th January 2025

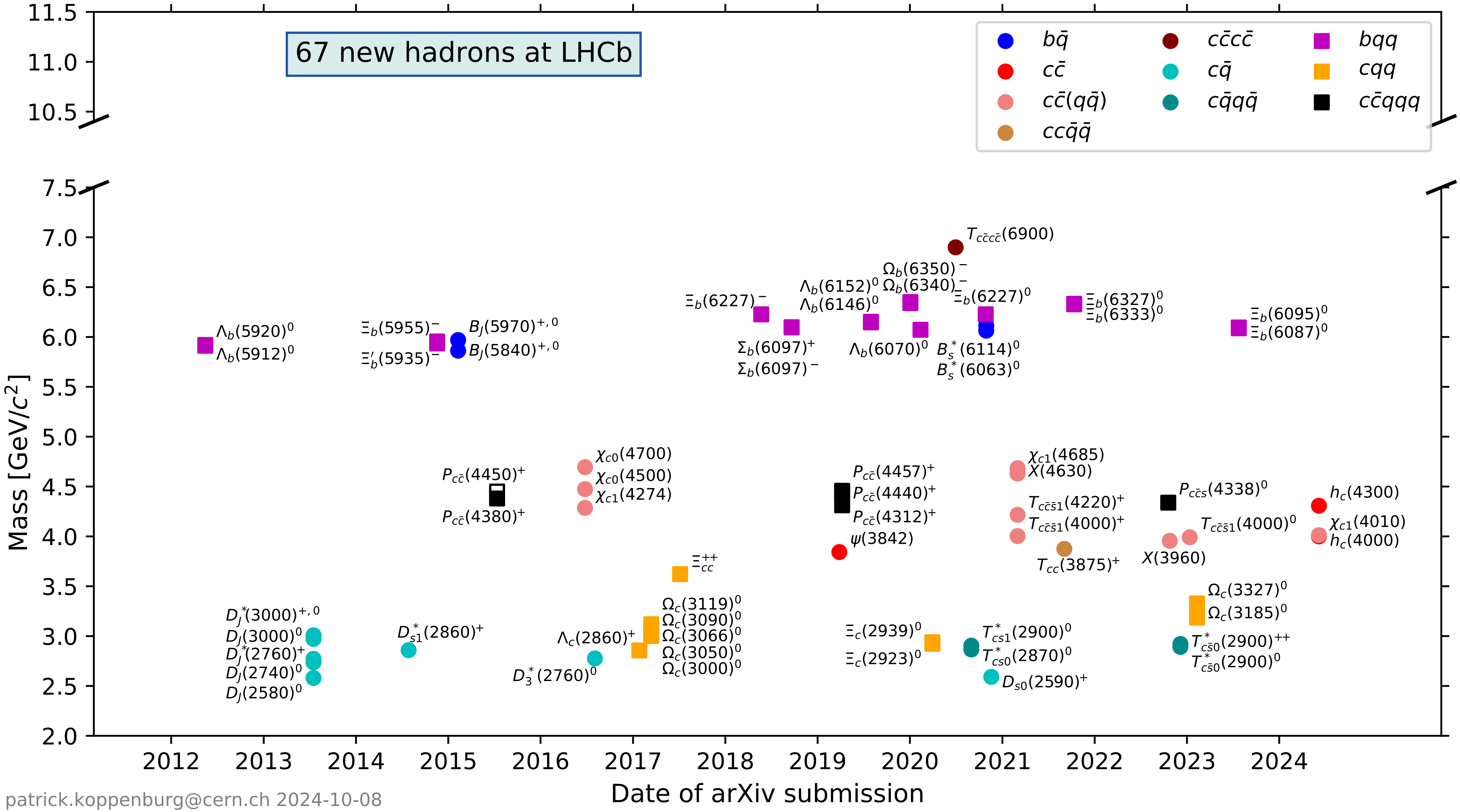
based on work:
PRL Editors' suggestion: arXiv: 2309.14070 (7 pages)
PRD Editors' suggestion: arXiv: 2309.14071 (55 pages)

UNIVERSITY OF CAMBRIDGE

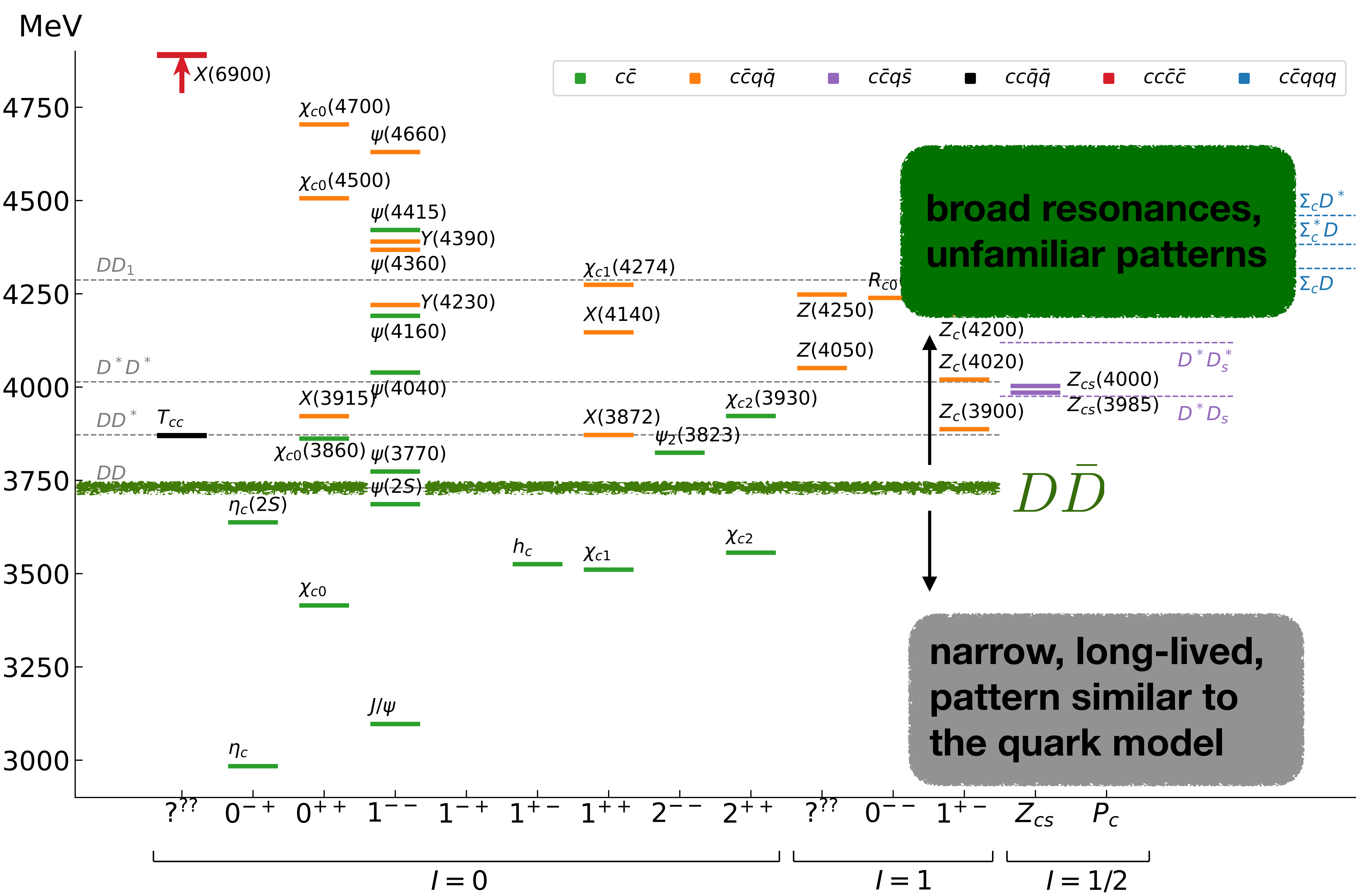
THE ROYAL SOCIETY



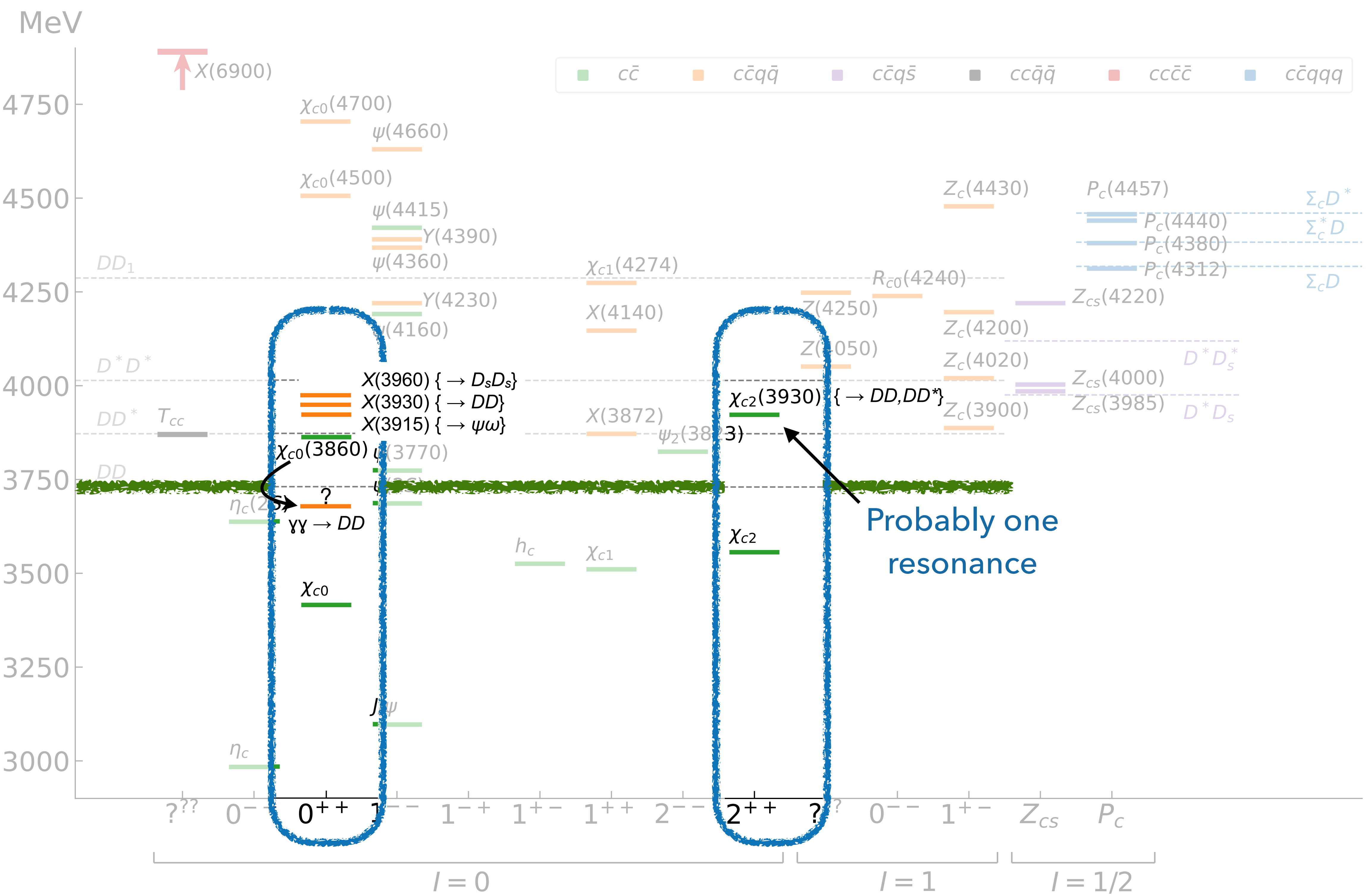
$\sim |t_{ij}|^2$
$D\bar{D} \to D^*\bar{D}^*$
$D\bar{D} \to D\bar{D}$
$D^*\bar{D}^* \to D^*\bar{D}^*$
E_{cm}
m
Γ
Contours of $|t_{ij}|^2$
resonance pole



67 new hadrons at LHCb
$b\bar{q}$
$c\bar{c}$
$c\bar{c}(q\bar{q})$
$cc\bar{q}\bar{q}$
$c\bar{c}c\bar{c}$
$c\bar{q}$
$c\bar{q}q\bar{q}$
bqq
cqq
$c\bar{c}qqq$
Mass [GeV/c^2]
Date of arXiv submission
$\Lambda_b(5920)^0$
$\Lambda_b(5912)^0$
$\Xi_b(5955)^-$
$\Xi_b'(5935)^-$
$B_J(5970)^{+,0}$
$B_J(5840)^{+,0}$
$\Xi_b(6227)^-$
$\Sigma_b(6097)^+$
$\Sigma_b(6097)^-$
$\Lambda_b(6152)^0$
$\Lambda_b(6146)^0$
$\Omega_b(6350)^-$
$\Omega_b(6340)^-$
$\Lambda_b(6070)^0$
$\Xi_b(6227)^0$
$B_s^*(6114)^0$
$B_s^*(6063)^0$
$T_{c\bar{c}c\bar{c}}(6900)$
$\Xi_b(6327)^0$
$\Xi_b(6333)^0$
$\Xi_b(6095)^0$
$\Xi_b(6087)^0$
$P_{c\bar{c}}(4450)^+$
$P_{c\bar{c}}(4380)^+$
$\chi_{c0}(4700)$
$\chi_{c0}(4500)$
$\chi_{c1}(4274)$
$P_{c\bar{c}}(4457)^+$
$P_{c\bar{c}}(4440)^+$
$P_{c\bar{c}}(4312)^+$
$\psi(3842)$
$\chi_{c1}(4685)$
$X(4630)$
$T_{c\bar{c}\bar{s}1}(4220)^+$
$T_{c\bar{c}\bar{s}1}(4000)^+$
$T_{cc}(3875)^+$
$P_{c\bar{c}s}(4338)^0$
$T_{c\bar{c}\bar{s}1}(4000)^0$
$X(3960)$
$h_c(4300)$
$\chi_{c1}(4010)$
$h_c(4000)$
Ξ_{cc}^{++}
$D_J^*(3000)^{+,0}$
$D_J(3000)^0$
$D_J^*(2760)^+$
$D_J(2740)^0$
$D_J(2580)^0$
$D_{s1}^*(2860)^+$
$\Lambda_c(2860)^+$
$D_3^*(2760)^0$
$\Omega_c(3119)^0$
$\Omega_c(3090)^0$
$\Omega_c(3066)^0$
$\Omega_c(3050)^0$
$\Omega_c(3000)^0$
$\Xi_c(2939)^0$
$\Xi_c(2923)^0$
$T_{cs1}^*(2900)^0$
$T_{cs0}^*(2870)^0$
$D_{s0}(2590)^+$
$\Omega_c(3327)^0$
$\Omega_c(3185)^0$
$T_{c\bar{s}0}^*(2900)^{++}$
$T_{c\bar{s}0}^*(2900)^0$
patrick.koppenburg@cern.ch 2024-10-08



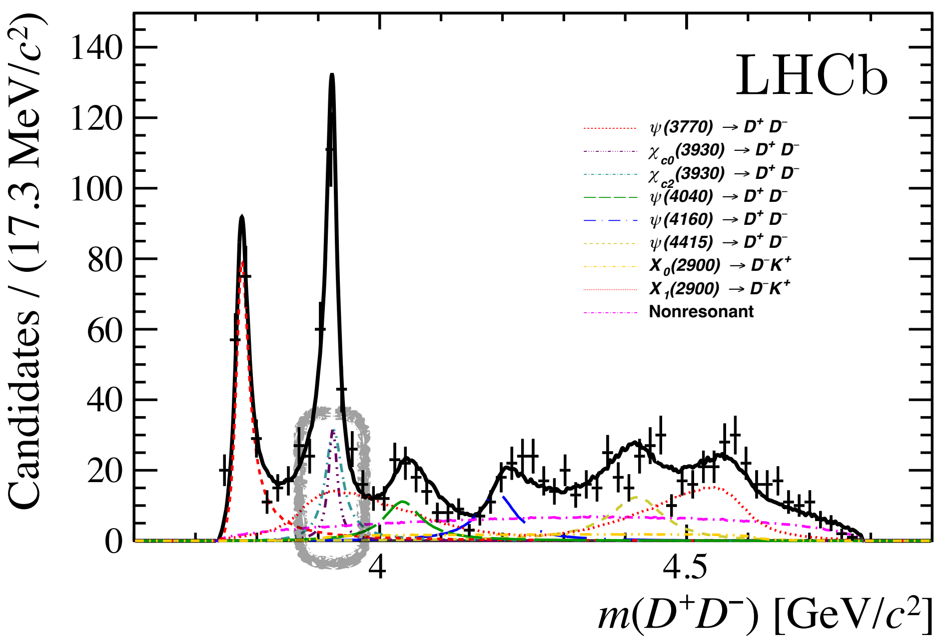
JPAC arXiv:2112.13436

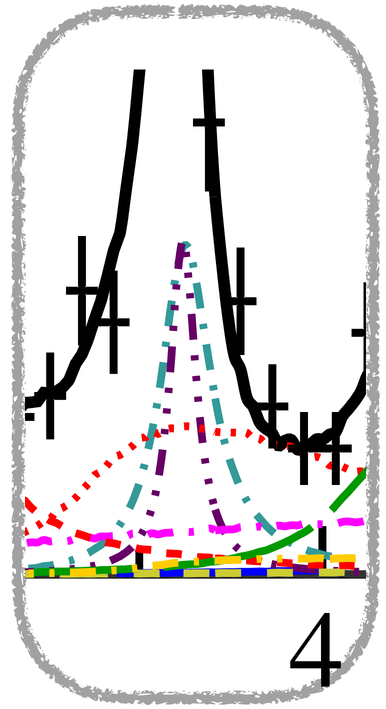


Level counting is not clear:

- Near threshold behaviours?
- Multiple decoupled resonances?

$B^+ \to D^+ D^- K^+$ arXiv:2009.00026

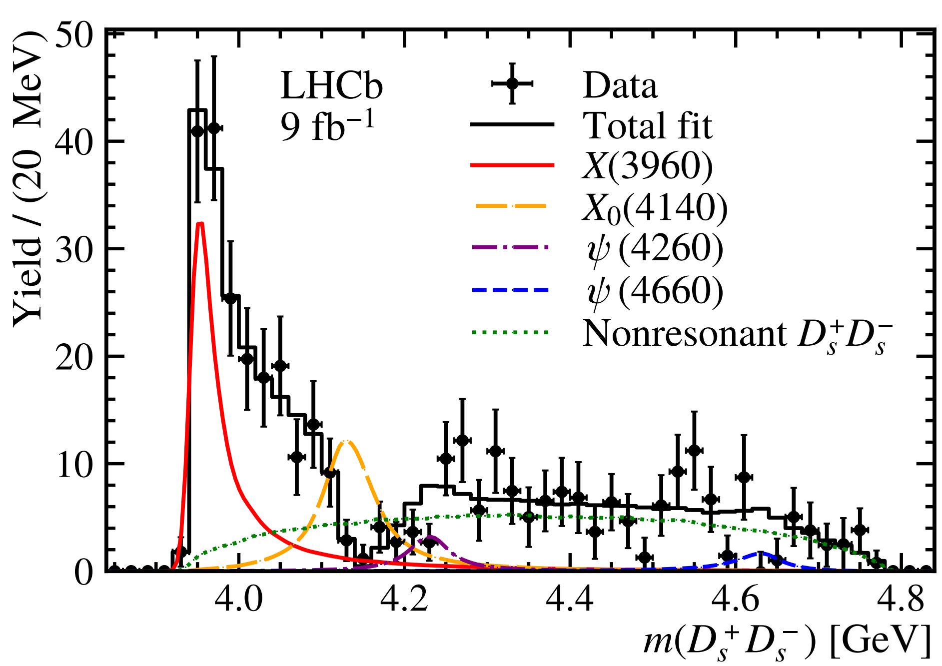




overlapping 0^{++} and 2^{++} resonances around 3925 MeV

Large effects from $X_1(2900) \to DK$ (3-body?)

$B^+ \to D_s^+ D_s^- K^+$ arXiv:2210.15153



Near threshold enhancement at $D_s\bar{D}_s$ threshold

Most likely S-wave - $k^{2\ell}$ threshold suppression

Relatively simple: only one large amplitude

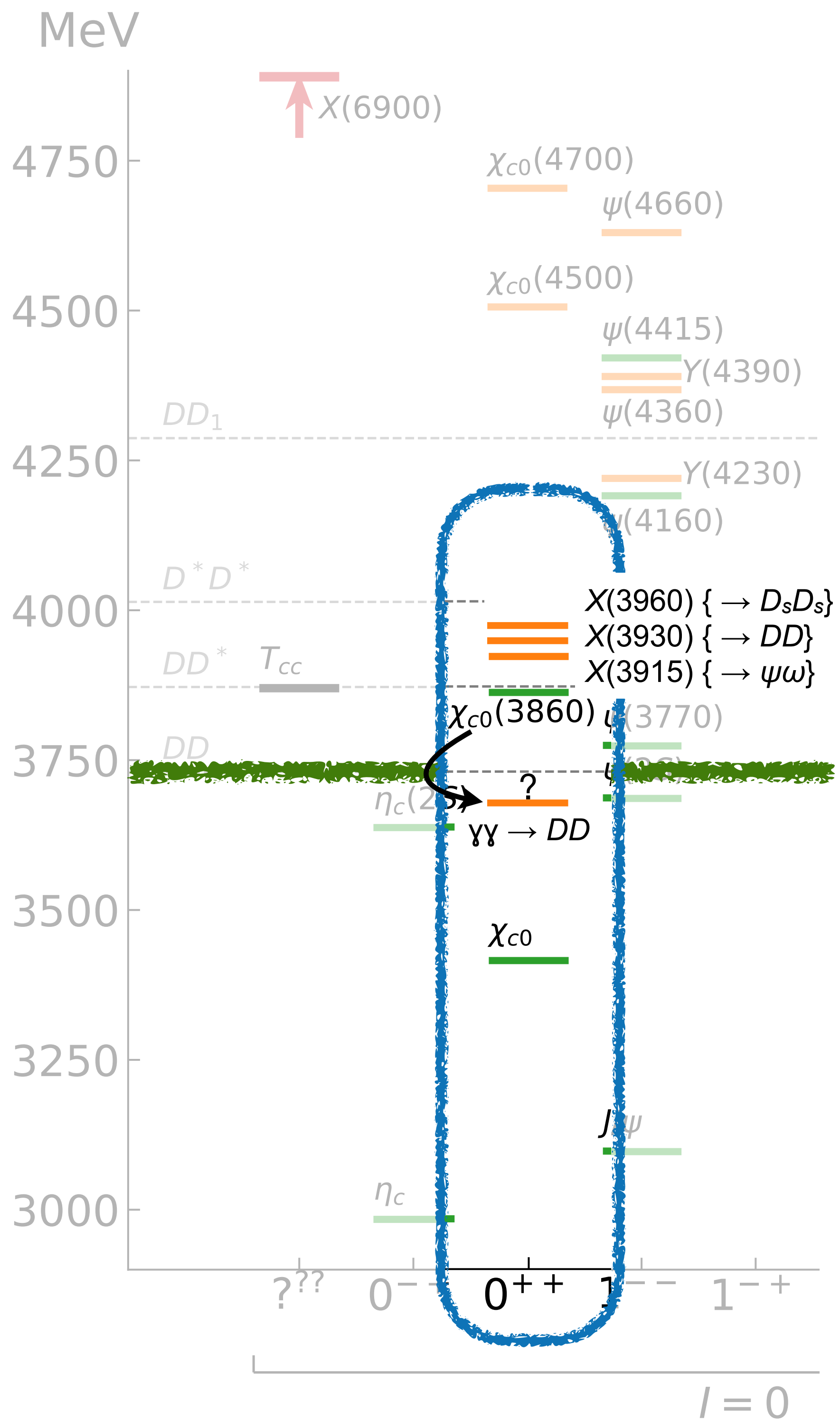
Also arXiv:2406.03156 (LHCb)

$B^+ \to D^{*\pm} D^{\mp} K^+$

Consistent $J^{PC} = 2^{++}$ state around 3925 MeV

Several new states, very complicated process

For $D\bar{D}^*$, $J^{PC} = 1^{++}$ also contributes



are all of these bumps resonances?

how are these enhancements related?

how many states are there in 0^{++} and 2^{++}?

can we understand how the quark-model-like states and meson-meson like states contribute to the observed features?

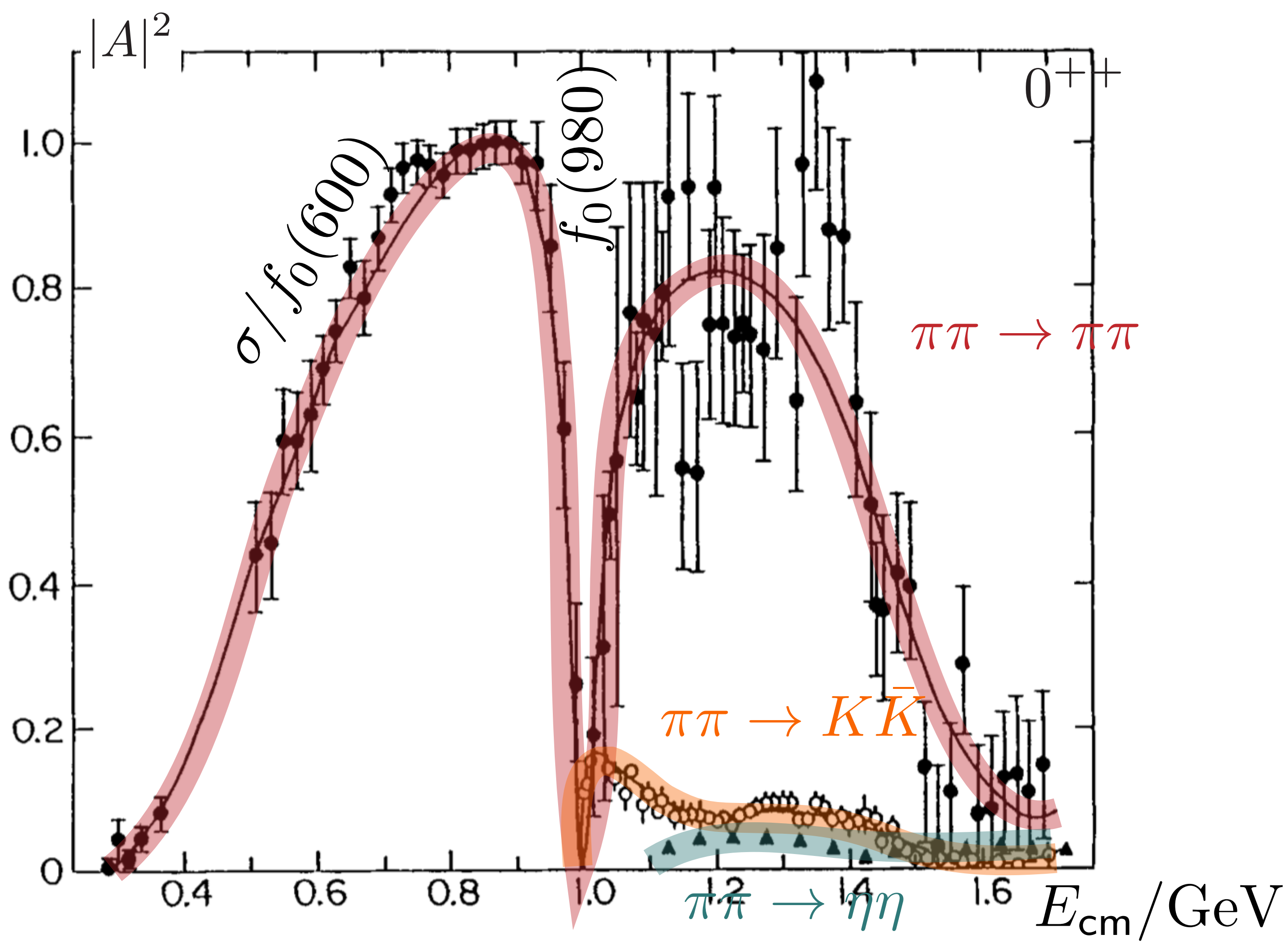
first principles Lattice QCD calculations can help understand this

Familiar example: $f_0(980)$

Features in different final states from the same resonance

S-wave, s-channel scattering

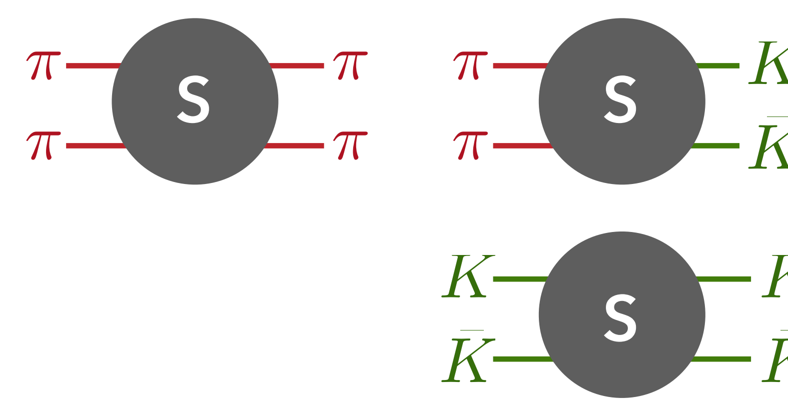
CERN-Munich, ANL, BNL

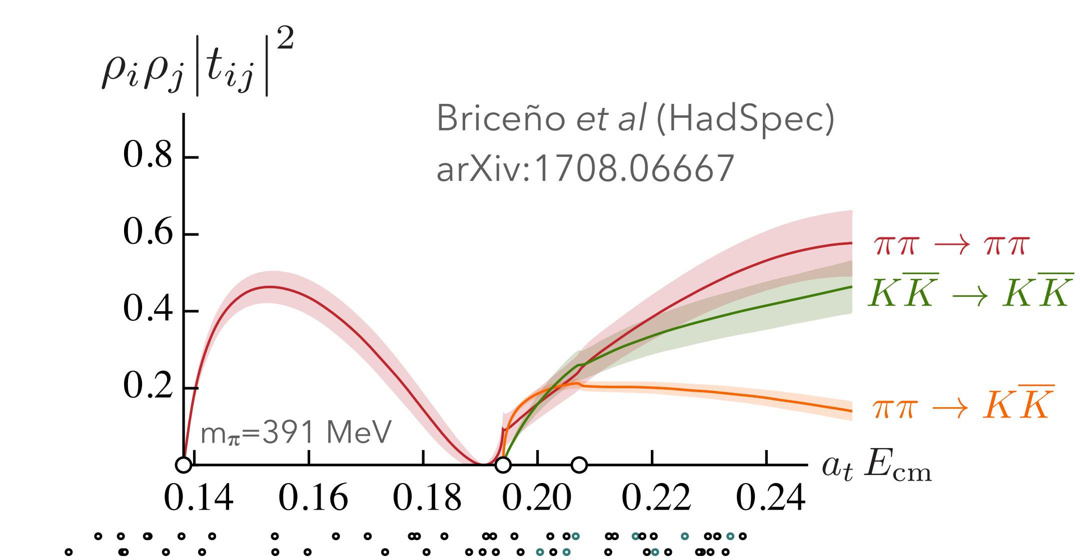


seen as a dip in $\pi\pi$

sharp turn-on at threshold in $K\bar{K}$

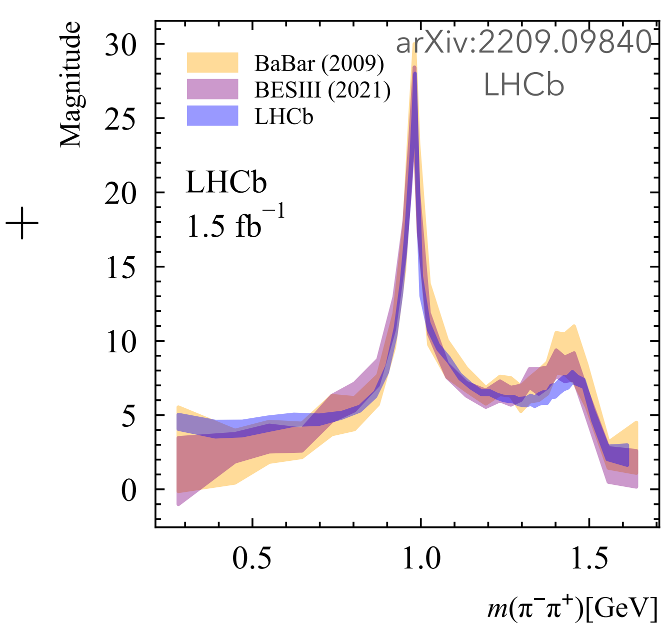
Coupled-channel S-matrix:



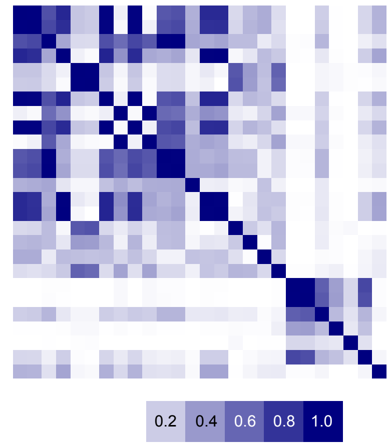


$D_s^+ \to \pi^-\pi^+\pi^+$

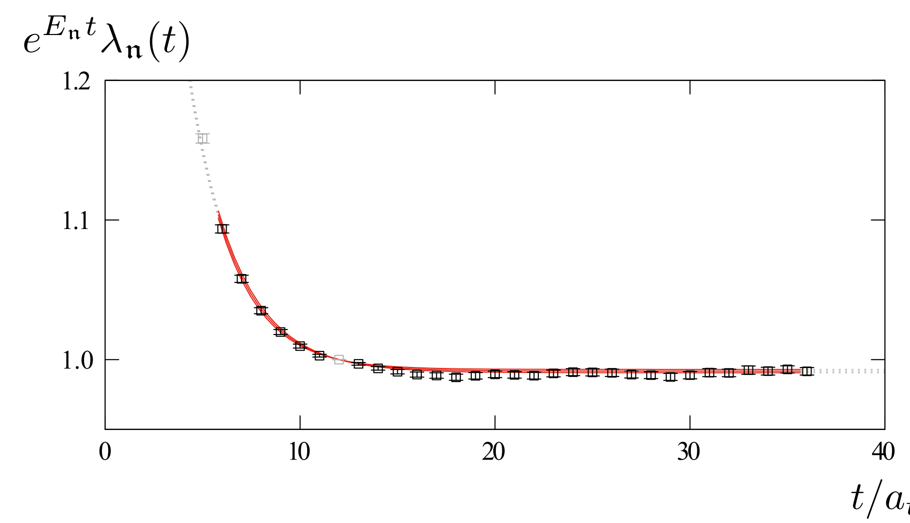
Can also appear as a peak in decays



Compute Correlation Matrix



Generalised Eigenvalue Problem



Operators

$$\mathcal{O}^\dagger \sim \bar{q}\Gamma q$$

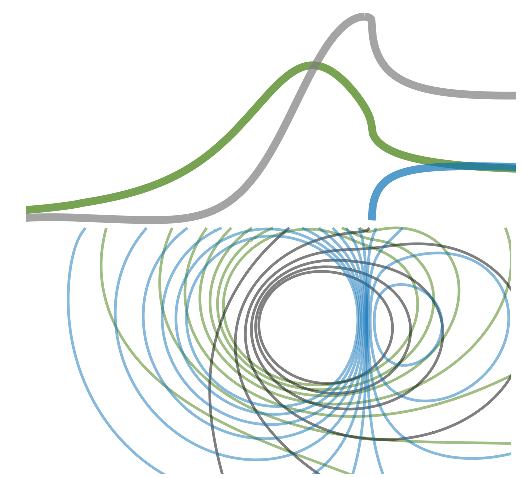
$$(\pi\pi)^\dagger \sim \sum_{\vec{p}} (\text{CGs})\ \pi^\dagger\pi^\dagger$$

$$\pi^\dagger \sim \sum_i v_i^{(\pi)} \mathcal{O}_i^\dagger$$

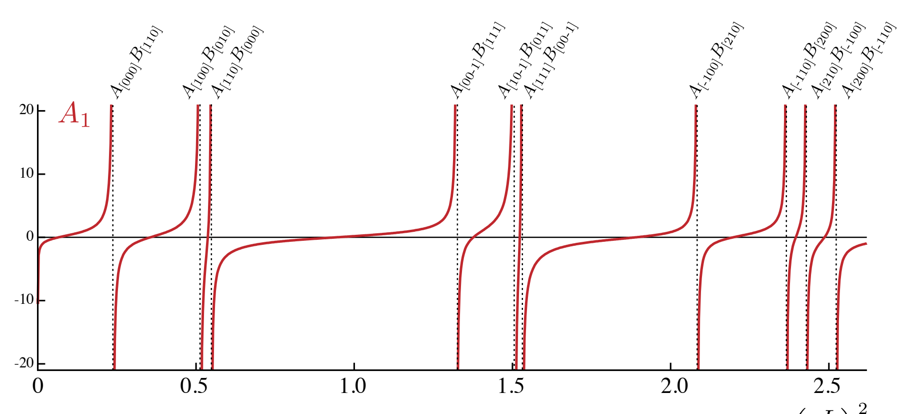
$$C_{ij}(t)v_j^{\mathfrak{n}} = \lambda_{\mathfrak{n}}(t)C_{ij}(t_0)v_j^{\mathfrak{n}}$$

Obtain Finite Volume Spectrum

Lüscher Quantisation Condition



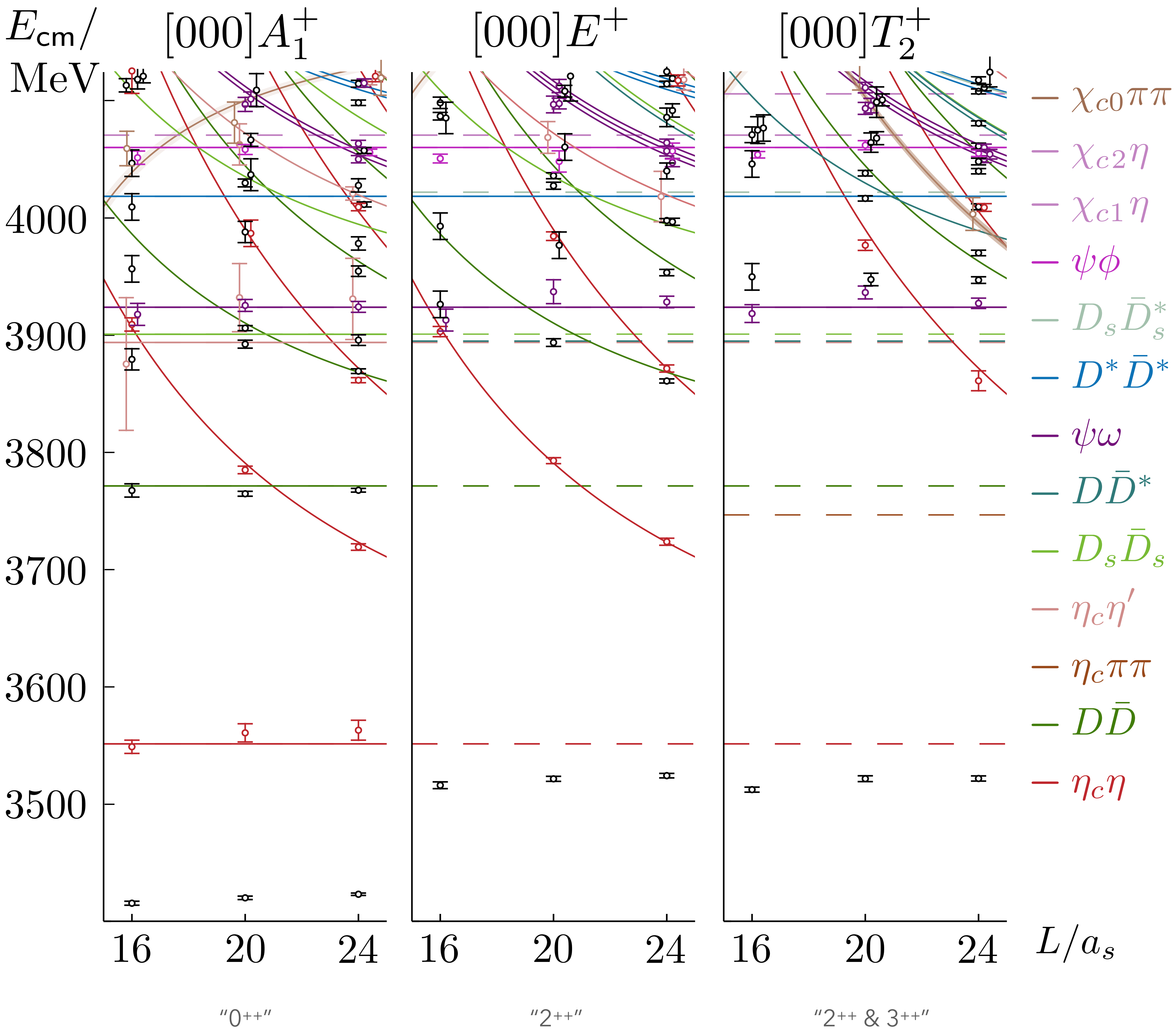
Determine Scattering Amplitudes

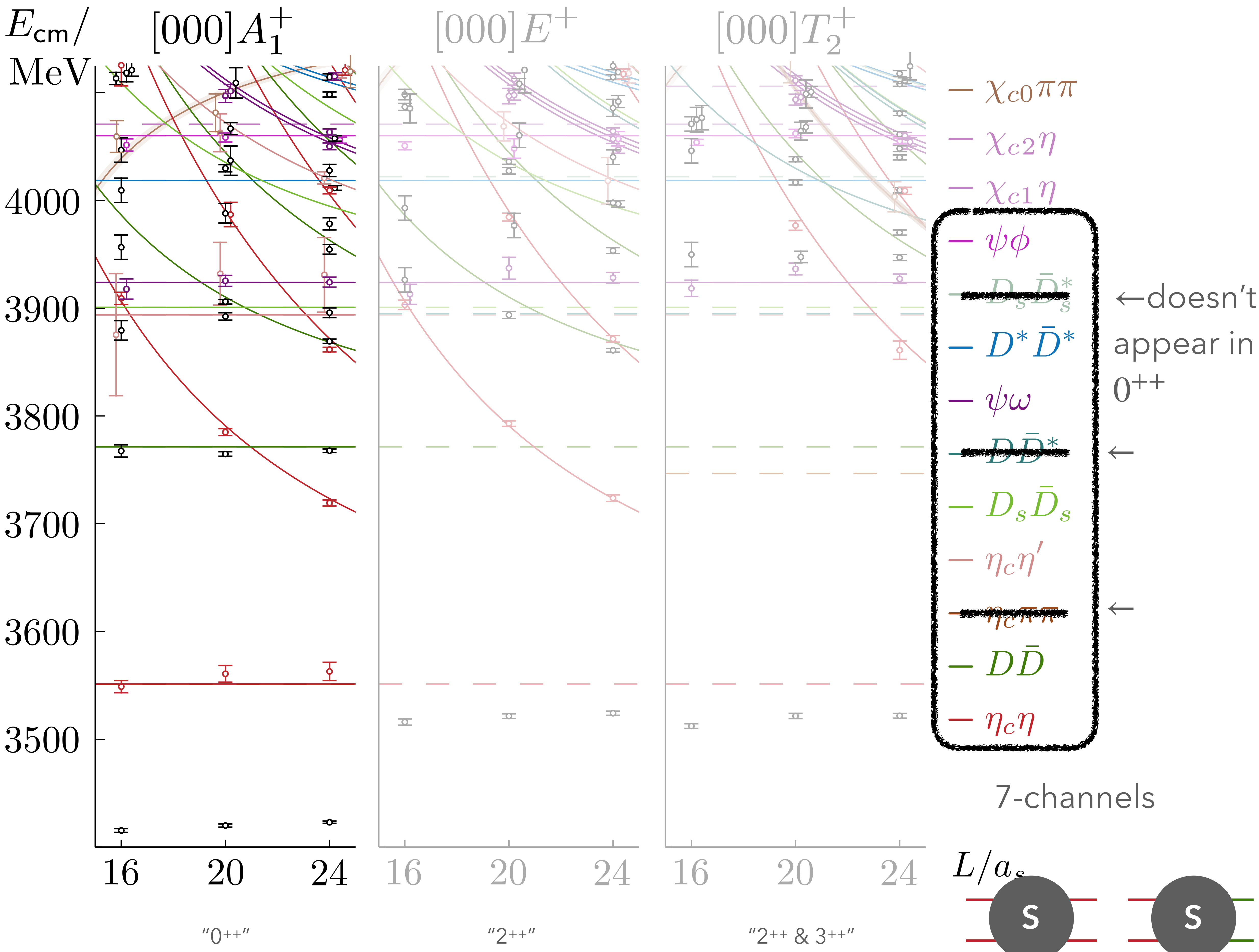


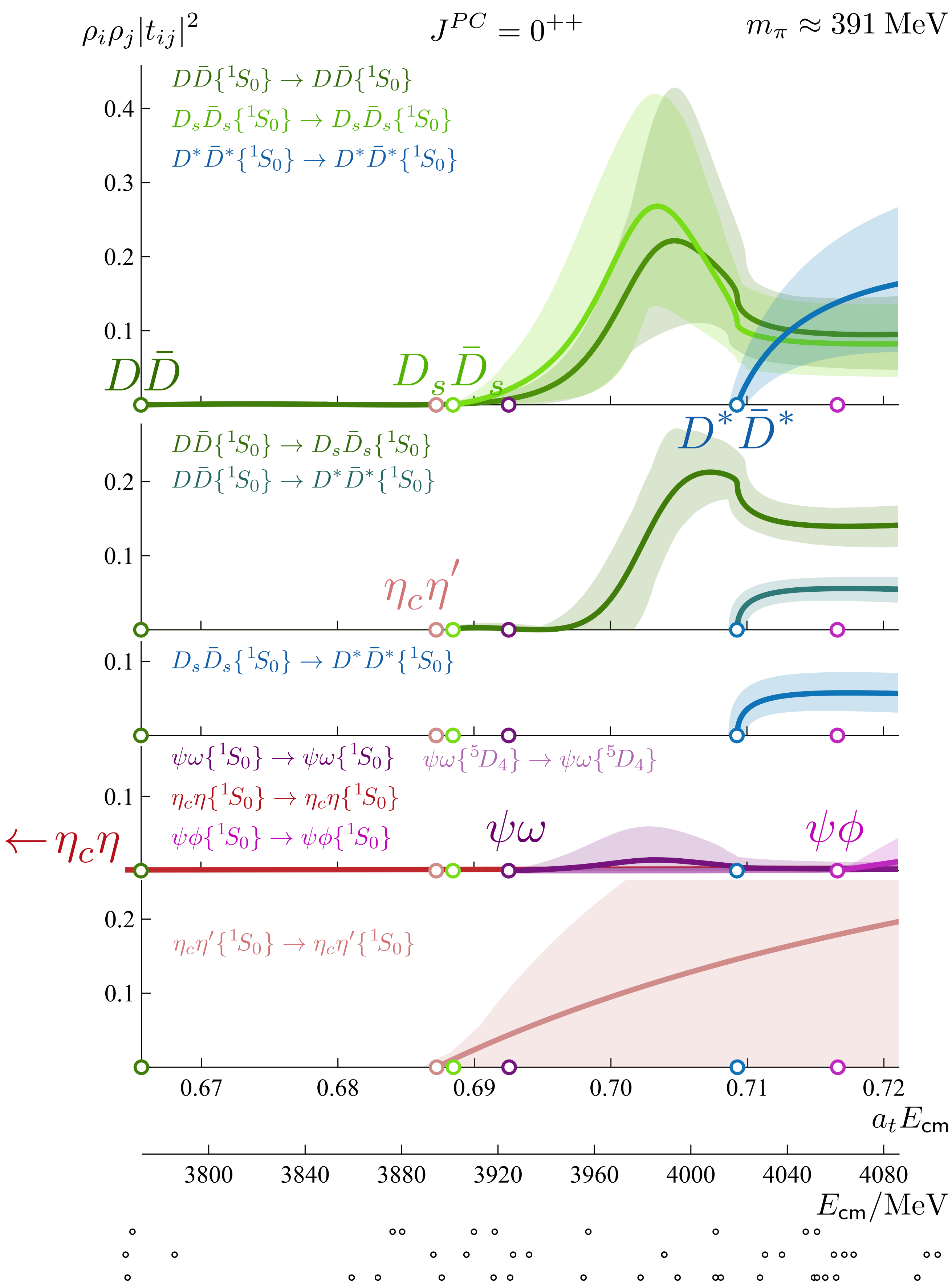
$$\det[\text{amp} + \text{FV}] = 0$$

Poles, Couplings

Review: Briceño, Dudek, Young, https://doi.org/10.1103/RevModPhys.90.025001


E_{cm}/ MeV
$[000]A_1^+$
$[000]E^+$
$[000]T_2^+$
4000
3900
3800
3700
3600
3500
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
"0++"
"2++"
"2++ & 3++"


$E_{\rm cm}$/ MeV
$[000]A_1^+$
$[000]E^+$
$[000]T_2^+$
3500
3600
3700
3800
3900
4000
16
20
24
L/a_s
"0++"
"2++"
"2++ & 3++"
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
←doesn't appear in 0^{++}
←
←
7-channels
S
S



consider 7-channel system

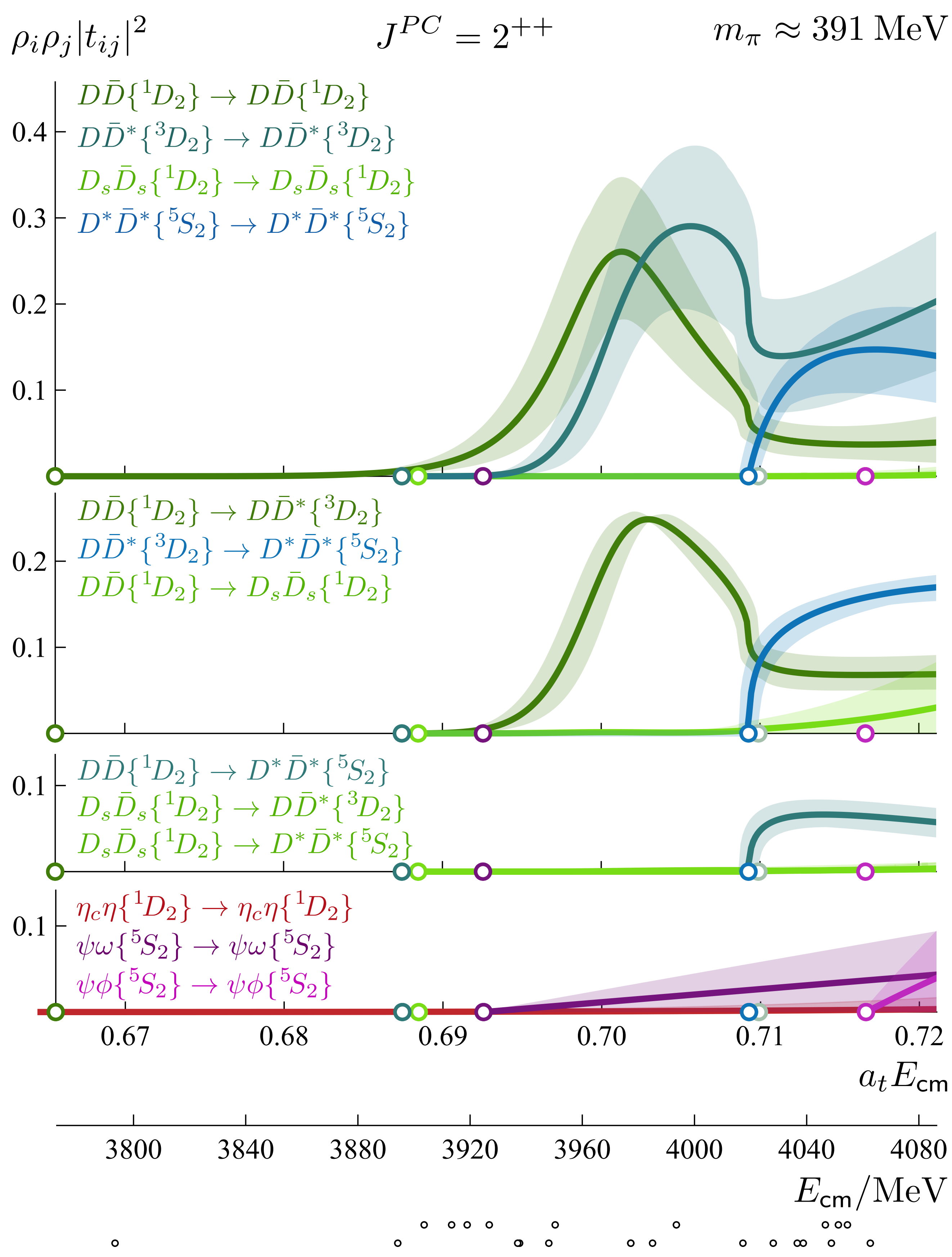
$$K_{ij} = \frac{g_i g_j}{m^2 - s} + \gamma_{ij}$$

$$\boldsymbol{S} = \boldsymbol{1} + 2i\boldsymbol{\rho}^{\frac{1}{2}}\cdot\boldsymbol{t}\cdot\boldsymbol{\rho}^{\frac{1}{2}}$$

$$\boldsymbol{t}^{-1} = \boldsymbol{K}^{-1} + \boldsymbol{I}$$

$$\mathrm{Im} I_{ij} = -\rho_i = 2k_i/\sqrt{s}$$

K-matrix pole terms are necessary to obtain a good quality of fit



7-channels, mixture of *S* and *D*

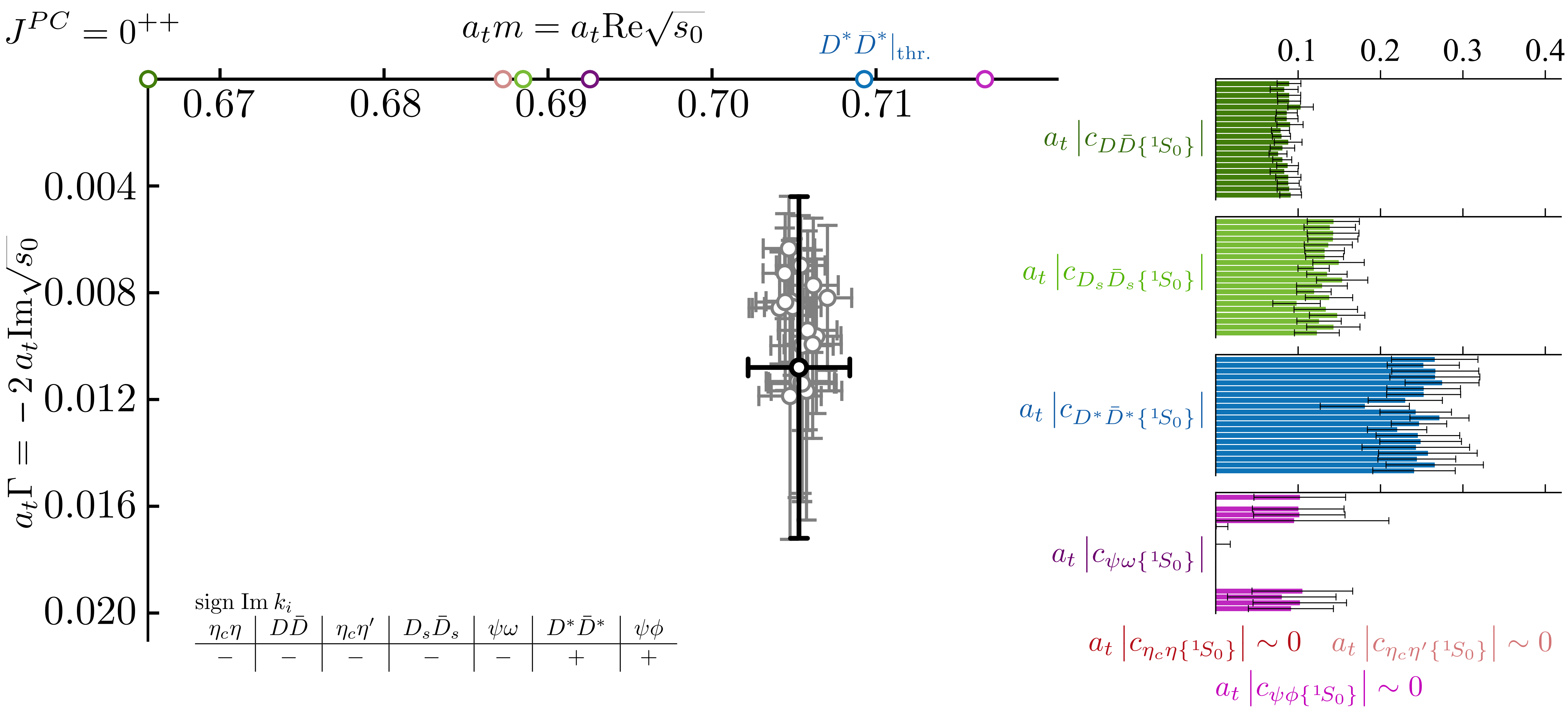
$D\bar{D}, D_s\bar{D}_s\{^1D_2\}$ $D\bar{D}^*\{^3D_2\}$ $D^*\bar{D}^*\{^5S_2\}$

$\eta_c\eta\{^1D_2\}$ $\psi\omega, \psi\phi\{^5S_2\}$

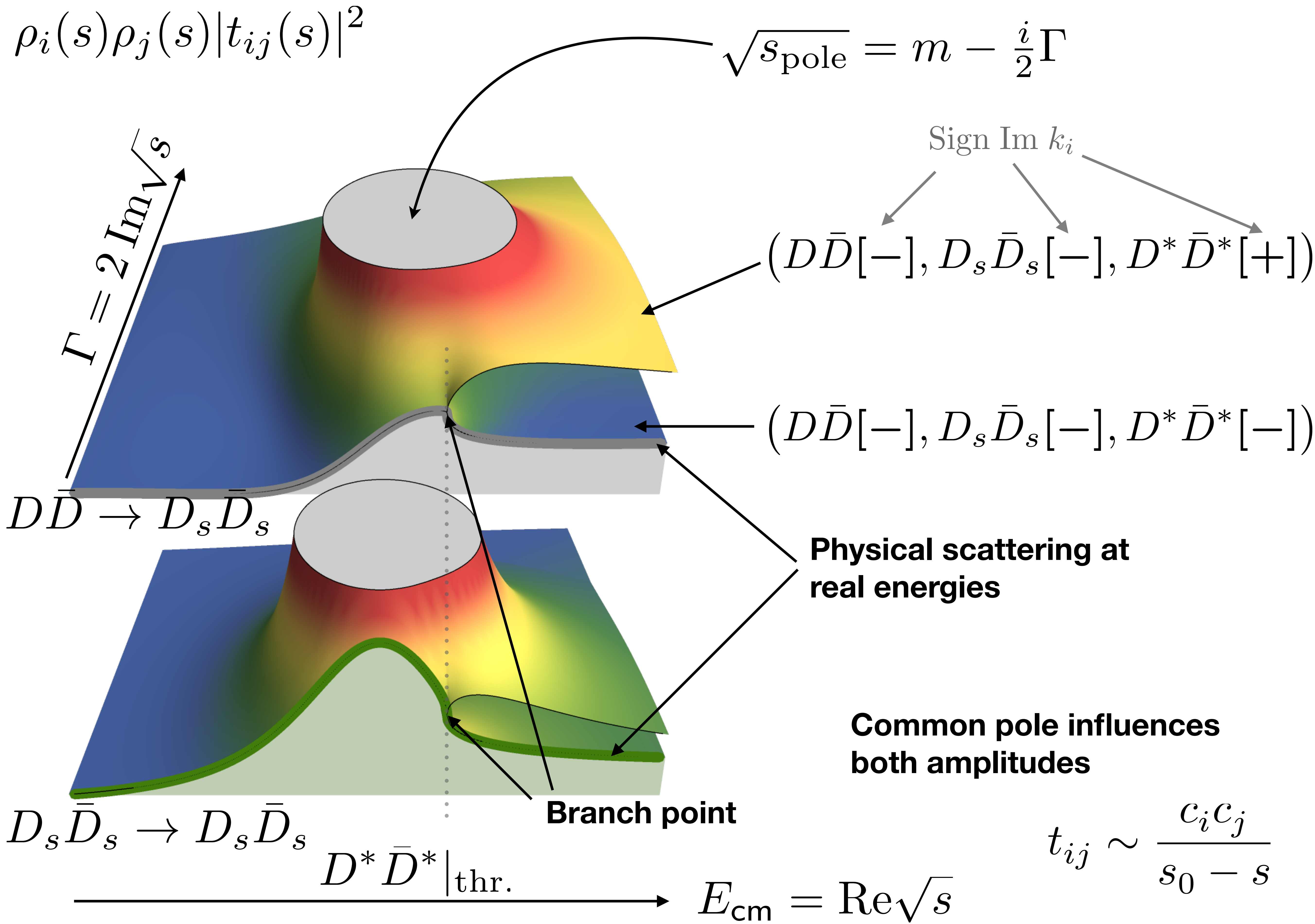
peaks at a similar energy

very small $D_s\bar{D}_s$ amplitudes -
some phase space suppression

$D\bar{D}^*$ is large -
similar phase space to $D_s\bar{D}_s$



$$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$$

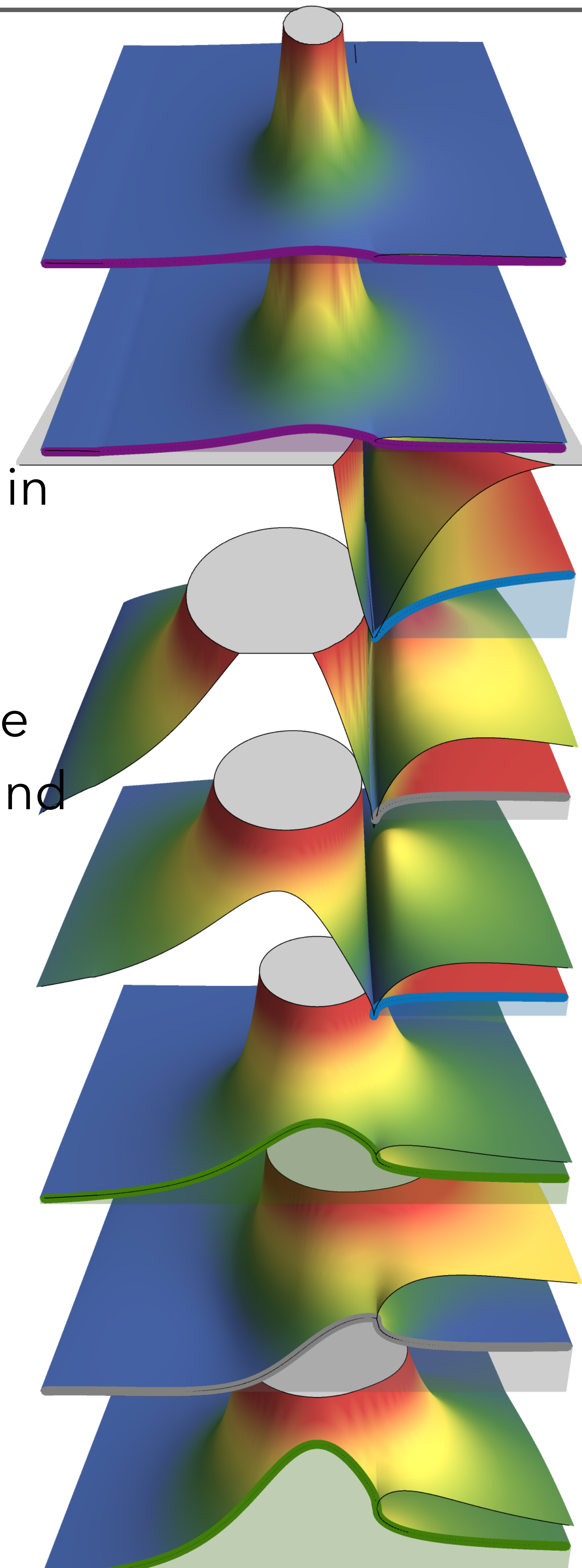

$\rho_i(s)\rho_j(s)|t_{ij}(s)|^2$
$\sqrt{s_{\text{pole}}} = m - \frac{i}{2}\Gamma$
Sign Im k_i
$(D\bar{D}[-], D_s\bar{D}_s[-], D^*\bar{D}^*[+])$
$(D\bar{D}[-], D_s\bar{D}_s[-], D^*\bar{D}^*[-])$
$\Gamma = 2\,\text{Im}\sqrt{s}$
$D\bar{D} \to D_s\bar{D}_s$
Physical scattering at real energies
Common pole influences both amplitudes
Branch point
$D_s\bar{D}_s \to D_s\bar{D}_s$
$D^*\bar{D}^*|_{\text{thr.}}$
$E_{\text{cm}} = \text{Re}\sqrt{s}$
$t_{ij} \sim \frac{c_i c_j}{s_0 - s}$

$$\rho_i(s)\rho_j(s)|t_{ij}(s)|^2$$

one resonance pole
– many different amplitudes

We don't need different poles in different coupled amplitudes

A single resonance pole can be responsible for many bumps and features



$J/\psi\omega \to J/\psi\omega$

$D\bar{D} \to J/\psi\omega$

$D^*\bar{D}^* \to D^*\bar{D}^*$

$D_s\bar{D}_s \to D^*\bar{D}^*$

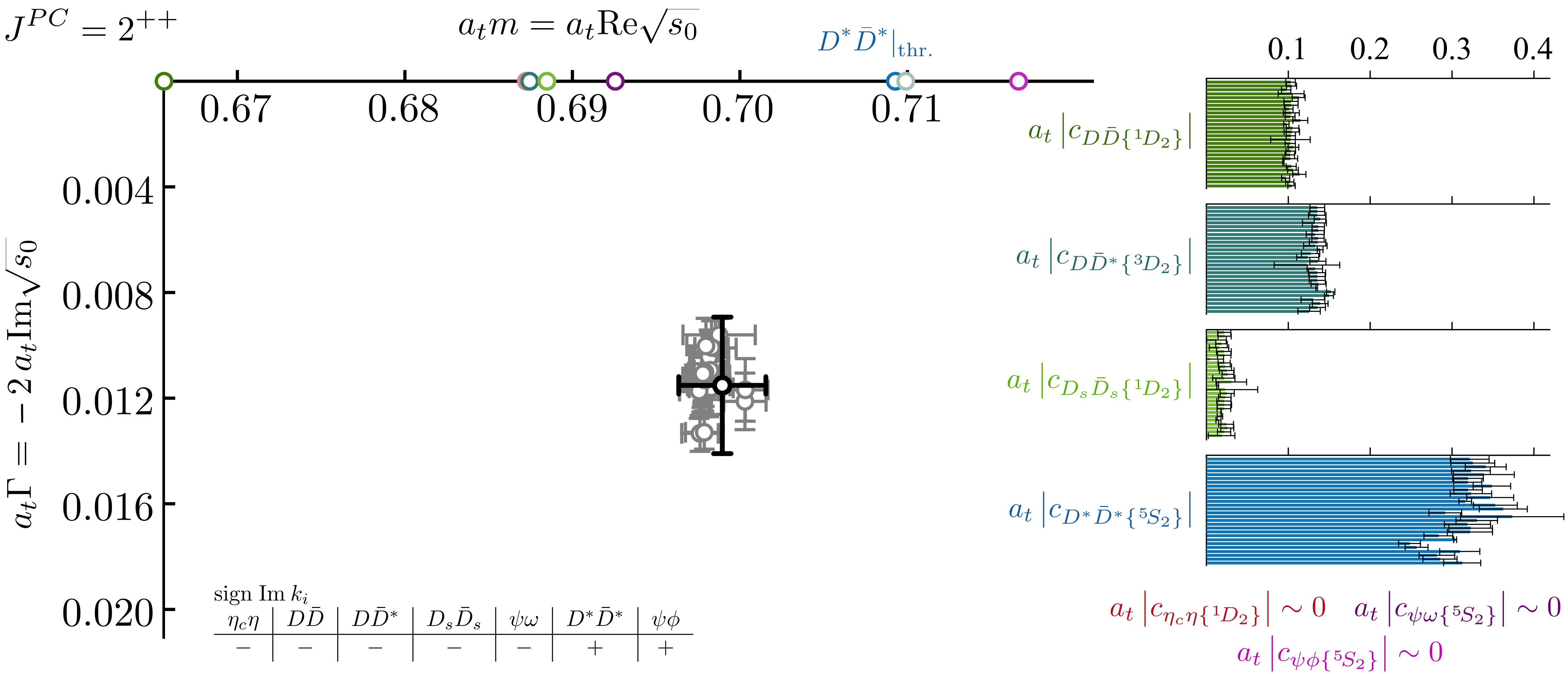
$D\bar{D} \to D^*\bar{D}^*$

$D\bar{D} \to D\bar{D}$

$D\bar{D} \to D_s\bar{D}_s$

$D_s\bar{D}_s \to D_s\bar{D}_s$

Similar story for 2^{++}

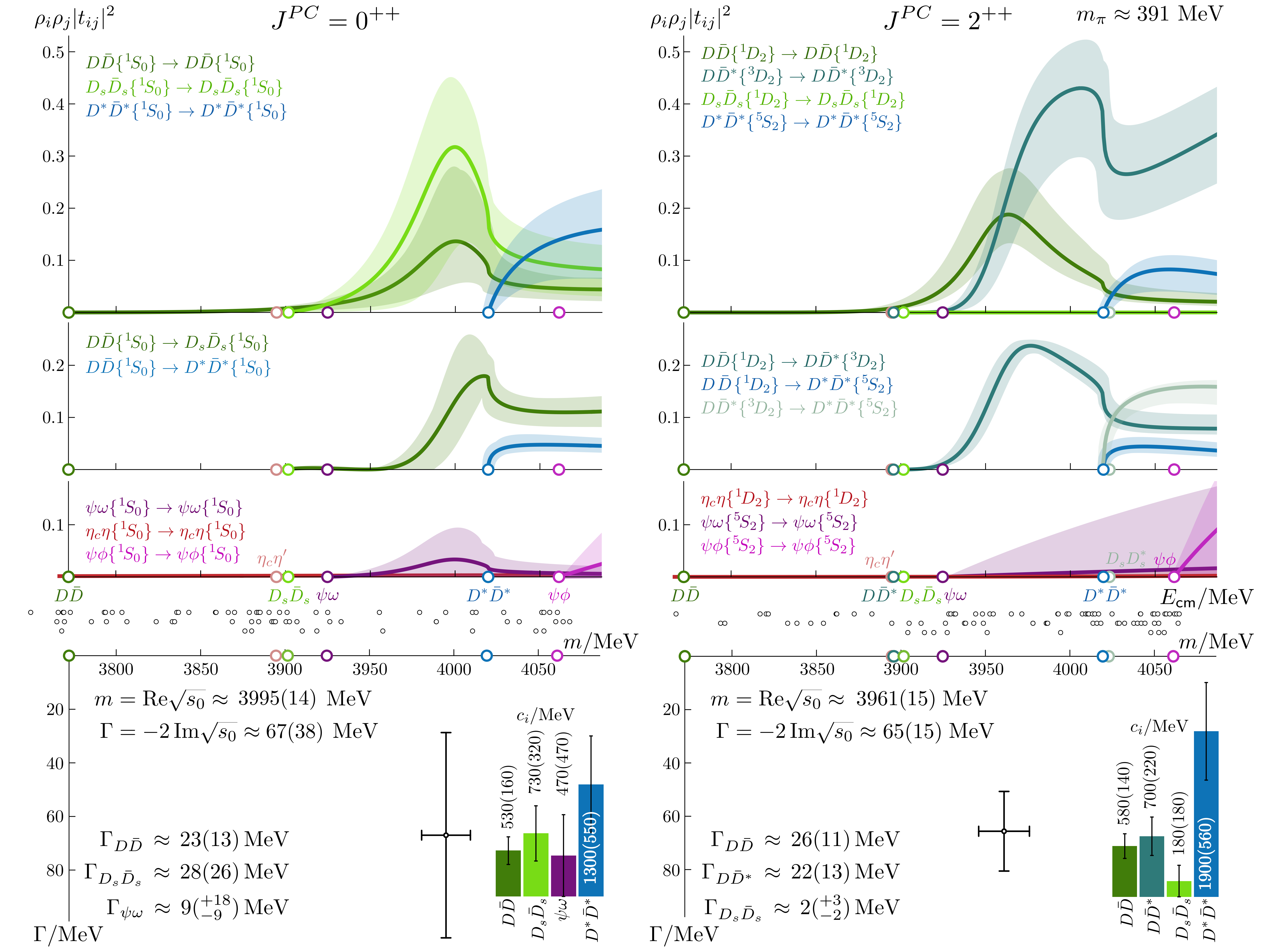


additional poles were found

- don't appear to be important

"coupling-ratio" phenomena seen in K-matrix pole parameters

- possible to rescale K-matrix g_i factors and obtain similar amplitudes
- t-matrix couplings are found to be well-determined



$\rho_i\rho_j|t_{ij}|^2$
$J^{PC} = 0^{++}$
$D\bar{D}\{^1S_0\} \to D\bar{D}\{^1S_0\}$
$D_s\bar{D}_s\{^1S_0\} \to D_s\bar{D}_s\{^1S_0\}$
$D^*\bar{D}^*\{^1S_0\} \to D^*\bar{D}^*\{^1S_0\}$
$D\bar{D}\{^1S_0\} \to D_s\bar{D}_s\{^1S_0\}$
$D\bar{D}\{^1S_0\} \to D^*\bar{D}^*\{^1S_0\}$
$\psi\omega\{^1S_0\} \to \psi\omega\{^1S_0\}$
$\eta_c\eta\{^1S_0\} \to \eta_c\eta\{^1S_0\}$
$\psi\phi\{^1S_0\} \to \psi\phi\{^1S_0\}$
$\eta_c\eta'$
$D\bar{D}$
$D_s\bar{D}_s$
$\psi\omega$
$D^*\bar{D}^*$
$\psi\phi$
m/MeV
$m = \mathrm{Re}\sqrt{s_0} \approx 3995(14)$ MeV
$\Gamma = -2\,\mathrm{Im}\sqrt{s_0} \approx 67(38)$ MeV
$\Gamma_{D\bar{D}} \approx 23(13)$ MeV
$\Gamma_{D_s\bar{D}_s} \approx 28(26)$ MeV
$\Gamma_{\psi\omega} \approx 9(^{+18}_{-9})$ MeV
c_i/MeV
530(160)
730(320)
470(470)
1300(550)
Γ/MeV
$\rho_i\rho_j|t_{ij}|^2$
$J^{PC} = 2^{++}$
$m_\pi \approx 391$ MeV
$D\bar{D}\{^1D_2\} \to D\bar{D}\{^1D_2\}$
$D\bar{D}^*\{^3D_2\} \to D\bar{D}^*\{^3D_2\}$
$D_s\bar{D}_s\{^1D_2\} \to D_s\bar{D}_s\{^1D_2\}$
$D^*\bar{D}^*\{^5S_2\} \to D^*\bar{D}^*\{^5S_2\}$
$D\bar{D}\{^1D_2\} \to D\bar{D}^*\{^3D_2\}$
$D\bar{D}\{^1D_2\} \to D^*\bar{D}^*\{^5S_2\}$
$D\bar{D}^*\{^3D_2\} \to D^*\bar{D}^*\{^5S_2\}$
$\eta_c\eta\{^1D_2\} \to \eta_c\eta\{^1D_2\}$
$\psi\omega\{^5S_2\} \to \psi\omega\{^5S_2\}$
$\psi\phi\{^5S_2\} \to \psi\phi\{^5S_2\}$
$\eta_c\eta'$
$D_sD_s^*$
$\psi\phi$
$D\bar{D}$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\psi\omega$
$D^*\bar{D}^*$
E_{cm}/MeV
m/MeV
$m = \mathrm{Re}\sqrt{s_0} \approx 3961(15)$ MeV
$\Gamma = -2\,\mathrm{Im}\sqrt{s_0} \approx 65(15)$ MeV
$\Gamma_{D\bar{D}} \approx 26(11)$ MeV
$\Gamma_{D\bar{D}^*} \approx 22(13)$ MeV
$\Gamma_{D_s\bar{D}_s} \approx 2(^{+3}_{-2})$ MeV
c_i/MeV
580(140)
700(220)
180(180)
1900(560)
Γ/MeV

Scalar and tensor charmonium resonances from lattice QCD

- at m_π=391 MeV, one scalar and one tensor resonance pole is found below 4 GeV
- The **level counting** is not obviously different from the **quark model**

- large **coupled-channel** effects in OZI **connected D-meson channels**
- OZI **disconnected** channels look **small everywhere**

- we have extracted a **complete** unitary **S-matrix** and this naturally **connects** features seen in **different channels** and simplifies the overall picture

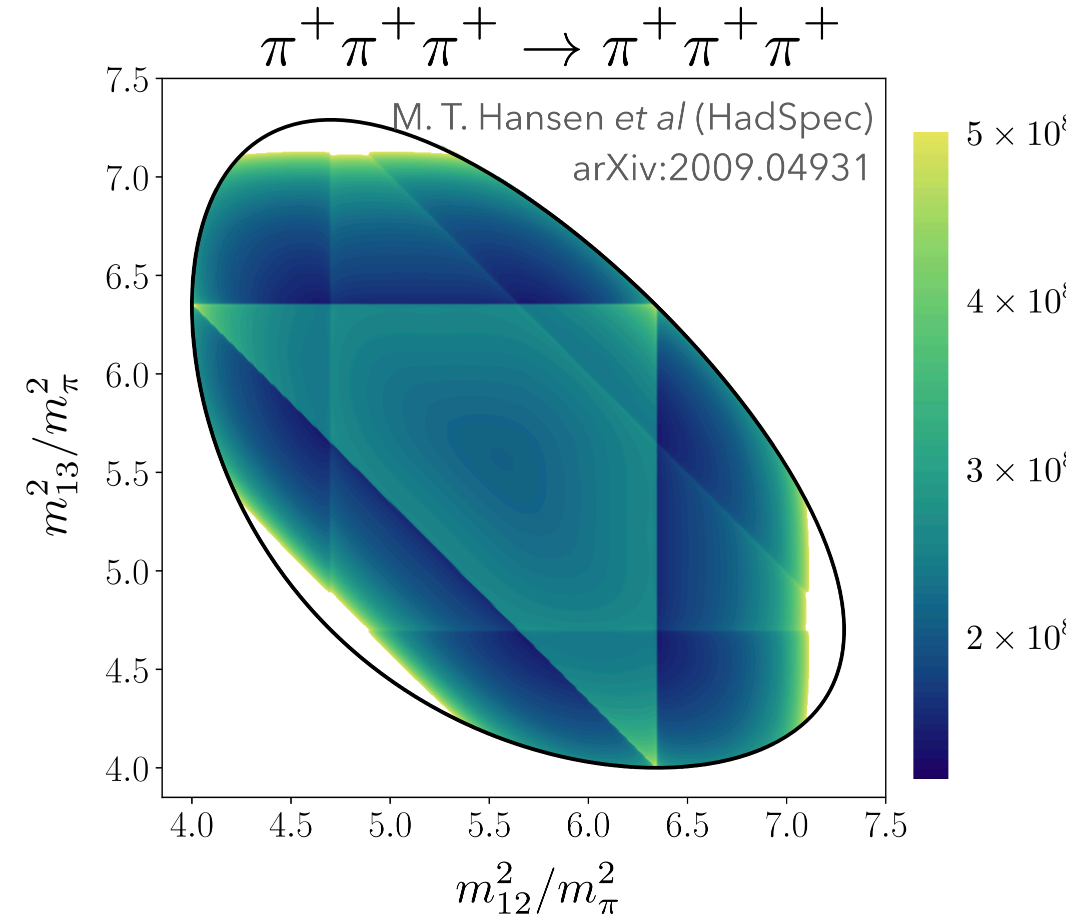
- some channels have enhancements **very different** to simple **Breit-Wigners**

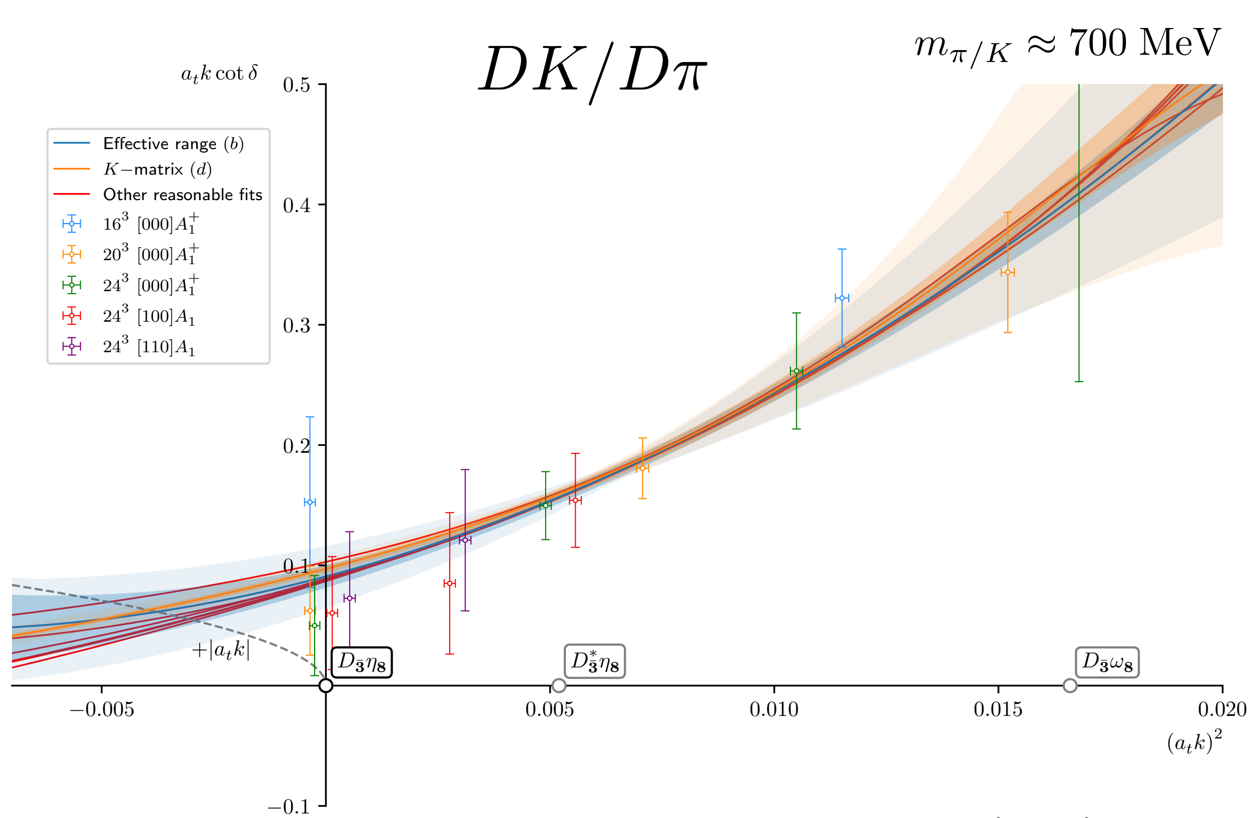
- a clear, as yet unobserved, 3^{++} resonance is present in $D\bar{D}^*$ & a bound state in 2^{-+}
- we **do not find** a **near-threshold S-wave** $D\bar{D}$ state (between 3700 and 3860 MeV)

- these methods can also be applied to the X(3872) 1^{++} channel and the vector channel

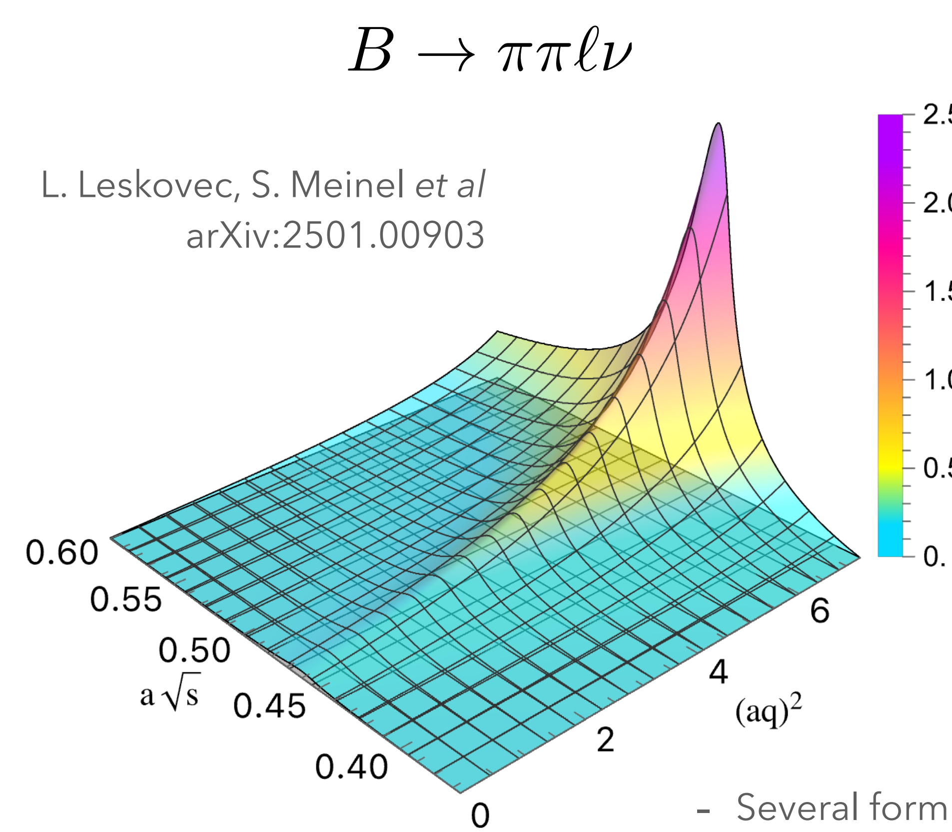
Full details and references:
arXiv: 2309.14070 (7 pages)
arXiv: 2309.14071 (55 pages)

- Big picture stuff with large light quark masses eg consider flavour SU(3) point
- At some point it should be possible to consider the full process: $B^+ \to D_{(s)}^{(*)}\bar{D}_{(s)}^{(*)}K^+$
- Much of the three-body theory exists but we need more practical experience (3-body is challenging)



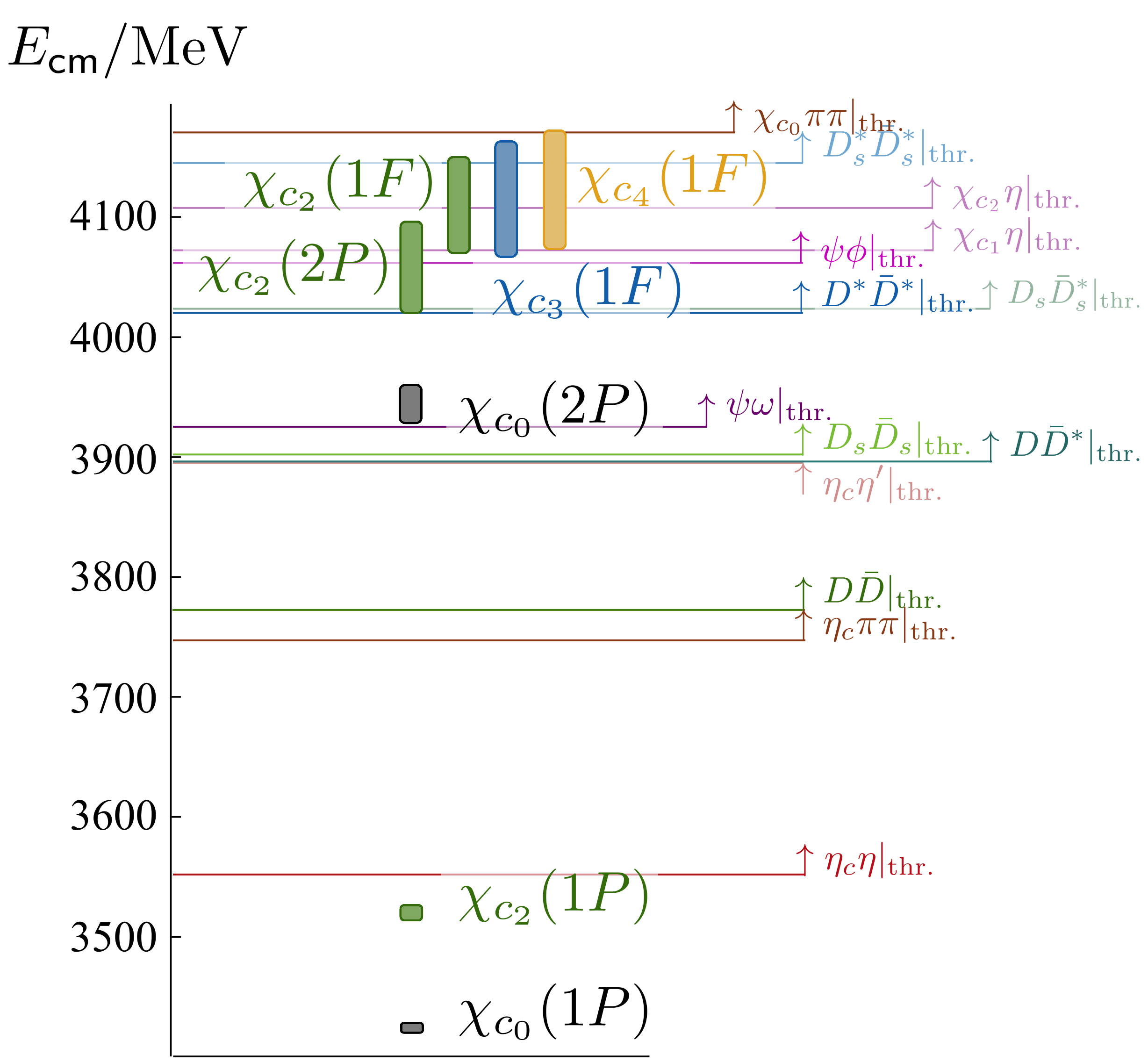


J. D. Yeo *et al* (HadSpec)
arXiv:2403.10498



- Several form factors
- ρ-resonance peak

Previously:



spectra from qqbar operators only,
Liu et al JHEP 1207 (2012) 126

"HadSpec" lattices

anisotropic (3.5 finer spacing in time)
Wilson-Clover

L/a_s=16, 20, 24
m_π = 391 MeV

rest and moving frames

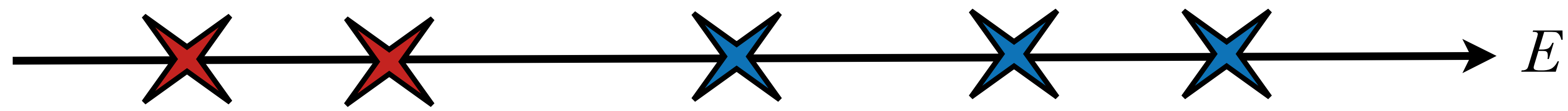
N_f = 2+1 flavours
all light+strange annihilations included
no charm annihilation

using *distillation* (Peardon *et al* 2009)
many channels, many wick contractions

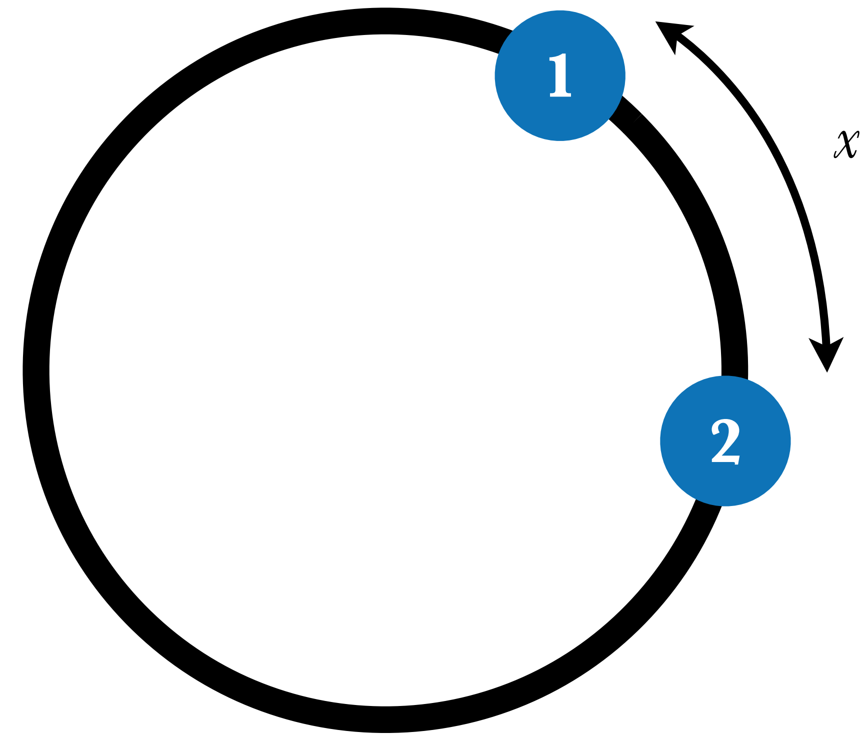
This study: Meson-meson + qqbar ops

Derivative ops - good overlap upto J=4

Variationally-optimised single meson ops



1-dimensional QM, periodic BC, two interacting particles: $V(x_1 - x_2) \neq 0$



$$\psi(0) = \psi(L), \quad \left.\frac{\partial\psi}{\partial x}\right|_{x=0} = \left.\frac{\partial\psi}{\partial x}\right|_{x=L}$$

$$\sin\left(\frac{pL}{2} + \delta(p)\right) = 0$$

$$p = \frac{2\pi n}{L} - \frac{2}{L}\delta(p)$$

Phase shifts via Lüscher's method:

$$\tan\delta_1 = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1;q^2)}$$

$$\mathcal{Z}_{00}(1;q^2) = \sum_{n\in\mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2}$$

Lüscher 1986, 1991

generalisation to a 3-dimensional strongly-coupled QFT

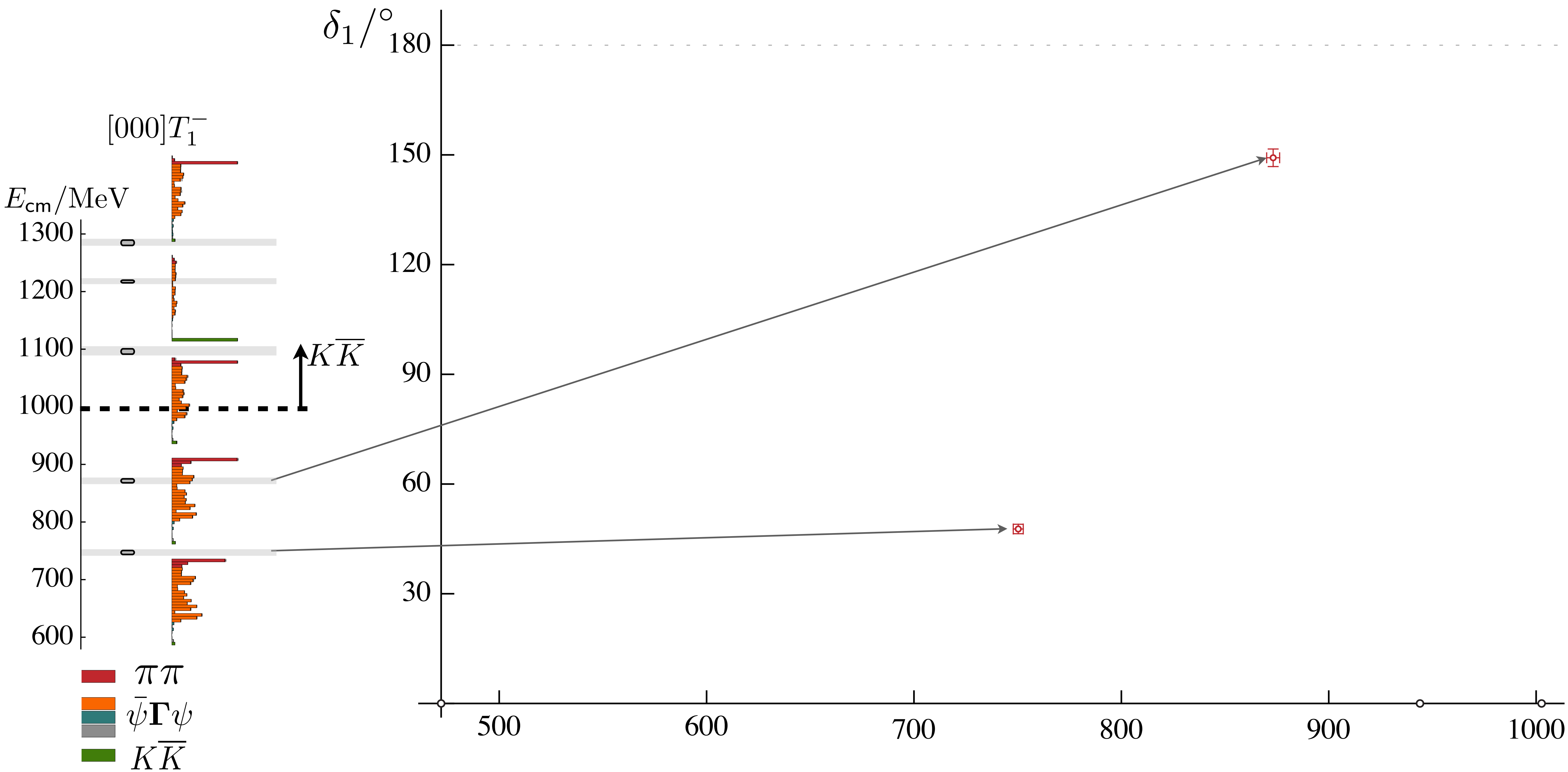
→ powerful non-trivial mapping from finite vol spectrum to infinite volume phase

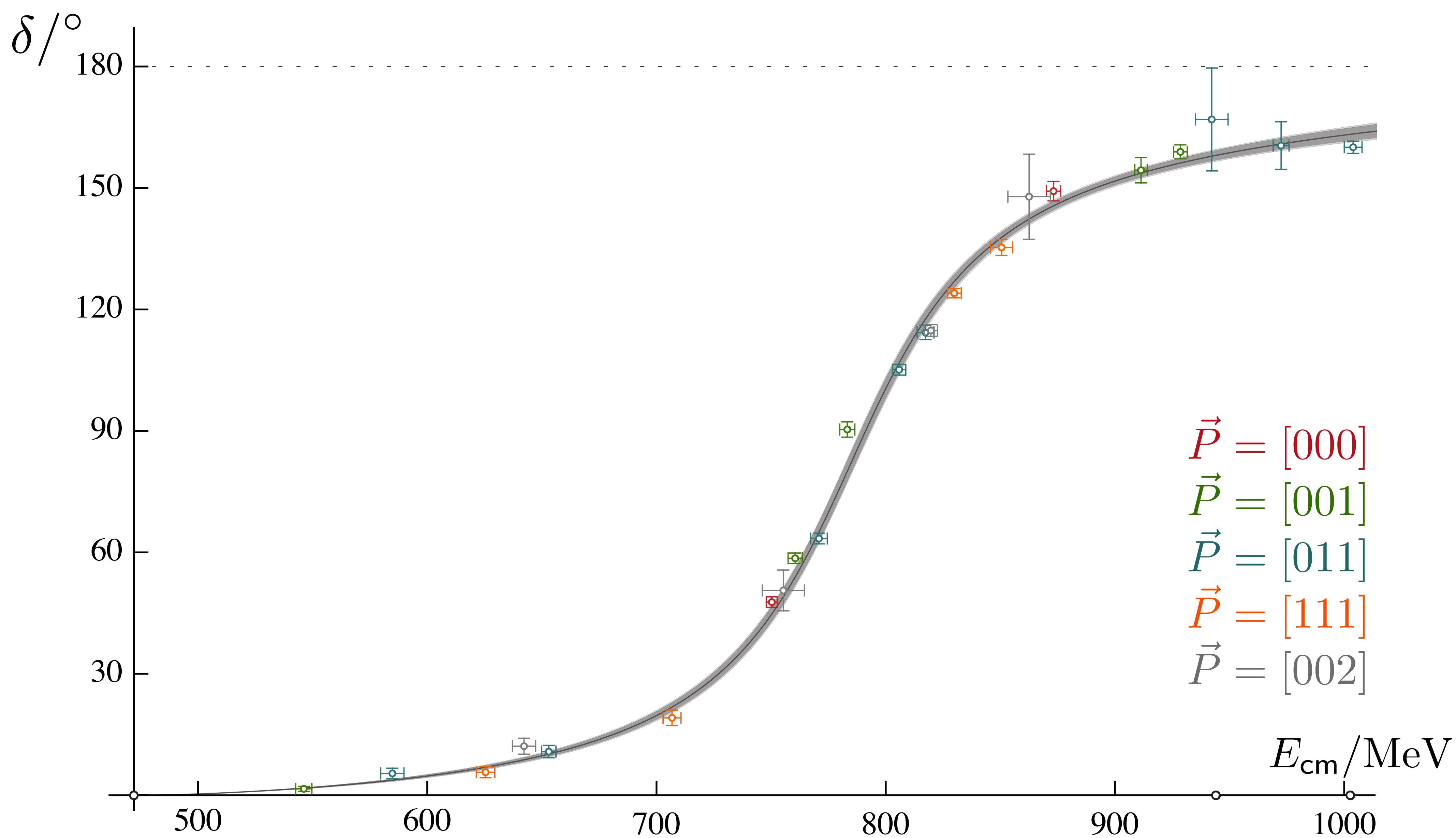
See also Kim, Sachrajda, Sharpe: Nucl. Phys. B727 (2005) (arXiv:hep-lat/0507006)

Review by Briceno, Dudek, Young: Rev. Mod. Phys. 90, 025001 (arXiv:1706.06223)

Phase shifts via the Lüscher method: $\tan\delta_1 = \frac{\pi^{3/2} q}{\mathcal{Z}_{00}(1;q^2)}$

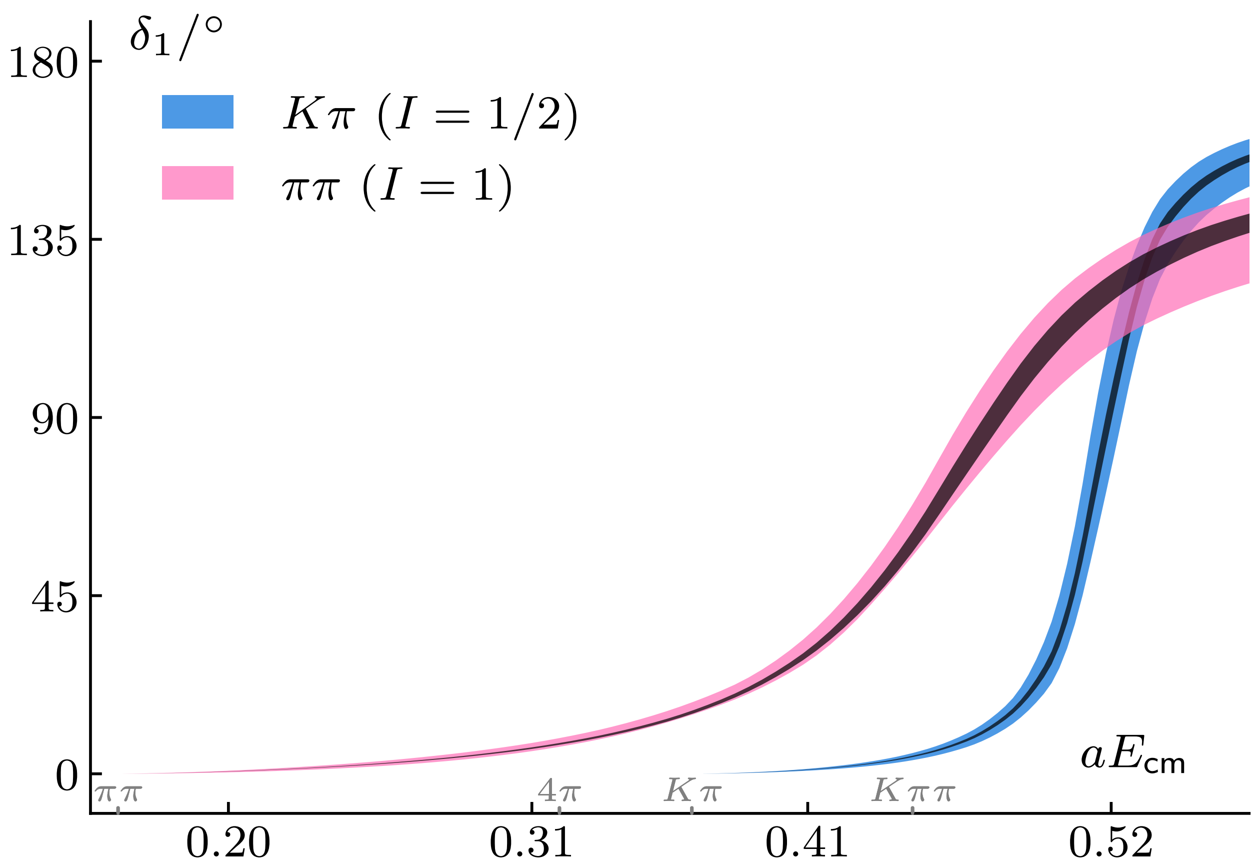
$$\mathcal{Z}_{00}(1;q^2) = \sum_{n\in\mathbb{Z}^3} \frac{1}{|\vec{n}|^2 - q^2}$$

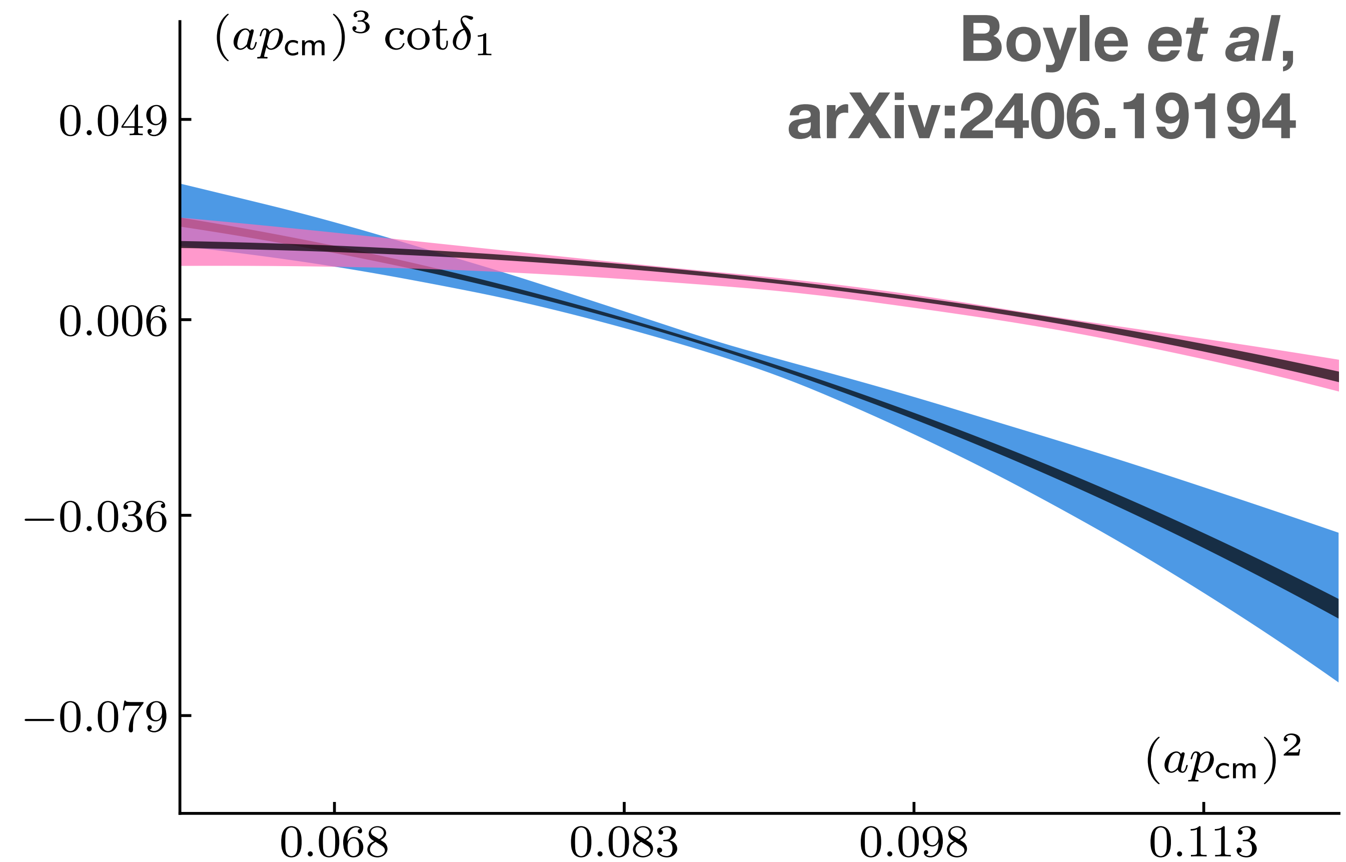


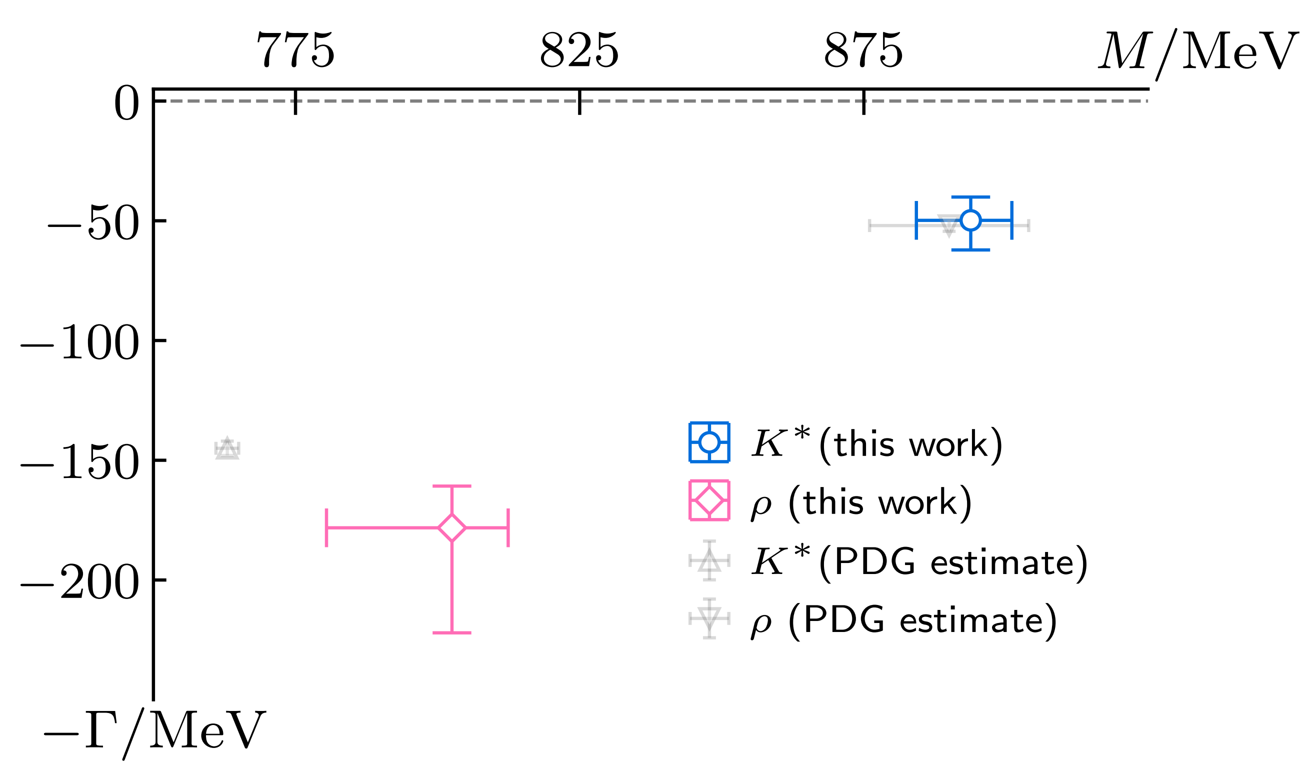


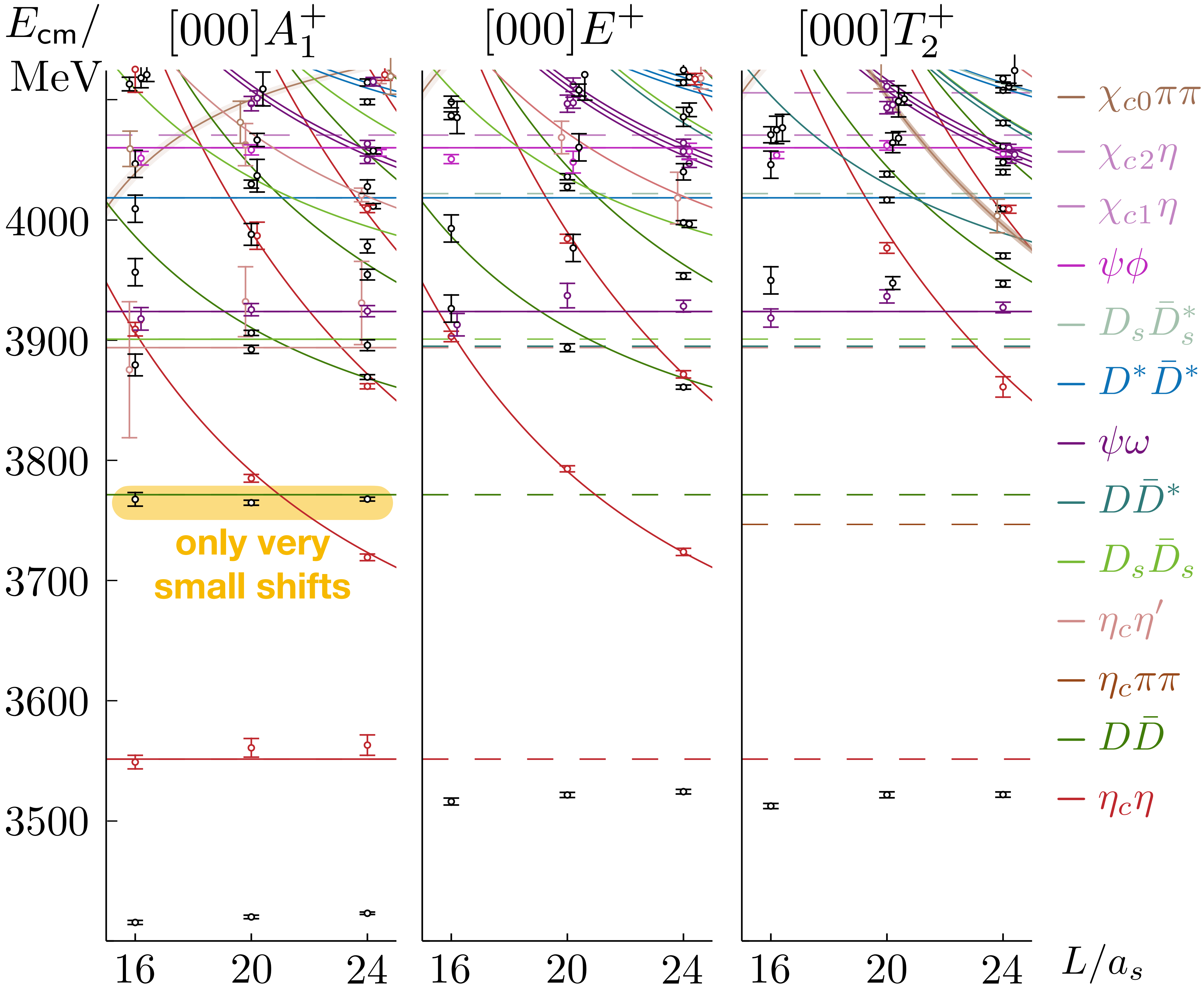
$\delta/^\circ$
180
150
120
90
60
30
500
600
700
800
900
1000
E_{cm}/MeV
$\vec{P} = [000]$
$\vec{P} = [001]$
$\vec{P} = [011]$
$\vec{P} = [111]$
$\vec{P} = [002]$

Boyle *et al*, arXiv:2406.19194

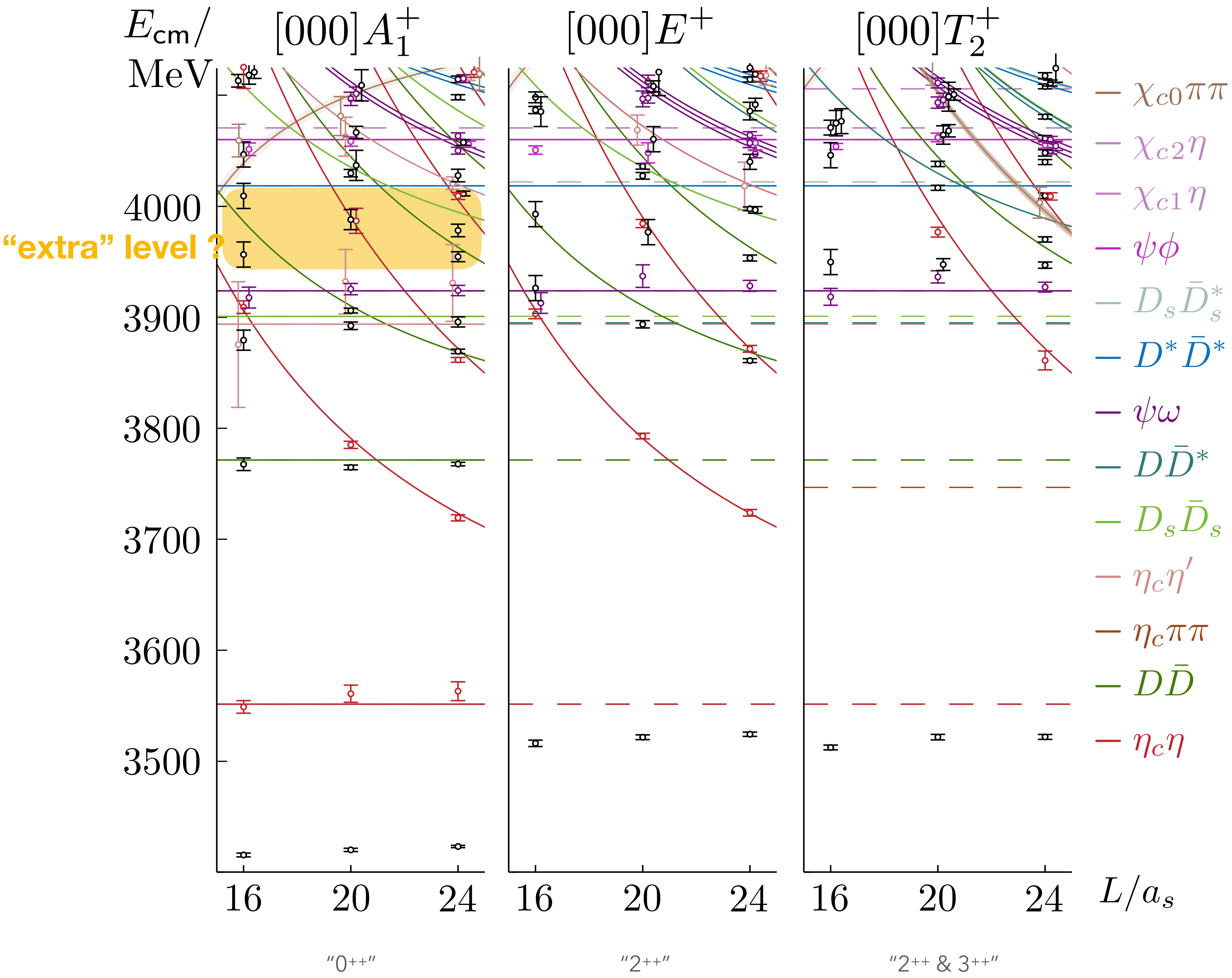




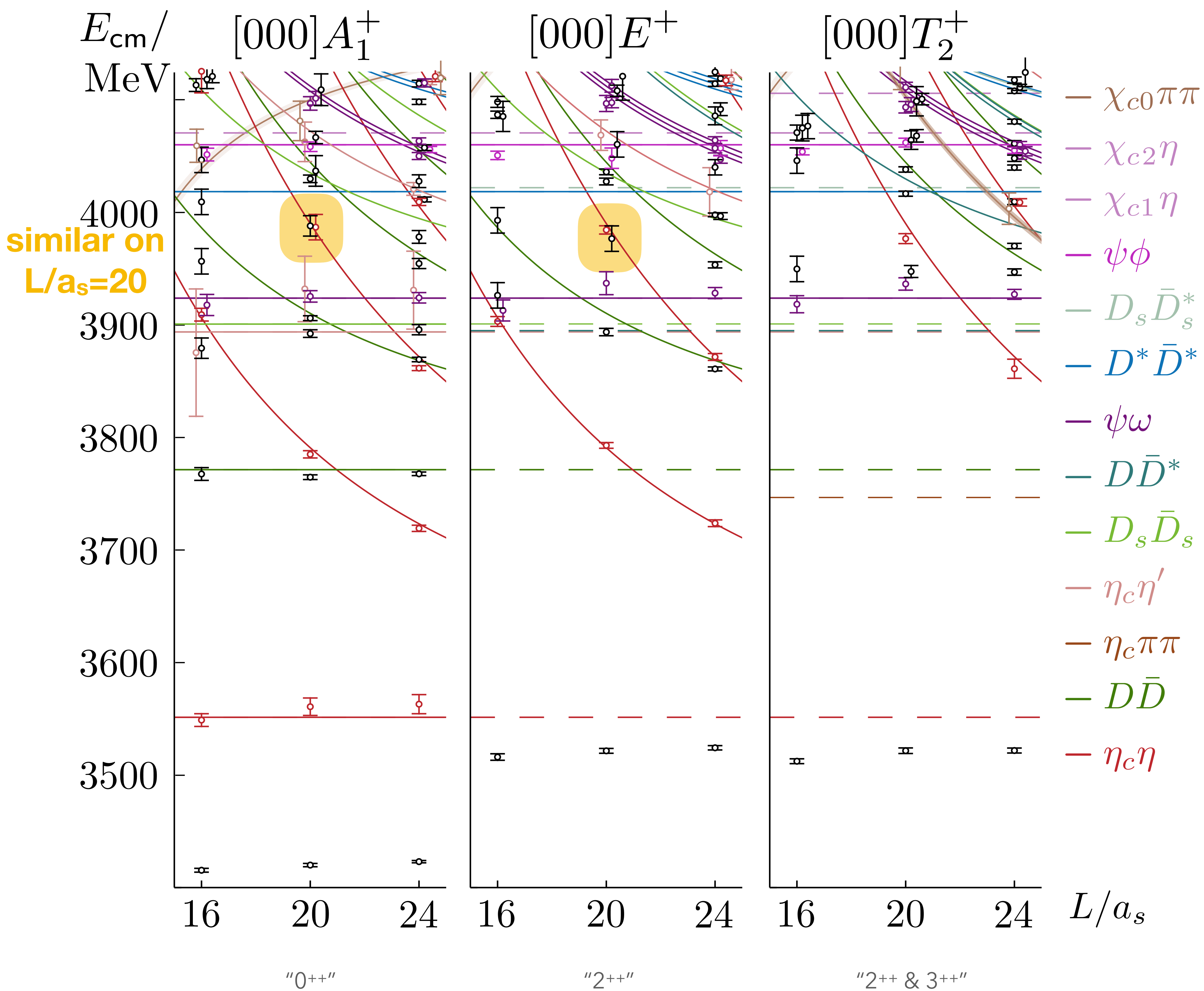



$E_{\mathrm{cm}}/$ MeV
$[000]A_1^+$
$[000]E^+$
$[000]T_2^+$
4000
3900
3800
3700
3600
3500
16 20 24
L/a_s
only very small shifts
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$

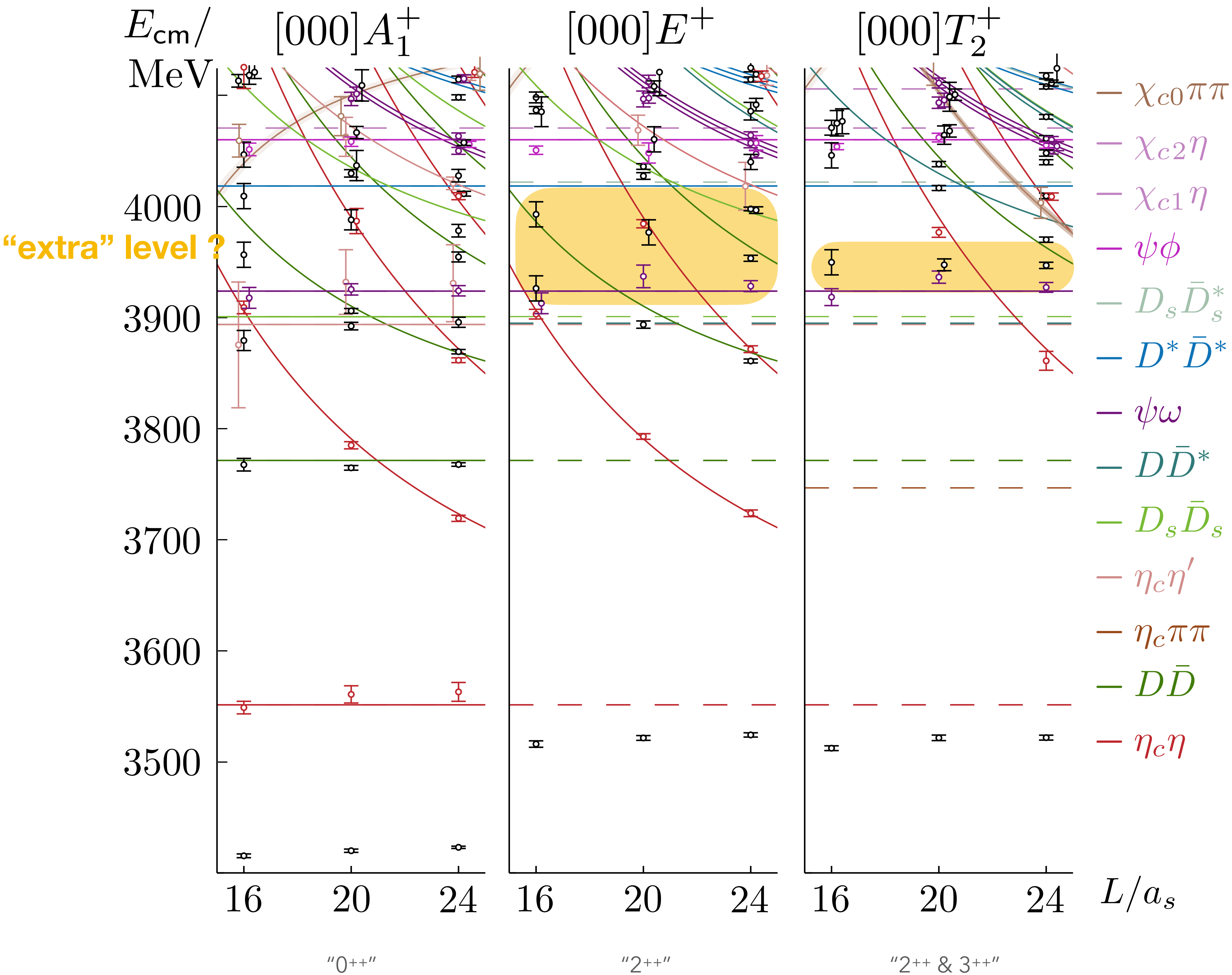
"0++" "2++" "2++ & 3++"



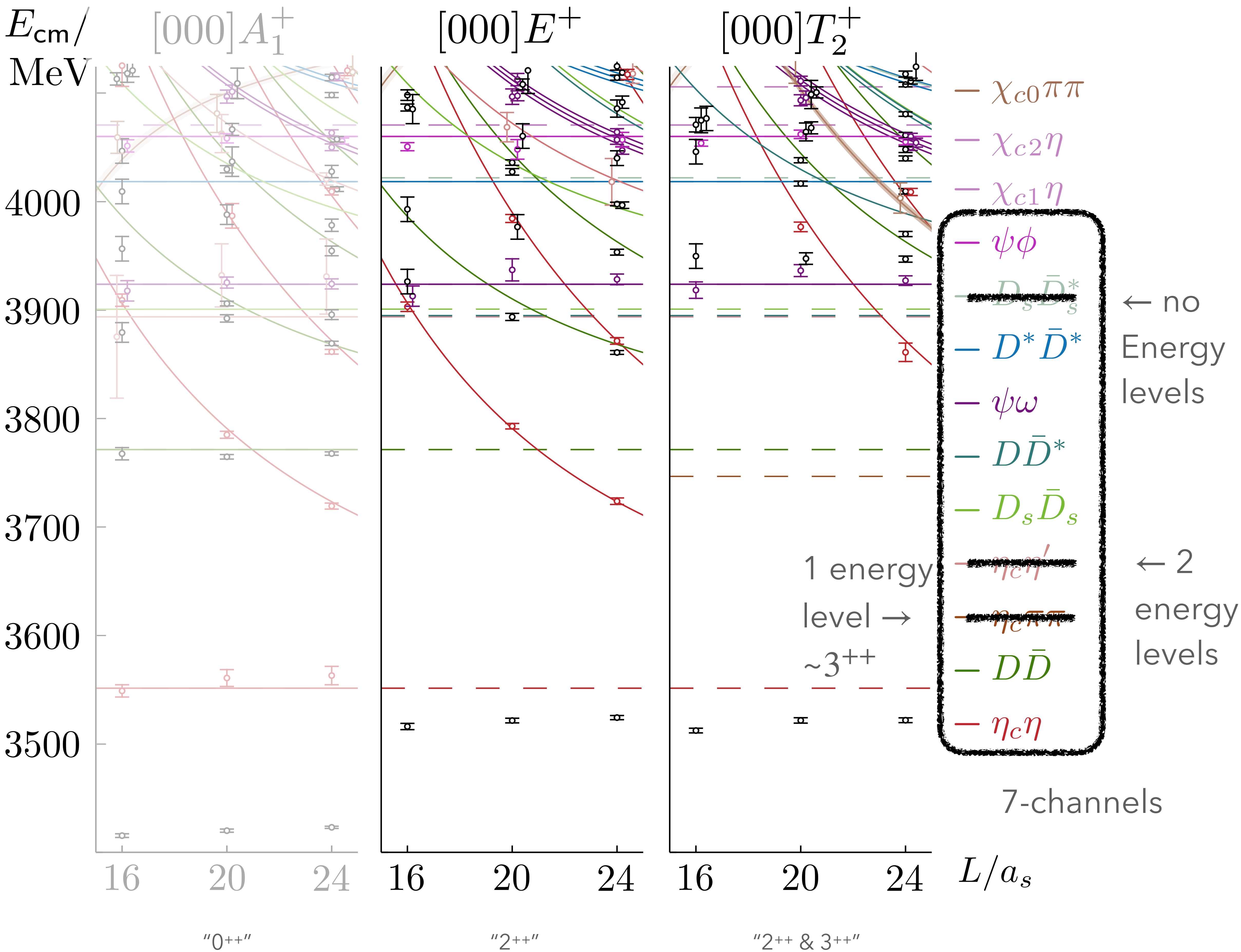
E_{cm}/ MeV
$[000]A_1^+$
$[000]E^+$
$[000]T_2^+$
3500
3600
3700
3800
3900
4000
"extra" level ?
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
"0++"
"2++"
"2++ & 3++"



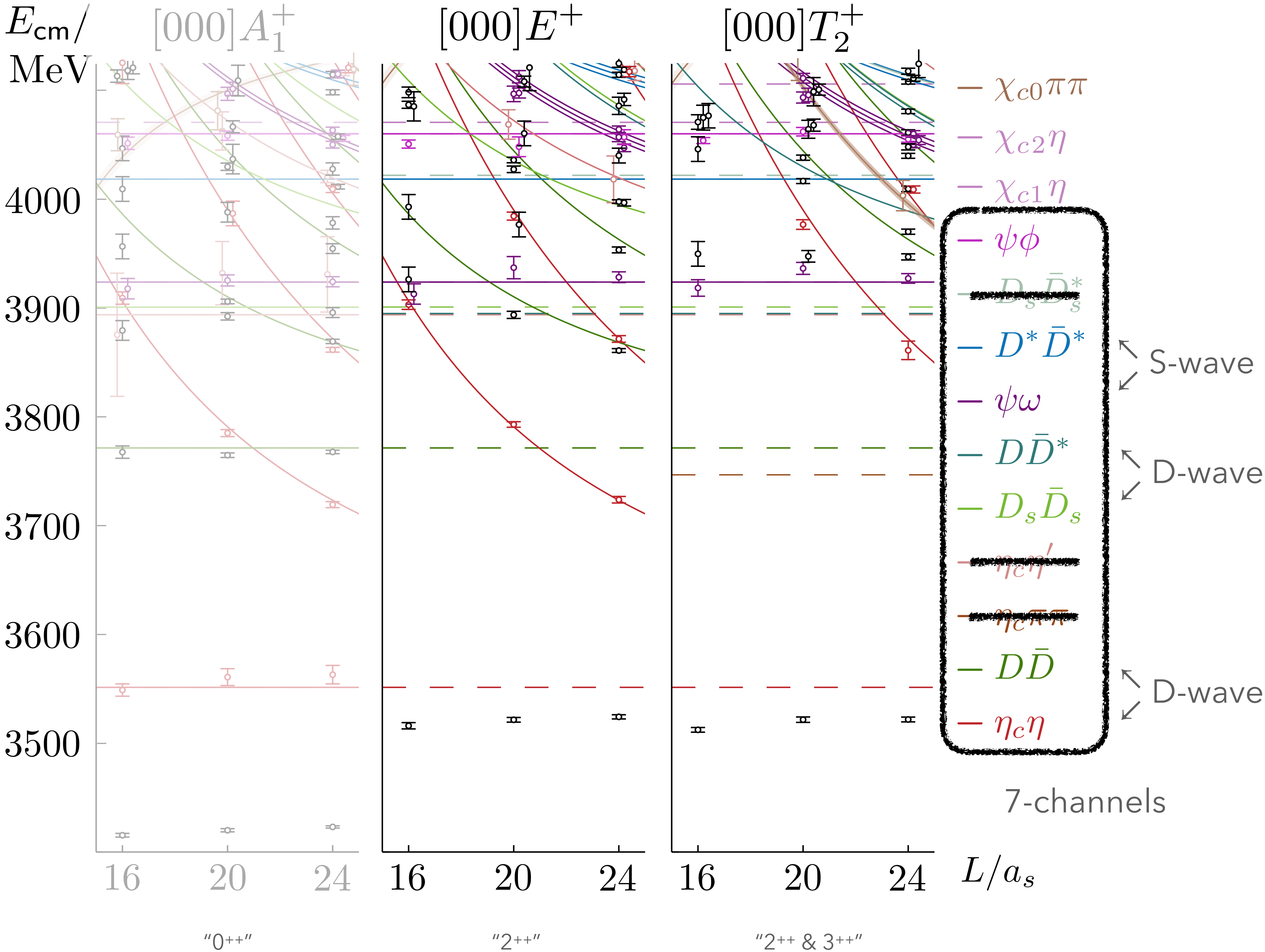
$E_{\mathsf{cm}}/$ MeV
$[000]\,A_1^+$
$[000]\,E^+$
$[000]\,T_2^+$
similar on L/a$_s$=20
4000
3900
3800
3700
3600
3500
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
"0++"
"2++"
"2++ & 3++"



E_{cm}/MeV
$[000]A_1^+$
$[000]E^+$
$[000]T_2^+$
"extra" level ?
3500
3600
3700
3800
3900
4000
16
20
24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
"0++"
"2++"
"2++ & 3++"

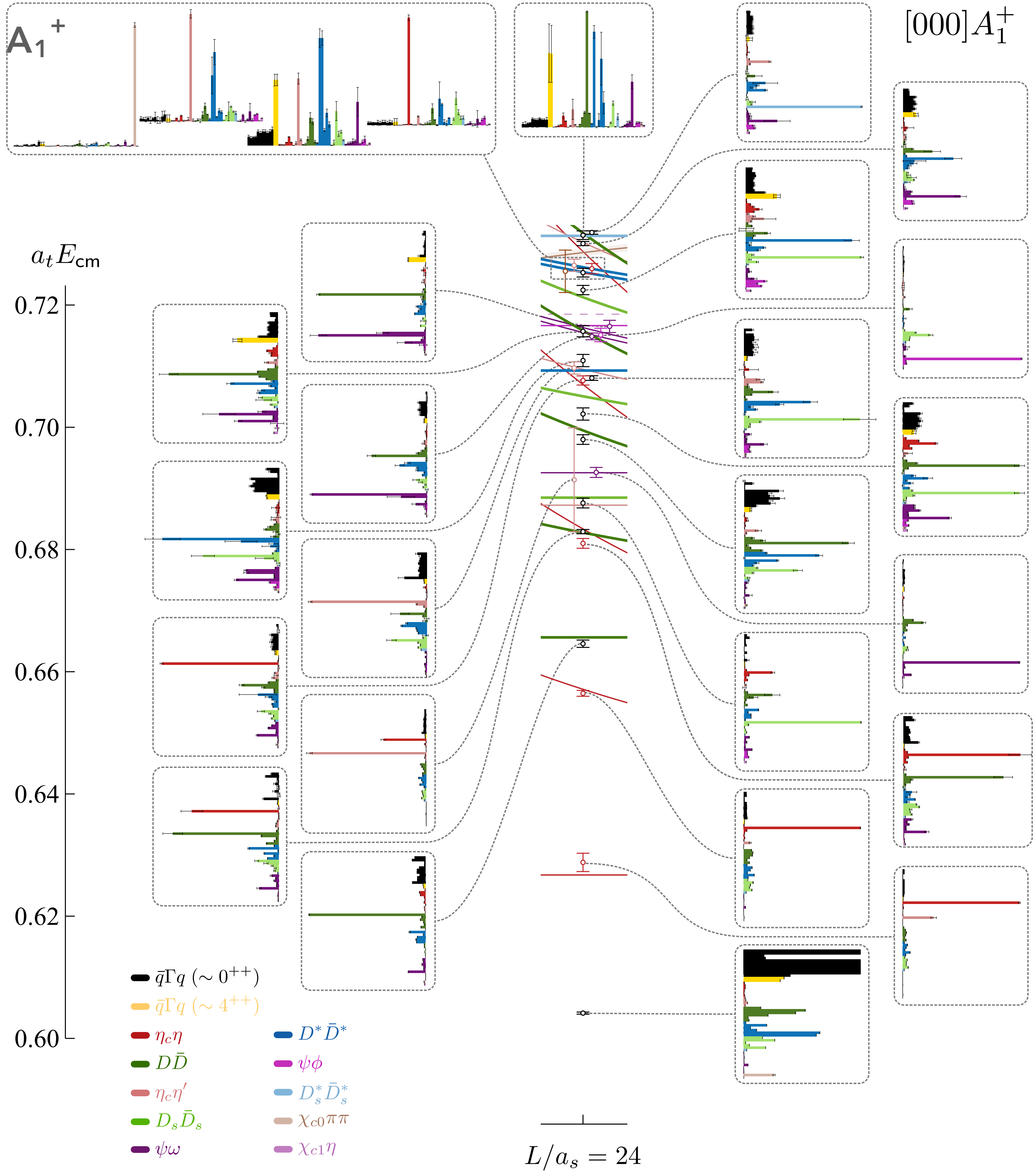


E_{cm}/ MeV
[000]A_1^+
[000]E^+
[000]T_2^+
4000
3900
3800
3700
3600
3500
16 20 24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
← no Energy levels
← 2 energy levels
1 energy level → ~3^{++}
7-channels
"0++"
"2++"
"2++ & 3++"

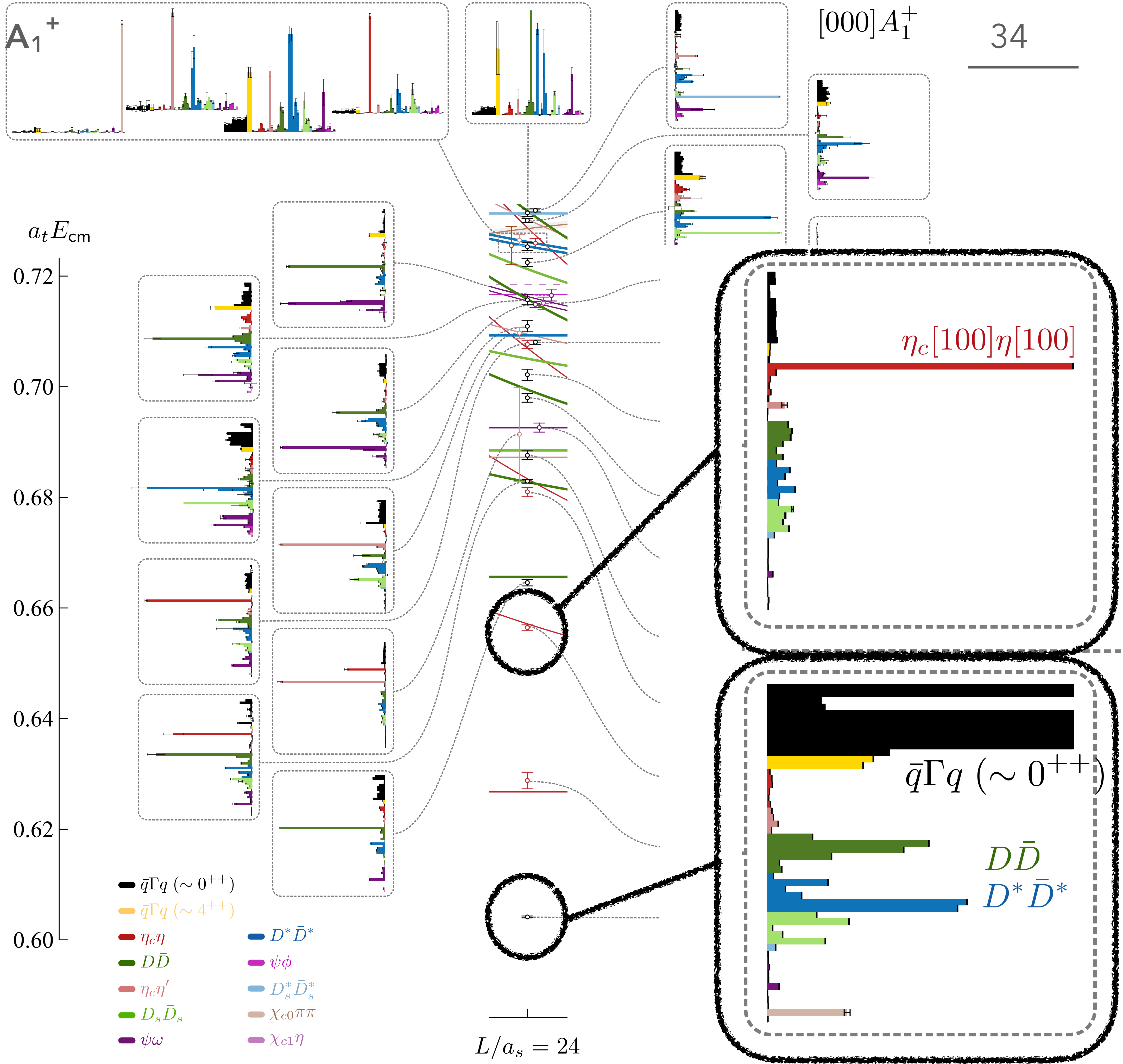

E_{cm}/ MeV
[000]A_1^+
[000]E^+
[000]T_2^+
4000
3900
3800
3700
3600
3500
16 20 24
16 20 24
16 20 24
L/a_s
$\chi_{c0}\pi\pi$
$\chi_{c2}\eta$
$\chi_{c1}\eta$
$\psi\phi$
$D_s\bar{D}_s^*$
$D^*\bar{D}^*$
$\psi\omega$
$D\bar{D}^*$
$D_s\bar{D}_s$
$\eta_c\eta'$
$\eta_c\pi\pi$
$D\bar{D}$
$\eta_c\eta$
S-wave
D-wave
D-wave
7-channels
"0++"
"2++"
"2++ & 3++"

S-wave - [000] A_1^+

m_π=391 MeV

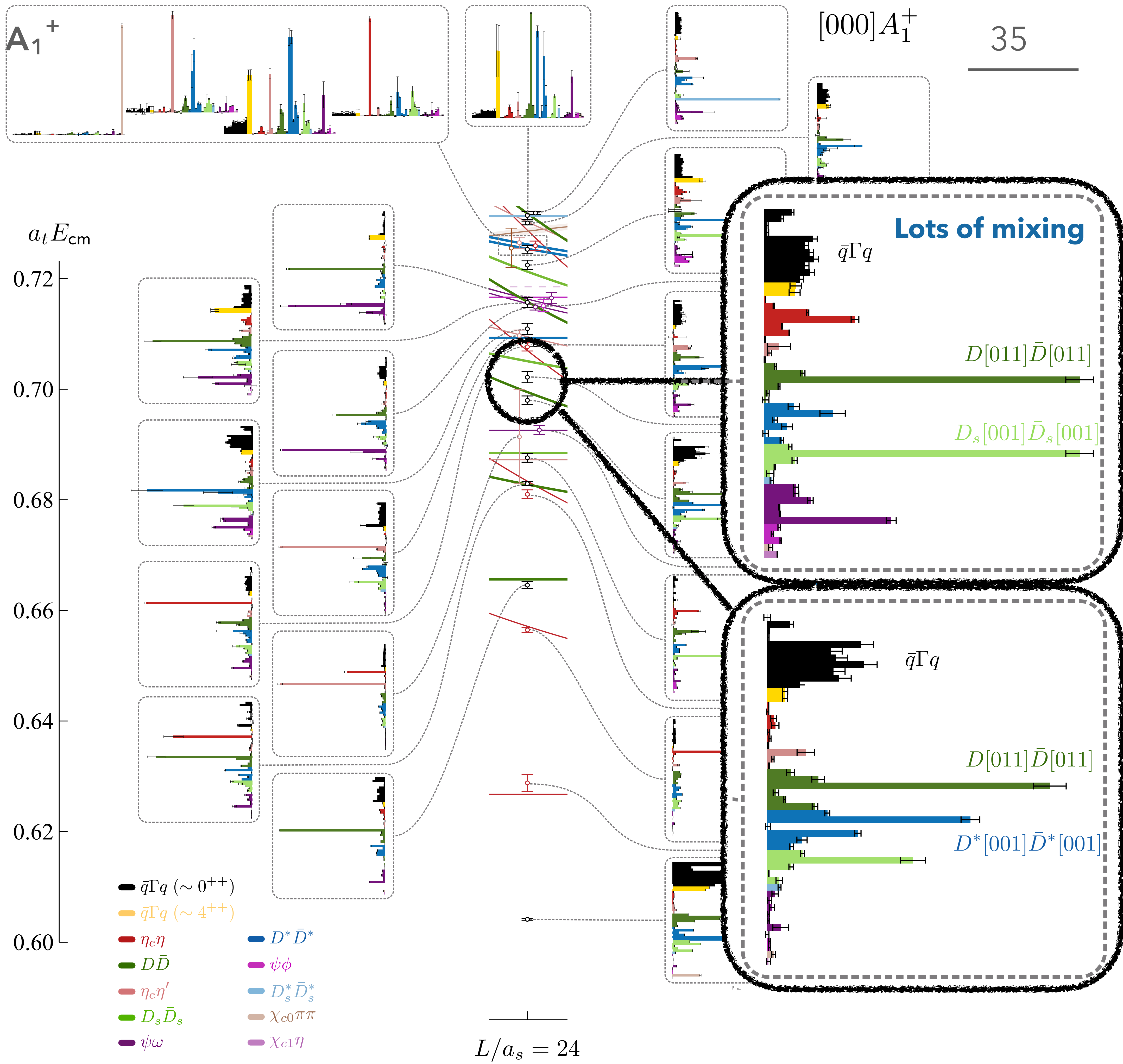


S-wave - [000] A_1^+



[000]A_1^+
$a_t E_{\mathrm{cm}}$
0.72
0.70
0.68
0.66
0.64
0.62
0.60
$\eta_c[100]\eta[100]$
$\bar{q}\Gamma q\ (\sim 0^{++})$
$D\bar{D}$
$D^*\bar{D}^*$
$\bar{q}\Gamma q\ (\sim 0^{++})$
$\bar{q}\Gamma q\ (\sim 4^{++})$
$\eta_c\eta$
$D\bar{D}$
$\eta_c\eta'$
$D_s\bar{D}_s$
$\psi\omega$
$D^*\bar{D}^*$
$\psi\phi$
$D_s^*\bar{D}_s^*$
$\chi_{c0}\pi\pi$
$\chi_{c1}\eta$
$L/a_s = 24$

S-wave - [000] A_1^+



$[000]A_1^+$
$a_t E_{\mathsf{cm}}$
0.72
0.70
0.68
0.66
0.64
0.62
0.60
$L/a_s = 24$
Lots of mixing
$\bar{q}\Gamma q$
$D[011]\bar{D}[011]$
$D_s[001]\bar{D}_s[001]$
$\bar{q}\Gamma q$
$D[011]\bar{D}[011]$
$D^*[001]\bar{D}^*[001]$
$\bar{q}\Gamma q\ (\sim 0^{++})$
$\bar{q}\Gamma q\ (\sim 4^{++})$
$\eta_c\eta$
$D\bar{D}$
$\eta_c\eta'$
$D_s\bar{D}_s$
$\psi\omega$
$D^*\bar{D}^*$
$\psi\phi$
$D_s^*\bar{D}_s^*$
$\chi_{c0}\pi\pi$
$\chi_{c1}\eta$

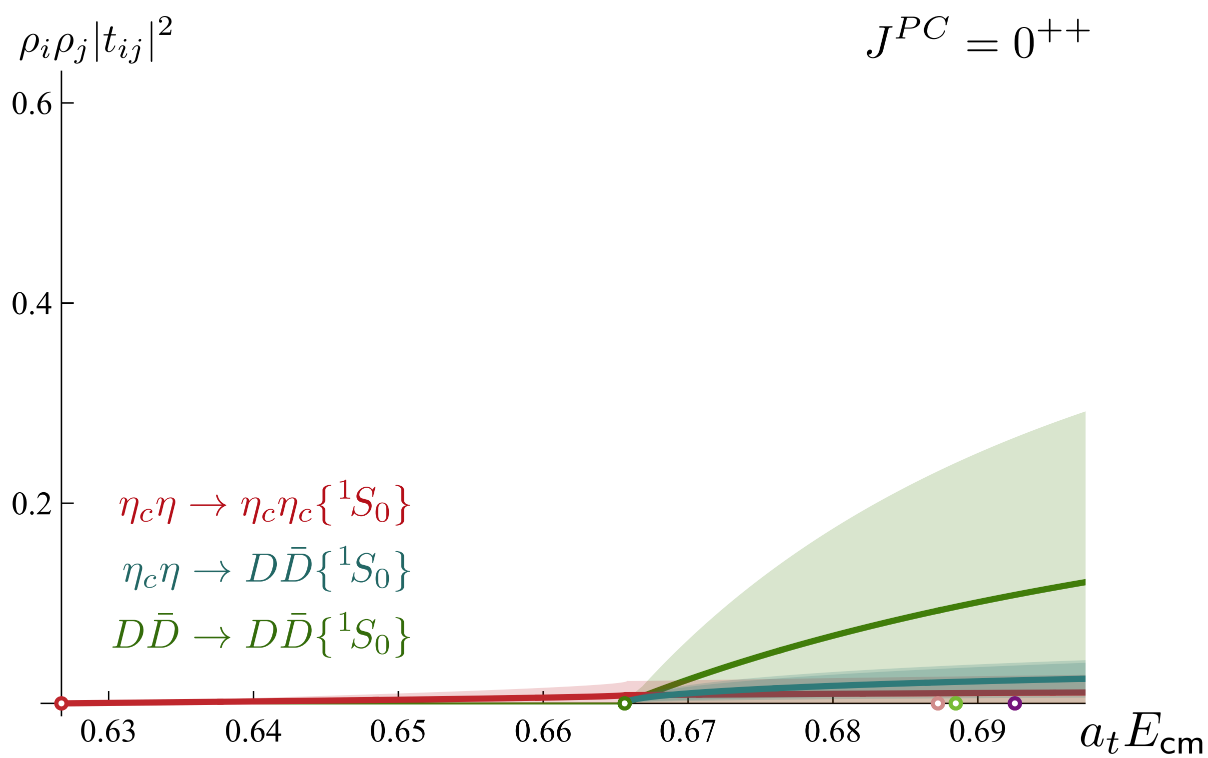
$$\boldsymbol{S} = \boldsymbol{1} + 2i\boldsymbol{\rho}^{\frac{1}{2}} \cdot \boldsymbol{t} \cdot \boldsymbol{\rho}^{\frac{1}{2}}$$

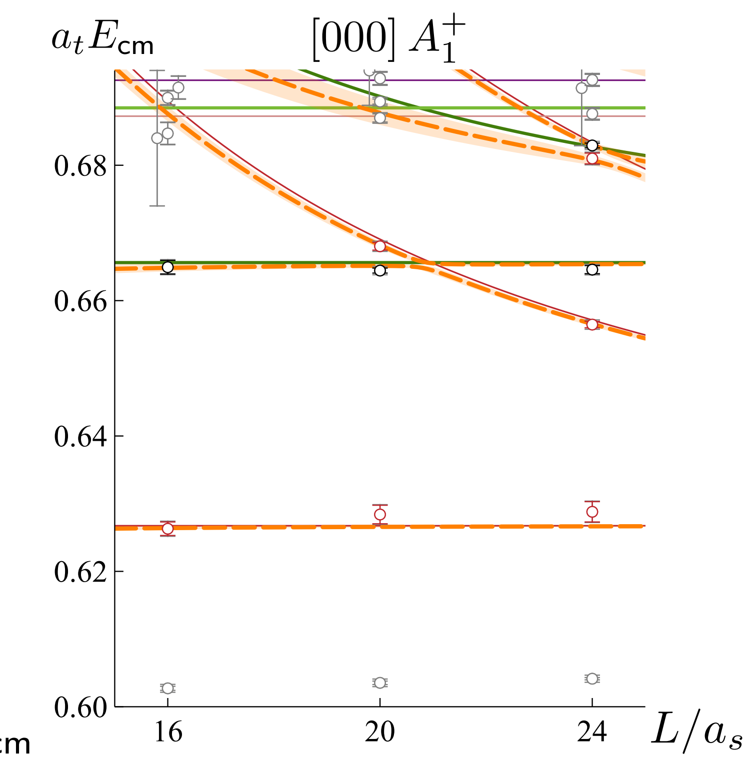
$$\boldsymbol{t}^{-1} = \boldsymbol{K}^{-1} + \boldsymbol{I}$$

$$\mathrm{Im} I_{ij} = -\rho_i = 2k_i/\sqrt{s}$$

$$\det[\boldsymbol{1} + i\boldsymbol{\rho} \cdot \boldsymbol{t}\,(\boldsymbol{1} + i\boldsymbol{\mathcal{M}}(L))] = 0$$

$$\boldsymbol{K} = \begin{bmatrix} \gamma_{\eta_c\eta\to\eta_c\eta} & \gamma_{\eta_c\eta\to D\bar{D}} \\ \gamma_{\eta_c\eta\to D\bar{D}} & \gamma_{D\bar{D}\to D\bar{D}} \end{bmatrix}$$



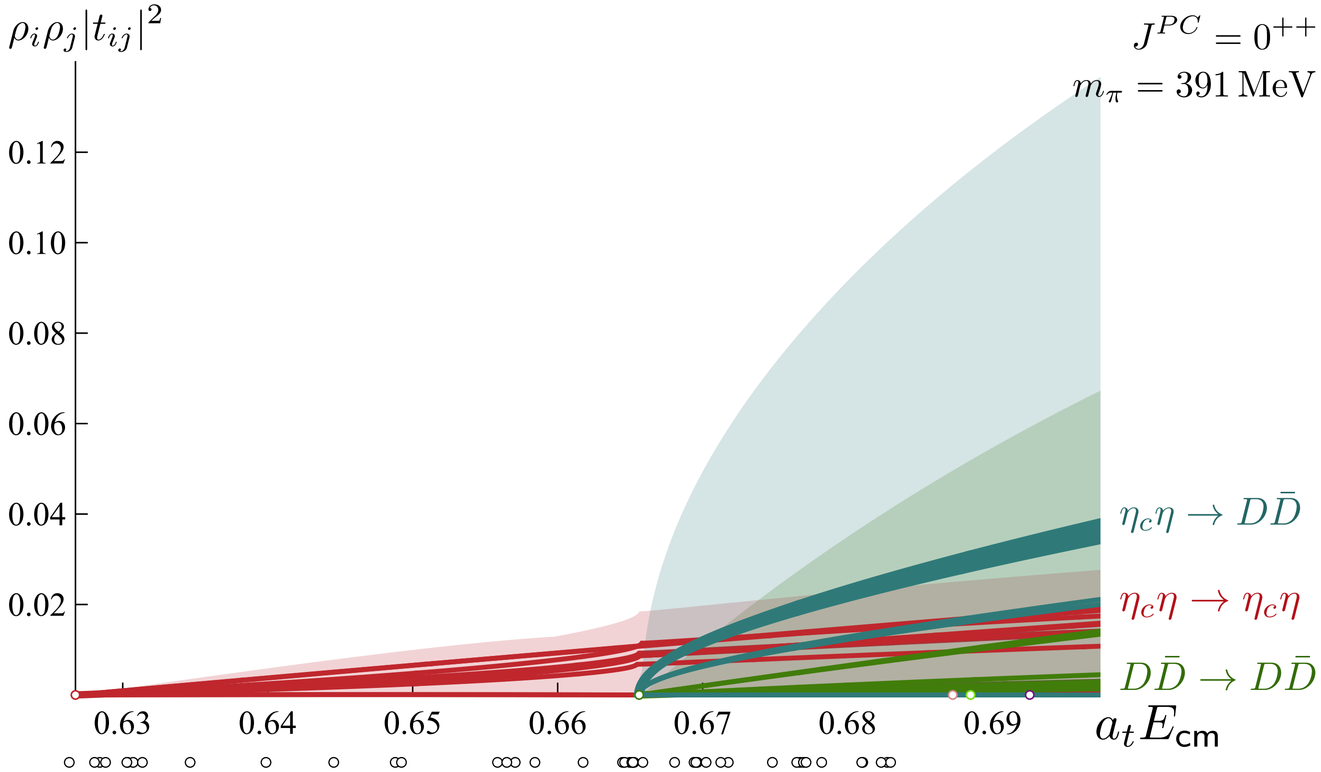


using rest-frame only

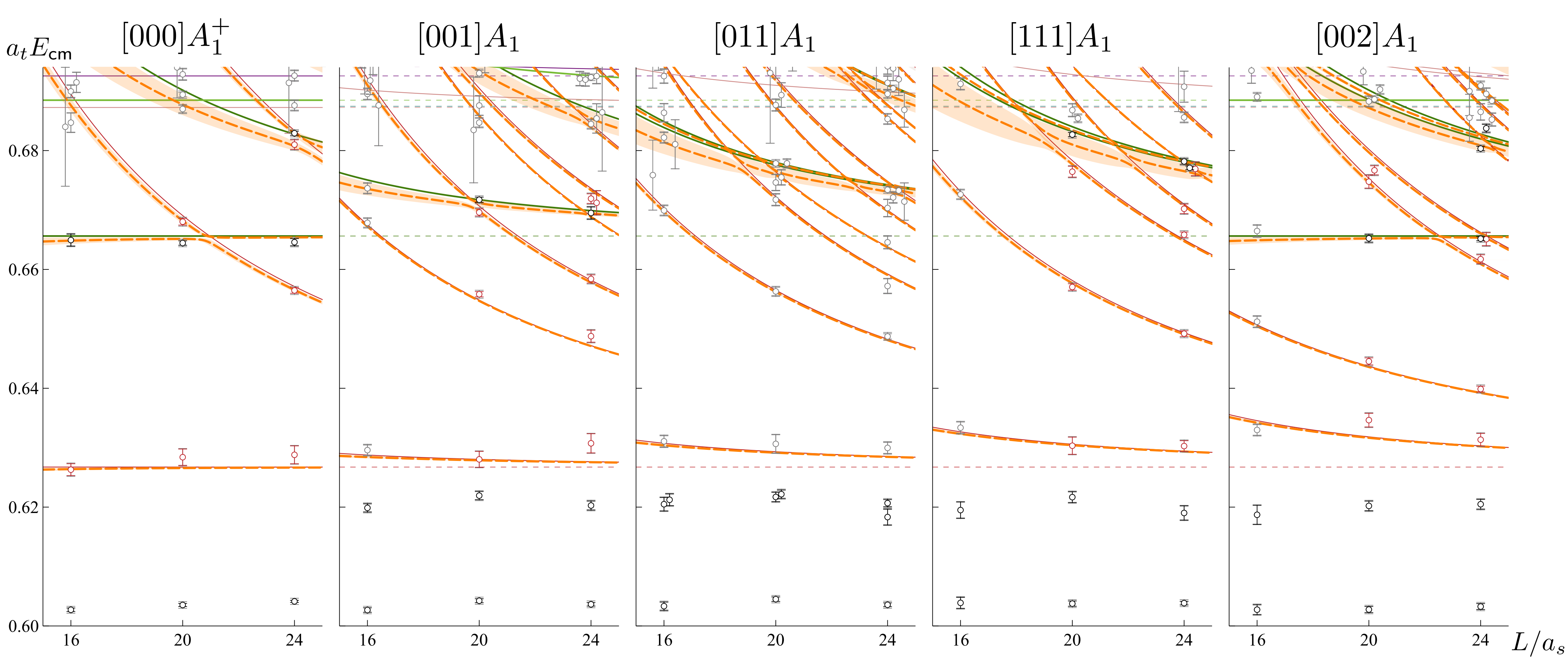
$$\begin{aligned} \gamma_{\eta_c\eta\to\eta_c\eta} &= (0.34 \pm 0.23 \pm 0.09) \\ \gamma_{\eta_c\eta\to D\bar{D}} &= (0.58 \pm 0.29 \pm 0.05) \\ \gamma_{D\bar{D}\to D\bar{D}} &= (1.39 \pm 1.19 \pm 0.24) \end{aligned} \quad \begin{bmatrix} 1.00 & 0.77 & -0.24 \\ & 1.00 & -0.22 \\ & & 1.00 \end{bmatrix}$$

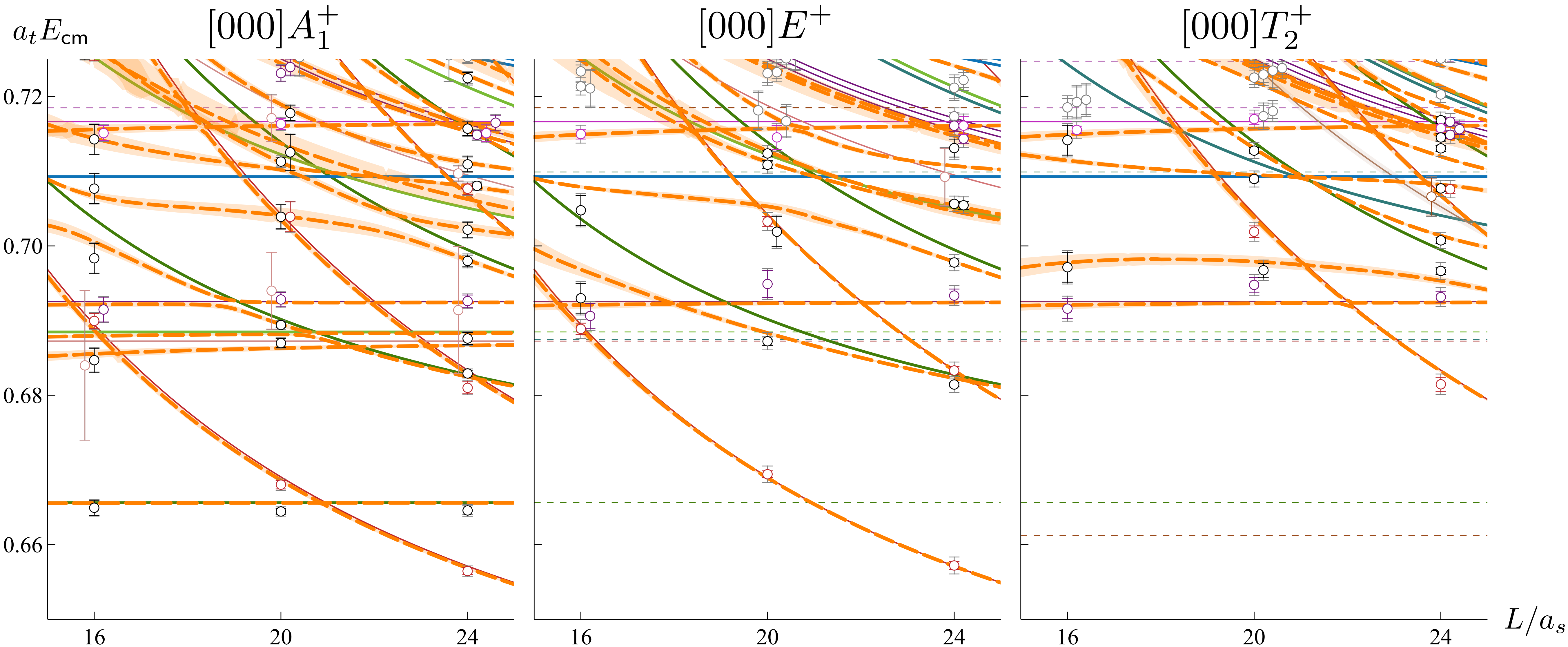
$$\chi^2/N_{\mathrm{dof}} = \frac{5.65}{10-3} = 0.81$$

(smaller range)



using zero and non-zero total momentum





$$\det[\mathbf{1} + i\boldsymbol{\rho} \cdot \boldsymbol{t}\,(\mathbf{1} + i\boldsymbol{\mathcal{M}}(L))] = 0$$

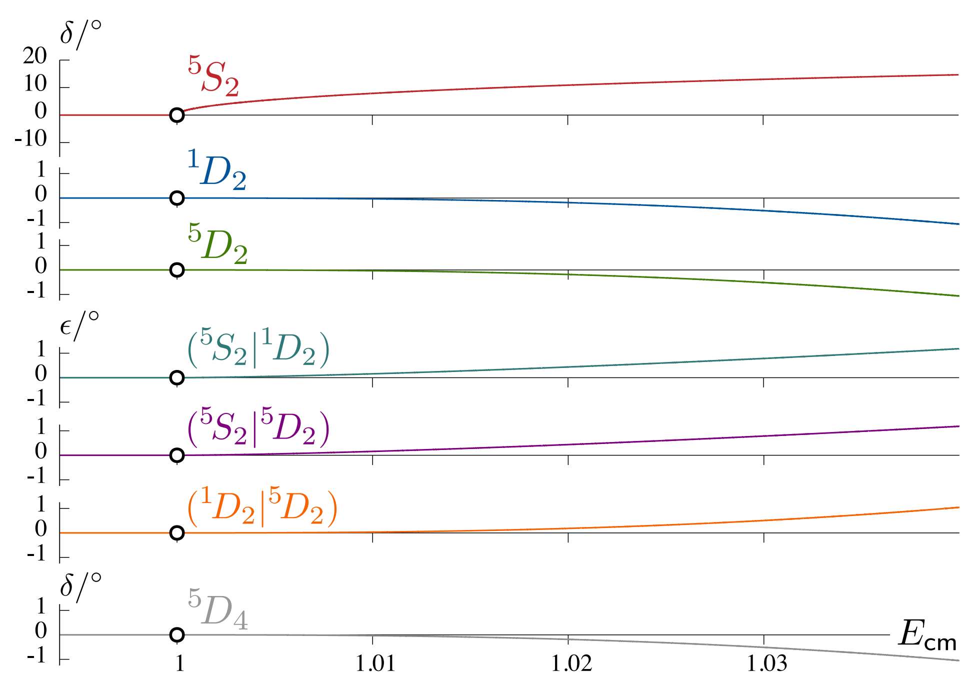
$$\det\left[\boldsymbol{D}(E_{\mathsf{cm}})\right] = 0$$

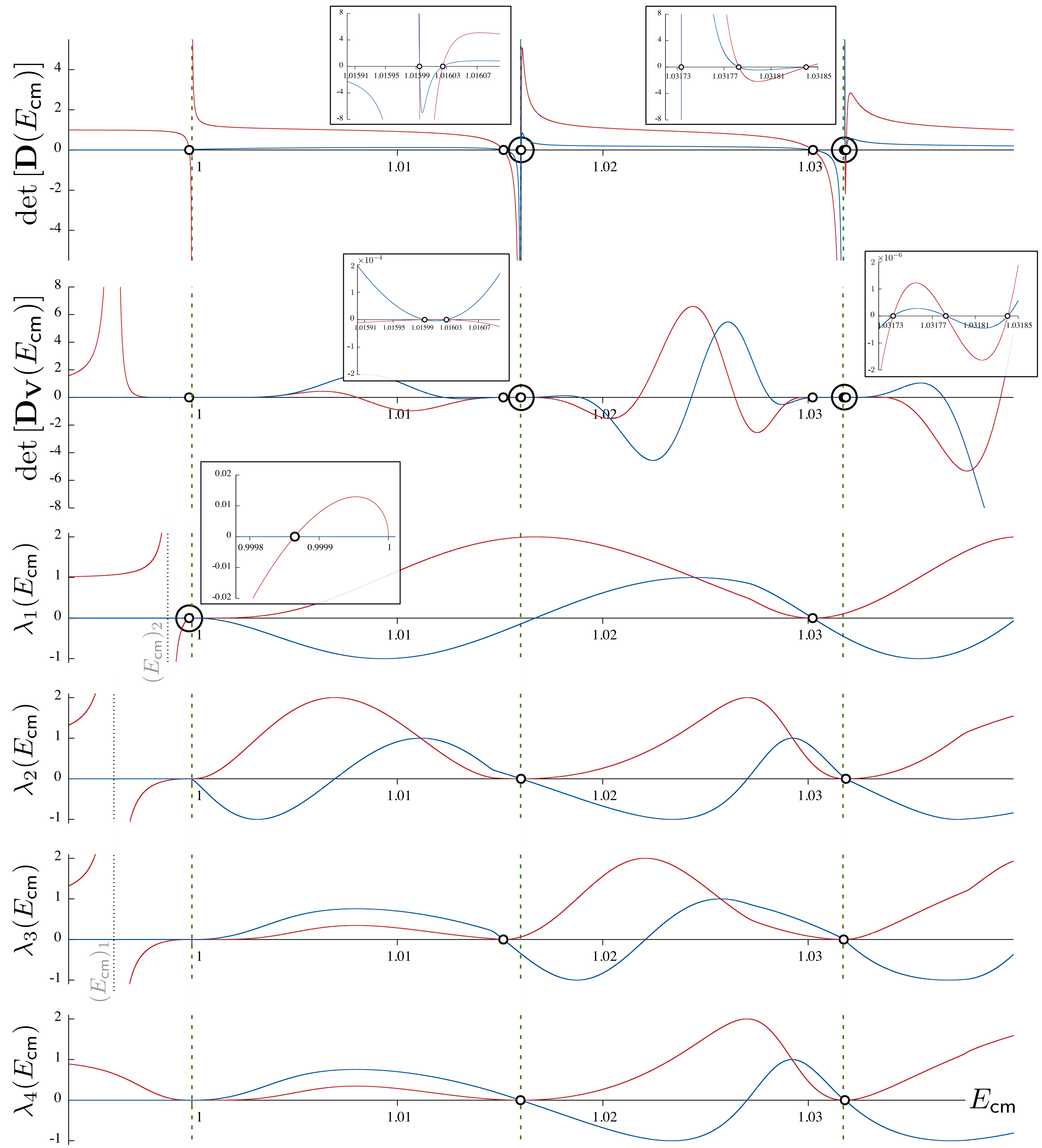
$$\boldsymbol{D}(E_{\mathsf{cm}}) = \boldsymbol{1} + i\boldsymbol{\rho}(E_{\mathsf{cm}})\cdot\boldsymbol{t}(E_{\mathsf{cm}})\cdot\big(\boldsymbol{1}+i\boldsymbol{\mathcal{M}}(E_{\mathsf{cm}},L)\big)$$

$$\boldsymbol{D}_V(E_{\mathsf{cm}}) = \boldsymbol{1} + \boldsymbol{S}(E_{\mathsf{cm}})\cdot\boldsymbol{V}(E_{\mathsf{cm}},L)$$

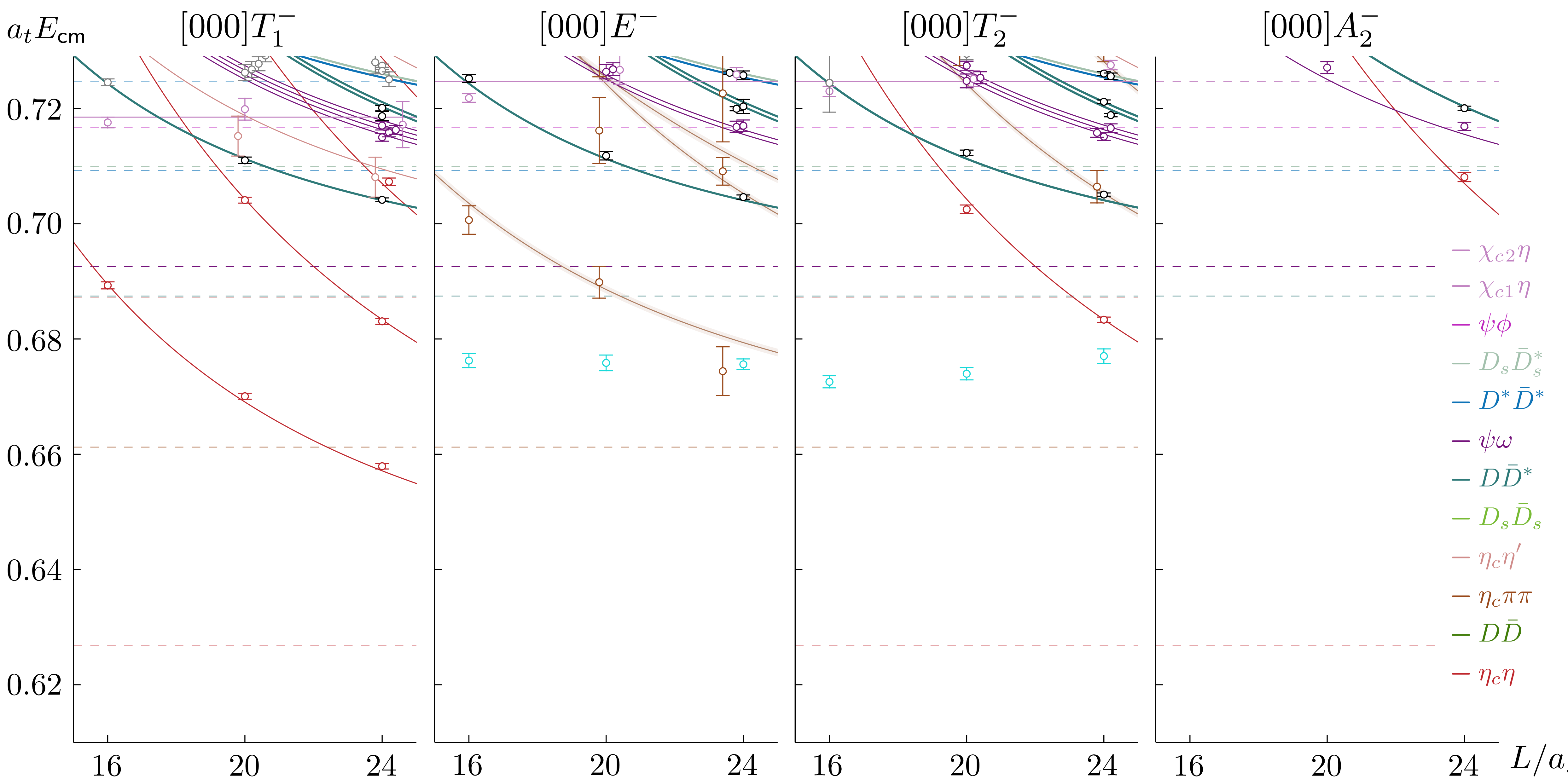
$$\det\left[\boldsymbol{D}_V(E_{\mathsf{cm}})\right] = \prod_{p=1}^{n}\lambda_p(E_{\mathsf{cm}})$$

$$\boldsymbol{D}_V(E_{\mathsf{cm}})\,\boldsymbol{v}^{(p)}(E_{\mathsf{cm}}) = \lambda_p(E_{\mathsf{cm}})\,\boldsymbol{v}^{(p)}(E_{\mathsf{cm}})$$





Woss, DW, Dudek, arXiv:2001.08474, PRD 101, 114505 (2020)

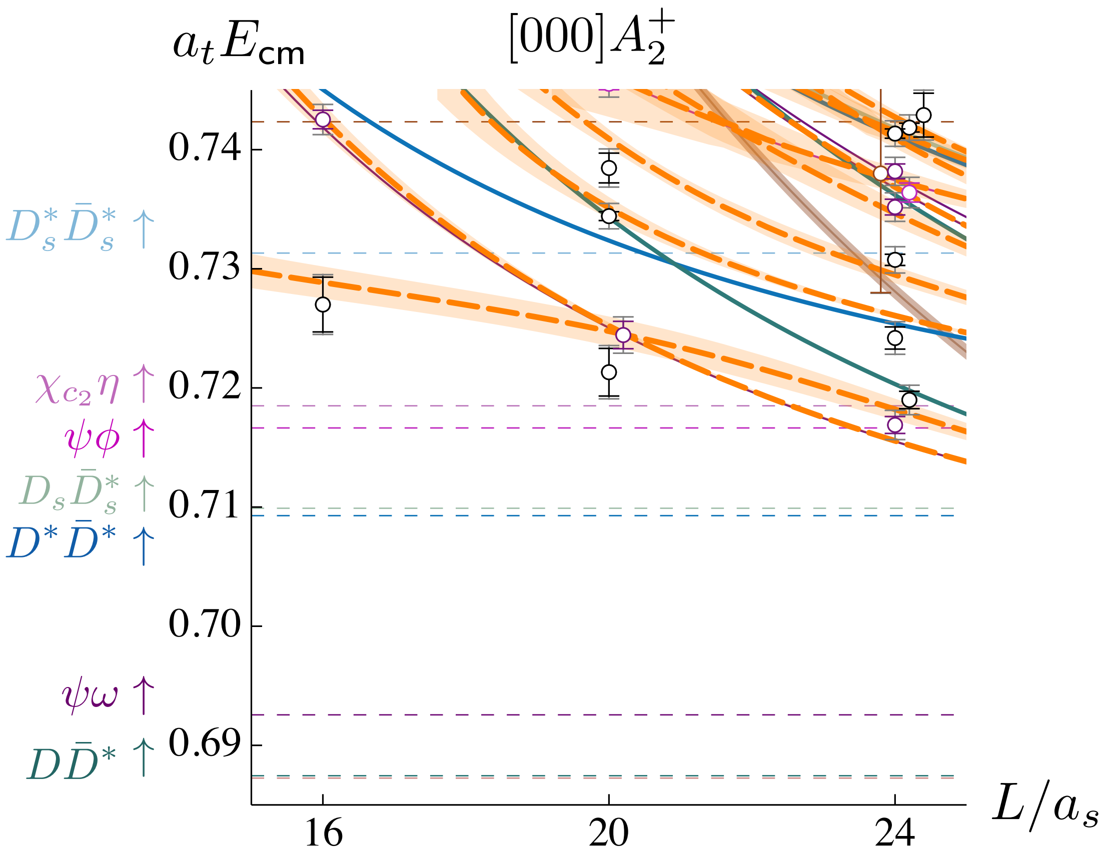


(we also computed lattice irreps with non-zero total momentum)

P=- partial waves can then contribute

very little going on here

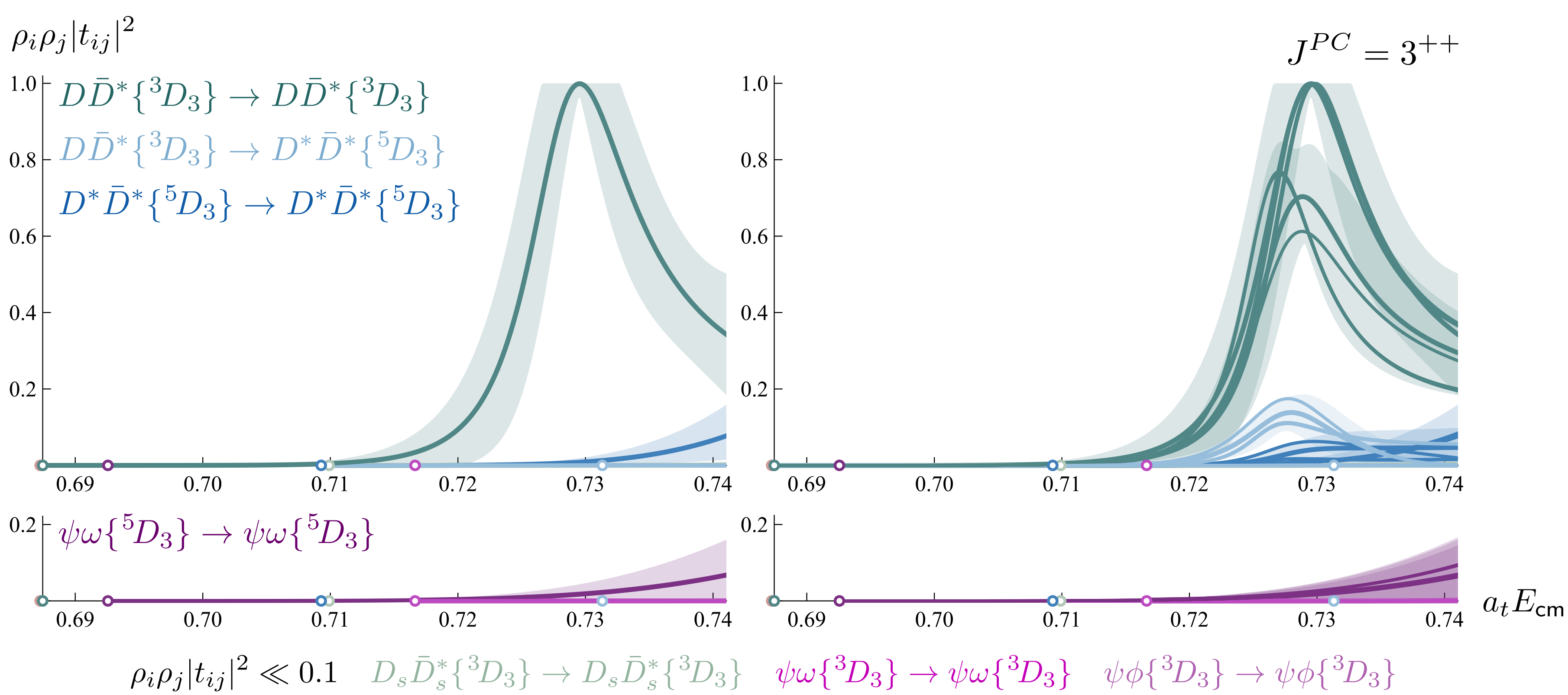
an η_{c2} 2^{-+} state arises below DD*

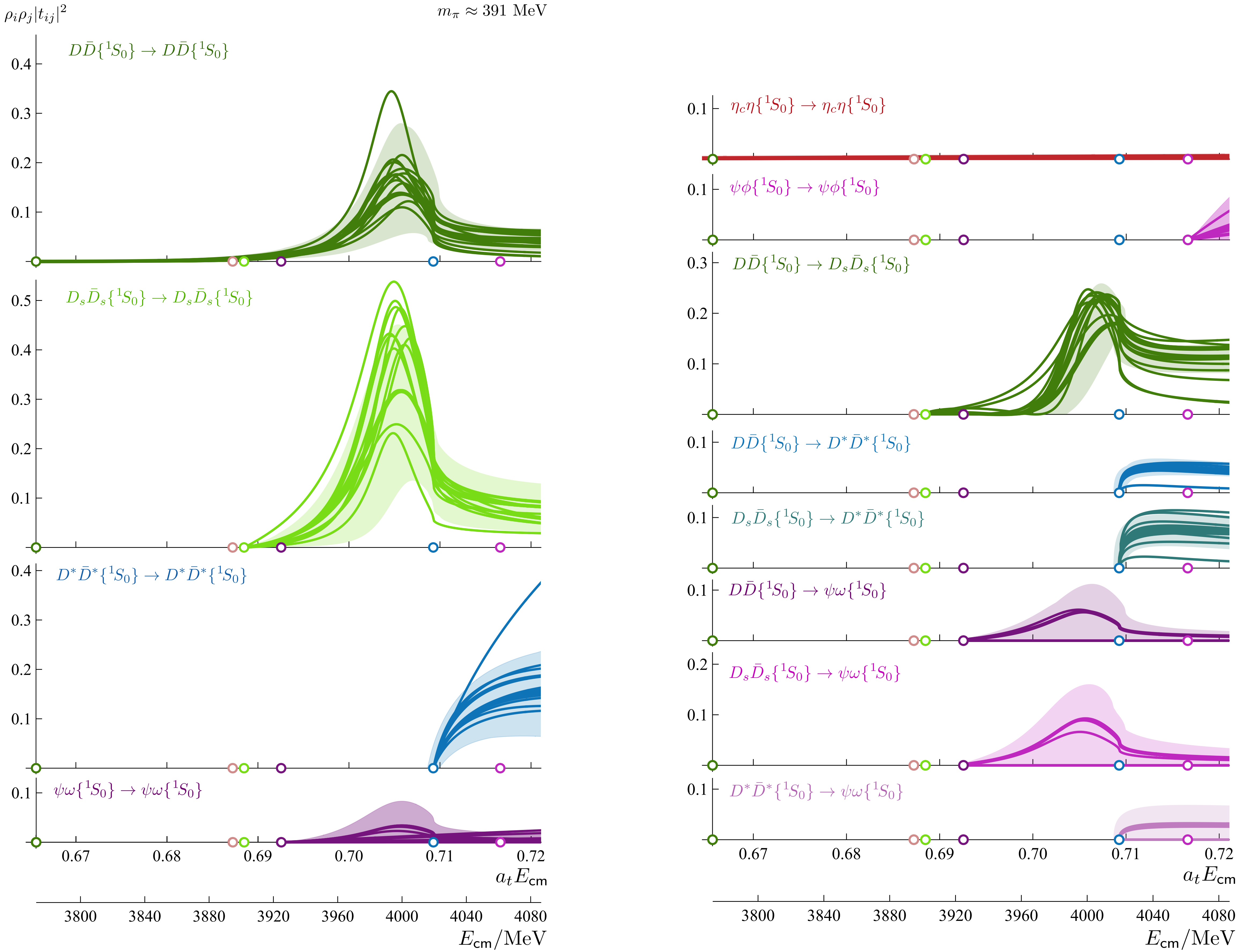


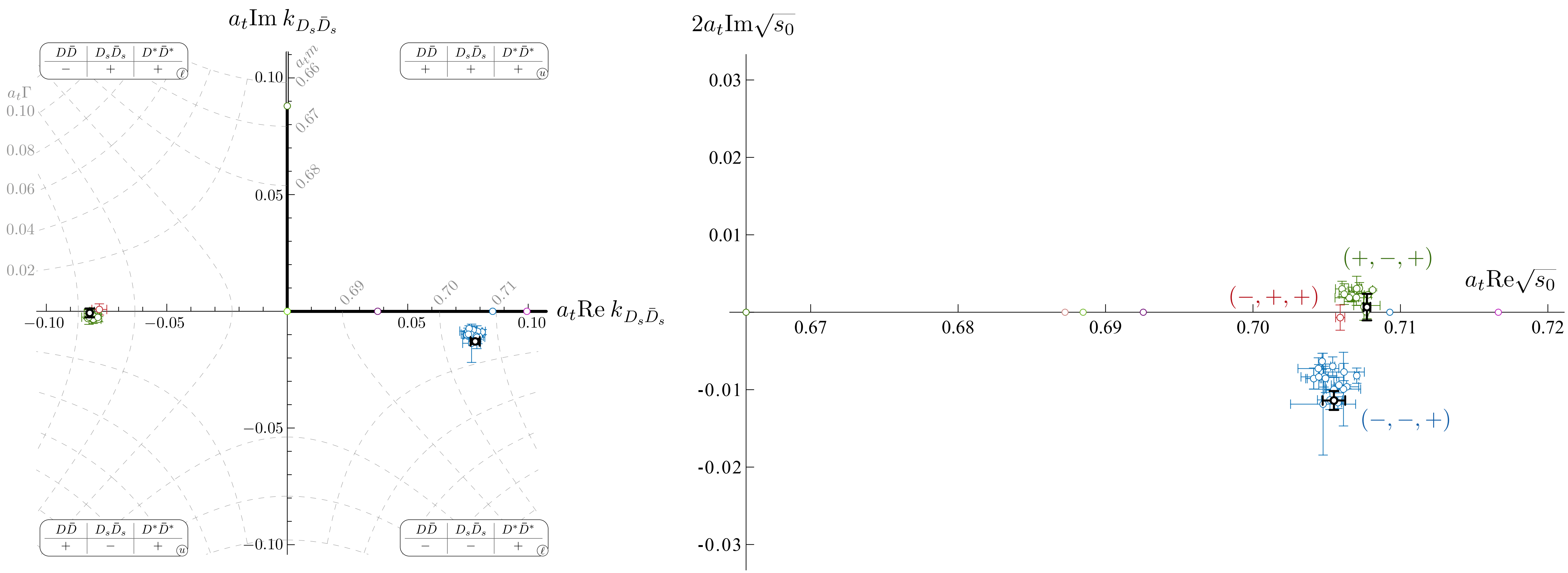
extra level and resonance higher up

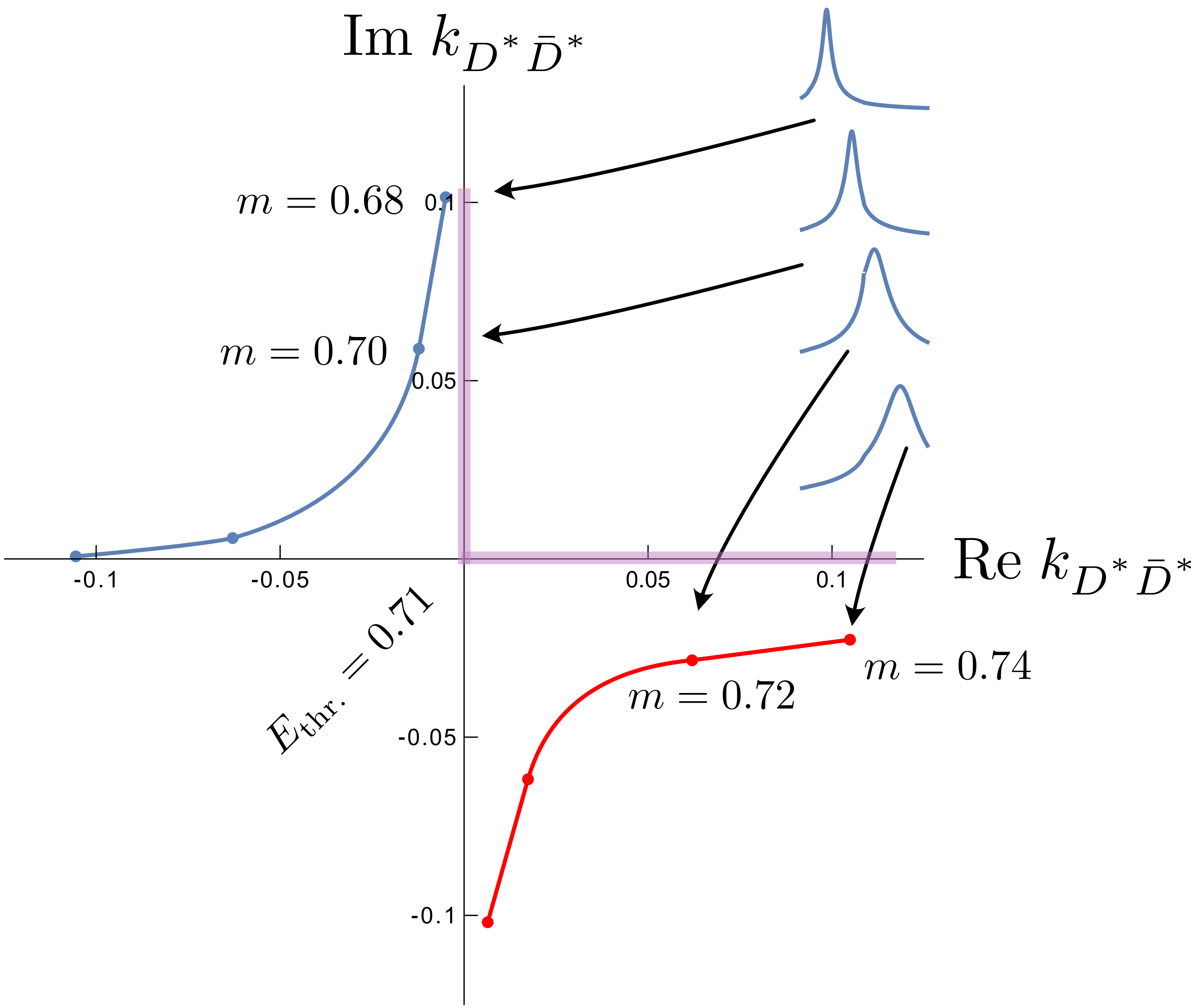
two classes of amplitudes were found:

- zero D*D* coupling
- finite D*D* coupling
- all had significant DD* coupling
- amps very small below 4050 MeV ($a_t E_{cm}$=0.715)

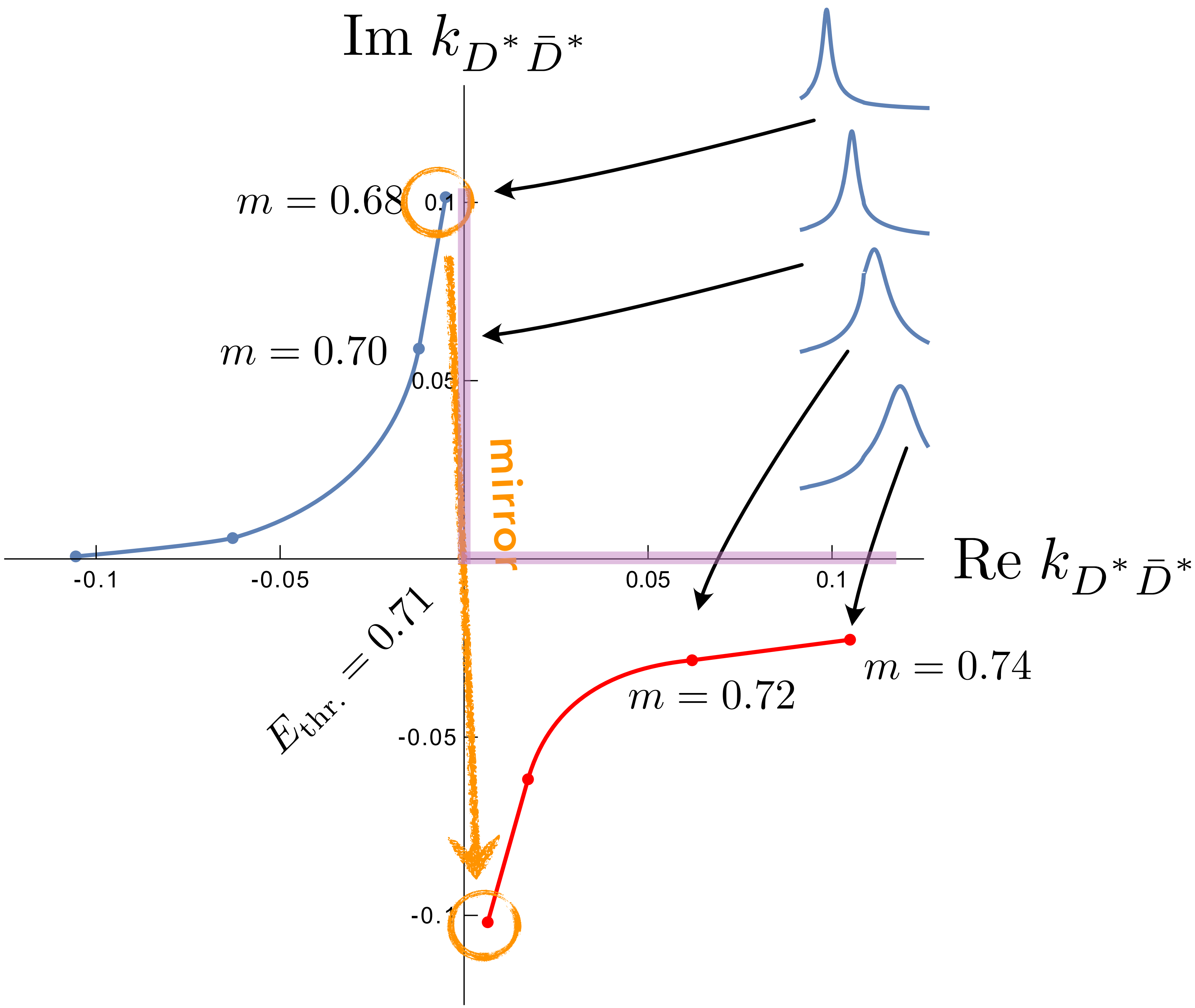



$\rho_i \rho_j |t_{ij}|^2$
$m_\pi \approx 391$ MeV
$D\bar{D}\{^1S_0\} \to D\bar{D}\{^1S_0\}$
$D_s\bar{D}_s\{^1S_0\} \to D_s\bar{D}_s\{^1S_0\}$
$D^*\bar{D}^*\{^1S_0\} \to D^*\bar{D}^*\{^1S_0\}$
$\psi\omega\{^1S_0\} \to \psi\omega\{^1S_0\}$
$\eta_c\eta\{^1S_0\} \to \eta_c\eta\{^1S_0\}$
$\psi\phi\{^1S_0\} \to \psi\phi\{^1S_0\}$
$D\bar{D}\{^1S_0\} \to D_s\bar{D}_s\{^1S_0\}$
$D\bar{D}\{^1S_0\} \to D^*\bar{D}^*\{^1S_0\}$
$D_s\bar{D}_s\{^1S_0\} \to D^*\bar{D}^*\{^1S_0\}$
$D\bar{D}\{^1S_0\} \to \psi\omega\{^1S_0\}$
$D_s\bar{D}_s\{^1S_0\} \to \psi\omega\{^1S_0\}$
$D^*\bar{D}^*\{^1S_0\} \to \psi\omega\{^1S_0\}$
0.67 0.68 0.69 0.70 0.71 0.72
$a_t E_{\rm cm}$
3800 3840 3880 3920 3960 4000 4040 4080
$E_{\rm cm}$/MeV

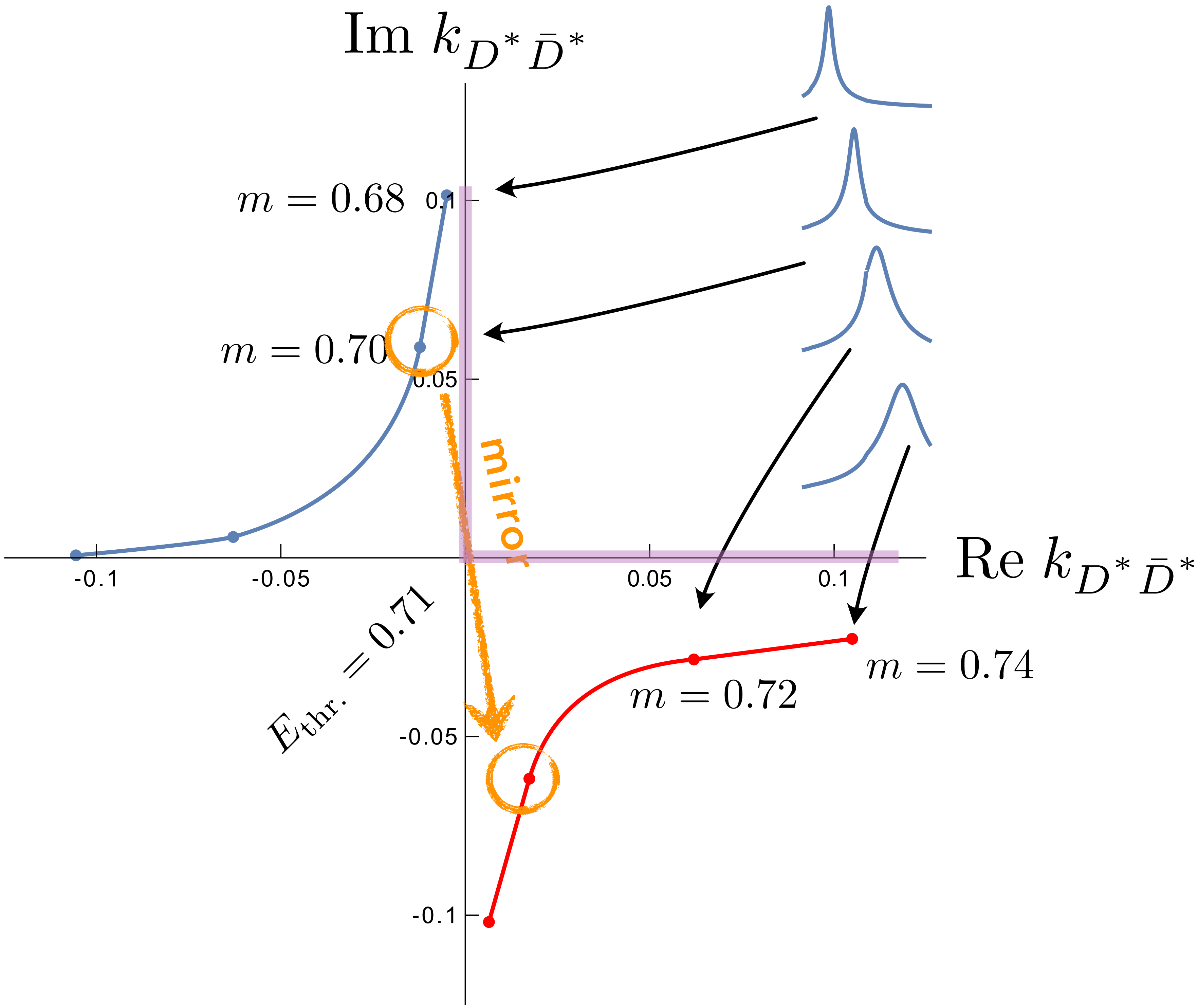

$a_t \mathrm{Im}\, k_{D_s\bar{D}_s}$
$a_t \mathrm{Re}\, k_{D_s\bar{D}_s}$
$2a_t \mathrm{Im}\sqrt{s_0}$
$a_t \mathrm{Re}\sqrt{s_0}$
$(+,-,+)$
$(-,+,+)$
$(-,-,+)$



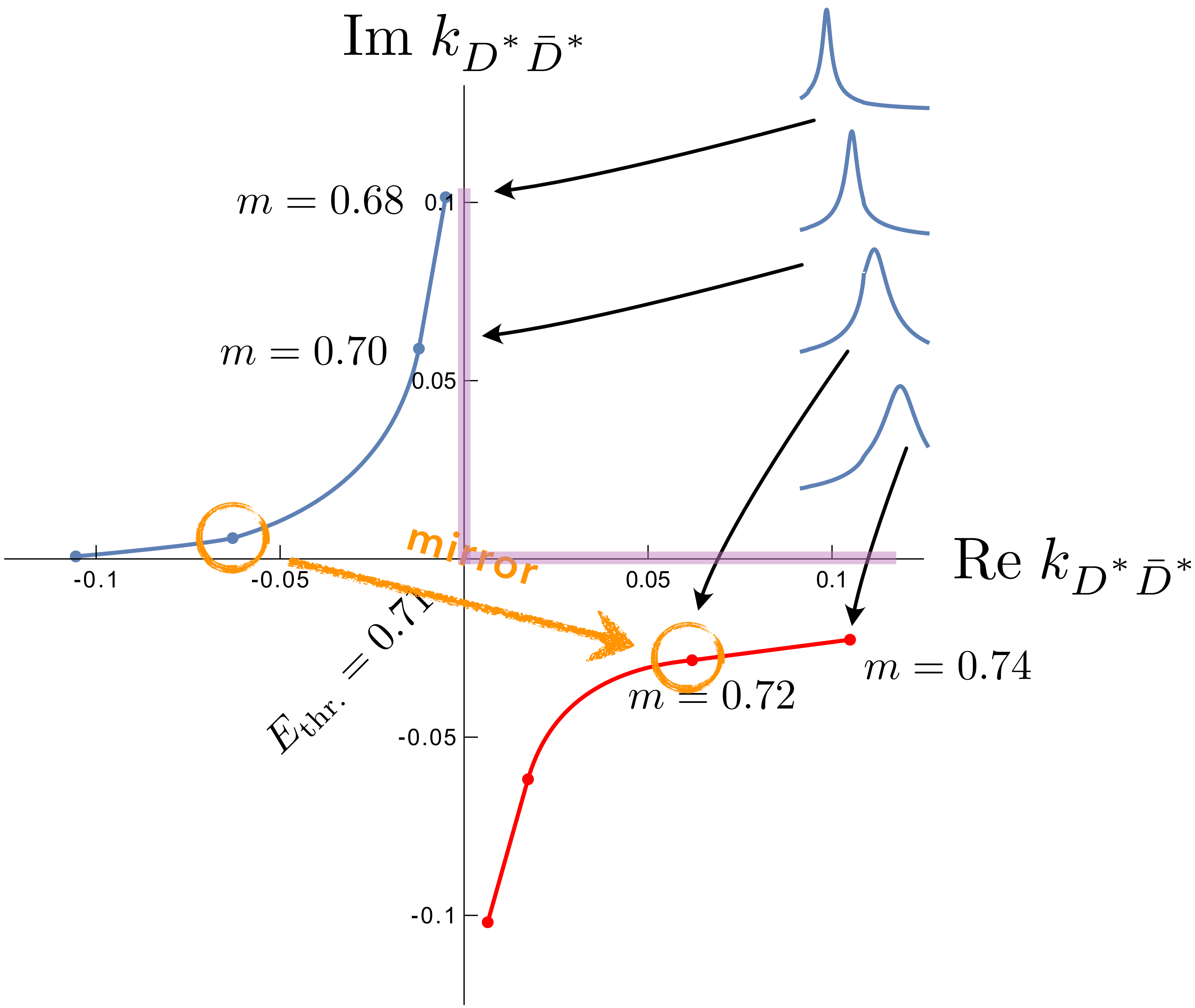
$$t_{ij} = \frac{g_i g_j}{m_0^2 - s - i g_{D\bar{D}}^2 \rho_{D\bar{D}} - i g_{D^*\bar{D}^*}^2 \rho_{D^*\bar{D}^*}}$$



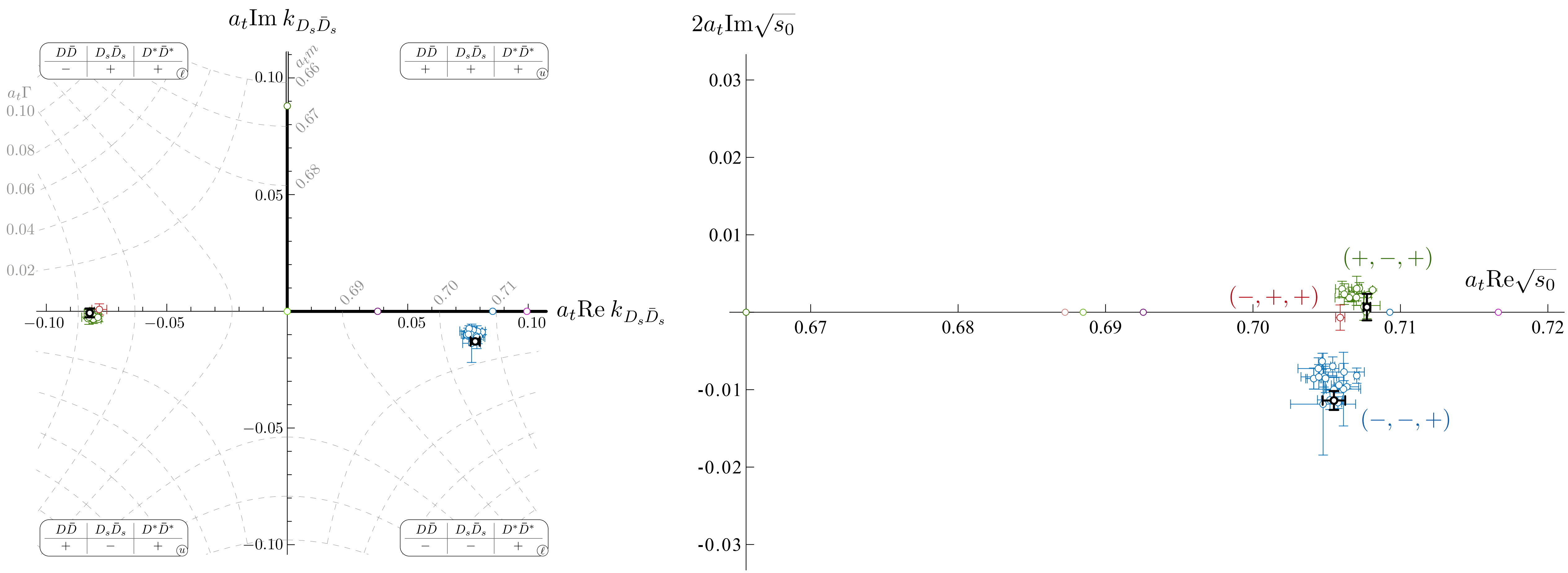
$$t_{ij} = \frac{g_i g_j}{m_0^2 - s - i g^2_{D\bar{D}} \rho_{D\bar{D}} - i g^2_{D^*\bar{D}^*} \rho_{D^*\bar{D}^*}}$$



$$t_{ij} = \frac{g_i g_j}{m_0^2 - s - i g_{D\bar{D}}^2 \rho_{D\bar{D}} - i g_{D^*\bar{D}^*}^2 \rho_{D^*\bar{D}^*}}$$

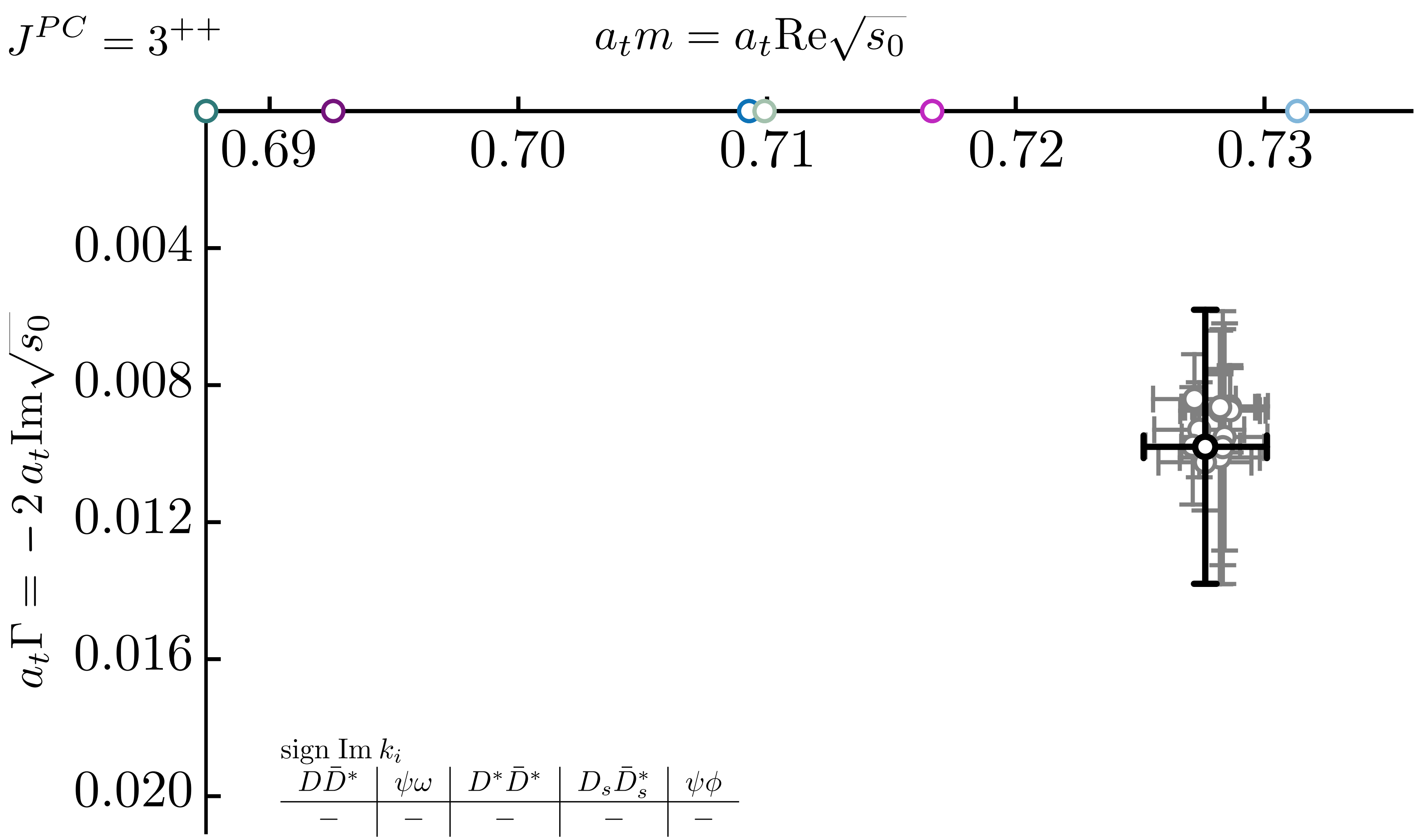


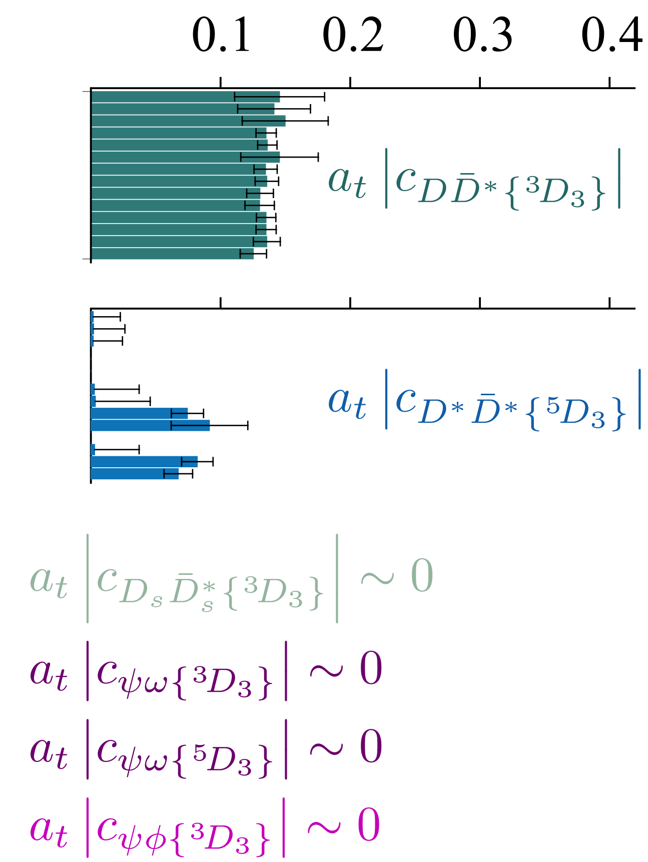
$$t_{ij} = \frac{g_i g_j}{m_0^2 - s - i g_{D\bar{D}}^2 \rho_{D\bar{D}} - i g_{D^*\bar{D}^*}^2 \rho_{D^*\bar{D}^*}}$$

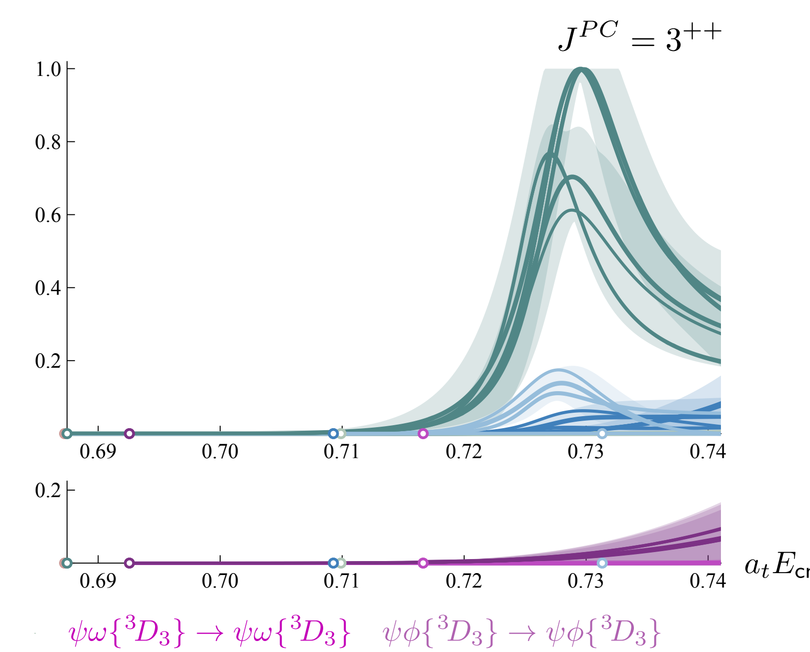


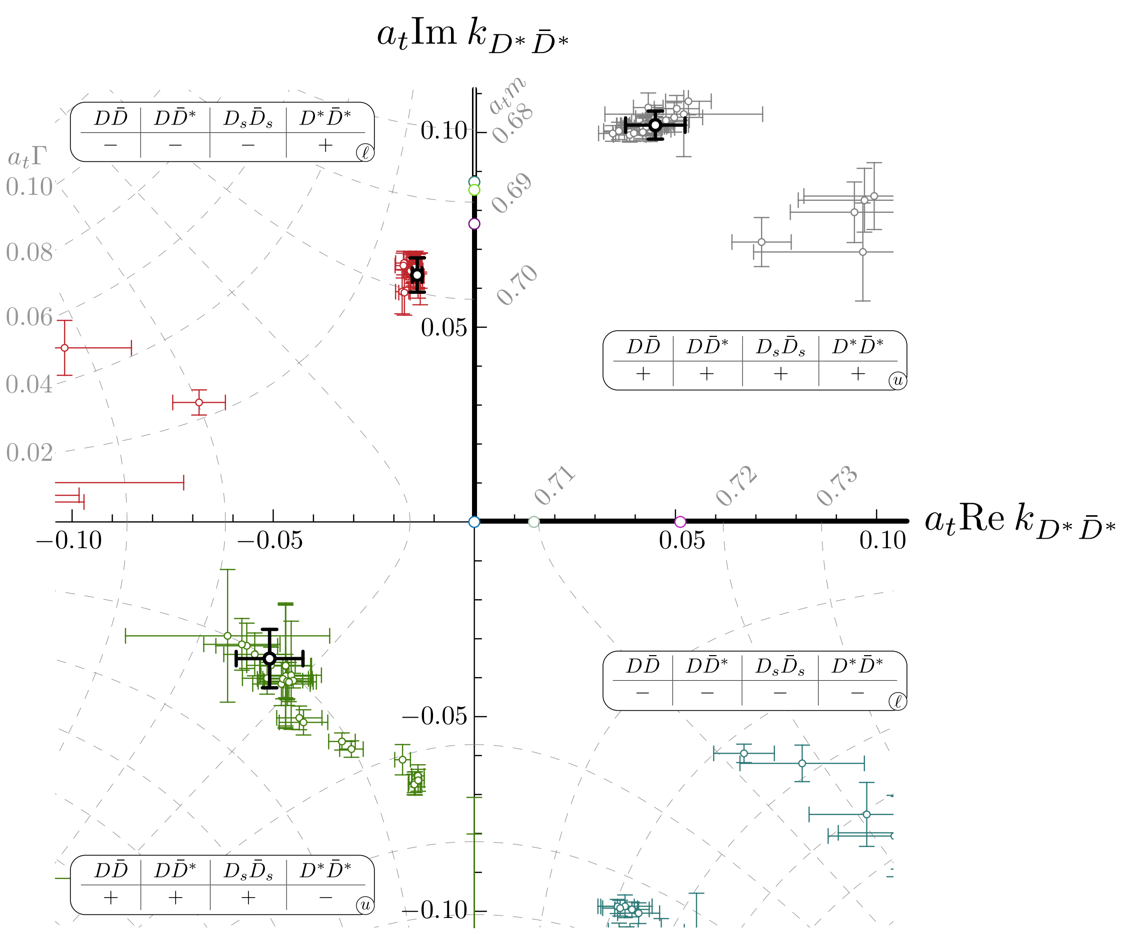
the green/red cluster of poles are just mirror poles

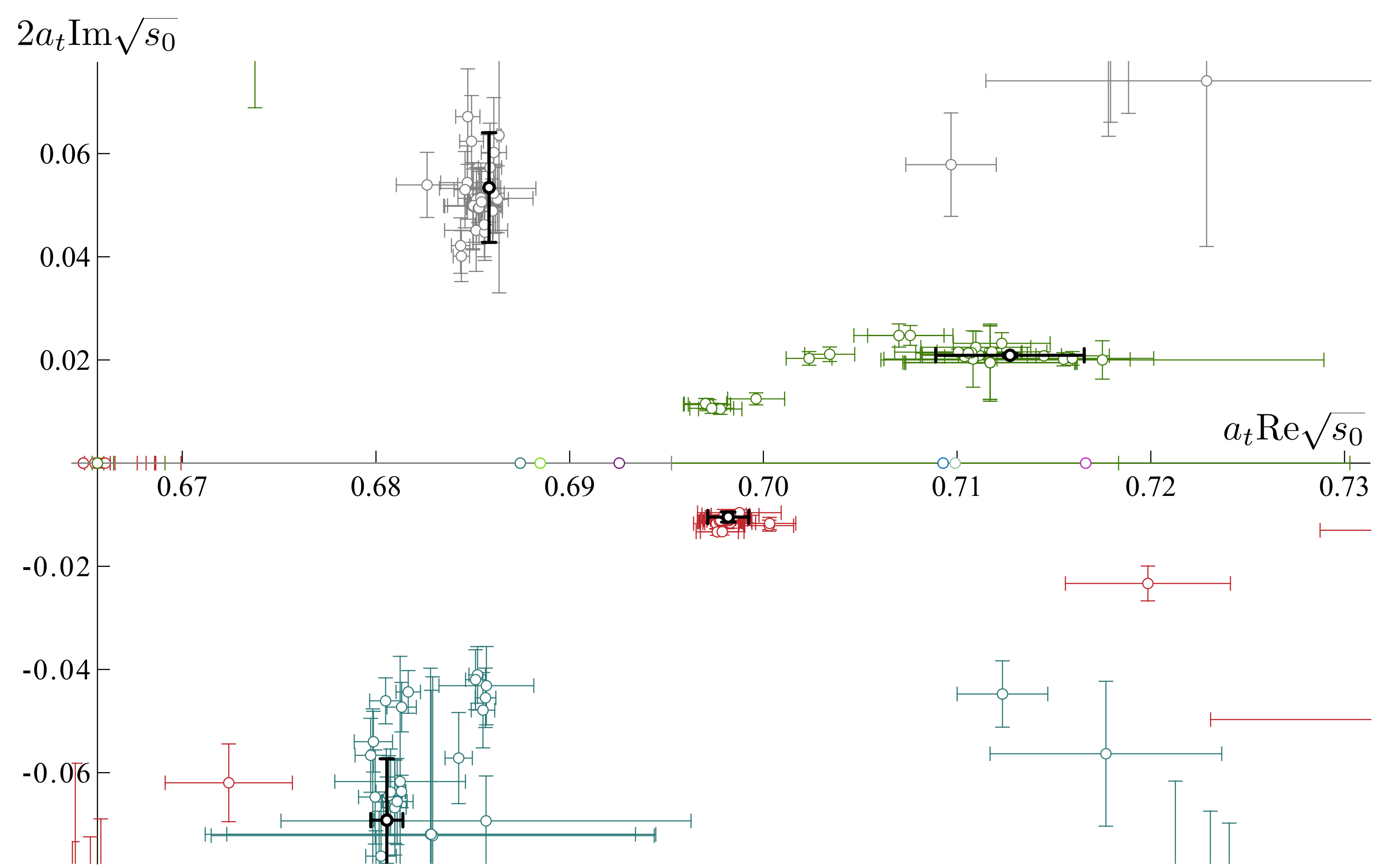
- amplitude is **dominated by a single resonance pole** in this energy region

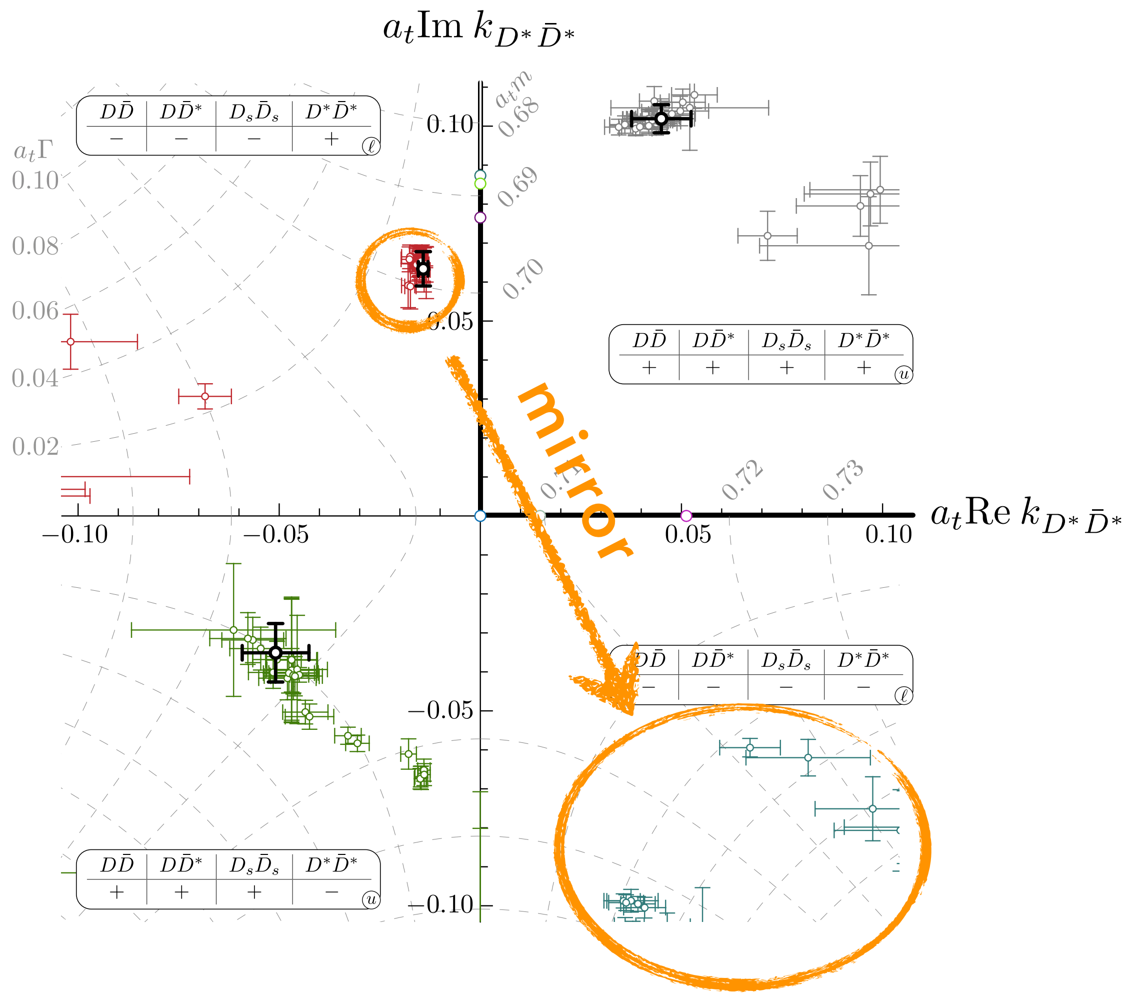

$J^{PC} = 3^{++}$
$a_t m = a_t \mathrm{Re}\sqrt{s_0}$
0.69
0.70
0.71
0.72
0.73
$a_t\Gamma = -2\, a_t \mathrm{Im}\sqrt{s_0}$
0.004
0.008
0.012
0.016
0.020
sign Im k_i
$D\bar{D}^*$ | $\psi\omega$ | $D^*\bar{D}^*$ | $D_s\bar{D}_s^*$ | $\psi\phi$
– | – | – | – | –

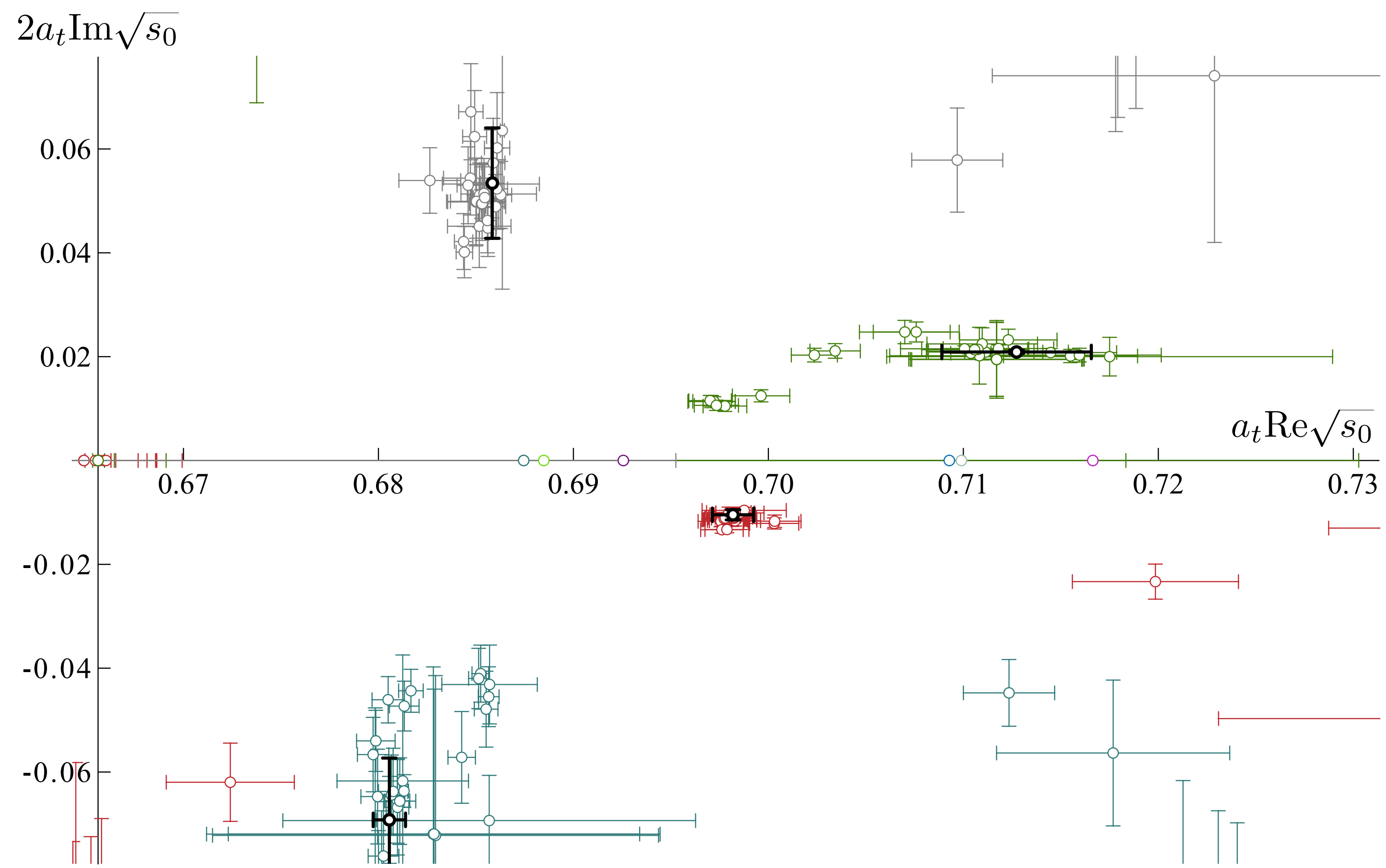

0.1
0.2
0.3
0.4
$a_t \left|c_{D\bar{D}^*\{^3D_3\}}\right|$
$a_t \left|c_{D^*\bar{D}^*\{^5D_3\}}\right|$
$a_t \left|c_{D_s\bar{D}_s^*\{^3D_3\}}\right| \sim 0$
$a_t \left|c_{\psi\omega\{^3D_3\}}\right| \sim 0$
$a_t \left|c_{\psi\omega\{^5D_3\}}\right| \sim 0$
$a_t \left|c_{\psi\phi\{^3D_3\}}\right| \sim 0$


$J^{PC} = 3^{++}$
1.0
0.8
0.6
0.4
0.2
0.69
0.70
0.71
0.72
0.73
0.74
$a_t E_{\mathsf{cm}}$
$\psi\omega\{^3D_3\} \to \psi\omega\{^3D_3\}$
$\psi\phi\{^3D_3\} \to \psi\phi\{^3D_3\}$


$a_t \mathrm{Im}\, k_{D^*\bar{D}^*}$
$a_t \mathrm{Re}\, k_{D^*\bar{D}^*}$
$a_t\Gamma$
$a_t m$
0.10
0.08
0.06
0.04
0.02
0.05
-0.05
-0.10
0.68
0.69
0.70
0.71
0.72
0.73
$D\bar{D}$ $D\bar{D}^*$ $D_s\bar{D}_s$ $D^*\bar{D}^*$

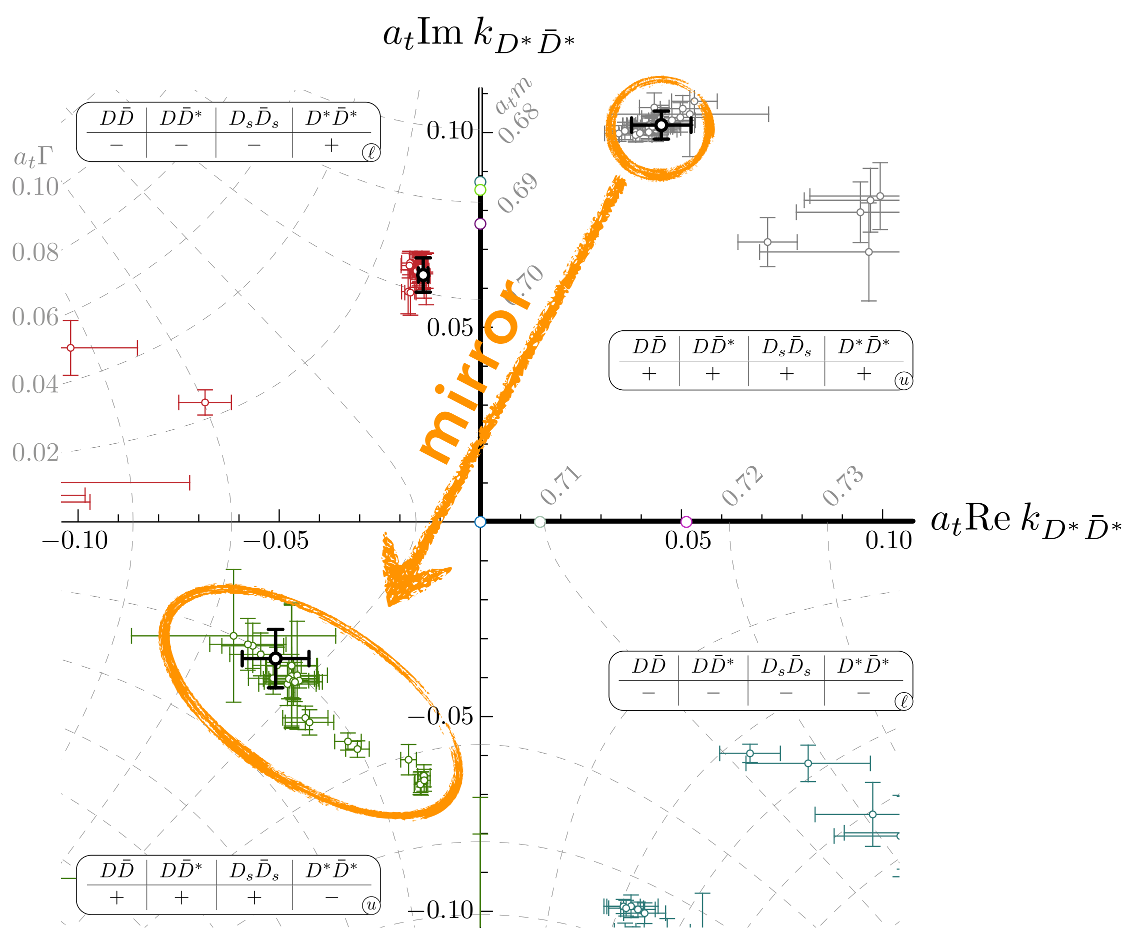

$2a_t \mathrm{Im}\sqrt{s_0}$
$a_t \mathrm{Re}\sqrt{s_0}$
0.06
0.04
0.02
-0.02
-0.04
-0.06
0.67
0.68
0.69
0.70
0.71
0.72
0.73

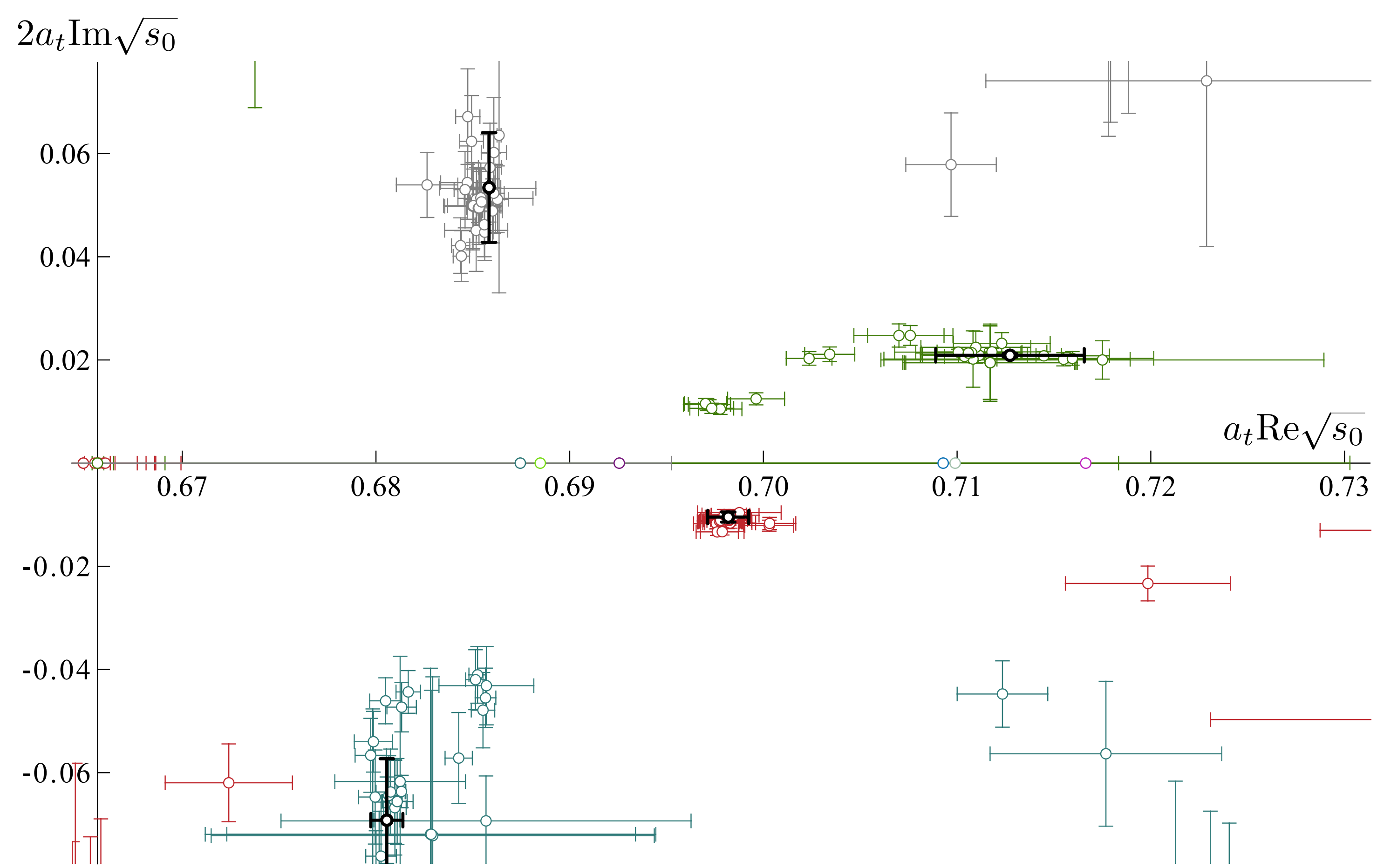




mirror pole - similar to a Flatté

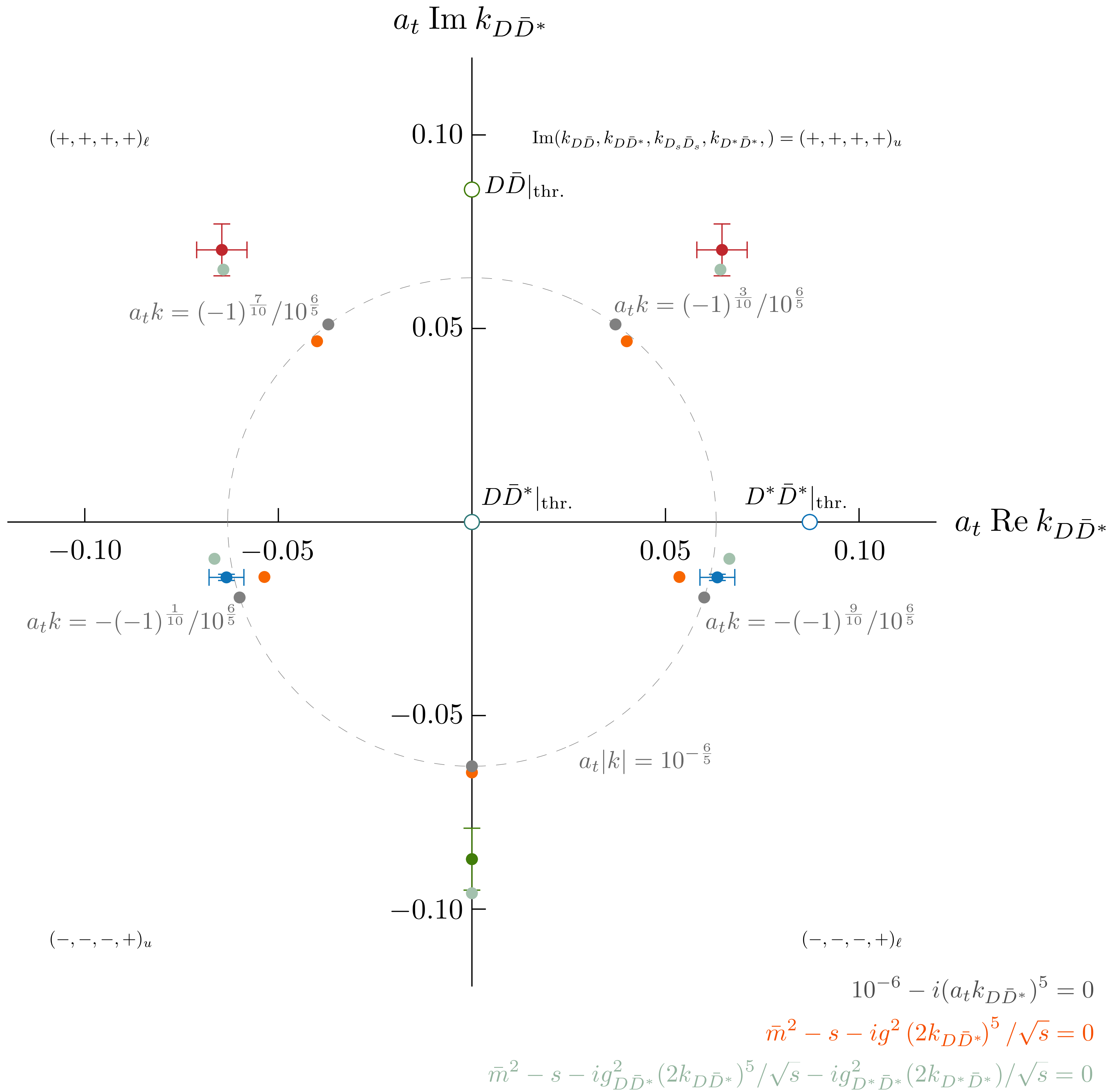
poles - tensor - more going on here, but mostly not physics



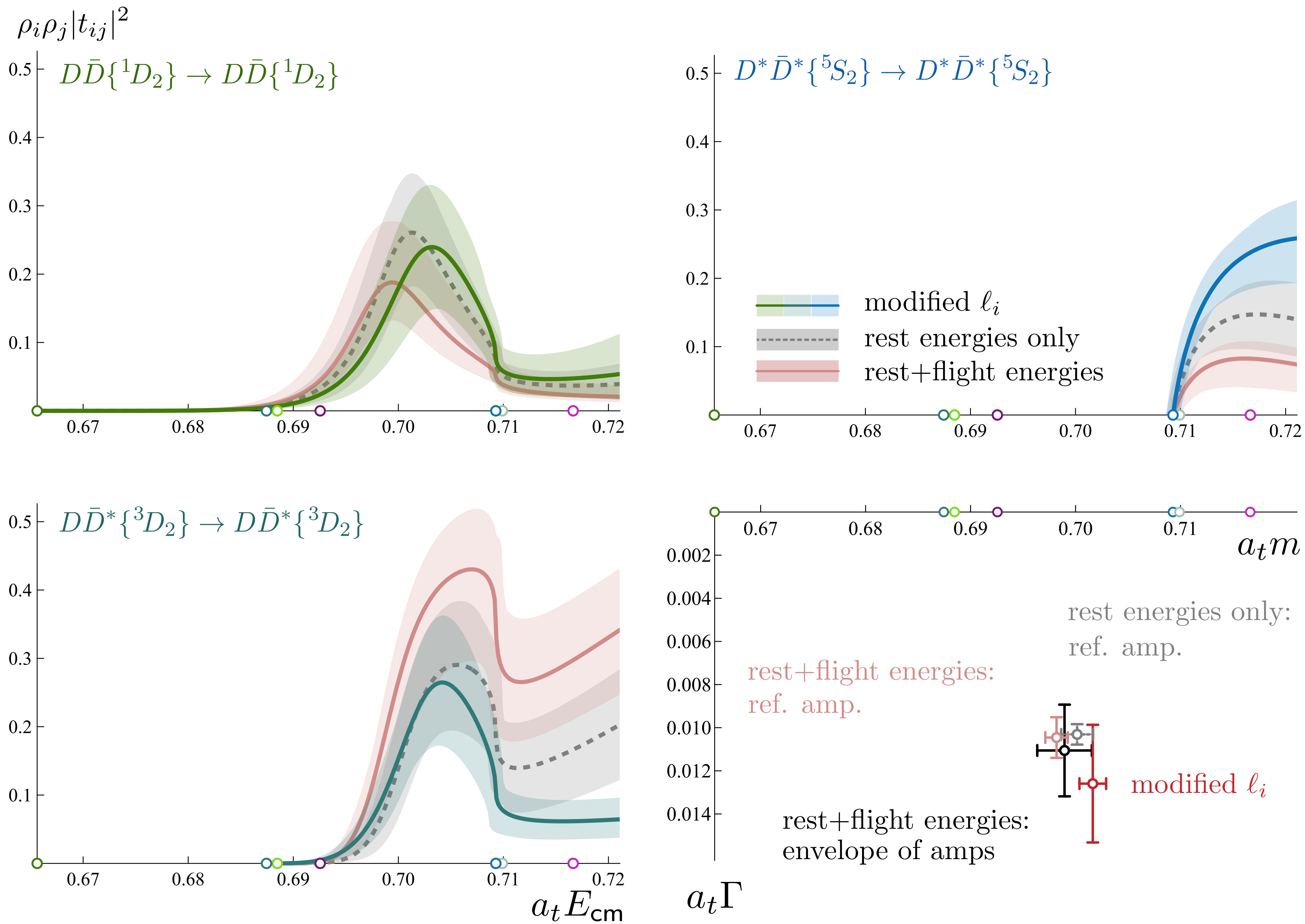


“green” pole is a mirror of the physical sheet pole

physical sheet pole arises because of the large g_{DD^*}


$a_t \,\mathrm{Im}\, k_{D\bar{D}^*}$
$a_t \,\mathrm{Re}\, k_{D\bar{D}^*}$
0.10
0.05
−0.05
−0.10
−0.10
−0.05
0.05
0.10
$(+,+,+,+)_\ell$
$\mathrm{Im}(k_{D\bar{D}}, k_{D\bar{D}^*}, k_{D_s\bar{D}_s}, k_{D^*\bar{D}^*},) = (+,+,+,+)_u$
$D\bar{D}|_{\mathrm{thr.}}$
$D\bar{D}^*|_{\mathrm{thr.}}$
$D^*\bar{D}^*|_{\mathrm{thr.}}$
$a_t k = (-1)^{\frac{7}{10}}/10^{\frac{6}{5}}$
$a_t k = (-1)^{\frac{3}{10}}/10^{\frac{6}{5}}$
$a_t k = -(-1)^{\frac{1}{10}}/10^{\frac{6}{5}}$
$a_t k = -(-1)^{\frac{9}{10}}/10^{\frac{6}{5}}$
$a_t|k| = 10^{-\frac{6}{5}}$
$(-,-,-,+)_u$
$(-,-,-,+)_\ell$
$10^{-6} - i(a_t k_{D\bar{D}^*})^5 = 0$
$\bar{m}^2 - s - ig^2\left(2k_{D\bar{D}^*}\right)^5/\sqrt{s} = 0$
$\bar{m}^2 - s - ig^2_{D\bar{D}^*}(2k_{D\bar{D}^*})^5/\sqrt{s} - ig^2_{D^*\bar{D}^*}(2k_{D^*\bar{D}^*})/\sqrt{s} = 0$

using a modified DDbar* threshold factor



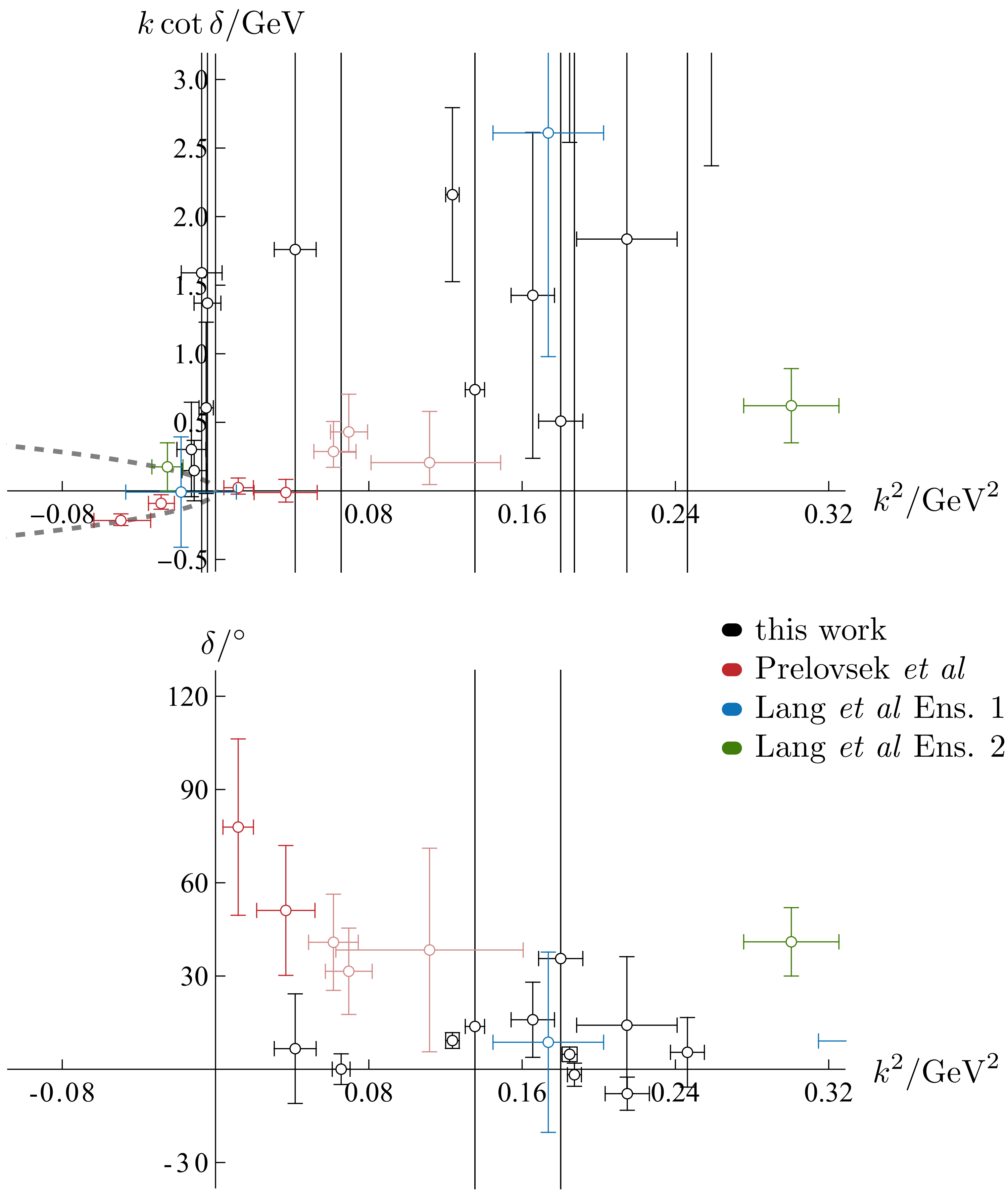
- different physical sheet pole
- no obvious nearby (+,+,+,-) sheet pole (there are some with a_t E >0.74)

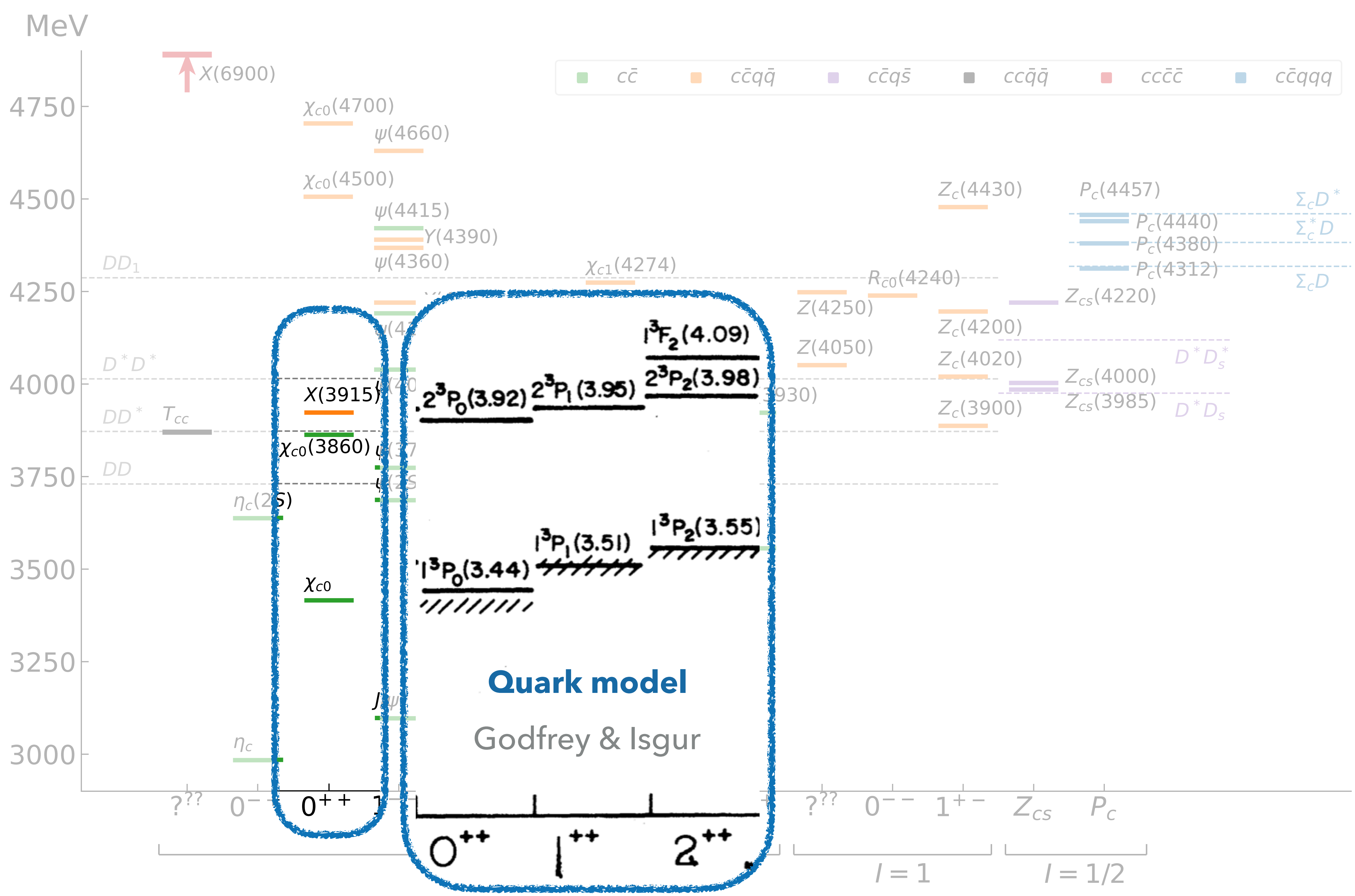
Results from Prelovsek, Padmanath et al, suggest effects at DDbar and DsDsbar thresholds

- pion mass ~ 280 MeV

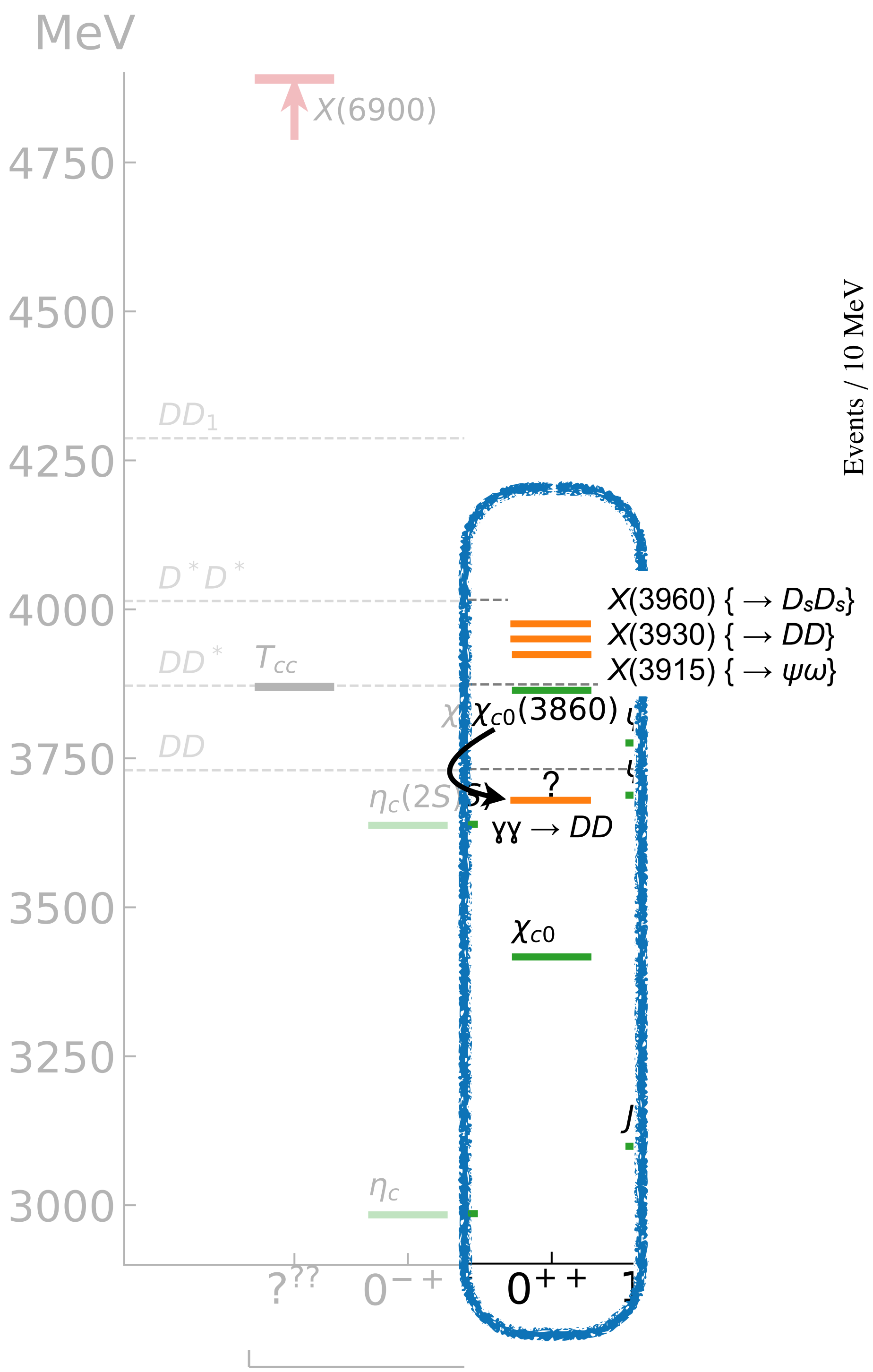
- light quark heavier than physical, strange quark lighter than physical

hard to justify such a large change due to the light quark mass (no one-pion-exchange term)

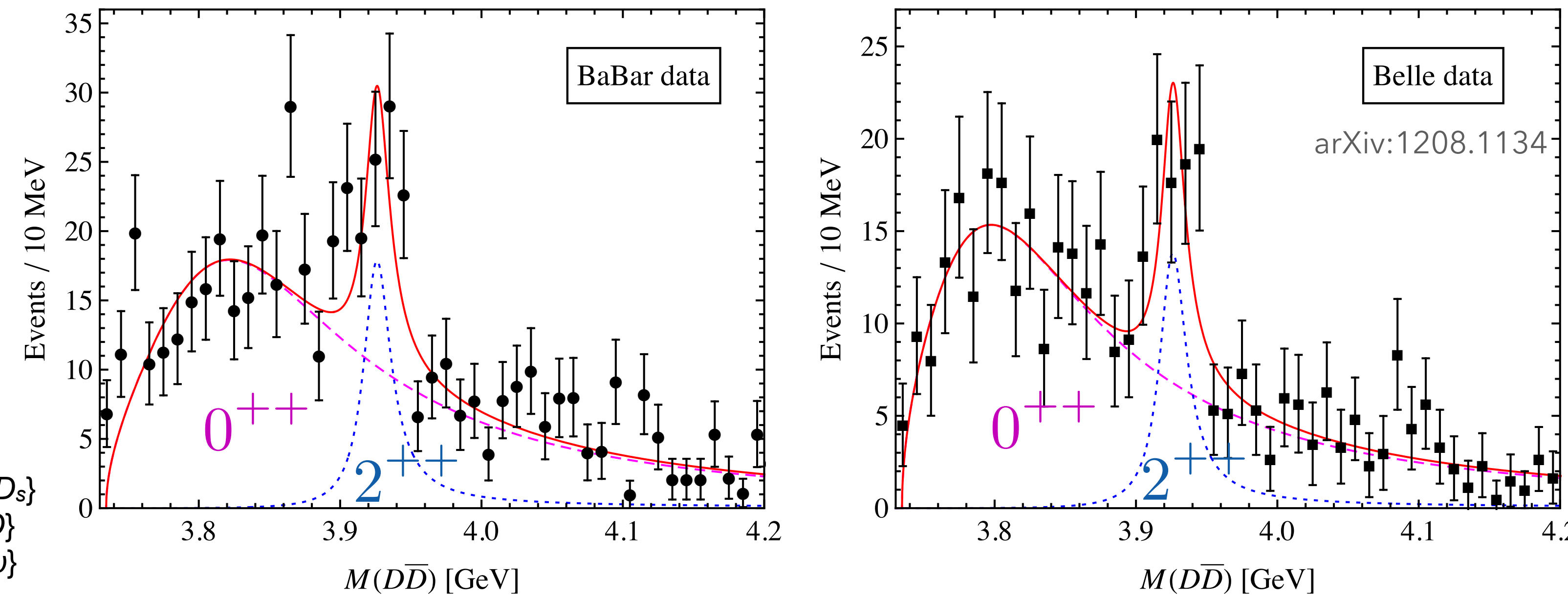




JPAC arXiv:2112.13436

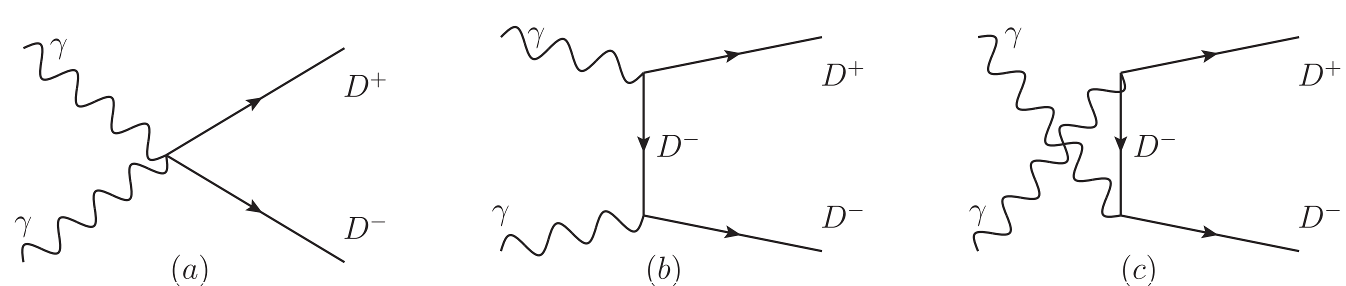


- BaBar, Belle - resonance around 3860 MeV $\gamma\gamma \to D\bar{D}$



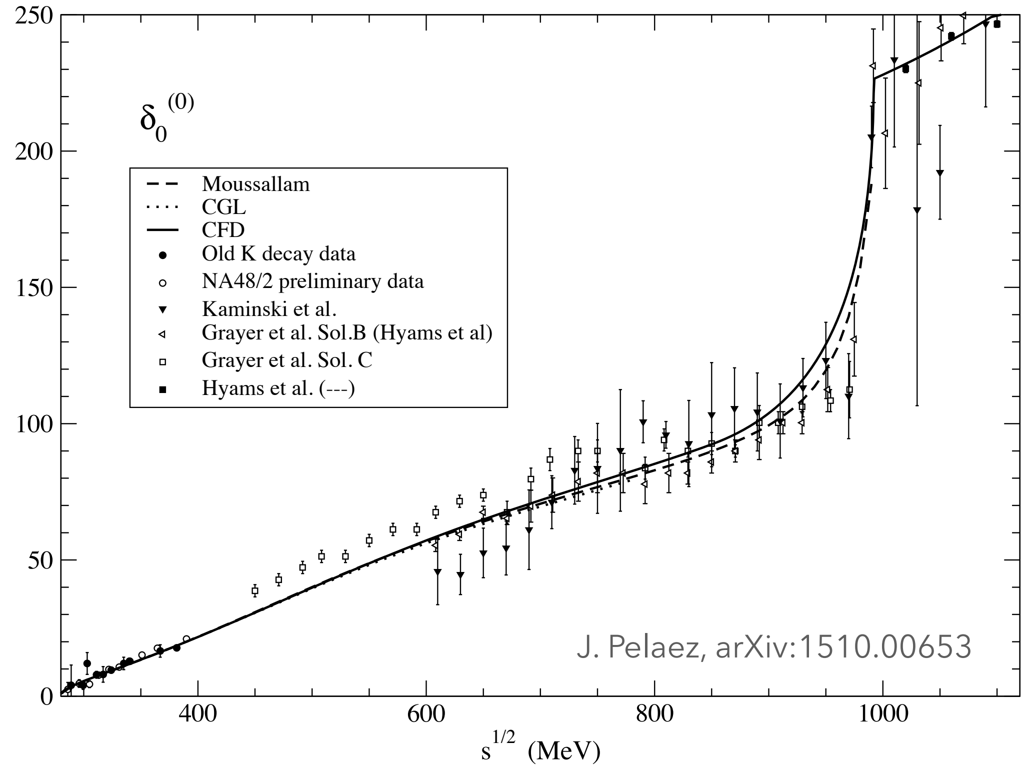
- Guo & Meissner (2012)
  m = 3840 MeV, Γ = 220 MeV

- Wang et al (2021), Daneika et al (2022):
  Complications from Born exchanges lead to a lower state around 3700 MeV

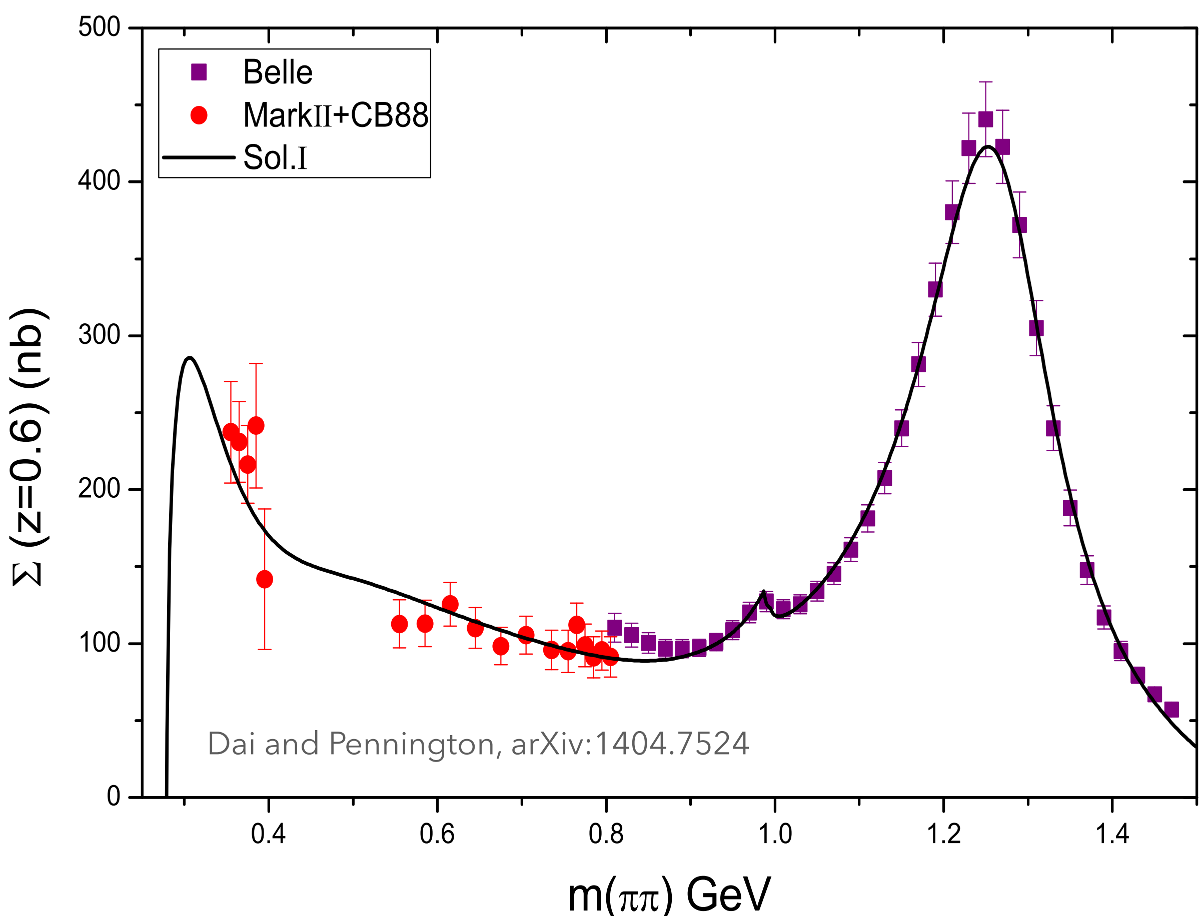


arXiv:2010.15431

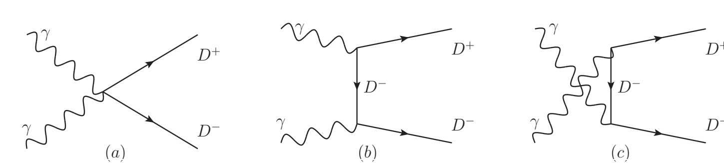
no state around 3840-3860 MeV (?)



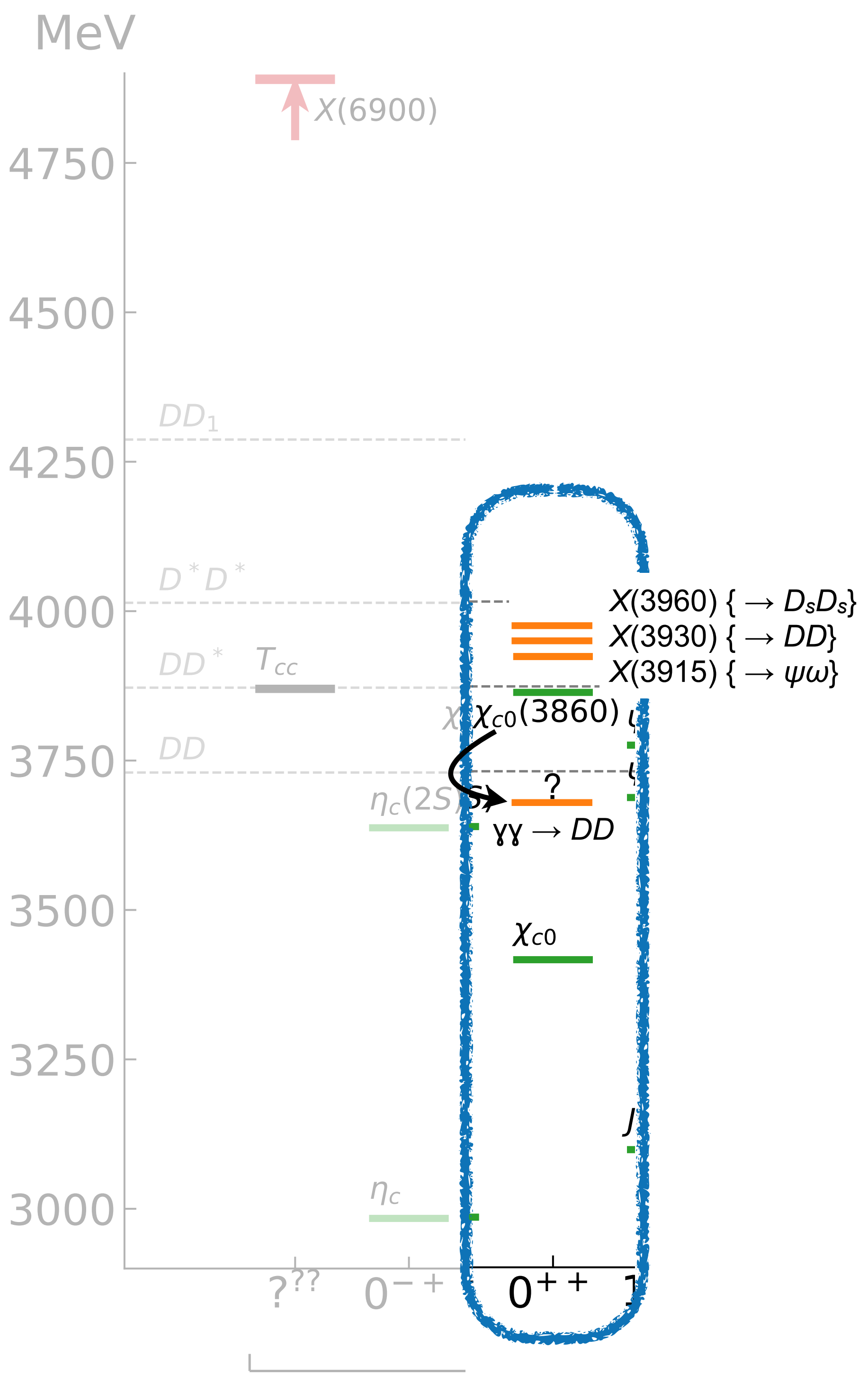
$$\pi\pi \to \pi\pi \quad (S - \text{wave})$$



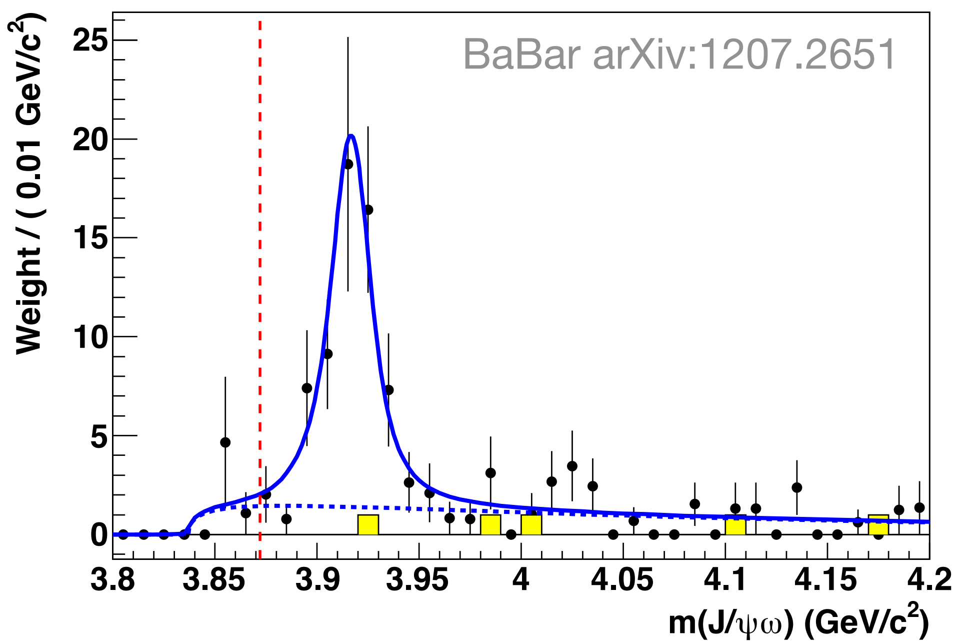
$$\gamma\gamma \to \pi\pi$$



extra structure at threshold, not linked to a resonance or bound state



- BaBar, Belle - resonance around 3915 MeV in $J/\psi\omega$

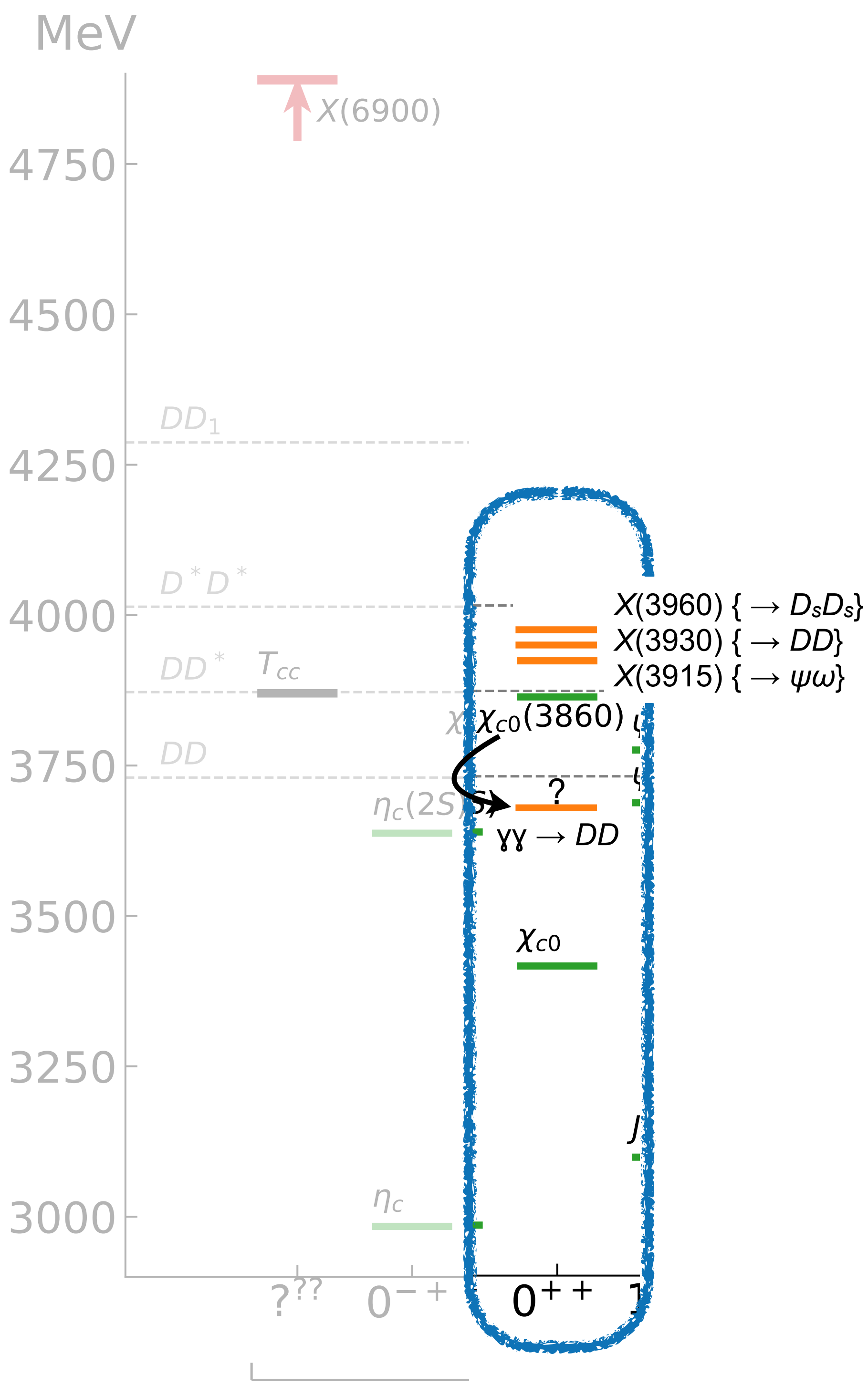


$m = (3919.4 \pm 2.2 \pm 1.6)$ MeV

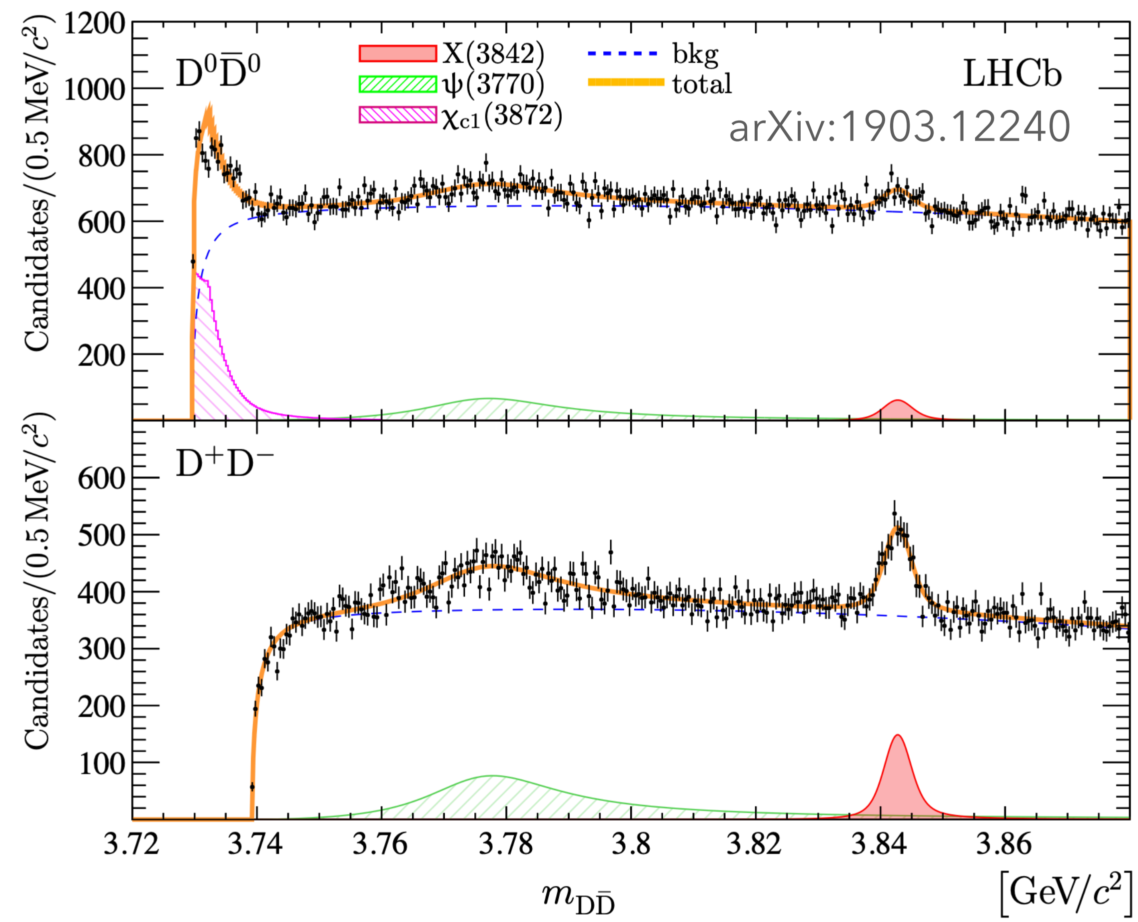
$\Gamma = (13 \pm 6 \pm 3)$ MeV

$J^P = 0^+$

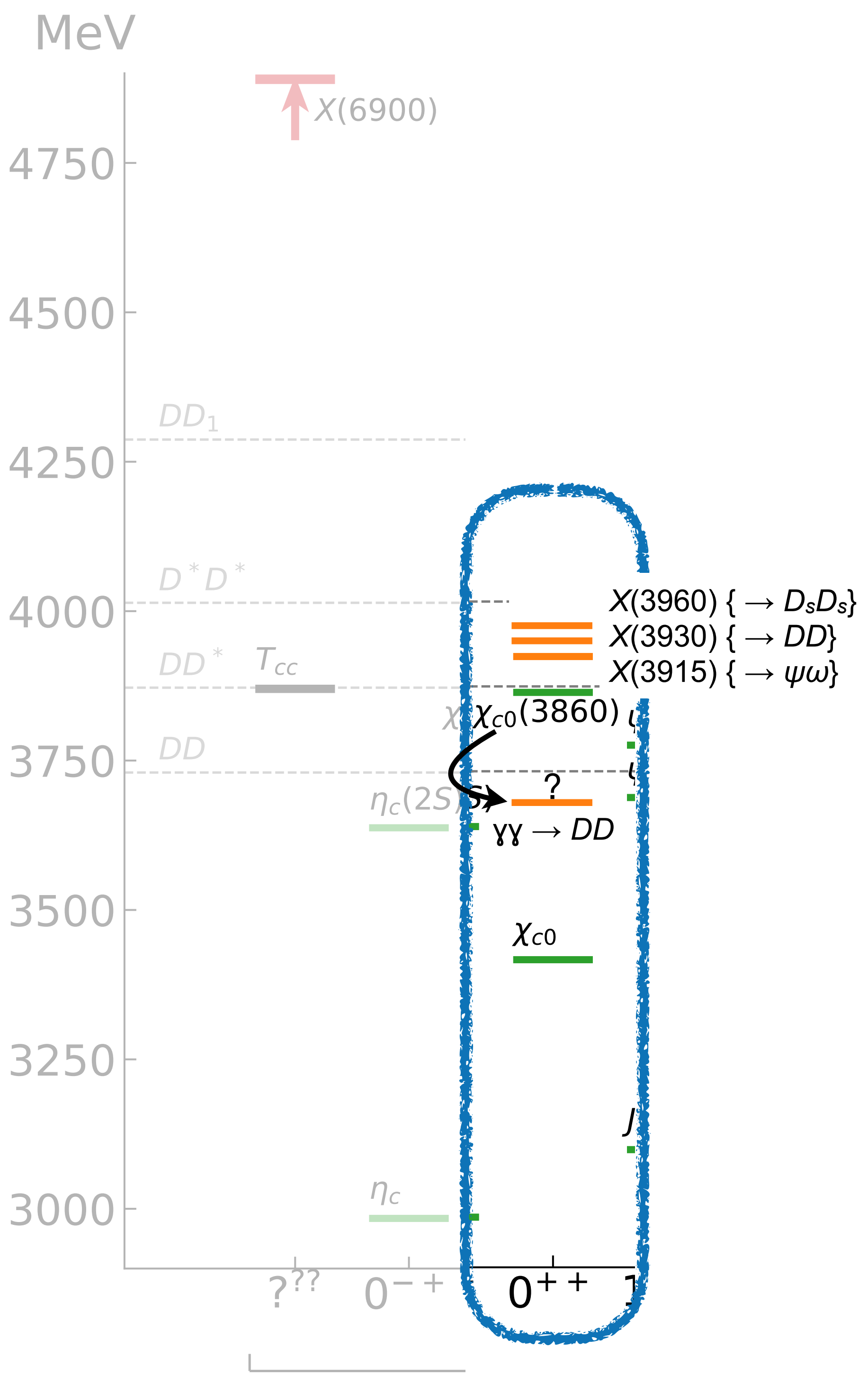
(several other studies of this)



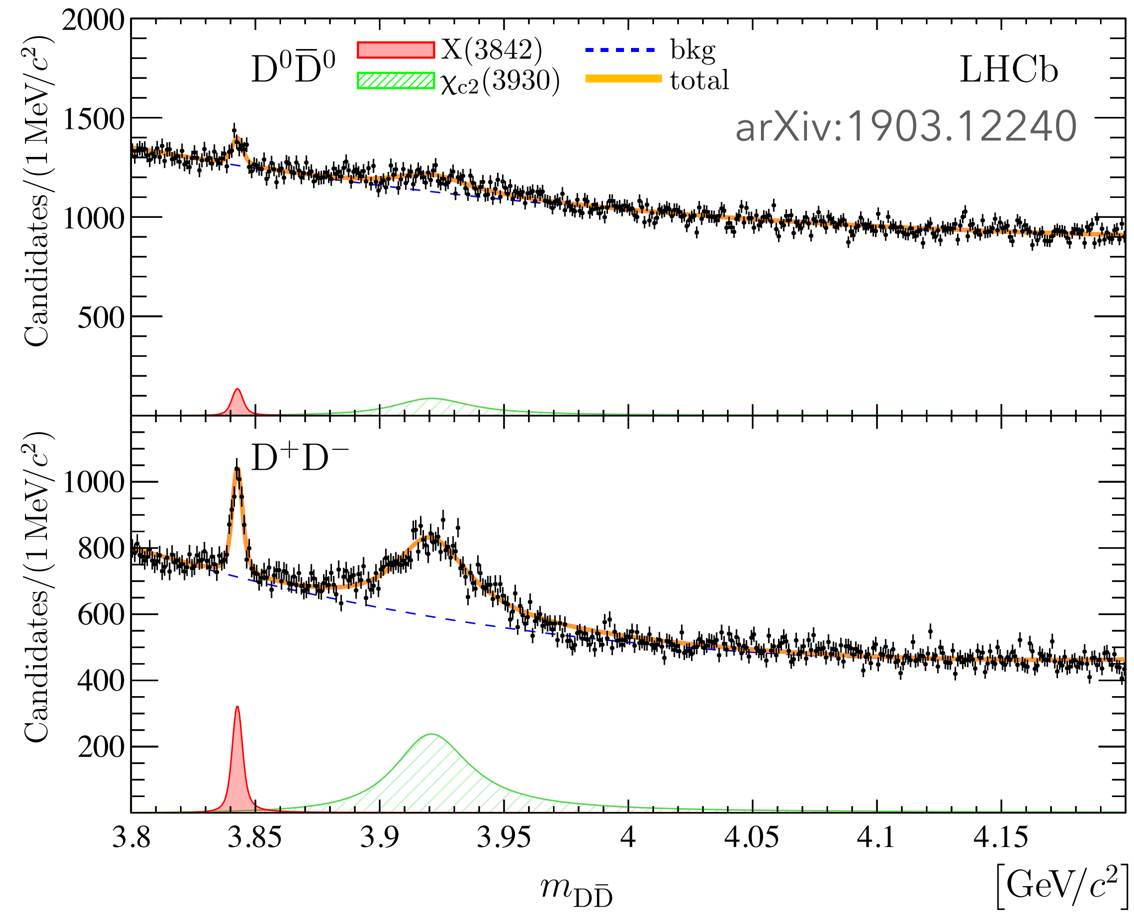
DDbar at LHCb
hadronic production process



Peak at DDbar threshold attributed to "feed-down" from X(3872) decays



Same study from LHCb, higher energies

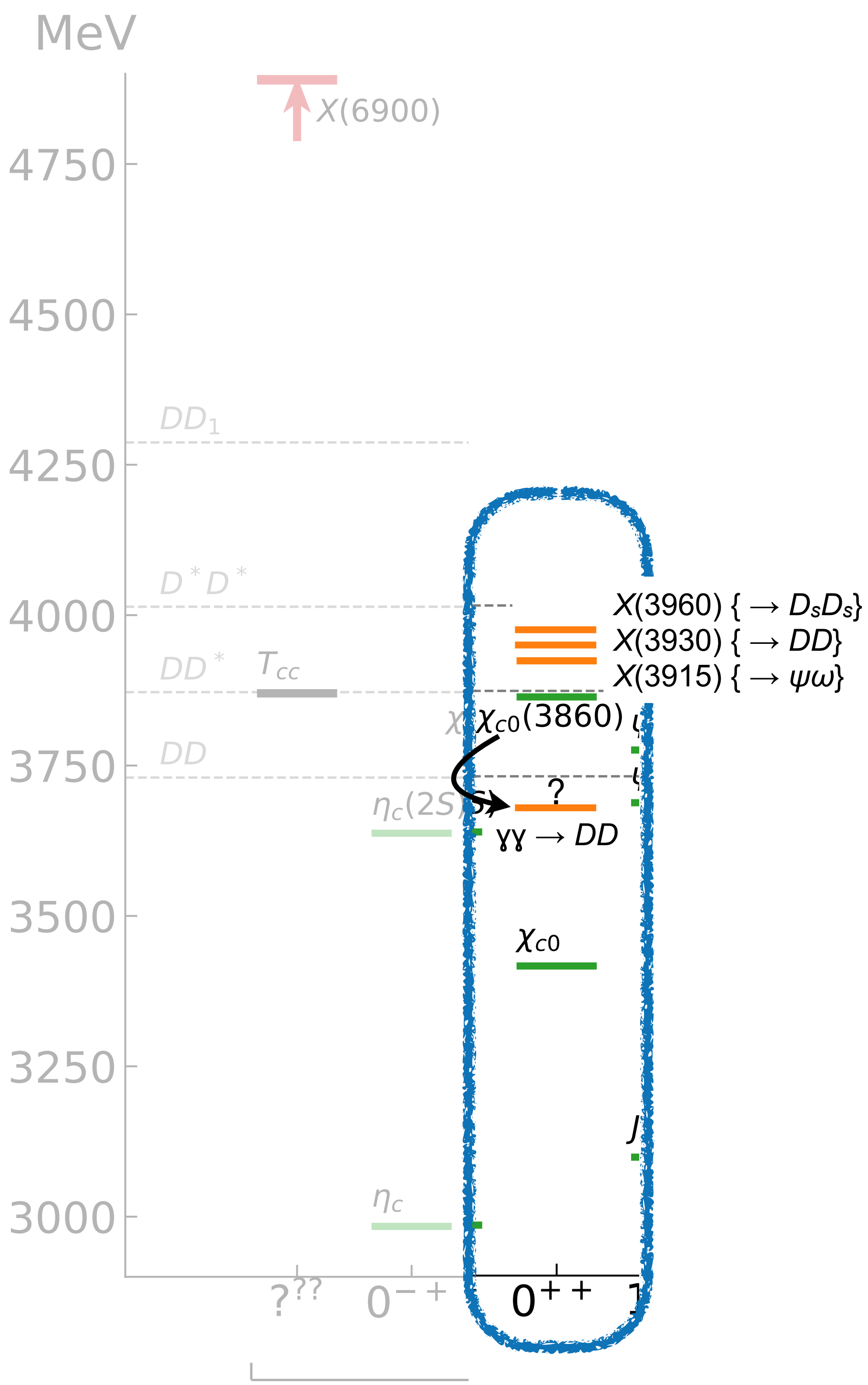


$\chi_{c2}(3930)$

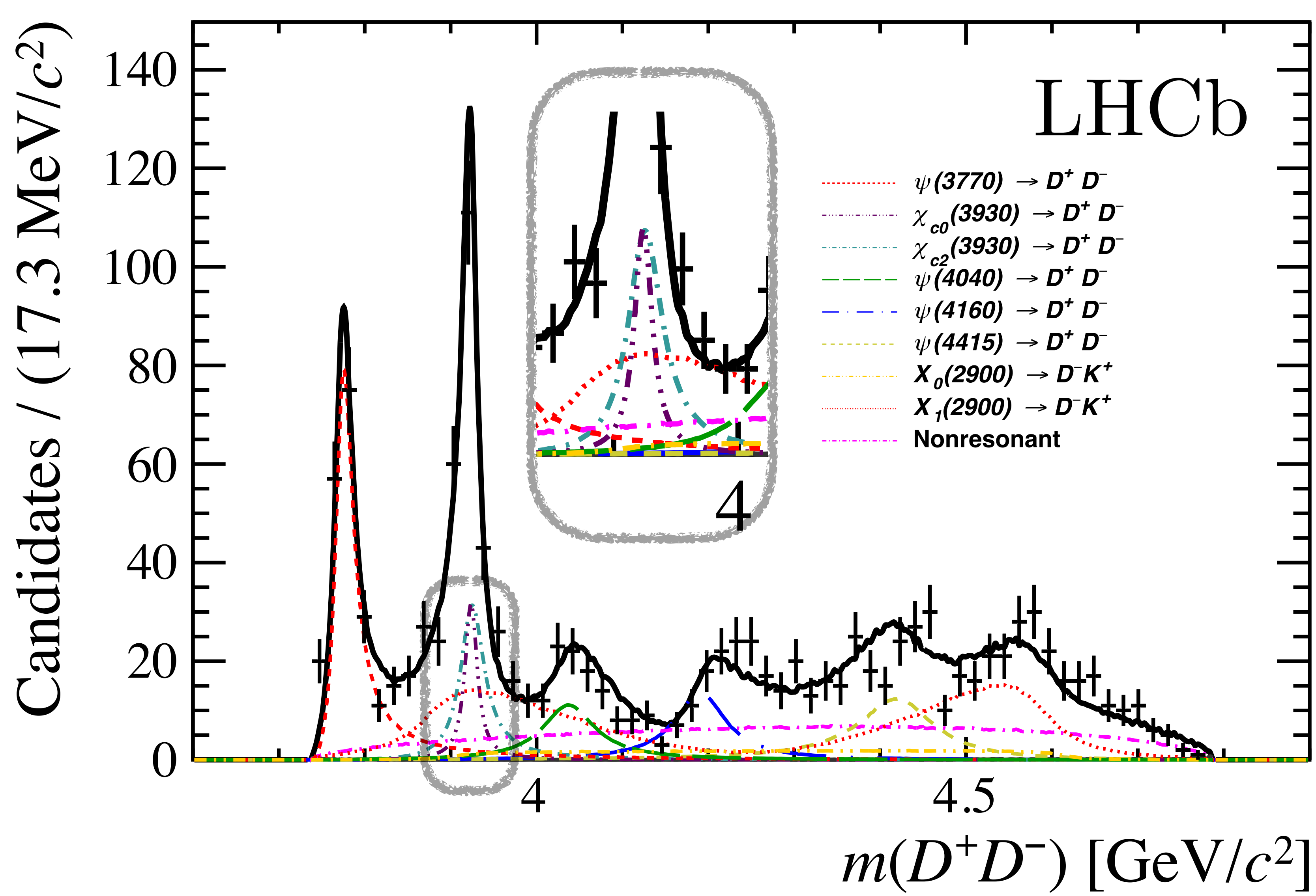
$m \approx 3922$ MeV

$\Gamma \approx 37$ MeV

not obviously inconsistent with earlier Belle & BaBar results

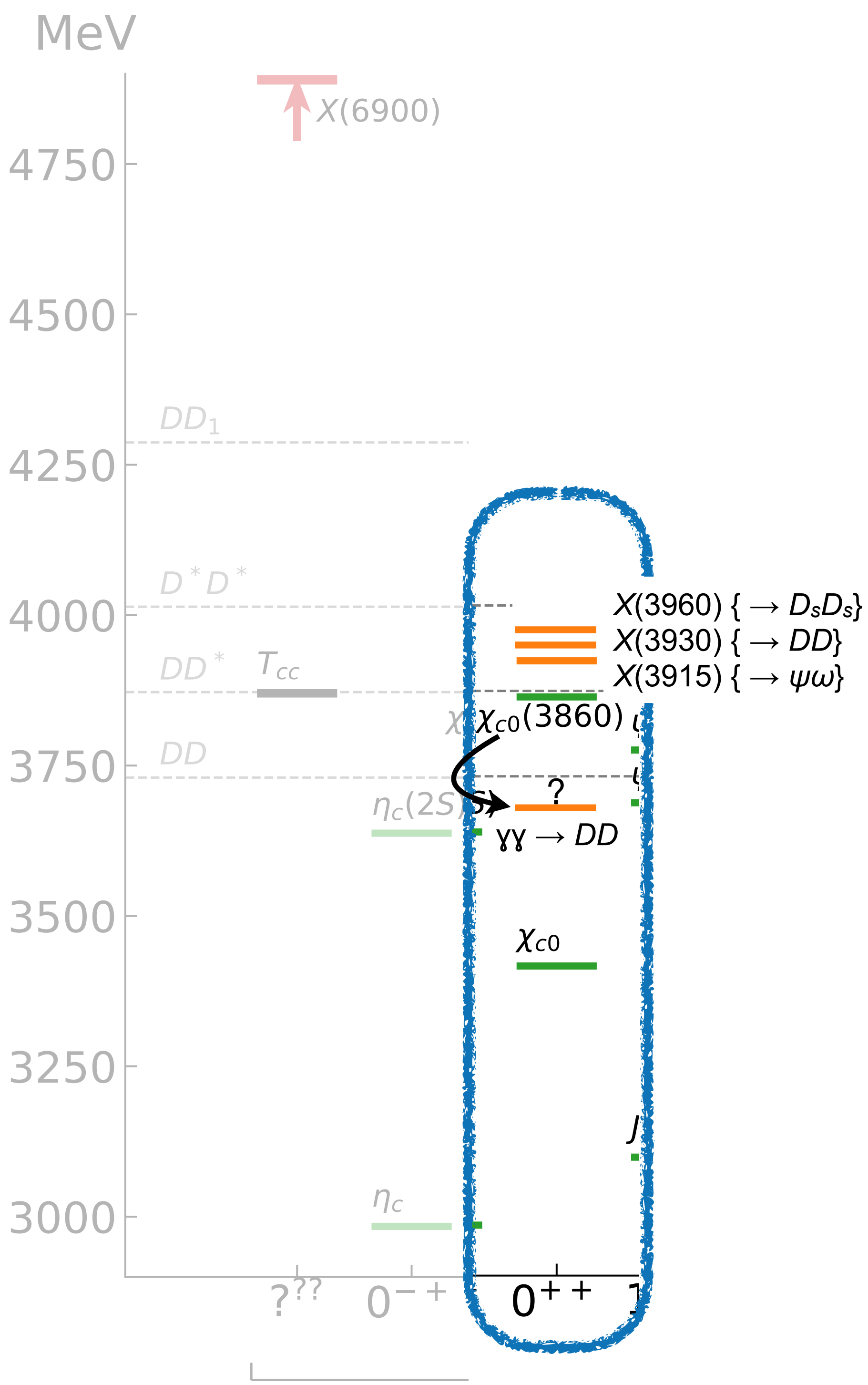


$B^+ \to D^+ D^- K^+$ arXiv:2009.00026



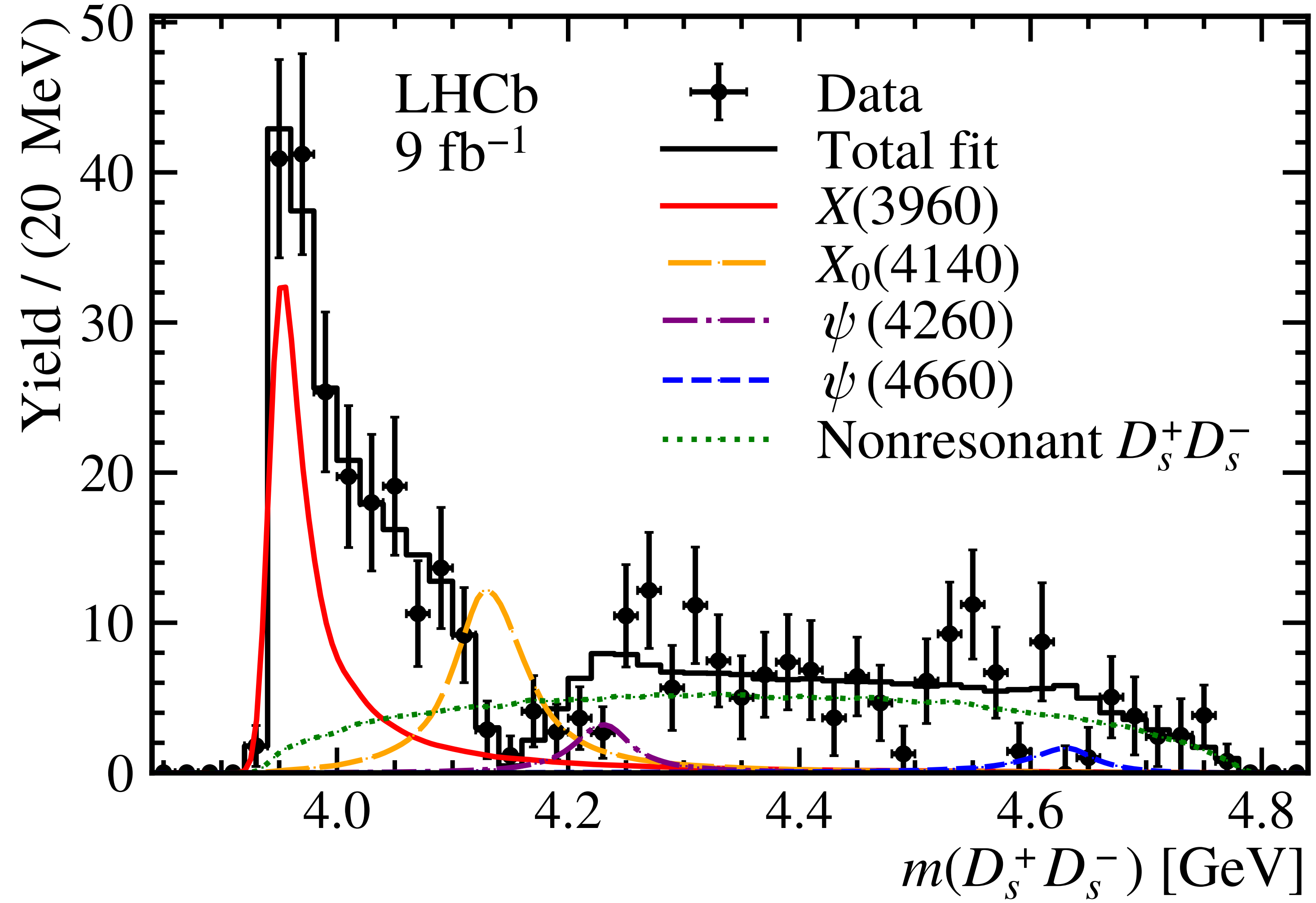
overlapping 0++ and 2++ resonances around 3925 MeV

no need for a low 0++ resonance



$$B^+ \to D_s^+ D_s^- K^+$$

arXiv:2210.15153
LHCb



enhancement in DsDs at threshold "X(3960)"

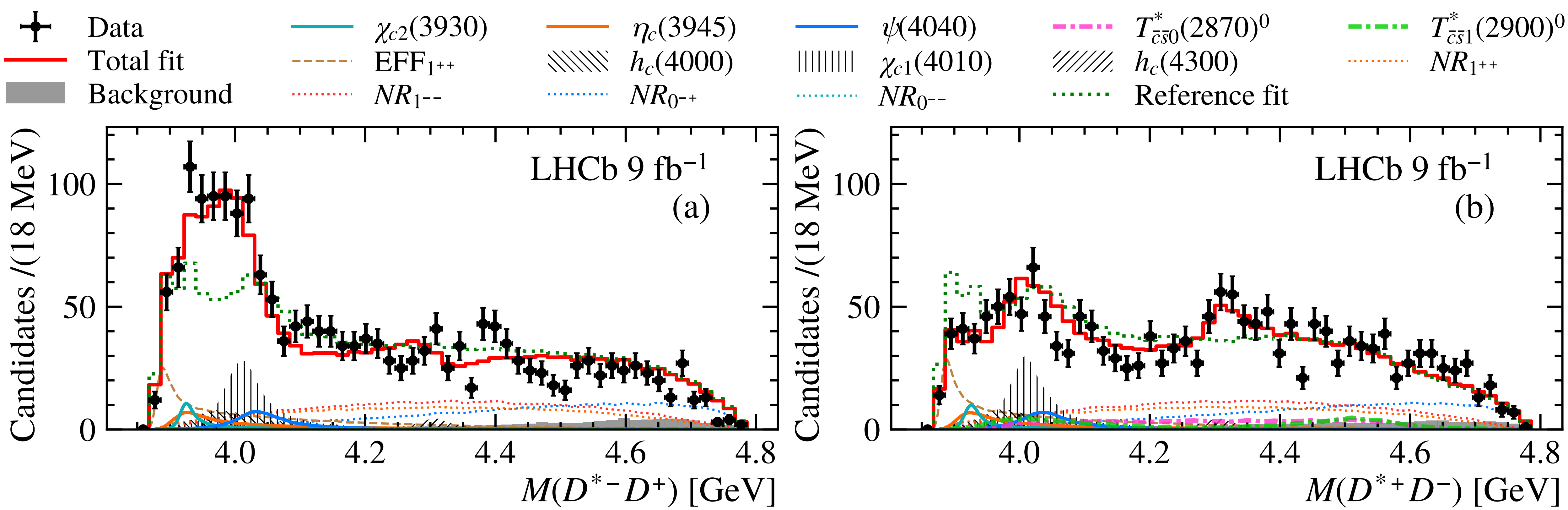
$$m \approx 3956 \text{ MeV}$$

$$\Gamma \approx 43 \text{ MeV}$$

$$J^{PC} = 0^{++}$$

$B^+ \to D^{*\pm} D^{\mp} K^+$

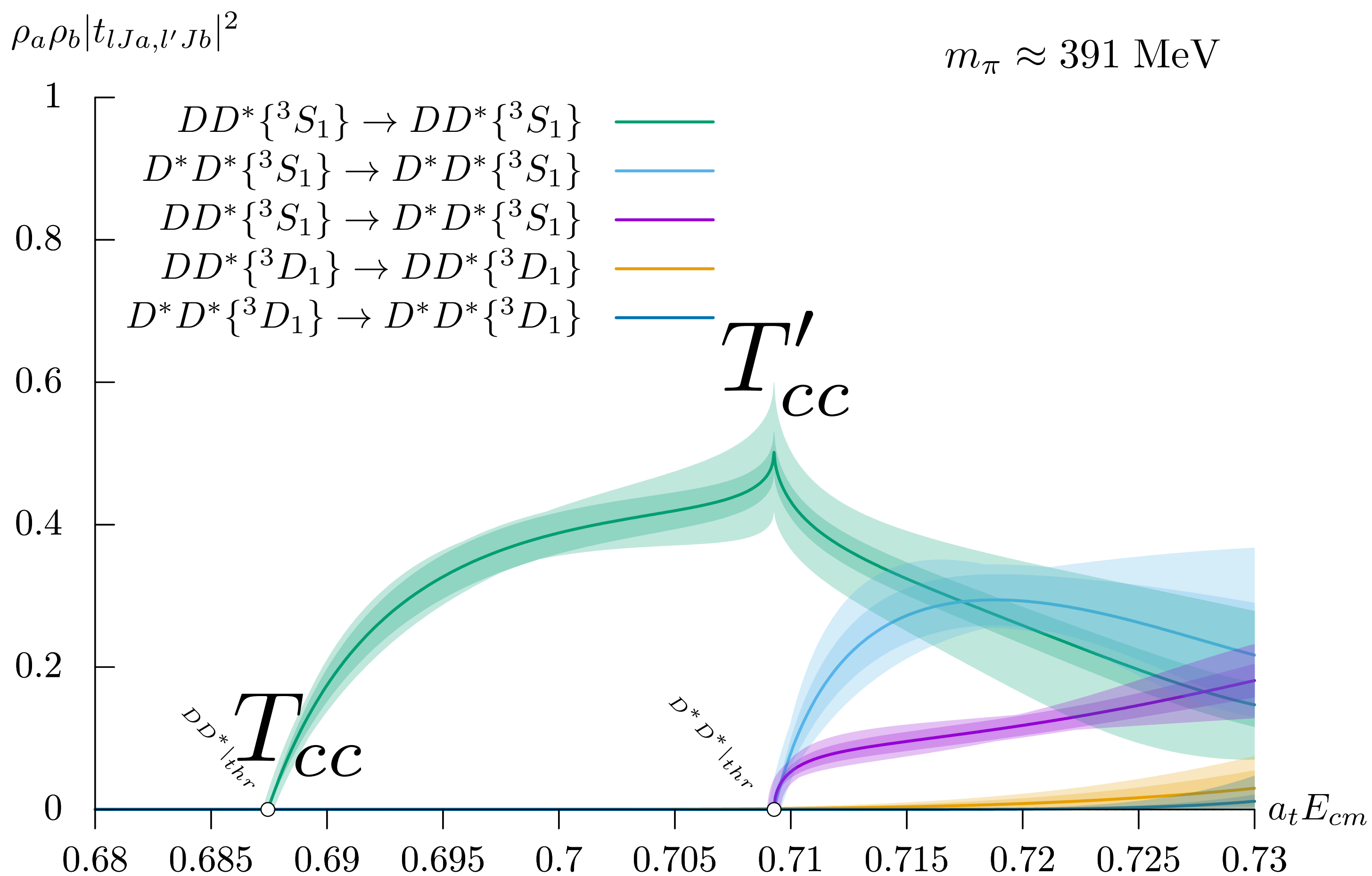
LHCb arXiv:2406.03156

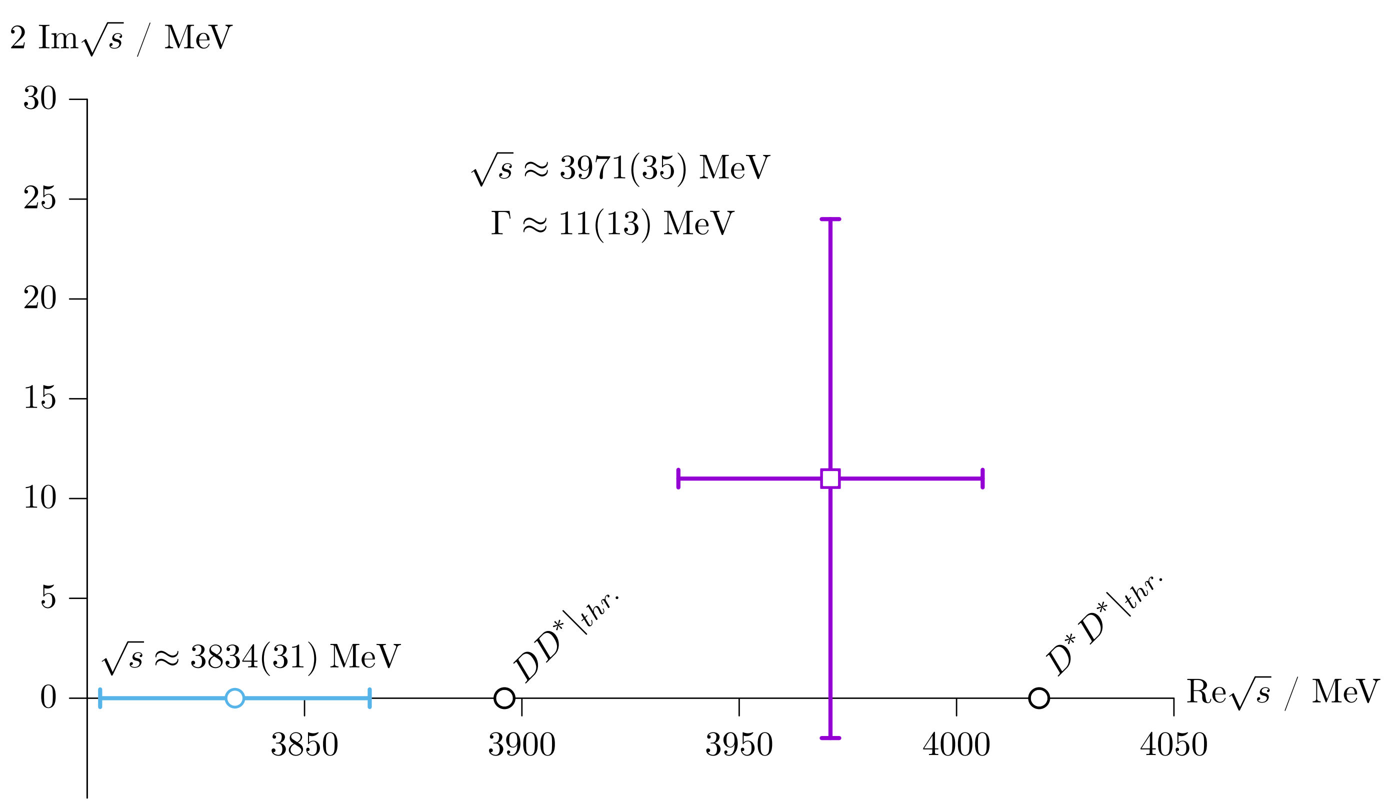


Very complicated

Many amplitudes contribute with similar strength

New resonances proposed around 4000 MeV

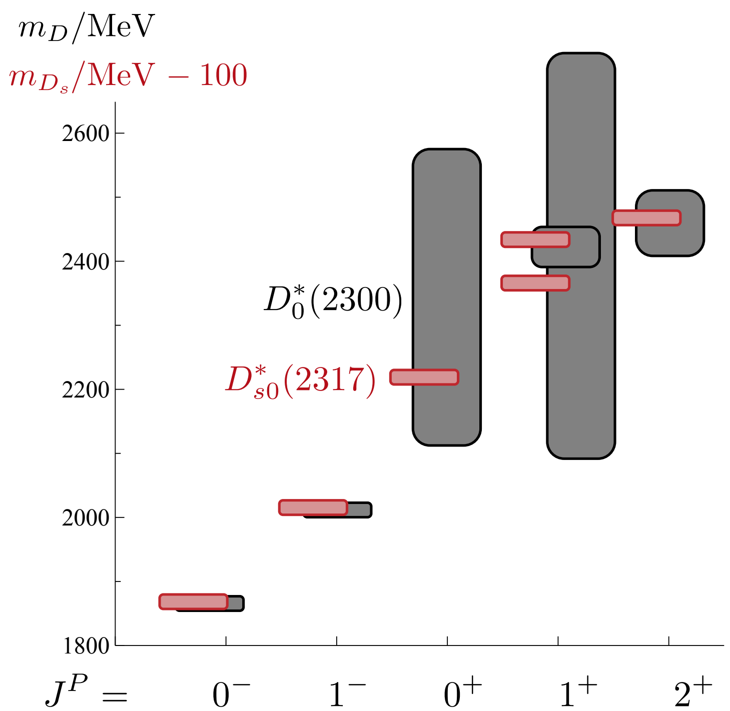


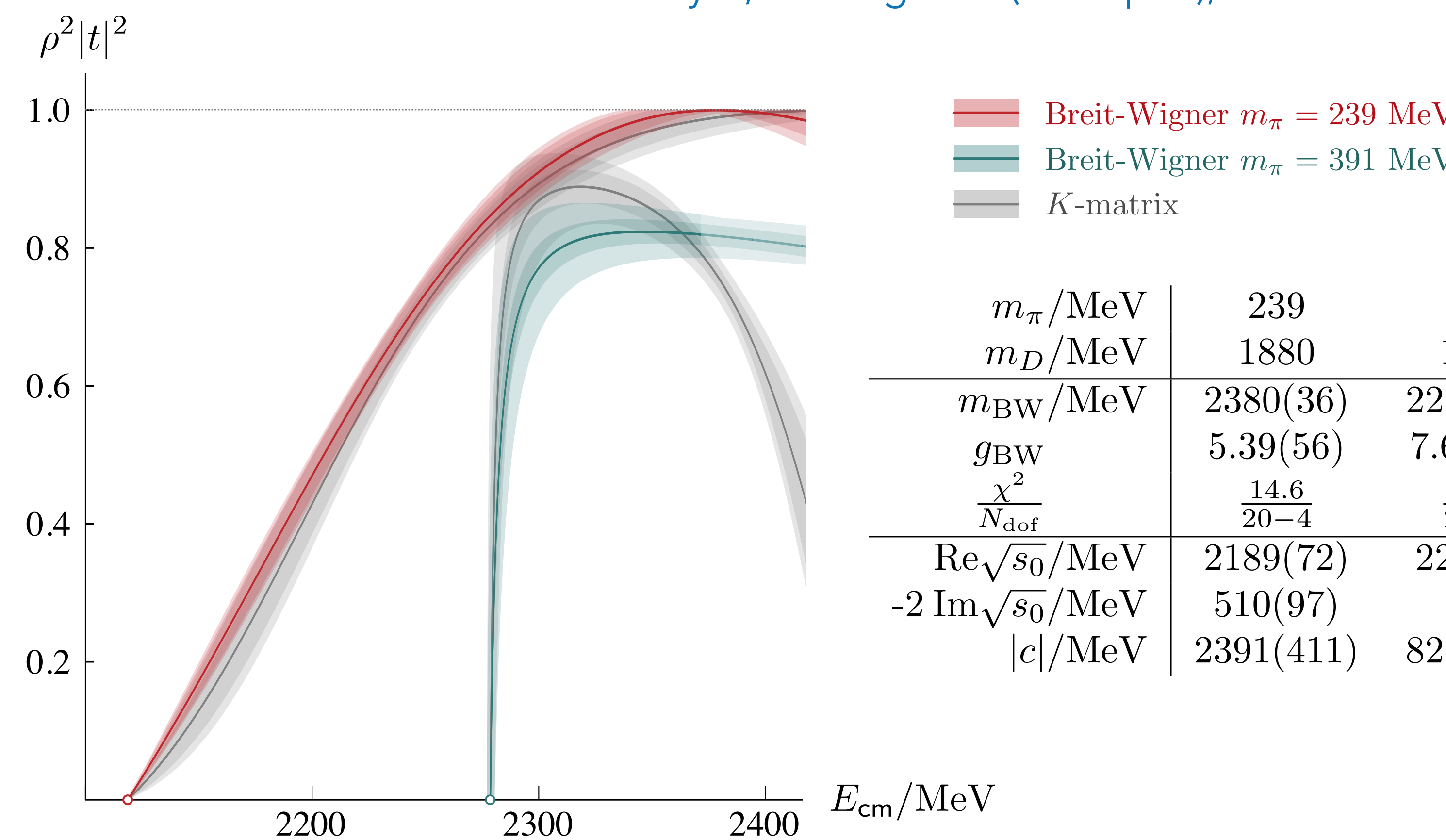


DD*-D*D* coupled channel
Whyte, Wilson, Thomas
arXiv:2405.15741

- *S* and *D*-wave in $J^P=1^+$
- virtual bound state below DD* and resonance below D*D*
- (neglecting left cuts)

L. Gayer, N. Lang et al (HadSpec), arXiv:2102.04973





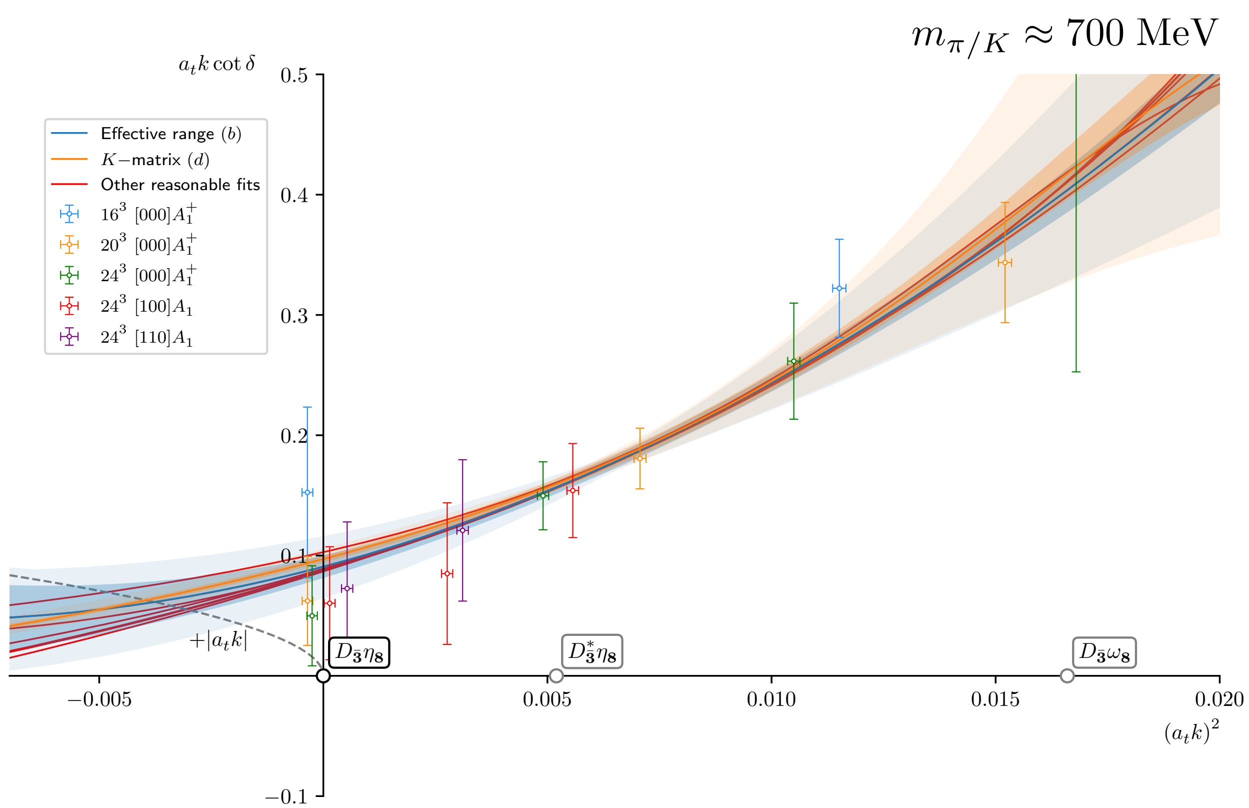
m_π/MeV	239	391
m_D/MeV	1880	1887
m_{BW}/MeV	2380(36)	2206(32)
g_{BW}	5.39(56)	7.62(75)
$\frac{\chi^2}{N_{\mathrm{dof}}}$	$\frac{14.6}{20-4}$	$\frac{36.0}{29-5}$
$\mathrm{Re}\sqrt{s_0}$/MeV	2189(72)	2275(1)
-2 $\mathrm{Im}\sqrt{s_0}$/MeV	510(97)	-
$\lvert c\rvert$/MeV	2391(411)	826(133)

$D_0^*(2300)$ & $D_{s0}^*(2317)$

what is the mass ordering?

why are the masses so close?

why are the widths so different?

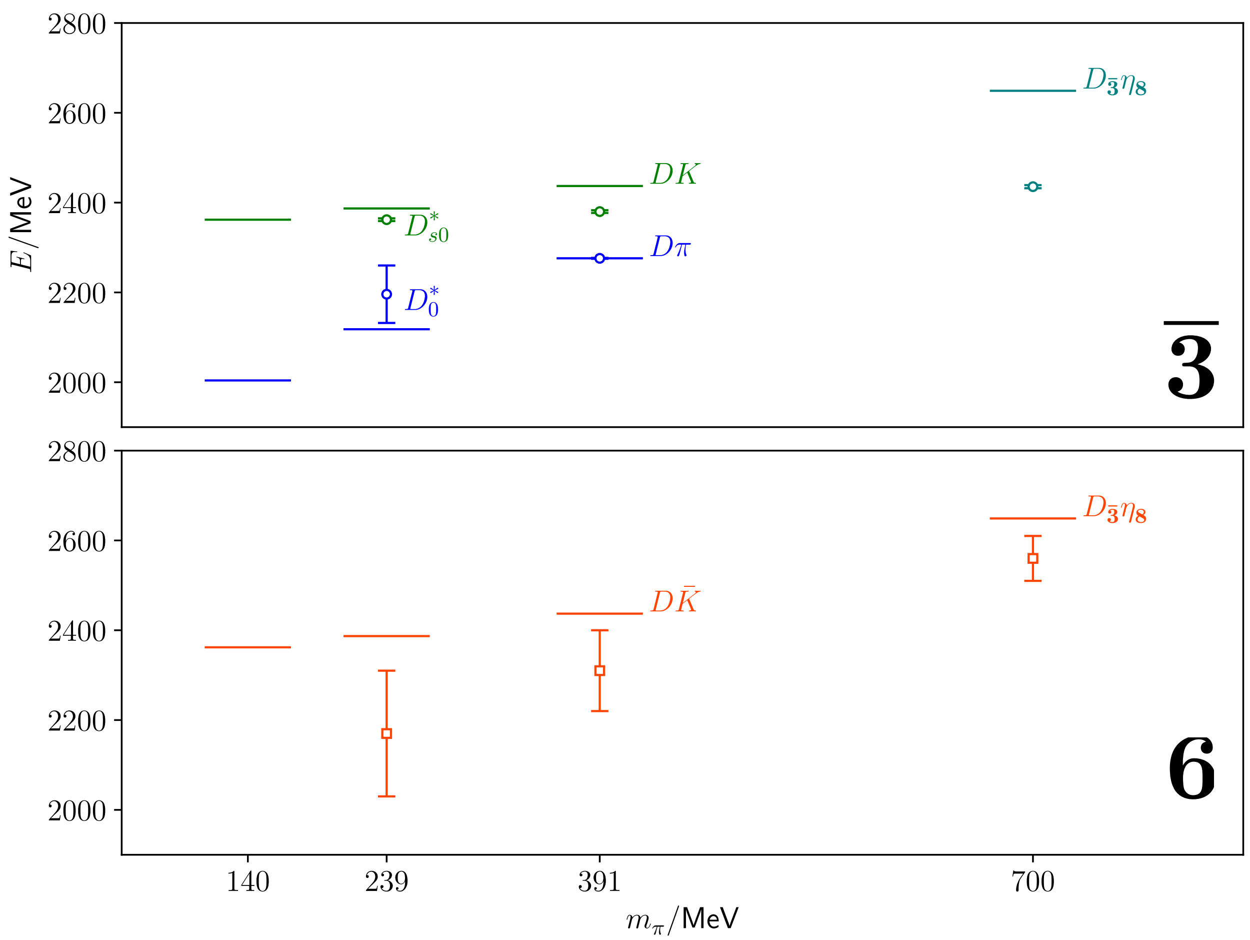


Dπ/DK scattering with SU(3) flavour symmetry

Yeo, Thomas, Wilson

arXiv:2403.10498

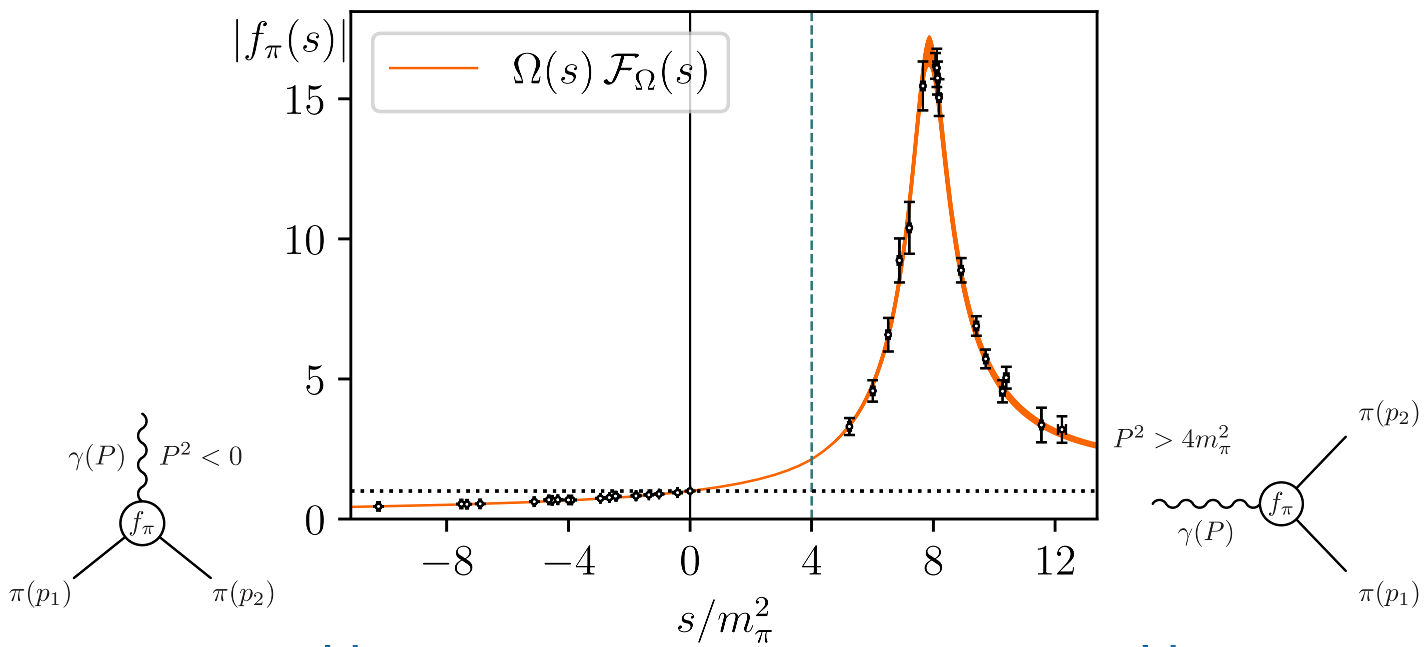
- S-wave interactions in flavour SU(3) 3bar, 6, 15bar
- Virtual bound state sextet pole
- Also deeply bound 3bar state, similar to Ds0(2317), much greater binding



SU(3) flavour:
D-meson and light meson

$$\bar{\mathbf{3}} \otimes \mathbf{8} \rightarrow \bar{\mathbf{3}} \oplus \mathbf{6} \oplus \overline{\mathbf{15}}$$

$$\bar{\mathbf{3}} \otimes \mathbf{1} \rightarrow \bar{\mathbf{3}}$$



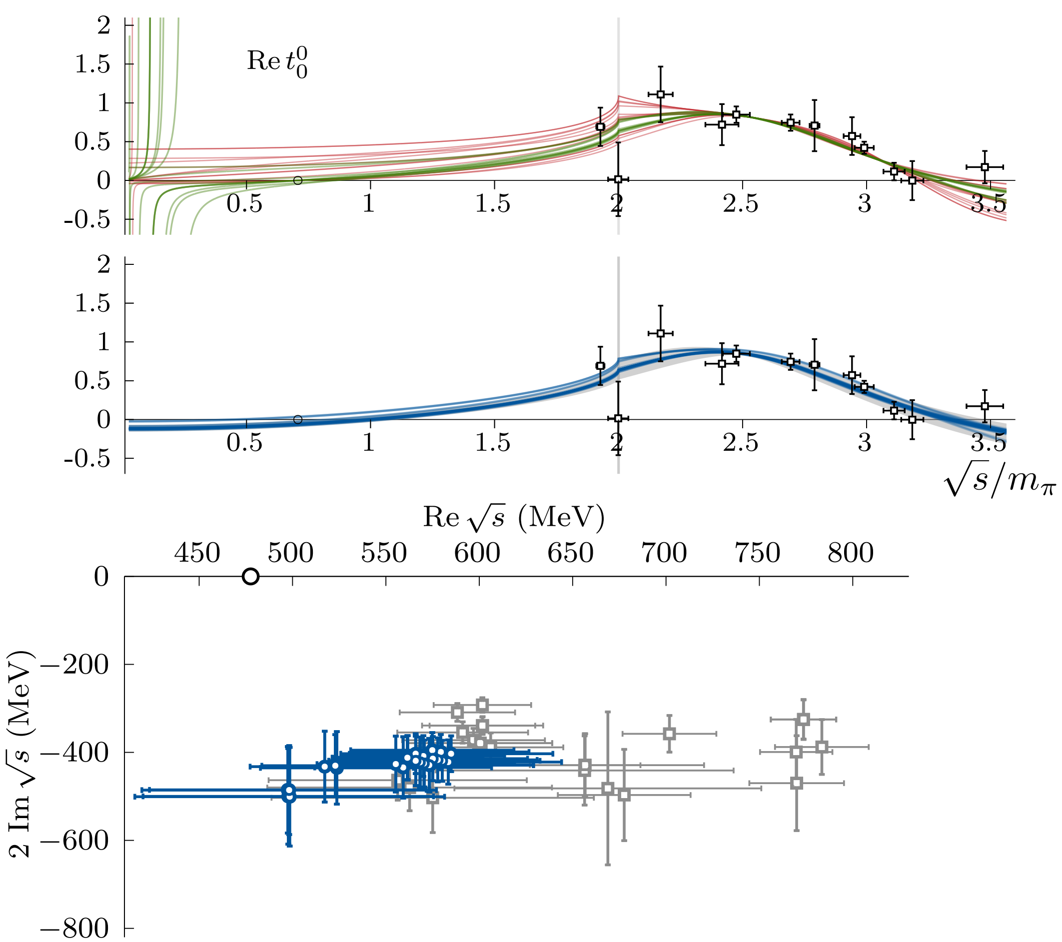
spacelike timelike

Timelike meson form-factors

Ortega-Gama, Dudek, Edwards

arXiv:2407.20617

- Lellouch-Lüscher analysis to extract infinite volume scattering amplitude
- Extended to coupled-channel region (KKbar)



Simple K-matrices

Dispersive amps

ππ dispersive + light quark mass-dependence

Rodas, Dudek, Edwards

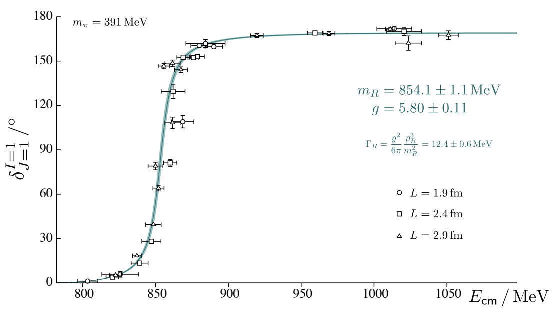
arXiv:2303.10701 & arXiv:2304.03762

- pins down σ-pole position
- Adler zero arises naturally
- interesting interplay with bound-state σ and Adler zeros when considering the light-quark mass dependence

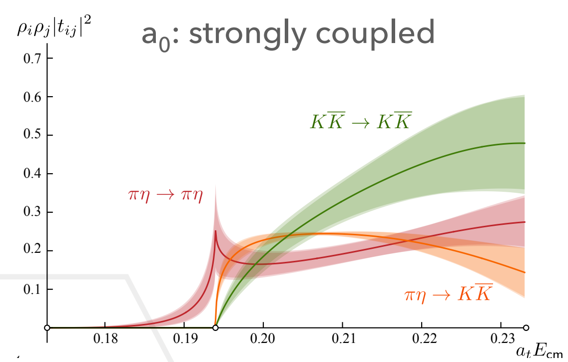
hadspec
hadspec.org

2012 2014 2016 2018 2020

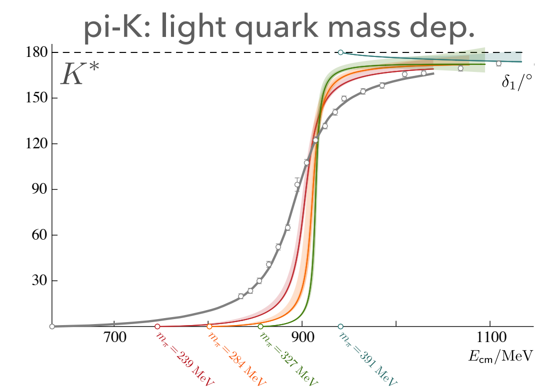
applications: (a biased sample)

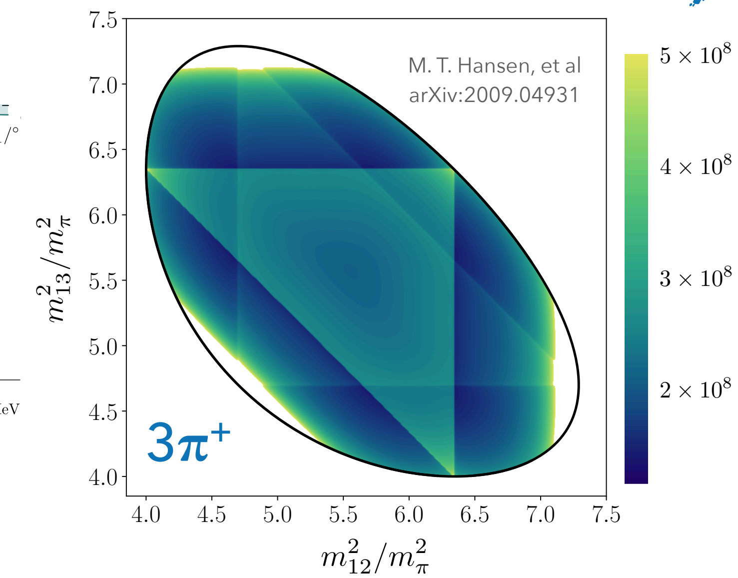


elastic scattering:
rho resonance

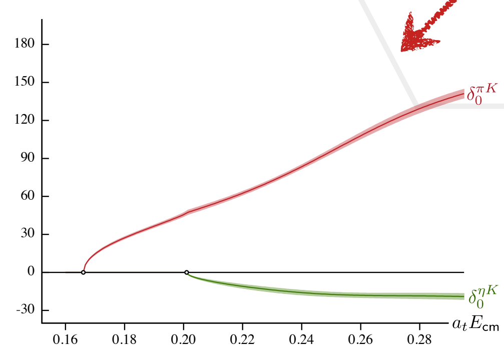


J. Dudek et al, PRD 93 (2016) 9, 094506

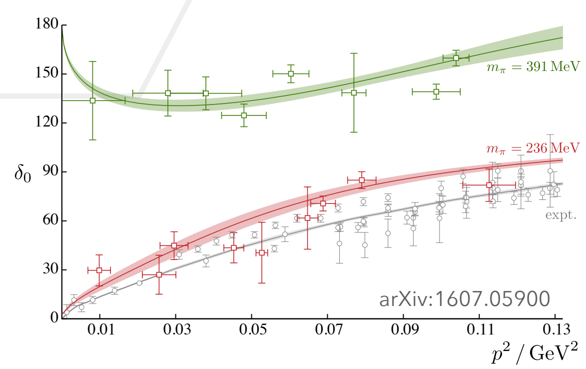




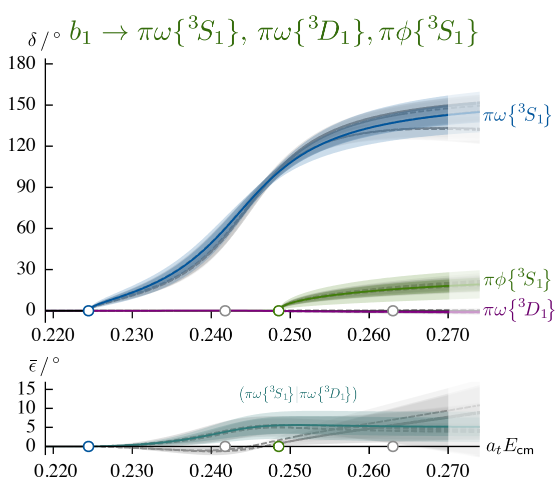
coupled-channel
scattering



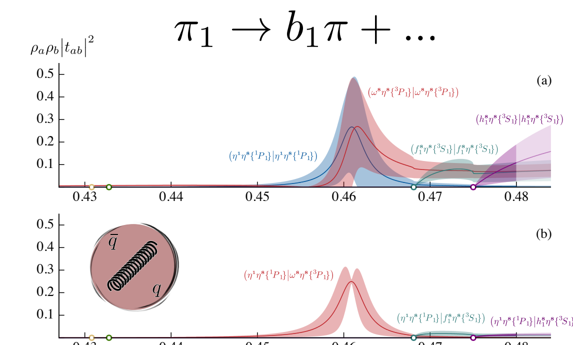
pi-K: almost decoupled,
first ever application



elastic scattering:
sigma resonance,
light quark mass dep.



scattering of
hadrons with spin



π_1 decays, SU(3) flavour,
11 active channels

formalism/theory developments:

pseudocalar two-body
coupled-channel
scattering

resonance
transition FFs

scattering of
hadrons with spin

three-body
scattering

form factors
of resonances

general three-body
scattering

more general processes:
two currents, ...