

**Gravitational Wave**  
**with**  
**Domain Wall Dominance**

**Sungwoo Hong**

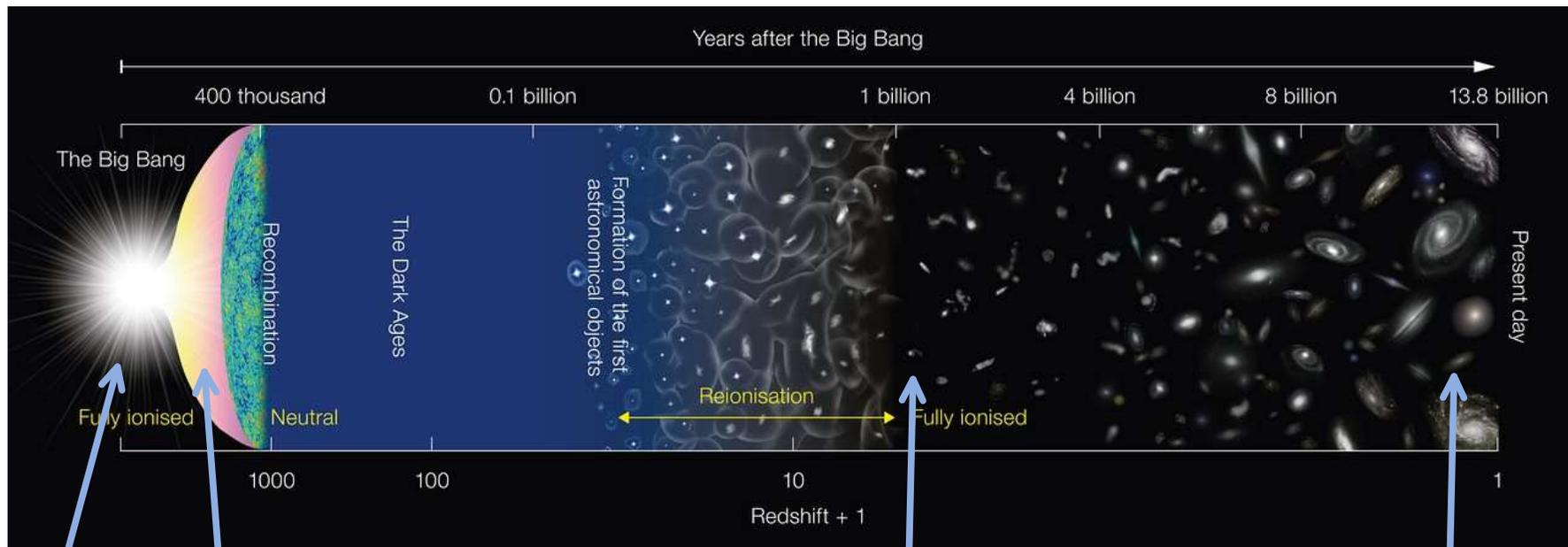
KAIST

2502.xxxxx: SH, Sung Mook Lee (CERN), Qiuyue Liang (IPMU)

2025 CAU-IBS BSM Workshop

# I. Introduction

[1] Study of "**Phases of Universe**" of universe is interesting and important!



Credit: <https://www.eso.org/public/images/eso1620a/>

**DED**

**RD**

**MD**

**DED**

Inflation

Thermalization  
BBN, BAU, DM

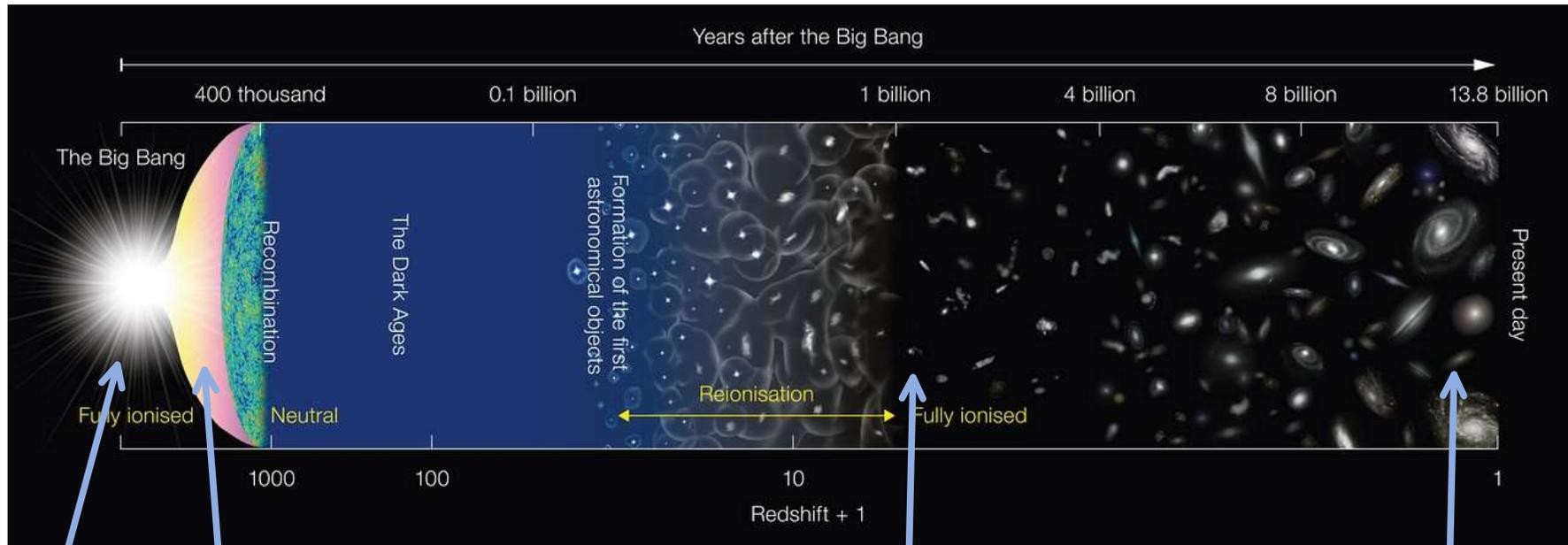
LSS

?

(causality violating question)

# I. Introduction

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Credit: <https://www.eso.org/public/images/eso1620a/>

DED → RD



MD



DED

Reheating

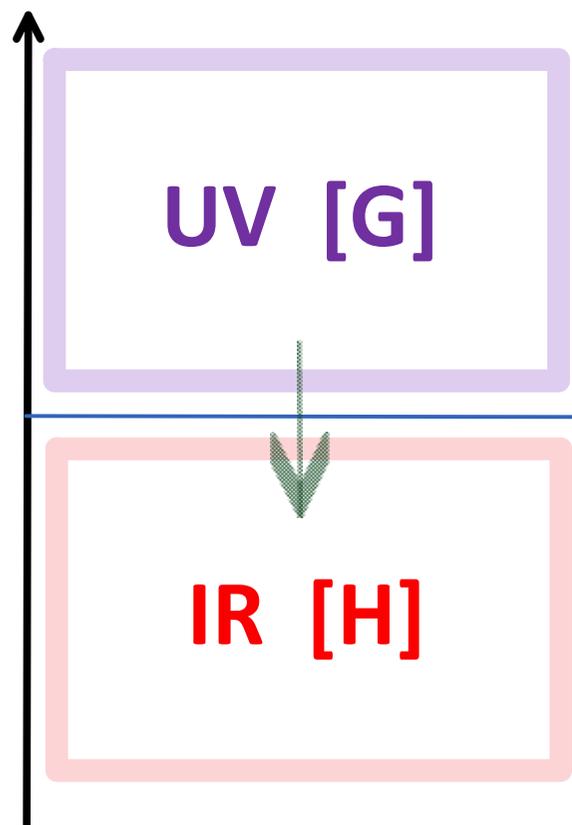
CMB

?

# I. Introduction

[1] Study of "**Phases of Universe**" of universe is interesting and important!

[2] In addition to **particle** energy densities,  
There exist various **topological defects**, whose existence is rather ubiquitous.



$\Pi_0 (G/H) \Rightarrow$  Domain Wall

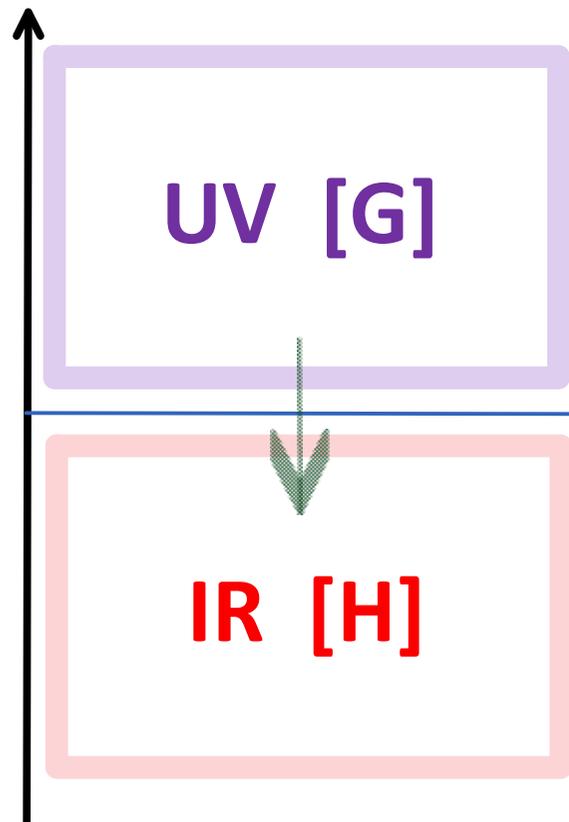
$\Pi_1 (G/H) \Rightarrow$  Cosmic String

$\Pi_2 (G/H) \Rightarrow$  Monopole

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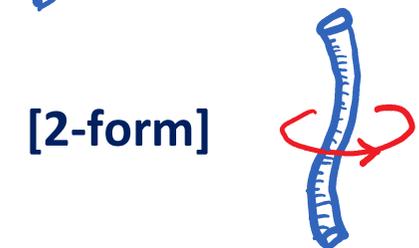
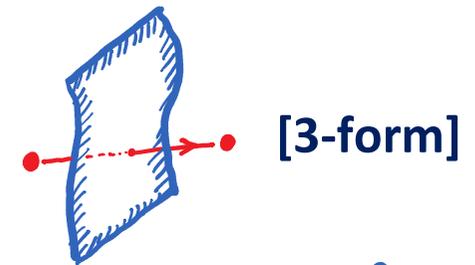
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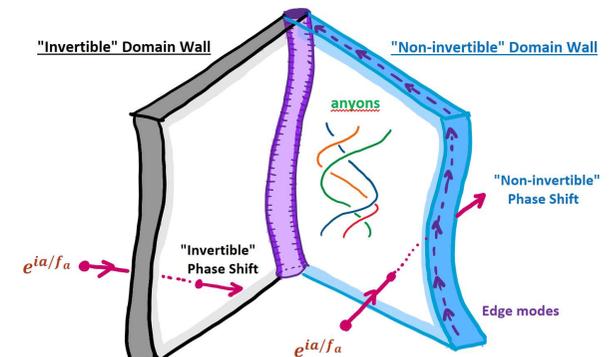
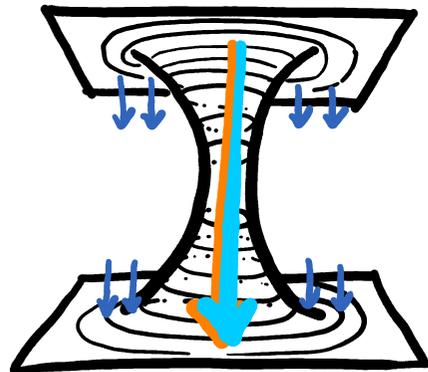
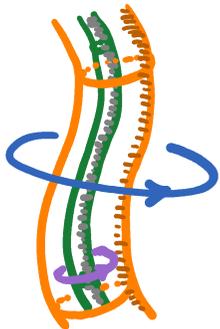
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## "Generalized Symmetries and Topological Defects"

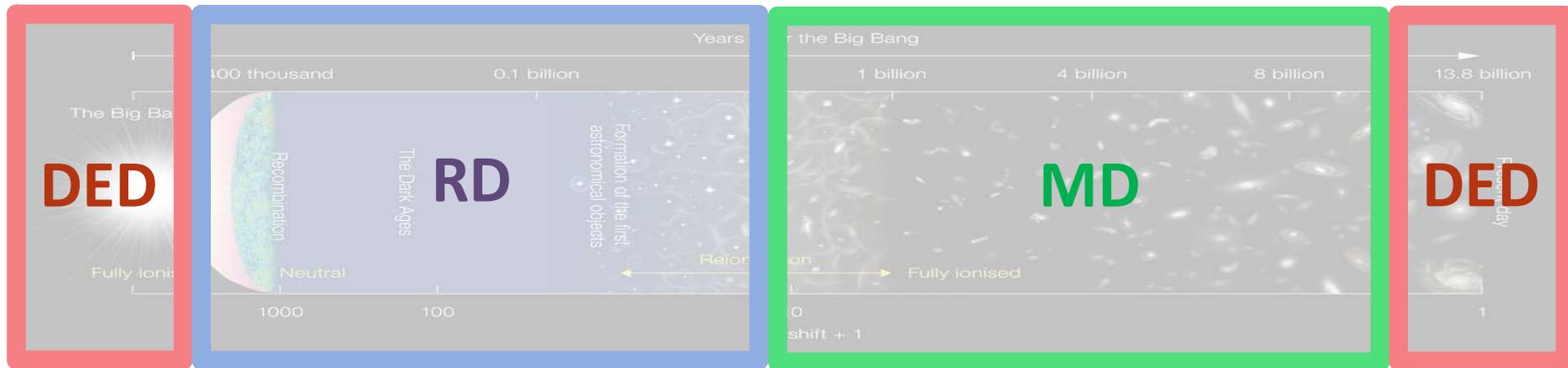
- Physics with generalized symmetry is new, promising possibility.
- In general, topological defects, often **new types**, appear in those cases.
- Top. Defects might be the best probe for some physics.  
E.g. TQFT-couplings, global structure of SM, ...



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**Topological Defect Dominance**

# I. Introduction



## [3] Topological Defect Dominance

$$\rho_{mon} \sim \rho_{matt} \sim \frac{1}{a^3}$$

$$\rho_{st} \sim \frac{\mu t}{t^3} \sim \frac{\mu}{t^2} \sim \rho_{rad} \quad (\text{infinite string in scaling regime})$$

$$\rho_w \sim \frac{\sigma t^2}{t^3} \sim \frac{\sigma}{t} \quad (\text{infinite wall in scaling regime})$$

# I. Introduction



## [3] Topological Defect Dominance

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**monopole Problem**

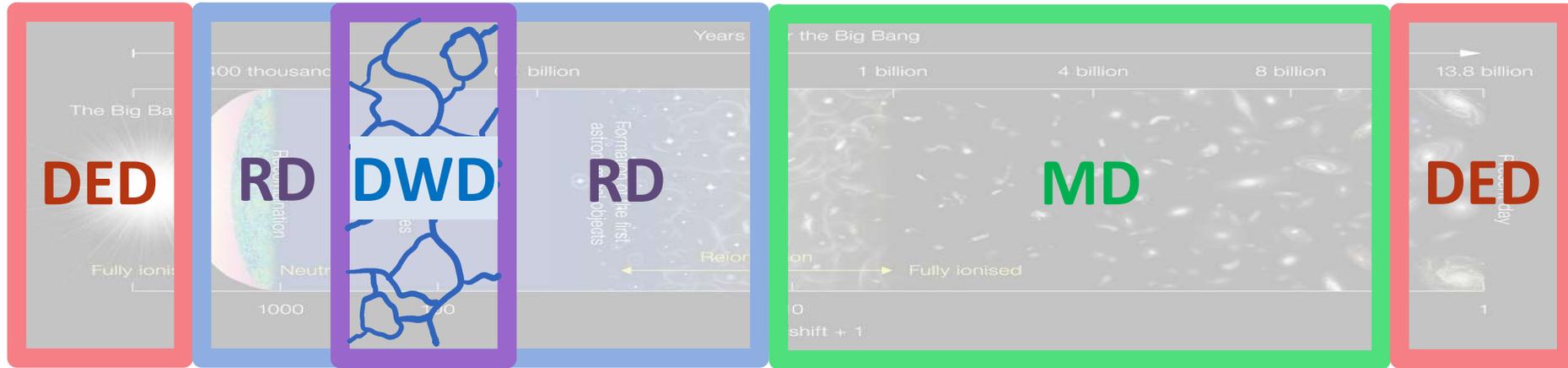
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(infinite string in scaling regime)

$$\rho_w \sim \frac{\sigma t^2}{t^3} \sim \frac{\sigma}{t}$$

**Domain Wall Problem**

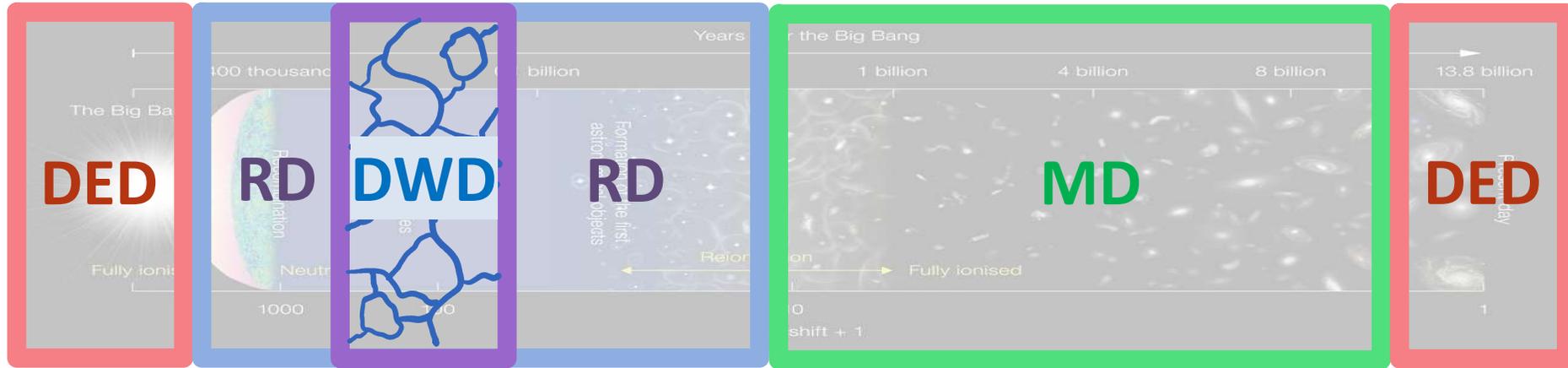
# I. Introduction



## [4] Domain Wall Dominance (DWD)

- DWD is natural to occur: in RD,  $\frac{\rho_w}{\rho_{rad}} \sim t$
- DWD is stopped by making DWs unstable
  - (i) explicit breaking of discrete symmetry  $\rightarrow V_{bias}$
  - (ii) walls bounded by strings
  - (iii) symmetry restoration
  - (iv) Lazarides-Shafi mechanism

# I. Introduction



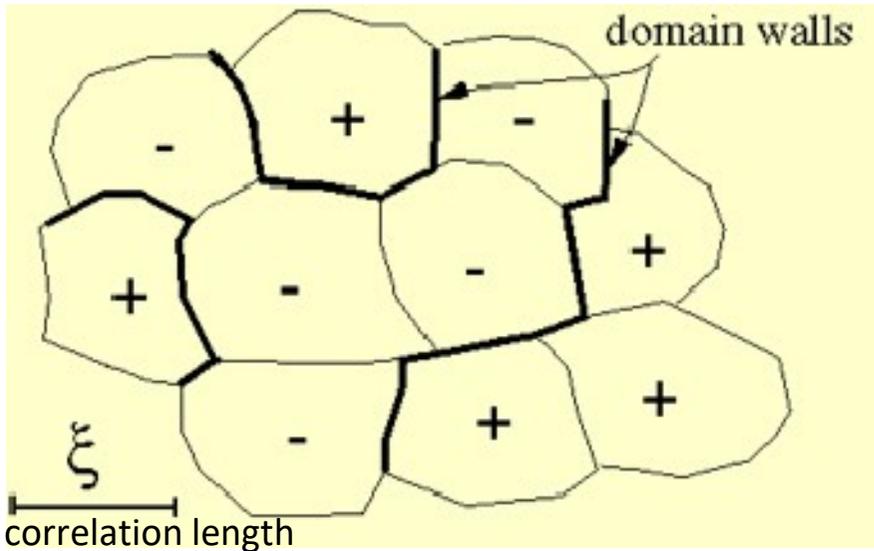
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- Several **fascinating features of DWD** (this talk)!

## II. Domain Walls in RD

[1] DW Formation (Kibble-Zurek mechanism)

Consider  $Z_2$  case for concreteness.



- 2nd-order PT:  $\xi \sim T_c^{-1}$
- 1st-order PT:  $\xi$  can be larger

Either case, by causality

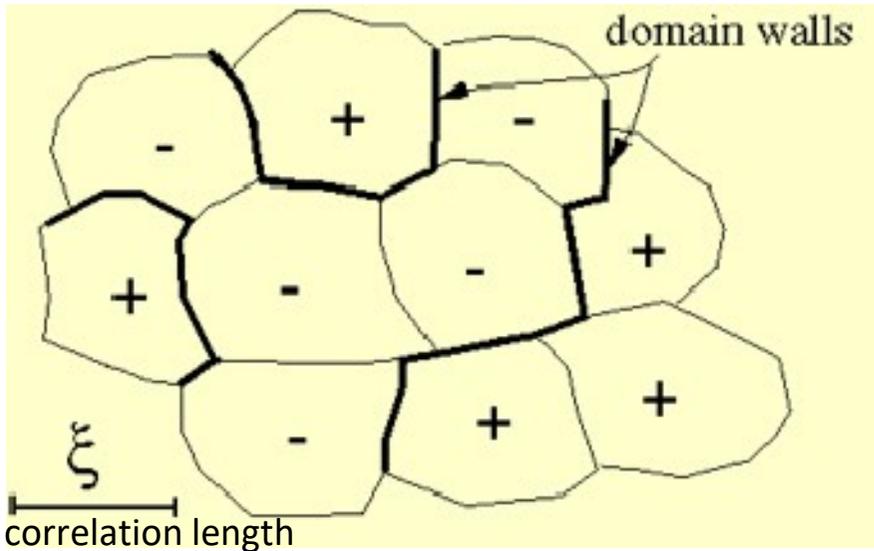
$$\xi \leq a(t) \int_0^t \frac{dt'}{a(t')} = a(t) \int_0^t (\mathbf{aH})^{-1} d \ln a$$

[https://www.ctc.cam.ac.uk/outreach/origins/cosmic\\_structures\\_two.php](https://www.ctc.cam.ac.uk/outreach/origins/cosmic_structures_two.php)

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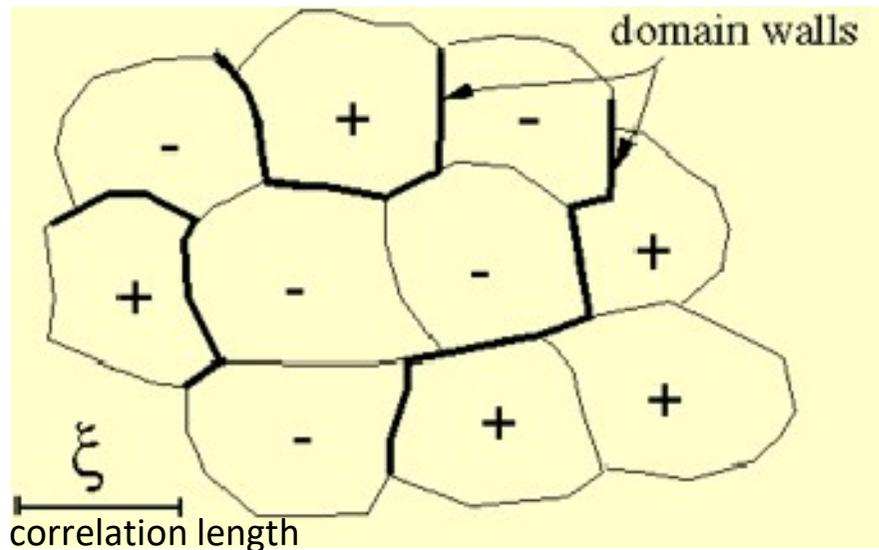
According to Percolation Theory,

- If  $p_{\pm} > p_c$  then infinite (plus/minus)-cluster appears.
- Typically,  $p_c < 0.5$  for 3d case ( $p_c < 0.31$  for cubic lattice)
- Finite clusters of size :  $n_s \propto s^{-\alpha} e^{-\beta s^{2/3}}$  ( $\alpha, \beta$  depends on  $p_{\pm}$ )

## II. Domain Walls in RD

[2] DW network

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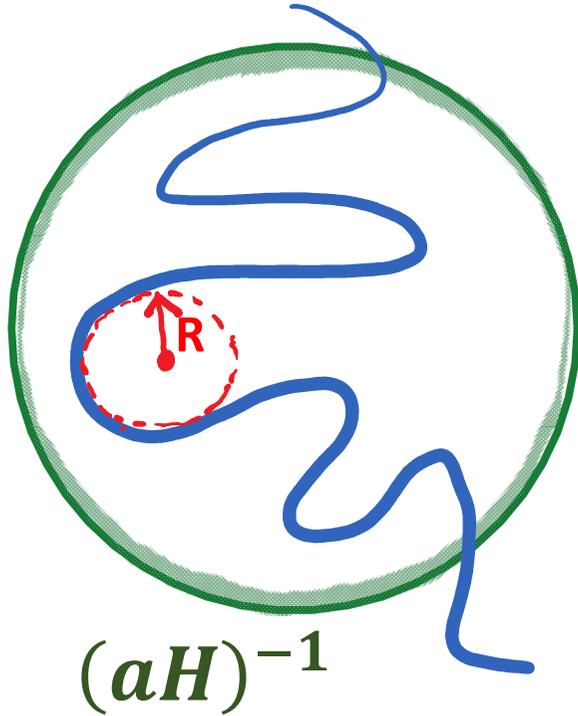


[https://www.ctc.cam.ac.uk/outreach/origins/cosmic\\_structures\\_two.php](https://www.ctc.cam.ac.uk/outreach/origins/cosmic_structures_two.php)

- DW-network dominated by **one-infinite wall** of complicated topology
- Some finite closed walls, with typical size  $R \sim \xi$
- Walls with  $R \gg \xi$  exponentially rare

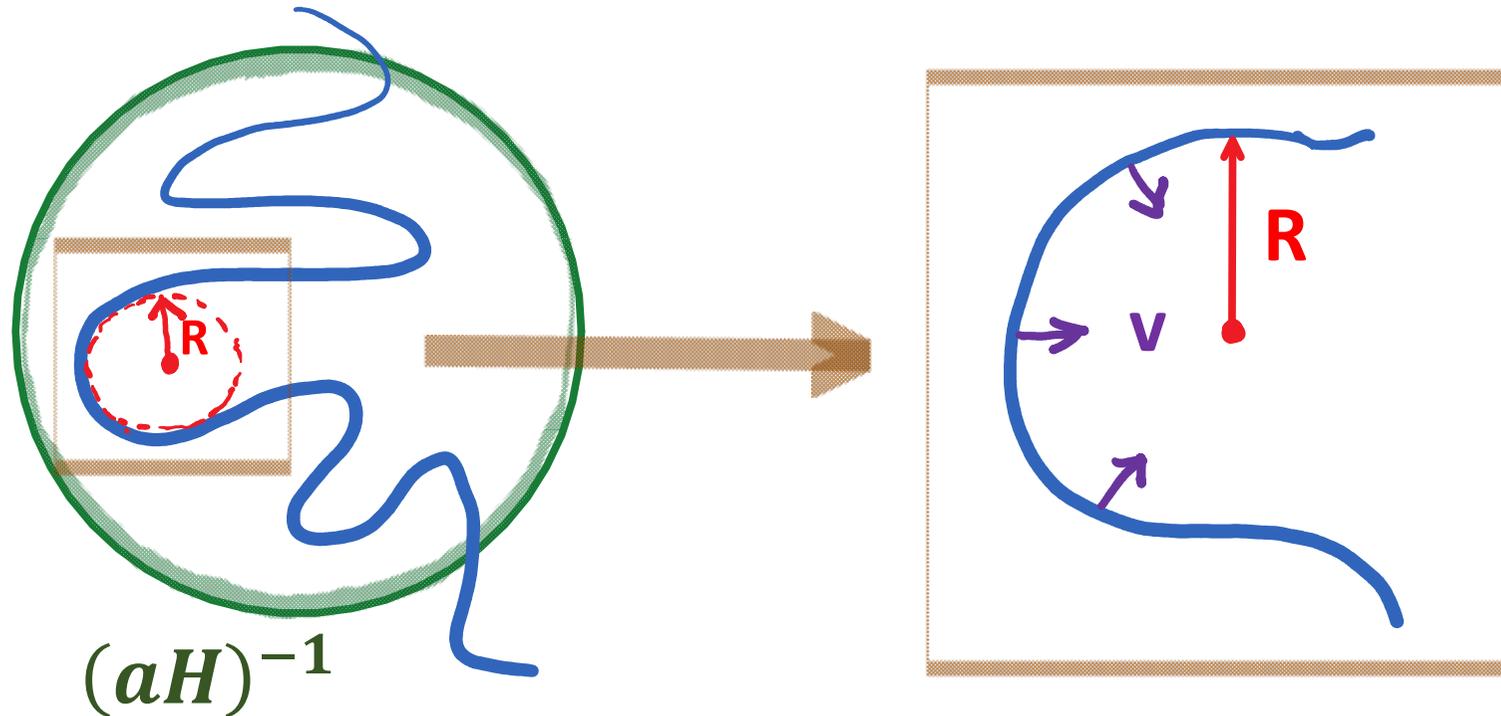
## II. Domain Walls in RD

[3] Evolution of DW network



## II. Domain Walls in RD

### [3] Evolution of DW network



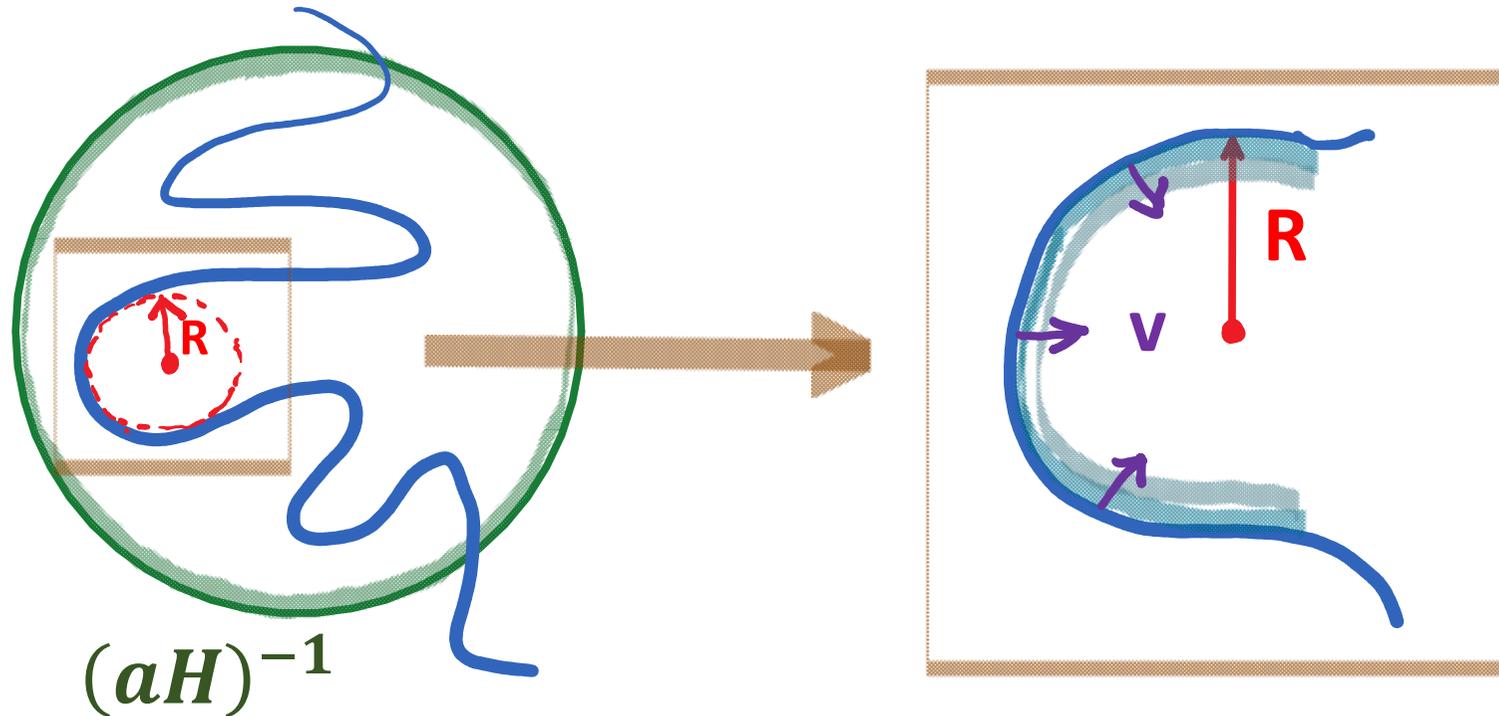
(1) tension:  $f_T \sim \frac{\sigma}{R}$

: stretch out the irregularities

speed-up the wall in the cosmic plasma

## II. Domain Walls in RD

### [3] Evolution of DW network



(2) Friction (from wall-particle interactions) [cf.  $\rho_{rad} \sim \left(\frac{3}{8\pi G}\right) H^2 \sim \frac{1}{Gt^2}$ ]

$$f_{fric} \sim N_f n \Delta p \sim N_f \left(\frac{T^3}{\pi^2}\right) (Tv) \sim T^4 v \sim \frac{v}{Gt^2}$$

## II. Domain Walls in RD

[3] Evolution of DW network

(3) For sufficiently large walls,  $v$  is determined by the balance

$$f_T \sim f_{ric} : \quad \frac{\sigma}{R} \sim \frac{v}{Gt^2} \quad \Rightarrow \quad v \sim \frac{G\sigma t^2}{R}$$

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(4) If the dissipation time  $t_d \sim R/v$  is  $t_d \sim t$  (Hubble time)

$$v \sim (G\sigma t)^{1/2} \sim \frac{(\sigma t)^{1/2}}{M_p} \propto t^{1/2}$$

$$R \sim vt \sim (G\sigma)^{1/2} t^{3/2} \sim \frac{\sigma^{1/2} t^{3/2}}{M_p} \propto t^{3/2}$$

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(5) DW becomes relativistic at  $t_r \sim \frac{1}{G\sigma}$ ,  $R \sim t_r \sim H(t_r)^{-1}$

## II. Domain Walls in RD

[3] Evolution of DW network

(6) As  $t \rightarrow t_r$

$$\rho_w \sim \frac{\sigma R^2}{R^3} \sim \frac{\sigma}{R} \sim \begin{cases} \text{(a) } \rho_w \sim \frac{M_p^2}{t_r^2} \sim \rho_c(t_r) \\ \text{(b) } \rho_w \sim \frac{\sigma}{t_r} \end{cases}$$

(7)  $\rho_w \sim \frac{M_p^2}{t_r^2} \sim \rho_c(t_r)$  means DW energy starts dominating the universe

(8)  $\rho_w \sim \frac{\sigma}{t_r}$  shows the **scaling (self-similar) solution**

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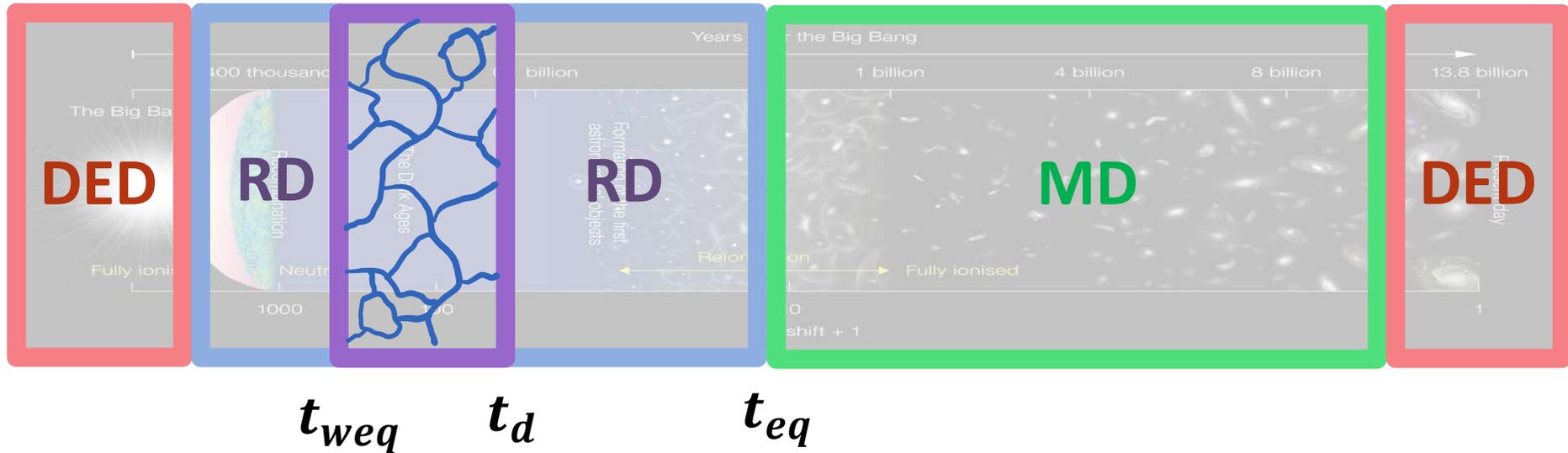
(8)  $\rho_w \sim \frac{\sigma}{t_r}$  shows the **scaling (self-similar) solution**

(9) On **super-horizon**:

Both amplitude and wavelength **conformally stretched out**.

Shape of DW remaining unchanged.

### III. Domain Wall Dominance



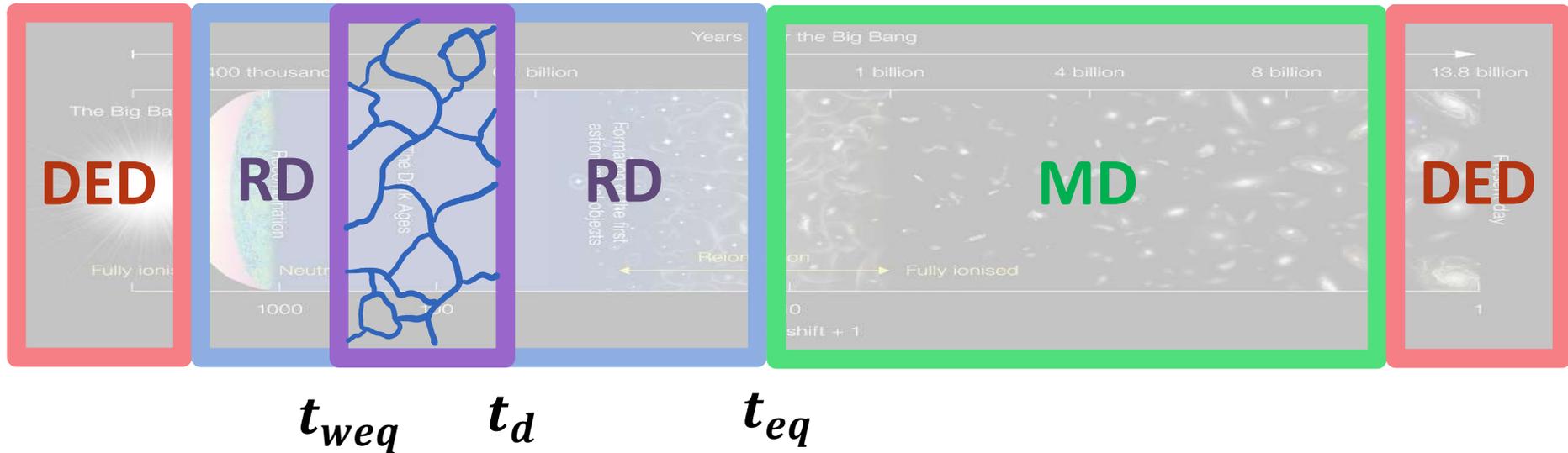
[1] RD  $\rightarrow$  DWD at  $t_{weq}$

$$(1) \rho_w \sim \frac{\sigma}{t} \sim \rho_c \sim \frac{1}{Gt^2} \Rightarrow t_{weq} \sim \frac{1}{G\sigma}$$

(2)  $R \rightarrow t$  during  $t < t_{weq}$ , after that  $R \propto a(t)$

(3)  $\rho_w \sim \frac{\sigma}{R} \sim \frac{\sigma}{a(t)}$ :  $\rho_w$  scales like single- $\infty$ -wall.

### III. Domain Wall Dominance



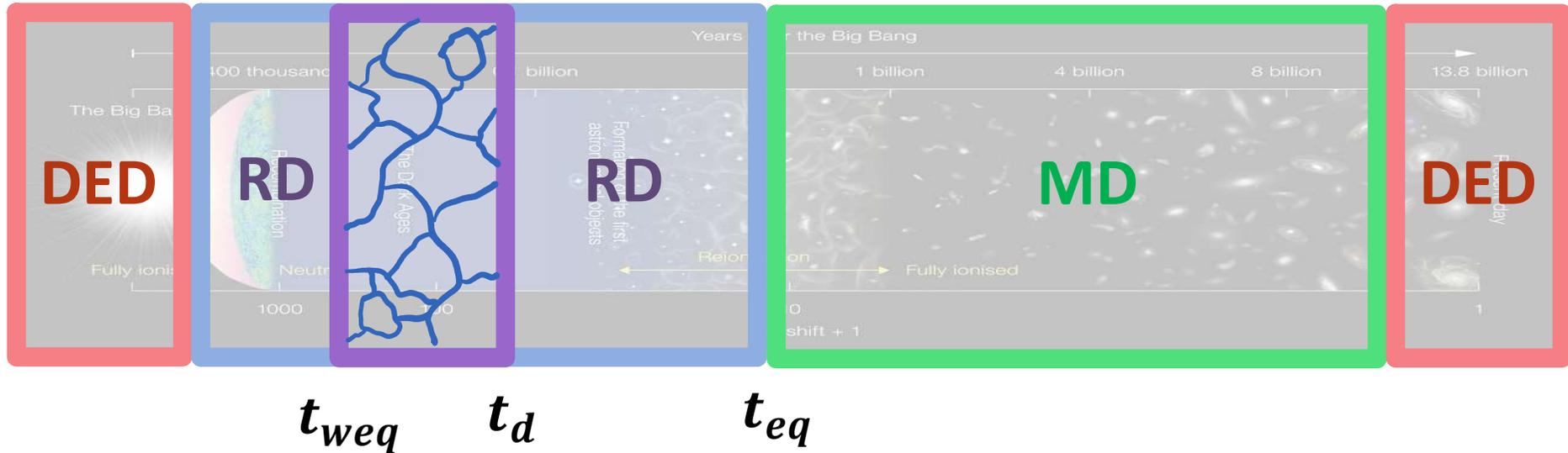
#### [2] Evolution in DWD

(1)  $\rho_w \sim \frac{\sigma}{R} \sim \frac{\sigma}{a(t)}$ :  $\rho_w$  scales like single- $\infty$ -wall.

(i)  $H^2 = \frac{8\pi G}{3} \rho \Rightarrow a(t) \propto t^2$  ( $a \sim t^{1/2}$  [RD],  $a \sim t^{2/3}$  [MD])

(ii)  $\dot{\rho} + 3H(1+w)\rho = 0 \Rightarrow w_{dwd} = -\frac{2}{3}$  ( $w_{rd} = \frac{1}{3}$ ,  $w_{md} = 0$ )

### III. Domain Wall Dominance



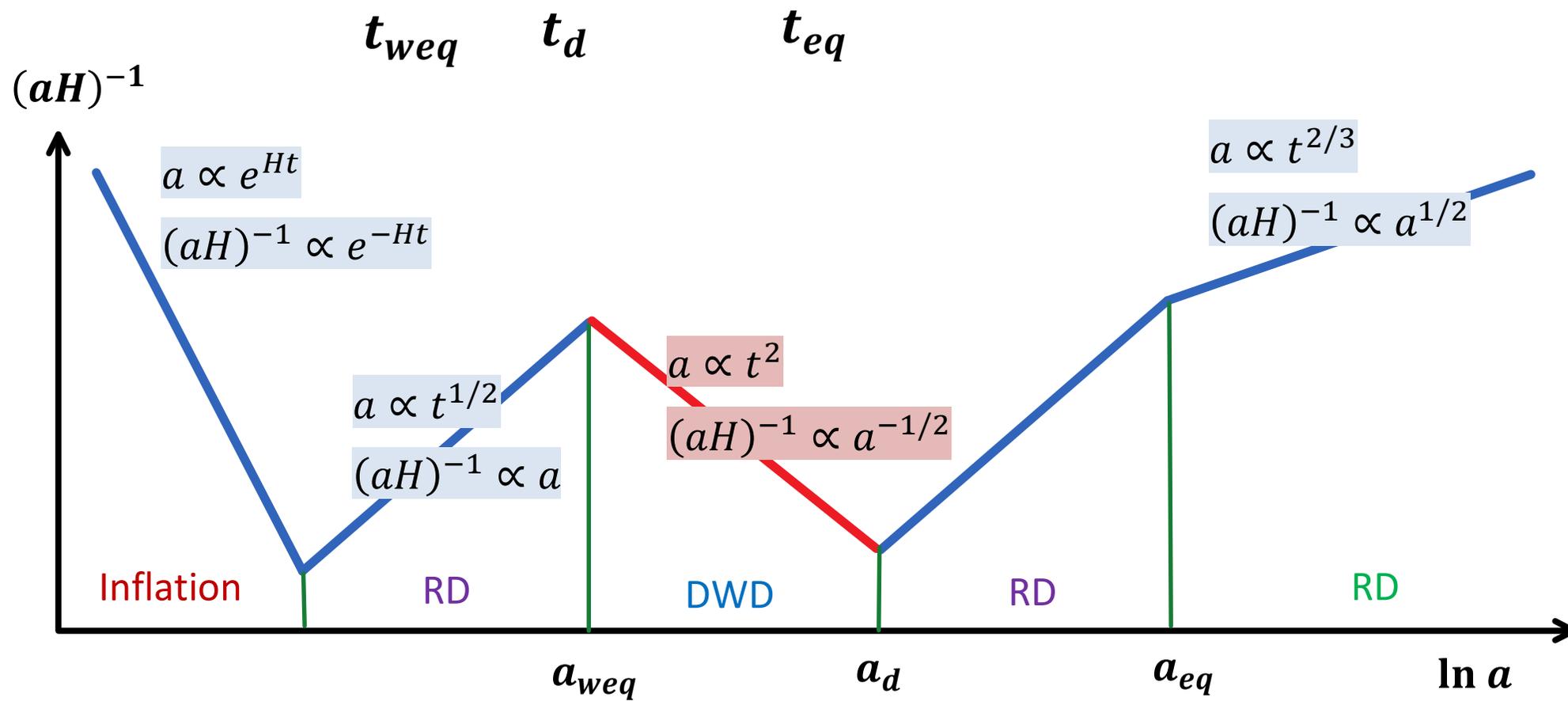
#### [2] Evolution in DWD

(2) comoving Hubble radius

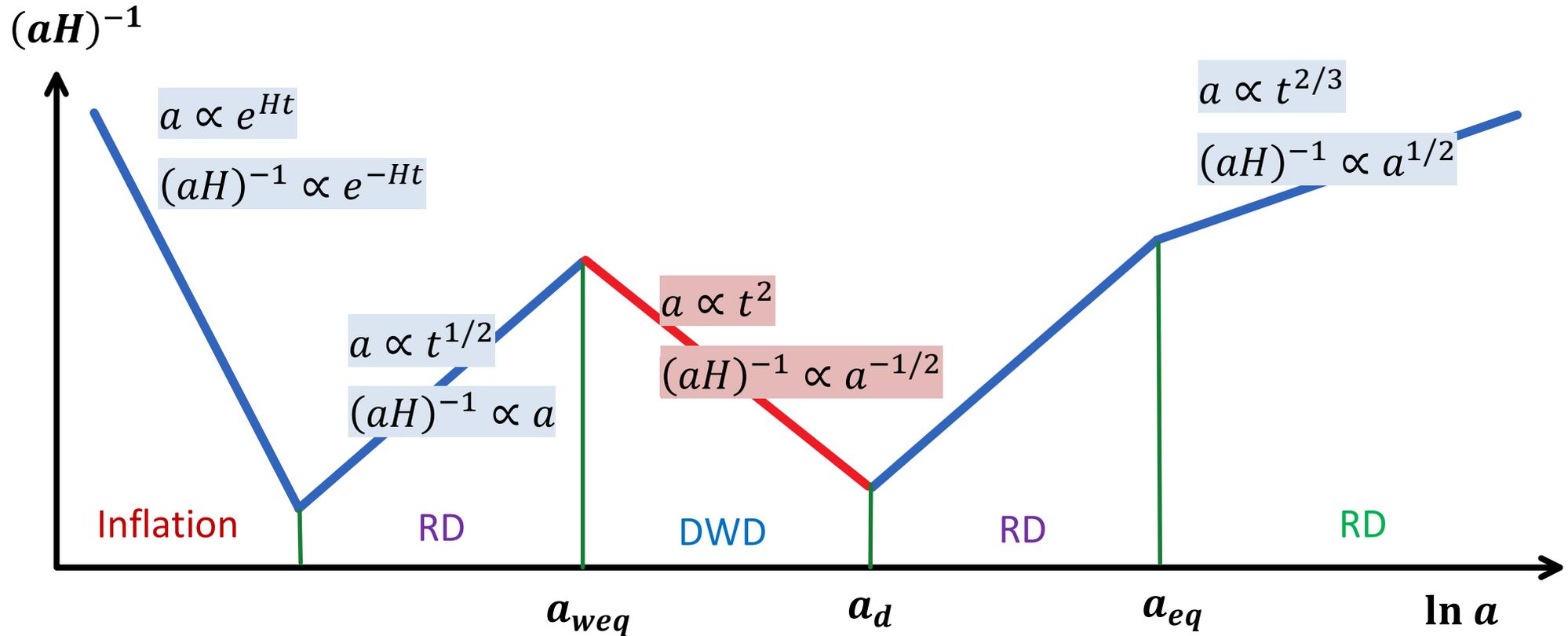
$$(aH)^{-1} \propto a^{(1+3w)/2} \sim \begin{array}{l} a \quad [RD] \\ a^{1/2} \quad [MD] \\ a^{-1/2} \quad [DWD] \end{array}$$

For  $a \sim t^\lambda \rightarrow \ddot{a} \sim \lambda(\lambda - 1)t^{\lambda-2}$  ( $> 0$  for DWD,  $< 0$  for RD/MD)

# III. Domain Wall Dominance



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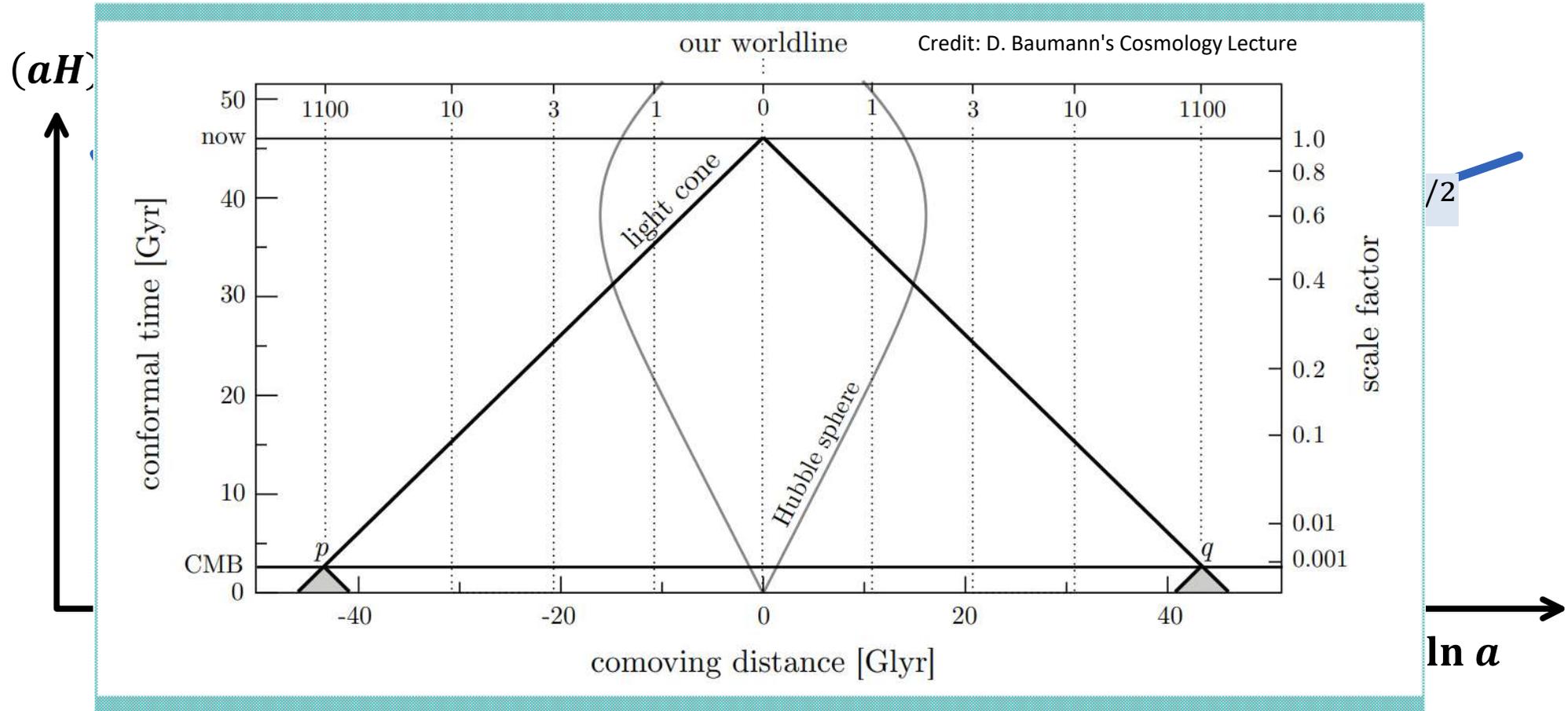


#### [3] Key features of DWD

(1)  $(aH)^{-1}$  at early times, e.g.  $t_{weq} < t < t_d$ , can be **(much) larger** than naïve extrapolation based on RD universe only

⇒ No. of Hubble patches and characteristic wavelength observable today can be quite different.

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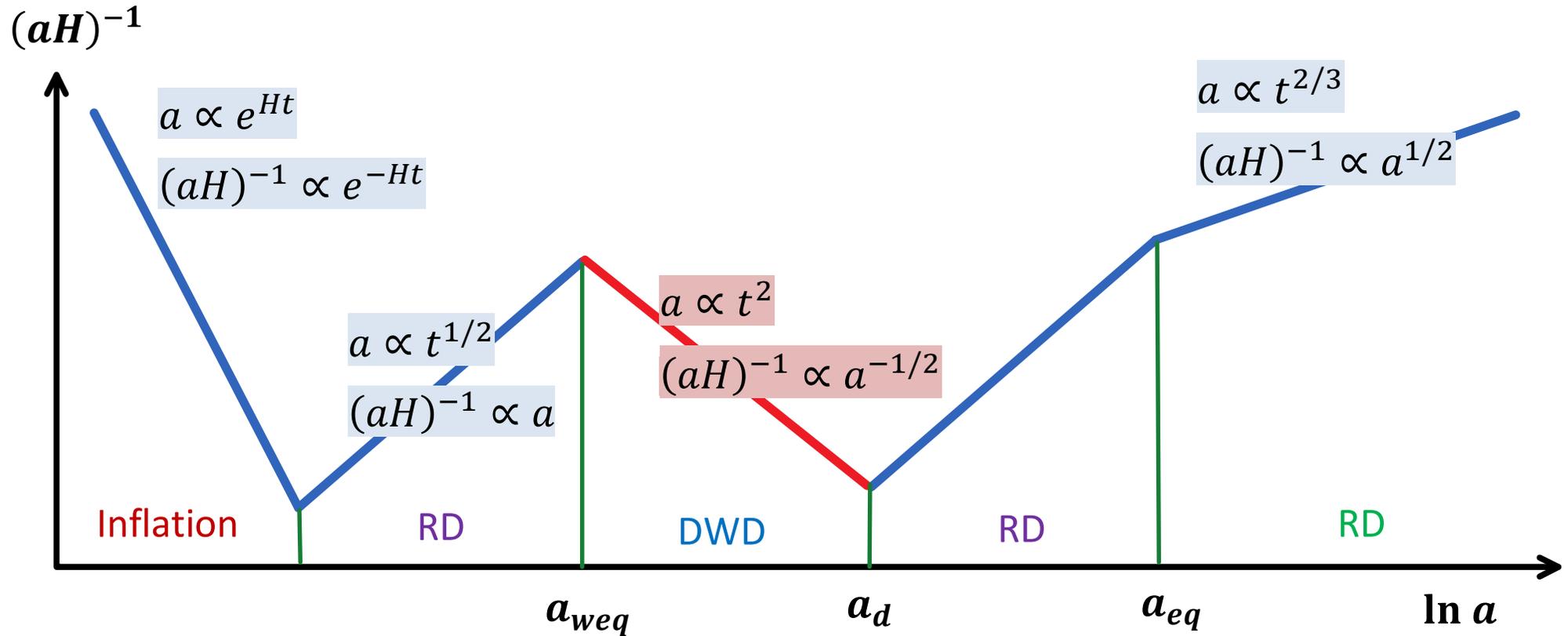


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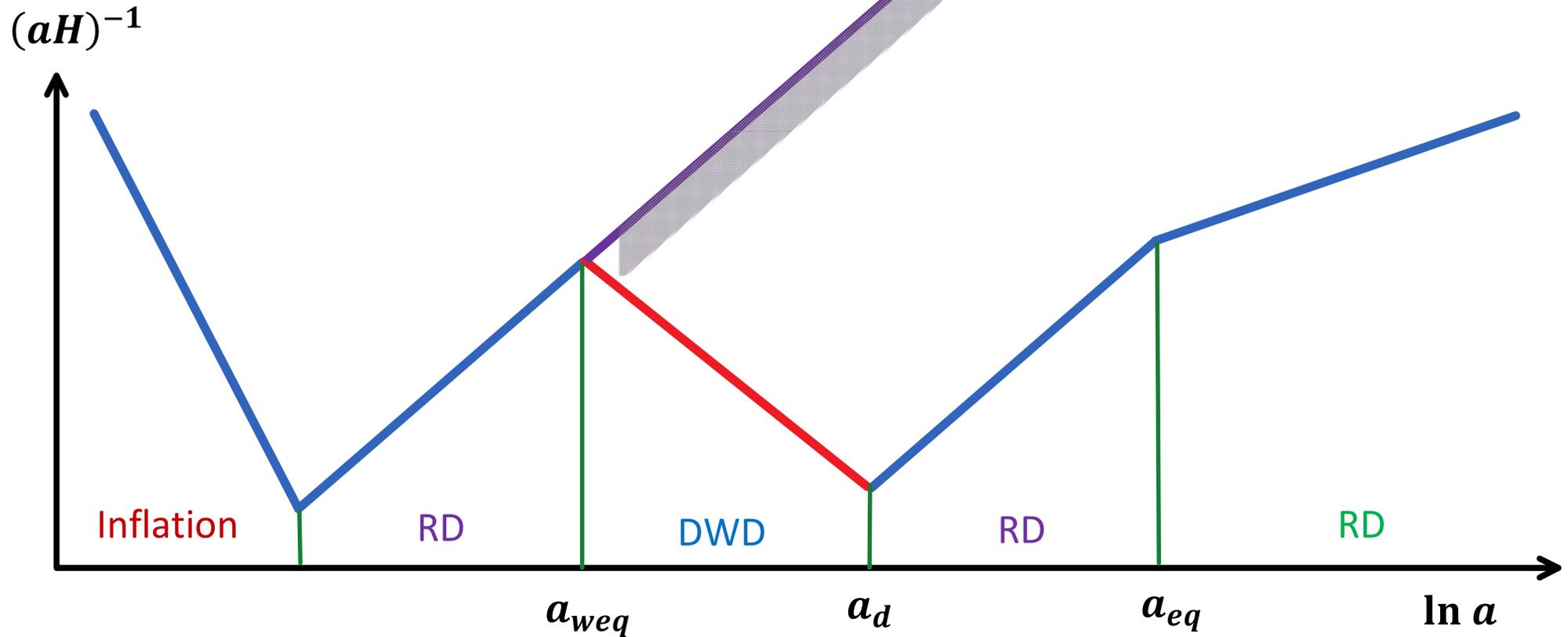
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(2) **Decrease in  $(aH)^{-1}$**  during DWD-phase

⇒ **preservation** of GW produced before and during DWD!

⇒ **New probes** of early universe.

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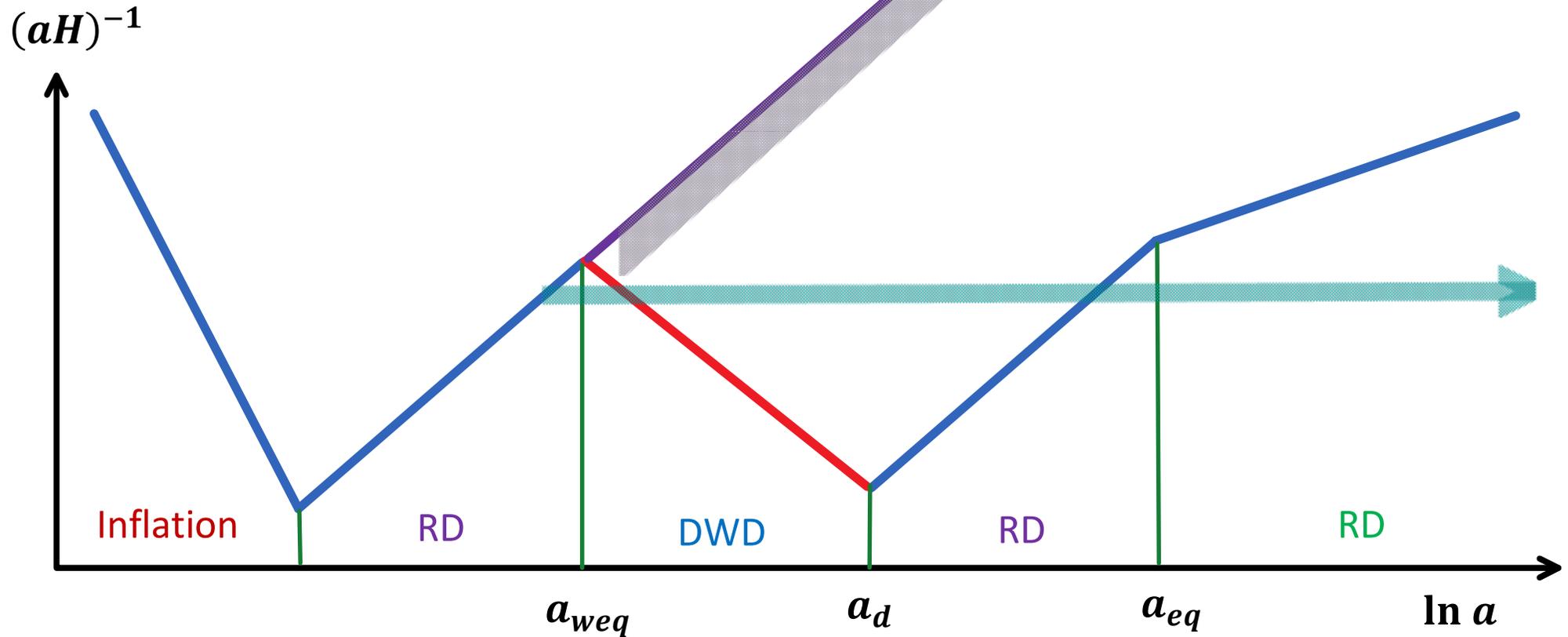
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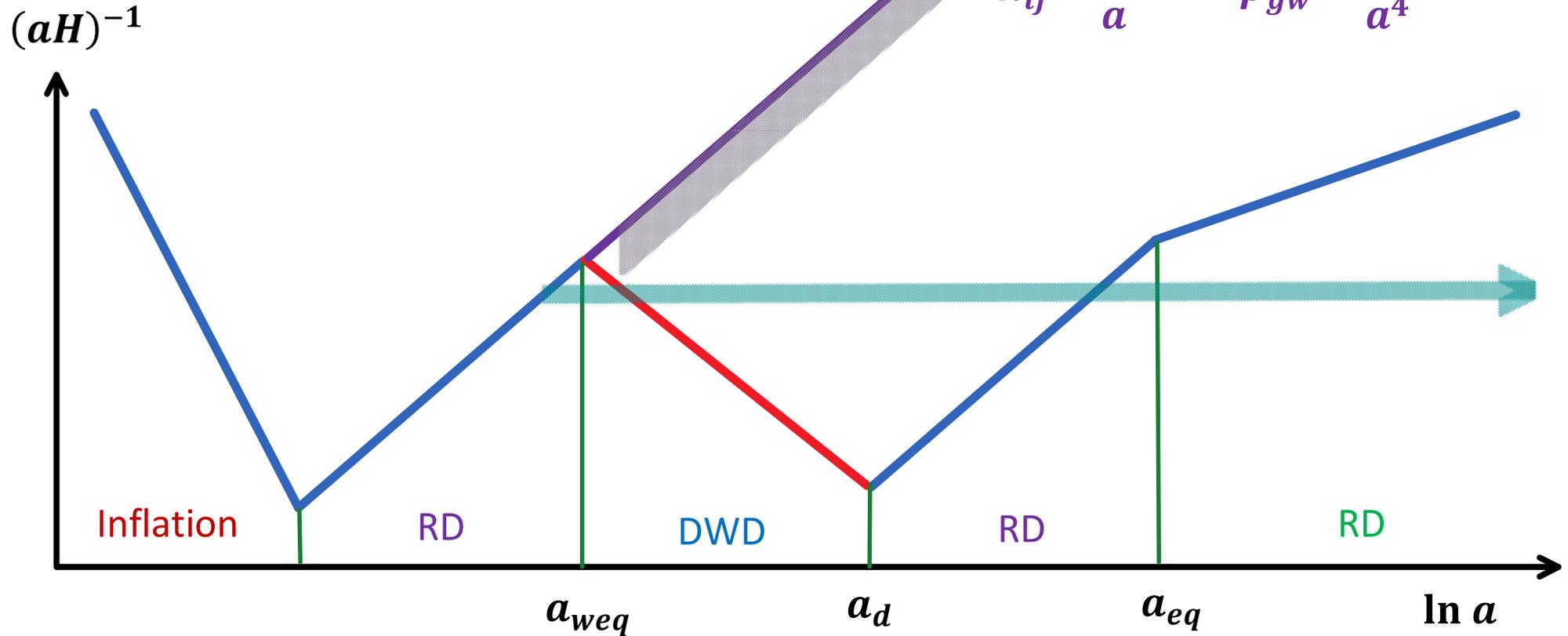
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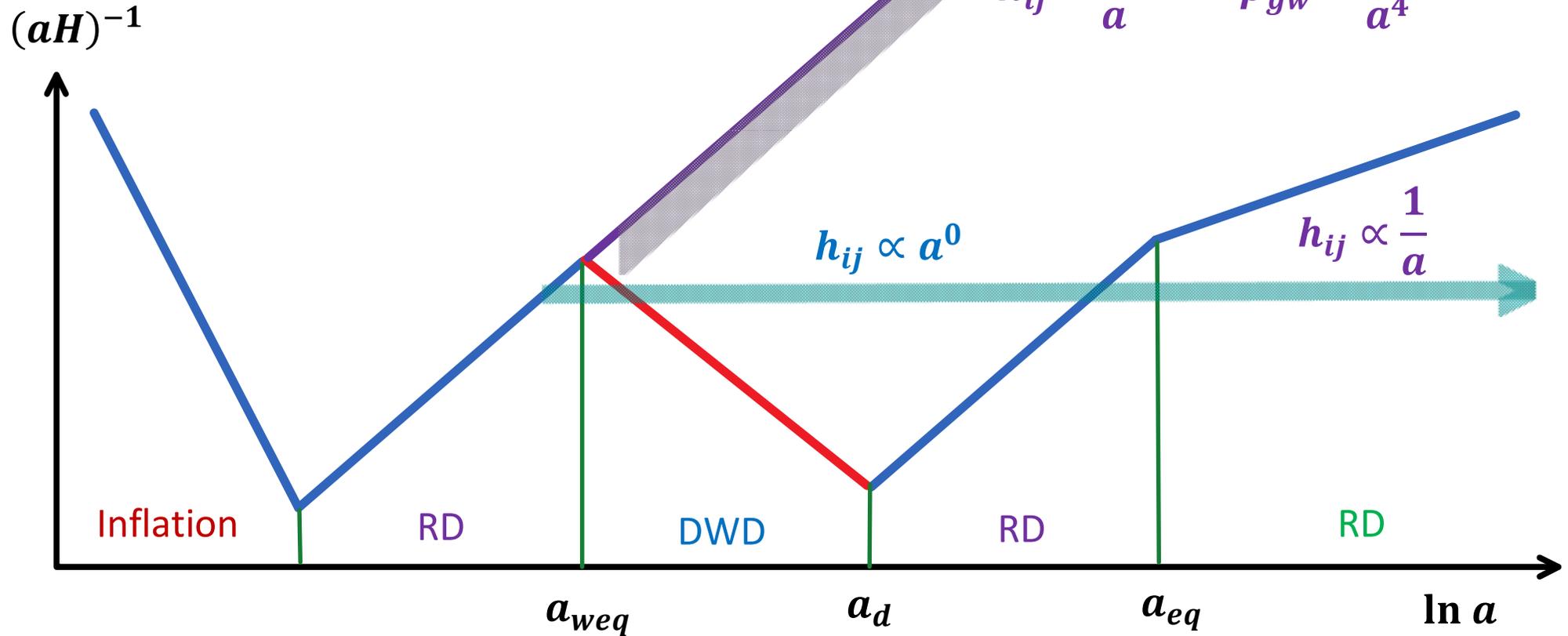
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$\Rightarrow$  **New sensitive probes** of early universe.

# IV. Gravitational Waves with Domain Wall Dominance

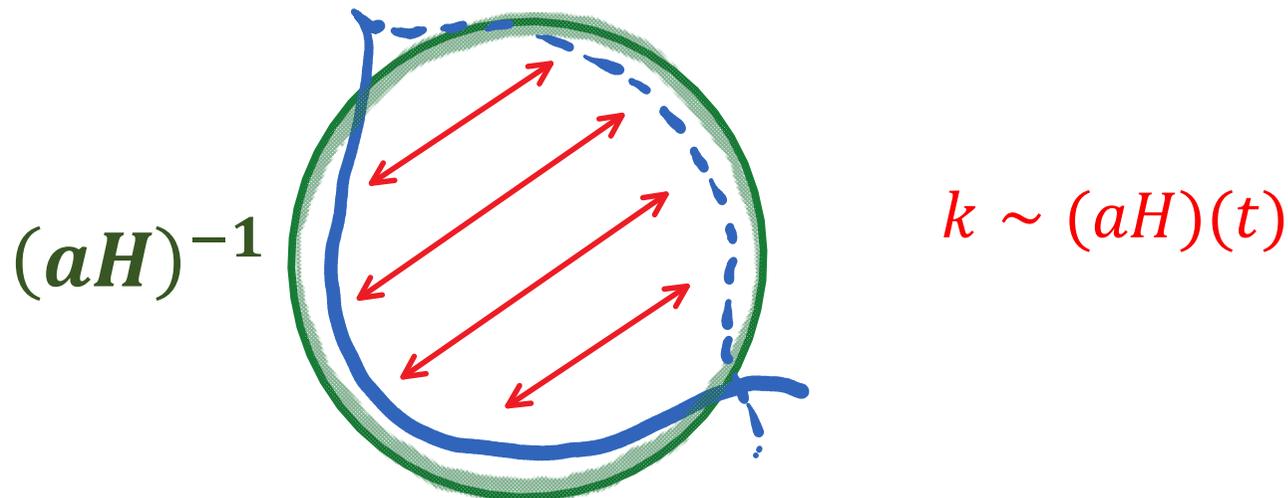
[1] GW production

(1) In scaling regime (SR) during RD

$$\text{GW Power: } P_{gw}^{SR} \sim G \langle \ddot{Q}_{ij} \ddot{Q}_{ij} \rangle \sim G \sigma^2 t^2 \quad [\text{Quadrupole formula}]$$

$$\text{cf. Since } R \sim t, \quad Q_{ij} \sim M_w t^2, \quad M_w \sim \sigma t^2 \quad \Rightarrow \quad \ddot{Q}_{ij} \sim \sigma t$$

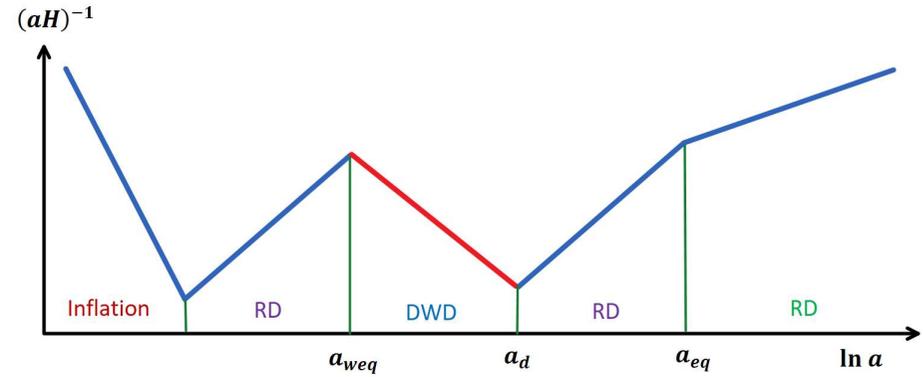
$$p_{gw}^{SR} \sim n P_{gw}^{SR} \sim G \sigma^2 t^2 / t^3 \sim G \sigma^2 / t$$



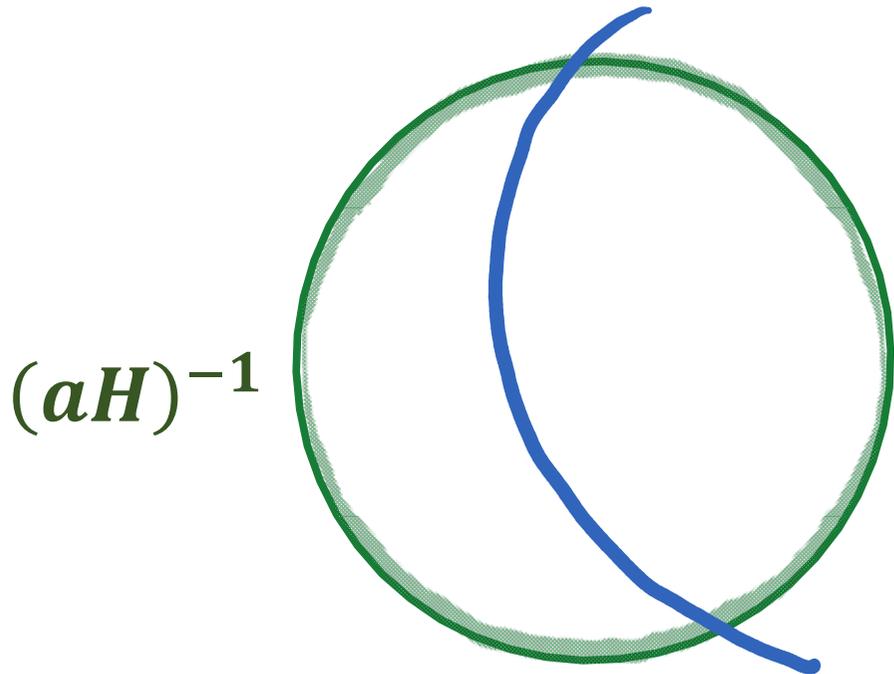
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[1] GW production

(2) During DWD



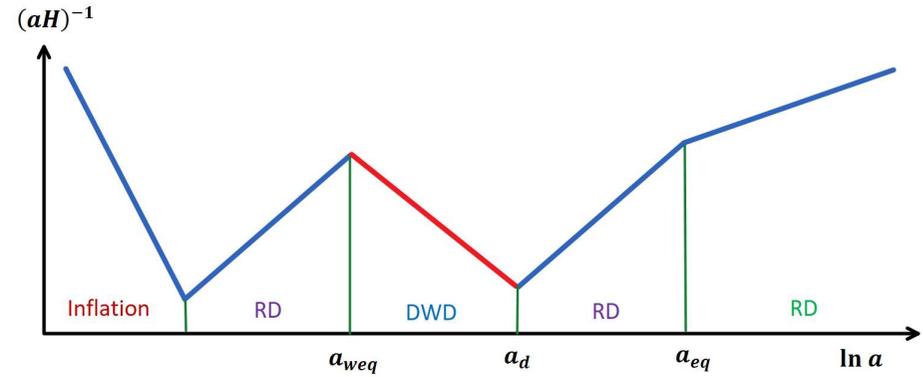
$$\text{GW Power: } P_{gw}^{DWD} \sim G \langle \ddot{Q}_{ij} \ddot{Q}_{ij} \rangle \sim G \sigma^2 \frac{H^{-6}}{R^4} \ll P_{gw}^{SR}$$



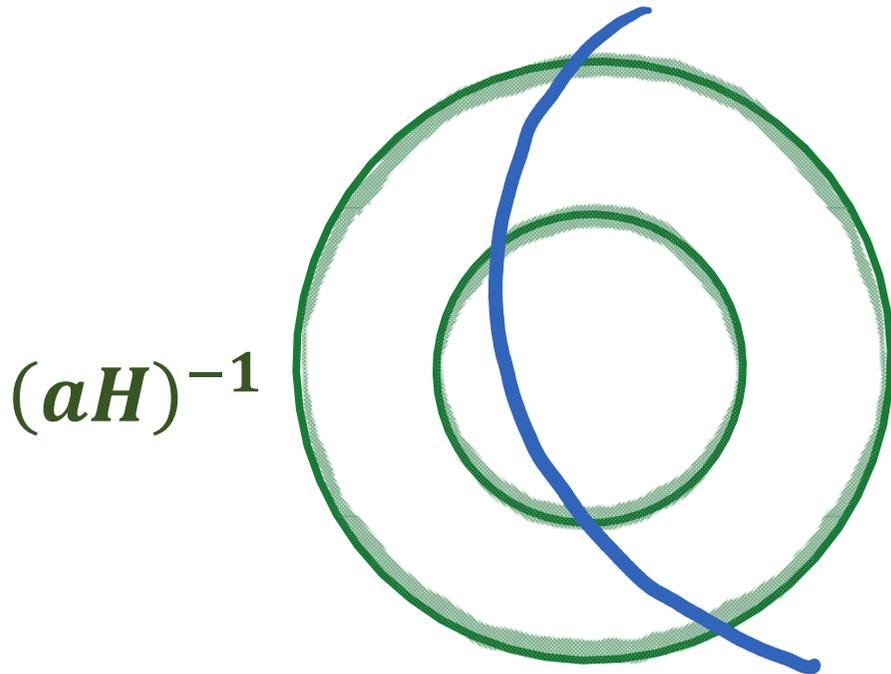
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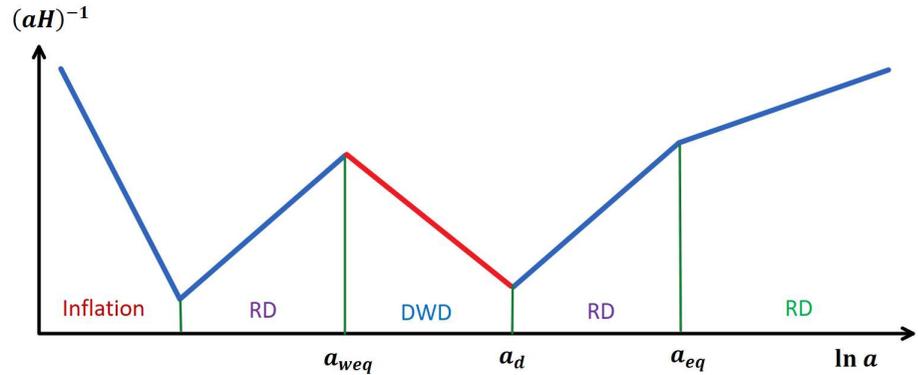
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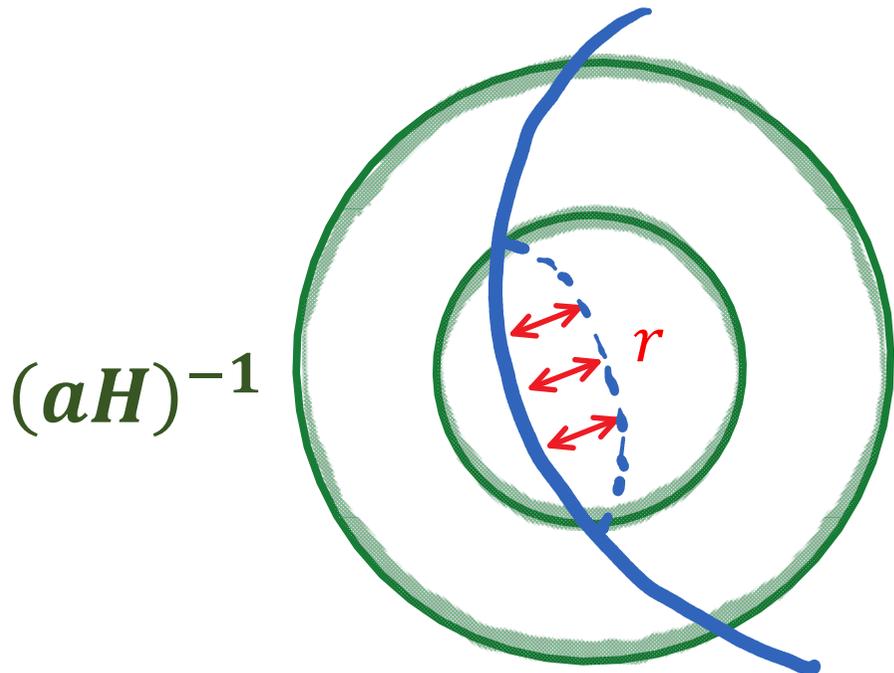
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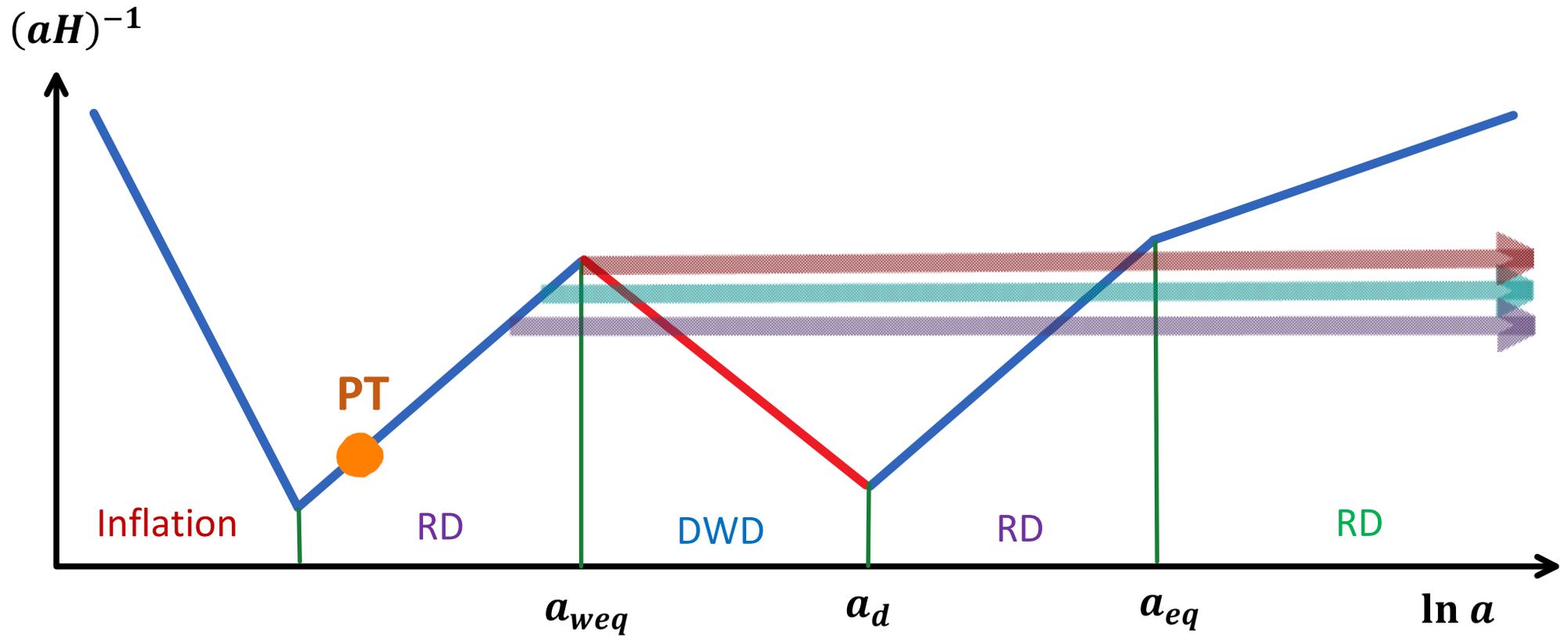
$$r \sim R \left( \frac{H^{-1}}{R} \right)^2 \sim \frac{H^{-2}}{R} \quad (\text{for } H^{-1} \ll R)$$

$$Q_{ij} \sim M_w r^2 \sim (\sigma H^{-2}) \left( \frac{H^{-2}}{R} \right)^2 \sim \frac{\sigma H^{-6}}{R}$$

$$\omega \sim H$$

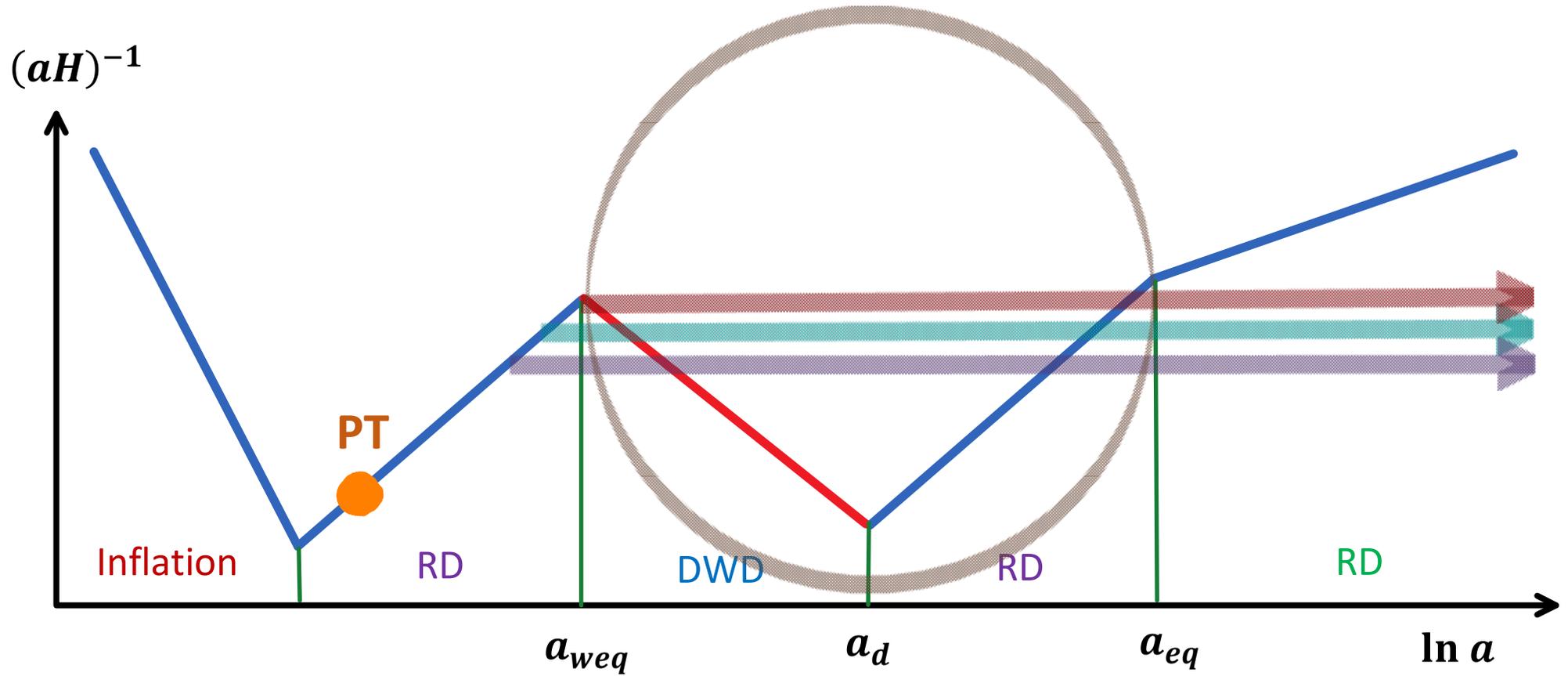
# IV. Gravitational Waves with Domain Wall Dominance

[1] GW production



# IV. Gravitational Waves with Domain Wall Dominance

[1] GW production



## IV. Gravitational Waves with Domain Wall Dominance

[2] Evolution of GW

Linearized Einstein eq:  $h''_{\lambda} + 2\mathcal{H}h'_{\lambda} - \nabla^2 h_{\lambda} = 16\pi G a^2 \Pi^{TT}$   
(cf.  $\lambda = \pm$ ,  $(\cdot)' = d/d\eta$ ,  $\mathcal{H} = aH$ )

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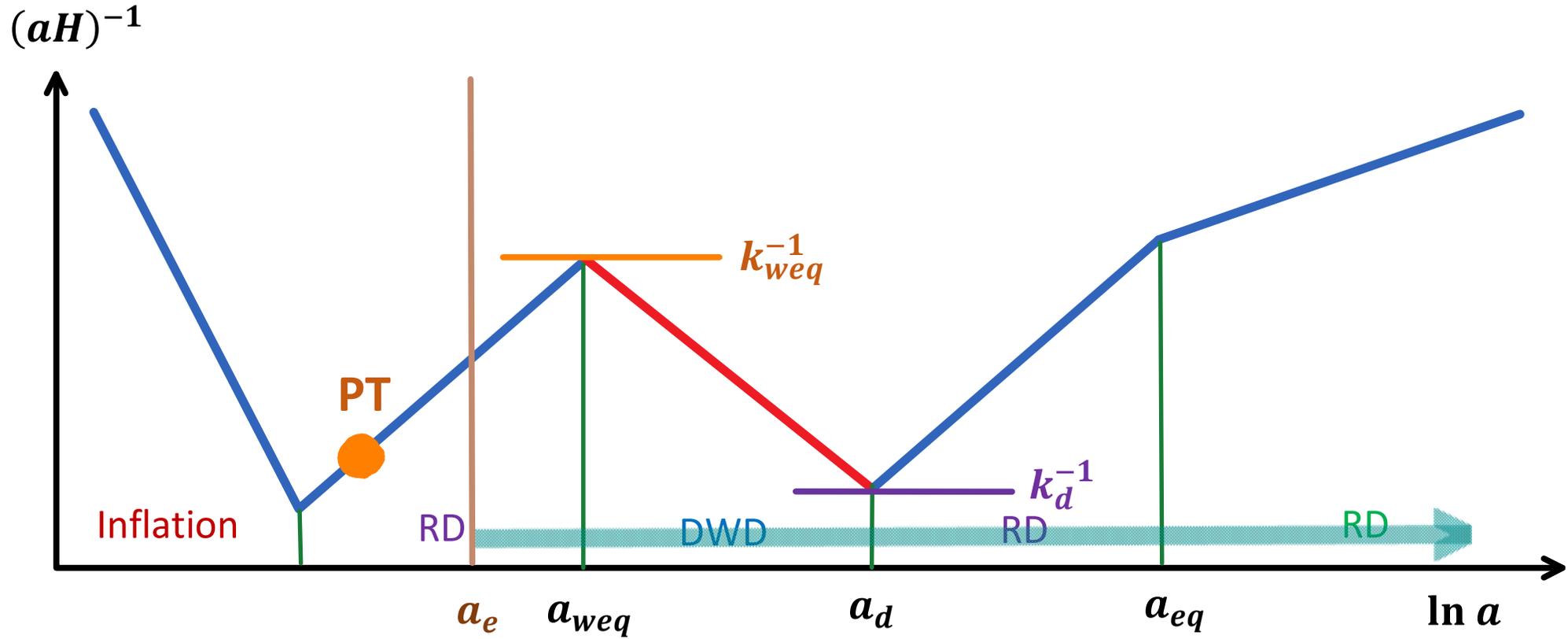
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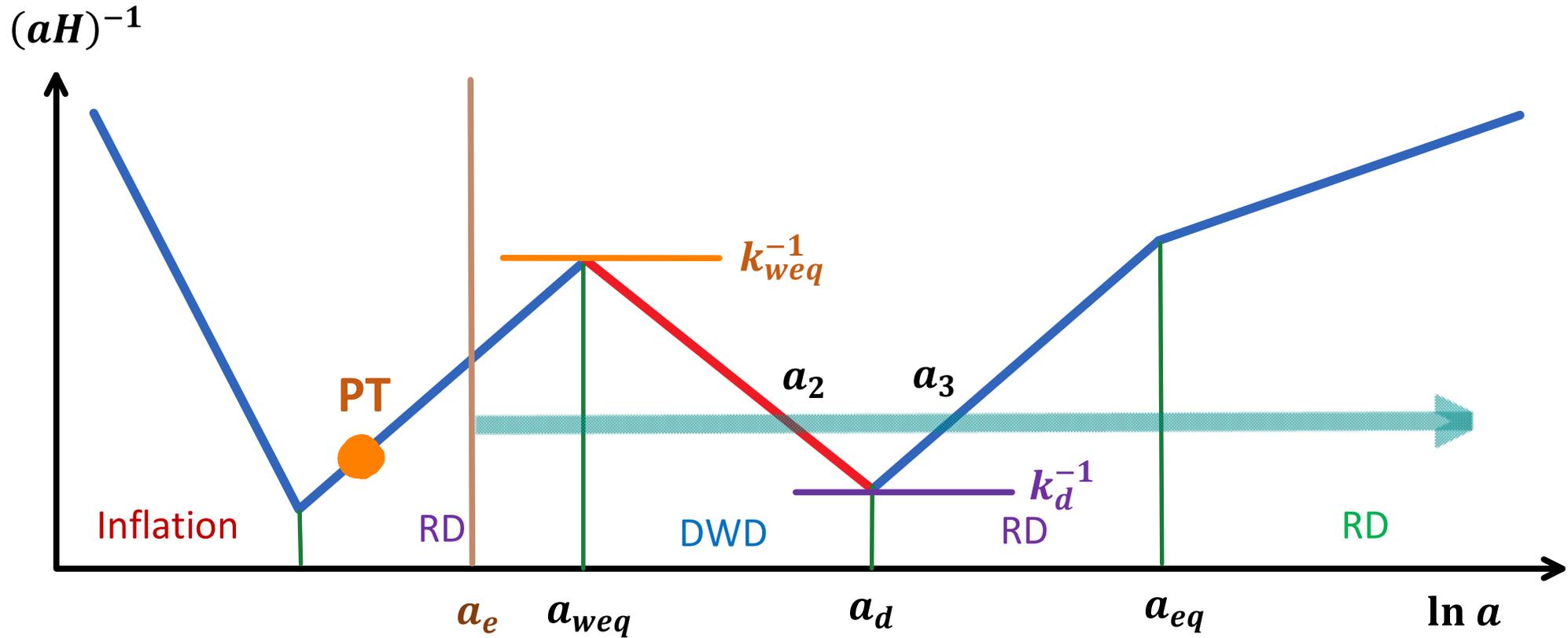
Transfer function:  $h_\lambda(a_0, k) = \mathbf{T}(\mathbf{a}_e, \mathbf{k}) h_\lambda(a_e, k)$

# IV. Gravitational Waves with Domain Wall Dominance



(i)  $k_d < k$  :  $T(a_e, k) = \frac{a_e}{a_0}$

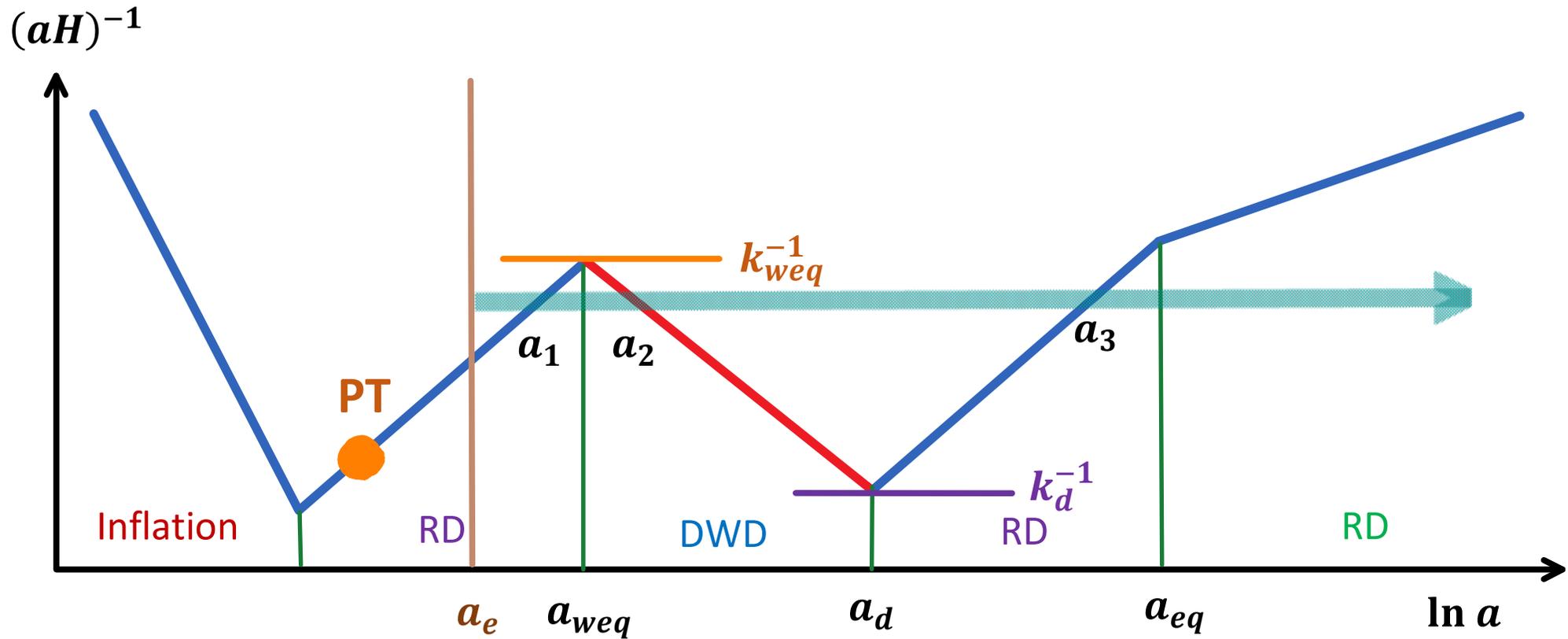
## IV. Gravitational Waves with Domain Wall Dominance



$$(i) \quad k_d < k : \quad T(a_e, k) = \frac{a_e}{a_0}$$

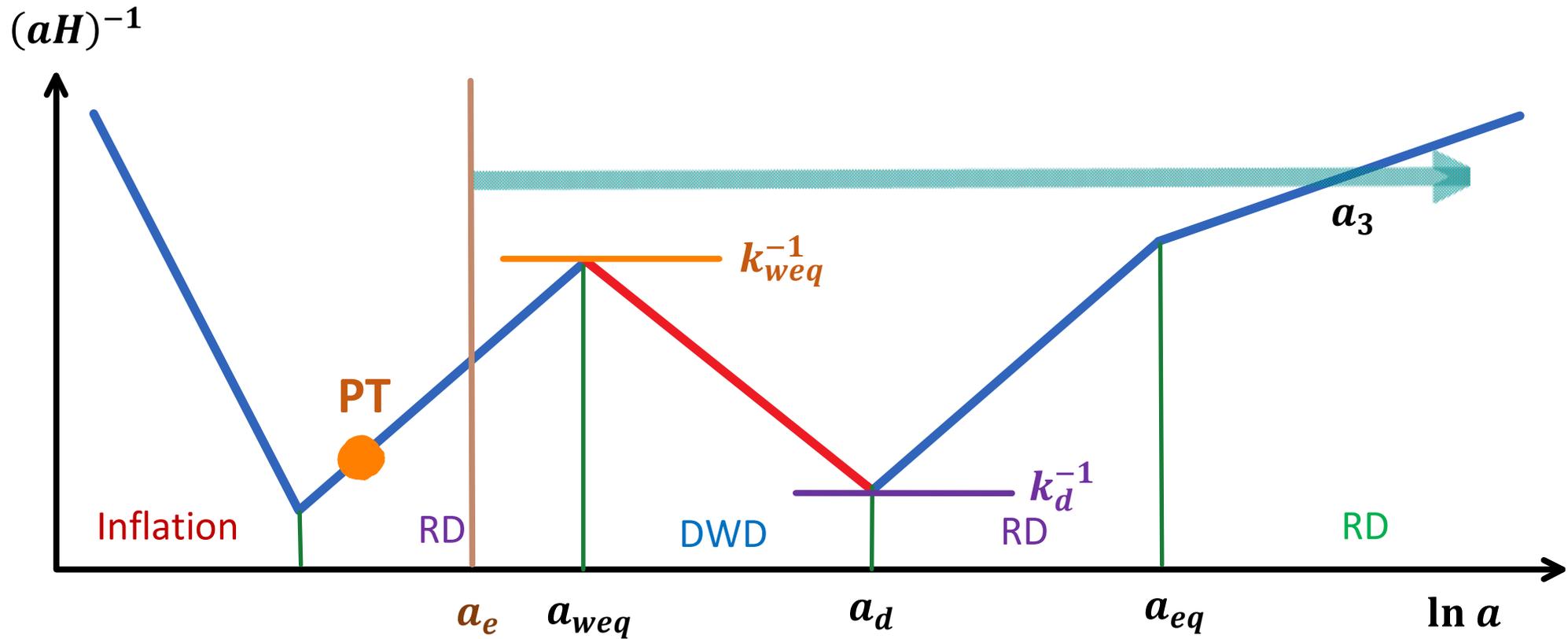
$$(ii) \quad a_e H_e < k < k_d : \quad T(a_e, k) = \left( \frac{a_e}{a_2} \right) \left( \frac{a_3}{a_0} \right)$$

## IV. Gravitational Waves with Domain Wall Dominance



(iii)  $k_{weq} < k < a_e H_e$  :  $T(a_e, k) = \left( \frac{a_1}{a_2} \right) \left( \frac{a_3}{a_0} \right)$

## IV. Gravitational Waves with Domain Wall Dominance



$$(iii) \quad k_{weq} < k < a_e H_e : \quad T(a_e, k) = \begin{pmatrix} a_1 \\ a_2 \end{pmatrix} \begin{pmatrix} a_3 \\ a_0 \end{pmatrix}$$

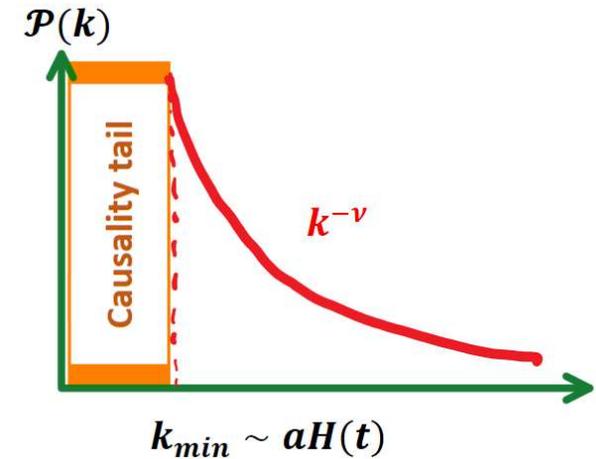
$$(iv) \quad k < k_{weq} : \quad T(a_e, k) = \begin{pmatrix} a_3 \\ a_0 \end{pmatrix}$$

# IV. Gravitational Waves with Domain Wall Dominance

[3] Instantaneous GW Power Spectrum

$$(1) \quad \frac{dp_{gw}}{dk_e}(t_e, k_e) = p_{gw}(t_e) \mathcal{P}(k_e)$$

$$\text{PDF: } \mathcal{P}(k_e) = \frac{\nu-1}{k_{min}^{-\nu+1}(t_e)} k_e^{-\nu} \Theta(k_e - k_{min}(t_e))$$

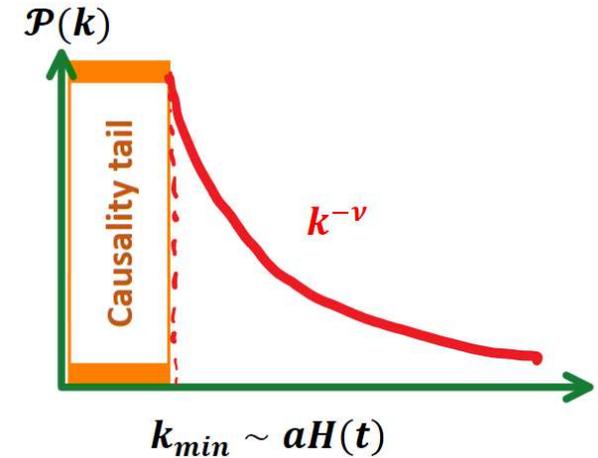


# IV. Gravitational Waves with Domain Wall Dominance

## [3] Instantaneous GW Power Spectrum

$$(1) \quad \frac{d\rho_{gw}}{dk_e}(t_e, k_e) = \rho_{gw}(t_e) \mathcal{P}(k_e)$$

$$\text{PDF: } \mathcal{P}(k_e) = \frac{\nu-1}{k_{min}^{-\nu+1}(t_e)} k_e^{-\nu} \Theta(k_e - k_{min}(t_e))$$



## (2) GW energy spectrum observed today

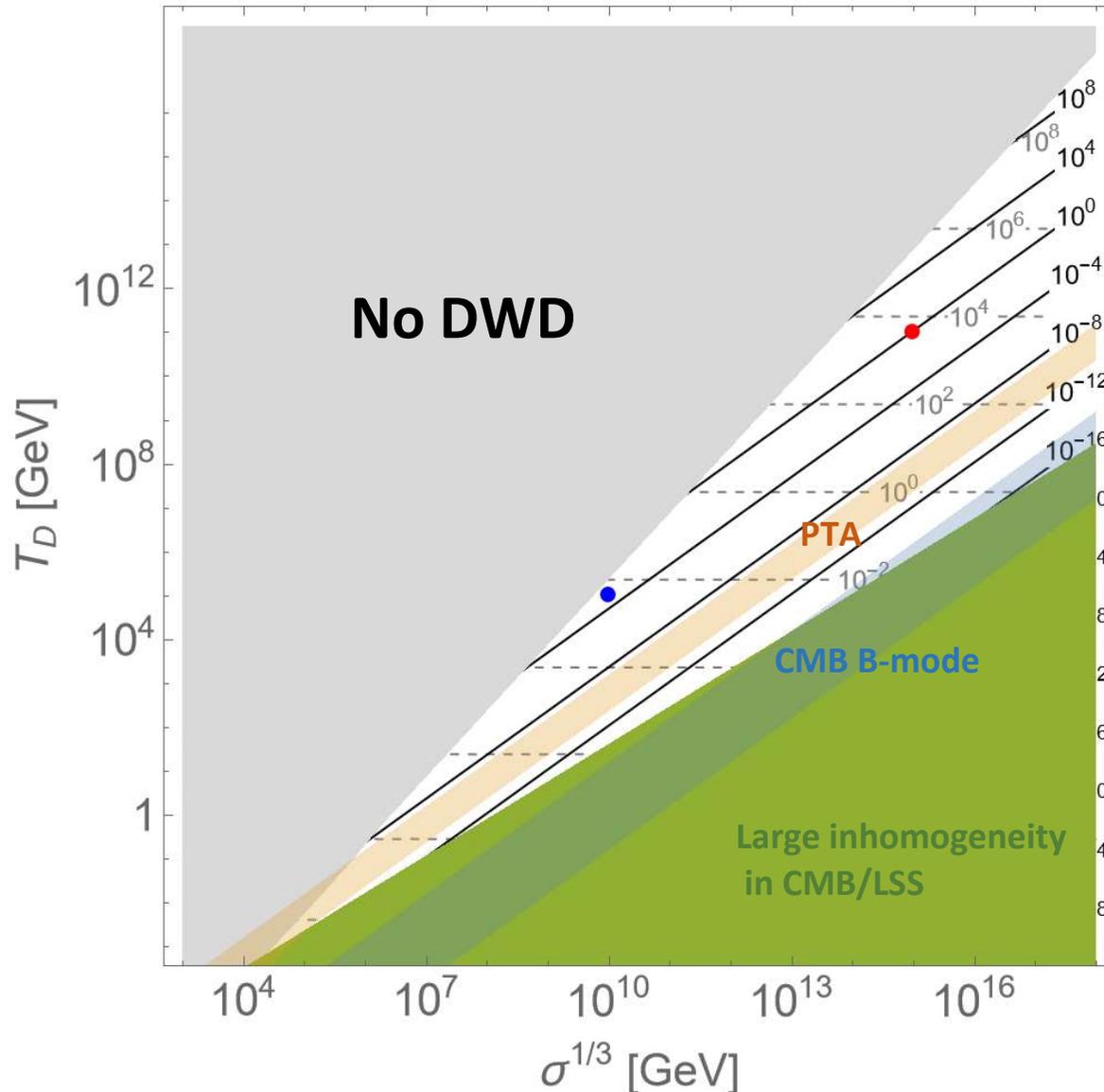
$$\frac{d\rho_{gw}}{dk}(t_0) = \int_{t_*}^{t_0} dt a(t) |T(a, k)|^2 \rho_{gw}(t) \mathcal{P}\left(\frac{k}{a}\right)$$

$$h^2 \Omega_{gw}(k) = \frac{h^2}{\rho_{c,0}} \frac{d\rho_{gw}}{d \log k}(t_0) \propto \begin{cases} k^{-\nu+1} & (\nu < 5) \\ k^{-4} & (\nu \geq 5) \end{cases}$$

Numerical simulations:  $\Omega_{gw} \propto k^{-1}$  vs  $k^{-1.7 \sim 1.8} \Rightarrow \nu \sim 2 - 3$

# IV. Gravitational Waves with Domain Wall Dominance

## [4] Results



**[BM1]**  $T_d = 10^5 \text{ GeV}$   
 $\sigma^{1/3} = 10^{10} \text{ GeV}$

**[BM1]**  $T_d = 10^{11} \text{ GeV}$   
 $\sigma^{1/3} = 10^{15} \text{ GeV}$

Black solid

$$f_{weq} = \frac{k_{weq}}{2\pi}$$

$$\approx 4.5 \times 10^{12} \left( \frac{T_d^3}{\sigma} \right) \text{ Hz}$$

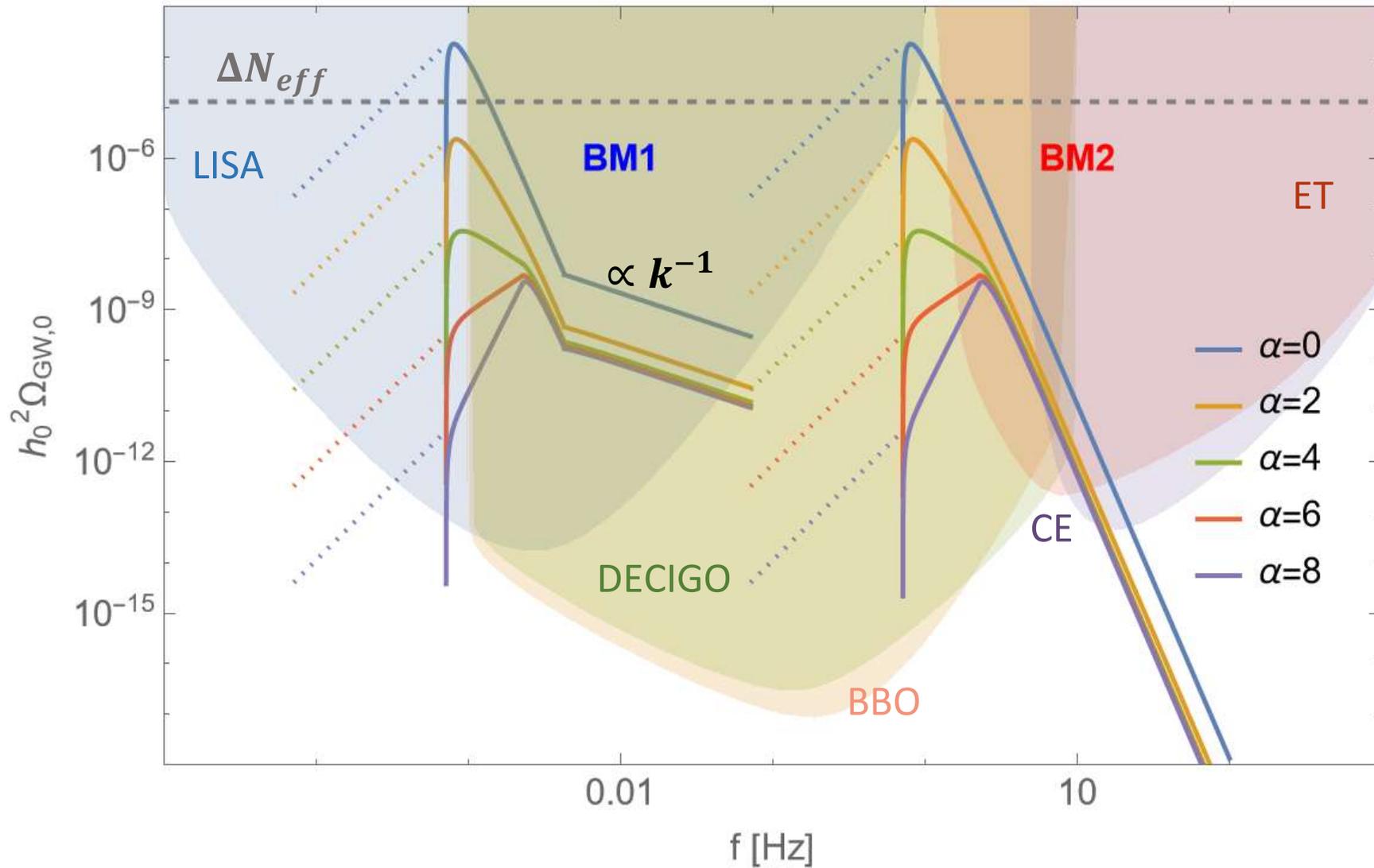
Gray dashed

$$f_d = \frac{k_d}{2\pi} \sim k_{eq} \left( \frac{t_{eq}}{t_d} \right)^{1/2}$$

$$\approx 2.7 \times 10^{-7} \left( \frac{T_d}{\text{GeV}} \right) \text{ Hz}$$

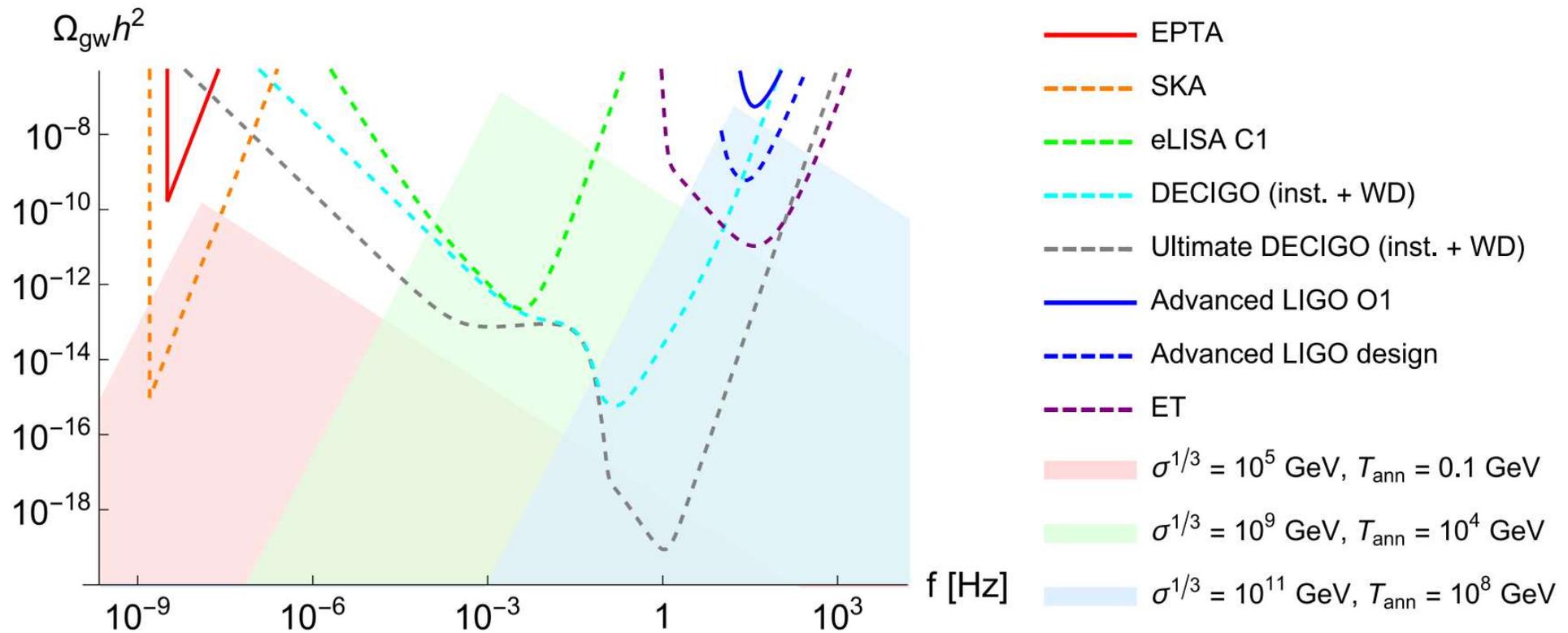
# IV. Gravitational Waves with Domain Wall Dominance

[4] Results:  $\nu = 2$



# IV. Gravitational Waves with Domain Wall Dominance

## [4] Results



<https://arxiv.org/abs/1703.02576>

**Thank you  
For  
Your Attention!**