

Aspects of axion dark matter

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Based on [arXiv:2207.06884](https://arxiv.org/abs/2207.06884) (PRD 2024) and upcoming papers with Sang Hui Im (CTPU); K.S. Jeong, D.-h.Yeom (PNU); TaeHun Kim (KIAS)

Momentum kick by axions

Axion-fermion couplings and a momentum kick

Atoms and axion dark matter

Atoms under axion dark matter

Chiral magnetic effects

Anomaly in Fermi liquid

Axial anomaly, CME in medium

Detecting axion dark matter

LACME

Heating up

Conclusions

1. Momentum kick by axions

momentum kick

- ▶ As axion is introduced to solve the strong CP problem, it couples to $G\tilde{G}$ of QCD that breaks CP and P. (Peccei-Quinn 1977; Weinberg, Wilczek 1978)
- ▶ Axions therefore couple to all SM particles via loops or at the tree level.
- ▶ Furthermore, if the axions are the main component of dark matter in our universe as CDM, it oscillates coherently:

$$a(t) = \frac{\sqrt{2\rho_{\text{DM}}}}{m_a} \sin(m_a t),$$

neglecting the axion velocity, $|\vec{\nabla}a/\dot{a}| \ll 1$, compared to the speed of light.

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momentum kick

- ▶ At the leading order, the axion-electron coupling behaves as an axial chemical potential, μ_5 , regulating the helicity in medium.

$$\mathcal{L}_{ae} = C_e \frac{\partial_\mu a}{f} \bar{\Psi} \gamma^\mu \gamma^5 \Psi \approx \mu_5 \bar{\Psi} \gamma^0 \gamma^5 \Psi, \quad \mu_5 = C_e \frac{\dot{a}}{f}$$

- ▶ By rewriting the gamma matrices

$$\gamma^0 \gamma^5 = \frac{2}{3} \vec{\gamma} \cdot \vec{\Sigma}, \quad \Sigma^i \equiv \frac{i}{4} \epsilon^{ijk} \gamma^j \gamma^k$$

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- ▶ The Lagrangian density of free electrons in the background (homogeneous) axion DM becomes

$$\mathcal{L} = \bar{\Psi} \left[\gamma_0 \cdot i\partial_t - \vec{\gamma} \cdot (i\vec{\nabla} + \frac{2}{3}\mu_5\vec{\Sigma}) - m \right] \Psi.$$

- ▶ The electron gets a momentum kick along its spin by ADM:

$$\vec{p} \mapsto \vec{p} + \delta\vec{p}, \quad \delta\vec{p} = \frac{2}{3}\mu_5\vec{\Sigma}$$

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Recap

- Being a coherent field, Axion DM behaves like a axial chemical potential:

$$\mathcal{L}_{ae} = C_e \frac{\partial_\mu a}{f} \bar{\Psi} \gamma^\mu \gamma^5 \Psi \approx \mu_5 \bar{\Psi} \gamma^0 \gamma^5 \Psi$$

- It adds a momentum kick to fermions:

$$\vec{p} \rightarrow \vec{p} + \frac{2}{3} C_f \frac{\dot{a}}{f} \vec{S}.$$

Momentum kick

- ▶ The momentum kick to electrons in vacuum has no effect in general due to the principle of relativity.
- ▶ It can have however interesting physical effects for a well-defined reference frame such as
- ▶ **Electrons in medium**: it has chiral magnetic effects (Fukushima-Kharzeev-Warringa '08; DKH-Jeong-Im-Yeom, '22)

$$\langle \vec{j} \rangle = v_F \frac{e^2}{2\pi^2} \mu_5 \vec{B}$$

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Electrons in atom: energy level split

- ▶ Since the electron momentum shifts along its spin by the axion DM, $\delta\vec{p} = 2\vec{S}\mu_5/3$, the energy spectrum of atoms gets modified:

$$H = \frac{(\vec{p} + \delta\vec{p})^2}{2m} - \frac{e^2}{4\pi r} + H' = H_0 + \delta H,$$

- ▶ The perturbation due to the axion DM is then

$$\delta H \simeq \frac{2\mu_5}{3m} \vec{p} \cdot \vec{S}$$

- ▶ The perturbation δH is parity odd and carries an angular momentum $l = 1$. It will therefore lift the energy degeneracy for states with $\Delta l = 1$.

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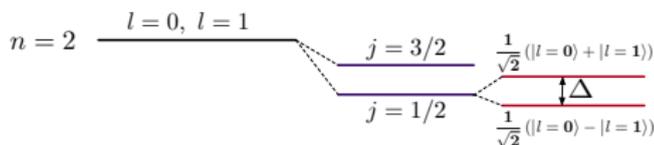
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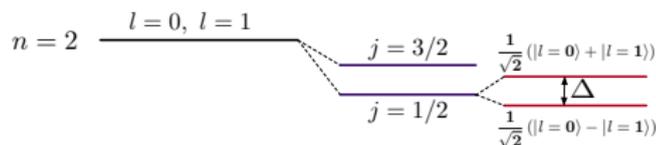
$$\Delta = \frac{\hbar^2}{8\sqrt{6}a} \cdot \frac{\mu_5}{m} \approx 2 \times 10^{-25} \text{ eV} \cos(m_{at}).$$

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Electrons in atom: atomic EDM

- ▶ The proton in the nucleus of hydrogen does not rest static in the medium of axions, getting a constant kick from coherent axions. With $\mu_5^p = c_{ap}\sqrt{2\rho_a}/f_{ap}$

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- ▶ Because of the jiggling proton, the electron Coulomb potential gets modified:

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$$d_p \approx 2 \times 10^{-20} e \text{ cm} \left(\frac{10^{-6} \text{ eV}}{m_a} \right) \cdot \left(\frac{\rho_a}{0.4 \text{ GeV/cm}^3} \right)^{1/2} \cdot \left(\frac{10^{10} \text{ GeV}}{f_a/c_{ap}} \right) \cdot$$

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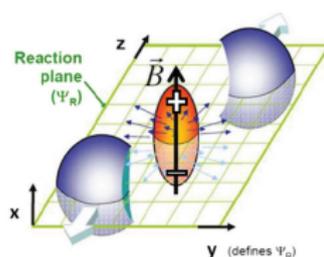
2. Chiral Magnetic Effects

Chiral magnetic effects

What is chiral magnetic effect?

- ▶ **Chiral magnetic effect** is that magnetic field generates a persistent electric current in Fermi liquid if its **axial chemical potential** $\mu_5 \neq 0$ (Fukushima+Kharzeev+Warringa '08):

$$\vec{J} = a\mu_5\vec{B}$$



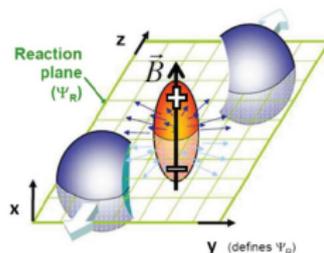
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Anomaly in Fermi liquid

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$$\langle \partial_\mu J_5^\mu \rangle_A = v_F \frac{e^2}{16\pi^2} \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma}$$

- ▶ and Fermi liquid exhibits CME if $\mu_5 \neq 0$:

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- ▶ Consider the three-point function of chiral currents with $q_i^2 = -Q^2$ (Coleman+Grossman 1982) :

$$\Gamma_{\mu\nu\lambda}\delta^{(4)}(q_1 + q_2 + q_3) \equiv \int \Pi_i d^4x_i e^{ix_i \cdot q_i} \langle 0 | T j_\mu(x_1) j_\nu(x_2) j_\lambda(x_3) | 0 \rangle$$

$$\Gamma_{\mu\nu\lambda}(q_1, q_2, q_3) = F(Q^2) [\epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta q_{3\lambda} + \dots]$$

$$q_3^\lambda \Gamma_{\mu\nu\lambda} = \frac{1}{8} a \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta.$$

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Anomaly in Fermi liquid

- ▶ Fermi liquid is gapless and consists of two chiral modes. Under the magnetic field only one of them becomes gapless, contributing to anomaly.
- ▶ The (anomalous) axial symmetry of Fermi liquid is the phase rotation of the spin-up/down component of ψ_+ . Namely the helicity of $\psi_+ = \vec{\alpha} \cdot \hat{v}_F \psi_+$ corresponds to the axial charge.

$$Q_A = \langle \bar{\psi} \gamma^0 \gamma_5 \psi \rangle = 2 \langle \psi_+^\dagger \vec{S} \cdot \hat{v}_F \psi_+ \rangle,$$

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- ▶ Under a magnetic field, electrons have the Landau levels:

$$E_n(p_z) = \pm \sqrt{p_z^2 + m^2 + 2|eB|n},$$

with $2n = 2n_r + 1 + |m_L| - \text{sign}(eB)(m_L + 2s_z)$.

- ▶ LLL electrons are moving along the B field, $\vec{p} = p_z \hat{z}$, with spins polarized **opposite** to the \vec{B} field, while spins are paired for all other electrons in $n \neq 0$.
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Axial anomaly in medium

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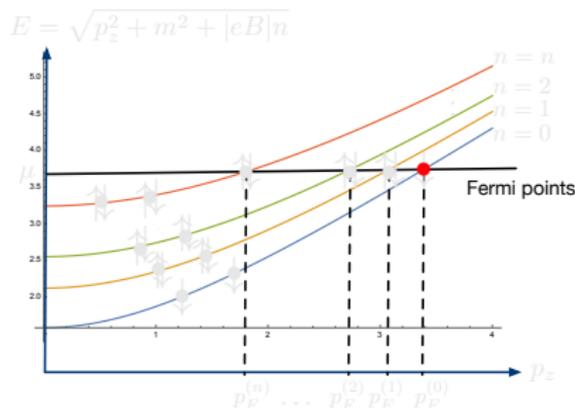


Figure: Fermi liquid under weak field

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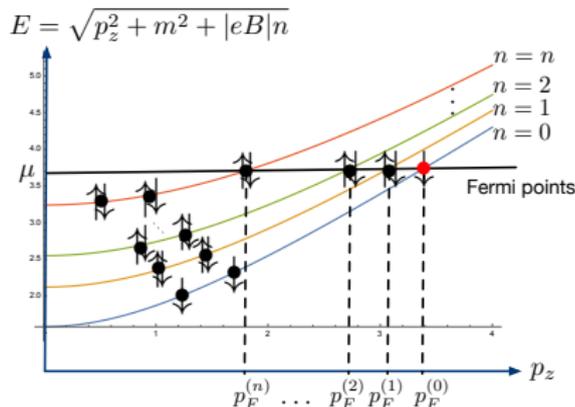


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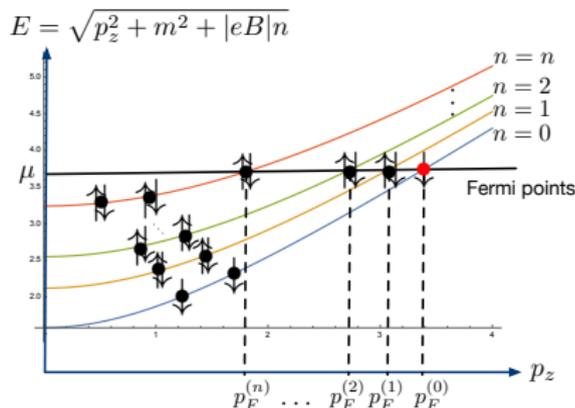


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Axial anomaly in medium

- ▶ To calculate the **ABJ anomaly in Fermi liquid** we consider the anomalous two-point function of LLL electrons in medium, which are **effectively 2-dimensional**:

$$\Gamma_5^{\mu\nu}(q_1)\delta^{(2)}(q_1 + q_2) \equiv \int \Pi_i d^2x_i e^{iq_i \cdot x_i} \langle 0 | T j^\mu(x_1) j_5^\nu(x_2) | 0 \rangle .$$

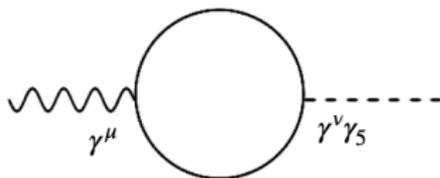


Figure: ABJ anomaly by LLL in Fermi Liquid

Axial anomaly in medium

- ▶ For $q/\mu \rightarrow 0$ (Manuel '96: DKH '98), using $j_5^\nu = \epsilon^{\nu\alpha} j_\alpha$, the anomalous two-point function of LLL becomes

$$\Gamma_5^{\mu\nu}(q) = \frac{eB}{2\pi^2 v_F} \left[-\eta^{\mu 0} \epsilon^{\nu 0} + \frac{q^0}{2} \left(\frac{V^\mu \epsilon^{\nu\alpha} V_\alpha}{V \cdot q} + \frac{\bar{V}^\mu \epsilon^{\nu\alpha} \bar{V}_\alpha}{\bar{V} \cdot q} \right) \right],$$

where $V^\mu = (1, 0, 0, v_F)$ and $\bar{V}^\mu = (1, 0, 0, -v_F)$.

- ▶ The vector current is conserved:

$$q_\mu \Gamma_5^{\mu\nu}(q) = 0.$$

- ▶ The axial current is however anomalous:

$$\langle \partial_\nu j_5^\nu \rangle_A = ie \int \frac{d^2 q}{4\pi^2} \lim_{q_0 \rightarrow 0} \lim_{q_3 \rightarrow 0} e^{iq \cdot x} q_\nu A_\mu(q) \Gamma_5^{\mu\nu}(q) = \frac{e^2 B}{4\pi^2} v_F \epsilon^{\mu\nu} F_{\mu\nu}.$$

Axial anomaly in medium

- ▶ The ABJ anomaly becomes in the rest frame of the medium

$$\langle \partial_\nu j_5^\nu \rangle_A = \frac{e^2}{16\pi^2} v_F \epsilon^{\mu\nu\rho\sigma} F_{\mu\nu} F_{\rho\sigma} .$$

- ▶ From the anomalous two-point function one can calculate the CME, in the leading order in μ_5 .

$$\langle j^3 \rangle_B = -e\mu_5 \lim_{q_0 \rightarrow 0} \lim_{q_3 \rightarrow 0} \Gamma_5^{30}(q) = \frac{e^2 B}{2\pi^2} v_F \mu_5 .$$

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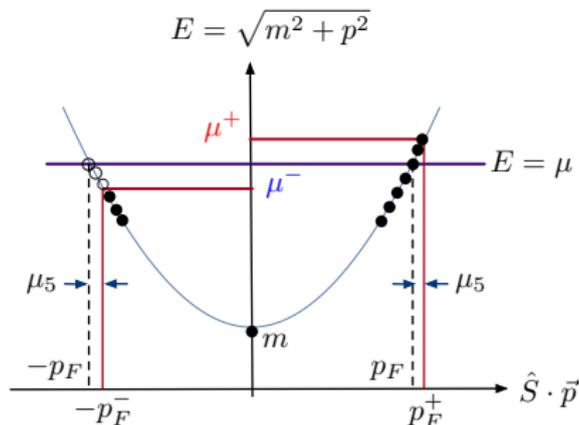
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Electrons in medium

- ▶ μ shifts the energy to create an imbalance in particle number:

$$E \rightarrow E + \mu$$

- ▶ μ_5 shifts the momentum along the spin direction to create an helicity imbalance in medium: $\hat{S} \cdot \vec{p} \rightarrow \hat{S} \cdot \vec{p} + \mu_5$

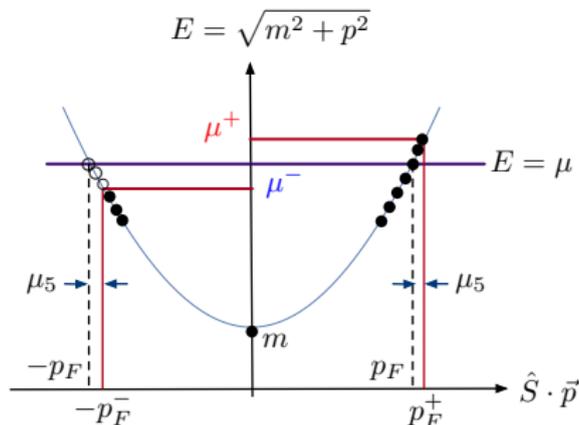


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chiral magnetic effects in chiral medium

- ▶ μ_5 creates the helicity imbalance in **medium under magnetic field, \vec{B}** :

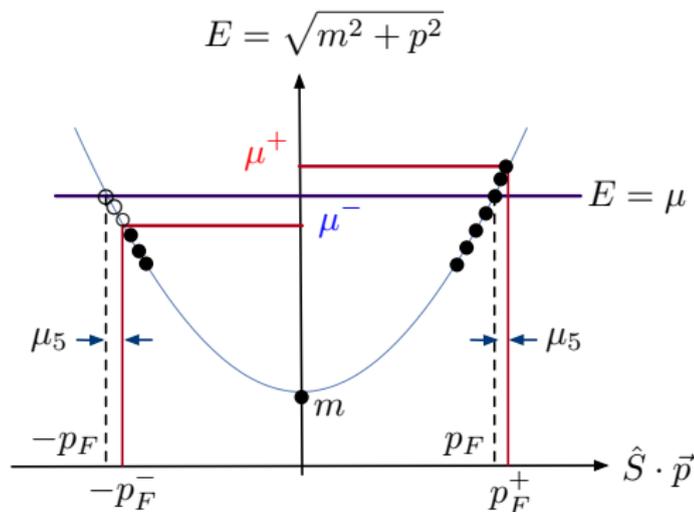


Figure: Polarized medium with μ_5 . ($\hat{S} \cdot \vec{p} = \hat{B} \cdot \vec{p}$)

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- Helicity imbalance in LLL electrons due to μ_5 :

$$\Delta\rho = \rho_{h=+1}^{n=0} - \rho_{h=-1}^{n=0} = \frac{|eB|}{4\pi^2} (p_F^+ - p_F^-) = \frac{|eB|}{2\pi^2} \mu_5.$$

- CME is a **current flow** due to the helicity imbalance in (**polarized**) medium by the axial chemical potential μ_5 and B .

$$\langle \vec{j} \rangle = v_F \frac{e^2}{2\pi^2} \mu_5 \vec{B}$$

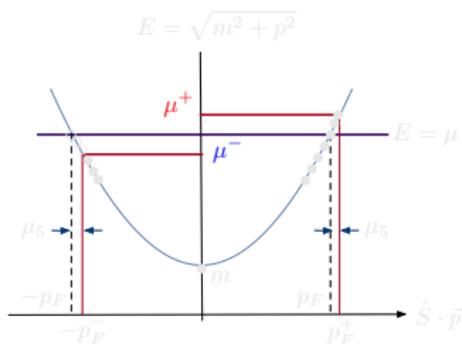


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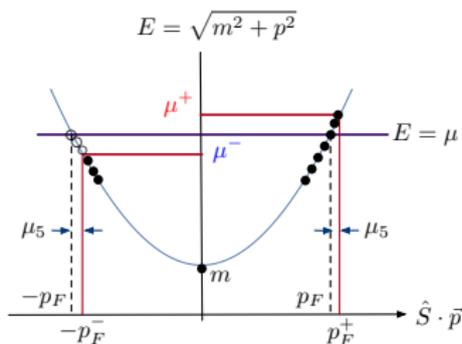


Figure: polarized medium

Chiral magnetic effects in medium

- ▶ At one-loop the current is given by

$$\langle j^\mu \rangle = e \langle \bar{\Psi} \gamma^\mu \Psi \rangle = -e \int \frac{d^4 p}{(2\pi)^4} \text{Tr} [\gamma^\mu S_F^{n=0}(p, \mu, \mu_5)] .$$

- ▶ The medium contribution is then ($r = \mu_5/m$, $v_F = p_F/m$)

$$\begin{aligned} \langle j^3 \rangle &= \int_0^\mu d\mu' \frac{\partial}{\partial \mu'} \langle j^\mu(\mu') \rangle \\ &= \frac{e^2 B}{4\pi^2} \left[\int_0^{\mu_+} d p_0 \int_{p_z > 0} |p_z| \delta(p_{\parallel}^2 - m^2) - \int_0^{\mu_-} d p_0 \int_{p_z > 0} |p_z| \delta(p_{\parallel}^2 - m^2) \right] \\ &= \frac{e^2 B}{4\pi^2} \left[\sqrt{(p_F + \mu_5)^2 + m^2} - \sqrt{(p_F - \mu_5)^2 + m^2} \right] \\ &= \frac{e^2 B}{2\pi^2} \mu_5 v_F [1 + \mathcal{O}(v_F^2, r^2)] . \end{aligned}$$

3. A new axion experiment (proposal)

Low temperature Axion Chiral Magnetic Effects (LACME)

(2207.06884)

LACME

- ▶ The axion couples to photons and electrons among others :

$$\mathcal{L}_{\text{axion}} \supset -C_{a\gamma\gamma} \frac{\alpha}{16\pi f} a F \cdot \tilde{F} + C_e \frac{\partial_\mu a}{f} \bar{\psi} \gamma^\mu \gamma_5 \psi$$

- ▶ The axion-photon coupling modifies the Maxwell equations :

$$\vec{\nabla} \times \vec{B} - \frac{\partial}{\partial t} \vec{E} = C_{a\gamma\gamma} \frac{\alpha}{f} \dot{a} \vec{B} = \vec{j}_{\text{vac}}.$$

- ▶ The axion-electron coupling creates CME currents in electron medium (chiral magnetic effects):

$$\vec{j}_{\text{cme}} = 4v_F C_e \frac{\alpha}{2\pi f} \dot{a} \vec{B}$$

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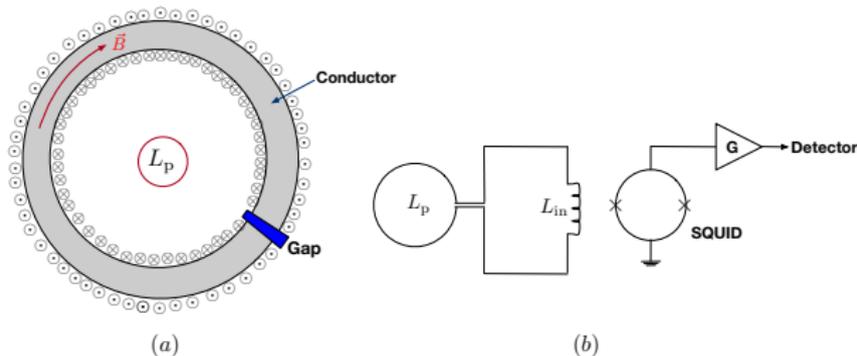
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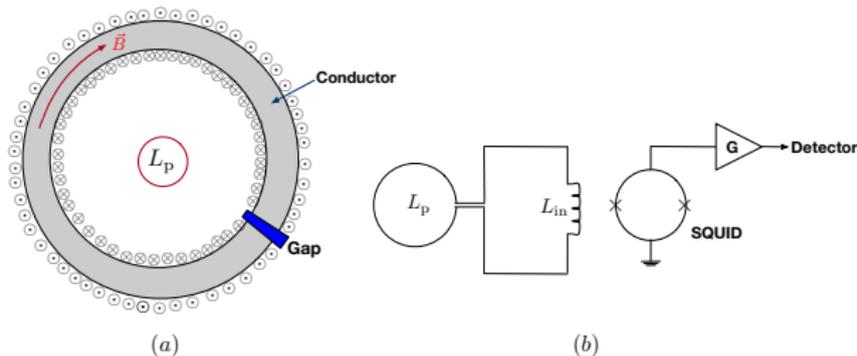


- ▶ Magnetic fields create a current under axion dark matter :

$$\vec{j} = [C_{a\gamma\gamma} + 4v_F C_e] \frac{\alpha}{2\pi f} \vec{B} \sqrt{2\rho_{DM}} \cos(m_a t)$$

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- ▶ With a conductor inserted, the magnetic field induce currents, the axion signal:

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- ▶ Since the signal power is proportional to the current squared, the LACME signal is therefore

$$\left| P_S^{\text{conductor}} - P_S^{\text{vacuum}} \right| \propto \left(\frac{4v_F C_e}{f} \right)^2 \left| 1 + \frac{C_{a\gamma\gamma}}{2v_F C_e} \right|$$

- ▶ Requiring the signal-to-noise $|P_S^{\text{conductor}} - P_S^{\text{vacuum}}|/P_N > 1$, we project the LACME sensitivity.

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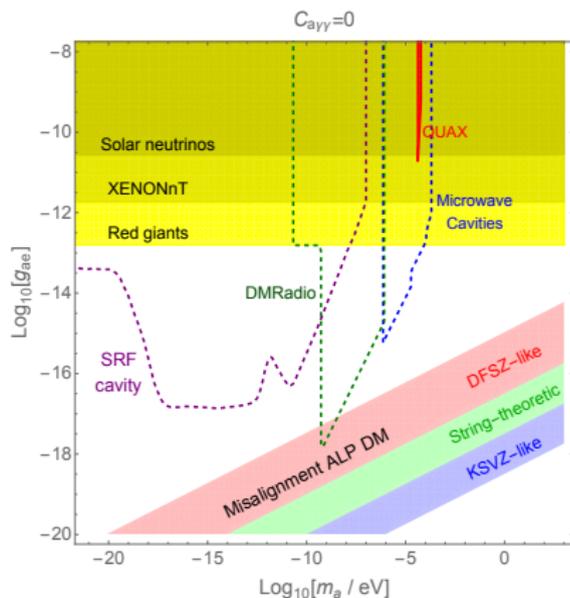
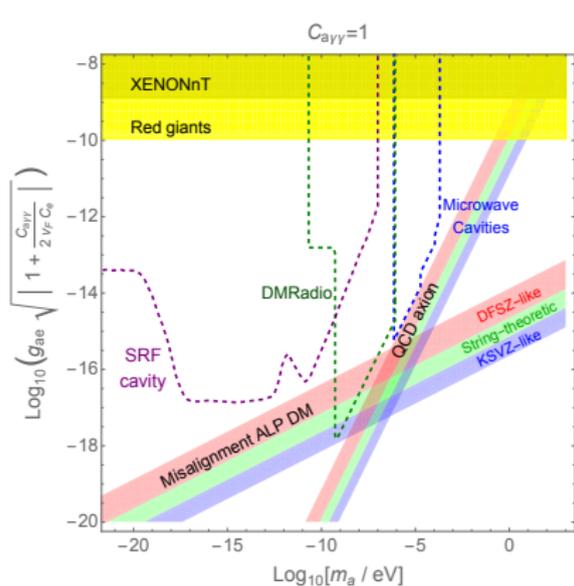
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LACME

- Projection of LACME from existing axion haloscopes, assuming $v_F = 0.01$ ($g_{ae} = 2C_e m_e / f$):



Axion-electron coupling

- ▶ The axion-electron coupling depends on the UV model.
- ▶ The strength of the axion-electron coupling varies as (See e.g. 2106.05816 by Choi+Im+Seong)

$$C_e \simeq \begin{cases} \mathcal{O}(1) & \text{DFSZ-like models} \\ \mathcal{O}(10^{-4} \sim 10^{-3}) & \text{KSVZ-like models} \\ \mathcal{O}(10^{-3} \sim 10^{-2}) & \text{String-theoretic axions.} \end{cases}$$

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4. Heating up compact stars

with Sang Hui Im and TaeHun Kim, to appear (arXiv:2503.xxxxx)

- ▶ Neutron stars are the densest objects observed in the sky, made mostly of neutrons, which are in β equilibrium with electrons and protons.
- ▶ The electrons in NS are described as Fermi liquid or gas with the Fermi momentum, determined by the β equilibrium and charge-neutrality,

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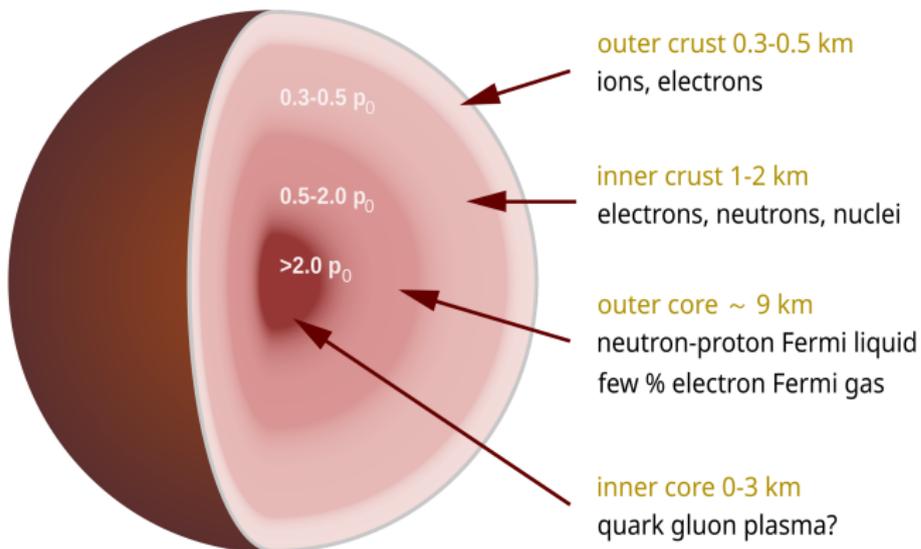
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Neutron star density profile



Heating up neutron stars

- ▶ NS carries a magnetic field. For magnetars, the magnetic field at the surface $B \gtrsim 10^{12}$ Gauss.
- ▶ Inside the NS, there will be currents induced by ADM:

$$\vec{j}_{\text{tot}} = \vec{j}_{\text{vac}} + \vec{j}_{\text{cme}} = \frac{e^2}{4\pi^2 m_e} g_{ae}^{\text{eff}} \sqrt{2\rho_{\text{DM}}} \cos(m_a t) \vec{B}$$

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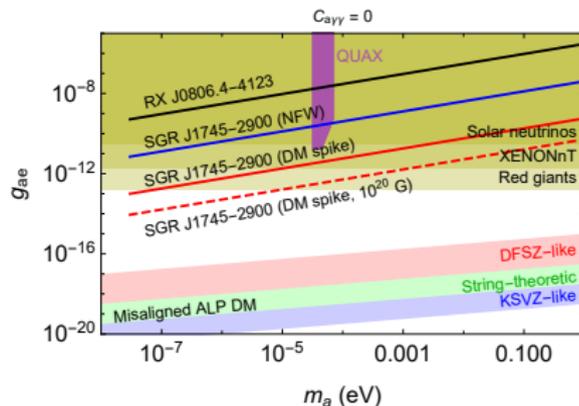
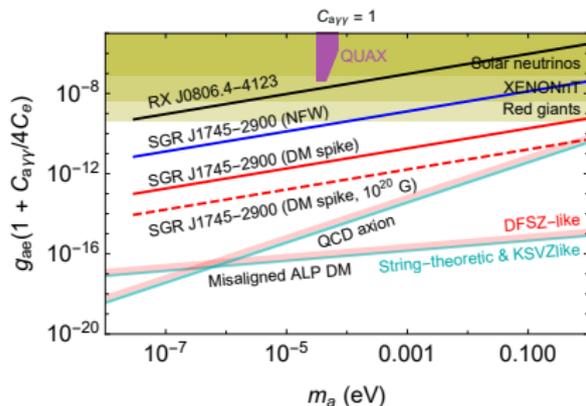
Heating up neutron stars

- ▶ Since the surface temperature of NS is measured to be between 0.1 – 1 keV, the power supplied by the currents are constrained to be

$$\bar{P}_{\text{tot}} \leq 4\pi r_0^2 \sigma T^4 \simeq 1.29 \times 10^{26} \text{ W} \times \left(\frac{r_0}{10 \text{ km}}\right)^2 \left(\frac{T}{100 \text{ eV}}\right)^4$$

Heating up neutron stars

- ▶ Constraint plot of g_{ae} from the heating. For solid lines, $B_{\max} = 10^{18}$ G and for the red dashed line, $B_{\max} = 10^{20}$ G in the core.



Conclusion

- ▶ Axion SM provides an axial chemical potential to electrons:

$$\mu_5 = \frac{C_f}{f} \dot{a} = \frac{C_f}{f} \sqrt{2\rho_a} \cos(m_a t)$$

- ▶ The axion DM therefore adds a kick to electrons:

$$\vec{p} \rightarrow \vec{p} + \frac{2}{3} \mu_5 \vec{S}, \quad \mu_5 = \frac{C_f}{f} \dot{a}.$$

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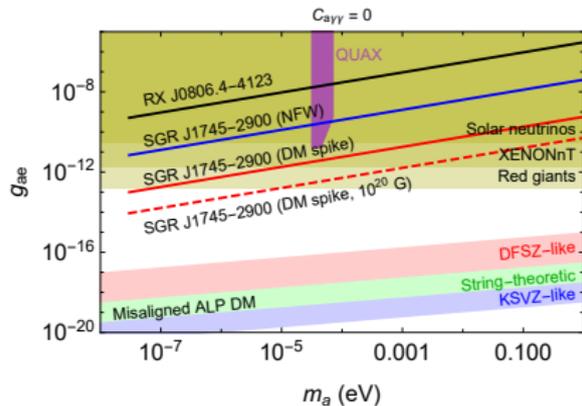
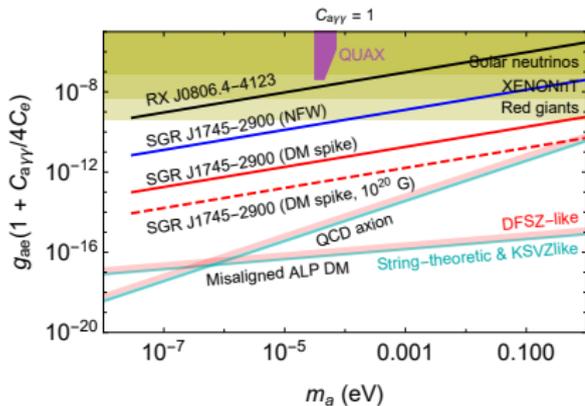
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Thank you for listening !