Searches for Power-law Warped Extra Dimensions

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SHI and Krzysztof Jodlowski, 2412.20913

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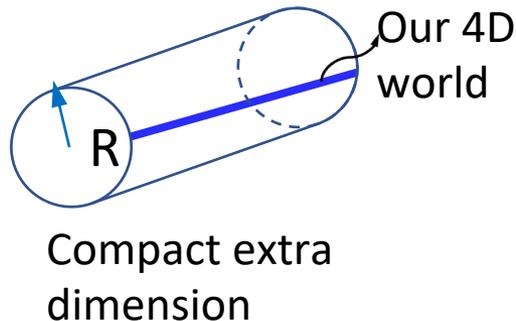Chung Ang University, Seoul, Korea / Feb 18, 2025

Outline

- Motivations and a model for a power-law warped extra dimension

- New KK graviton phenomenology

- Experimental and astrophysical limits

Extra spatial dimensions beyond 4D spacetime

- String theory (6 extra dimensions for consistency of theory)
- Flat N extra dimensions (ADD model)

N Arkani-Hamed, S Dimopoulos, G Dvali '98

Our 4D world

R

Compact extra dimension

$$V(r) \sim \begin{cases} \dfrac{1}{(M_{4+N})^{2+N}} \dfrac{m}{r^{1+N}}, & r \ll R \\[2ex] \dfrac{1}{(M_{4+N})^{2+N} R^N} \dfrac{m}{r}, & r \gg R \end{cases}$$

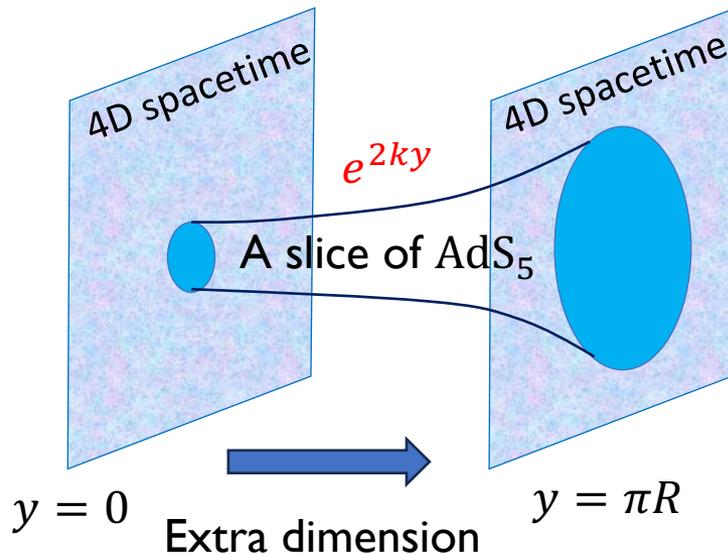M_{4+N} : Planck scale in $4 + N$ spacetime

Effective 4D Planck scale $M_P^2 = (M_{4+N})^{2+N} R^N$

➡ $M_{4+N} = \left(\dfrac{M_P^2}{R^N}\right)^{1/(2+N)} \sim$ TeV for $R \sim 10^{\frac{30}{N}-17}$ cm

The hierarchy problem can be solved by large extra dimensions (e.g. ~ 0.1 mm for $N = 2$).

Extra spatial dimensions beyond 4D spacetime

- A warped extra dimension (RS model) L Randall, R Sundrum '99

$$ds^2 = e^{2ky} g_{\mu\nu} dx^\mu dx^\nu + dy^2$$

Effective 4D Planck scale

$$M_P^2 \simeq \frac{M_5^3}{k} e^{2k\pi R}$$

M_5 : 5D Planck scale

$k\ (\sim M_5)$: AdS$_5$ curvature

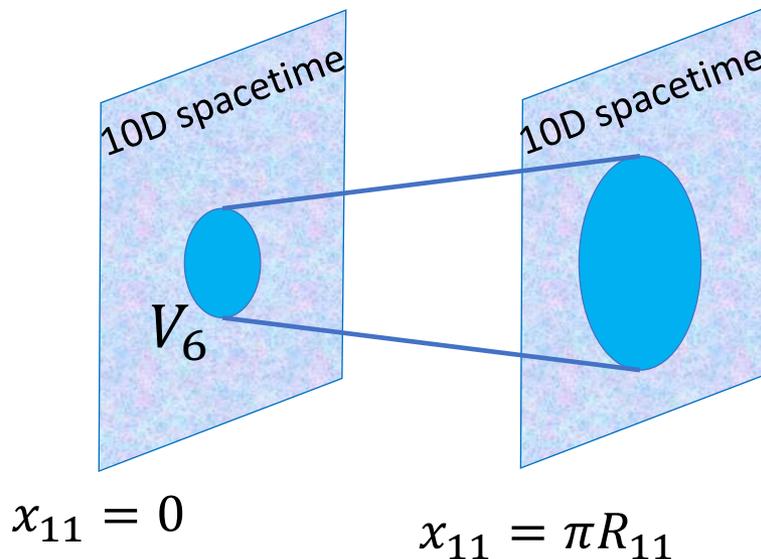$$M_5 \sim M_P e^{-k\pi R} \sim \text{TeV} \quad \text{for} \quad R \sim 10^{-3} \text{ fm}$$

The hierarchy problem can be solved by a small extra dimension.

Extra dimensional models from string theory

- 11D Heterotic M-theory

  Horava and Witten '96
  Lukas, Ovrut, Stelle, Waldram '98
  SHI, Nilles, Olechowski '19

  Dual theory of strongly coupled 10D heterotic string theory

  ➤ The 11th dimension can be parametrically larger than the other 6D extra dimensional space.

  ➤ Consequently, the theory can be described by 5D EFT after integrating out the 6D extra dimensions.

  ➤ The effective 5D metric is power-law warped due to the growing 6D extra dimensional space proportional to x_{11}.

10D spacetime

10D spacetime

V_6

$x_{11} = 0$

$x_{11} = \pi R_{11}$

$$ds^2_{5D} = (ky + 1)^{2q} g_{\mu\nu} dx^\mu dx^\nu + dy^2 \qquad q = \frac{1}{6}, \frac{1}{7}, \frac{1}{10}, \ldots$$

Extra dimensional models from string theory

- Little string theory

  String theory in zero string coupling limit $g_s \to 0$, which may address the hierarchy problem

  $$M_P^2 = \frac{1}{g_s^2} M_S^8 V_6$$

  M_S : fundamental string scale

  V_6 : the volume of 6D extra dimensional space

  Its dual theory is given by a 7D theory with a *linear dilaton* background, whose 5D approximate theory is described by a power-law warped extra dimension.

  $$ds_{5D}^2 = (ky + 1)^{2q} g_{\mu\nu} dx^\mu dx^\nu + dy^2 \qquad q = 1$$

  "Linear dilaton model"   Antoniadis, Arvanitaki, S Dimopoulos, Giveon '11
  Giudice, Kats, McCullough, Torre, Urbano '17

Phenomenology of power-law warped extra dimensions?

- Many 5D EFTs from string theory predict a power-law warped extra dimension with 0 (ADD limit) $< q < \infty$ (RS limit)

$$ds^2_{5D} = (ky + 1)^{2q} g_{\mu\nu} dx^\mu dx^\nu + dy^2$$

- Nevertheless, its phenomenology has not been studied except $q = 1$ (linear dilaton model; LD).

- In our work, we demonstrate that the KK gravitons from a power-law warped extra dimension with $0 < q < 1$ have quite distinct collider signatures compared with ADD, RS, and LD, while addressing the hierarchy problem.
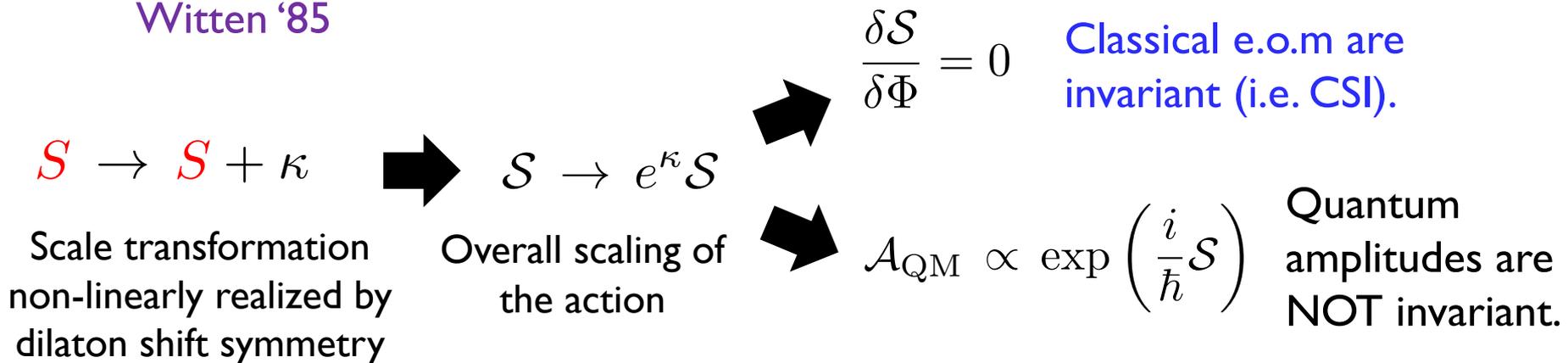
String-inspired 5D model

$$\mathcal{S} = \int d^5x \sqrt{-g}\, e^S \left(\frac{1}{2}\mathcal{R} + \frac{Z_S}{2}(\partial_M S)^2 - \Lambda \right)$$

This form of lagrangian can be generically obtained from string theory with the field S identified as the string dilaton or a Kähler modulus.

This is due to *classical scale invariance (CSI)* of EFTs from string theory.

Witten '85

$$S \rightarrow S + \kappa$$

$$\mathcal{S} \rightarrow e^\kappa \mathcal{S}$$

$$\frac{\delta \mathcal{S}}{\delta \Phi} = 0$$

Classical e.o.m are invariant (i.e. CSI).

$$\mathcal{A}_{\mathrm{QM}} \propto \exp\left(\frac{i}{\hbar}\mathcal{S} \right)$$

Quantum amplitudes are NOT invariant.

Scale transformation non-linearly realized by dilaton shift symmetry

Overall scaling of the action

5D CSI action

$$\mathcal{S} = \int d^5 x \sqrt{-g}\, e^S \left(\frac{1}{2}\mathcal{R} + \frac{Z_S}{2}(\partial_M S)^2 - \Lambda \right)$$

CSI : $\quad S \to S + \kappa$

$$\mathcal{S} \to e^\kappa \mathcal{S} \qquad \mathcal{A}_{\text{QM}} \propto \exp\left(\frac{i}{\hbar}\mathcal{S} \right) \qquad \text{Not respected by quantum effects}$$

Yet invariant under spurious transformation of Planck constant

$$\hbar \to e^\kappa \hbar$$

Selection rule for radiative corrections

$$\mathcal{S}_{1\mathrm{PI}} = \int d^5 x \sqrt{-g}\, e^S \left[\frac{1}{2}\mathcal{R} + \frac{Z_S}{2}(\partial_M S)^2 - \Lambda + \sum_{n=1}^{\infty} C_n \left(\frac{\hbar}{16\pi^2} e^{-S} \right)^n \right]$$

Perturbative radiative corrections
invariant under

Barring possible non-perturbative
corrections and assuming

$$\begin{cases} S \rightarrow S + \kappa \\ \hbar \rightarrow e^{\kappa}\hbar \end{cases}$$

$$e^{-S} < 1 \quad \text{(: normally true, since this serves as coupling.)}$$

the leading action would be taken to be
stable against quantum effects.

Green, Schwarz, Witten
Giudice, Kats, McCullough, Torre, Urbano 17'

Examples of UV origins

$$\mathcal{S} = \int d^5 x \sqrt{-g}\, e^S \left(\frac{1}{2}\mathcal{R} + \frac{Z_S}{2}(\partial_M S)^2 - \Lambda \right)$$

$$Z_S = \begin{cases} 1, & \text{String dilaton} \\ \frac{\mathcal{D}-1}{\mathcal{D}} < 1, & \mathcal{D}\text{-dim internal volume } V_{\mathcal{D}} \\ \geq \frac{23}{18}, & \text{Kähler modulus in Heterotic M-theory} \end{cases}$$

$$\Lambda = \begin{cases} \frac{D-10}{3\alpha'}, & \text{Non-critical string} \\ \text{stack of NS5-branes}, & \text{Little String Theory (LST)} \\ \text{c.c. in higher dimension}, & \mathcal{D}+5 \text{ dim theory} \\ \text{4-form flux}, & \text{Heterotic M-theory} \end{cases}$$

Background solution

$$\mathcal{S} = \int d^5 x \sqrt{-g}\, e^S \left(\frac{1}{2}\mathcal{R} + \frac{Z_S}{2}(\partial_M S)^2 - \Lambda \right)$$

Dilaton field $\quad S = k_s y \quad$ "Linear dilaton" $\quad k_s = \sqrt{\dfrac{-2Z_S\Lambda}{(4 - 3Z_S)(5 - Z_S)}}$

Metric in the Einstein frame
$$ds_E^2 = e^{\frac{2}{3}S}ds_J^2 = e^{\frac{2}{3}k_s y}\left(e^{2py}\eta_{\mu\nu}dx^\mu dx^\nu + dy^2 \right) \quad \text{AdS}_5 \text{ in the Jordan frame}$$

$$\equiv e^{2k_1 y}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2k_2 y}dy^2$$

$$\frac{p}{k_s} = 1 - Z_S \quad \left(\Leftrightarrow \frac{k_1}{k_2} = 4 - 3Z_S \right)$$

$Z_S = 1$: Linear Dilaton model (LD)

$Z_S \neq 1$: "General" Linear Dilaton model (GLD)

Choi, SHI, Shin '17

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Background solution

Metric in the Einstein frame

$$ds_E^2 = e^{2k_1 y}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2k_2 y}dy^2$$

$$dz \equiv e^{k_2 y}dy$$

$$= (k_2 z + 1)^{2q}\eta_{\mu\nu}dx^\mu dx^\nu + dz^2$$

where

$$k_2 = \frac{1}{3}\sqrt{\frac{-2\Lambda}{(4 - 3Z_S)(5 - Z_S)}} \qquad q = \frac{k_1}{k_2} = 4 - 3Z_S$$

The General Linear Dilaton model (GLD) gives rise to a power-law warped extra dimension with a power q determined by the dilaton wave function normalization.
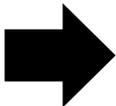
Power-law warping solution to the hierarchy problem

$$ds^2 = (kz+1)^{2q}\eta_{\mu\nu}dx^\mu dx^\nu + dz^2 \qquad z \in [0, L]$$

Effective 4D Planck scale $\qquad M_P^2 \sim M_5^3 (kL)^{2q} L$

power-law warping

size of the extra dimension

For $k \sim M_5$

$$M_5 \sim \left(\frac{M_P^2}{L^{1+2q}}\right)^{1/(3+2q)} \sim \text{TeV} \quad \text{for} \quad L \sim 10^{\frac{30}{1+2q}-17} \text{ cm}$$

The hierarchy problem can be solved by an intermediate size of the extra dimension (e.g. ~ 1 nm for $q = 1$).

KK graviton spectrum and couplings

$$ds^2 = (kz+1)^{2q}\eta_{\mu\nu}dx^\mu dx^\nu + dz^2 \qquad \Delta\mathcal{L}_{\text{int}} = c_n h^{(n)}_{\mu\nu} T^{\mu\nu}_{\text{SM}}$$

$q > 1$ $\qquad m_n \sim nk, \quad c_n \sim \dfrac{1}{M_5} \qquad n = 1, 2, 3, \cdots$

$q = 1$ $\qquad m_n \sim k\sqrt{1 + \left(\dfrac{n\pi}{\ln(M_P/M_5)}\right)^2}, \quad c_n \sim \dfrac{1}{M_5}$

RS-like

$q < 1$ $\qquad m_n \sim nk\left(\dfrac{M_5}{M_P}\right)^{\frac{2(1-q)}{1+2q}}, \quad c_n \sim \dfrac{1}{M_P} n^{\frac{3q}{2(1-q)}}$

Similar to ADD with $N = \dfrac{1+2q}{1-q}$

but having larger couplings

ADD with
N extra dimensions
$\qquad m_{n_1\ldots n_N} \sim \sqrt{n_1^2 + \cdots + n_N^2}\, M_{4+N}\left(\dfrac{M_{4+N}}{M_P}\right)^{2/N}, \quad c_{n_1\ldots n_N} = \dfrac{1}{M_P}$

Characteristic features of KK graviton phenomenology

$$ds^2 = (kz + 1)^{2q}\eta_{\mu\nu}dx^\mu dx^\nu + dz^2$$

- RS-like scenarios ($q \geq 1$) : heavy KK gravitons with sizable couplings
(determined by $M_5 > $ TeV)

  ➡ Visible KK gravitons at colliders (i.e. short-lived)

- ADD model : light & heavy KK gravitons with the small coupling $1/M_P$

  ➡ Invisible KK gravitons at colliders (i.e. long-lived),
strong astrophysical limits

- Scenarios with $0 < q < 1$: light & heavy KK gravitons with couplings
growing with KK graviton mass

  ➡ Both visible and invisible KK gravitons at colliders,
strong or moderate astrophysical limits
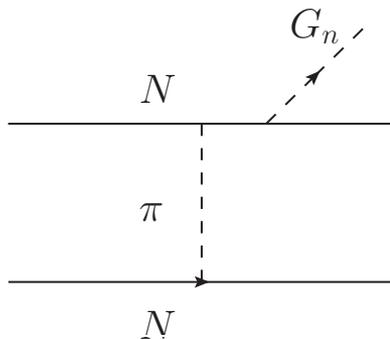
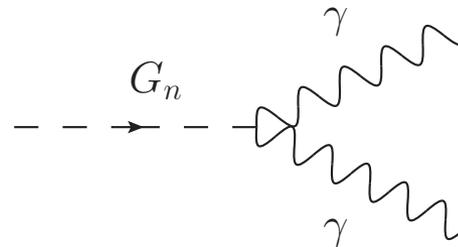# KK gravitons in colliders and astrophysical sources

Hadron colliders
(LHC)

Lepton colliders
(LEP, FCC-ee, CLIC)

Proton beam dumps
(SHiP, NA62, DUNE, …)
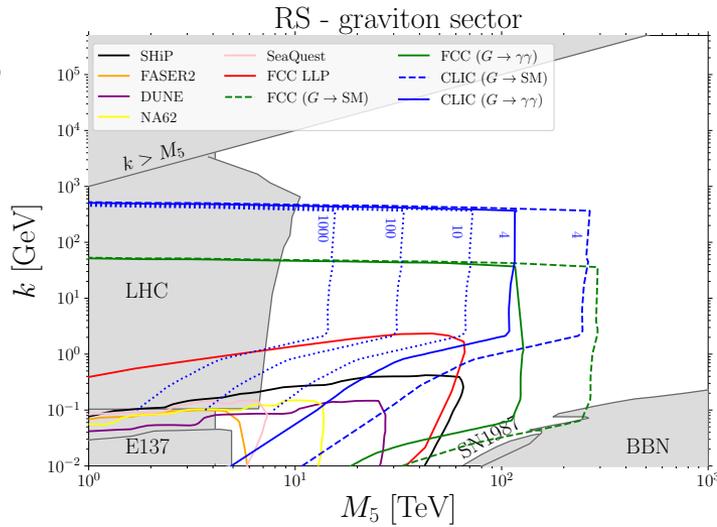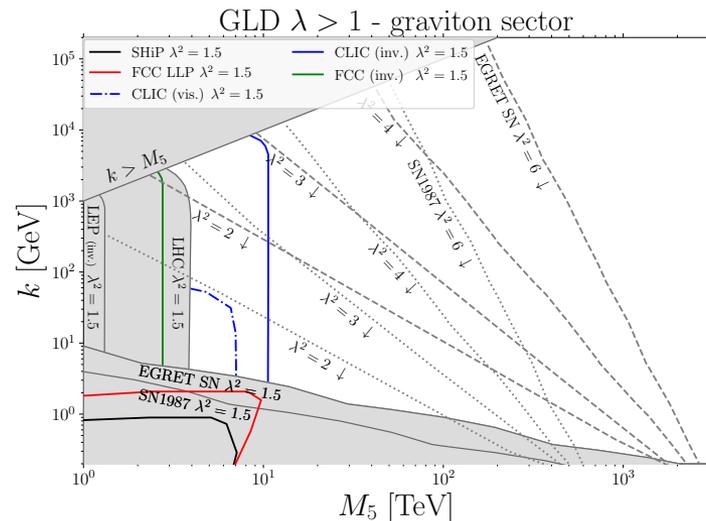
Neutron stars and
Supernovae

Decay channels

or long-lived

$$q \left(\equiv \frac{1}{\lambda^2} \right) = \frac{2}{3} < 1$$

Future colliders such ... warped extra dimension with both visible KK g... ...nergy from invisible KK gravitons.

Conclusions

- Power-law warped extra dimensions are common in string theory, realized by a dilaton field propagating in extra dimensions and the *classical scale invariance*.

- Their phenomenology has never been seriously studied so far except the power $q = 1$ (LD model).

- A power-law warped extra dimension can address the hierarchy problem with an intermediate size of the extra dimension (~ nm).

- The associated KK graviton spectrum and couplings show a distinctive pattern compared with the conventional models such as ADD, RS, and LD.

- Future colliders can probe a power-law warped extra dimension by visible KK graviton signatures and missing energy from invisible KK gravitons.