Primordial black hole (PBH) formation from an interrupted phase transition

Tae Hyun Jung
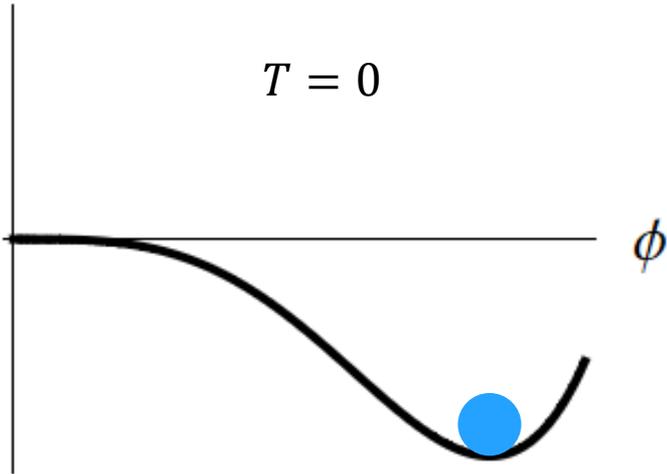
IBS CTPU PTC

Outline

1. **Cosmological (first-order) phase transition**

2. **Interrupted phase transition**

3. **Primordial black hole formation**

4. **Mass and abundance**

5. **Summary**

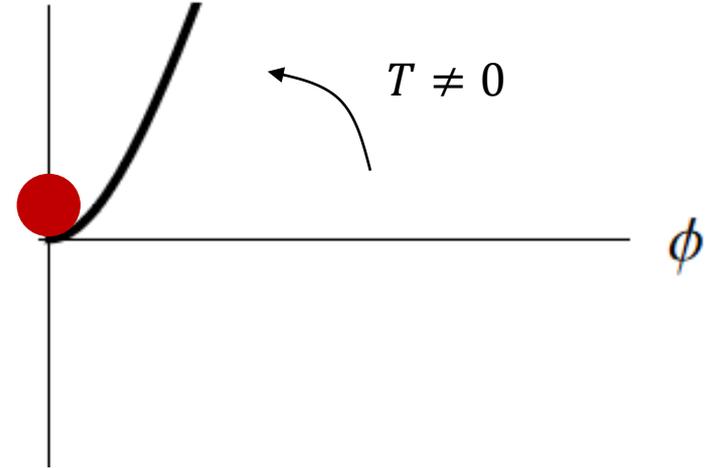# Cosmological first-order phase transitions

Spontaneously **broken** symmetry

   e.g. EW sym, chiral sym

$\longleftrightarrow$

Phase transition

restored at high temperature

$T = 0$

$T \neq 0$

ϕ

ϕ

Effective potential

$V_{\text{eff}}(\phi) \sim (\text{vol})^{-1} \ln Z \sim (\text{vol})^{-1} E[\phi]$

(vacuum energy density)
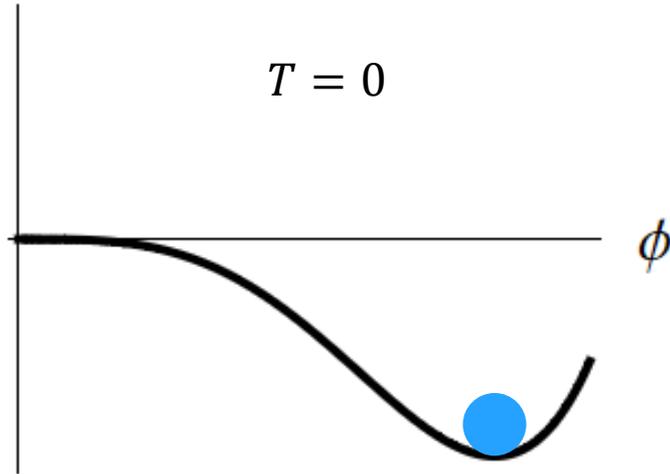
Effective potential at finite temperature

$V_T(\phi) \sim (\text{vol})^{-1} T \ln Z(T)$

(free energy density)

$= \rho - Ts$

Cosmological first-order phase transitions

Spontaneously **broken** symmetry   **restored** at high temperature

e.g. EW sym, chiral sym

$\leftrightarrow$

Phase transition

$T = 0$

$T = T_c$

First-order
if barrier exists

$T \neq 0$

Effective potential
$V_{\text{eff}}(\phi) \sim (\text{vol})^{-1} \ln Z \sim (\text{vol})^{-1} E[\phi]$
(vacuum energy density)

Effective potential at finite temperature
$V_T(\phi) \sim (\text{vol})^{-1} T \ln Z(T)$
(free energy density)
$= \rho - Ts$

Examples: Electroweak PT, QCD PT, …
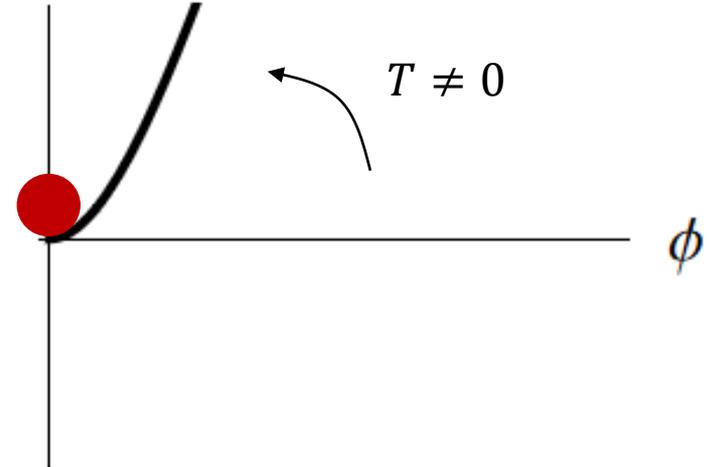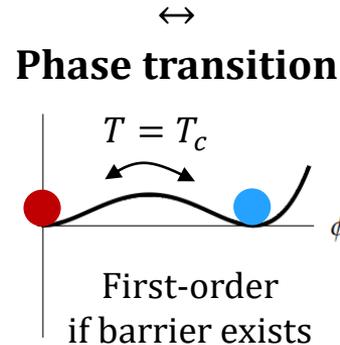$\rightarrow$ They are known to be cross-over in SM but it can be easily changed in BSM

Cosmological first-order phase transitions

- BSM involving spontaneous symmetry breaking → **phase transitions** in the early Universe
  e.g. PQ, B-L, GUT

- If some of them were first-order, stochastic gravitational waves can be produced.

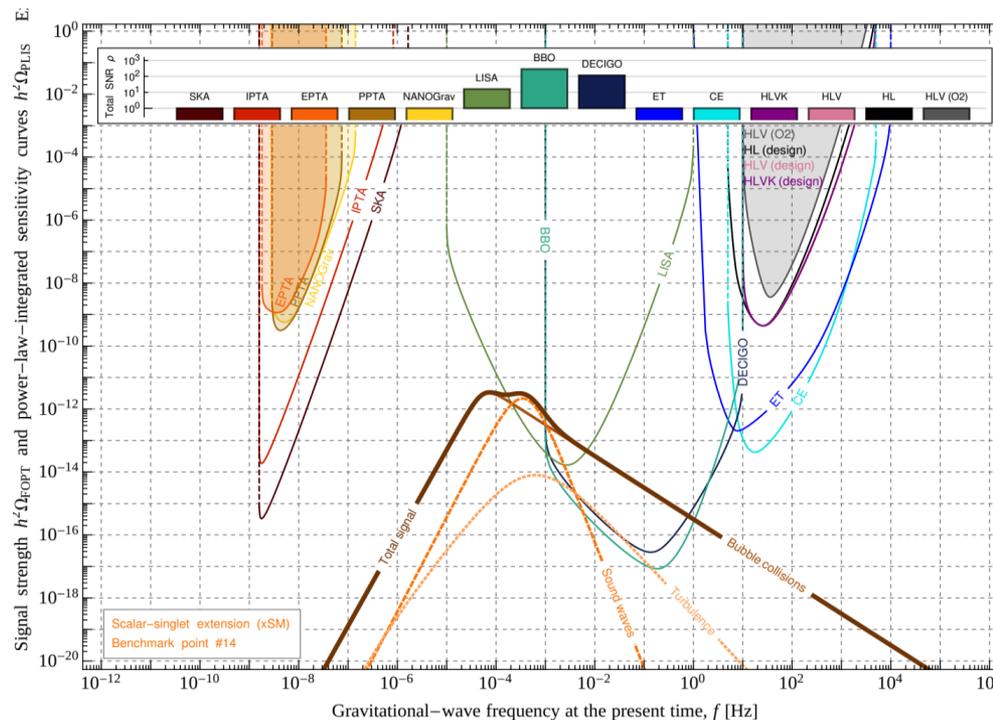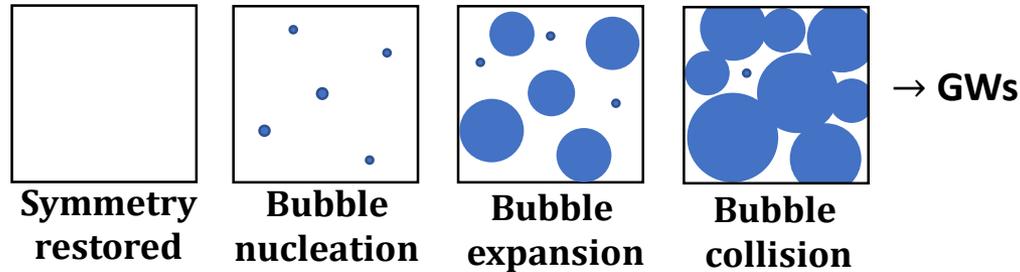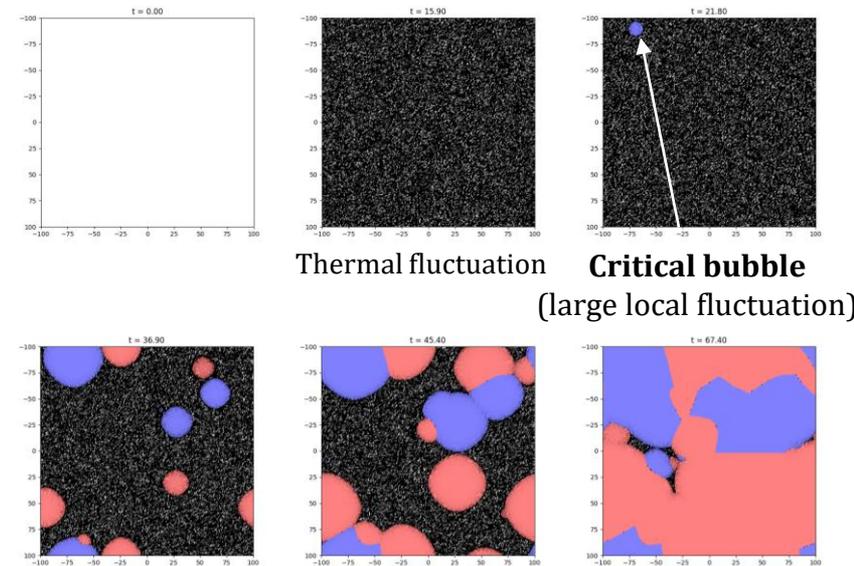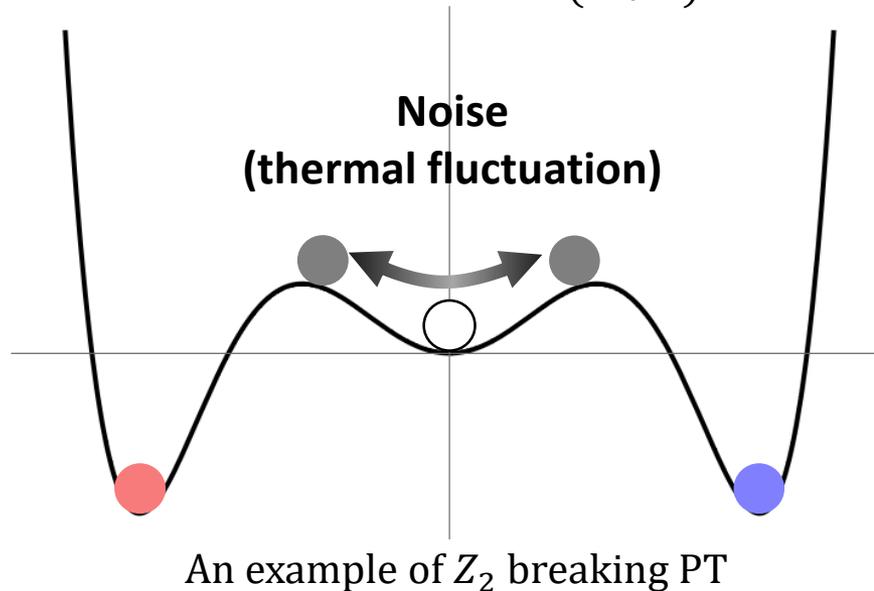**Symmetry restored**   **Bubble nucleation**   **Bubble expansion**   **Bubble collision**   → **GWs**

Fig from
K. Schmitz, 2002.04615

Cosmological first-order phase transitions

- Bubble nucleation at finite temperature: escape from the local minimum by **thermal fluctuation**

  Kramers (1940), Langer (1967), Linde (1981)

- **Nucleation rate $\Gamma_n = A_{\mathrm{dyn}} \times P_{\mathrm{stat}} \simeq T^4 \exp(-S_3/T)$**

- $(3+1)$D numerical simulation   T. Dutka, **THJ**, C. S. Shin, (2412.15864)

**Noise
(thermal fluctuation)**

An example of Z_2 breaking PT

Thermal fluctuation   **Critical bubble**
(large local fluctuation)

- Phase transition proceeds when $\color{red}{\Gamma_n(T) > H^4}$
  (otherwise, spacetime expansion dilutes bubbles, and PT does not proceed)

- Quick and rough estimation
  $\Gamma_n/H^4 \sim$ **# of bubbles nucleated in one Hubble patch in one Hubble time**

- Similarly, the history of PT can be studied by looking at the time dependence of Γ_n
  $\rightarrow$ the time dependence of temperature matters.
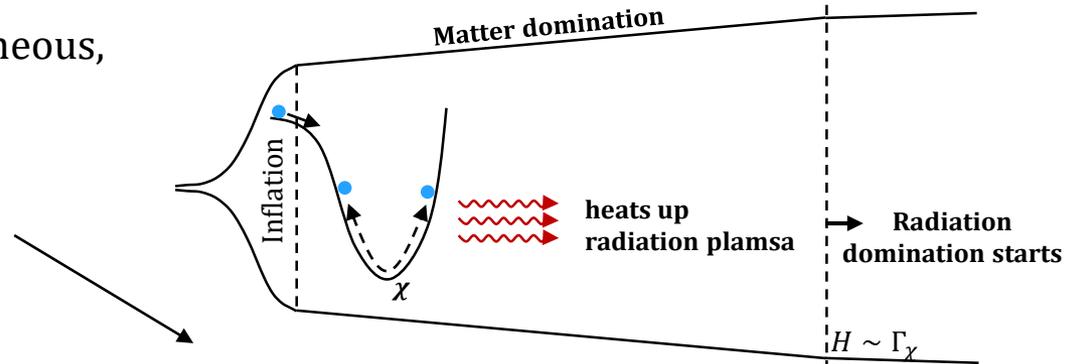
How temperature changes in the early Universe

- After inflation (when the inflaton exits from the slow-roll phase), it starts coherent oscillation. The decay of this oscillation continuously heats up the radiation plasma.

- Assuming the thermalization is instantaneous,
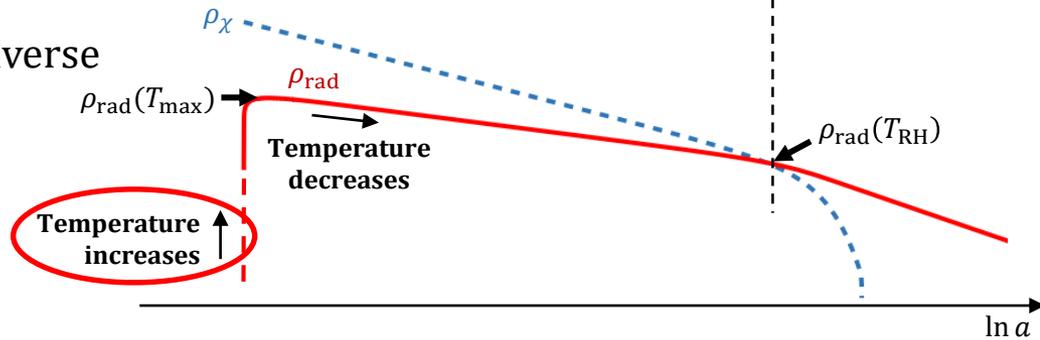$$\dot{\rho}_\chi + 3H\rho_\chi = -\Gamma_\chi \rho_\chi$$
$$\dot{\rho}_{\rm rad} + 4H\rho_{\rm rad} = \Gamma_\chi \rho_\chi$$

Initial condition: $T_i \sim H_I/2\pi \ (\ll T_{\rm max})$
$$\rho_{\chi,i} \sim 3M_{Pl}^2 H_I^2$$

- $T_{\rm max}$: maximum temperature that the Universe has ever reached.

$$T_{\rm max} \sim H_I \left(\frac{M_{Pl}^2 \Gamma_\chi}{H_I^3}\right)^{1/4}$$

- After inflation, T increases, reaches $T_{\rm max}$ and then decreases.

Phase transitions during this temperature evolution?

Matter domination

Inflation

heats up
radiation plamsa

Radiation
domination starts

$H \sim \Gamma_\chi$

χ

ρ_χ

$\rho_{\rm rad}$

$\rho_{\rm rad}(T_{\rm max})$

$\rho_{\rm rad}(T_{\rm RH})$

Temperature
decreases

Temperature
increases

$\ln a$

Heating and cooling phase transitions

When the temperature increases,
("Heating" phase transition)

- The symmetry was broken during inflation (assuming H_I is not too large).

- As T increases, symmetry-breaking vacuum gets lifted up due to temperature effects.

- When $T > T_c$, symmetry-restoring vacuum becomes more preferred.

- "Heating" phase transition occurs arround T_n.
    - T_n: **nucleation temperature** defined by the temperature when $\Gamma_n(T_n) = H^4(T_n)$

- If $T_{\max} > T_n$, the heating PT can be completed.

- (symmetry-broken phase) → (symmetry-**restored** phase)
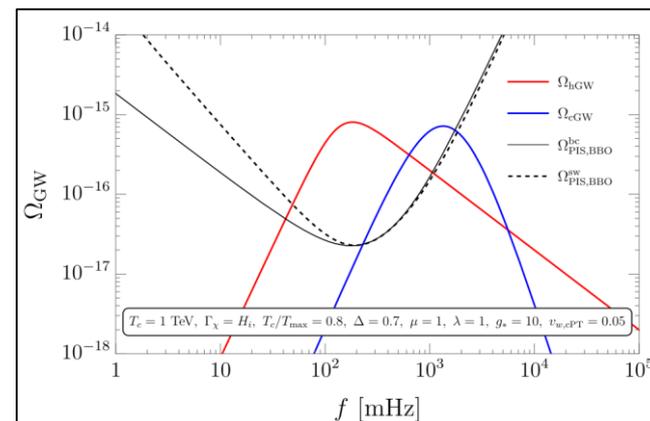
After $T_{\max}$, the temperature decreases back.

As the temperature decreases...
("Cooling" phase transition)

- Usual "cooling" PT occurs.

- (symmetry-**restored** phase) → (symmetry-broken phase)

⇒ GW is produced from both "heating" and "cooling" PTs

Doubly peaked GW spectrum
(Testable if $T_{\max} \sim T_c$)

$V_T(\phi) - V_T(0)$

First-order PT

T_c

T_n

Temperature increases

Ω_{hGW}
Ω_{cGW}
$\Omega^{bc}_{PIS,BBO}$
$\Omega^{sw}_{PIS,BBO}$

$T_c = 1$ TeV, $\Gamma_\chi = H_i$, $T_c/T_{\max} = 0.8$, $\Delta = 0.7$, $\mu = 1$, $\lambda = 1$, $g_* = 10$, $v_{w,c}PT = 0.05$

Buen-Abad, Chang, Hook, 2305.09712

Interrupted phase transition
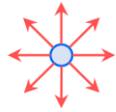
- $T_{\max} > T_n$: heating and cooling phase transitions

- $T_{\max} < T_c$: no phase transition

- $T_c < T_{\max} < T_n$: "**interrupted**" phase transition

    ✓ At $T > T_c$, symmetry-restoring bubbles can be nucleated in principle.

    ✓ Temperature turns around at $T_{\max} < T_n$ and Γ never reaches H^4.

    $\Rightarrow$ Bubbles exist, but they are far away from each other ($\Gamma/H^4 < 1$), and **bubbles never collide**.
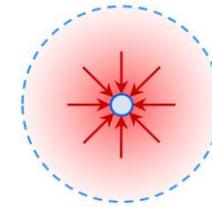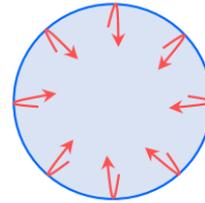
PT is not completed, but interrupted in the middle.

What happens to those bubbles?

Fate of bubbles nucleated in aborted PT

Fate of bubbles nucleated in aborted PT

Symmetry-restoring bubble

Temperature increase

T_max

(Γ is maximized at T_max)

Temperature decrease

time

$T = T_\text{c}$

$T < T_\text{c}$

In region of the bubble

| Thermal energy → Vacuum energy |

Thermal energy **(redshifted)** + Vacuum energy **(no redshift)**

| Vacuum energy → Thermal energy |

Other region

Thermal energy

Continuously redshifted

Thermal energy

$\Rightarrow$ **A positive density contrast is generated in the macroscopic region**

Fate of bubbles nucleated in aborted PT

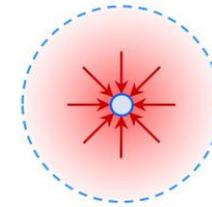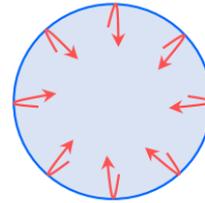Symmetry-restoring bubble

Temperature increase → T_{max} — **Temperature decrease** → time

(Γ is maximized at T_{max})

$T = T_c$ $(a = a_{c,2})$

$T < T_c$

ϕ ϕ ϕ

In region of the bubble	Thermal energy → Vacuum energy	Thermal energy **(redshifted)** + Vacuum energy **(no redshift)**		Vacuum energy → Thermal energy
Other region	Thermal energy	Continuously redshifted		Thermal energy
Background (in both)	**Matter-like reheaton**	Continuously decay		**Matter-like reheaton**

⇒ **A positive density contrast is generated in the macroscopic region**

(but suppressed by $\rho_{\text{rad}}/\rho_\chi$)

Positive density contrast → Gravitational collapse

- Critical density contrast in matter domination = 0 (if the system is spherically symmetric)

NR simulation de Jong, Aurrekoetxea, Lim, 2109.04896

SA: de Jong, Aurrekoetxea, Lim, 2306.11810
Luca et al, 2112.02534

Simulation vs Our case

- **Numerical simulation is not completely aligned to our case.**

- In the previous simulation,
  - ✓ They put spherically symmetric and positive-definite perturbation (not realistic in most cases)

  → Satisfied in our case; bubbles are spherical, and generated density perturbation is positive.

  - ✓ It was not a bubble, but it was a massless scalar wave moving toward the center
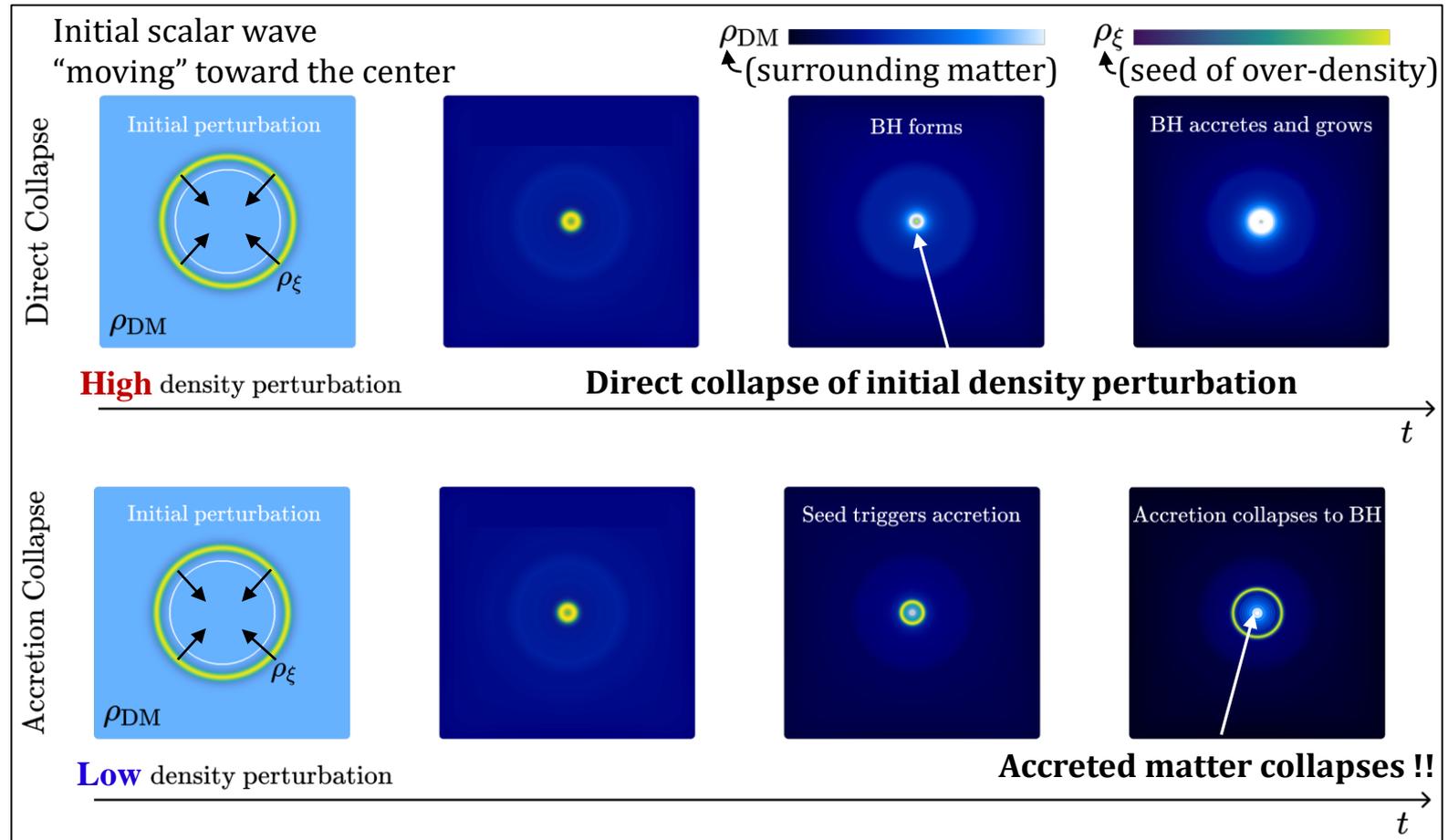 (no vacuum energy involved).

  → Not an issue; it just changes the interpretation of the initial condition.

  - ✓ Initial density perturbation was somewhat large.

  → In our case, there must be a long period of linear growth (!).

  - ✓ The surrounding matter did not decay (!).

  → In our case, reheaton decays and there's an end of matter domination.
  PBH formation must be finished before a_{RH}.

> Small $\delta_i \propto \rho_{\mathrm{rad}}/\rho_\chi$
> → requires a longer
>   time to grow
>
> Delay of a_{RH}
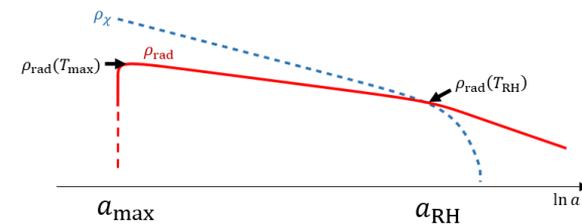> → makes δ_i smaller

∗Recall this:

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  → requires a longer
     time to grow

  Delay of a_{RH}
  → makes δ_i smaller

  ⇒ **A strong self-consistency condition** for our PBH formation.

$$\left(\frac{a_{\mathrm{RH}}}{a_{\max}}\right) < \left(\frac{10 g_{\star,\phi}(T_c)}{3 g_\star(T_{\mathrm{RH}})}\right)^2 \left(\frac{a_{c,2}}{a_{\max}}\right)\left[2 - \left(\frac{a_{c,2}}{a_{\max}}\right)^{-\frac{1}{2}}\right]^8$$

$\frac{g_{\star,\phi}}{g_\star}$: ratio of effective d.o.f in PT sector and total $\frac{a_{c,2}}{a_{\max}} \sim 2$ or 3 is realizable

∗Recall this:

$\frac{a_{c,2}}{a_{\max}} \gtrsim 2\text{-}3$ and $\frac{a_{\mathrm{RH}}}{a_{\max}} \lesssim 10\text{-}100$ would be needed

realizable!!
See our paper
for an example model...

PBH mass

- After formation, the PBH keeps accreting surrounding matter and the mass grows quickly.

- The maximally allowed mass is one Hubble mass (total energy inside one Hubble patch).

de Jong, Aurrekoetxea, Lim, 2109.04896

⇒ **The final PBH mass is determined by around one Hubble mass at T_{RH}**

$$\Rightarrow M_{PBH} \sim 3.5 \times 10^{-12} M_\odot \; \alpha \left(\frac{10^5 \text{GeV}}{T_{RH}}\right)^2 \left(\frac{100}{g_*(T_{RH})}\right)^{1/2}$$

$O(0.1 - 1)$ efficiency parameter
(we don't know the exact evolution)

- # of bubbles = # of PBHs (all bubbles become seeds of PBHs)

$$\mathbf{d}N_{\mathbf{PBH}} = \mathbf{d}N_{\mathbf{bubble}} = \mathrm{d}t\, V\, \Gamma$$

- Bubble nucleation rate Γ is maximized at $T_{\max}$ (when the potential barrier becomes smallest)
  $\Rightarrow$ Most bubbles are nucleated around $T_{\max}$ and we can only integrate over t around $T = T_{\max}$

expand at $T_{\max}$

$$\Gamma(T) \simeq T^4 \mathrm{e}^{-S_3/T} \Rightarrow \quad \Gamma(T) \simeq \Gamma(T_{\max}) \left(\frac{T}{T_m}\right)^{\widehat{\beta}_{\max}+4}$$

$$\hat{\beta} \equiv -\frac{\mathrm{d}\,(S_3/T)}{\mathrm{d}\ln T} : \text{PT rapidity parameter}$$

$\sim 10^4 - 10^6$ is realizable

This represents
incompleteness of PT

$$f_{\mathrm{PBH}} = \frac{M_{\mathrm{PBH}} n_{\mathrm{PBH}}/s}{\rho_{\mathrm{DM}}/s} \sim 1\,\alpha \left(\frac{T_{\mathrm{RH}}}{10^5\,\mathrm{GeV}}\right) \left(\frac{\left(\frac{\Gamma(T_{\max})}{H_{\max}^4}\right)}{10^{-15}}\right) \left(\frac{10^4}{\hat{\beta}_{\max}}\right)^{\frac{1}{2}} \left(\frac{a_{\mathrm{RH}}/a_{\max}}{10}\right)^{\frac{3}{2}}$$

(f_{PBH}: PBH relic density normalized by the observed DM relic)
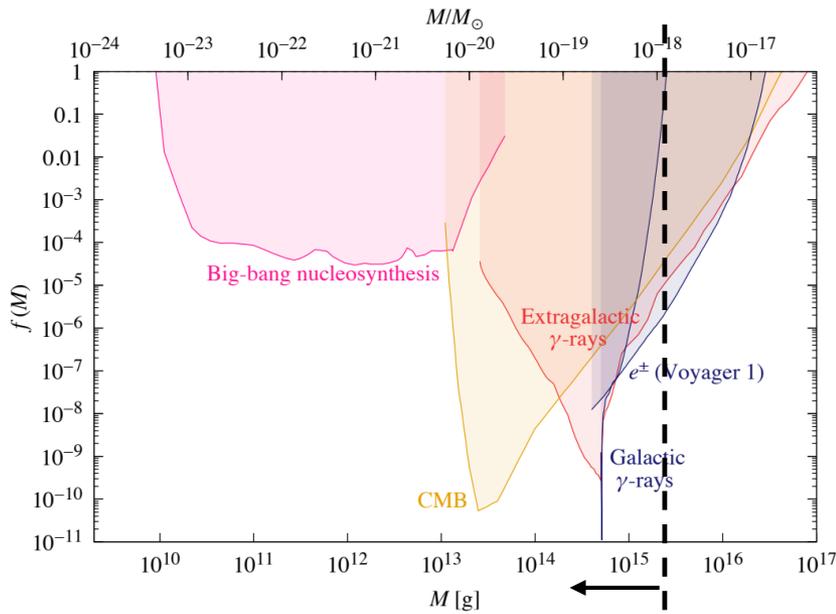
Remember;
this cannot be too large
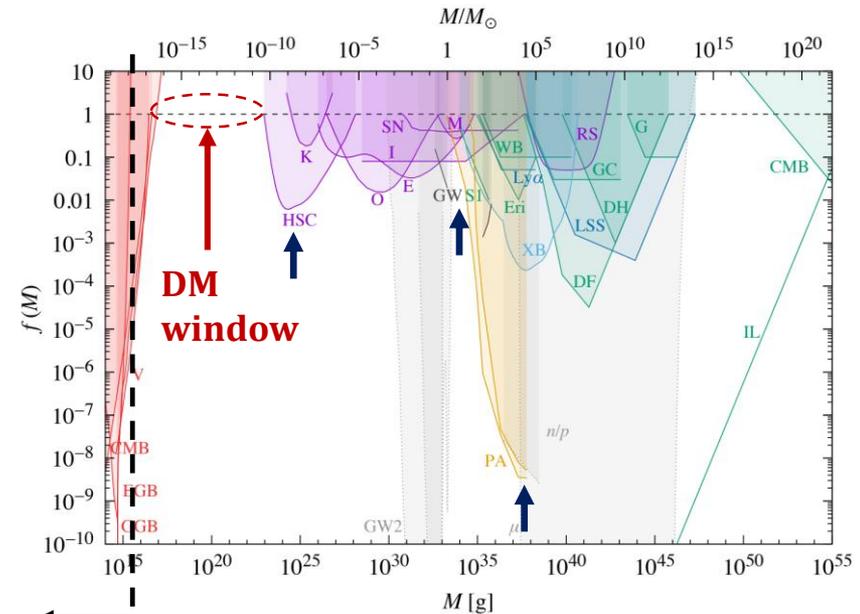(because it suppresses δ_i)

PBH phenomenology and constraints

$$M_{PBH} \sim 3.5 \times 10^{-12} M_\odot \, \alpha \left(\frac{10^5 \text{GeV}}{T_{RH}} \right)^2 \left(\frac{100}{g_*(T_{RH})} \right)^{1/2}$$

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Fully evaporate
before present

Fully evaporate
before present

Figs from Carr, Kohri, Sendouda, Yokoyama, 2002.12778

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$\alpha \sim 0.1, g_* \sim 100$

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$\hat{\beta}_{\mathrm{max}} \sim 10^5$

$\dfrac{a_{\mathrm{RH}}}{a_{\mathrm{max}}} \sim 10$

T_{RH} is too low
It masses up BBN.
(ruled out)

This represents incompleteness of PT — $\dfrac{\Gamma(T_{\mathrm{max}})}{H(T_{\mathrm{max}})^4}$

Fully evaporate before present

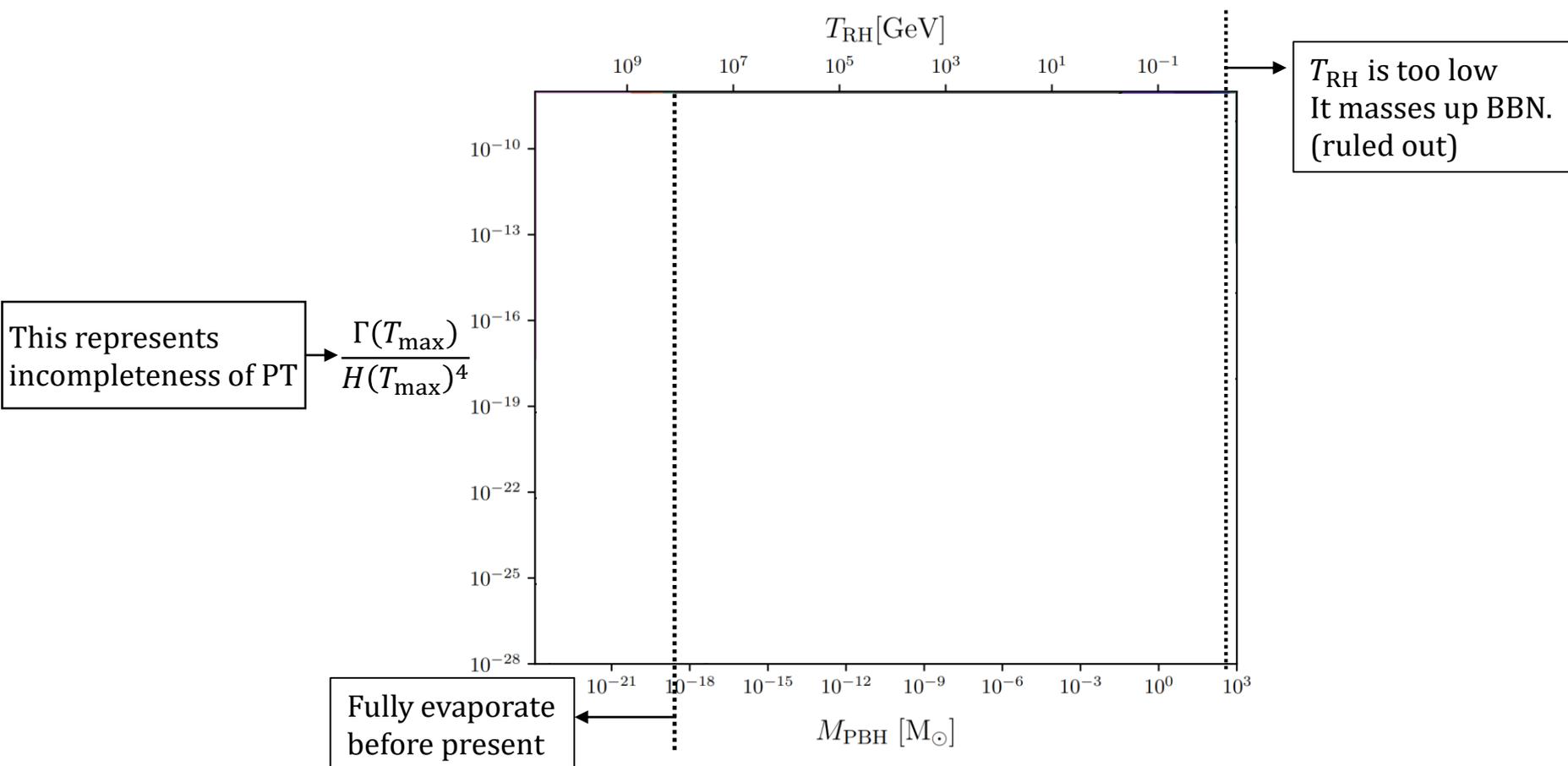$$M_{PBH} \sim 3.5 \times 10^{-12} M_\odot \, \alpha \left(\frac{10^5 \mathrm{GeV}}{T_{RH}}\right)^2 \left(\frac{100}{g_*(T_{RH})}\right)^{1/2}$$
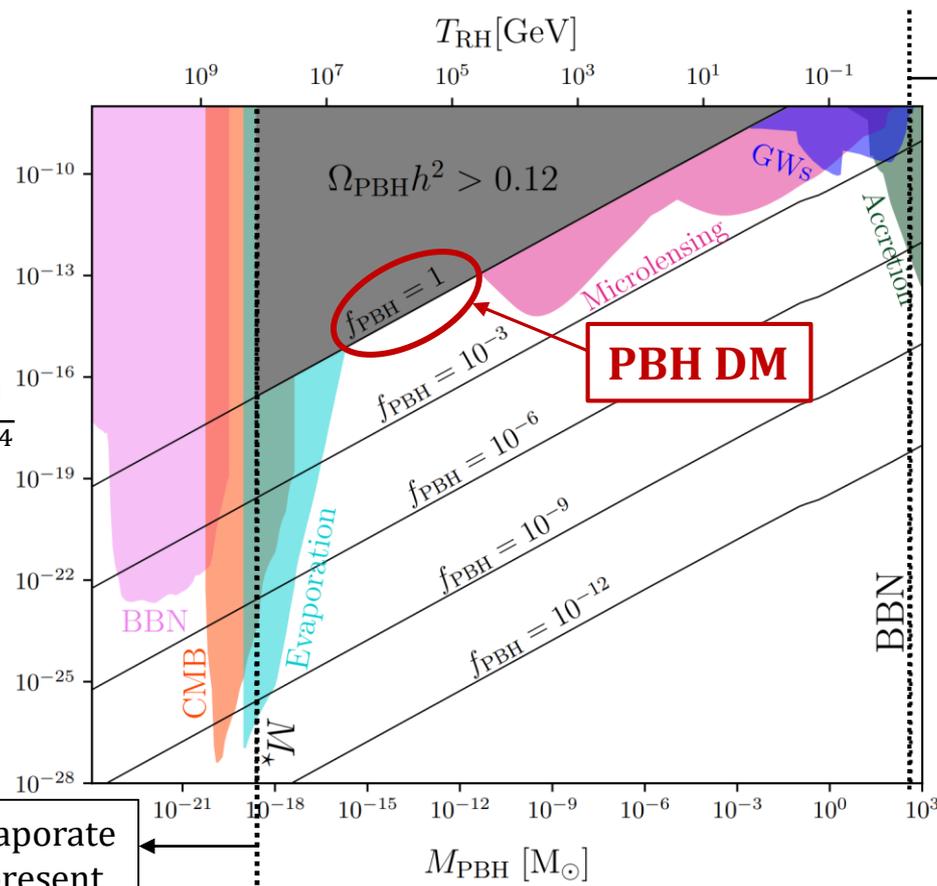
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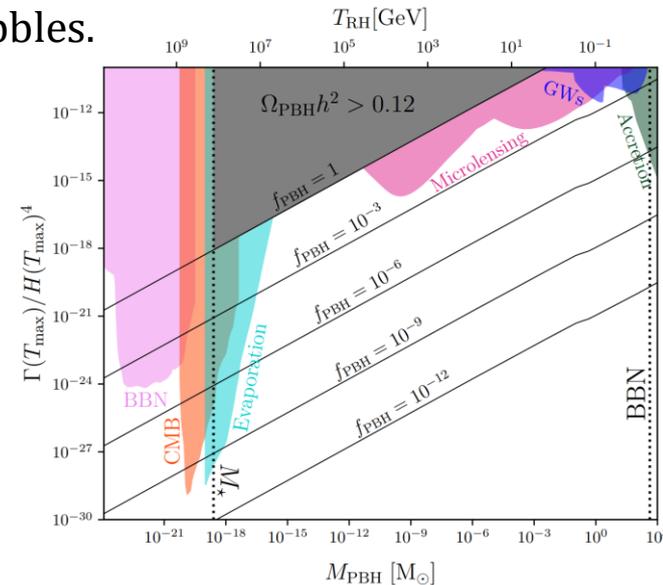
Fully evaporate
before present

- During reheating after inflation, a heating PT can be "interrupted" if $T_c < T_{max} < T_n$.

- Symmetry-restoring bubbles are still nucleated but they never collide.

- These bubbles shrink and disappear at $T < T_c$ while a positive density contrast is generated on a macroscopic scale.

- They trigger accretion of surrounding matter (reheaton) and collapse into BHs.

- The PBH mass grows until the end of reheating, and its final value is determined by around one Hubble mass at the reheating temperature.

- PBH abundance is estimated by counting the number of bubbles.

Thank you!

- (3 + 1)D numerical simulation of bubble nucleation

  T. Dutka, **THJ**, C. S. Shin, (2412.15864)

t = 0.00