

# Gravitational Waves from a First-Order Phase Transition of the Inflaton

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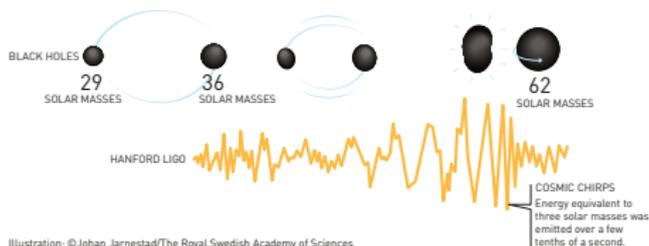
UNIVERSITY OF BERGEN

Based on JK, Seong Chan Park, Yeji Park, Juhoon Son, Liliana Velasco Sevilla,  
[arXiv:2412.17278](https://arxiv.org/abs/2412.17278)

# A New Window to the Universe



Virgo collaboration/CCO 1.0



Caltech/MIT/LIGO Lab

- Discovery of **gravitational waves** (GW) by LIGO & Virgo  
LIGO/Virgo, PRL 116 (2016), PRL 118 (2017)
- Source: mergers of black holes and neutron stars
- GW **astronomy** and **cosmology**

# A Window to the Very Early Universe?

- Pulsar Timing Arrays
  - Evidence for **Stochastic GW Background**

NANOGrav, ApJL **951** (2023)

EPTA, InPTA, A&A **678** (2023)

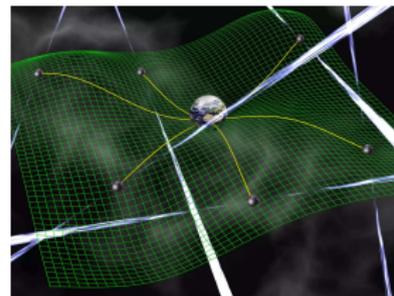
Parkes PTA, ApJL **951** (2023)

CPTA, RAA **23** (2023)

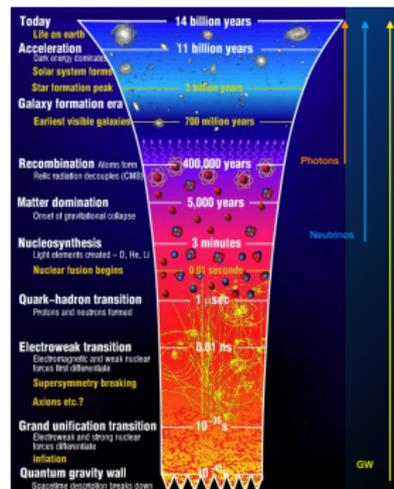
- Lower frequency than LIGO/Virgo events
  - Mergers of **supermassive** black holes?

- More interesting: **particle physics origin**
  - **First-Order Phase Transition (FOPT)**
  - Cosmic strings or domain walls
  - Inflation

See talks by Hong, Branchina, Tang, Gong, Maji

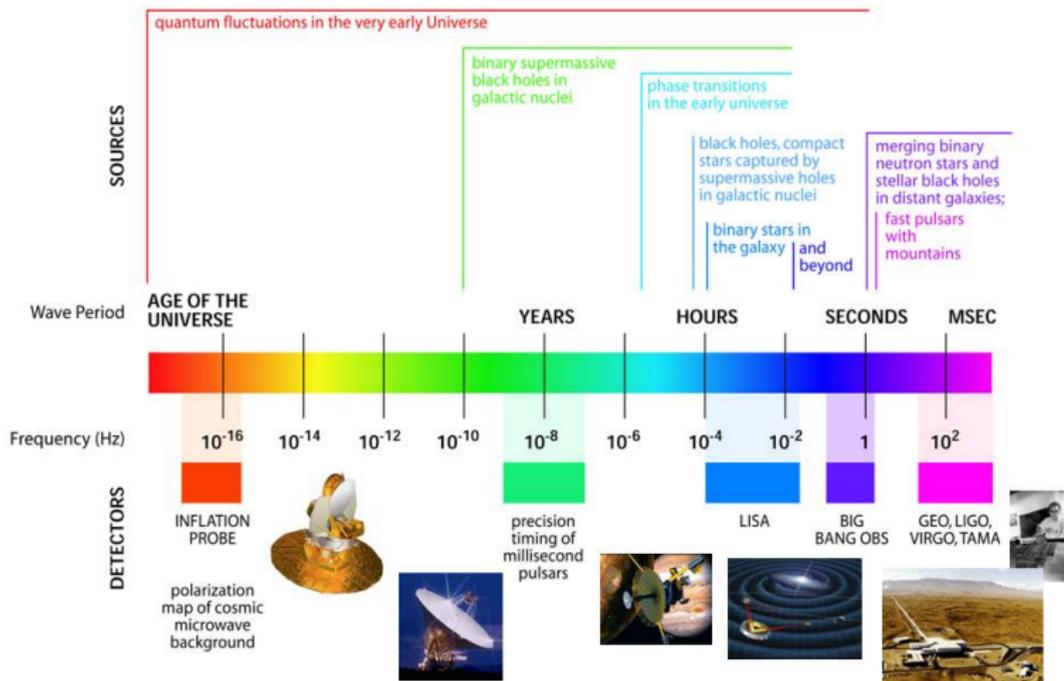


D. Champion/MPI for Radio Astronomy



# Looking Forward to More Discoveries

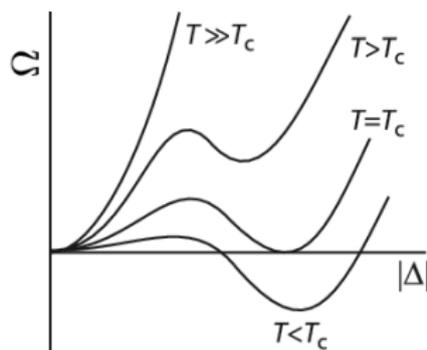
## THE GRAVITATIONAL WAVE SPECTRUM



<https://www.astro.gla.ac.uk/users/martin/powersof60/images/gwspectrum.jpg>

# First-Order Phase Transitions

See talks by Branchina, Jung



Kinnunen et al., Rep. Prog. Phys. **81** (2018)

- 1 High temperature: potential minimum at  $\phi = 0$
- 2  $T < T_c$ : deeper minimum at  $\phi = v$ , separated by **barrier**
- 3 Tunneling  $\leadsto$  bubbles of true vacuum
- 4 **GW** from expanding bubbles and surrounding plasma

# Higgs Inflation

Bezrukov & Shaposhnikov, PLB 659 (2008)

$$\mathcal{L}_J = \sqrt{-g_J} \left[ \frac{M_{\text{P}}^2}{2} R_J + \xi \phi^\dagger \phi R_J - g_J^{\mu\nu} (D_\mu \phi)^\dagger (D_\nu \phi) - V_J(\phi) + \dots \right]$$

- J: Jordan frame
- R: Ricci scalar
- $V_J$ : SM Higgs potential
- $\xi$ : non-minimal coupling to gravity
  - Consistent with all symmetries
  - Required for renormalization in curved spacetime
  - Not necessarily small

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- $\xi$ : non-minimal coupling to gravity
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  - Not necessarily small
- Def.: conformal factor  $\Omega^2(\phi) \equiv 1 + 2\xi \frac{\phi^\dagger \phi}{M_{\text{P}}^2}$

$$\mathcal{L}_J = \sqrt{-g_J} \left[ \frac{M_{\text{P}}^2}{2} \Omega^2(\phi) R_J - g_J^{\mu\nu} (D_\mu \phi)^\dagger (D_\nu \phi) - V_J(\phi) + \dots \right]$$

# Higgs Inflation

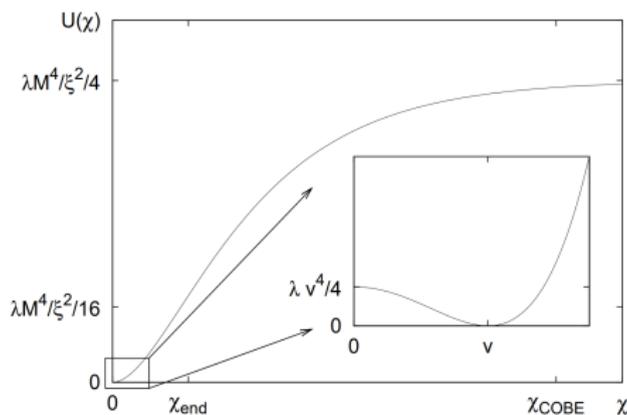
- Unitary gauge:  $\phi = \begin{pmatrix} 0 \\ (\varphi + v)/\sqrt{2} \end{pmatrix}$
- Weyl transformation to **Einstein frame**:  $g_J \rightarrow g_E \equiv \Omega^2(\varphi) g_J$
- Canonical normalization:  $\chi = \int_0^\varphi d\varphi \sqrt{\frac{3}{2} \frac{(\Omega^2_{,\varphi})^2}{\Omega^4} + \frac{1}{\Omega^2}}$

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$$\mathcal{L}_E = \sqrt{-g_E} \left[ \frac{M_{\text{P}}^2}{2} R_E - \frac{1}{2} g_E^{\mu\nu} (\partial_\mu \chi)(\partial_\nu \chi) - V_E(\chi) + \dots \right]$$

- Minimal coupling to gravity but modified potential
- **Potential flat for large field values  $\leadsto$  slow-roll**

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# Higgs Inflation in a Dark Sector

$$\mathcal{L}_J = \sqrt{-g_J} \left[ \frac{M_{\text{P}}^2}{2} \Omega^2(\phi) R_J + g_J^{\mu\nu} (D_\mu \phi)^\dagger (D_\nu \phi) - \frac{1}{4} g_J^{\mu\rho} g_J^{\nu\sigma} X_{\mu\nu} X_{\rho\sigma} - V_J(\phi) \right]$$
$$V_J(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 = \lambda \left( \phi^\dagger \phi - \frac{v^2}{2} \right)^2 + \text{const.}$$

- $U(1)_X$  gauge symmetry
- SM singlet scalar  $\phi$  with  $U(1)_X$  charge
  - Spontaneously breaks  $U(1)_X \rightsquigarrow$  FOPT  $\rightsquigarrow$  GW
  - Non-minimal coupling to gravity  $\rightsquigarrow$  Higgs-inflation-like inflation

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  - Spontaneously breaks  $U(1)_X \rightsquigarrow \text{FOPT} \rightsquigarrow \text{GW}$
  - Non-minimal coupling to gravity  $\rightsquigarrow$  Higgs-inflation-like inflation
- Optional: fermions with mass from Yukawa coupling
- Weak coupling to SM  $\rightsquigarrow$  reheating

$$\mathcal{L}_\psi = \sqrt{-g_J} \sum_{i=1}^{n_\psi} \left[ \bar{\psi}_{Li} i \not{D} \psi_{Li} + \bar{\psi}_{Ri} i \not{D} \psi_{Ri} - \frac{y}{2} (\bar{\psi}_{Li} \phi \psi_{Li}^c + \bar{\psi}_{Ri} \phi \psi_{Ri}^c + \text{h.c.}) \right]$$

# Gravitational Waves from Phase Transition

Scalar potential with 1-loop and finite-temperature corrections

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# Gravitational Waves from Phase Transition

Scalar potential with **1-loop** and **finite-temperature** corrections

- ↪ Potential **barrier** around  $T \sim v$
- ↪ **FOPT possible**

GW spectrum determined by

- Nucleation temperature  $T_n$
- $\alpha \leftrightarrow$  strength of PT
- $\beta \leftrightarrow$  duration

Calculated with help from `CosmoTransitions`

Wainwright, Comput. Phys. Commun. **183** (2012)

# Effective Scalar Potential at Finite Temperature

$$V(h_c, T) = V_{\text{tree}}(h_c) + V_{1\text{-loop}}(h_c) + V_{\text{th}}(h_c, T)$$

$$V_{\text{tree}}(h_c) = -\frac{\mu^2}{2} h_c^2 + \frac{\lambda}{4} h_c^4$$

$$V_{1\text{-loop}}(h_c) = \sum_{i=h,\chi,g,f} \frac{n_i}{64\pi^2} m_i^4(h_c) \left[ \ln \frac{|m_i^2(h_c)|}{v^2} - C_i \right]$$

$$V_{\text{th}}(h_c, T) = \sum_{i=h,\chi,g} \frac{n_i}{2\pi^2} T^4 \text{Re} J_b\left(\frac{m_i^2(h_c)}{T^2}\right) + \frac{n_f}{2\pi^2} T^4 J_f\left(\frac{m_f^2(h_c)}{T^2}\right)$$

$$m_h^2(h_c) = -\mu^2 + 3\lambda h_c^2$$

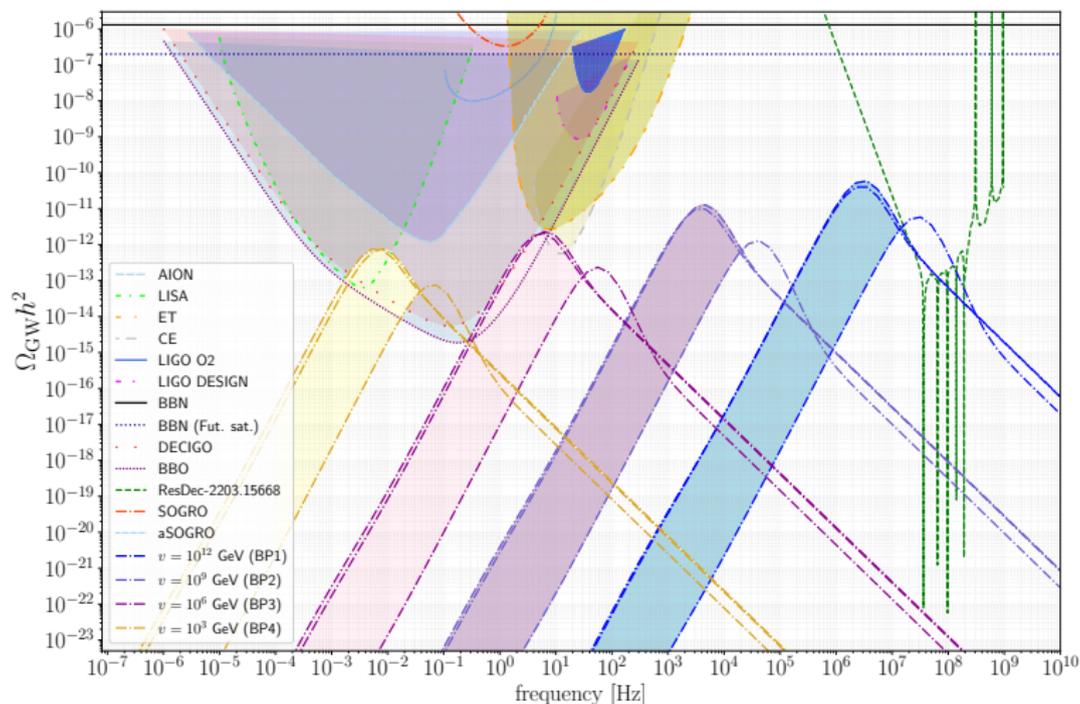
$$m_\chi^2(h_c) = -\mu^2 + \lambda h_c^2$$

$$m_g^2(h_c) = g^2 h_c^2$$

$$m_f^2(h_c) = \frac{y^2}{2} h_c^2$$

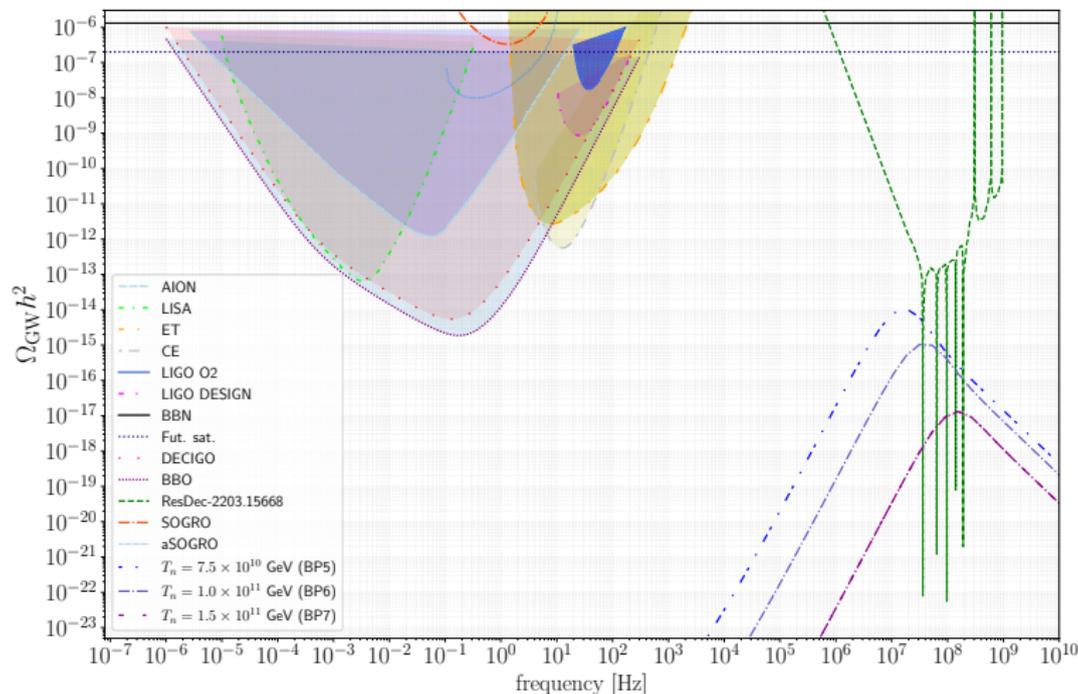
# Gravitational Wave Spectrum I

No fermions,  $g = 0.95$ ,  $\lambda = 10^{-3}$   
Bubble wall velocity  $v_w = 1$ ,  $v_{\text{det}} = 0.1$

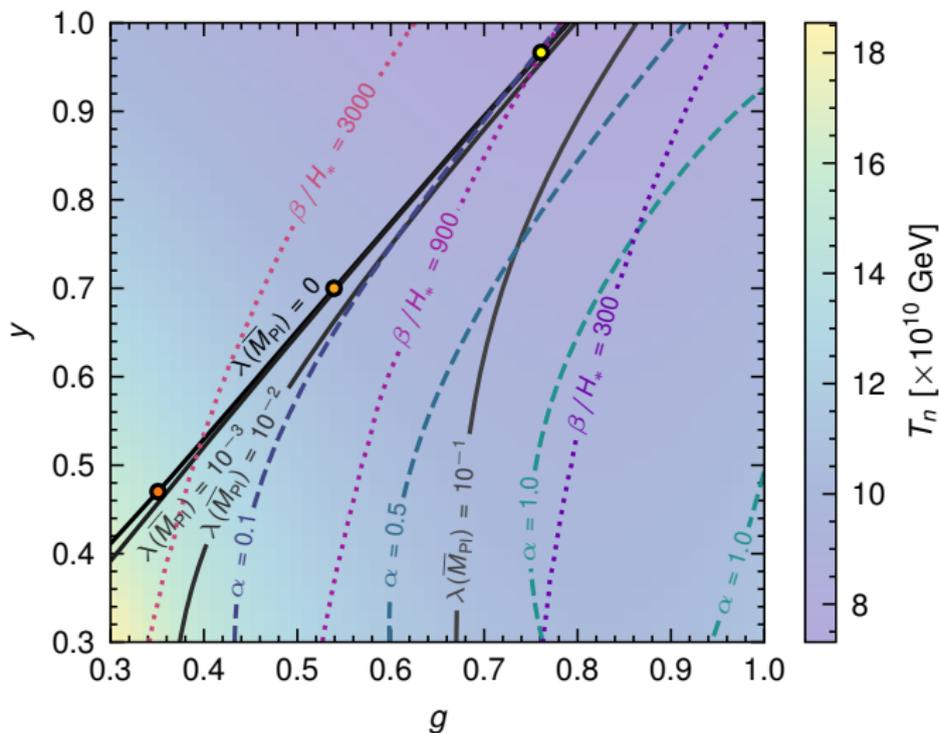


# Gravitational Wave Spectrum II

1 fermion pair,  $v_w = v_{\text{det}}$ ,  $v = 10^{12}$  GeV,  $\lambda = 10^{-3}$   
( $g, y$ )  $\simeq$  (0.76, 0.97), (0.54, 0.70), (0.35, 0.47)



# Parameter Space with 1 Fermion Pair



## CMB measurements

Scalar power spectrum amplitude	$A_s = (2.098 \pm 0.023) \times 10^{-9}$
Scalar spectral index	$n_s = 0.9649 \pm 0.0042$
Tensor-to-scalar power ratio	$r < 0.036$ (95% CL)

Planck, A&A **641** (2020); BICEP/Keck, PRL **127** (2021) RG

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Planck, A&A 641 (2020); BICEP/Keck, PRL 127 (2021) RG

## Model results (60 e-foldings)

- $n_s \simeq 0.965$
- $r \simeq 0.003$
- $A_s \simeq 5.1 \frac{\lambda(M_P)}{\xi^2} \rightsquigarrow \xi \simeq 5 \times 10^4 \sqrt{\lambda(M_P)}$

# Inflationary Observables

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	$g(M_P)$	$\lambda(M_P)$	$y(M_P)$	$n_\psi$	$\xi$
BP1	0.98	0.37	-	0	$3 \times 10^4$
BP2	0.99	0.57	-	0	$4 \times 10^4$
BP3	1.00	0.92	-	0	$5 \times 10^4$
BP4	1.02	2.22	-	0	$7 \times 10^4$
BP5	0.79	$\simeq 0$	1.10	1	$\mathcal{O}(1)$
BP6	0.55	$\simeq 0$	0.74	1	$\mathcal{O}(1)$
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- With fermions: renormalization group running allows  $\xi \sim 1$
- Analogous to **Critical Higgs Inflation**

Hamada et al., PRL 112 (2014), PRD 91 (2015)

# Conclusions and Outlook

- Unified framework for inflation and first-order phase transition
- Dark  $U(1)_X$  and non-minimal coupling to gravity  
     $\leadsto$  gravitational waves and inflation from same scalar
- Peak GW frequencies between  $10^{-2}$  Hz and  $10^8$  Hz

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     $\leadsto$  gravitational waves and inflation from same scalar
- Peak GW frequencies between  $10^{-2}$  Hz and  $10^8$  Hz
- Future directions
  - Calculation of bubble wall velocity using `WallGo`  
    Ekstedt et al., [arXiv:2411.04970](https://arxiv.org/abs/2411.04970)
  - Inflaton coupling to SM  $\leadsto$  reheating
  - Dark Matter
  - Different gauge groups

# Parameters Determining the GW Signal

- $T_n$ : temperature at which probability of nucleating 1 bubble per horizon volume is  $\sim 1$
- $\alpha = \frac{1}{\rho_{\text{rad}}} \left[ \Delta V(h_c, T) - T \frac{dV(h_c, T)}{dT} \right] \Big|_{T=T_n}$
- $\frac{\beta}{H(T_n)} = T_n \frac{d}{dT} \frac{S_3}{T} \Big|_{T_n} \simeq \left( \frac{T_n}{T - T_n} \right) \left( \frac{S_3(T)}{T} - \frac{S_3(T_n)}{T_n} \right)$   
( $T$ : arbitrary reference temperature not too far from  $T_n$ )
- Expressions modified for large supercooling

# SM Extensions with First-Order Phase Transitions

- **SM: no FO electroweak PT** for Higgs mass  $\gtrsim 70$  GeV  
Kajantie et al., PRL **77** (1996); Karsch et al., NP Proc. Suppl. **53** (1997)  
Csikor et al., PRL **82** (1999)
- **Most famous: SUSY with light stop**  
Carena et al., PLB **380** (1996); Espinosa, NPB **475** (1996)  
Delepine et al., PLB **386** (1996); Cline & Kainulainen, NPB **482** (1996); ...
- **Minimal from model building perspective: 2HDM**  
Dorsch et al., JHEP **10** (2013); Basler et al., JHEP **02** (2017)  
Andersen et al., PRL **121** (2018); ...
- **Minimal from EFT perspective: higher-dimensional operators**  
Zhang, PRD **47** (1993); Grojean et al., PRD **71** (2005)  
Bödeker et al., JHEP **02** (2005); Chala et al., JHEP **07** (2018); ...
- **Many recent works** Review: Roshan & White, arXiv:2401.04388