

# Radiative corrections to the QCD $\theta$ parameter at the two-loop level

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Based on:

TB, J. Hisano, T. Kitahara and N. Osamura, JHEP 02 (2024) 195 (arXiv:2311.07817)

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# Introduction

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In the QCD sector, we can introduce CPV parameters:

$$\mathcal{L}_{CPV} = -\bar{q} (m_q P_R + m_q^* P_L) q + \theta_G \frac{\alpha_s}{8\pi} G\tilde{G}$$
$$\xrightarrow{\text{chiral rot.}} -|m_q| \bar{q}q + (\theta_G - \arg(m_q)) \frac{\alpha_s}{8\pi} G\tilde{G}$$

Physical parameter (= independent from field redefinition) :  $\bar{\theta}$

$$\bar{\theta} = \theta_G - \sum_{\# \text{ of quark}} \arg(m_q)$$

$\bar{\theta}$  generates neutron EDM [ J. Liang, et al. arXiv:2301.04331 ]

$$|d_n| \simeq 1.48 \times 10^{-16} \bar{\theta} \text{ e} \cdot \text{cm}$$

Current experimental bound of  $\bar{\theta}$  (and the nEDM):

$$|\bar{\theta}| \lesssim 1.2 \times 10^{-10} \quad (|d_n| < 1.8 \times 10^{-26} \text{ e} \cdot \text{cm})$$

Why  $\bar{\theta} \ll \delta_{\text{CKM}} \sim \mathcal{O}(1)$ ? → Strong CP Problem

Possible solutions:

- axion
- P or CP symmetric model ← today's talk

P/CP symmetric model (e.g. left-right model, Nelson-Barr model)

- P/CP symmetry forbids  $\theta$  term at tree level. ( $G\tilde{G}$  is P-odd and CP-odd.)
- At low energy, these symmetries have to be spontaneously broken.

→  $\theta$  is radiatively generated,

$$\bar{\theta} = \cancel{\theta}_{\text{tree}} + \delta\theta$$

Precise evaluation of  $\delta\theta$  is essential for testing these models.

## How to evaluate the radiative $\theta$

Conventional method:  $\theta$  is evaluated by the imaginary part of the renormalized quark mass using Fujikawa method.

[ J. R. Ellis and M. K. Gaillard, Nucl. Phys. B 150 (1979) ]

$$\mathcal{L}_{CPV} = -\bar{q} \underbrace{(m_q + \Delta m_q)}_{\text{renormalized mass}} P_R q + \text{h.c.} \rightarrow \bar{\theta} = -\arg(m_q + \Delta m_q)$$

Problems:

- Contributions other than  $\arg(m_q + \Delta m_q)$  cannot be evaluated.  
     $\longleftrightarrow$  dipole operators contribute to  $\delta\theta$  by RGE. [ E. E. Jenkins, et al. JHEP 01 (2018) ]
- The renormalization condition of  $m_q$  is not clear.

We studied the 2-loop correction to the QCD  $\theta$  by calculating Feynman diagrams.

- Diagrammatic evaluation of  $\theta$  includes contributions beyond  $\arg(m_q + \Delta m_q)$ .
- We confirmed that when using the conventional method, defining the renormalized  $m_q$  at zero momentum is appropriate.

1. One-loop evaluation in an EFT (review)
  - Confirmed the contribution of the dipole operator to  $\theta$ .
2. Two-loop evaluation in a toy model with CP-violating Yukawa interactions
  - Clarified the definition of the renormalized  $m_q$  and compared with the conventional method.
  - Identified cases where the conventional method is insufficient.

## Light quark EFT & 1-loop $\theta$

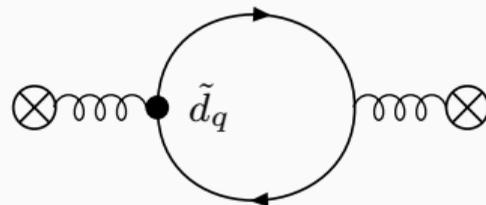
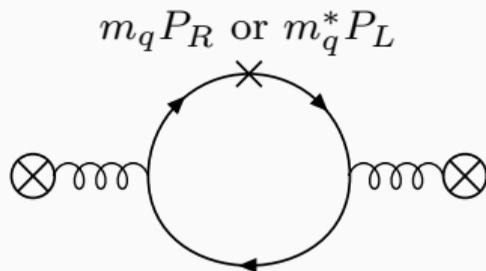
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## 1-loop $\theta$ (review)

Light quark EFT with effective operators up to dimension-5. (one-flavor,  $\theta_G=0$ )

$$\mathcal{L}_{\text{eff}} = \bar{q} [i\not{D} - (m_q^* P_L + m_q P_R)] q - \frac{1}{2} g_s \mu_q \bar{q} (\sigma \cdot G) q - \frac{i}{2} g_s d_q \bar{q} (\sigma \cdot G) \gamma_5 q$$

Feynman diagrams contributing to  $\theta$ :



## 1-loop $\theta$ (review)

Evaluating the effective action ( $\text{Tr log}$ ):

$$\begin{aligned}\Delta S &= -i \text{Tr log} \left\| i \not{D} - (m_q^* P_L + m_q P_R) \right\| \\ &= - \left( \frac{i}{2} \log \frac{m_q^*}{m_q} + 2 |m_q| \tilde{d}_q \log \frac{|m_q|}{\mu} \right) \int d^4 x \frac{\alpha_s}{8\pi} G \tilde{G}\end{aligned}$$

- 1st term is consistent with Fujikawa method:  $\frac{i}{2} \log \frac{m_q^*}{m_q} = \arg(m_q)$
- 2nd term could be evaluated by diagrammatic calculation: a contribution from the chromo EDM  $\tilde{d}_q$ . It is consistent with the RG effect  $\left( \mu \frac{d\theta}{d\mu} = 4m_q \tilde{d}_q \right)$ .

With this approach, we can evaluate not only the contribution of  $\arg(m_q)$  but also other contributions (e.g. CEDM).

However the evaluation of  $\text{Tr log}$  becomes difficult at 2-loop level or higher.

→ We need alternative method.

## Evaluation of the 2-loop $\theta$

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2-loop level  $\theta$  in a toy model with CP-violating Yukawa.

## 1. EFT approach (review)

- quark mass phase + RG effect

## 2. Direct diagram calculations

- Check consistency with EFT results in the light quark mass limit.
- Clarify the definition of the renormalized quark mass when using the Fujikawa method ( $\bar{\theta} = \arg(m_q + \Delta m_q)$ ).
- See that  $\arg(m_q + \Delta m_q)$  may not be sufficient.

We studied the 2-loop  $\theta$  in a simplified model.

The simplified model with a virtual quark, a real-scalar, and a CPV Yukawa:

$$-\mathcal{L} = \bar{q} (\text{Re}[m_q] + i \text{Im}[m_q] \gamma_5) q + \frac{1}{2} m_\phi^2 \phi^2 + y_q \phi \bar{q} P_R q + y_q^* \phi \bar{q} P_L q$$

Symmetry:  $\phi \rightarrow -\phi$ ,  $y_q \rightarrow -y_q$

Invariant quantities under the chiral rotation:  $\theta_G - \arg(m_q)$ ,  $y_q m_q^*$

Assumptions:  $m_q > \Lambda_{\text{QCD}}$ ,  $\text{Im}(m_q) \ll \text{Re}(m_q)$ ,  $\theta_G = 0$

$\Rightarrow$  Complex phases contributing to the  $\theta$ :

$$\arg(m_q) \simeq \frac{\text{Im}(m_q)}{\text{Re}(m_q)}$$

$$\text{Im}((y_q m_q^*)^2) \simeq \text{Im}(y_q^2) \text{Re}(m_q)^2$$

# Light quark EFT (1)

The case of  $m_q \ll m_\phi$

→ the correction to the  $\theta$  can be understood in terms of a light quark EFT.

Integrating out the heavy  $\phi$  at the scale  $\mu = m_\phi$ , we obtain effective CPV operators up to dim-6:

$$-\mathcal{L}_{\text{eff}} = \bar{q}i \text{Im}[m_q + \Delta m_q(\mu)]\gamma_5 q + \frac{i}{2}g_s \tilde{d}_q \bar{q}\sigma^{\mu\nu}\gamma_5 G_{\mu\nu} q - C_4^q(\bar{q}q)(\bar{q}i\gamma_5 q) \\ - \cancel{C_5^q(\bar{q}\sigma^{\mu\nu}q)(\bar{q}i\sigma_{\mu\nu}\gamma_5 q)} + \Delta\theta_{\text{th}} \frac{\alpha_s}{8\pi} G_{\mu\nu}^a \tilde{G}^{a\mu\nu} + \cancel{\frac{1}{3}\omega f^{abc} G_{\mu\nu}^a G_\rho^{b\nu} \tilde{G}^{c\rho\mu}}$$

- $\Delta m_q(\mu)$ : Correction to quark mass at zero momentum.
- $\tilde{d}_q$ : chromo-EDM ,  $C_4^q$  : 4-fermi operator
- $\Delta\theta_{\text{th}}$  : Threshold correction to  $\theta$  (undetermined in EFT)
- Slashed operator: Negligible because they are higher-order corrections.

RG effects at ( $m_q \leq \mu \leq m_\phi$ ):

- Mixing  $\tilde{d}_q$  and  $C_4^q$ :  $\tilde{d}_q, C_4^q \rightarrow \tilde{d}_q(\mu)$  [ J. Hisano, et al. Phys. Lett. B 713 (2012) ]

$$\mu \frac{d\tilde{d}_q}{d\mu} = \frac{4m_q}{16\pi^2} C_4^q$$

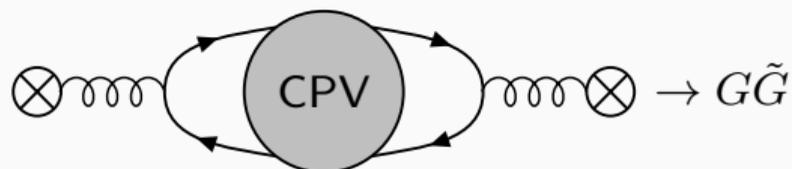
- contribution of  $\tilde{d}_q$  [ E. E. Jenkins, et al. JHEP 01 (2018) ]

$$\mu \frac{d\theta}{d\mu} = 4m_q \tilde{d}_q(\mu)$$

Combining with the conventional method ( $\arg(m_q + \delta m_q)$ ), we obtain:

$$\bar{\theta} = \delta\theta_{\text{EFT}} = -\frac{\text{Im}[m_q + \Delta m_q]}{\text{Re}[m_q]} - 2 \int_{\log m_\phi^2}^{\log m_q^2} \text{Re}[m_q] \tilde{d}_q(\mu) d \log \mu^2 + \Delta\theta_{\text{th}}$$

Next, we consider diagrammatic evaluations of the radiative  $\theta$ .



Problems:

- Evaluating  $\text{Tr} \log$  is difficult at 2-loop order or higher.
- The standard diagram calculations cannot handle total derivative term  $G\tilde{G} \propto \partial [A\partial A + \frac{2}{3}A^3]$ .

→ Fock-Schwinger gauge method

## Fock-Schwinger gauge method

We evaluate  $\theta$  with the background gluon field in Fock-Schwinger gauge.

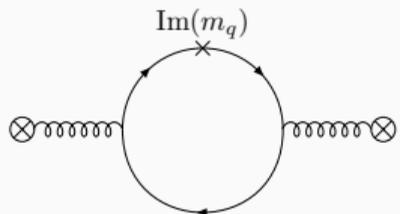
Fock-Schwinger gauge :  $(x - x_0)^\mu A_\mu(x) = 0$

- It breaks translational symmetry (momentum conservation), allowing treatment of the total derivative term  $G\tilde{G} \propto \partial [A\partial A + \frac{2}{3}A^3]$ .
- Gauge-invariant quantities restore translational symmetry.
- The gauge field can be expanded in the field-strength.

$$A_\mu(q) = -\frac{i(2\pi)^4}{2} G_{\nu\mu}(0) \frac{\partial}{\partial q_\nu} \delta^{(4)}(q) + \dots$$

The bellow 1-loop diagram corresponds to the contribution of  $\text{Im}(m_q)$  to the  $\theta$ . (It consistent with Fujikawa method.)

[ J. Hisano, et al. JHEP 03 (2023) ]

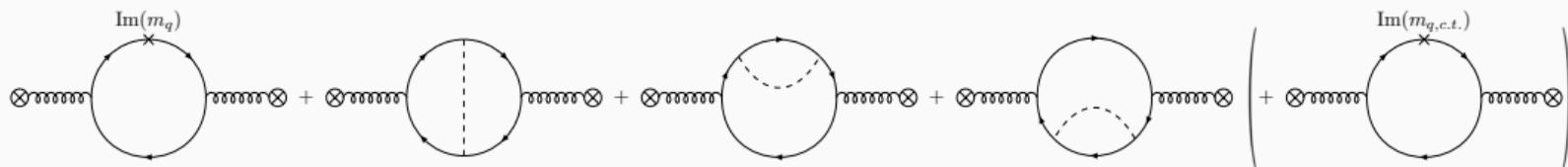

$$\rightarrow \delta\theta_{1\text{-loop}} = -\frac{\text{Im}(m_q)}{\text{Re}(m_q)}$$

## Diagrams contributing to the $\theta$ (2-loop level)

Model:

$$-\mathcal{L} = \bar{q} (\text{Re}[m_q] + i \text{Im}[m_q] \gamma_5) q + \frac{1}{2} m_\phi^2 \phi^2 + y_q \phi \bar{q} P_R q + y_q^* \phi \bar{q} P_L q$$

Diagrams up to 2-loop:



They correspond to contributions of  $\text{Im}(m_q)$  + 2-loop diagram ( + quark mass counterterm)

→ We evaluated the radiative theta by calculating these diagrams using the Fock-Schwinger gauge method.

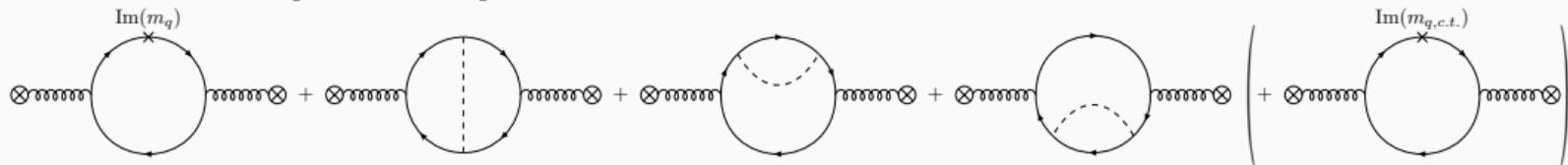
## Result

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## The case of $m_q \ll m_\phi$

Evaluation of the QCD  $\theta$  term at the 2-loop level using the Fock-Schwinger gauge method:

$$\bar{\theta} = \delta\theta = \delta\theta_{1\text{-loop}} + \delta\theta_{2\text{-loop}}$$



Diagrammatic calculations is consistent with the EFT approach when  $m_q \ll m_\phi$  (where  $\Delta\theta_{\text{th}}$  is determined here).

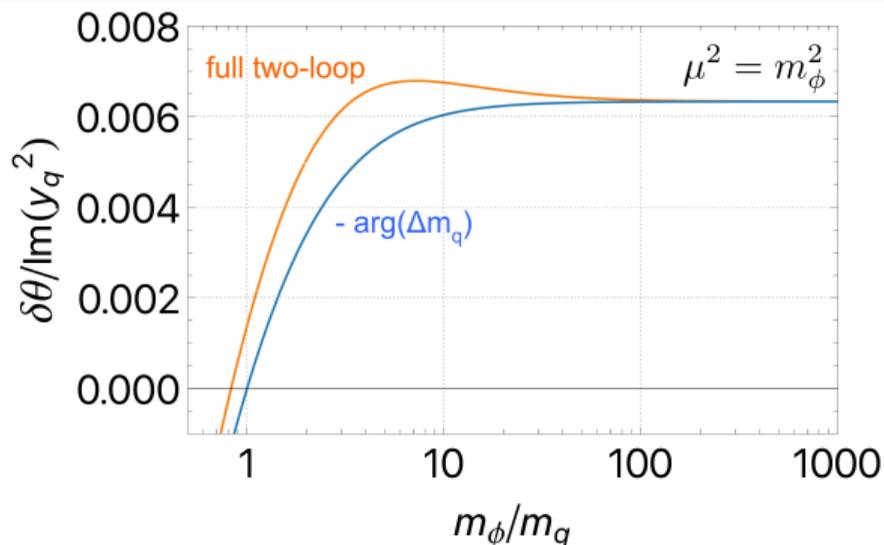
$$\delta\theta_{1\text{-loop}} + \delta\theta_{2\text{-loop}} = \delta\theta_{\text{EFT}} = -\frac{\text{Im}[m_q + \Delta m_q]}{\text{Re}[m_q]} - 2 \int_{\log m_\phi^2}^{\log m_q^2} \text{Re}[m_q] \tilde{d}_q(\mu) d \log \mu^2 + \Delta\theta_{\text{th}}$$

- We can evaluate the contributions not only  $\arg(m_q + \Delta m_q)$  but also  $\tilde{d}_q$ ,  $\Delta\theta_{\text{th}}$ .
- Contributions of  $\tilde{d}_q$ ,  $\Delta\theta_{\text{th}}$  are suppressed by  $m_q^2/m_\phi^2$ .

What happens when  $m_q \sim m_0$ ?

## The case of $m_q \leq m_\phi$

Analytical behavior of loop functions for  $m_q \leq m_\phi$



Blue: Complex phase of the 1-loop correction to quark mass (using Fujikawa method):

$$\delta\theta = -\arg(\Delta m_q)$$

Here,  $\Delta m_q(\mu)$  is the quark mass correction defined at zero momentum.

Orange: Analytical solution of the 2-loop diagram:

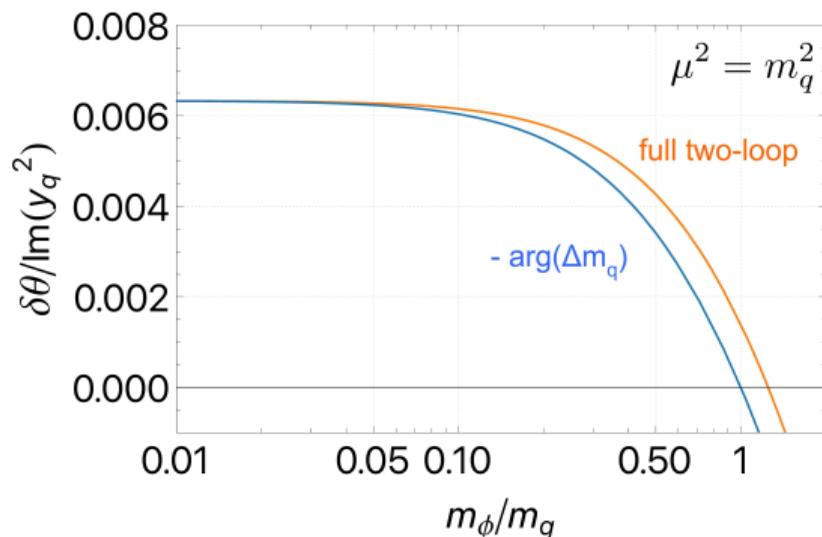
$$\delta\theta = \delta\theta_{2\text{-loop}}$$

We take the renormalization scale  $\mu = m_\phi$ .

When  $m_\phi/m_q \lesssim 10$ , Fujikawa method does not cover full contributions.

## The case of $m_q \geq m_\phi$

Analytical behavior of loop functions for  $m_q \leq m_\phi$



Blue: Complex phase of the 1-loop correction to quark mass (using Fujikawa method):

$$\delta\theta = -\arg(\Delta m_q)$$

$\Delta m_q(\mu)$  is the quark mass correction defined at zero momentum.

Orange: Analytical solution of the 2-loop diagram:

$$\delta\theta = \delta\theta_{2\text{-loop}}$$

We take the renormalization scale  $\mu = m_q$ .

When  $m_\phi/m_q \lesssim 10$ , Fujikawa method does not cover full contributions.

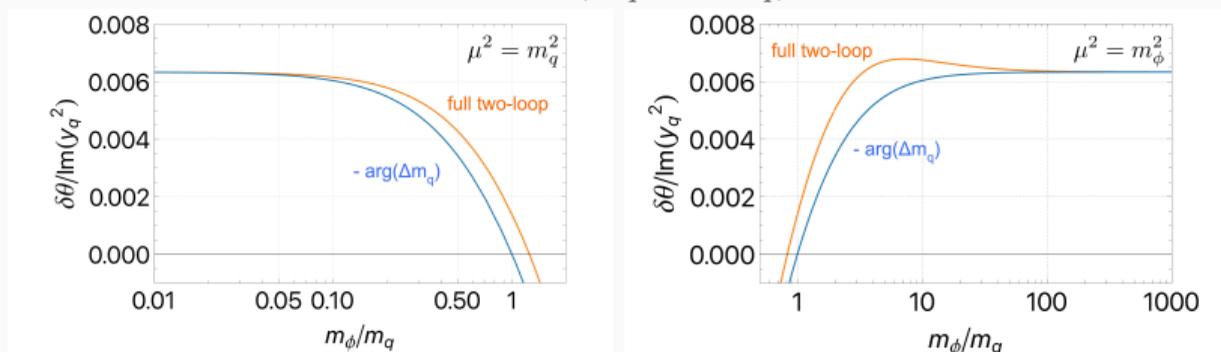
The coincidence for  $m_\phi/m_q \ll 1$  is cannot be explained by EFT.

# Summary

- With the diagrammatic method, we can evaluate full contributions to the radiative  $\theta$ .

$$\bar{\theta} = \delta\theta_{1\text{-loop}} + \delta\theta_{2\text{-loop}} = \text{[diagram 1]} + \text{[diagram 2]} + \text{[diagram 3]} + \text{[diagram 4]}$$

- If the renormalized  $m_q$  is defined at zero momentum,  $\arg(m_q + \Delta m_q)$  gives the correct evaluation for large mass hierarchies.
- When the mass hierarchy is small,  $\arg(m_q + \Delta m_q)$  is insufficient.



The Fock-Schwinger gauge method can evaluate higher-order corrections to the QCD  $\theta$ .

Measurements of the nEDM impose constraints on QCD  $\theta$ :

- Current constraint:  $\bar{\theta} \lesssim 10^{-10}$
- Future experiments may improve by 1-2 orders of magnitude.

In the minimal left-right model, we will study leading (3-loop) contributions to the radiative  $\theta$ .

- Previous estimates suggest  $\theta_{3\text{-loop}}$  is several orders below experimental constraints.

[ J. Hisano, et al. JHEP 03 (2023) ]

- A precise evaluation of  $\theta_{3\text{-loop}}$  can impose constraints on model parameters.

## Backup

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## Weinberg operator

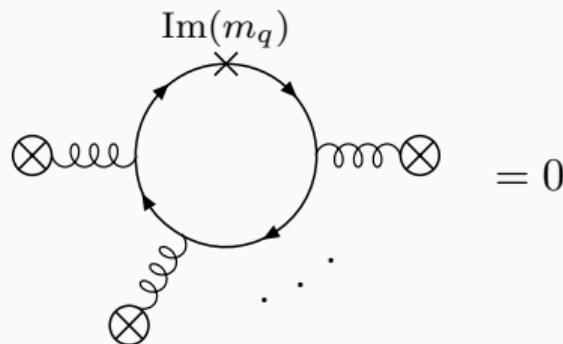
The contribution to the Weinberg operator from 1-loop diagrams involving  $\text{Im}(m_q)$ :

$$\begin{aligned}\frac{d}{dm_q} \Delta S &\supset \text{tr} \left[ \left\{ \frac{1}{p^2 - |m_q|} \left( \frac{1}{2} g_s (\sigma \cdot G) \right) \right\}^3 \frac{1}{p^2 - |m_q|} m_q^* P_R \right] \\ &\supset i \int d^4x \frac{c}{m_q |m_q|^2} GG\tilde{G} \quad (c : \text{real number})\end{aligned}$$

Integrating over  $m_q$ , and considering  $\frac{d}{dm_q^*} \Delta S$

$$\Delta S \supset i \int d^4x \left[ \frac{c}{|m_q|} - \frac{c}{|m_q|} \right] GG\tilde{G} = 0$$

Higher-order CP-violating gluon operators do not appear at the 1-loop level.



## Two-flavor case

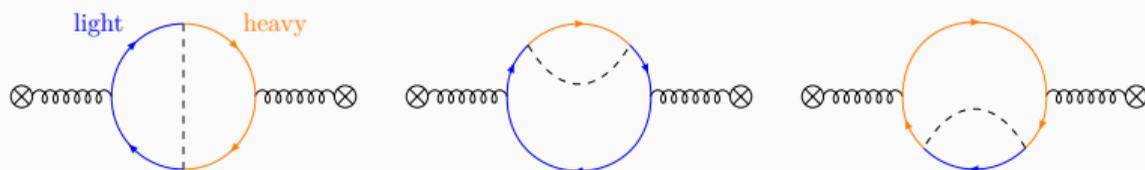
A toy model with CP-violating Yukawa interactions (two-flavor:

$i = l, h, m_l \ll m_h \sim m_\phi$ )

$$-\mathcal{L} = \bar{q}_i (\text{Re}[m_i] + i \text{Im}[m_i] \gamma_5) q_i + \frac{1}{2} m_\phi^2 \phi^2 + y_{ij} \bar{q}_i P_R q_j \phi + \text{h.c.}$$

- $q_l$ : light quark,  $q_h$ : heavy quark
- $\text{Im}(y_{ll} = \text{Im}(y_{hh})) = 0, \text{Im}(y_{lh}) \neq 0$

2-loop contribution to the  $\theta$ :



Leading contribution:  $-\text{Im}(\Delta m_l) / \text{Re}(m_l) \sim \mathcal{O}(m_h/m_l)$

Other contributions:  $\mathcal{O}(m_l/m_h), \mathcal{O}(m_l/m_\phi)$

# Nelson-Barr model

minimal BBP model (+ real scalar  $S$ )

[ L. Bento et al. Phys. Lett. B 267 (1991) ]

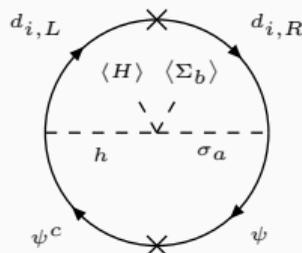
	chirality	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$	$\mathbb{Z}_N$
$Q$	L	<b>3</b>	<b>2</b>	1/6	0
$u$	R	<b>3</b>	<b>1</b>	2/3	0
$d$	R	<b>3</b>	<b>1</b>	-1/3	0
$H$	-	<b>1</b>	<b>2</b>	1/2	0
$\psi$	L	<b>3</b>	<b>1</b>	-1/3	$k$
$\psi^c$	R	<b>3</b>	<b>1</b>	-1/3	$k$
$\Sigma_a$	-	<b>1</b>	<b>1</b>	0	$k$
$S$	-	<b>1</b>	<b>1</b>	0	0

$$-\mathcal{L}_Y^d = y_u^{ij} \tilde{H} \bar{Q}_i u_j + y_d^{ij} H \bar{Q}_i d_j \\ + g^{ai} \Sigma_a \bar{\psi}_L d_i + f S \bar{\psi} \psi^c + \text{h.c.}$$

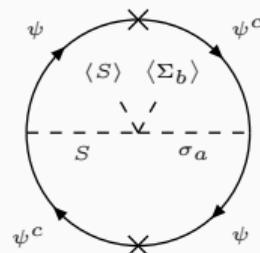
$$V(\Sigma_a, H) = \gamma_{ab} \Sigma_a^* \Sigma_b |H|^2 + \frac{1}{2} \tilde{\gamma}_{ab} \Sigma_a^* \Sigma_b S^2$$

- Relative phases of  $\Sigma_a$  lead to CPV.
- $f \langle S \rangle$  leads to the vector-like quark mass.

$$\delta\theta = \Delta\theta_\Sigma + \Delta\theta_S$$



$$\Delta\theta_\Sigma \simeq \frac{1}{16\pi^2} \gamma_{ab} g^{ak} g^{ck} \frac{\text{Im}[\langle \Sigma_b \rangle \langle \Sigma_c \rangle^*]}{m_h^2 - m_\Sigma^2}$$



$$\Delta\theta_S \simeq \frac{1}{8\pi^2} \tilde{\gamma}_{ab} g^{ak} g^{ck} \frac{\langle S \rangle}{m_\psi} \frac{\text{Im}[\langle \Sigma_b \rangle \langle \Sigma_c \rangle^*]}{m_S^2 - m_\Sigma^2}$$

Dominated by contributions of  $m_l$ . Involving only heavy particles (small mass hierarchy)

When  $m_\psi \sim m_S$ ,  $\langle \Sigma_a \rangle$ , we have to evaluate  $\Delta\theta_S$  using the diagrammatic method.

We calculated the radiative  $\theta$  using dimensional-regularization. However, definitions of  $\gamma_5$  and  $\varepsilon^{\mu\nu\lambda\kappa}$  are ambiguous in  $d$ -dimensional spacetime.

Breitenlohner-Maison-'t Hooft-Veltman (BMHV) scheme clarifies these definitions.

$$\gamma_5 = -\frac{i}{4}\bar{\varepsilon}^{\mu\nu\lambda\kappa}\gamma_\mu\gamma_\nu\gamma_\lambda\gamma_\kappa, \quad \hat{\varepsilon} = 0, \quad \{\bar{\gamma}_\mu, \gamma_5\} = 0, \quad [\hat{\gamma}_\mu, \gamma_5] = 0$$

Bar denotes 4-dim. vector, and hat denotes  $d - 4$ -dim. vector. It is often used to calculate anomalies or CPV operator effects.

What happens in the radiative  $\theta$  calculation?  $\rightarrow$  We will submit to arXiv.