

Quantum aspects of inflationary GWs

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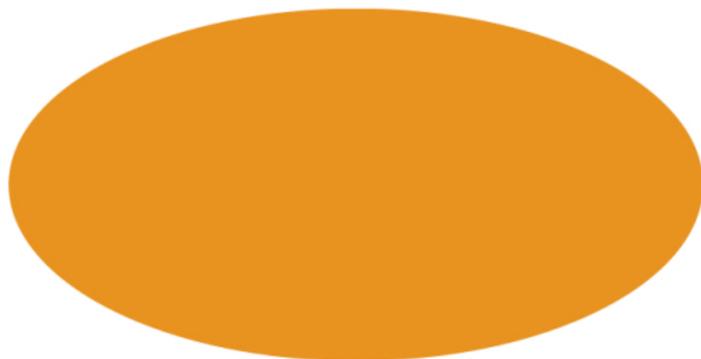
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Outline

- 1 Introduction
- 2 From quantum to classical: Decoherence
 - Decoherence
 - Decoherence during inflation
- 3 From quantum to classical: Large-scale solutions
 - Canonical conjugate pair
 - Large-scale behaviour
- 4 Conclusions

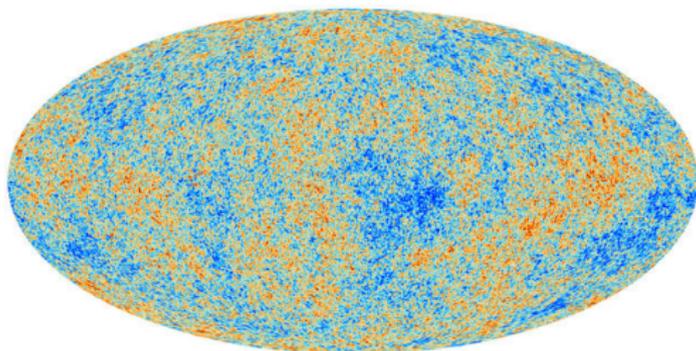
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Cosmic microwave background (CMB)



- Strong observational support for big bang cosmology
- Decoupling of photons and electrons
- Earliest universe we can observe by photons (380,000 yrs old)
- Almost perfect blackbody radiation of $T_0 = 2.725$ K

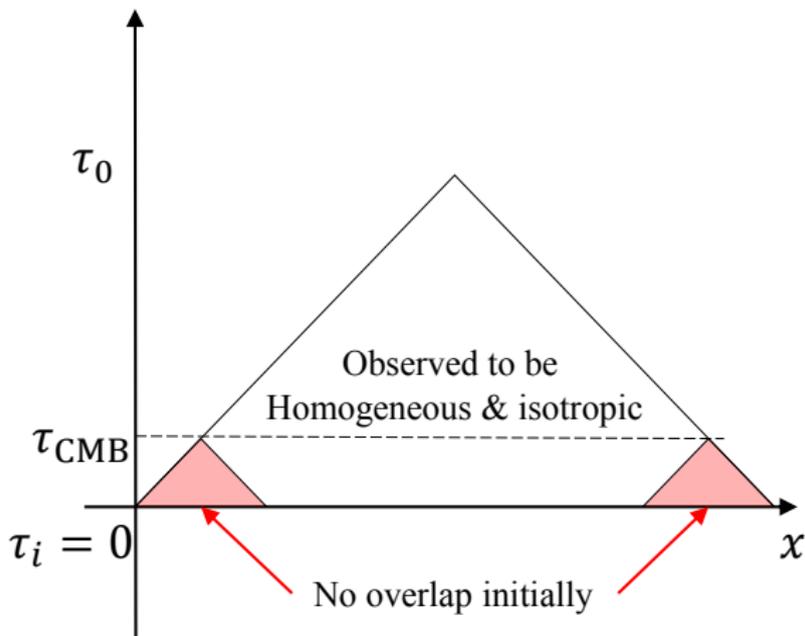
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- $\mathcal{O}(10^{-5})$ fluctuations: $T = T_0 \pm \mathcal{O}(10) \mu\text{K}$

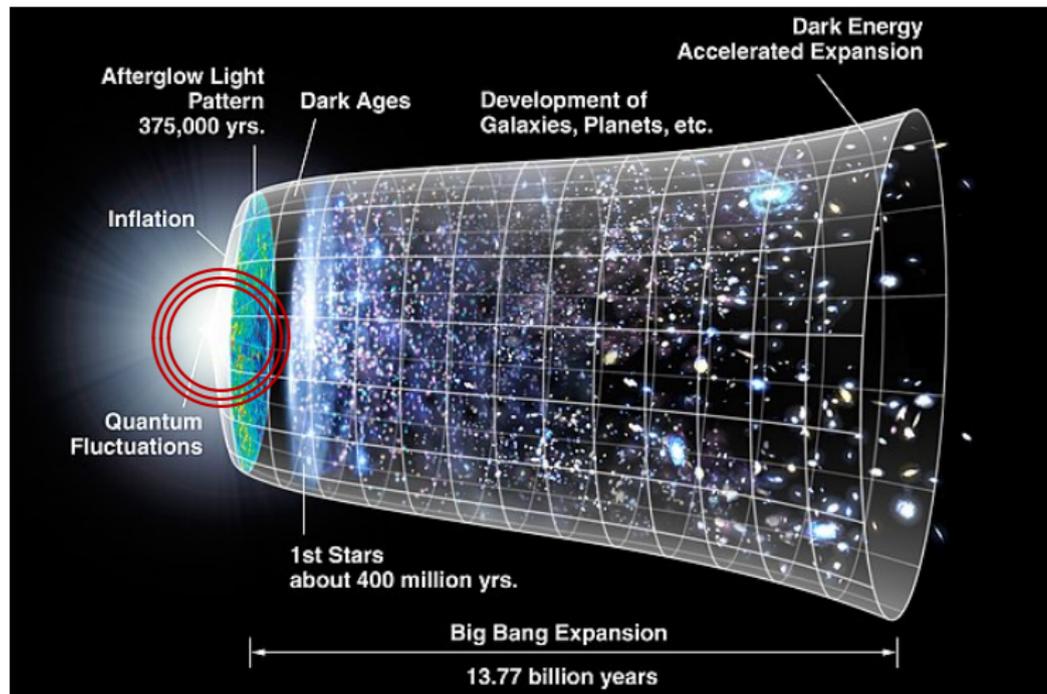
What is wrong with CMB?

Why is CMB so homogeneous and isotropic?



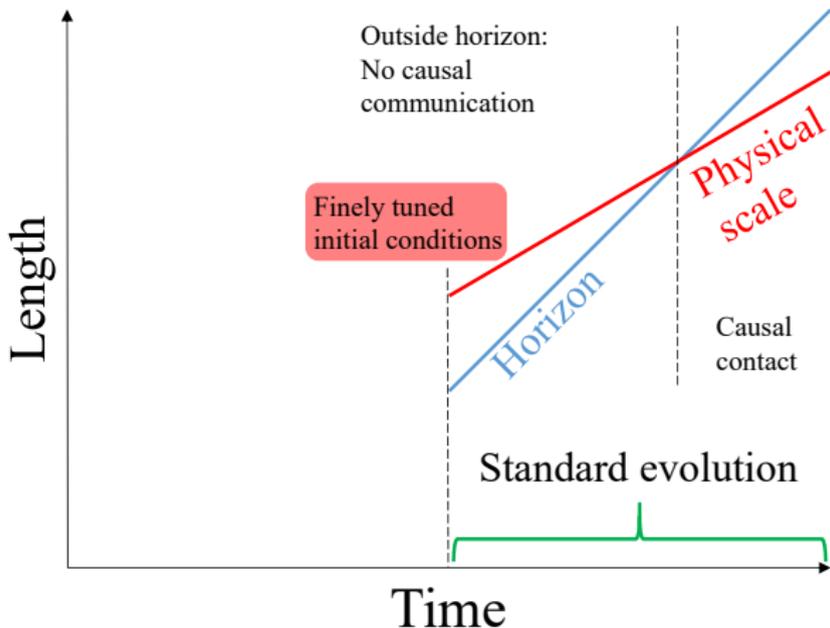
“Horizon problem”

Where is the origin of horizon problem?



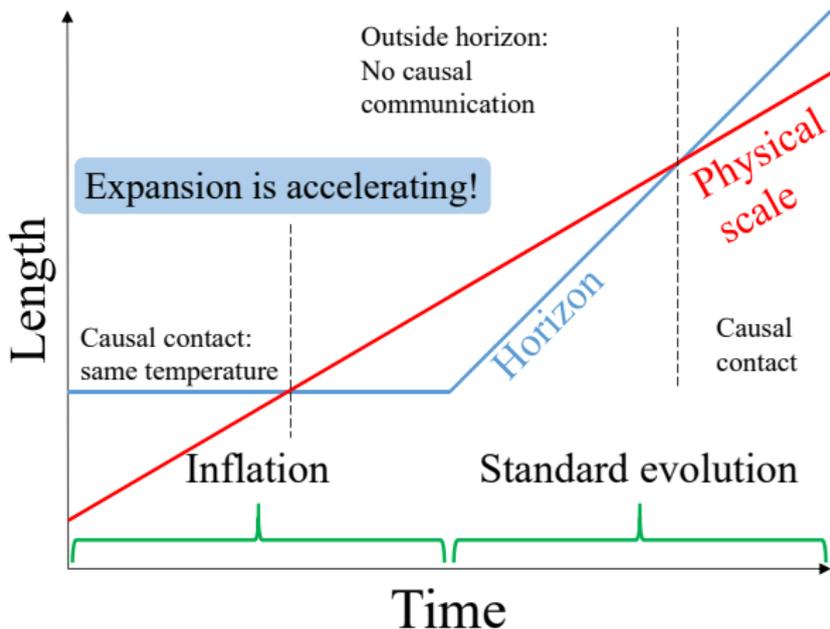
What is inflation?

Inflation = **accelerated expansion** before hot big bang evolution



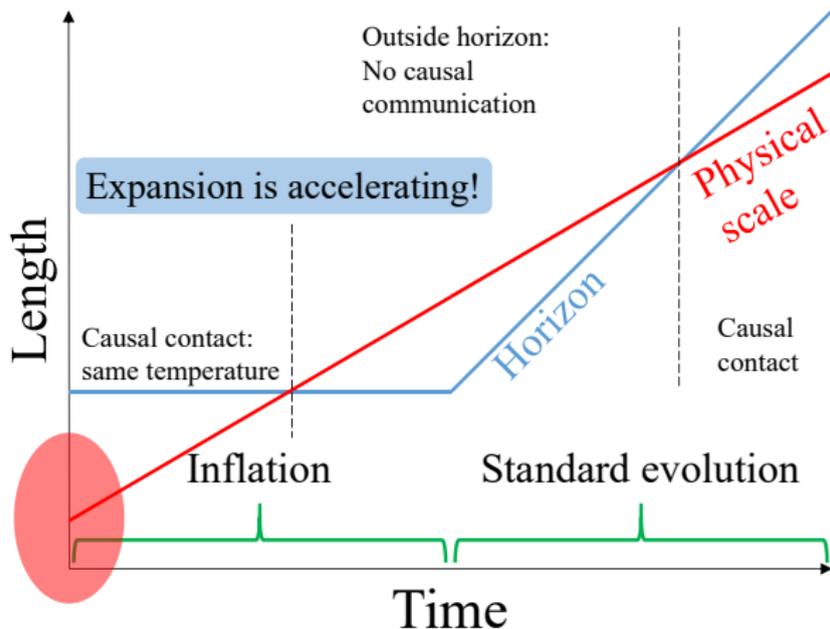
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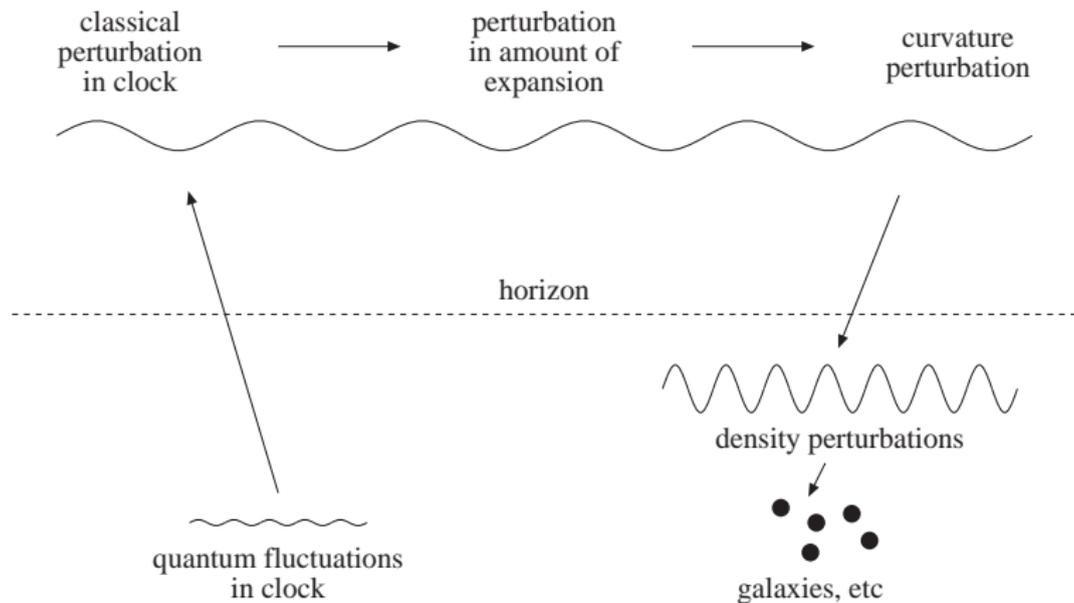
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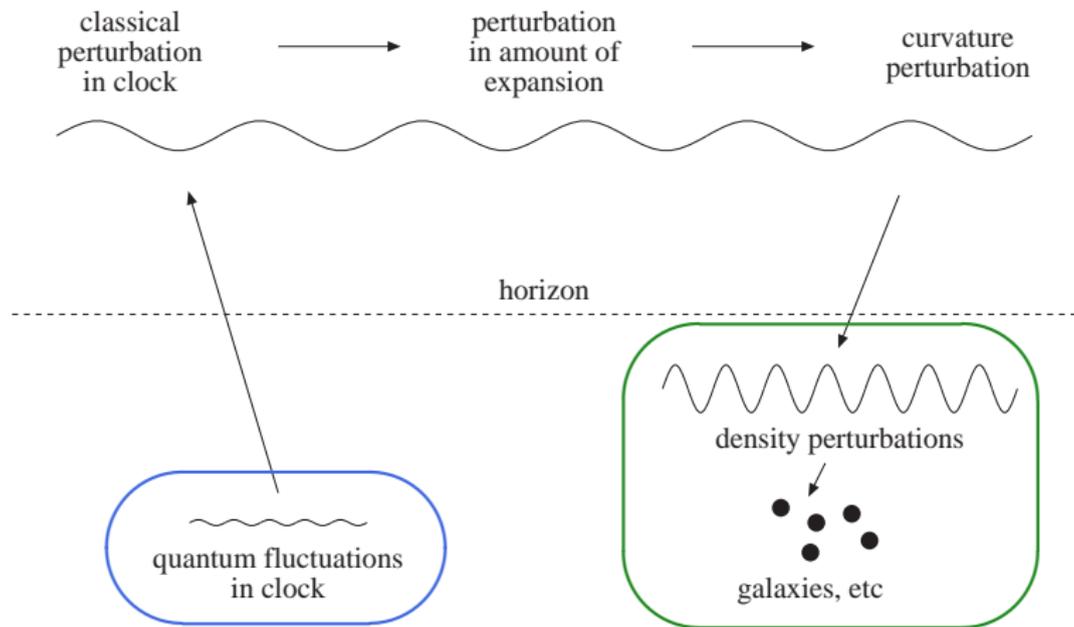


Quantum mechanics is relevant during inflation

Generation and evolution of perturbations



Generation and evolution of perturbations



QM signatures on cosmic scales, e.g. $\delta T/T_0$ of CMB

Scalar and tensor perturbations

Distance between 2 points in space

$$dl^2 = a^2(t) \{ [1 + 2\mathcal{R}(t, \mathbf{x})] \delta_{ij} + h_{ij}(t, \mathbf{x}) \} dx^i dx^j$$

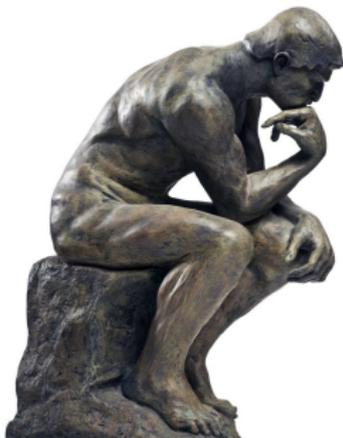
2 geometric perturbations constrained by observations

- $\mathcal{R}(t, \mathbf{x})$: curvature perturbation (scalar)
- $h_{ij}(t, \mathbf{x})$: **gravitational waves** (tensor)

We treat these perturbations quantum mechanically

Quantum aspects of inflationary GWs?

- How do inflationary GWs become classical?
- How to quantify the “classicality”?
- What are possible observable signatures?



Important for our understanding of early universe

1 Introduction

2 From quantum to classical: Decoherence

- Decoherence
- Decoherence during inflation

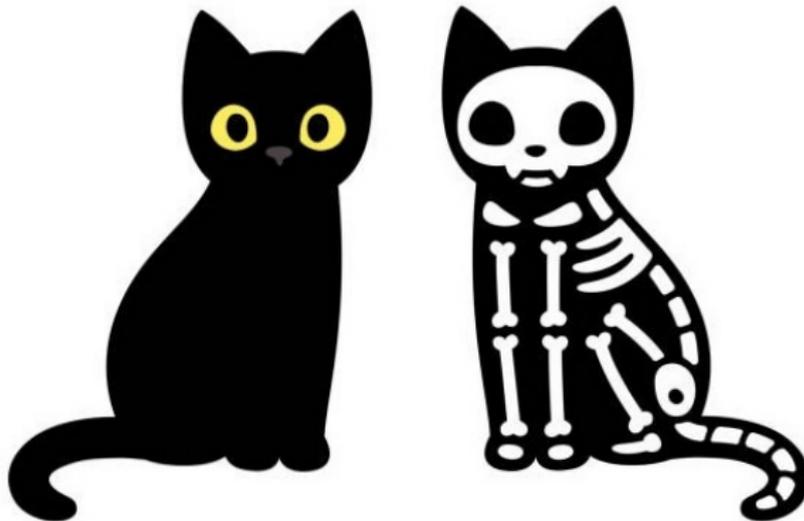
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- Large-scale behaviour

4 Conclusions

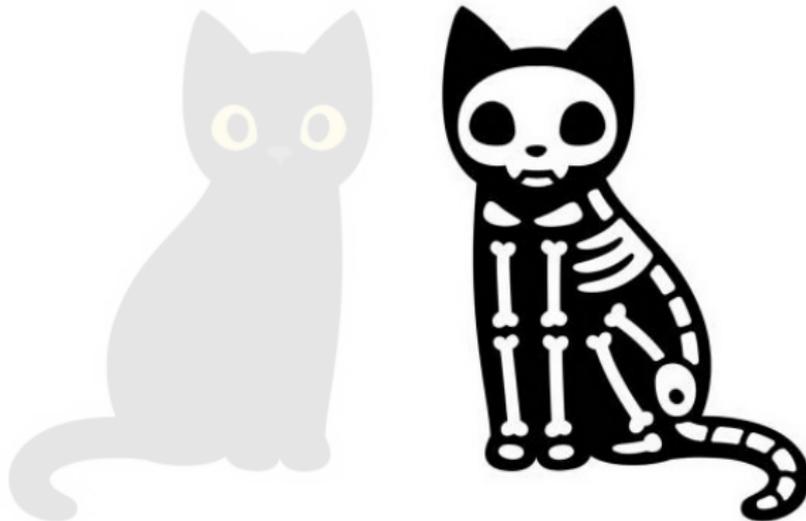
Emergence of classical world

In QM: A cat is alive **and** dead,



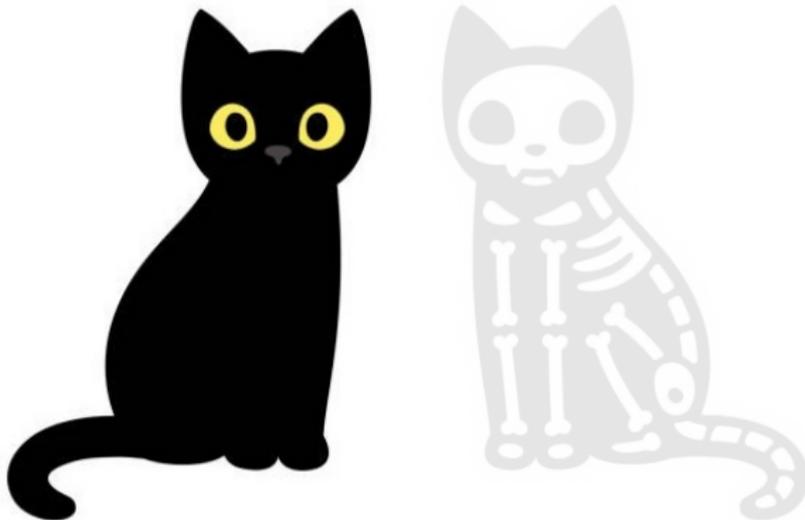
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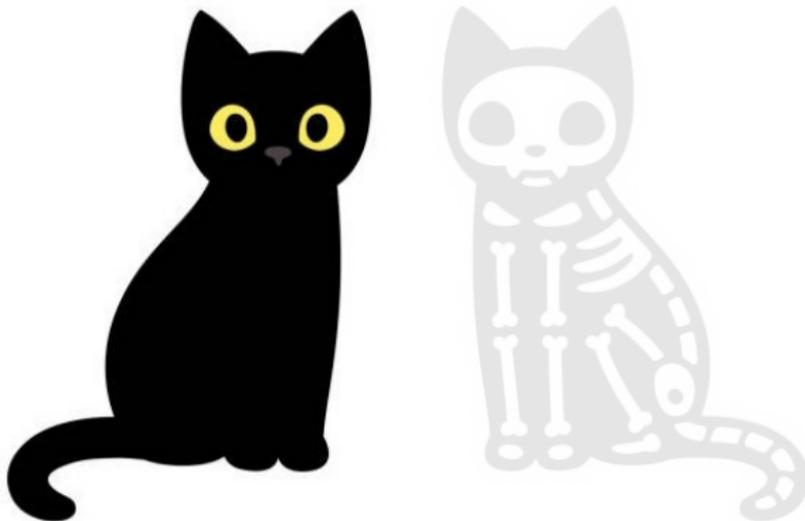
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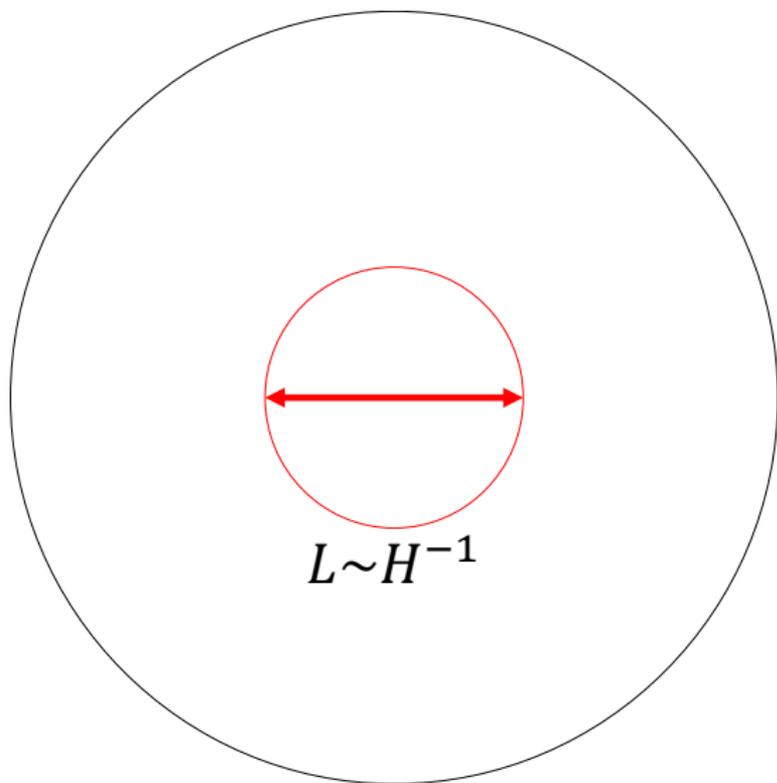


“Decoherence”

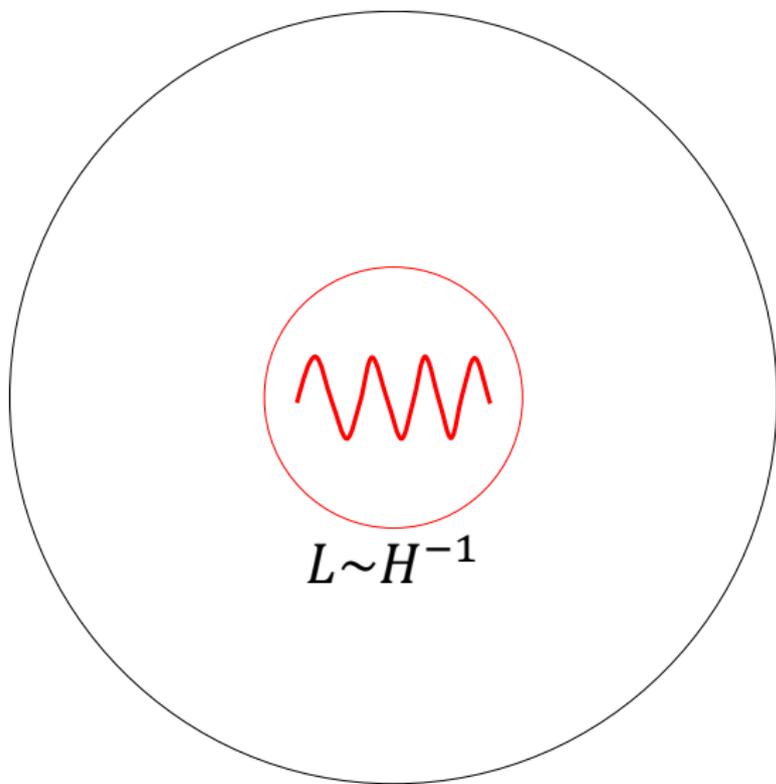
Quantum decoherence

- If $|\psi_1\rangle$ and $|\psi_2\rangle$ are quantum states, so is $\alpha|\psi_1\rangle + \beta|\psi_2\rangle$
- In larger systems, it is very difficult (no alive *and* dead cat)
- Quantum decoherence: Environmental influences
- We need non-linear interactions

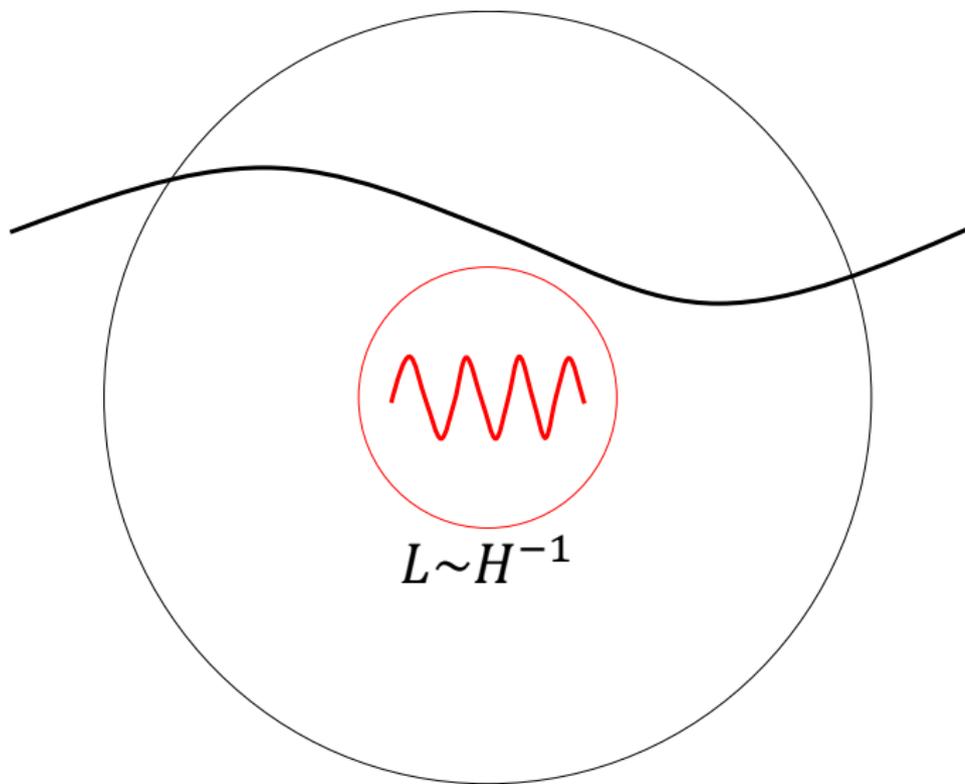
System and environment during inflation



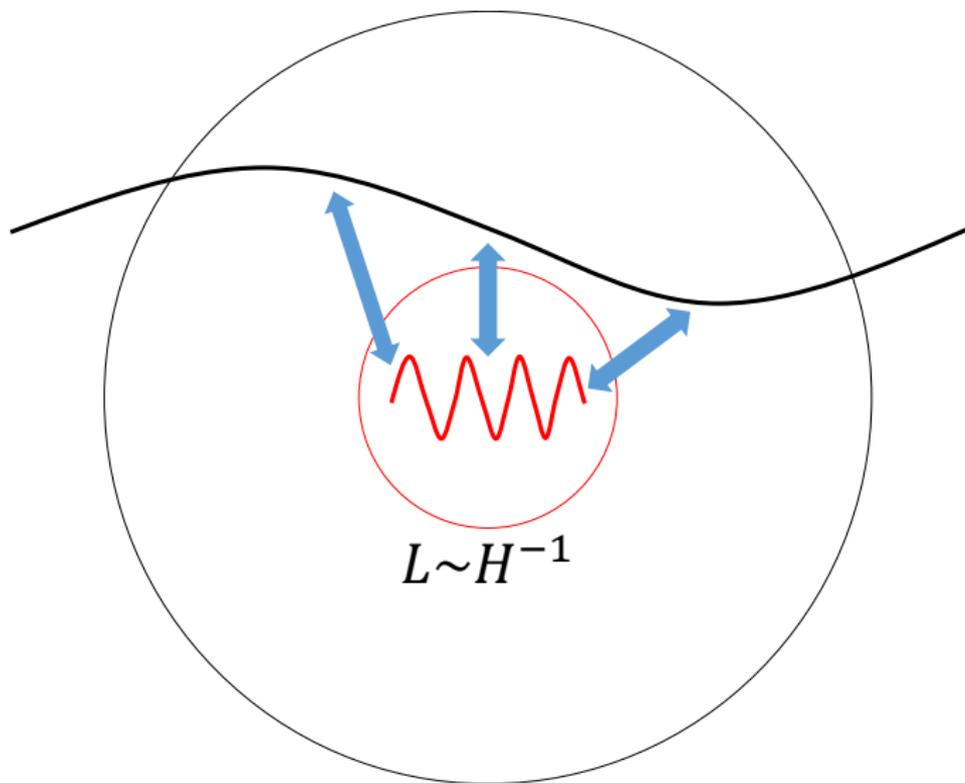
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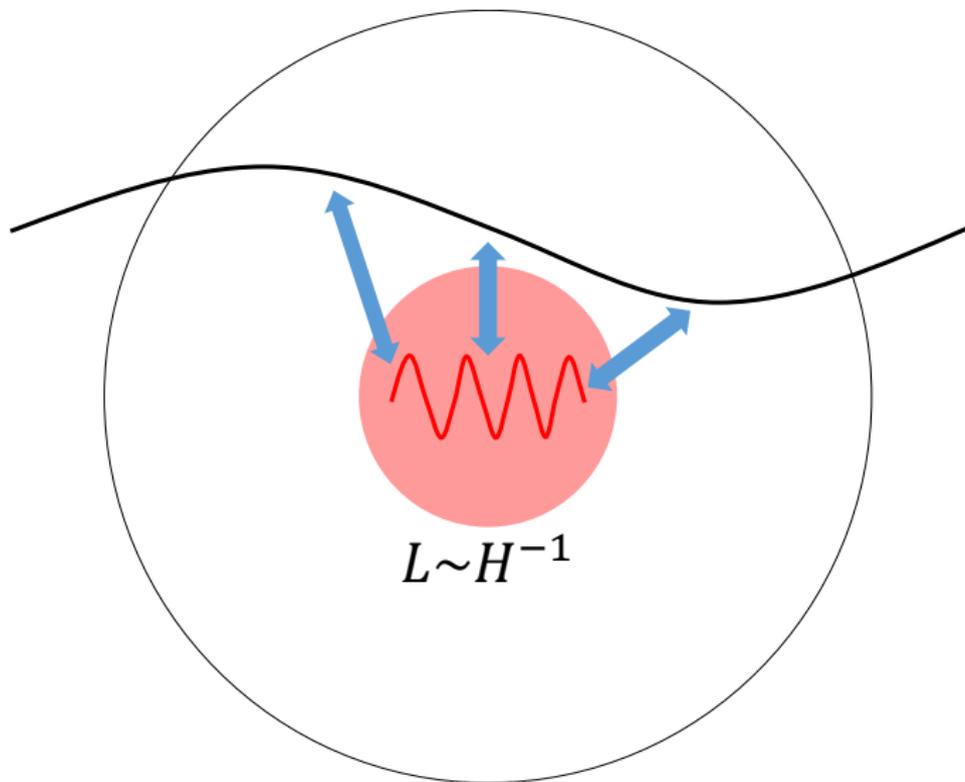
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Decoherence due to non-linear interactions

- Initially Bunch-Davies vacuum (“pure” state)
- Due to interactions, states other than vacuum are populated

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$$\rho_{\text{red}} \equiv \sum_i \langle \mathcal{E}_i | \Psi \rangle \langle \Psi | \mathcal{E}_i \rangle = \text{Tr}_{\mathcal{E}} \rho$$

- How do quantum states evolve? Linear in cubic interaction...

$$\frac{d\rho_{\text{red}}}{d\tau} = - \left(\begin{array}{ccc|c} \mathfrak{E}_{00} & 0 & \mathfrak{E}_{02} & \\ 0 & \mathfrak{E}_{11} & 0 & 0_{3 \times 4} \\ \mathfrak{E}_{20} & 0 & 0 & \\ \hline & 0_{4 \times 3} & & 0_{4 \times 4} \end{array} \right)$$

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Many states emerge (“mixed” states)

Deviation from unitary evolution

- $\rho_{\text{red}}|_{00}$ describes the evolution of system from BD vacuum
- $\rho_{\text{red}}|_{00}$ is reduced from 1: Non-unitary evolution
- Decoherence time: The e -folds for $\rho_{\text{red}}|_{00}$ to change by e^{-1}

$$\Delta N_{\text{dec}} \approx \frac{1}{3} \log \left[\frac{(2\pi)^2}{\Delta_{\mathcal{R}}^2} \frac{9}{2} (r \mathcal{C}_{\mathcal{J}\mathcal{E}})^{-1} \right] \approx 8.38689 - \frac{1}{3} \log(r \mathcal{C}_{\mathcal{J}\mathcal{E}})$$

Typically $5 \lesssim \Delta N_{\text{dec}} \lesssim 10$ for a wide range of r and $\mathcal{C}_{\mathcal{J}\mathcal{E}}$

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Alternative to decoherence

- Full consideration of decoherence is complicated
- Alternative: *Decoherence without decoherence* [Polarski & Starobinsky \(1996\)](#)
- Quickly commutation relation vanishes on large scales
- Equivalent to ignoring decaying mode (see later)

Why is it so? And what does it mean?

Quadratic interaction and conjugate pair

For both polarizations, $v_{\mathbf{k}} \equiv am_{\text{pl}} h_{(s)} / \sqrt{2}$

$$H = \int d^3x \frac{1}{2} \left[\pi_{\mathbf{k}} \pi_{-\mathbf{k}} + k^2 v_{\mathbf{k}} v_{-\mathbf{k}} + \frac{a'}{a} (\pi_{\mathbf{k}} v_{-\mathbf{k}} + v_{\mathbf{k}} \pi_{-\mathbf{k}}) \right]$$

Being canonical conjugate pair, $v(\mathbf{x})$ and $\pi(\mathbf{x})$ always satisfy

$$[v(\mathbf{x}, \tau), \pi(\mathbf{y}, \tau)] = i\delta^{(3)}(\mathbf{x} - \mathbf{y})$$

$$[a_{\mathbf{k}}(\tau), a_{\mathbf{q}}^\dagger(\tau)] = (2\pi)^3 \delta^{(3)}(\mathbf{k} - \mathbf{q})$$

Why commutation relations?

- Fundamental QM relations
- Blind to non-linear interaction
- Classicality from different viewpoint

Bogoliubov transformation

Due to time-dep BG, operators at different times are related by

$$a_{\mathbf{k}}(\tau) = \alpha_k(\tau) a_{\mathbf{k}}(\tau_0) + \beta_k(\tau) a_{-\mathbf{k}}^\dagger(\tau_0)$$

$$a_{-\mathbf{k}}^\dagger(\tau) = \alpha_k^*(\tau) a_{-\mathbf{k}}^\dagger(\tau_0) + \beta_k^*(\tau) a_{\mathbf{k}}(\tau_0)$$

By equal-time commutation relation, α_k and β_k always satisfy

$$[v_{\mathbf{k}}(\tau), \pi_{\mathbf{q}}(\tau)] = (|\alpha_k|^2 - |\beta_k|^2) [v_{\mathbf{k}}(\tau_0), \pi_{\mathbf{q}}(\tau_0)]$$

$$\therefore |\alpha_k|^2 - |\beta_k|^2 = 1$$

Equal-time commutation relation is guaranteed to hold always

Mode functions

By Bogoliubov x-form we can define mode functions $f_{\mathbf{k}}(\tau)$ and $g_{\mathbf{k}}(\tau)$

$$\begin{aligned}
 v_{\mathbf{k}}(\tau) &= \frac{1}{\sqrt{2k}} \left[a_{\mathbf{k}}(\tau) + a_{-\mathbf{k}}(\tau) \right] \\
 &= f_{\mathbf{k}}(\tau) a_{\mathbf{k}}(\tau_0) + f_{\mathbf{k}}^*(\tau) a_{-\mathbf{k}}(\tau_0) \quad \left[f_{\mathbf{k}} \equiv \frac{1}{\sqrt{2k}} (\alpha_k + \beta_k^*) \right] \\
 \pi_{\mathbf{k}}(\tau) &= -i \sqrt{\frac{k}{2}} \left[a_{\mathbf{k}}(\tau) - a_{-\mathbf{k}}(\tau) \right] \\
 &= -i g_{\mathbf{k}}(\tau) a_{\mathbf{k}}(\tau_0) + i g_{\mathbf{k}}^*(\tau) a_{-\mathbf{k}}(\tau_0) \quad \left[g_{\mathbf{k}} \equiv \sqrt{\frac{k}{2}} (\alpha_k - \beta_k^*) \right]
 \end{aligned}$$

Large-scale solutions of mode functions

f_k and g_k respectively satisfy the equations:

$$f_k'' + \left(k^2 - \frac{a''}{a} \right) f_k = 0$$

$$g_k = i \left(f_k' - \frac{a'}{a} f_k \right)$$

On very large scales ($k^2 \ll a''/a$) the solutions are

$$f_k = c_1 a + c_2 a \int \frac{d\tau}{a^2} \quad \left\{ \begin{array}{l} c_1 : \text{Growing mode} \\ c_2 : \text{Decaying mode} \end{array} \right.$$

$$g_k = i \frac{c_2}{a}$$

c_1 and c_2 are k -dependent constants of integrations

Squeezing parameter

Due to the constraint $|\alpha_k|^2 - |\beta_k|^2 = 1$, we may write

$$\alpha_k = e^{-i\theta_k} \cosh r_k$$

$$\beta_k = e^{i(\theta_k + 2\phi_k)} \sinh r_k$$

The “squeezing parameter” r_k satisfies the equation

$$r'_k = \frac{a'}{a} \cos(2\phi_k)$$

In terms of f_k and g_k , we can write $\cos(2\phi_k)$ as

$$\cos(2\phi_k) = \frac{k|f_k|^2 - \frac{1}{k}|g_k|^2}{\sqrt{\left(k|f_k|^2 - \frac{1}{k}|g_k|^2\right)^2 + 4\left[\Im(f_k g_k^*)\right]^2}}$$

Evolution of squeezing parameter

Using the large-scale solutions of f_k and g_k , during inflation we have

$$k|f_k|^2 - \frac{1}{k}|g_k|^2 = \underbrace{k|c_1|^2 a^2}_{\propto a^2} - \underbrace{\frac{|c_2|^2}{ka^2}}_{\propto a^{-2}} + \underbrace{2k\Re(c_1 c_2^*) a^2 \int \frac{d\tau}{a^2}}_{\propto a^{-1}} + \underbrace{k|c_2|^2 a^2 \left(\int \frac{d\tau}{a^2} \right)^2}_{\propto a^{-4}}$$

N.B. During RD and MD, 1st term is still always dominant

N.B.2 $\Im(f_k g_k^*)$ is at most constant

Large squeezing limit and classicality

Thus, very quickly c_1 dominates time evolution of r_k exponentially

$$\phi_k \rightarrow 0 \quad \text{and} \quad \cosh r_k (\approx \sinh r_k) \rightarrow \frac{e^{r_k}}{2} \sim \frac{1}{-k\tau}$$

This means $\alpha_k = \beta_k^*$ so that $|\alpha_k|^2 - |\beta_k|^2 = 0$

(equivalently $c_2 = 0$ so that $f_k \rightarrow c_1 a$ and $g_k \rightarrow 0$)

$$\left[u_k(\tau), \pi_q(\tau) \right] \xrightarrow{\tau \rightarrow 0} 0$$

As inflation proceeds, classicality emerges on large scales

Correlation between growing and decaying modes

Bogoliubov constraint reads from large-scale solutions of f_k and g_k

$$\begin{aligned} \cosh^2 r_k - \sinh^2 r_k = 1 &= f_k g_k^* + f_k^* g_k = 2 \left(\Re f_k \Re g_k + \Im f_k \Im g_k \right) \\ &= 2 \Im(c_1 c_2^*) = 2 \begin{vmatrix} \Re c_2 & \Re c_1 \\ \Im c_2 & \Im c_1 \end{vmatrix} \end{aligned}$$

- c_1 and c_2 are regarded as independent integration constants
- They are *not* independent but must always satisfy the above
- Especially, if $\Im c_1 = \Re c_2 = 0$ (BD case) $c_2 = -i/(2c_1)$
- **Non-zero c_2 means non-vanishing inflationary quantumness**
- Matrix relation looks very similar to SU(2), means something?

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Conclusions

- Inflation provides QM origin of GWs
- How do they become classical perturbations?
 - Decoherence: Non-linear interactions
 - Decoherence w/o decoherence: Vanishing commutator
- Classicality is always guaranteed
- Correlation bet (seemingly) adiabatic & isocurvature modes