



Heavy Dark Matter **and** the Evolution of the Early Universe

Chang Sub Shin
Chungnam National University

**based on Phys. Rev. D 101, 095019 (2020) by D. Chway, T.H. Jung, and C.S. Shin
and arXiv:2412.15864 by Tomasz Dutka(KIAS), Tae Hyun Jung(IBS), and Chang Sub Shin**

2025 CAU-IBS Beyond the Standard Model Workshop
Feb. 20, 2025 @ CAU

Outline

General Intro to Heavy Dark Matter Scenarios

- Unitarity Bound, Constraints, and Production Mechanisms

Phase Transition for Dark Matter Productions

- DM from Filtering effect and/or High energetic bubble collision etc.

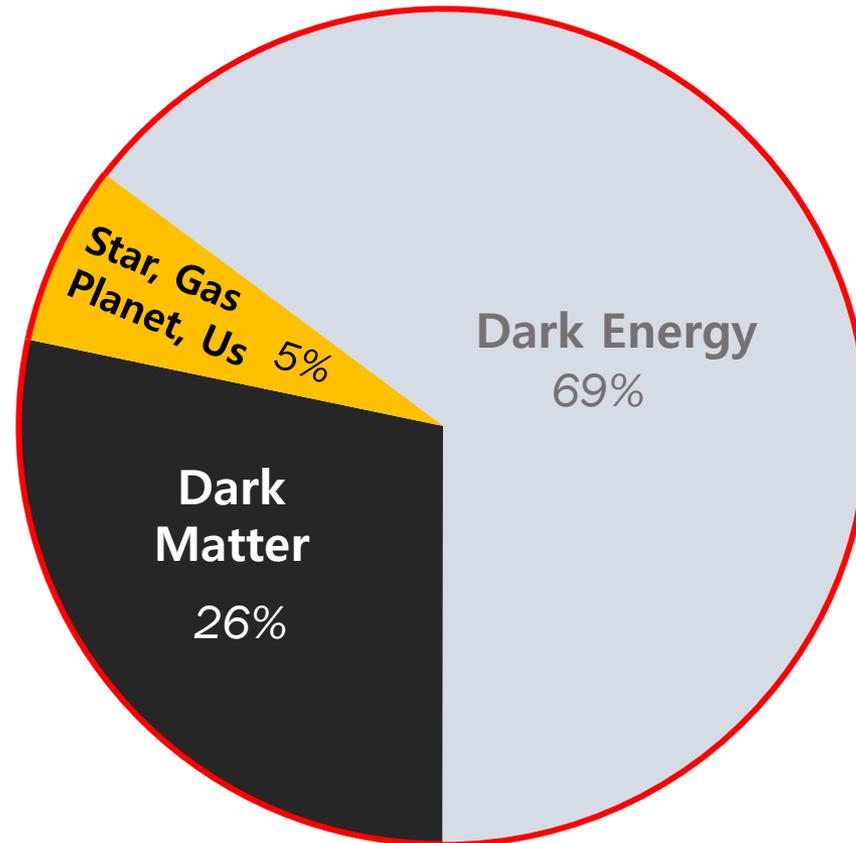
Fate of Supercooled Universe and First Order Phase Transition

- Phase mixing (Spinodal decomposition) vs Nucleation and Growth
- Lattice Simulations

Summary

Introduction to Heavy Particle Dark Matter

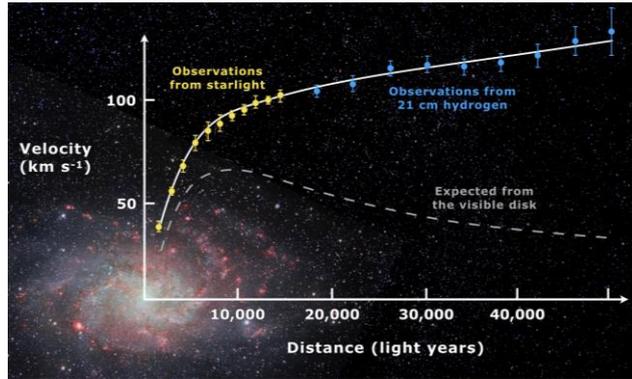
Content of the Universe



It is very difficult to properly understand the origin of the each content from **GR** and the list of particles of **the SM**

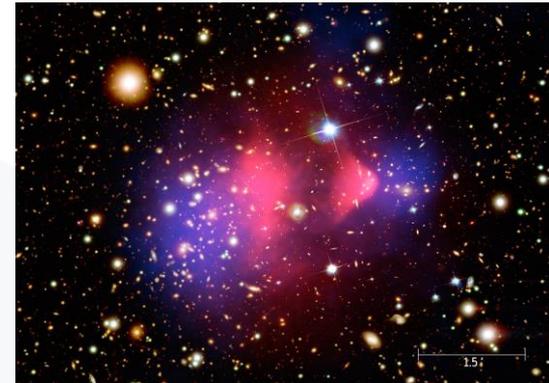
Content of the Universe - DM

Image of Galaxy Messier 33



Galaxy Scale

Image of Bullet Cluster 1E 0657-558



Galaxy Cluster Scale

Dark
Matter
26%

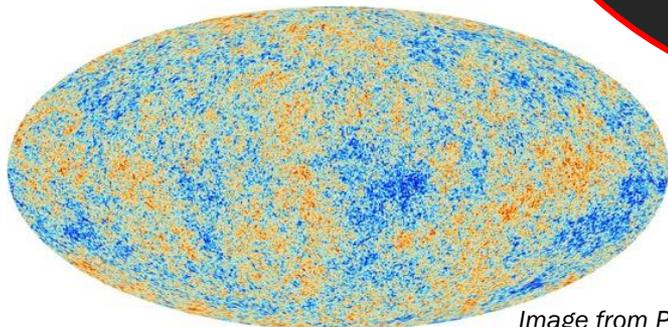


Image from PLACK

Cosmic Microwave Background

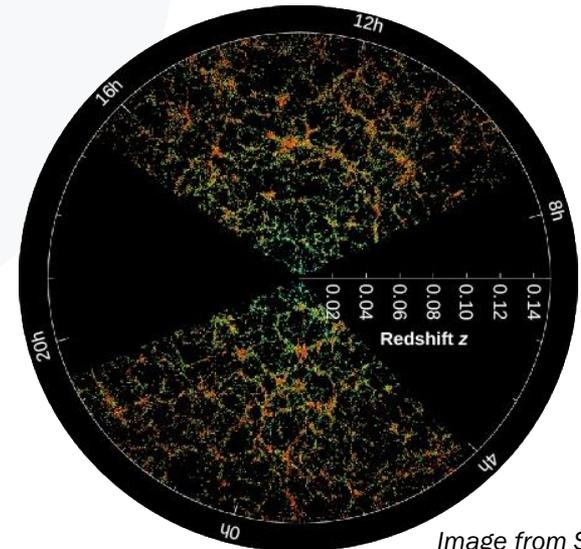
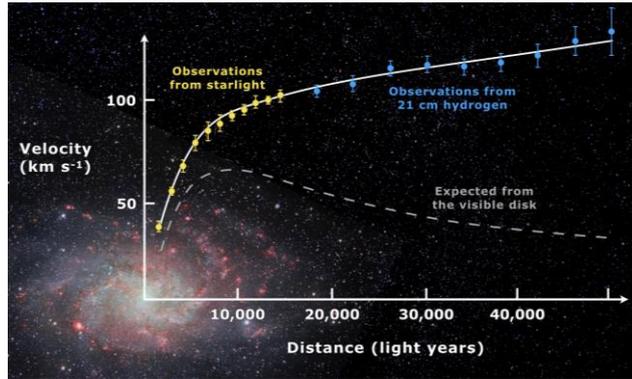


Image from SDSS

Large Scale Structure of the Universe

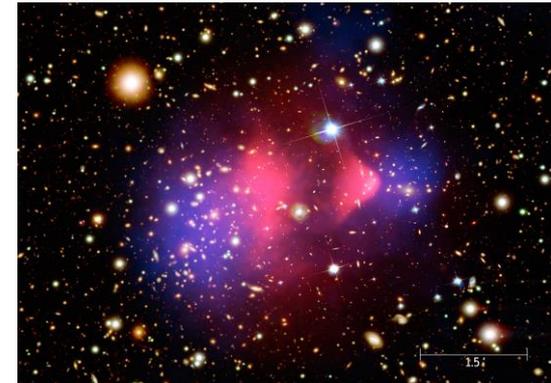
Content of the Universe - DM

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Galaxy Cluster Scale

<Dark Matter>
Feels Gravity,
Cosmologically Stable,
No Light Emission, No EM Charge
CANNOT be explained by the
particle contents of the SM

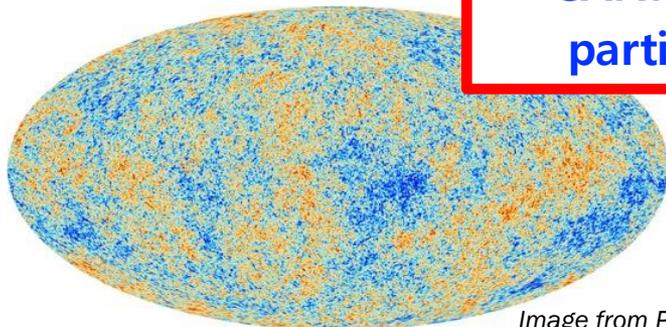


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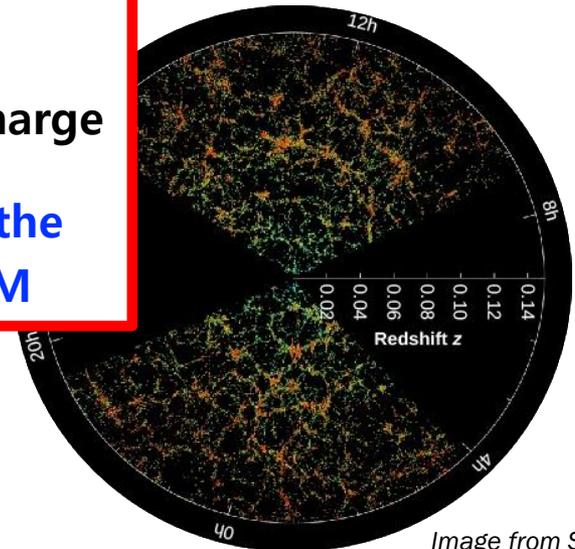
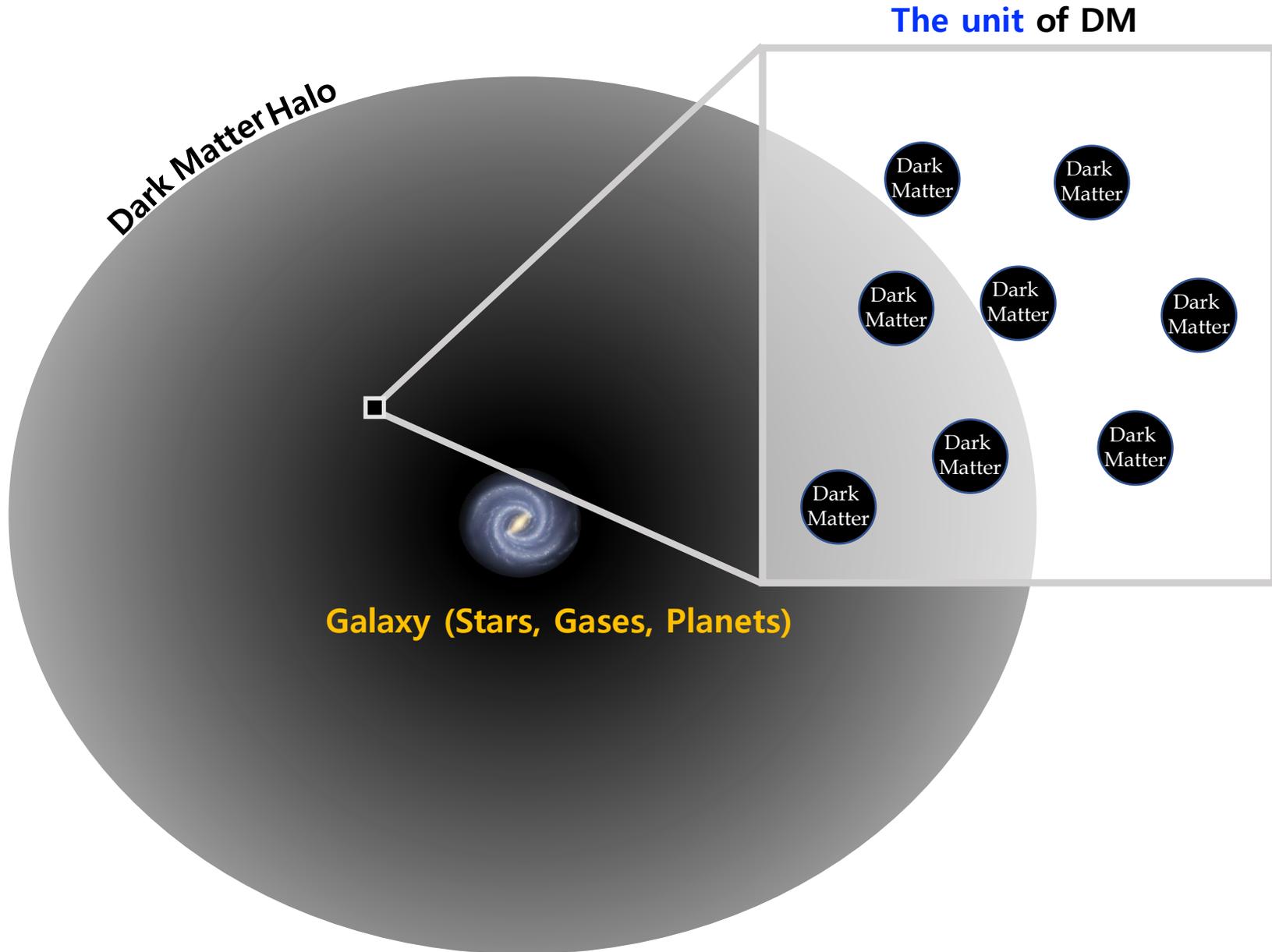


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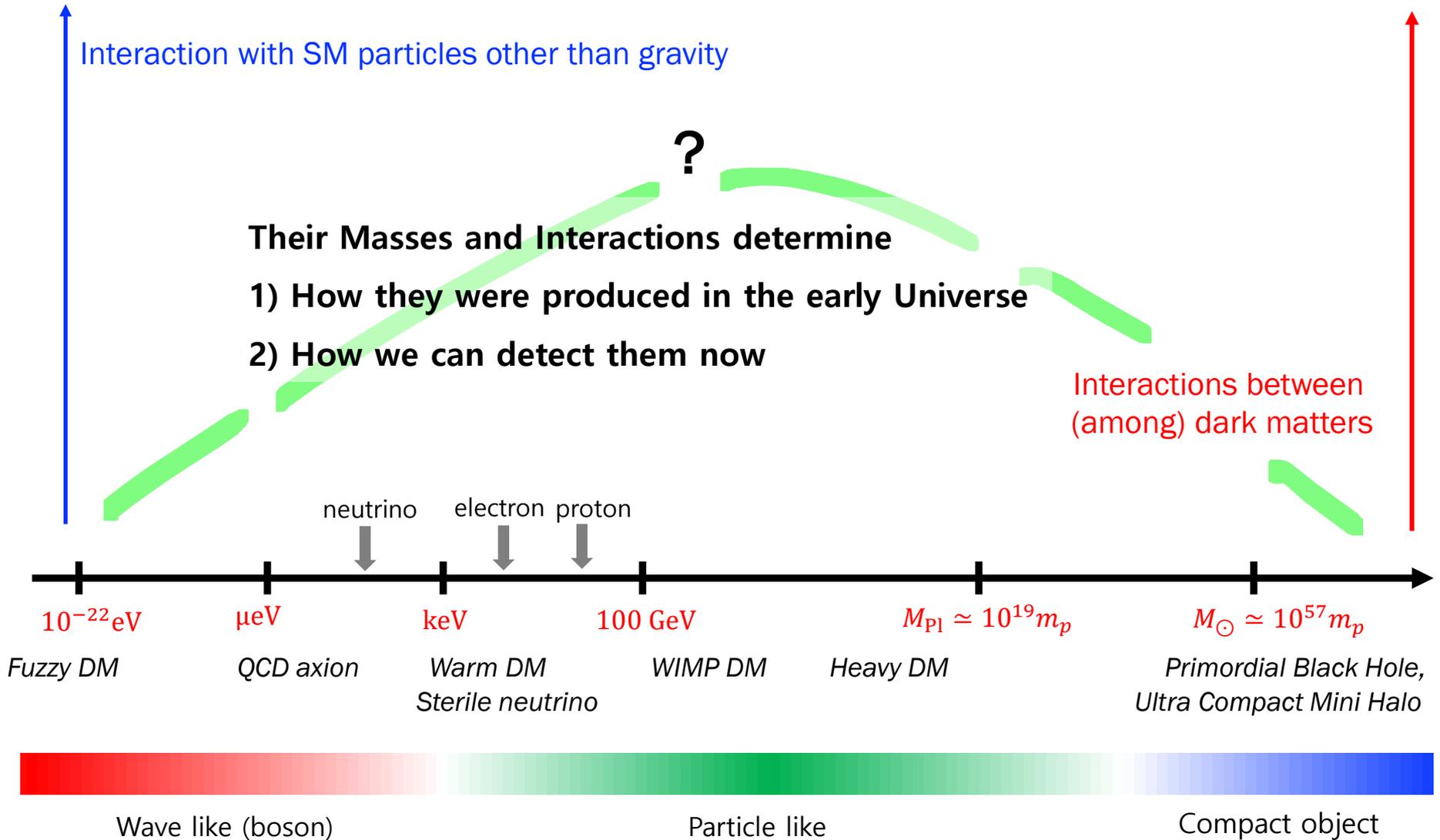
Large Scale Structure of the Universe

What is **the nature** of dark matter?



Candidates of DM for its mass and interactions

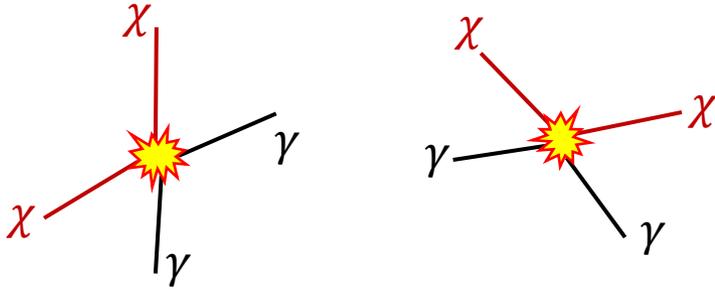
$$\bar{\rho}_{\text{DM}} = M_{\text{DM}} \bar{n}_{\text{DM}} = (0.25 - 0.27) \bar{\rho}_{\text{tot}} \simeq 1.2 \times 10^{-6} \text{GeV/cm}^3$$



Thermal Dark Matter

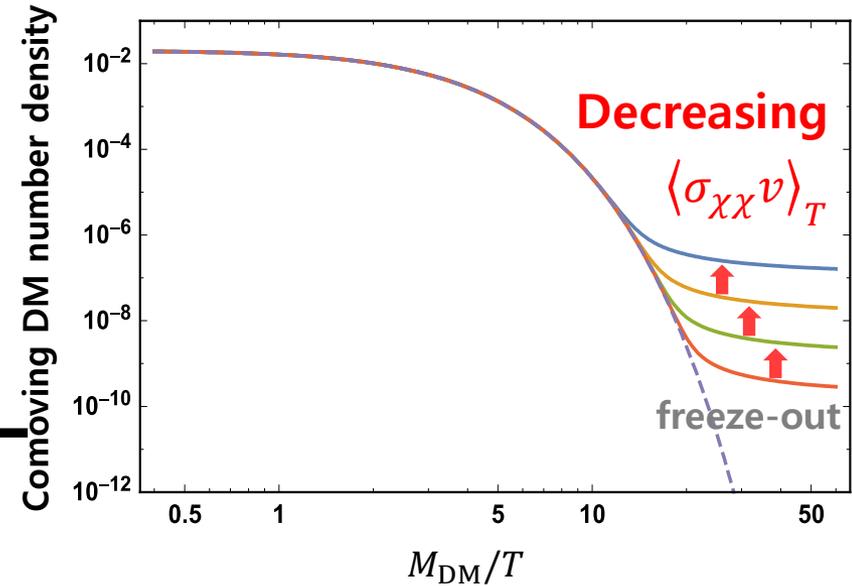
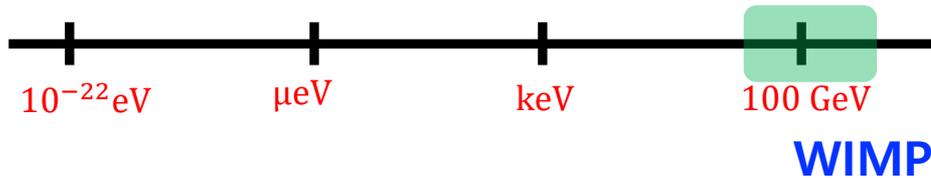
Weakly Interacting Massive Particle (WIMP) : (0.1 ~ 1000) GeV

Dark matter density is determined by its annihilation cross-section



$$\Omega_{\text{DM}} h^2 = 0.1 \left(\frac{3 \times 10^{-26} \text{ cm}^3/\text{s}}{\langle \sigma_{\chi\chi} v \rangle_T} \right)$$

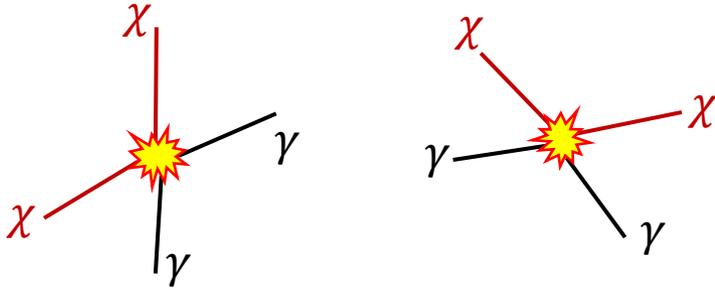
WIMP ($M_{\text{DM}} \sim 100 \text{ GeV}$, $\alpha_{\chi} \sim 0.01$)
is one of the best candidates for DM



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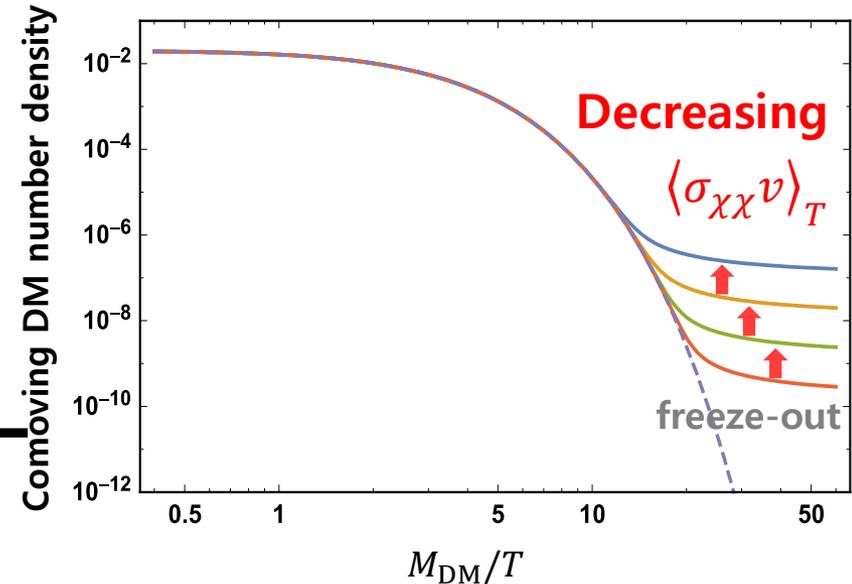
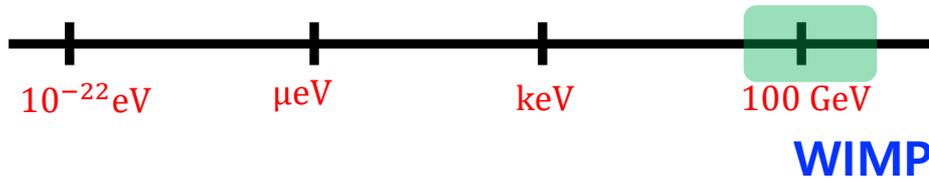
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Dark matter cross-section is limited by its mass and the velocity

The perturbative Unitarity bound:

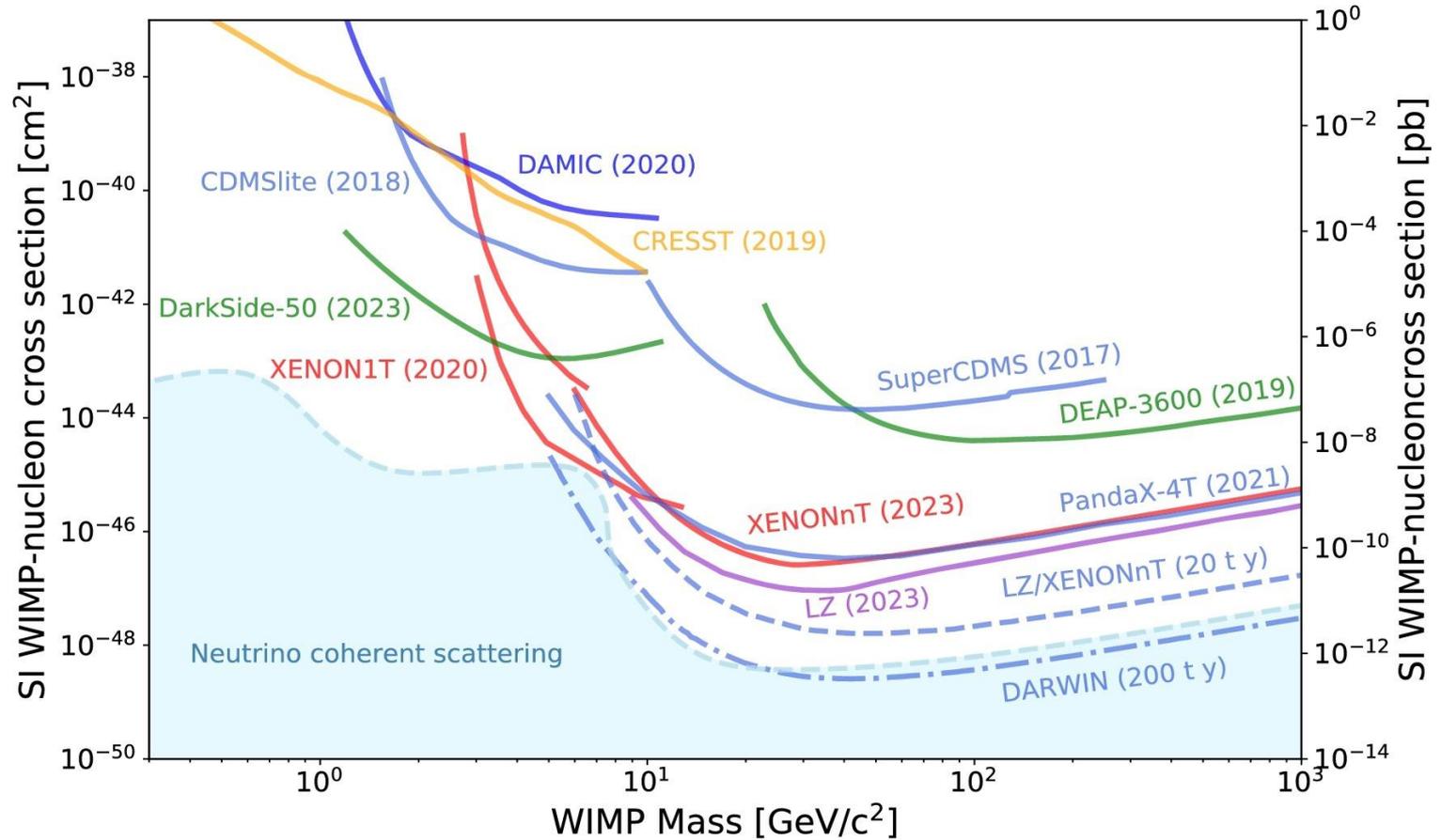
[Griest, Kamionkowski PRL64 (615) 1990]

$$\langle \sigma_{\chi\chi} v \rangle_T \leq \frac{4\pi}{M_{\text{DM}}^2 \langle v \rangle_T}$$

$$\Omega_{\text{DM}} h^2 \geq 0.1 \left(\frac{M_{\text{DM}}}{130 \text{ TeV}} \right)^2$$

Thermal Dark Matter

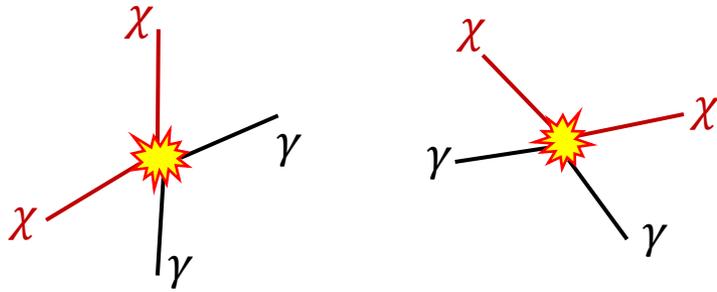
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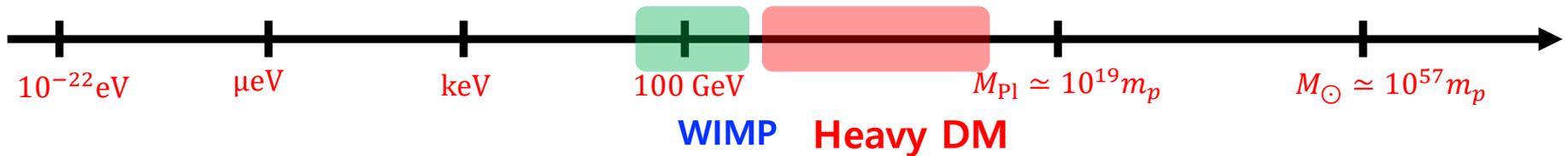
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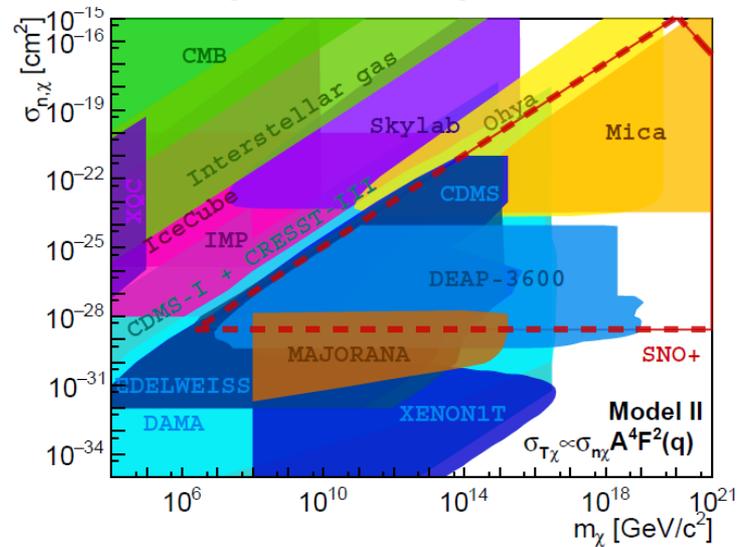
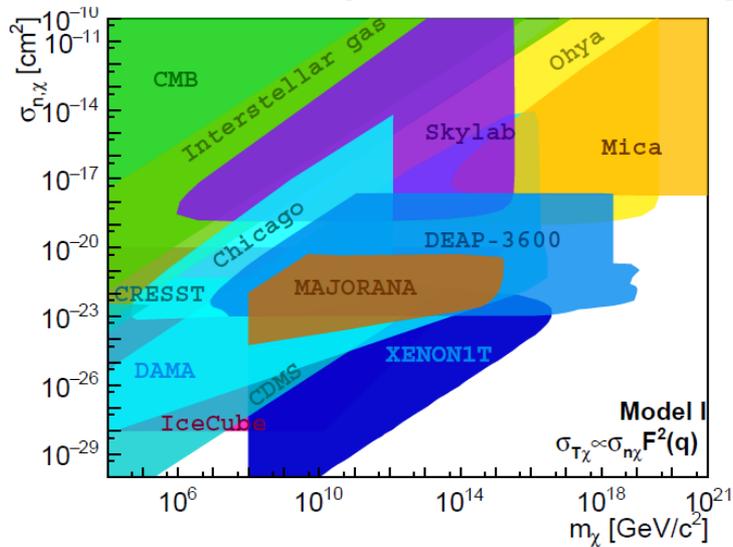


HOWEVER, No hints for WIMP DM so far:
Strong motivation of the beyond WIMP paradigm

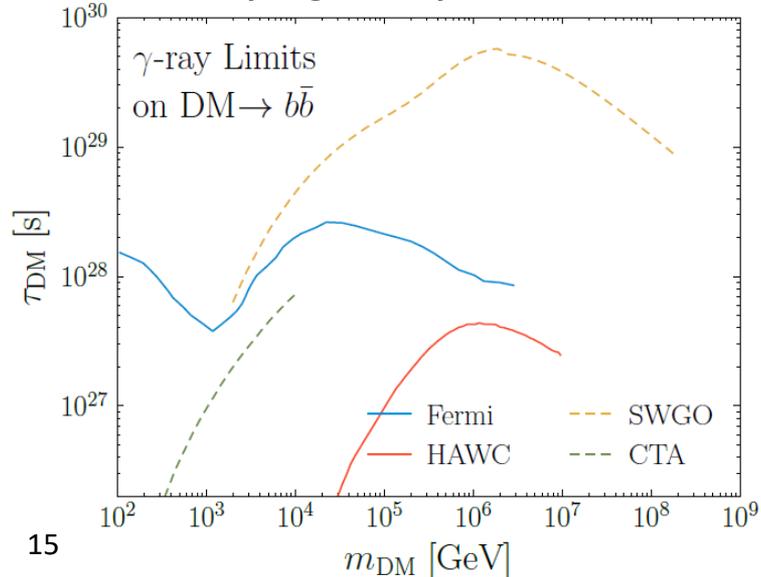
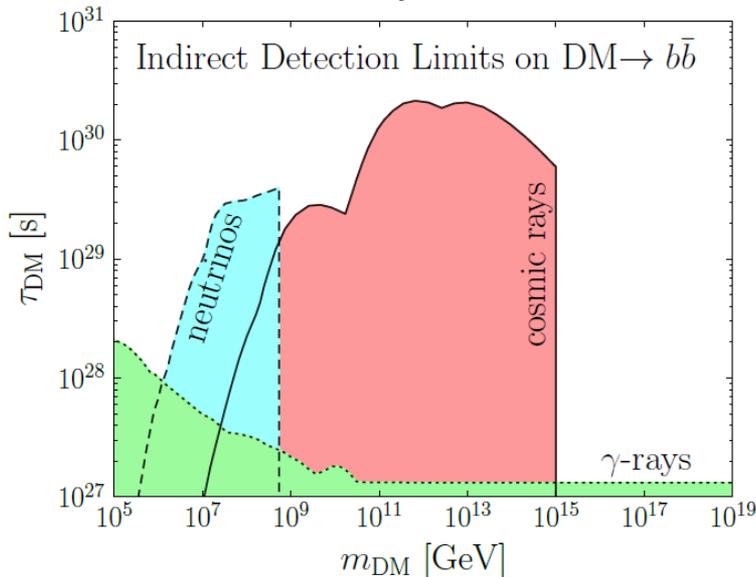
Direct/Indirect Detection of Heavy DM

Snowmass2021 Ultra-heavy particle dark matter arXiv:2203.06508

multi-scattering effect, tracklike signature for large scattering cross-sections



Cosmic-rays, neutrinos, photons from decaying heavy DMs



Heavy DM Beyond the Unitarity Bound

How can the Unitarity bound be overcome to allow various DM masses?

$$\Omega_{\text{DM}} h^2 \geq 0.1 \left(\frac{M_{\text{DM}}}{130 \text{ TeV}} \right)^2$$

- Generated **nonthermally** during inflation, the end of inflation, preheating by quantum fluctuations, scatterings, or parametric resonances for $M_{\text{DM}} >, < H_{\text{inf}}$
[hep-ph/9805473, 9802238, 9809547, 0104100, 1804.07471, 2011.13458, 2206.08940, 2405.13883, ...]

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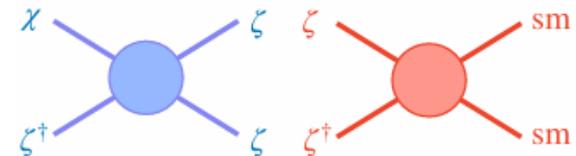
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- Dilution by **entropy production** after thermal freeze-out of heavy DM
[1701.05859, 1811.03608, 2010.09762, ...]

$$\Omega_{\text{DM}} h^2 \rightarrow \Omega_{\text{DM}} h^2 \times \frac{S_{\text{ini}}}{S_{\text{fin}}}$$

- Sommerfeld enhancement** of annihilation cross-section with a light mediator
[hep-ph/0610249, 1407.7874, 1703.00478, 1904.11503, 2110.13926, ...]
- Coannihilation with lighter particles: **enhancement of annihilation rate** for DM
[1906.00981, 2003.04900, ...]

$$\Gamma_{\text{ann}} = \langle \sigma v \rangle n_\chi \rightarrow \Gamma_{\text{ann}} = \langle \sigma v \rangle n_\zeta : \Gamma_{\text{ann}} \rightarrow \Gamma_{\text{ann}} \times \frac{n_\zeta}{n_\chi} \quad \text{for e.g. } m_\zeta < m_\chi < 3m_\zeta$$

- Freeze-out in **dark sector thermal bath**
[1609.02555, 1602.08490, 1905.05191, 1905.05191, 1912.05572, ...]



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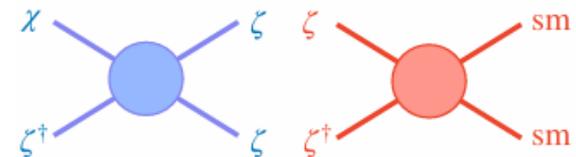
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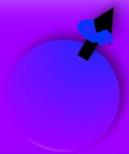
- Freeze-out in **dark sector thermal bath**
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- Phase Transition**



First Order Phase Transition for Dark Matter

Origin of Mass for particle DM: Expectation value of scalar field



Higgs field (giving mass to the elementary particles)

Nonzero expectation value of the scalar field imposes DM mass



Scalar Field (giving DM Mass)

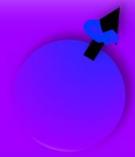
Temperature drops: Vapor → Dew



Universe expands → Temperature decreases → **Bubbles of scalar condensation form!**



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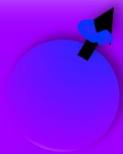
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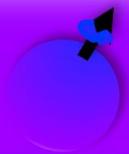
Scalar Field (giving DM Mass)

Temperature drops: Vapor → Dew



Universe expands → Temperature decreases → **Bubbles of scalar condensation collide!**

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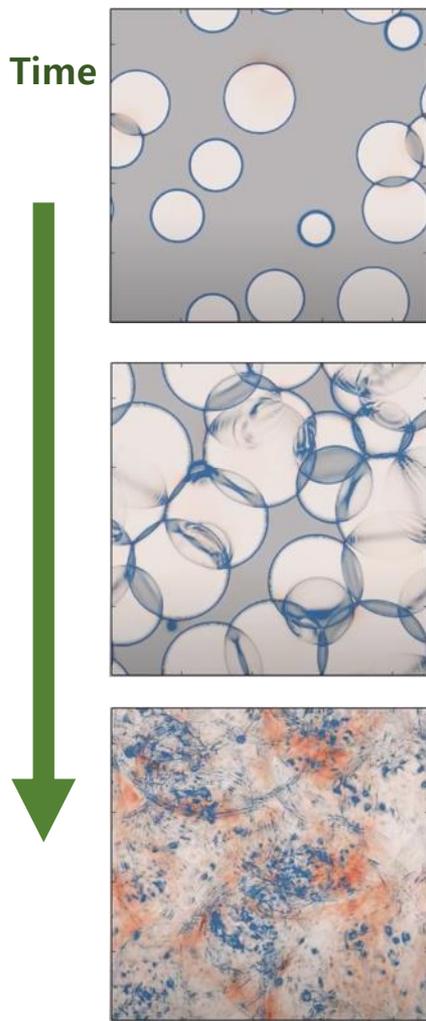
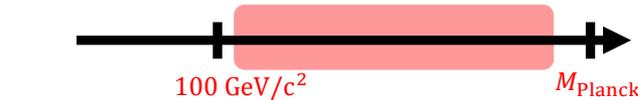


Universe expands → Temperature decreases → **Bubbles of scalar condensation fill the Universe** → **Cosmic 1st order phase transition**

Origin of DM mass & its abundance

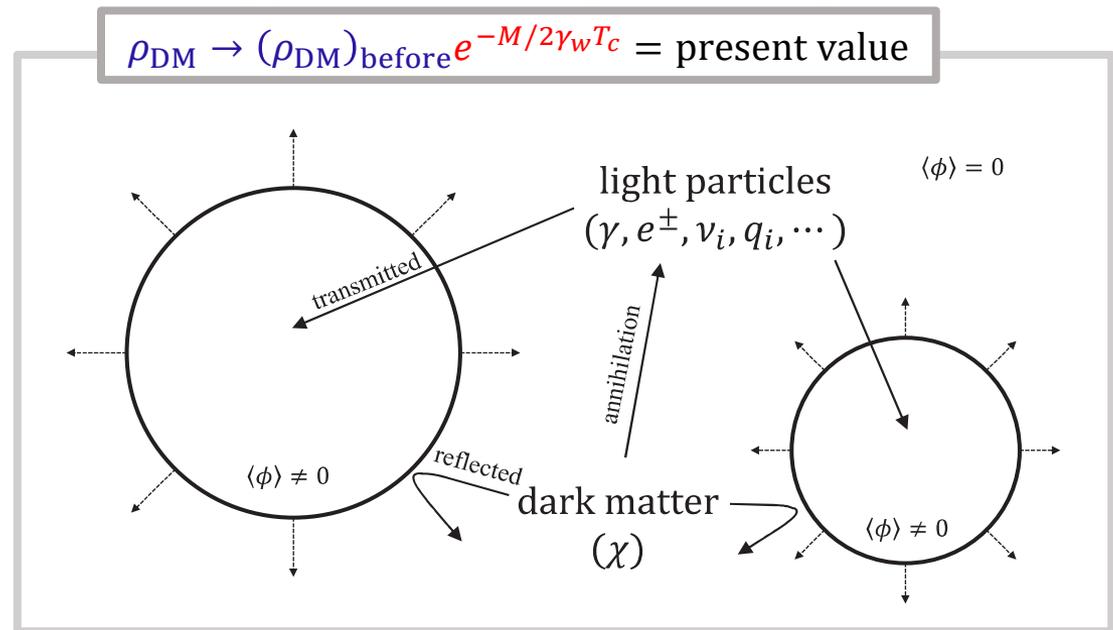
Proposing the mechanism working in a wide range of DM mass

D. Chway, T. H. Jung, CSS
Phys. Rev. D 101, 095019 (2020)



Filtering-out Mechanism

Exponential suppression of heavy DM relic by Filtering Effects

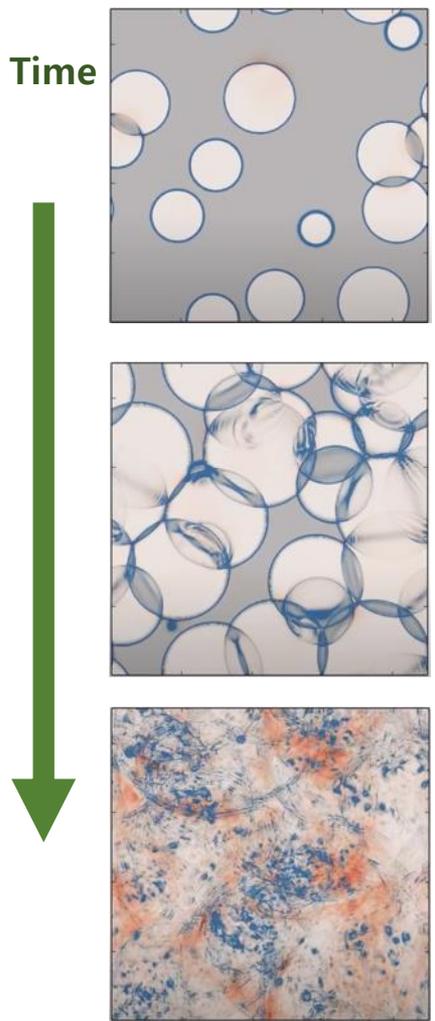
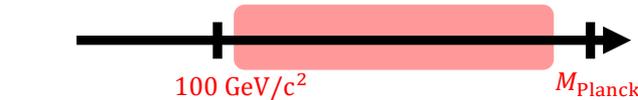


**Cosmic 1st order phase transition
→ Production of Gravitational Waves**

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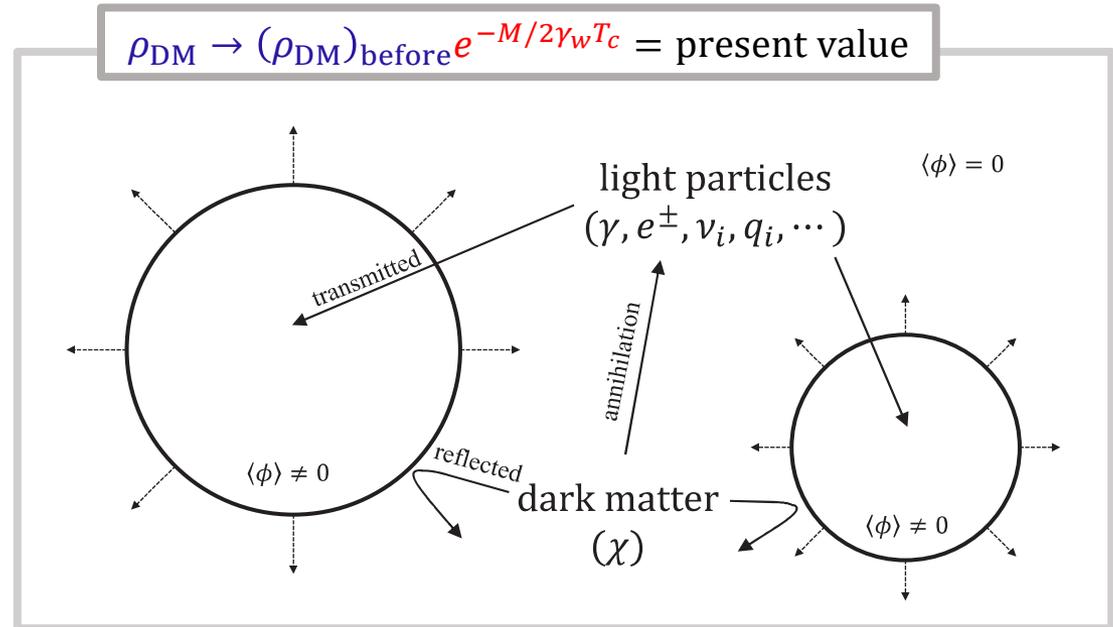
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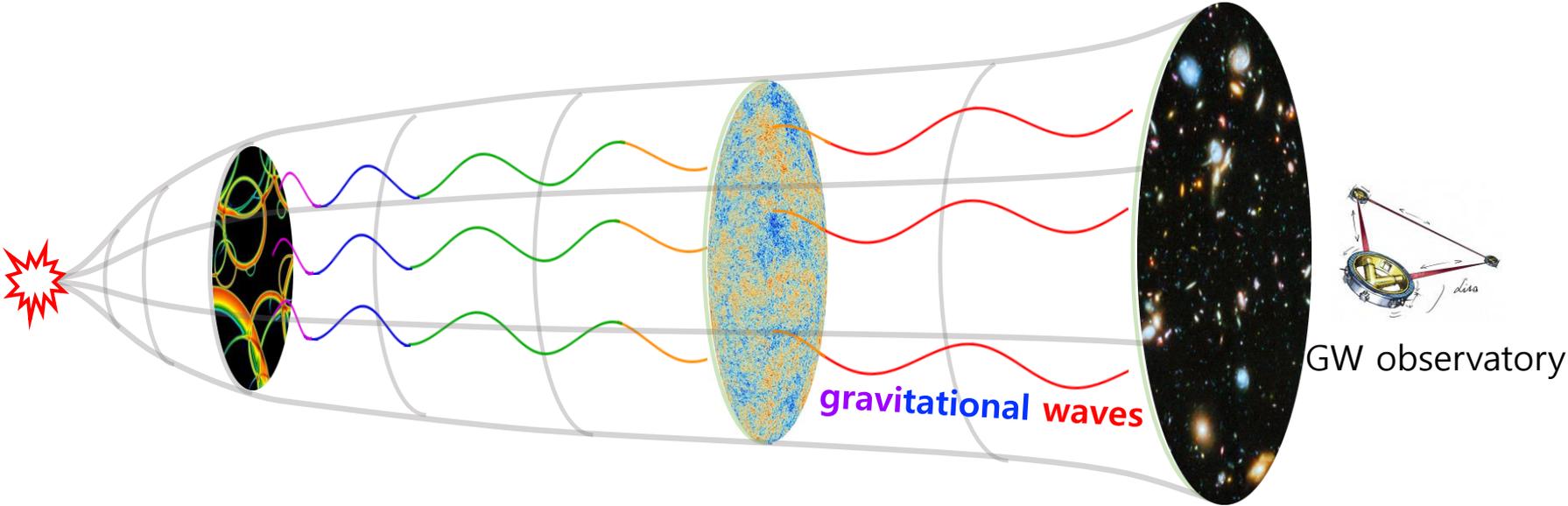


**Cosmic 1st order phase transition
→ Production of Gravitational Waves**

**Understanding the origin of DM
by GW observations**

Stochastic Gravitational Waves

Proposing the mechanism working in a wide range of DM mass



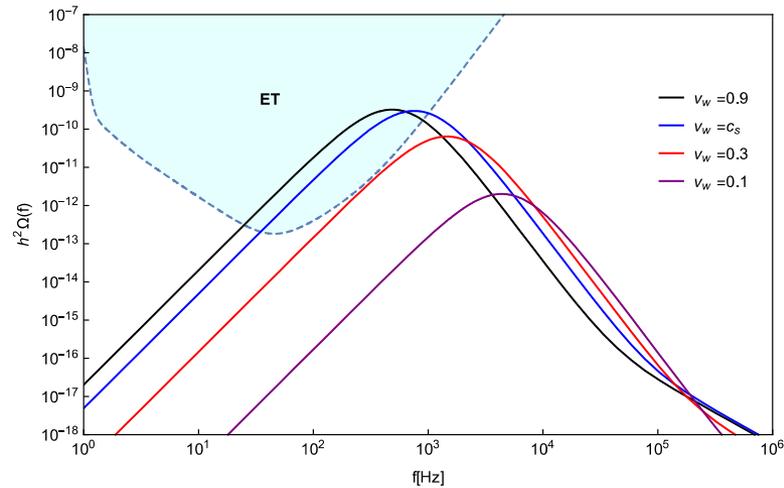
Understanding the origin of DM by GW observations

Origin of DM mass & GW observations

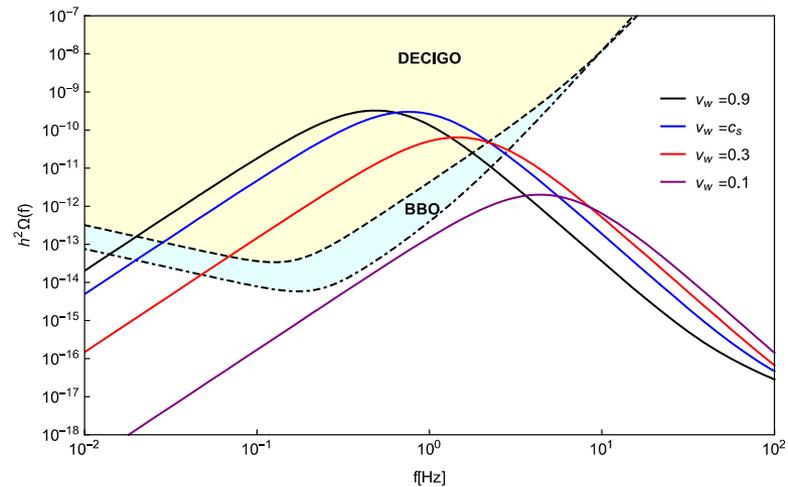
Proposing the mechanism working in a wide range of DM mass



Mass=100PeV



Mass=100TeV



GW observatory

M. Ahmadvand 2108.00958

Understanding the origin of DM by GW observations

Heavy DM Production from First Order PT

DM Filtering Mechanism

Baker, Kopp, Long, [arXiv:1912.02830] for strong coupling and non-rel. bubble

D. Chway, T. H. Jung, **CSS** [arXiv:1912.04238] for weak coupling and rel. bubble

DM dilution/production from Supercooling related with confining/conformal sector

Hambye, Strumia, Teresi [arXiv:1805.01473], Baldes, Garcia-Cely [arXiv:1809.01198],

Baldes, Gouttenoire, Sala [arXiv: 2007.08440], Kierkla, Karam, Swiezewska [arXiv:2210.07075]

DM from Ultra-relativistic Bubble Wall and Plasma Interactions ($\phi \rightarrow \text{DM}+\text{DM}$)

Azatov, Vanvlasselaer, Yin [arXiv:2101.05721], Ai, Fairbairn, Mimasu, You [arXiv:2406.20051]

DM from Ultra-rel. Bubble Collisions (local recovering of symmetry, high energetic pt collisions)

Falkowski, No [arXiv:1211.5615], Baldes, Dichtl, Gouttenoire, Sala [2306.15555]

Giudice, H. M. Lee, Pomarol, and Shakya [arXiv:2403.03252]

DM from the isolated Symmetric Phase (asymmetric DM, effect of squeezing)

Pontón, Bai, Jain [arXiv:1906.10739], Hong, Jung, Xie [arXiv:2008.04430],

Asadi, Kramer, Kuflik, Ridgway, Slatyer, Smirnov [arXiv: 2103.09827],

Gouttenoire, Kuflik, Liu [arXiv:2311.00029]

And many others

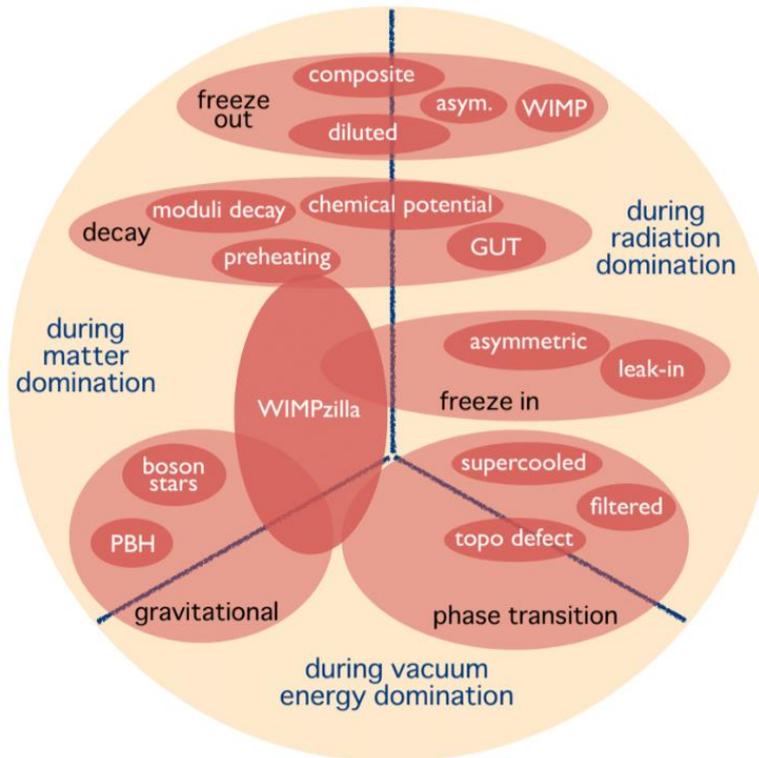
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What are the predictions for observables?

production of ultra-heavy dark matter



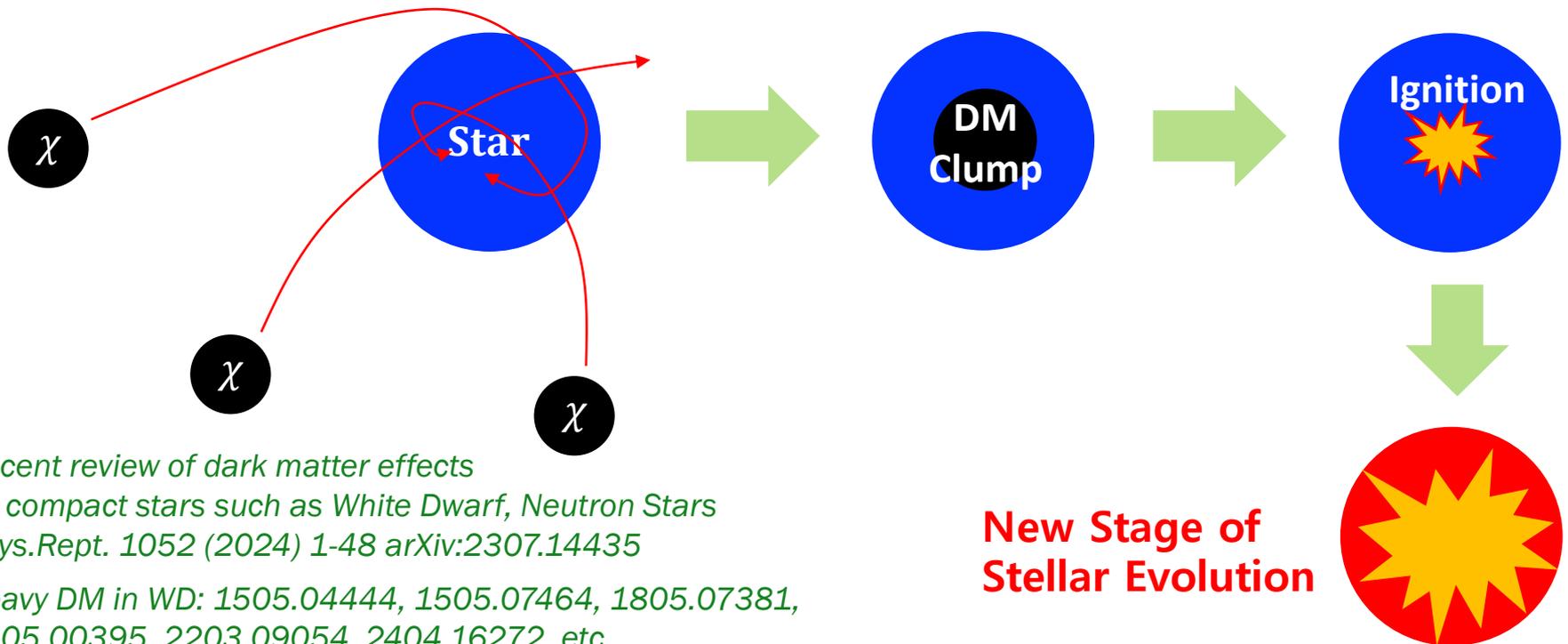
Snowmass2021 Ultra-heavy particle dark matter arXiv:2203.06508

DM Probe via Stellar Evolution

The mechanism working in a wide range of DM mass : **Interaction with the SM particles**



Dark Matter around the Star can be captured and accumulated in the core of the star because of **negligible annihilation cross-section**. It can trigger **the earlier nuclear reaction** in a certain stage of stellar evolution **by new heating sources**.



*Recent review of dark matter effects
on compact stars such as White Dwarf, Neutron Stars
Phys.Rept. 1052 (2024) 1-48 arXiv:2307.14435*

*Heavy DM in WD: 1505.04444, 1505.07464, 1805.07381,
1905.00395, 2203.09054, 2404.16272, etc.*

Heavy DM in Red Giant (work in progress) with S. Ganguly, M. He, S. Yun (IBS), O. Straniero (INAF)

Order of Phase Transition and Simulation

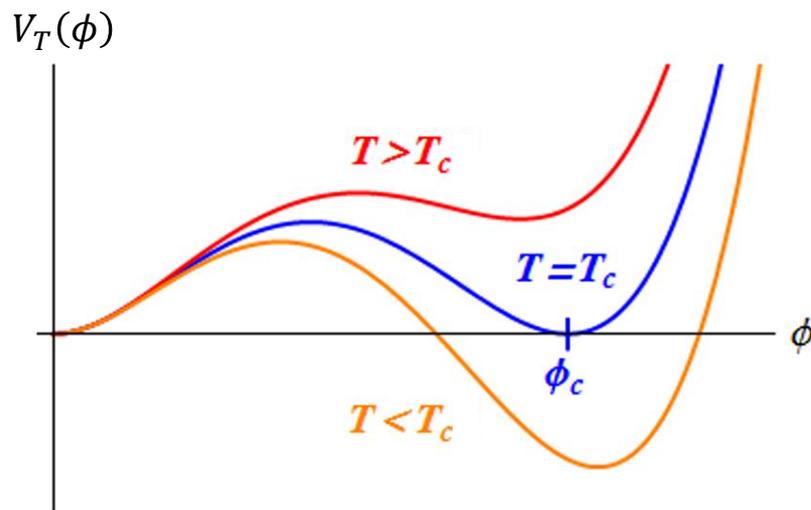
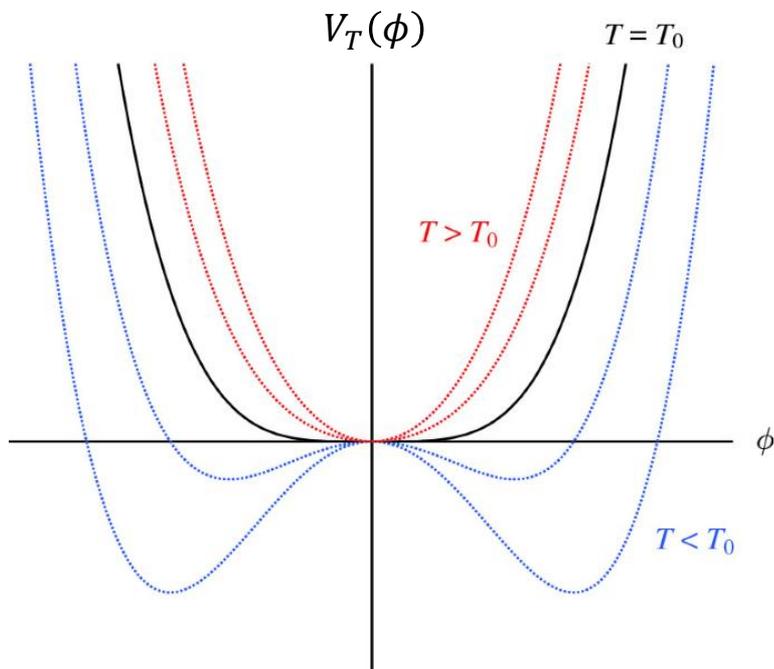
Tomasz Dutka, Tae Hyun Jung, and CSS [arXiv:2412.15864](https://arxiv.org/abs/2412.15864)

Condition for First Order Phase Transition

Equilibrium favors the state that minimizes the free energy at a given temperature T

$$f(T, \phi) = V_0(\phi) \pm \frac{g_i}{2\pi^2} T^4 J_{B,F} \left(\frac{m_i^2(\phi)}{T^2} \right) \quad \text{where} \quad J_{B,F}(y^2) = \int_0^\infty dx x^2 \ln \left(1 \mp e^{-\sqrt{x^2+y^2}} \right)$$

As the temperature decreases, a transition can occur from the symmetric phase to the broken phase (or vice versa). For the finite temperature effective potential $V_T(\phi) = f(T, \phi) - f(0, \phi)$

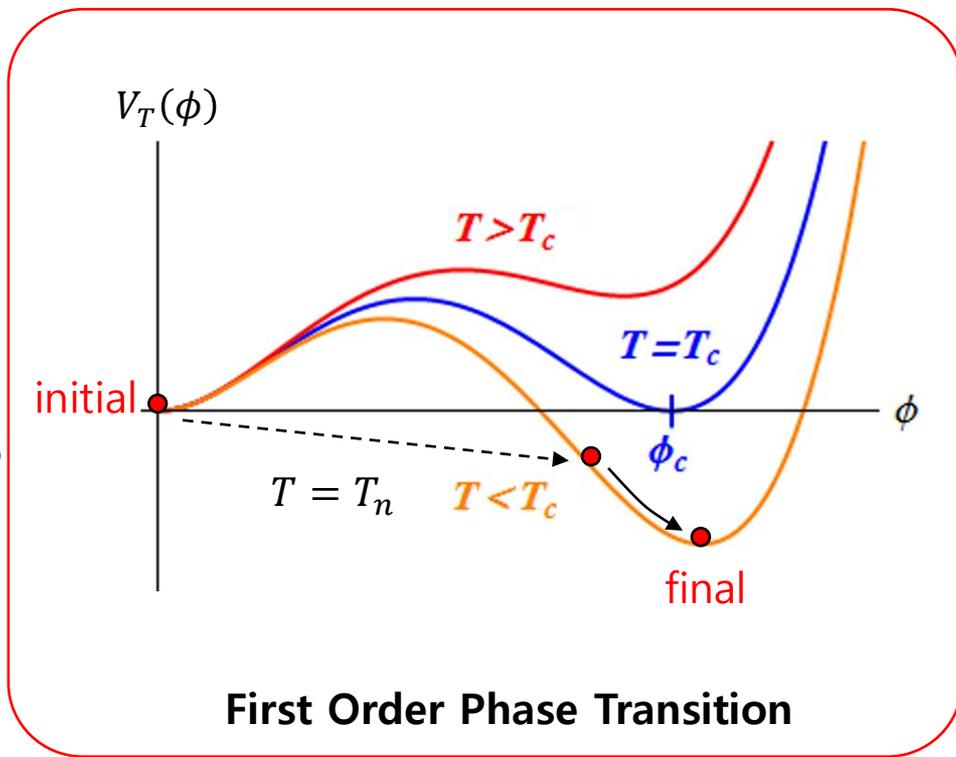
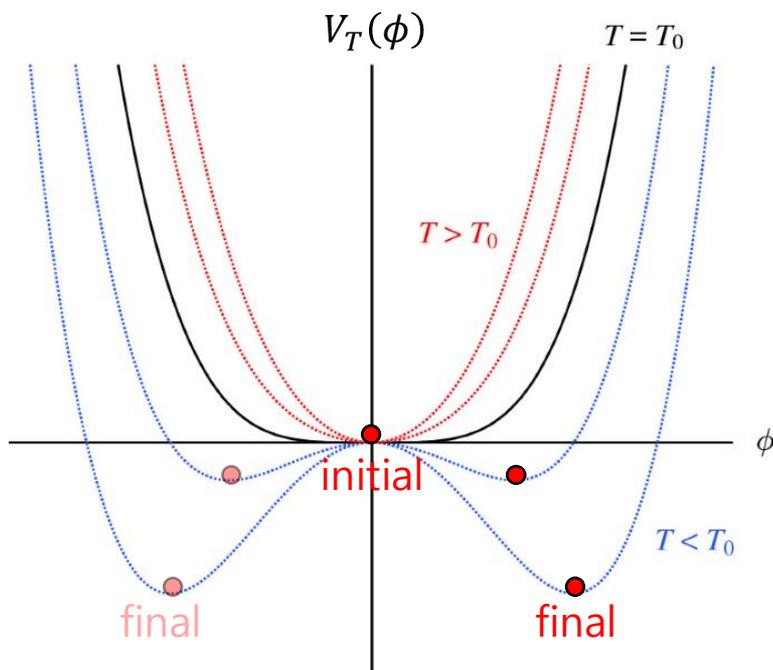


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As the temperature decreases, a transition can occur from the symmetric phase to the broken phase (or vice versa). For the finite temperature effective potential $V_T(\phi) = f(T, \phi) - f(0, \phi)$

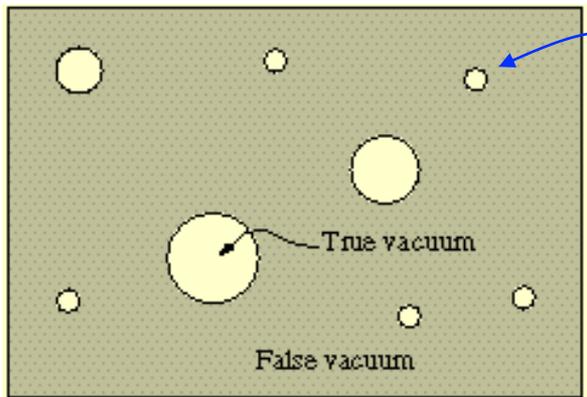


Condition for First Order Phase Transition

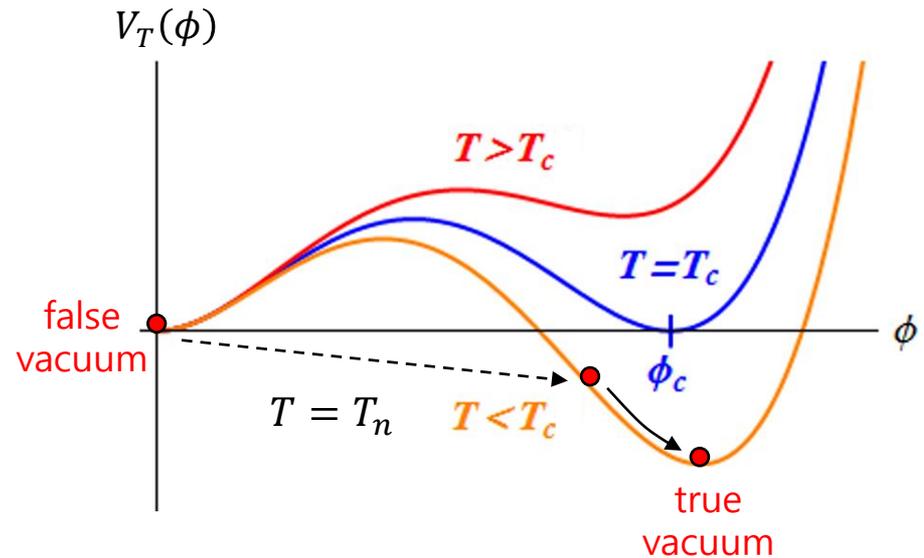
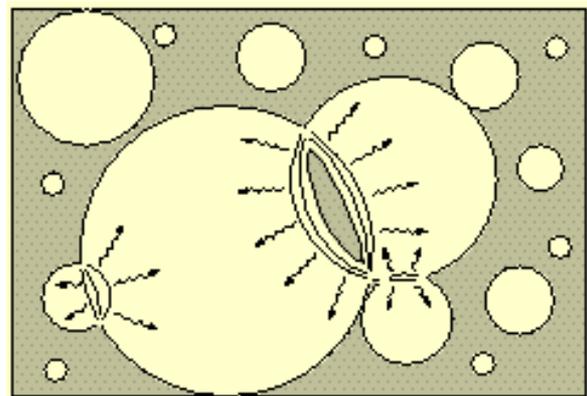
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Critical bubble



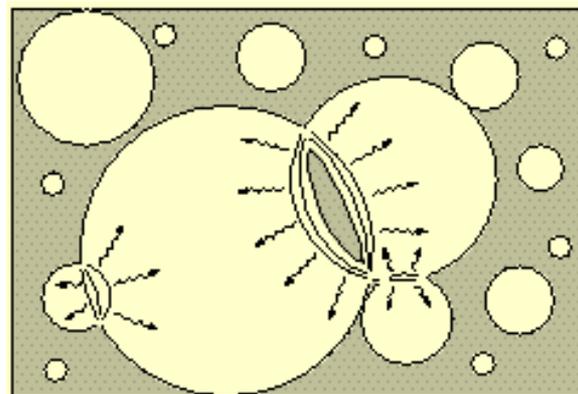
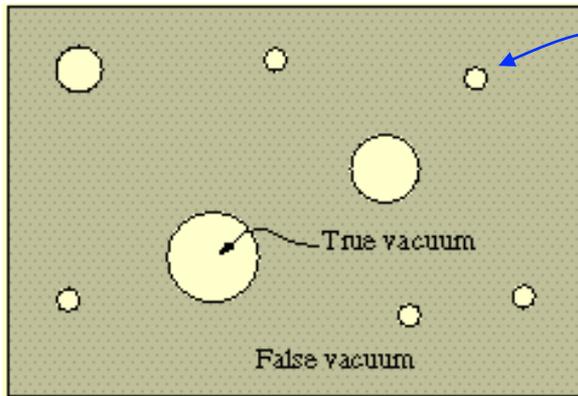
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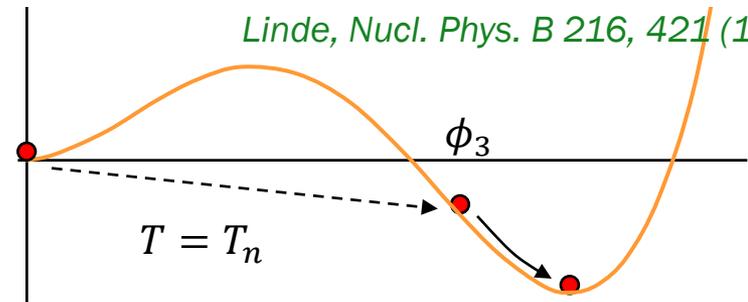
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Critical bubble with the field profile
to give the bounce action S_3

$$\frac{\Gamma_n(T)}{\text{Vol}} \simeq \left(\frac{T}{2\pi} \right) \left(\frac{S_3}{2\pi T} \right)^{\frac{3}{2}} \left(\frac{\det[-\partial^2 + V_T''(0)]}{\det[-\partial^2 + V_T''(\phi_3)]} \right)^{1/2} \exp \left(-\frac{S_3}{T} \right)$$

Linde, Nucl. Phys. B 216, 421 (1983)



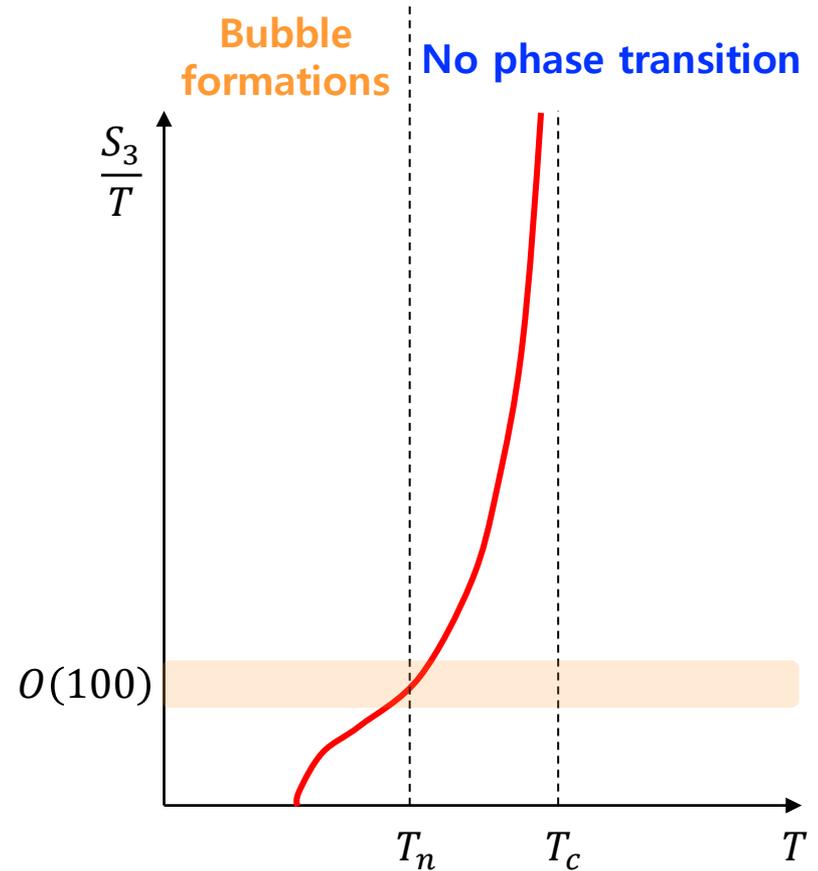
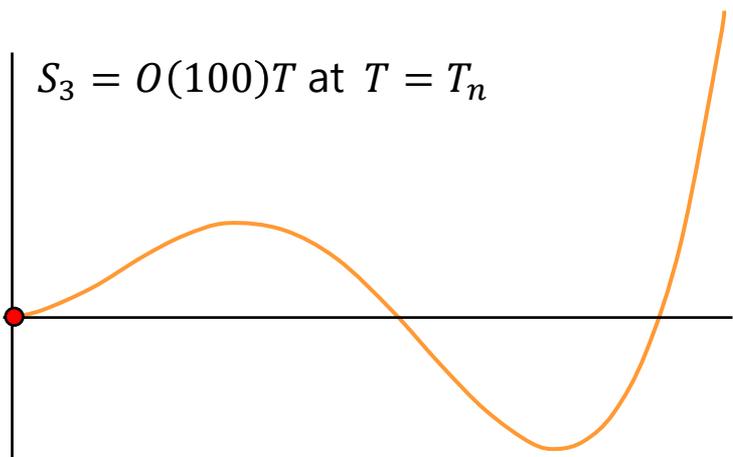
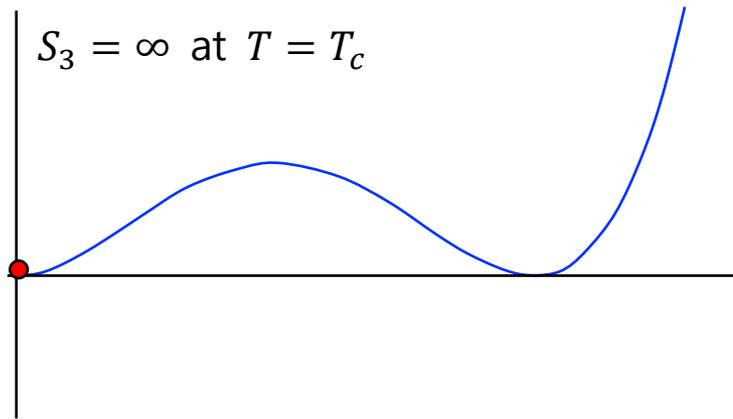
Nucleation temperature

$$\frac{\Gamma(T_n)}{V} \sim T_n^4 \exp \left(-\frac{S_3}{T_n} \right) = H^4(T_n)$$

Condition for First Order Phase Transition

The nucleation rate is exponentially sensitive to the bounce action S_3 of the critical bubble.

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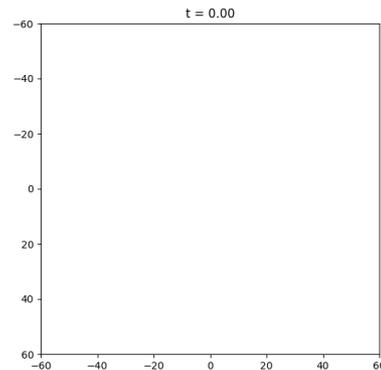
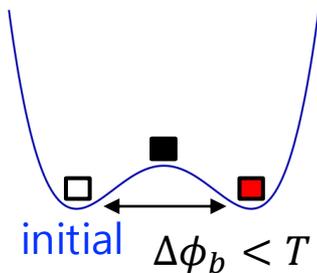
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For instance, if **the field distance of the potential barrier is smaller than the background temperature** ($\Delta\phi_b < T$), thermal fluctuations that cannot be captured by the critical bubble formation can induce **the phase mixing between two phases**, even at the critical temperature.



$t=0$

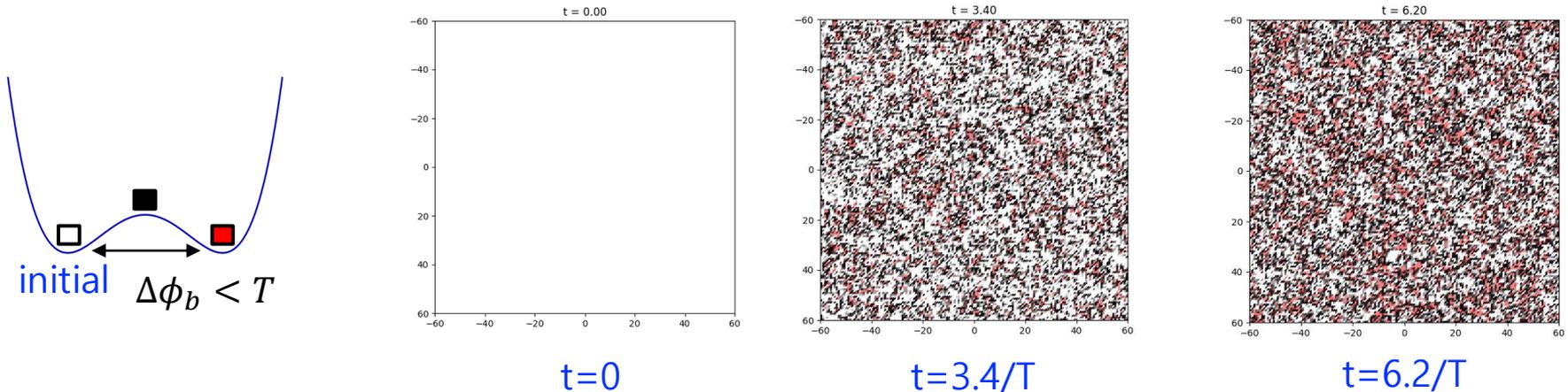
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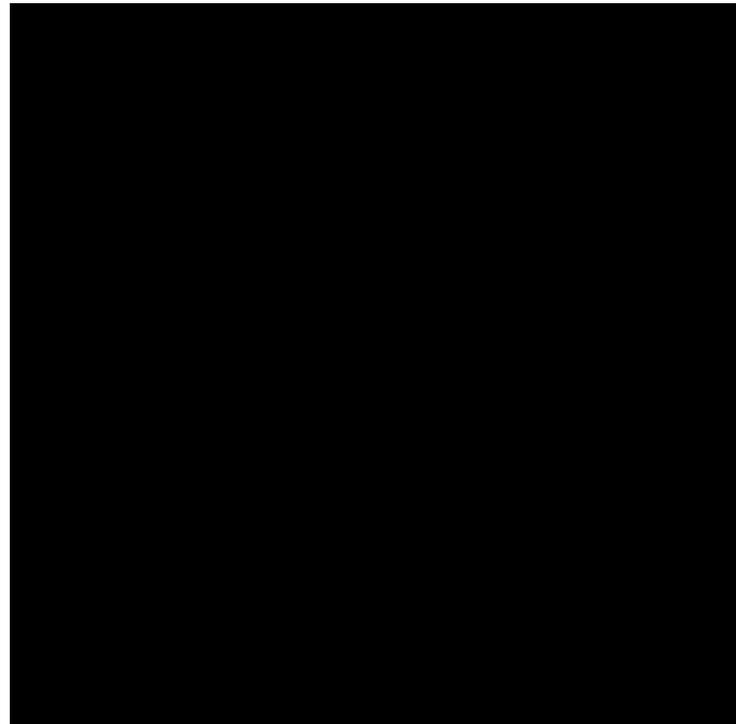
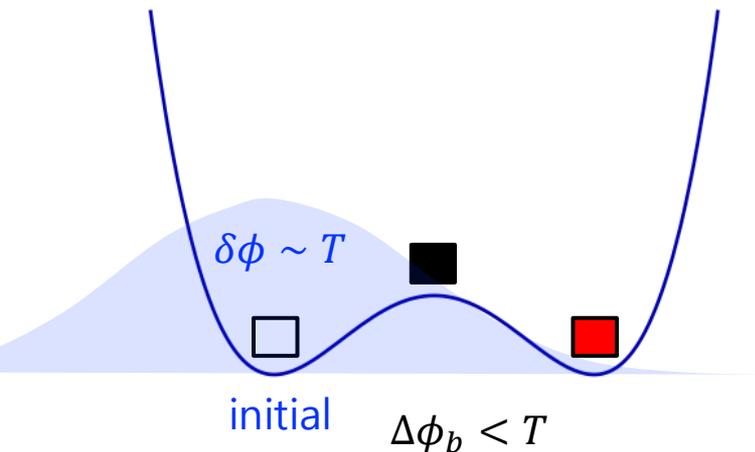
the phase mixing between **symmetry** and **broken** phases

Borrill, Gleiser Phys.Rev. D51 (1995) 4111-4121

Condition for First Order Phase Transition

Non-equilibrium thermal fluctuation becomes important when $\Delta\phi_b < T$

In this case, **no bubbles form**, and the two phases undergo strong mixing before the transition occurs—a process known as **spinodal decomposition**. **As $T < T_c$, the broken phase gradually and smoothly fills the Universe.**



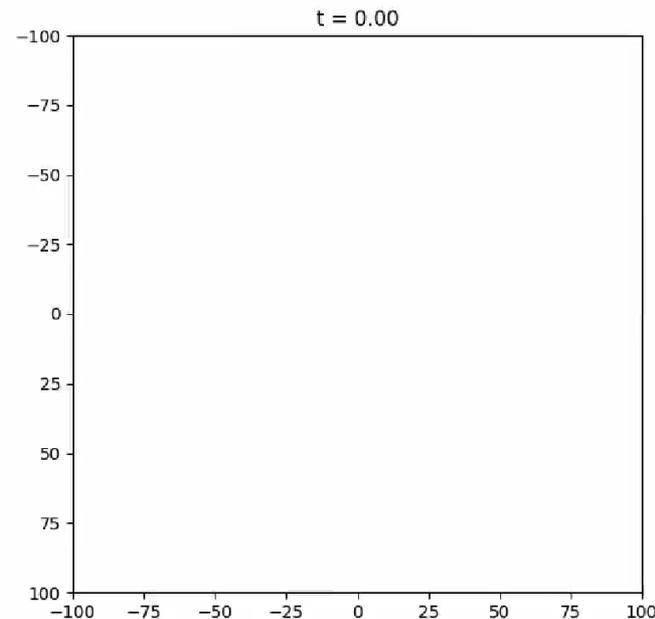
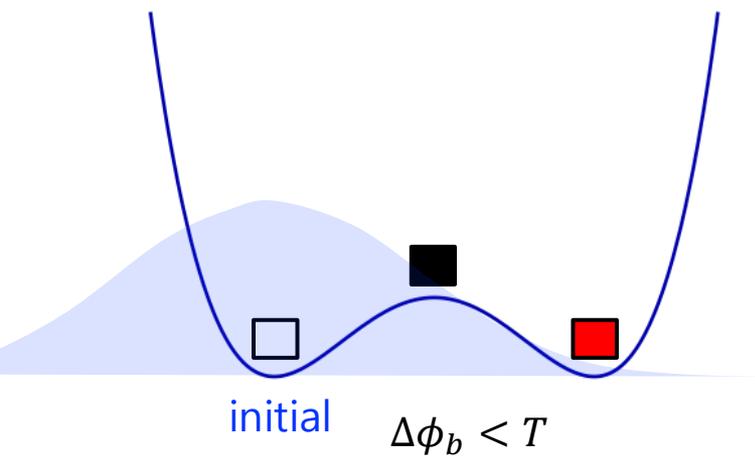
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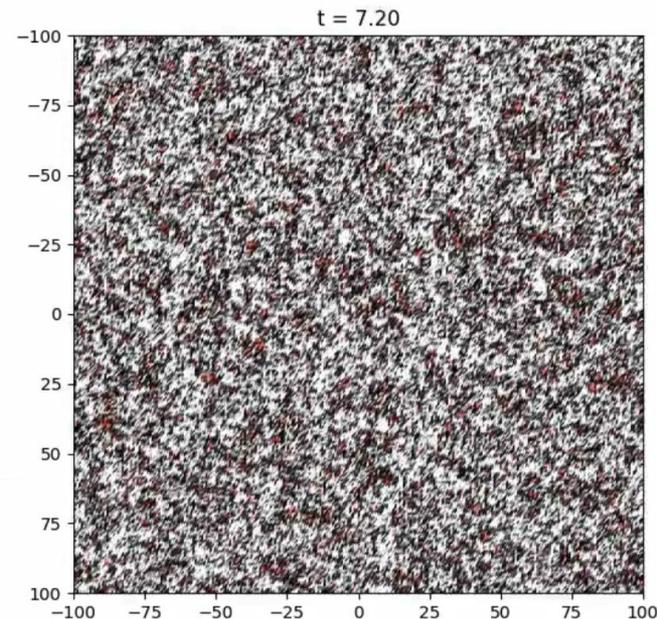
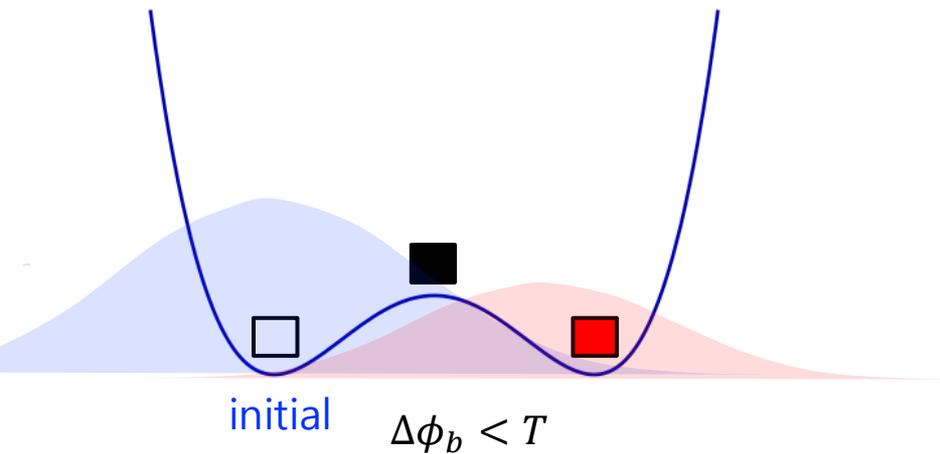
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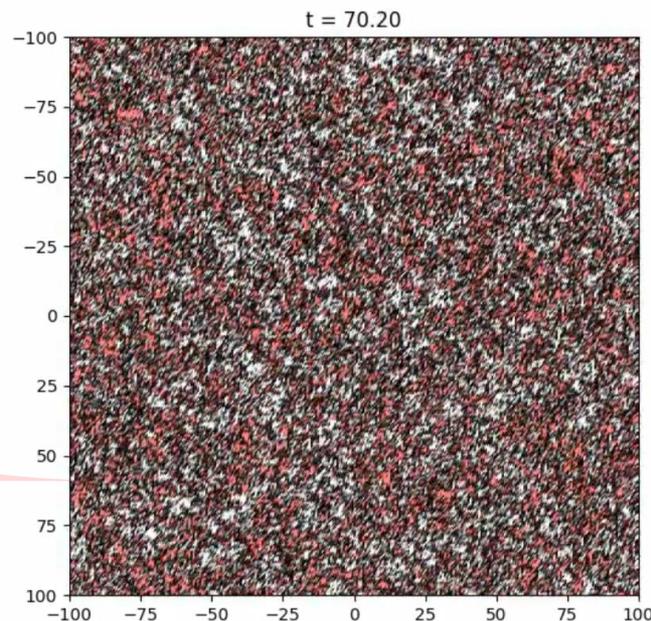
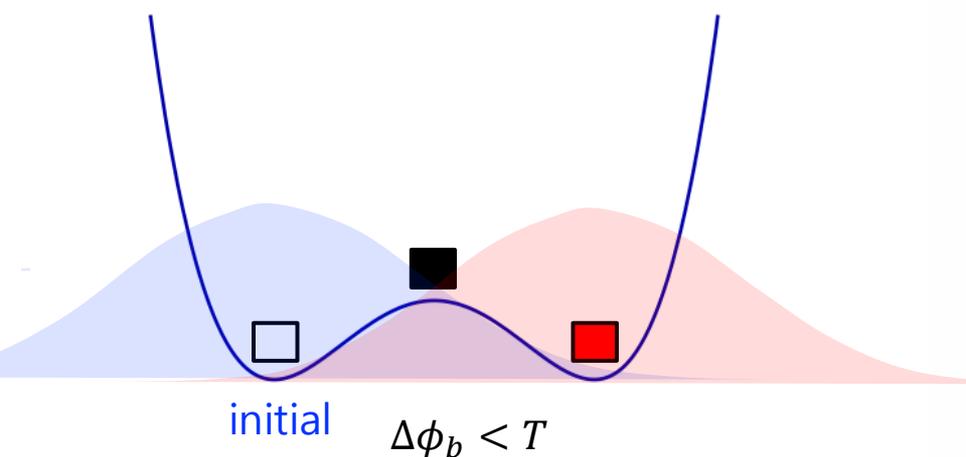
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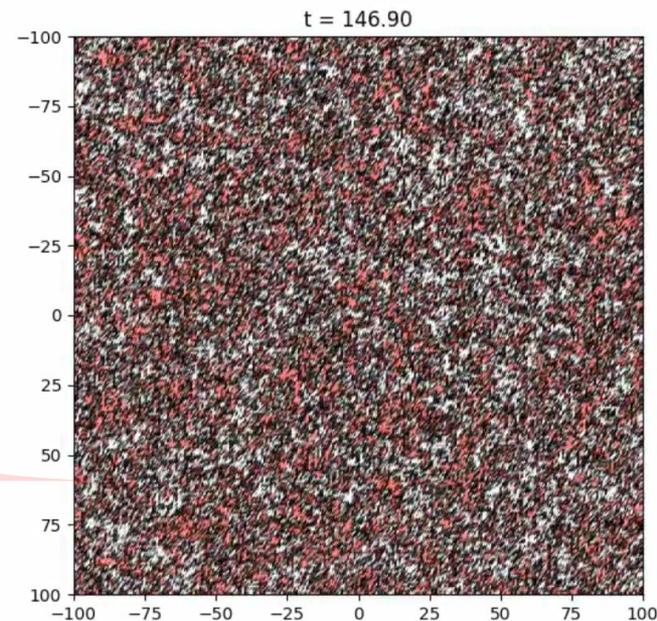
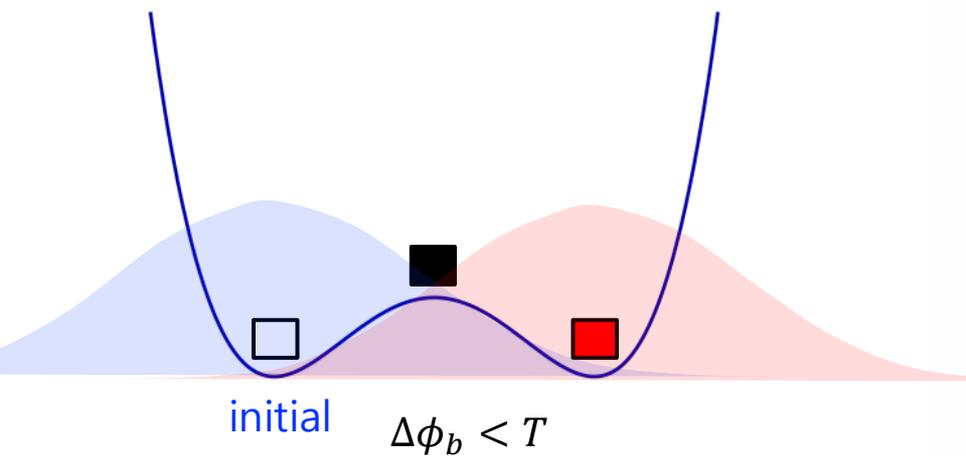
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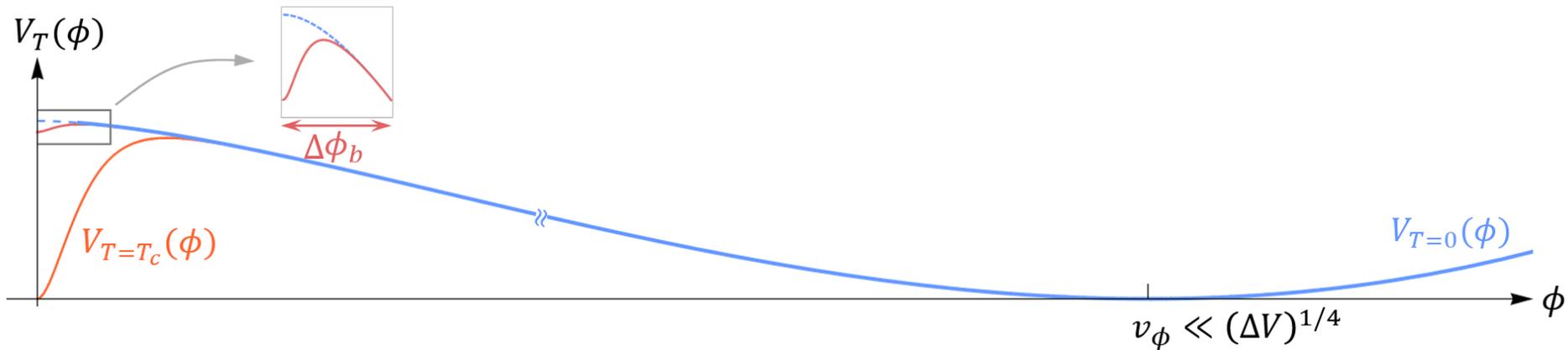


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Supercooled Phase Transition

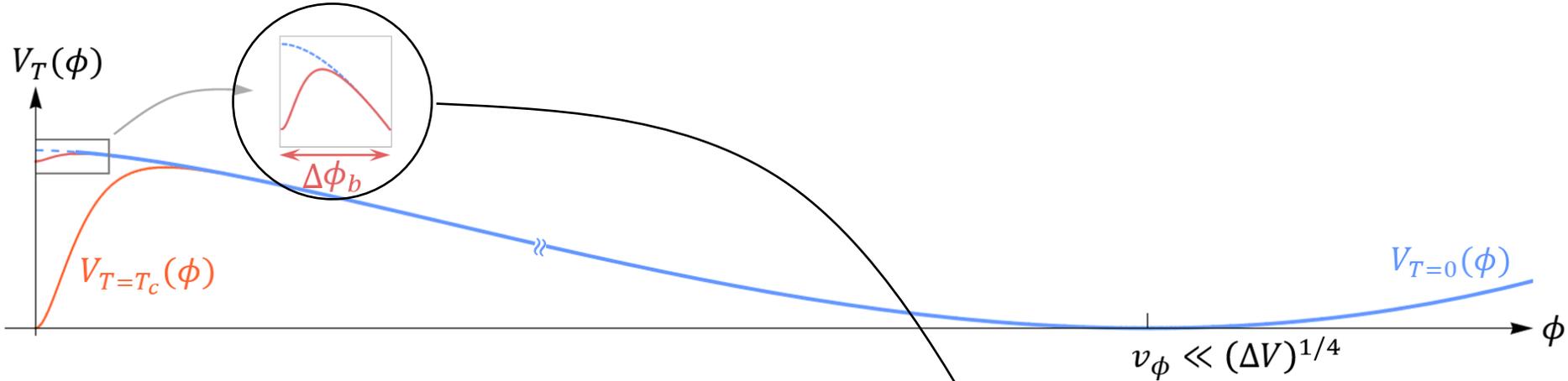
For a strong first-order PT—favorable for mechanisms like **heavy DM** generation via filtering or energetic collisions—a **supercooled transition with a flat potential is typically preferred**. In this scenario, **the true vacuum field value is significantly larger than the critical/nucleation temp**.



It is quite natural to expect **this phase transition to be first-order**, as the transition occurs before the potential barrier disappears.

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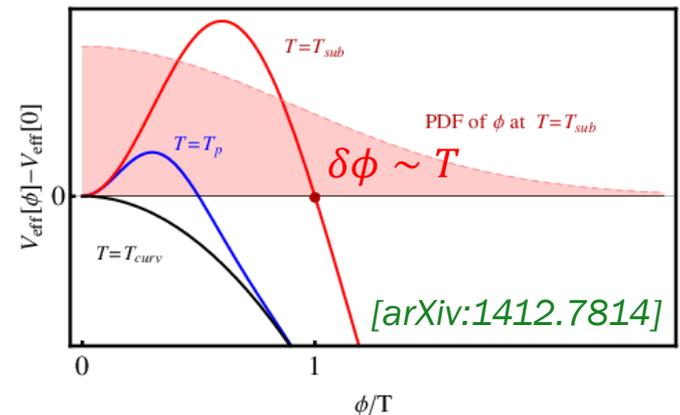
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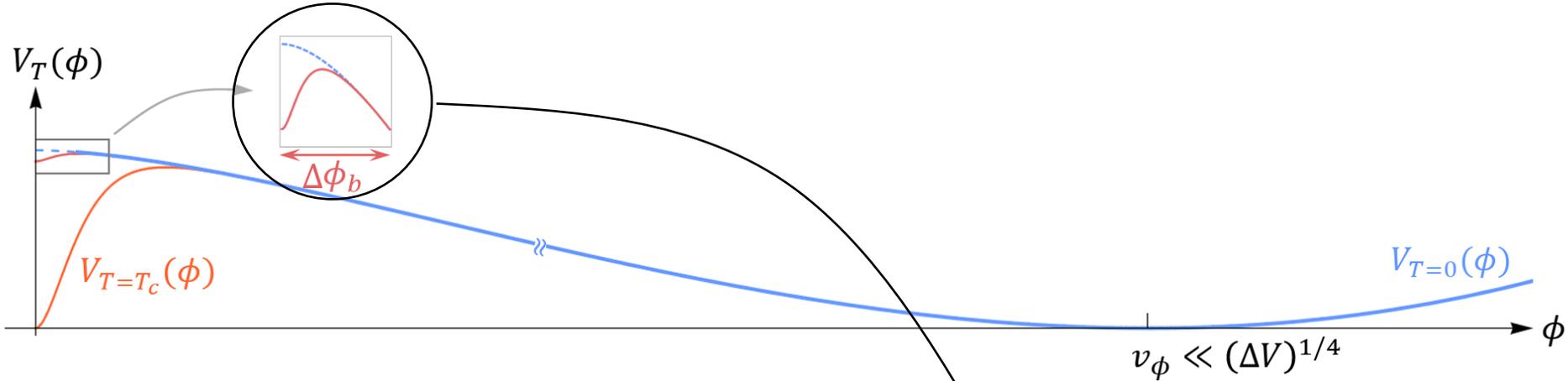
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schematic summary of potential shapes around the origin (not to scale)



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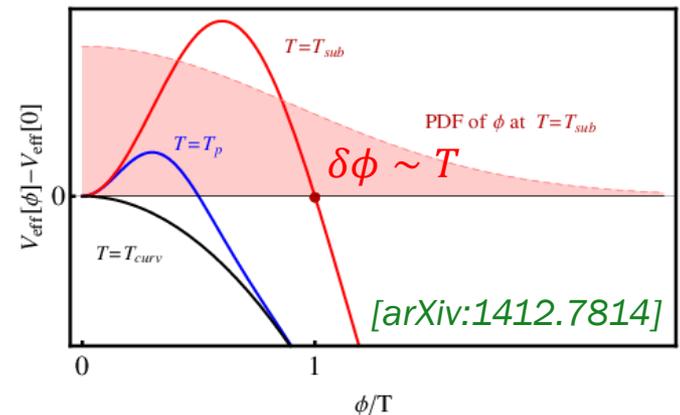
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However, there is some ambiguity, as the field distance for the potential barrier can be smaller than the temperature, $\Delta\phi_b < T$. In this case, thermal fluctuations can easily overcome the barrier. The question: **Is the transition still first order?** It may seem as if the field evolves from the top of the unstable potential without forming bubbles.

schematic summary of potential shapes around the origin (not to scale)



Simulation

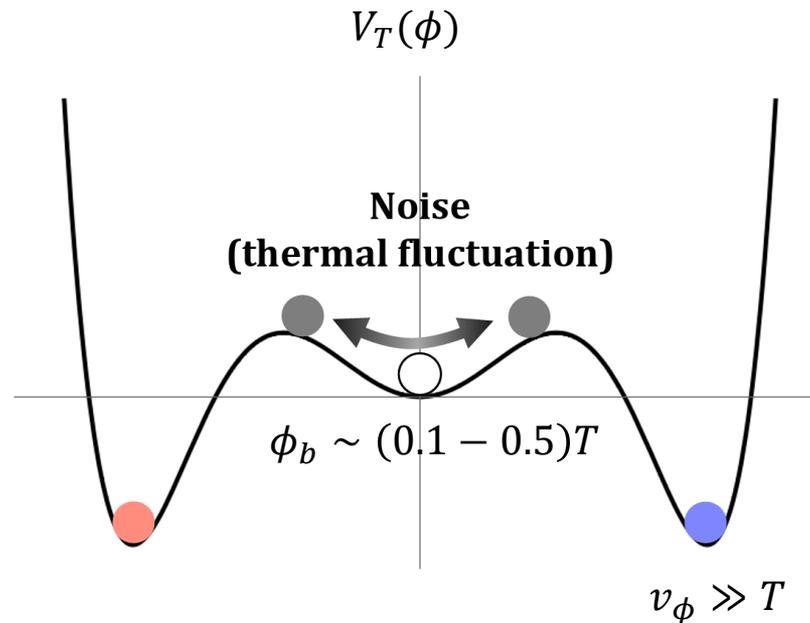
We directly solve the 3+1dimensional **Langevin equation** in the lattice ($N_{\text{lattice}}^3 \sim (200 - 400)^3$)

$$\partial_t^2 \phi(\mathbf{x}, t) + \eta \partial_t \phi(\mathbf{x}, t) - \nabla^2 \phi(\mathbf{x}, t) + \partial_\phi V_T(\phi) = \xi(\mathbf{x}, t)$$

with the **fluctuation-dissipation** theorem

$$\langle \xi(\mathbf{x}, t) \xi(\mathbf{x}', t') \rangle_T = D \delta(t - t') \delta^3(\mathbf{x} - \mathbf{x}')$$

$$D = 2\eta T, \quad \eta \sim T$$



See Yamaguchi, Yokoyama, [hep-ph/9707502](https://arxiv.org/abs/hep-ph/9707502) for EWPT

Simulation

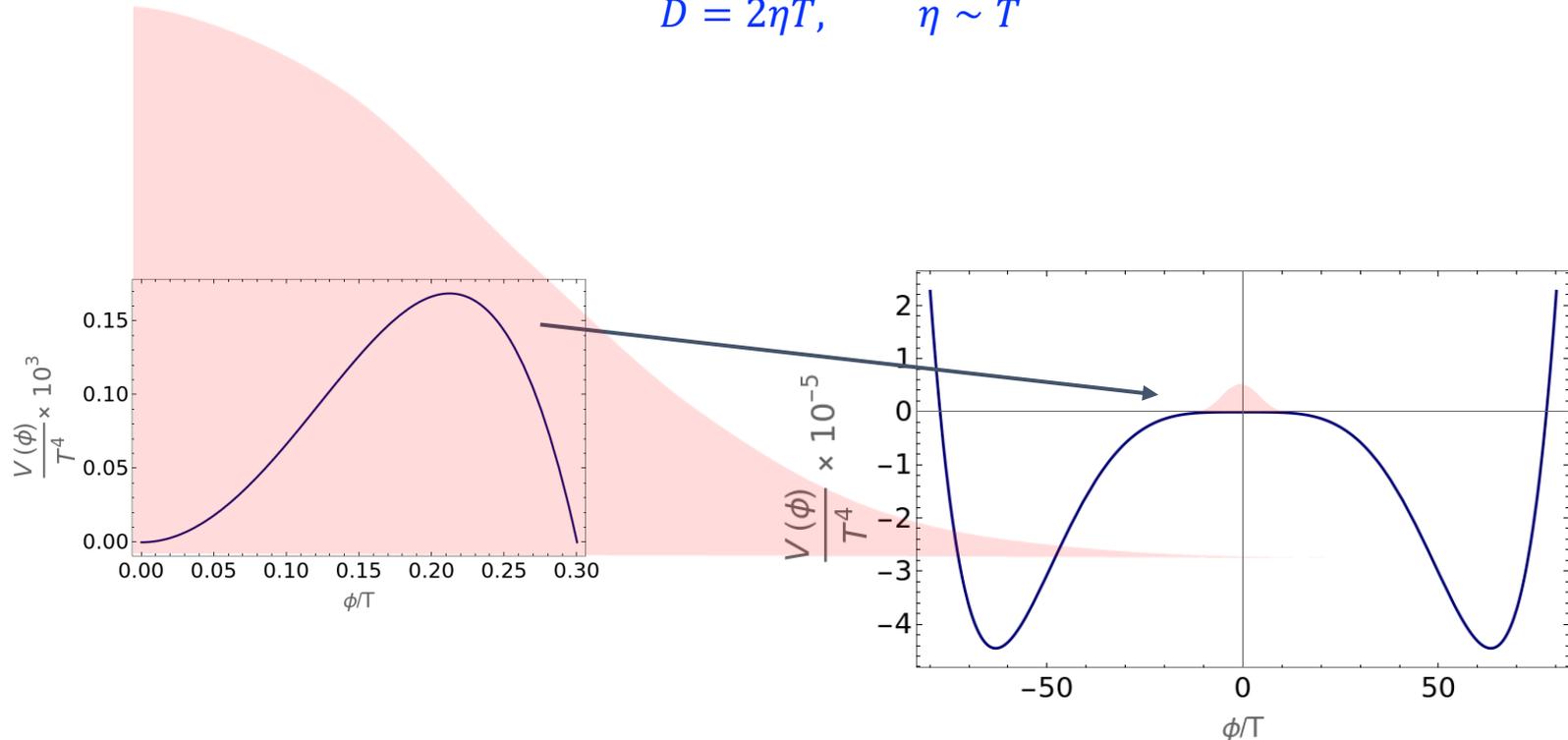
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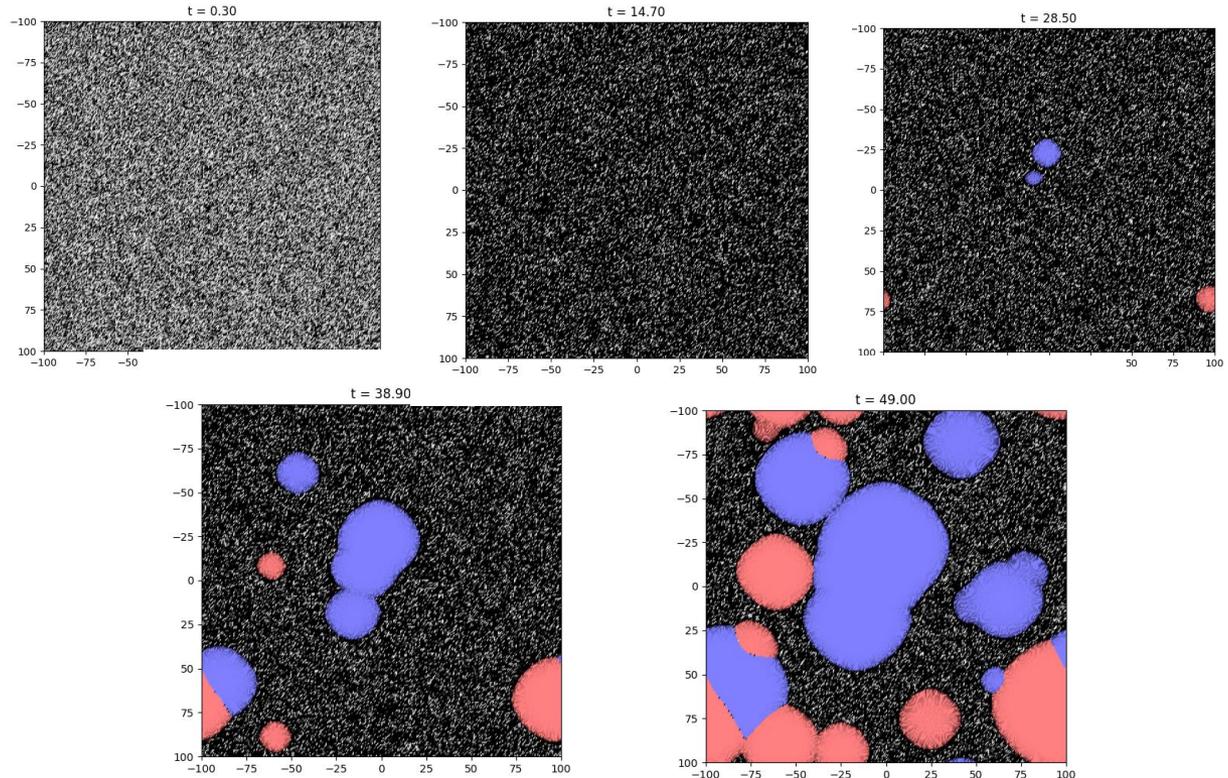
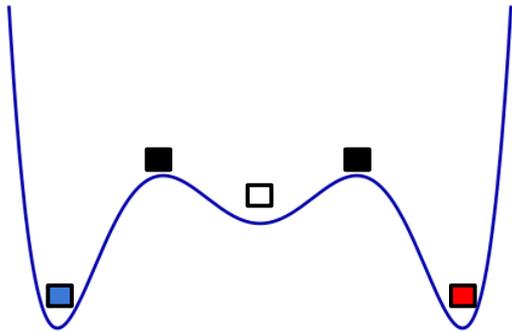
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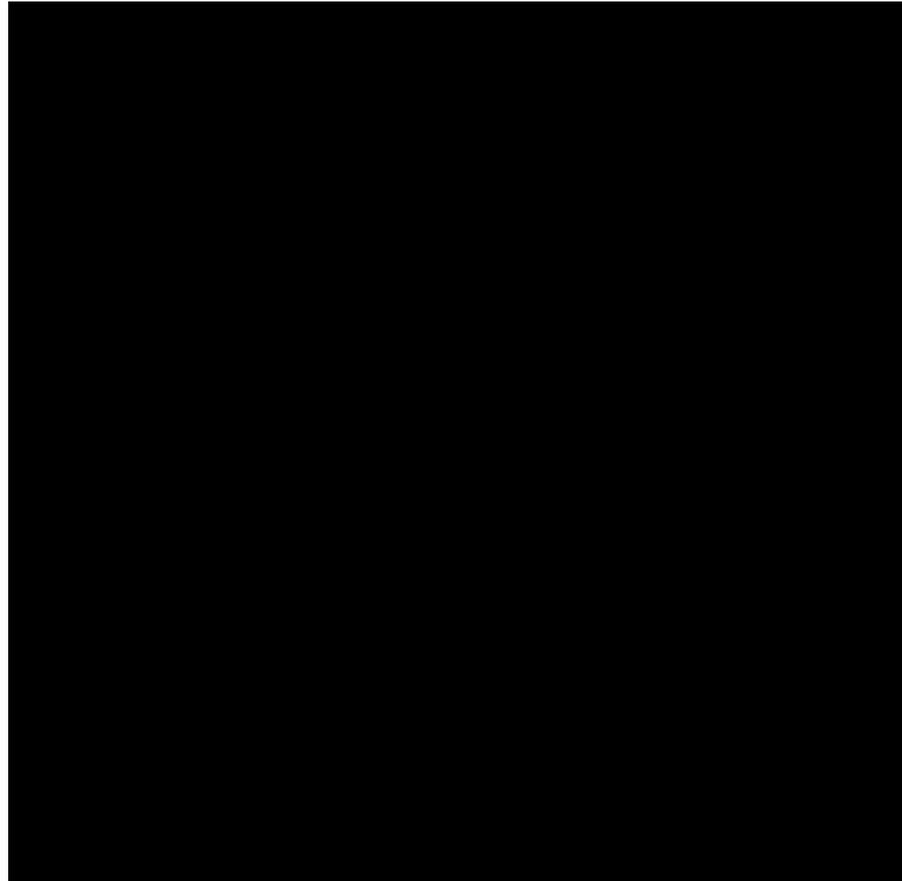
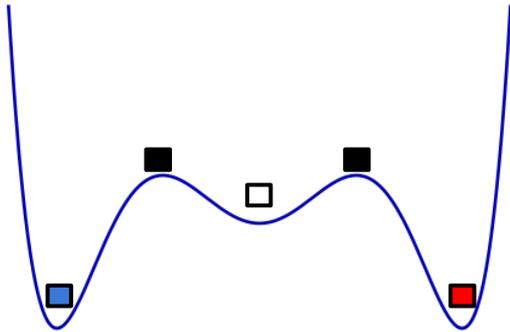
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Simulation supports bubble nucleation prediction

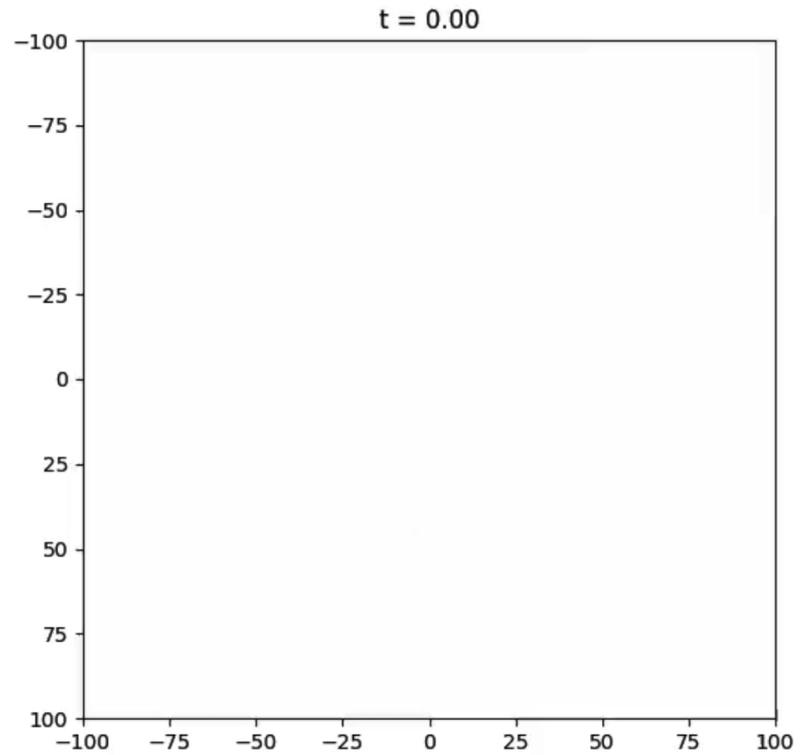
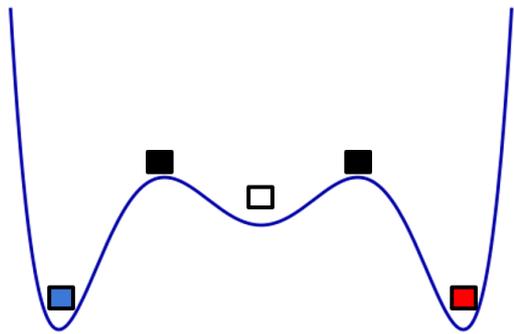
Simulation



Tomasz Dutka, Tae Hyun Jung, and CSS arXiv:2412.15864

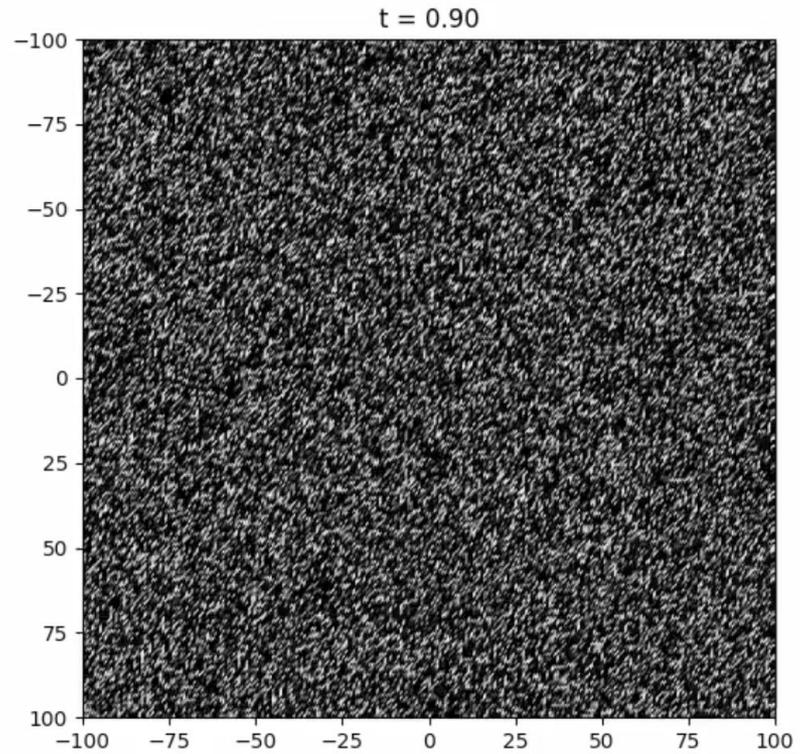
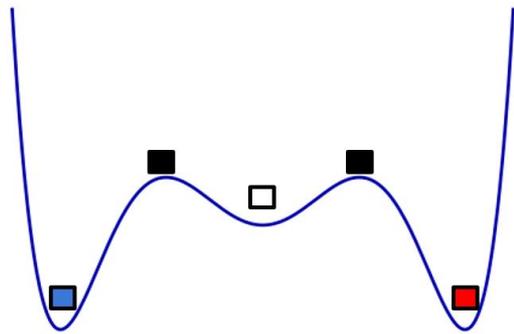
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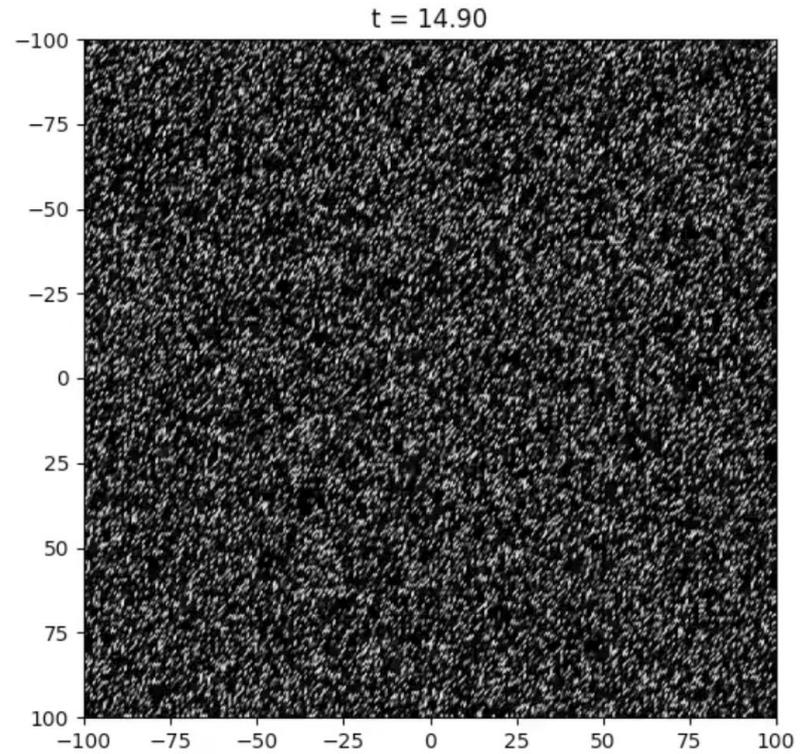
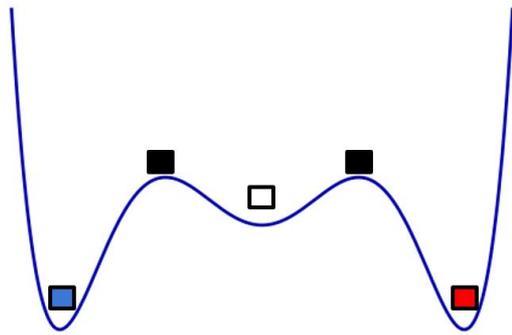
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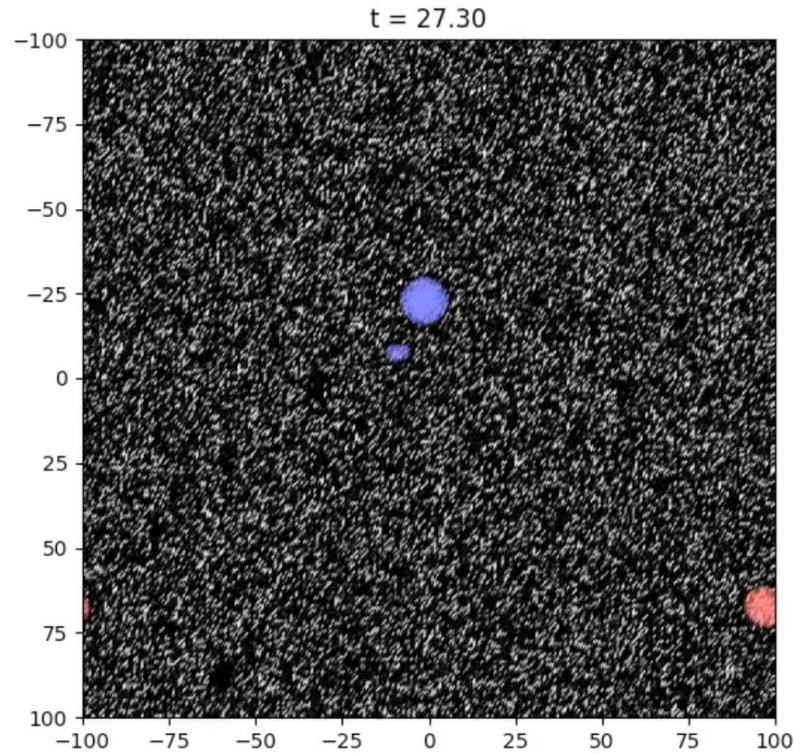
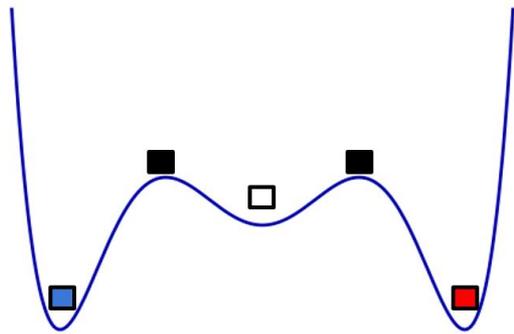
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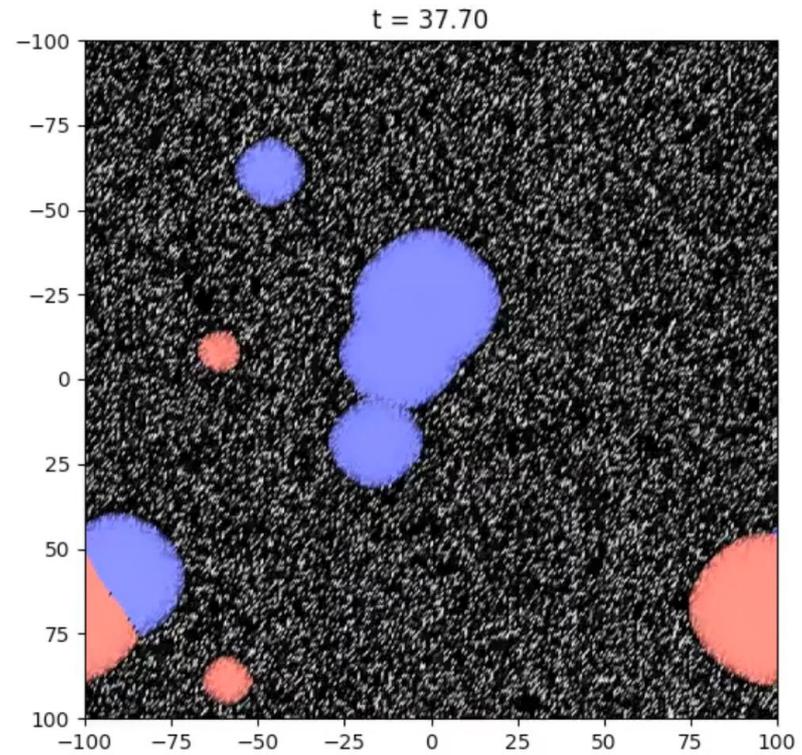
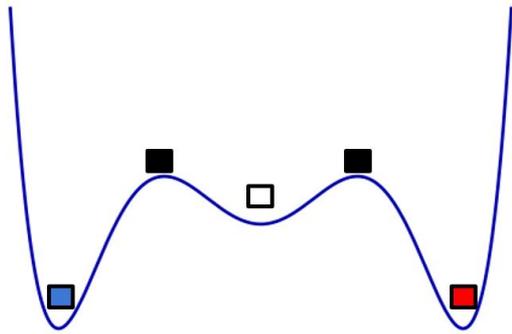
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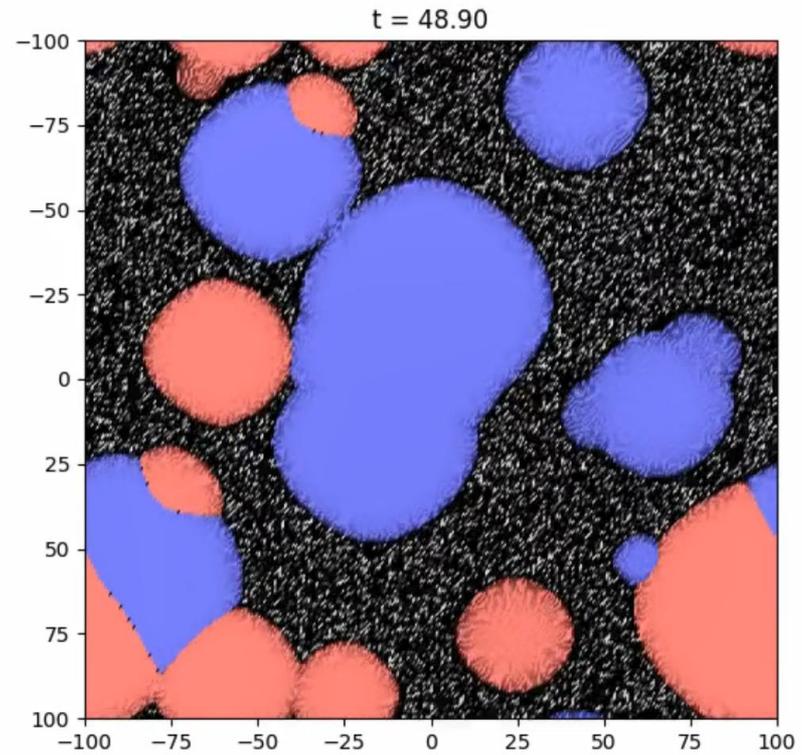
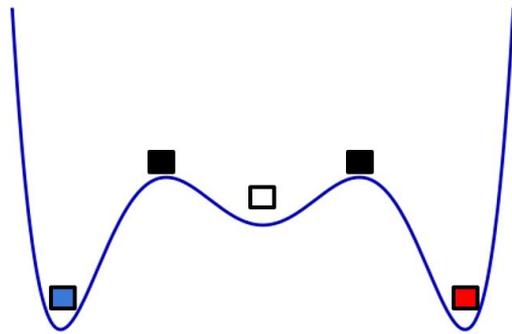
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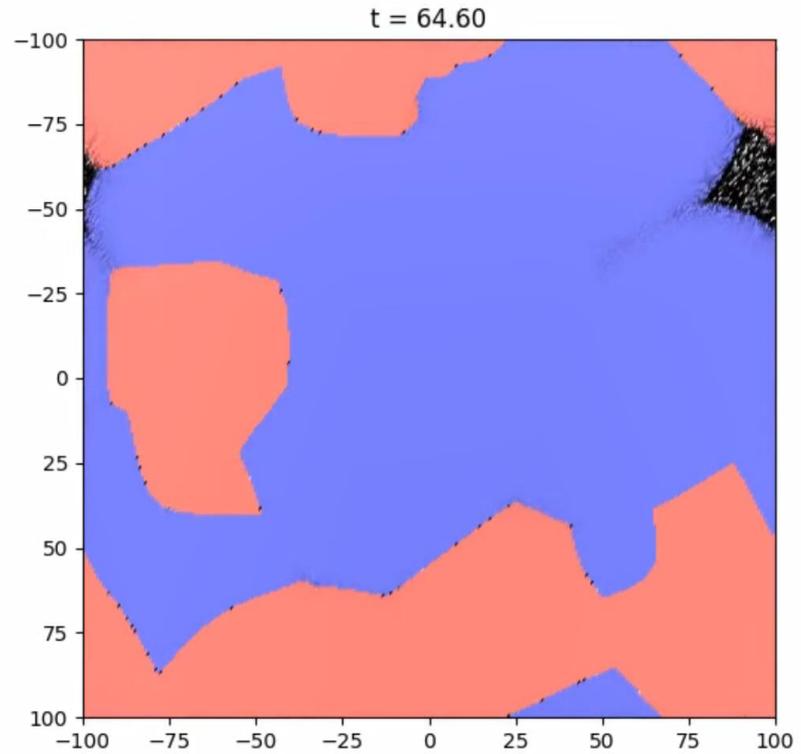
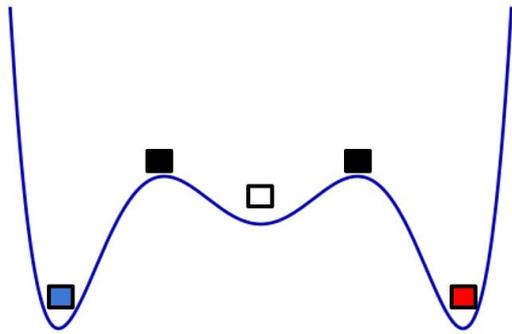
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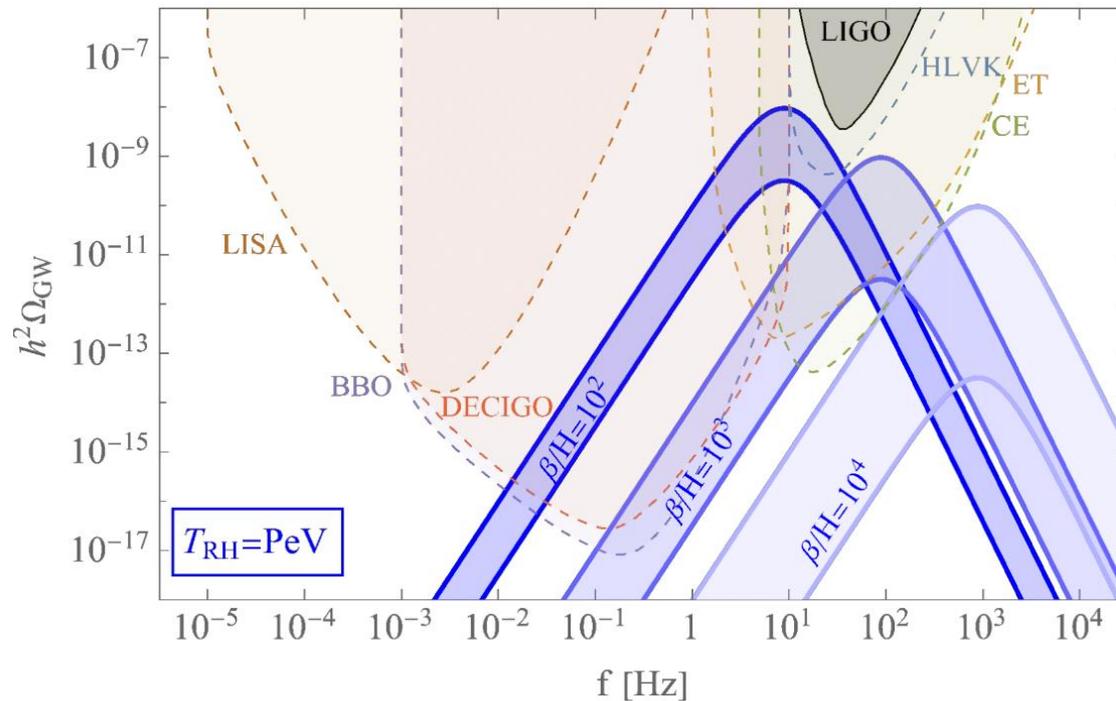


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Expected GW Signal

Further study is needed to determine whether the nucleation rate, the evolution of the true vacuum fraction, and the bubble wall velocity are consistent with our analytical understanding.

Nevertheless, we have strong evidence that a first-order phase transition is a generic feature of such a flat potential. The expected gravitational wave signal is as follows:



Summary

Heavy dark matter and its phenomenological implications are gaining significant attention.

We have explored various heavy dark matter production scenarios when sizable interactions with Standard Model particles exist, highlighting the role of a first-order phase transition.

Through simulations, we investigated whether thermal fluctuations—such as noise and friction in field dynamics—can actually induce bubble formation and expansion, i.e. first order phase transition, rather than spinodal decomposition, for various examples

A systematic study of first-order phase transitions enables more accurate predictions of dark matter phenomenology and gravitational wave signals.