

# Quintessential inflation

2412.08833

Seong Chan Park  
[sc.park@yonsei.ac.kr](mailto:sc.park@yonsei.ac.kr)

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# Goal

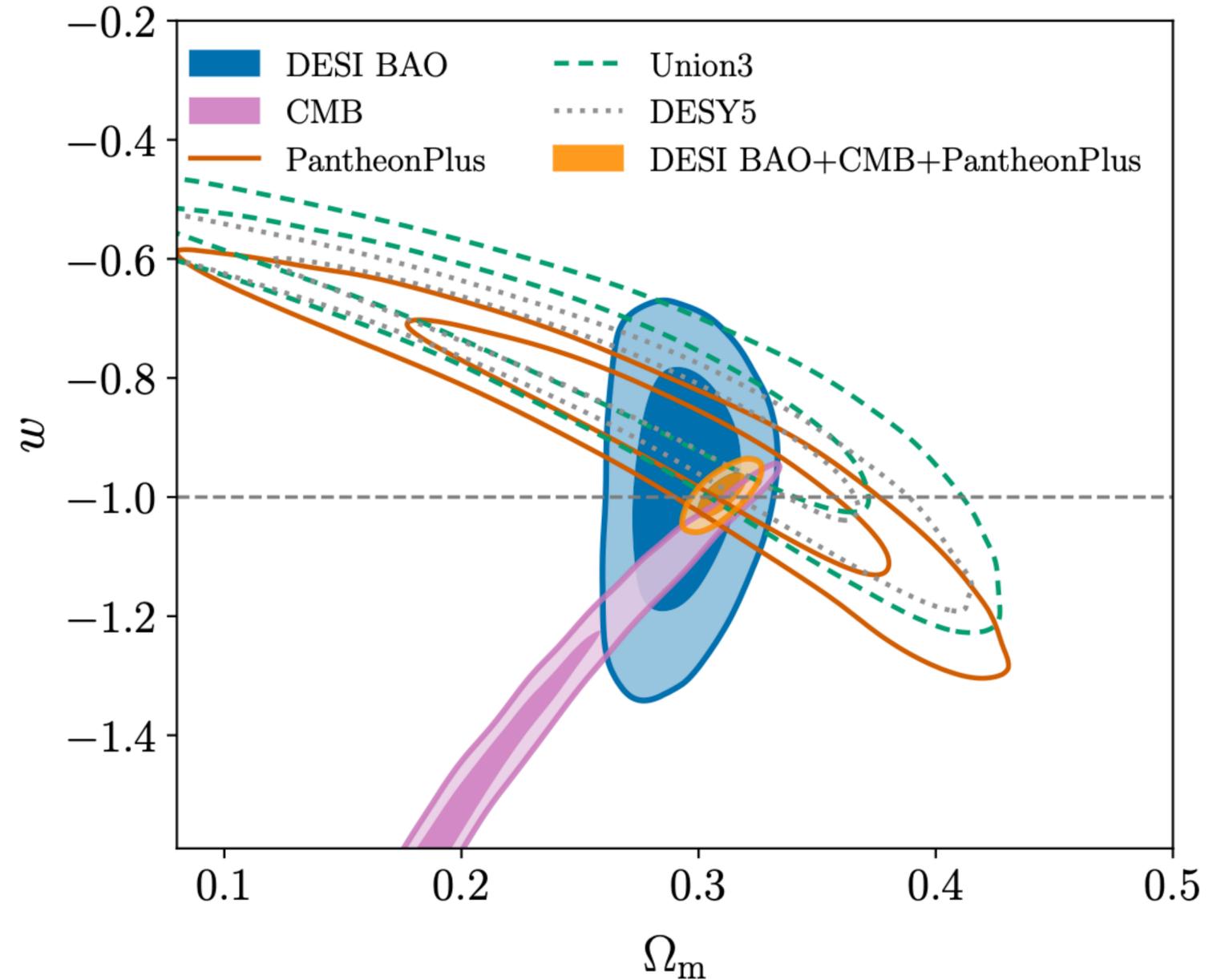
- To explain the early time inflation and the late time acceleration on the same footing
- A single field,  $\phi$ , is responsible for
  - $V_{\text{inf}} \sim \epsilon M_P^4 \delta_s^2 \sim 10^{-12} M_P^4$
  - $V_Q \sim 10^{-123} M_P^4 \sim m_\nu^4$

# Not $\Lambda$ but Quintessence ?

DESI 2024 [2404.03002]

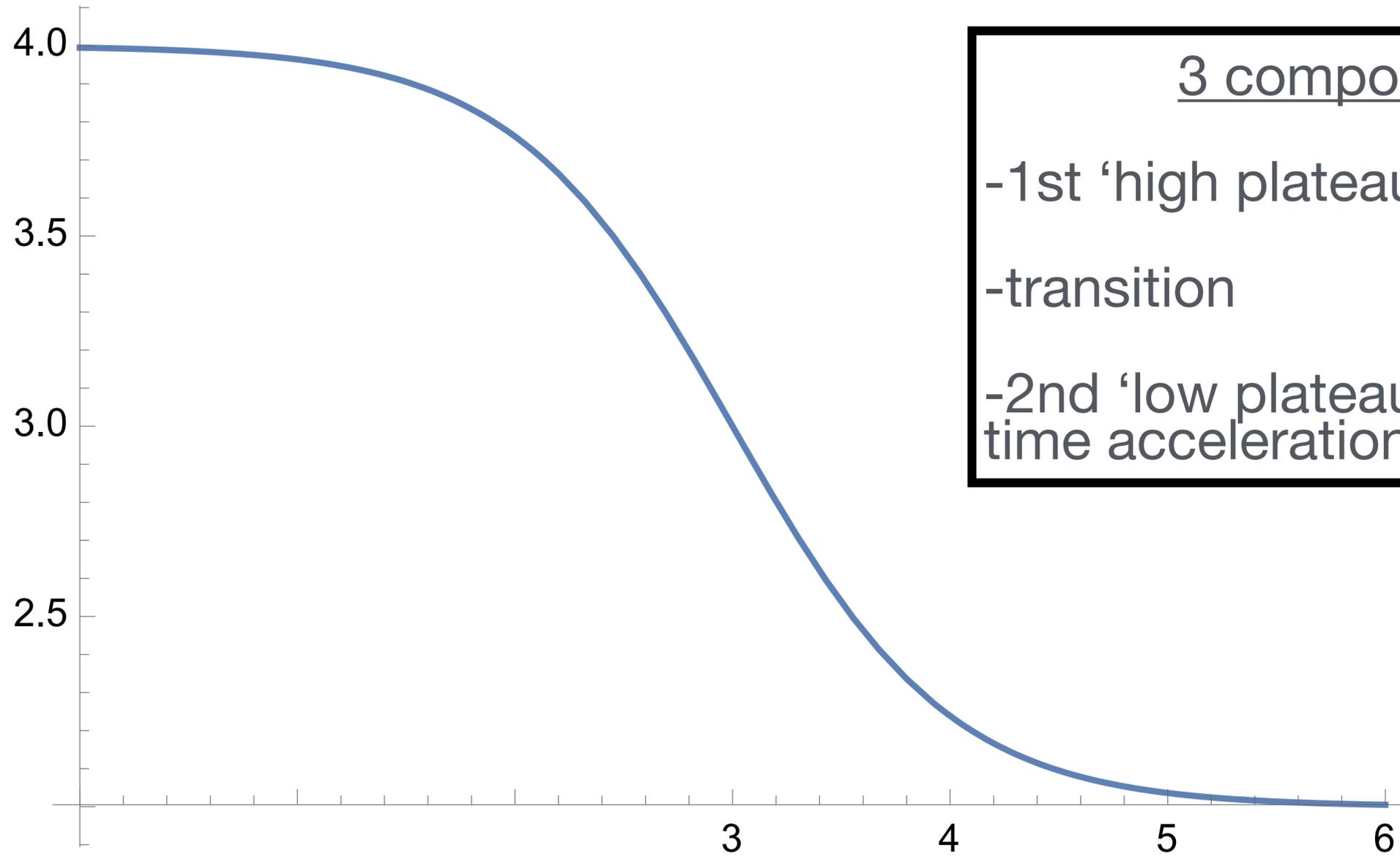
$$w = \begin{cases} -0.99^{+0.15}_{-0.13} & \text{DESI BAO,} \\ -1.122^{+0.062}_{-0.054} & \text{DESI BAO+CMB,} \\ -0.997 \pm 0.025 & \text{DESI BAO+CMB+PantheonPlus} \end{cases}$$

cf)  $w_{\Lambda} = -1$  for the cosmological constant



"SOMETHING like THIS"

Potential



3 components  
-1st 'high plateau' for Inflation  
-transition  
-2nd 'low plateau' for the late time acceleration

Quintessence field

# A suggested framework

$$S_J = \int d^4x \sqrt{-g_J} \left( \frac{M_P^2}{2} \Omega^2(\phi) R_J - \frac{1}{2} (\partial\phi)^2 - V_J(\phi) \right)$$

where  $\Omega^2 = 1 + K(\phi)$  and  $V_J = V_0 K(\phi)^2$

and *request*  $K(\phi) = \begin{cases} \text{Large} & \phi \ll M_P, \\ \text{Small} & \phi \gg M_P \end{cases}$

# The Einstein Frame action

- Weyl Transformation:

- $g_{J\mu\nu} \rightarrow g_{E\mu\nu} = \Omega^2(\phi)g_{J\mu\nu}$

- $R_J = \Omega^2(R_E - (3/2)g_J^{\mu\nu}\partial_\mu \log \Omega^2 \partial_\nu \log \Omega^2 + 3 \square \log \Omega^2)$

$$S_E = \int d^4x \sqrt{-g_E} \left[ \frac{M_P^2}{2} R_E - \frac{1}{2} (\partial h)^2 - V_E(h) \right]$$

with  $V_E(h(\phi)) = \frac{V_J(h(\phi))}{\Omega^4(h(\phi))} = \frac{V_0 K^2}{(1+K)^2}$  and the canonical

normalization is  $\frac{dh}{d\phi} = \sqrt{\frac{1}{\Omega^2} + \frac{\frac{3}{2}M_P^2(\Omega^2_{,\phi})^2}{\Omega^4}} = \sqrt{\frac{1+K+\frac{3}{2}M_P^2 K^2_{,\phi}}{(1+K)^2}} =: F(\phi)$

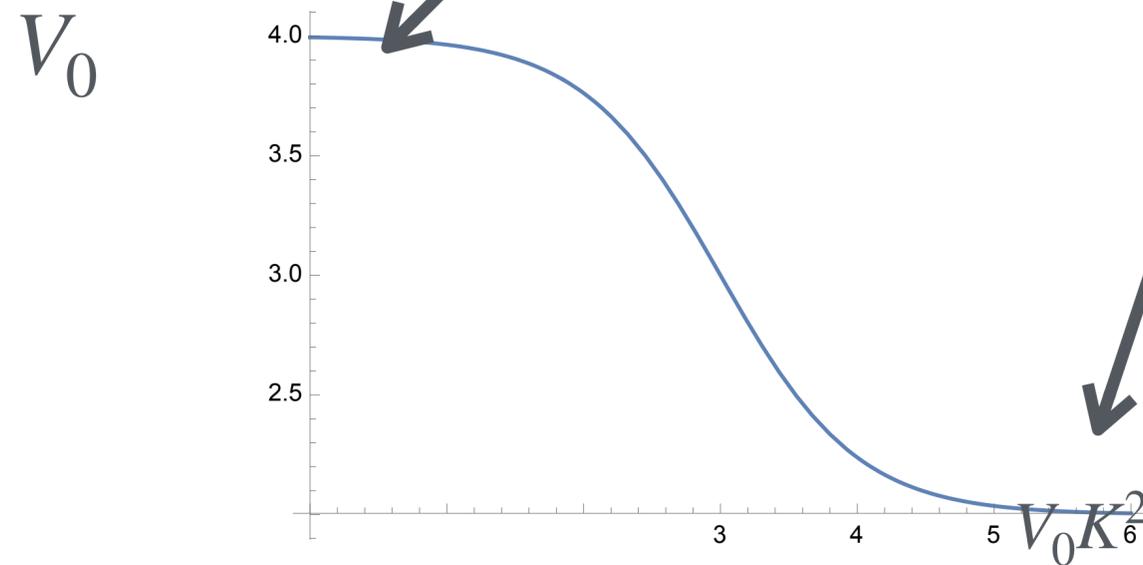
# The Potential

$$V_E(h(\phi)) = \frac{V_J(h(\phi))}{\Omega^4(h(\phi))} = \frac{V_0 K^2}{(1+K)^2}$$

Remember : we requested  $K(\phi) = \begin{cases} \text{Large} & \phi \ll M_P, \\ \text{Small} & \phi \gg M_P \end{cases}$

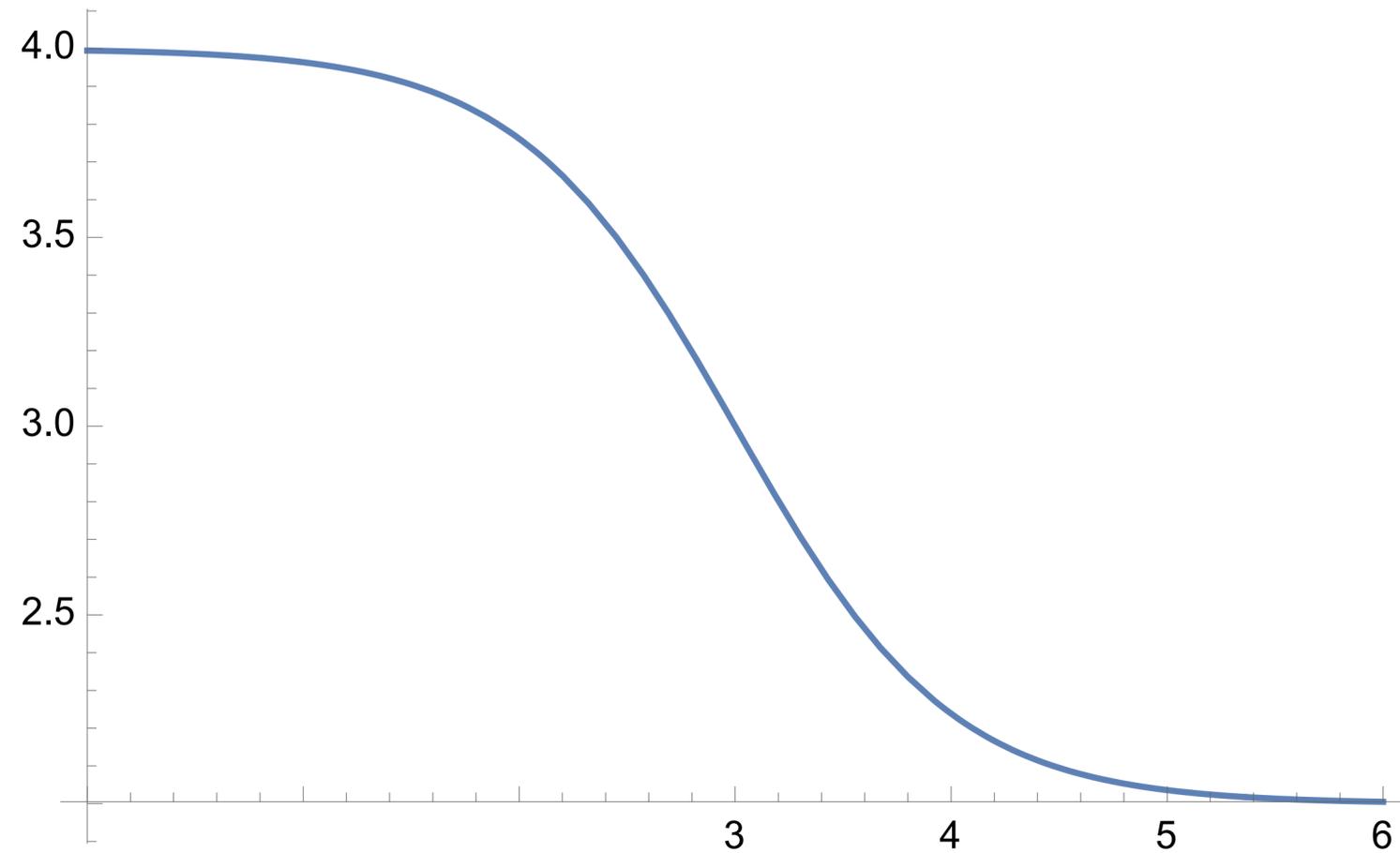
At a large field limit,  $K$  is small  $\Rightarrow V_E \sim (\text{small})^2 \sim (\text{very small})$

At a small field limit,  $K$  is large  $\Rightarrow V_E \sim V_0 \sim (\text{constant})$



Monotonically decreasing  $K(\phi)$

$$V_E(h(\phi)) = \frac{V_J(h(\phi))}{\Omega^4(h(\phi))} = \frac{V_0 K^2}{(1+K)^2} \quad \Downarrow \quad \Rightarrow \quad V_E = \begin{cases} V_0(1 - 2K^{-1}) \sim \rho_{\text{inf}}, \\ V_0 K^2 \sim \rho_{\text{DE}}, \end{cases}$$



# Excesice for students

$$M_P = 1 \quad \frac{dh}{d\phi} = \sqrt{\frac{1}{\Omega^2} + \frac{\frac{3}{2}M_P^2(\Omega^2, \phi)^2}{\Omega^4}} = \sqrt{\frac{1 + K + \frac{3}{2}M_P^2K^2, \phi}{(1 + K)^2}} =: F(\phi)$$

$$\epsilon(h(\phi)) := \frac{1}{2} (V_{,h}/V)^2 \quad \begin{matrix} \uparrow \\ = \end{matrix} \frac{1}{2} \left( \frac{1}{F(\phi)} V_{,\phi}/V \right)^2 = \frac{1}{F^2(\phi)} \hat{\epsilon}(\phi)$$

$$V_{,h} = \frac{dV}{dh} = \frac{dV}{d\phi} \underbrace{\frac{d\phi}{dh}}_{=: 1/F(\phi)}$$

$$\eta(h(\phi)) := \frac{V_{,hh}}{V} \quad \begin{matrix} \uparrow \\ = \end{matrix} \frac{1}{F^2(\phi)} \hat{\eta}(\phi) - \frac{F'}{F^2} \sqrt{2\hat{\epsilon}(\phi)}$$

$$V_{,hh} = \frac{1}{F(\phi)} \frac{d}{d\phi} \left( \frac{1}{F(\phi)} \frac{d}{d\phi} \right)$$

# What's your $K(\phi)$ ?

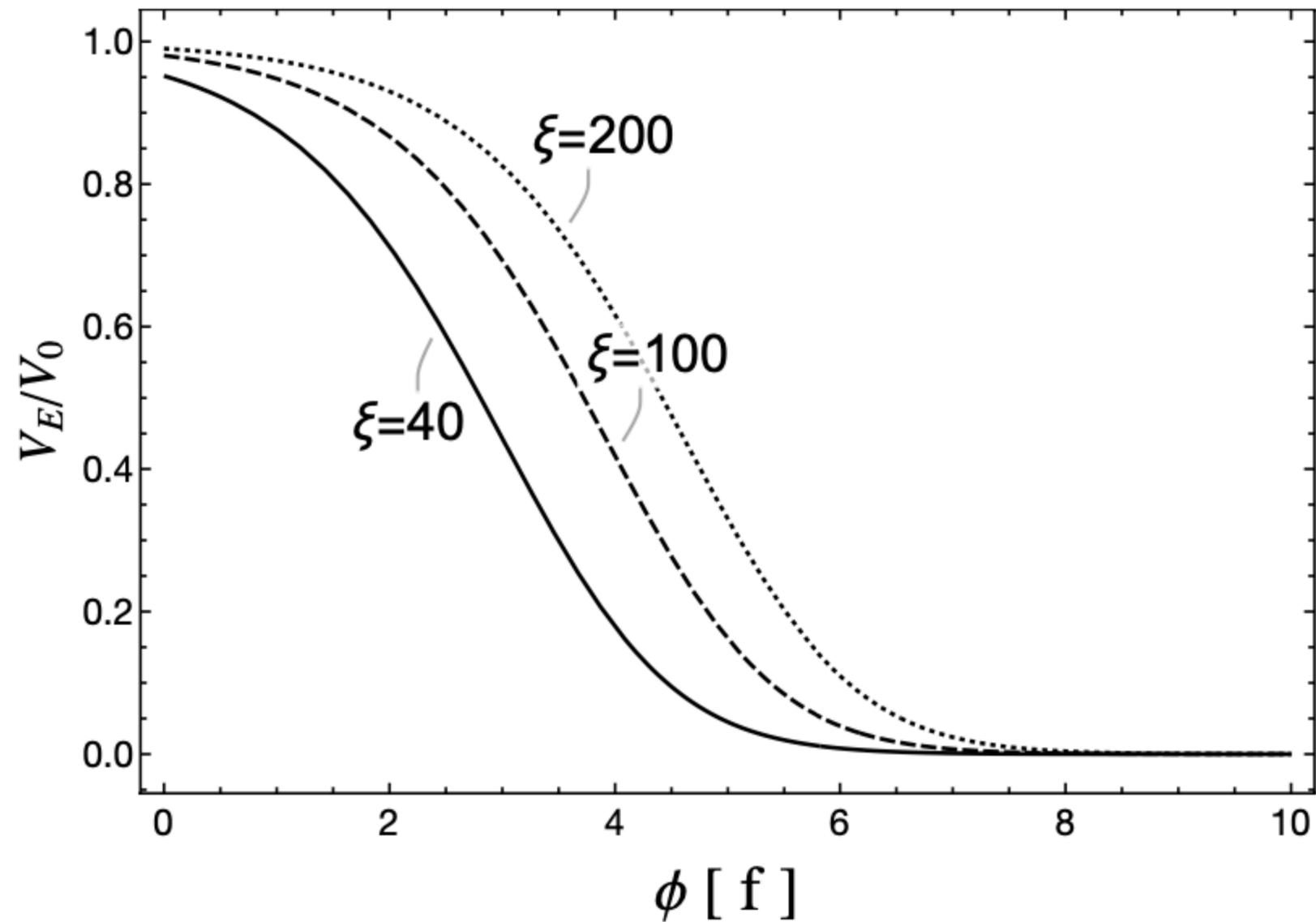
- $K(\phi) = \begin{cases} \text{Large} & \phi \ll M_P, \\ \text{Small} & \phi \gg M_P \end{cases}$

=> Monotonically decreasing functions are good candidates

- Infinite possibilities (ex)  $K(\phi) \sim \xi e^{-\phi/f}, \xi(\phi/M_P)^{-n}, \dots$
- ...NONE OF THEM looks more attractive than others...
- In any case, I want to check a simple one and see what I can learn more.
- **There will be better models..**

**My example:**

$$K = \xi e^{-\phi/f}, \quad V_E = V_0 \left( 1 + \frac{e^{\phi/f}}{\xi} \right)^{-2}.$$

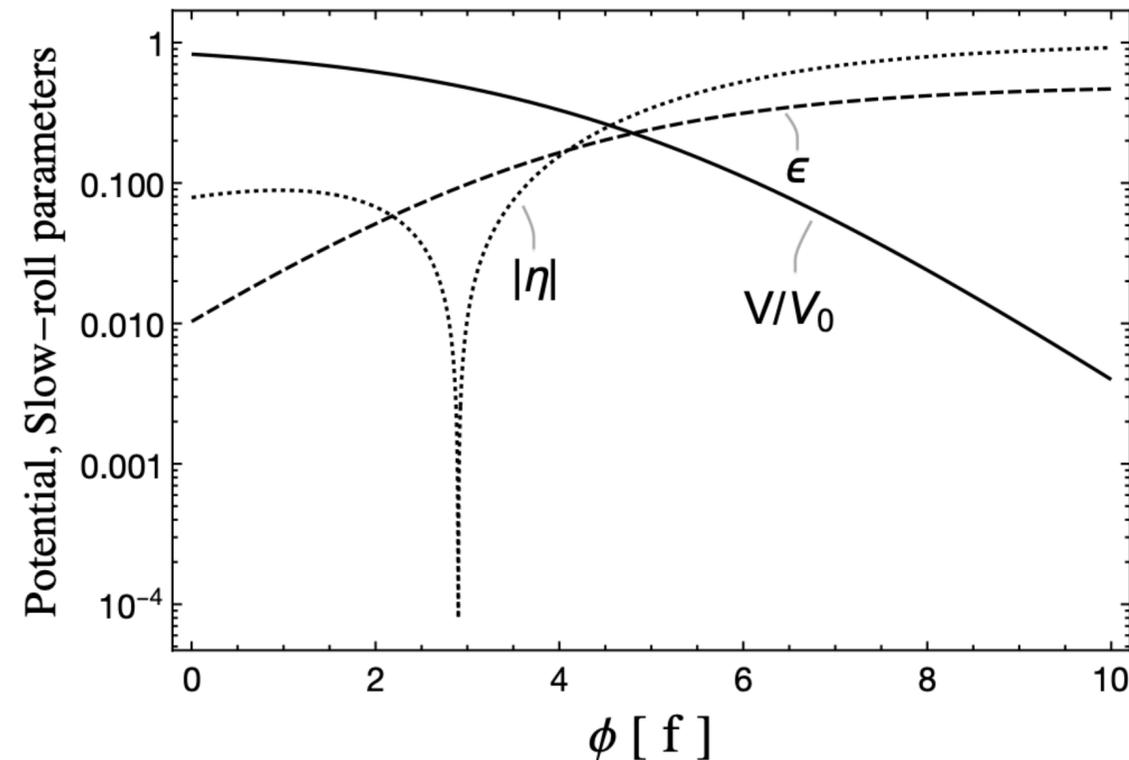


# Slow-Roll Parameters

- $$V_E = \begin{cases} V_0(1 - \frac{2}{\xi}e^{\phi/f}) & \phi \ll f \ln \xi, \\ V_0\xi^2 e^{-2\phi/f} & \phi \gg f \ln \xi, \end{cases}$$

- $$\epsilon = \frac{2M_P^2}{f^2(1 + \xi e^{-\phi/f}) + \frac{3}{2}\xi^2 M_P^2 e^{-2\phi/f}}$$

- $$\eta = -\frac{2M_P^2 f(1 + \xi e^{-\phi/f})}{f^2(1 + \xi e^{-\phi/f}) + \frac{3}{2}\xi^2 M_P^2 e^{-2\phi/f}}$$



**NOTE:**

The slow-roll parameters are calculated using the canonical field  $h$  then expressed by the Jordan frame field  $\phi$ .

# Early Time Inflation : $\phi \ll f \log \xi$

$$\epsilon \approx \frac{4}{3K^2} = \frac{4}{3\xi^2 e^{-2\phi/f}}, \quad \eta \approx -\frac{4}{3K} = -\frac{3}{3\xi e^{-\phi/f}}.$$

$$\bullet N_e = \frac{1}{M_P^2} \int_{h_{\text{end}}}^{h_*} \frac{V_E(h(\phi)) dh(\phi)}{dV_E/dh} \approx \frac{3\xi}{4} (e^{-\phi_*/f} - e^{-\phi_{\text{end}}/f})$$

$$\bullet n_s = 1 - 6\epsilon_* + 2\eta_* \approx 1 - \frac{2}{N_e} - \frac{9}{2N_e^2}, \quad r = 16\epsilon_* \approx \frac{12}{N_e^2}$$

$\implies n_s = 0.965, r = 0.003$ . : Consistent with the WMAP data!!

$$\bullet A_s = \frac{1}{12\pi^2 M_P^2} \frac{V_E(s_*)^3}{V_{E,s}(s_*)^2}$$

$$\implies \frac{V_0}{fM_P^3} \approx (1.67 - 1.71) \times 10^{-10} \times \left(\frac{N_e}{60}\right)^{-2} \quad . : \text{fixing } V_0/f$$

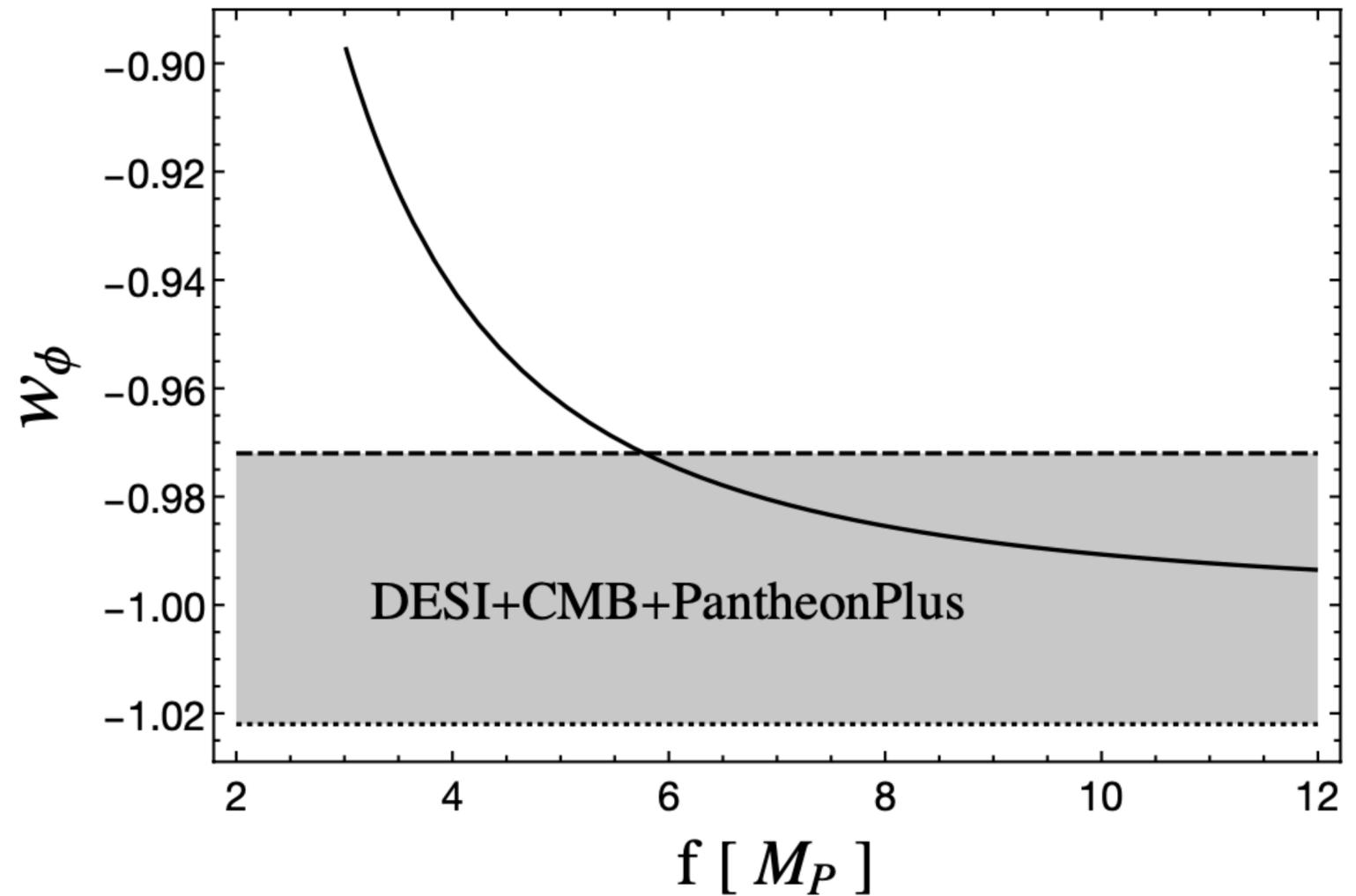
# Late Time DE. : $\phi \gg f \log \xi$

$$\epsilon \simeq \frac{2M_P^2}{f^2}, \eta \simeq \frac{4M_P^2}{f^2}.$$

$$w(\phi) = \frac{\frac{1}{2}\dot{\phi}^2 - V_E(\phi)}{\frac{1}{2}\dot{\phi}^2 + V_E(\phi)}$$

$$\approx -1 + \frac{2\Omega_\phi}{3}\epsilon$$

$$\dot{\phi}^2 \approx \left( -\frac{V'_E(\phi)}{3H} \right)^2 \approx \frac{\Omega_\phi M_P^2 V_E'^2}{3V_E} = \frac{2}{3}V_E \Omega_\phi \epsilon,$$



# Dark Energy

$$\rho_\phi(N) \simeq V_E(\phi(N)) \simeq \frac{V_0 \xi^2}{e^{2\phi(N)/f}}.$$

- Matching the Hubble parameter

$$V_E \approx 3M_P^2 H^2 \Omega_\phi \Rightarrow \phi_0 = \phi(N=0) \approx \frac{f}{2} \ln \left[ \frac{V_0 \xi^2}{3M_P^2 H^2 \Omega_\phi} \right].$$

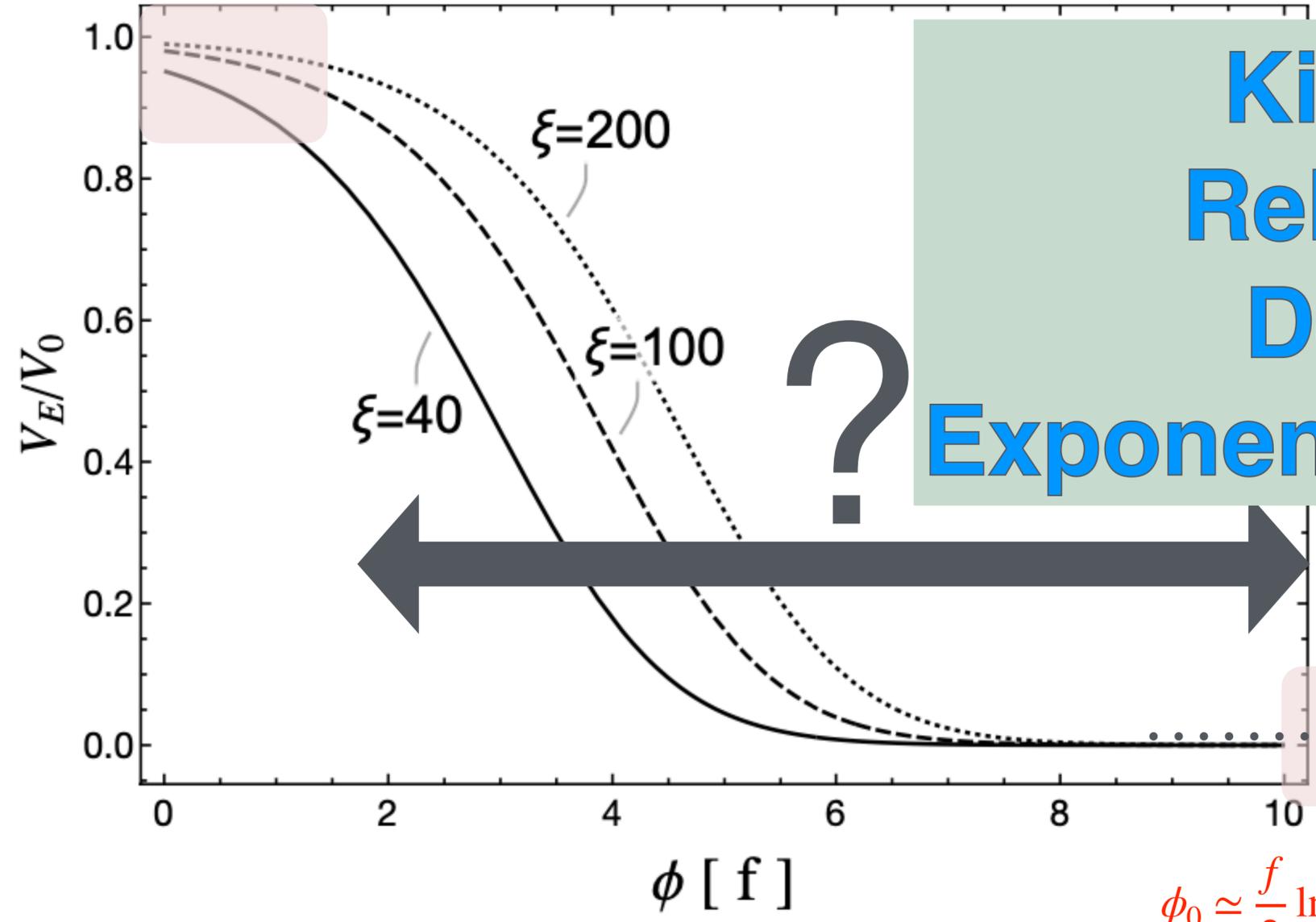
- Taking  $\rho_{\text{crit}} = 3M_P^2 H^2 \sim m_\nu^4$ , and  $V_0 \sim 10^{-10} f M_P^3$  from the inflation data we can determine the field value at the current universe:

$$\phi_0 \simeq \frac{f}{2} \ln \left[ \frac{10^{-10} f M_P^3 \xi^2}{\Omega_\phi m_\nu^4} \right].$$

- Q. Can I realize this? Probably, yes. Dynamics, Reheating, and all the details should be fixed to answer this question.

# Filling the gap

inflaton here



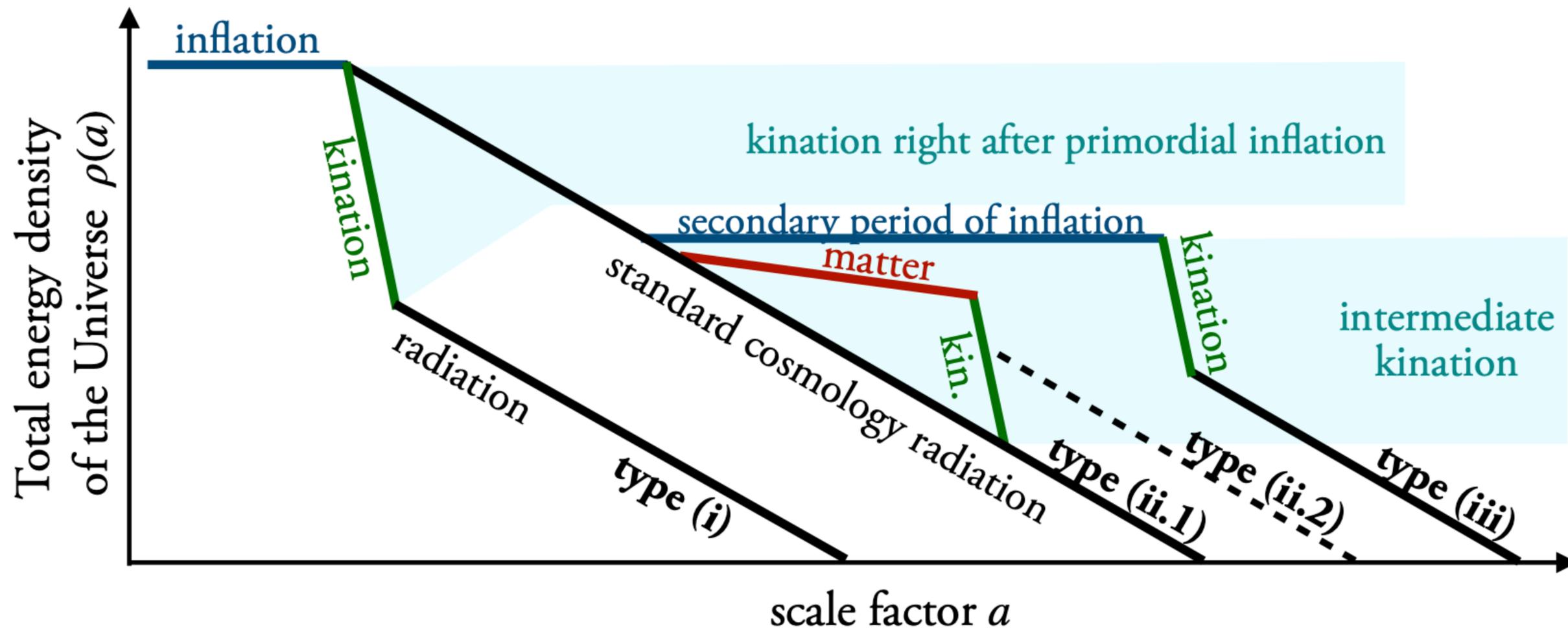
**Kination?**  
**Reheating?**  
**Decay??**  
**Exponential Attractor?**

$$\phi_0 \simeq \frac{f}{2} \ln \left[ \frac{10^{-10} f M_P^3 \xi^2}{\Omega_\phi m_\nu^4} \right].$$

DE down there

# I can imagine all complications.

Y. Gouttenoire et. al. 2111.01150



# Future?

- I think **the framework itself is interesting:**

- But, underlying physics should be better understood

✓ which  $K(\phi)$  is the true one?

✓ “fill the gap” **with more details** of dynamical evolution & reheating

✓ quantum loop effects ?? CC problem????

- **THERE ARE MANY THINGS TO DO!**

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where  $\Omega^2 = 1 + K(\phi)$  and  $V_J = V_0 K(\phi)^2$

$$\text{and } K(\phi) = \begin{cases} \text{Large} & \phi \ll M_P, \\ \text{Small} & \phi \gg M_P \end{cases}$$