

# Late-Time Cosmology without Dark Sector but with Closed String Massless Sector

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# Overview of Our Results

- **Based on Double Field Theory & String Frame:**  
**Jeong-Hyuck Park et al. PLB 860 (2025)**
- **Modified Friedmann Equations**
- **Define the Density Parameters**
- **Match with Type Ia Supernovae & Quasar Absorption Spectra**
- **Find the Most Probable Values**

# Overview of Our Results

$$\Omega_k + \Omega_{\epsilon} + \Omega_{\Lambda} + \Omega_h \sim 1$$

$$H_0 = 71.22 \text{ km/s/Mpc}$$

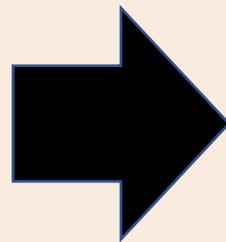
$$\Omega_k = 0.9925$$

$$\Omega_{\epsilon_0}^{w=1/3} = 0.0075$$

$$\Omega_{\epsilon_0}^{w=0} < 10^{-7}$$

$$|\Omega_{\Lambda}| < 10^{-6}$$

$$\Omega_h < 10^{-10}$$



Open Universe:

$$\Omega_k \sim 1$$

# Overview of Our Results

## Low Energy Effective ( $O(D, D)$ Symmetric) Action

$$S = \int d^4x \sqrt{-g} e^{-2\phi} \left( \underbrace{R - 2\Lambda}_{\text{Einstein Gravity \& Cosmological Constant}} + \underbrace{4\nabla_\rho \phi \nabla^\rho \phi}_{\text{Dilaton}} - \frac{1}{12} \underbrace{H_{\rho\sigma\lambda} H^{\rho\sigma\lambda}}_{\text{H-Flux}} + \underbrace{L_m}_{\text{Matter}} \right)$$

String Frame

Einstein Gravity & Cosmological Constant

Dilaton

H-Flux

Matter

# Motivation: Cosmological Constant Problem

*\*S. Weinberg, Rev. Mod. Phys. 61 (1989)*

Theory

$$\Lambda_{th} \sim \ell_P^{-2}$$

Observation

$$\Lambda_{ob} \sim \mathbf{10^{-120}} \ell_P^{-2}$$

$$\Lambda_{ob} \sim \mathbf{10^{-120}} \Lambda_{th}$$

**120-Orders-of-Magnitude Discrepancy**

# Motivation: Hubble Tension (James Webb Space Telescope)



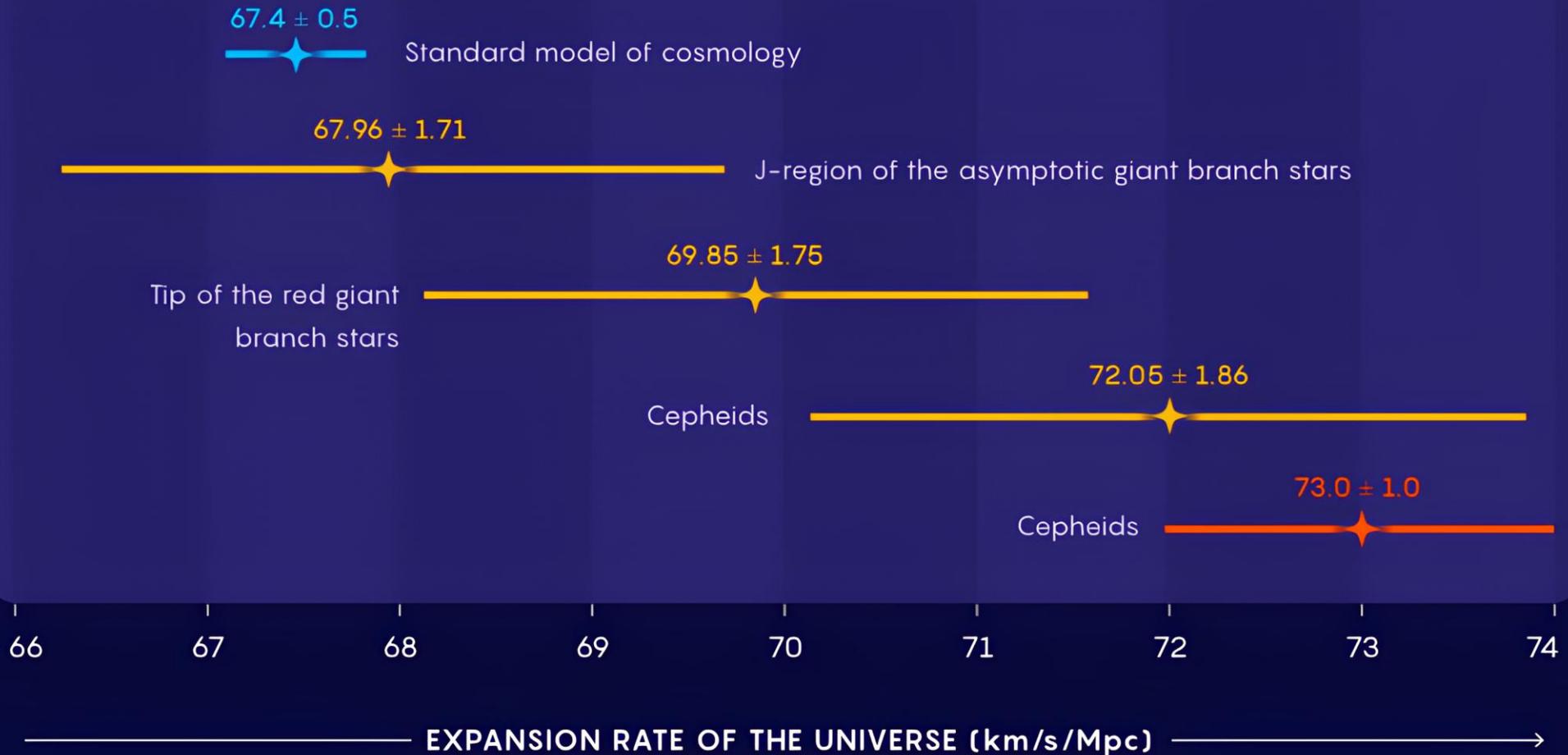
Prediction  
(Planck Collaboration, 2020)



Webb data  
(Freedman et al., 2024)



Hubble and Webb data  
(Riess et al., 2024)



# Double Field Theory

*\*W. Siegel, Phys. Rev. D 48 (1993)*

*\*C. Hull, B. Zwiebach, JHEP 909 (2009)*

**DFT = Stringy Completion of GR**

$$S_{DFT} = \int_{\Sigma_D} e^{-2d} \mathcal{S}(\mathbf{0})$$

**Integral of a Scalar Curvature Defined in Stringy Differential Geometry**

*\*I. Jeon, K. Lee, J.-H. Park, PRD 84 (2011)*



# Double Field Theory: $O(D,D)$ Symmetry

**$O(D,D)$  Symmetry:**  $H_{AB} = H_{BA}$      $H_A^C H_B^D J_{CD} = J_{AB}$      $J_{AB} = \begin{pmatrix} \mathbf{0} & \mathbf{I} \\ \mathbf{I} & \mathbf{0} \end{pmatrix}$

**Metric (Riemannian):**  $H_{AB} = \begin{pmatrix} g^{-1} & -g^{-1}B \\ Bg^{-1} & g - Bg^{-1}B \end{pmatrix}$

**Covariant Derivative:**  $\nabla = \partial + \Gamma$

**Compatibility:**  $\nabla H_{AB} = 0$      $\nabla d = 0$

**Torsionless:**  $\Gamma_{[ABC]} = 0$

**Curvature:**  $\delta S_{ABCD} = \nabla_{[A} \delta \Gamma_{B]CD} + \nabla_{[C} \delta \Gamma_{D]AB}$

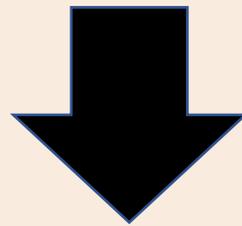
# Further Details in the Upcoming Presentation...

Time	12/18 (Wed)
10:00 - 11:00	Euihun Joung (20)
	Tabasum Rahnuma (20)
	Discussion (20)
11:00 -12:00	Jeong-Hyuck Park (30)
	Discussion (30)

# Hidden Symmetry Inside the Low Energy Effective Action

$$S_{DFT} = \int_{\Sigma_D} e^{-2d} \mathcal{S}_{(0)}$$

$O(D, D)$   
Symmetry



$\tilde{\partial}^\mu = 0$   
Riemannian  
Reduction

$$S = \int d^D x \sqrt{-g} e^{-2\phi} \left( R + 4\nabla_\rho \phi \nabla^\rho \phi - \frac{1}{12} H_{\rho\sigma\lambda} H^{\rho\sigma\lambda} \right)$$

# Einstein Double Field Equation

\*S. Angus, K. Cho & J. H. Park, Eur. Phys. J. C 78 (2018)

$$\mathbf{T}_{MN} = \mathbf{G}_{MN}$$

$$K^{(\mu\nu)} = e^{2d} \frac{\delta(e^{2d} L_{\text{matter}})}{\delta g_{\mu\nu}}$$
$$K^{[\mu\nu]} = -e^{2d} \frac{\delta(e^{2d} L_{\text{matter}})}{\delta B_{\mu\nu}}$$
$$T_{(0)} = \frac{1}{2} e^{2d} \frac{\delta(e^{2d} L_{\text{matter}})}{\delta d}$$

- $K_{(\mu\nu)} = R_{\mu\nu} + 2\nabla_{\mu}\nabla_{\nu}\phi - \frac{1}{4}H_{\mu\rho\sigma}H_{\nu}{}^{\rho\sigma}$
- $K_{[\mu\nu]} = \frac{1}{2}e^{2\phi}\nabla^{\rho}(e^{-2\phi}H_{\rho\mu\nu})$
- $T_{(0)} = R + 4\nabla_{\rho}\nabla^{\rho}\phi - 4\nabla_{\rho}\phi\nabla^{\rho}\phi - \frac{1}{12}H_{\rho\mu\nu}H^{\rho\mu\nu}$

# Friedmann Equation & Continuity Equation

*\*S. Angus, K. Cho, G. Franzmann, S. Mukohyama & J. H. Park, Eur. Phys. J. C 80 (2020)*

$$\frac{2}{N} \frac{d\phi}{dt} = 3H \pm \sqrt{3H^2 + 2\epsilon e^{2\phi} + T_{(0)} - \frac{6k^2}{a^2} + \frac{h^2}{2a^6}}$$

$$\frac{1}{N} \frac{dH}{dt} = pe^{2\phi} - \frac{2k}{a^2} + \frac{h^2}{2a^6} \pm H \sqrt{3H^2 + 2\epsilon e^{2\phi} + T_{(0)} - \frac{6k^2}{a^2} + \frac{h^2}{2a^6}}$$

$$\frac{1}{N} \frac{d\epsilon}{dt} = -3(\epsilon + p)H - \frac{e^{-2\phi}}{2N} \frac{dT_{(0)}}{dt}$$

For Simplicity:  
 $T_{(0)} \equiv 2\Lambda$

# Density Parameters

$$\Omega_k + \Omega_\epsilon + \Omega_\Lambda + \Omega_h = 1 + \frac{2}{3H^2} \left( \frac{\dot{\phi}}{N} \right)^2 - \frac{2\dot{\phi}}{HN}$$

$$\Omega_k = -\frac{k}{a^2 H^2}$$

$$\Omega_\epsilon = \frac{\epsilon e^{2\phi}}{3H^2}$$

$$\Omega_\Lambda = \frac{\Lambda}{3H^2}$$

$$\Omega_h = \frac{h^2}{12a^6 H^2}$$

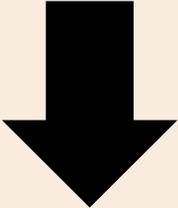
# Friedmann Equations (Density Parameters & Redshift)

$$\phi'(z) = -\frac{1}{2(1+z)} \left[ 3 \pm \sqrt{3 + 6(\Omega_k + \Omega_\epsilon + \Omega_\Lambda + \Omega_h)} \right]$$

$$H'(z) = -\frac{H}{1+z} \left[ 2\Omega_k + 3w\Omega_\epsilon + 6\Omega_h \pm \sqrt{3 + 6(\Omega_k + \Omega_\epsilon + \Omega_\Lambda + \Omega_h)} \right]$$

# String vs. Einstein Frame

String Frame: 
$$S = \frac{1}{2\kappa} \int d^D x \sqrt{-g} e^{-2\phi} \left( R + 4\nabla_\rho \phi \nabla^\rho \phi - \frac{1}{12} H_{\rho\sigma\lambda} H^{\rho\sigma\lambda} \right)$$

 
$$(g_E)_{\mu\nu} = e^{-\frac{4\phi_E}{D-2}} g_{\mu\nu}$$

Einstein Frame: 
$$S = \frac{1}{2\kappa_E} \int d^D x \sqrt{-g_E} \left( R_E - \frac{4}{D-2} \nabla_\rho \phi_E \nabla^\rho \phi_E - \frac{1}{12} e^{-\frac{8\phi_E}{D-2}} H_{\rho\sigma\lambda} H^{\rho\sigma\lambda} \right)$$

# String vs. Einstein Frame: Hubble Parameter

## String Frame

$$H^2 = \frac{1}{3} \epsilon e^{2\phi} - \frac{k}{a^2} + \frac{h^2}{12a^6} + \frac{1}{6} T_{(0)} - 2H \left( \frac{1}{N} \frac{d\phi}{dt} \right) - \frac{2}{3} \left( \frac{1}{N} \frac{d\phi}{dt} \right)^2$$

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## Einstein Frame

$$N_E = N e^{-\phi}$$

$$a_E = a e^{-\phi}$$

$$H_E^2 = \frac{1}{3} \epsilon e^{4\phi_E} - \frac{k}{a_E^2} + \frac{h^2 e^{-4\phi_E}}{12a_E^6} + \frac{1}{6} T_{(0)} e^{2\phi_E} + \frac{1}{3} \left( \frac{1}{N_E} \frac{d\phi_E}{dt} \right)^2$$

# String vs. Einstein Frame: Deceleration Parameter

## String Frame

$$q = -\frac{1}{H^2 a} \left( \frac{d}{N dt} \right)^2 \quad a = - \left( 1 + 2\Omega_k + 3w\Omega_\epsilon + 6\Omega_h \pm \sqrt{3 + 6(\Omega_k + \Omega_\epsilon + \Omega_\Lambda + \Omega_h)} \right)$$

$(\Omega_k + \Omega_\epsilon + \Omega_\Lambda + \Omega_h \sim 1)$

$$q_0 \simeq 1 - \Omega_k - 5\Omega_h + \frac{\Lambda}{3H_0^2} + \sum_w (1 - 3w)\Omega_{\epsilon_0^w}$$

➡ Easy to Realize Acceleration  $q_0 < 0$ :  $\Omega_k > 1$  or Negative  $\Lambda$

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## Einstein Frame

$$q_E = -\frac{1}{H_E^2 a_E} \left( \frac{d}{N_E dt} \right)^2 \quad a_E = \frac{1}{6H_E^2} \left[ \left( \frac{2}{N_E} \frac{d\phi_E}{dt} \right)^2 + (\epsilon + 3p)e^{4\phi_E} - 2\Lambda e^{2\phi_E} + \frac{h^2}{a_E^6} e^{-4\phi_E} \right]$$

For  $q_E < 0$ , it is necessary {

- Violate Strong Energy Condition ( $\epsilon + 3p < 0$ )
- Positive  $\Lambda$

# String vs. Einstein Frame: Equivalence Principle

## String Frame

$$S = \int d\tau \sqrt{-g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} \quad \rightarrow \quad S = \int d\tau e^\phi \sqrt{-g_{\mu\nu}^E \dot{x}^\mu \dot{x}^\nu}$$

String to Einstein

$$\ddot{x}^\mu + \Gamma_{\rho\sigma}^\mu \dot{x}^\rho \dot{x}^\sigma = 0 \quad \rightarrow \quad \ddot{x}^\mu + (\Gamma_E)_{\rho\sigma}^\mu \dot{x}^\rho \dot{x}^\sigma \propto \dot{\phi}$$

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## Einstein Frame

$$S = \int d\tau \sqrt{-g_{\mu\nu}^E \dot{x}^\mu \dot{x}^\nu} \quad \rightarrow \quad S = \int d\tau e^{-\phi} \sqrt{-g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu}$$

Einstein to String

$$\ddot{x}^\mu + (\Gamma_E)_{\rho\sigma}^\mu \dot{x}^\rho \dot{x}^\sigma = 0 \quad \rightarrow \quad \ddot{x}^\mu + \Gamma_{\rho\sigma}^\mu \dot{x}^\rho \dot{x}^\sigma \propto \dot{\phi}$$

# String vs. Einstein Frame: Equivalence Principle

## String Frame

$$S = \int d\tau \sqrt{-g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu} \quad \Rightarrow \quad \ddot{x}^\mu + \Gamma_{\rho\sigma}^\mu \dot{x}^\rho \dot{x}^\sigma = 0$$

- In general, one can choose any frame.
- However, in **Double Field Theory**, the  $O(D, D)$  Symmetry leads to **minimal coupling** for point particle. It predicts that the **Equivalence Principle** holds in the **String Frame** rather than in the Einstein Frame.
- Therefore, the **String Frame** becomes the natural choice in **Double Field Theory**.

- **Type Ia Supernova**

*\*D. Scolnic et al., Astrophys. J. 938 (2022)*

*\*A. G. Riess et al., Astrophys. J. Lett. 934 (2022)*

- **Type Ia Supernovae with Pantheon+ Analysis**
- **1583 Data Points ( $0.01 \leq z \leq 2.26$ )**

- **Absorption Spectra of Quasar**

*\*J. A. King et al, Mon. Not. R. Astron. Soc. 422 (2012)*

*\*M. R. Wilczynska et al, Mon. Not. R. Astron. Soc. 454 (2015)*

*\*C. J. A. P. Martins et al, Phys. Rev. D 95 (2017)*

*\*M. R. Wilczynska et al, Sci. Adv. 6 (2020)*

- **Measurement of Fine-Structure Constant,  $\alpha$**
- **199 Data Points ( $0.22 \leq z \leq 7.06$ )**

$$e^{2\phi} = \frac{\alpha_{eff}(z)}{\alpha_{eff}(0)}$$

# Bayesian Inference

- **Ensemble Sampler for Markov Chain Monte Carlo (MCMC): EMCEE**  
*\*D. Foreman-Mackey et al, Publ. Astron. Soc. Pac. 125 (2013)*
- **100 Walkers,  $10^6$  Steps with Supercomputer (KISTI)**  
*\*Total 100 Million Steps* *\*한국과학기술정보연구원*
- **Discard 5% of Initial Steps as Burn-In Phase**

**Theory:**

**Double Field Theory**

**Equation:**

**Evolution of  $\phi$  &  $H$**

**Parameters:**

$H_0$     $\Omega_k$     $\Omega_\epsilon$     $\Omega_\Lambda$     $\Omega_h$

**Data:**

1583 Data Points of  $\mu$  (Supernova)  
199 Data Points of  $\phi$  (Quasar)

**Method:**

**Bayesian Inference: EMCEE**

# Results: FIG. 1

$$H_0 = 71.22$$

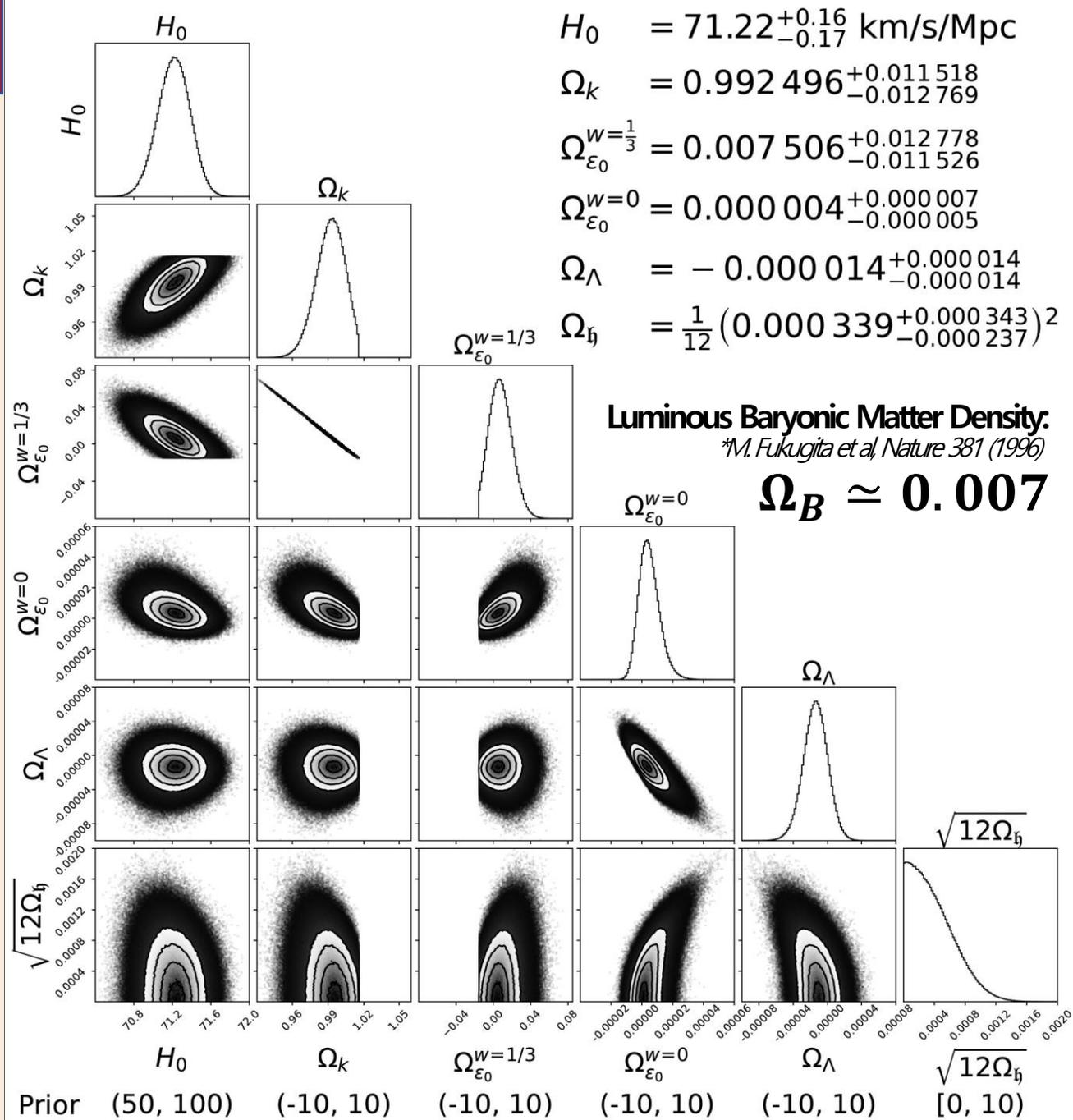
$$\Omega_k = 0.9925$$

$$\Omega_{\epsilon}^{w=1/3} = 0.0075$$

$$\Omega_{\epsilon}^{w=0} \sim 10^{-6}$$

$$\Omega_{\Lambda} \sim -10^{-6}$$

$$\Omega_h \sim 10^{-8}$$



# Results: FIG. 2

$$H_0 = 71.29$$

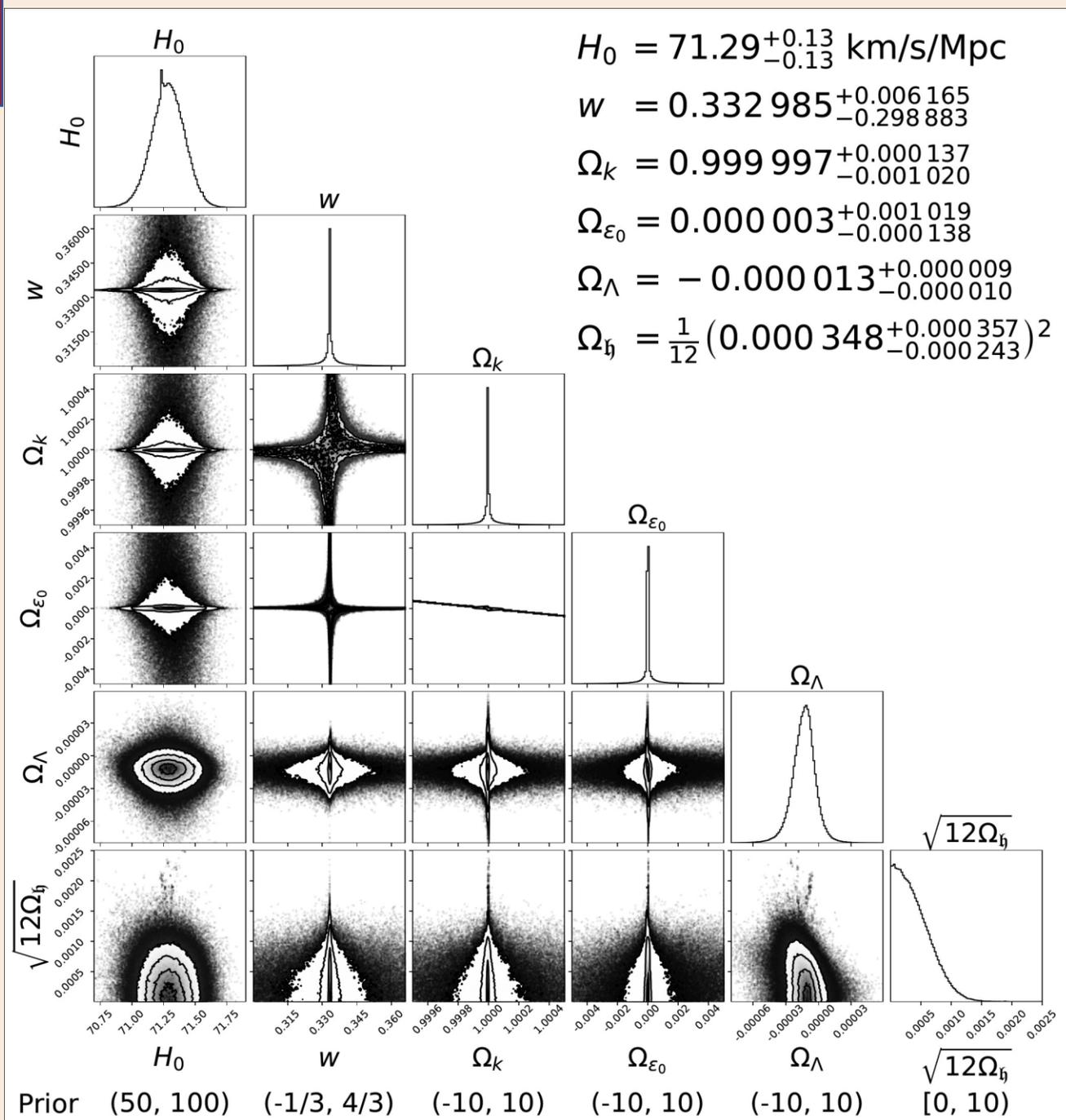
$$\Omega_k = 0.9999$$

$$w = 0.3329$$

$$\Omega_{\epsilon_0} \sim 10^{-6}$$

$$\Omega_\Lambda \sim -10^{-5}$$

$$\Omega_h \sim 10^{-8}$$



# Results: FIG. 3 (Vacuum)

$$H_0 = 71.29$$

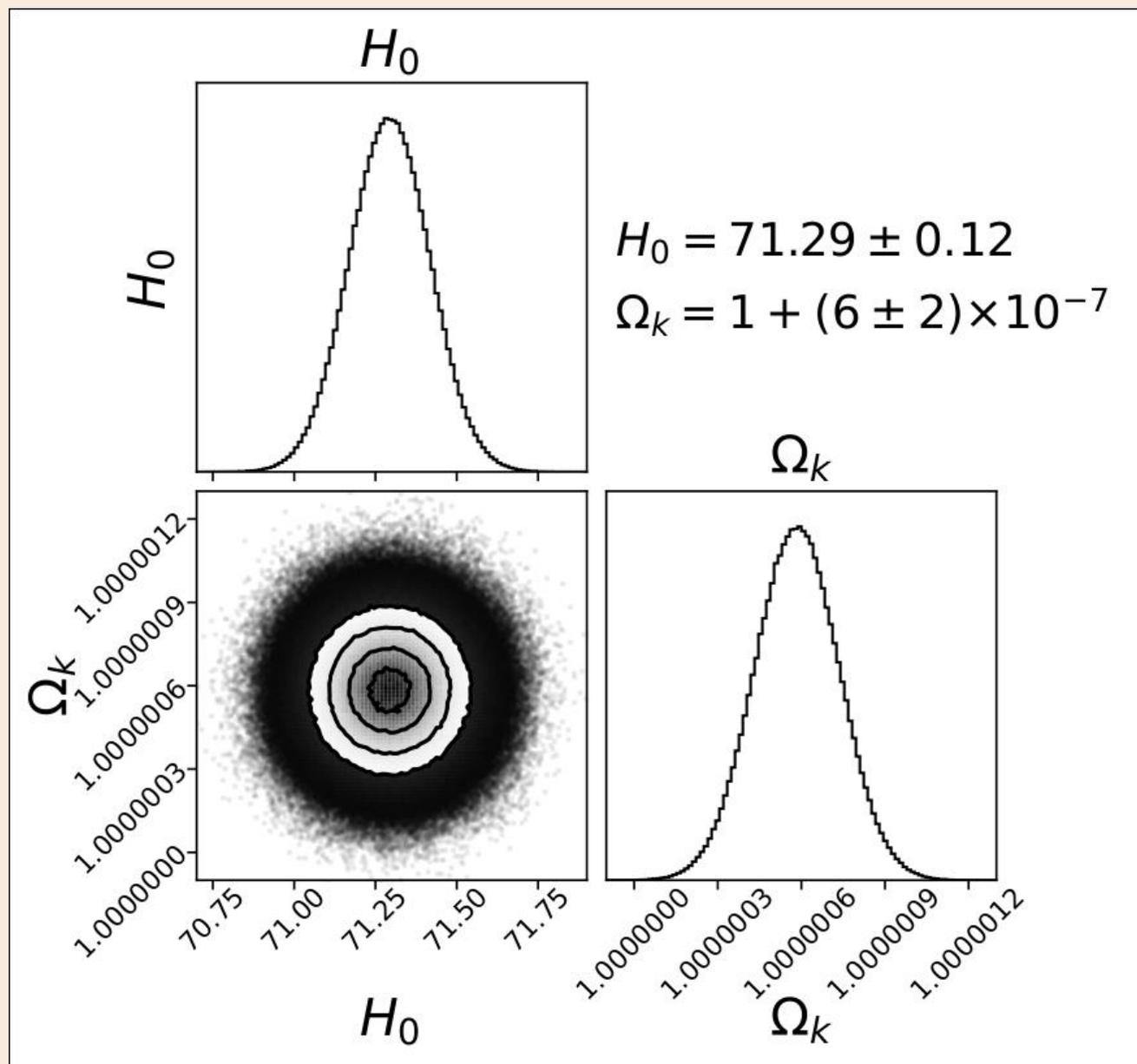
$$\Omega_k = 1 + 6 \times 10^{-7}$$

$$\Omega_{\epsilon}^{w=1/3} = 0$$

$$\Omega_{\epsilon}^{w=0} = 0$$

$$\Omega_{\Lambda} = 0$$

$$\Omega_h = 0$$



# Results: Reduced Chi-Square

$$\chi^2_{\nu}$$

$$\chi^2_{\nu} = \frac{\chi^2}{N_D - N_p}$$

$N_D$ : # of Data  
 $N_p$ : # of Parameters

**Fig. 1: 1.32**

**$\chi^2_{\nu} \gg 1$ : Poor Fit**

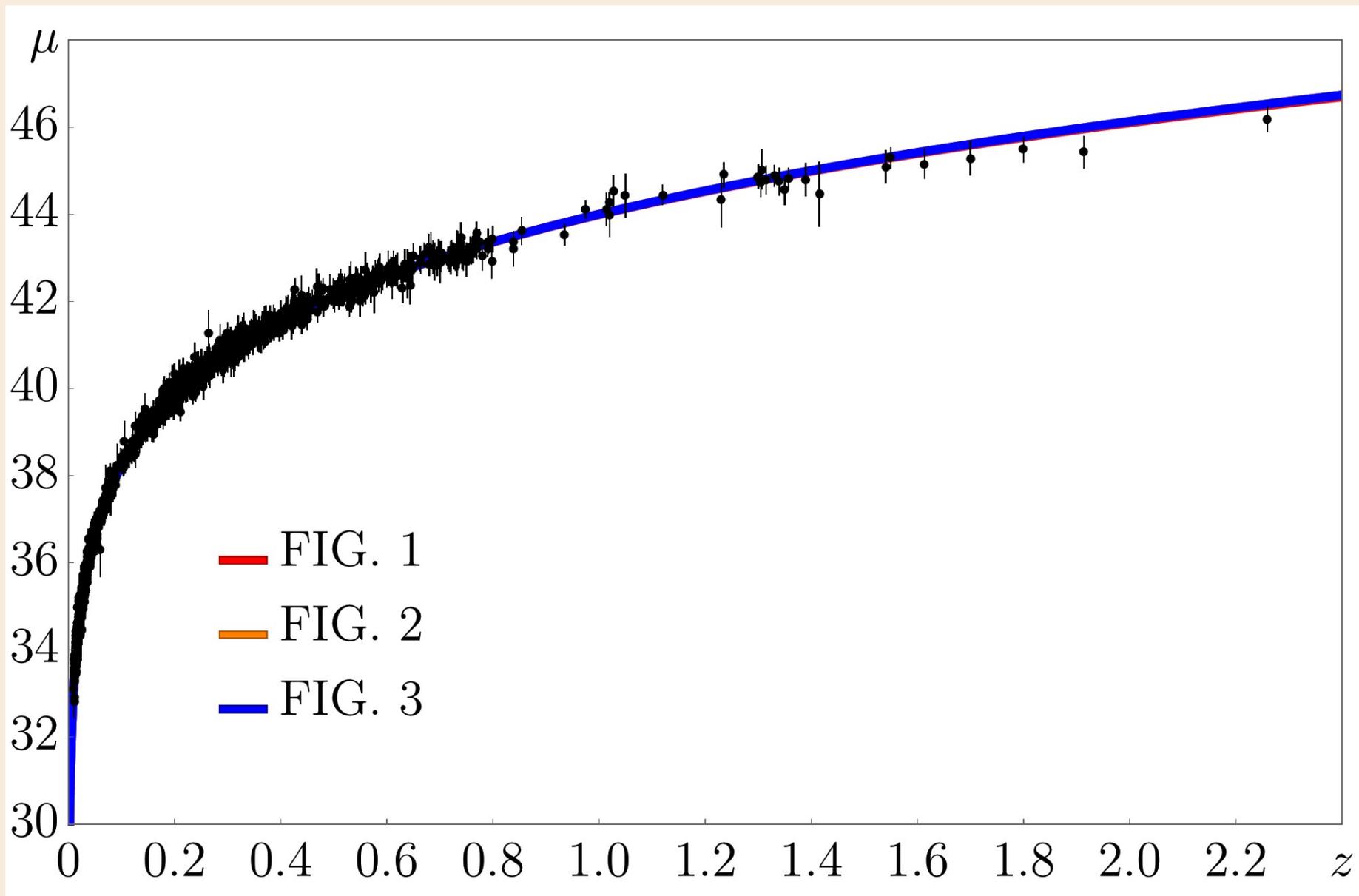
**Fig. 2: 1.35**

**$\chi^2_{\nu} \sim 1$ : Good**

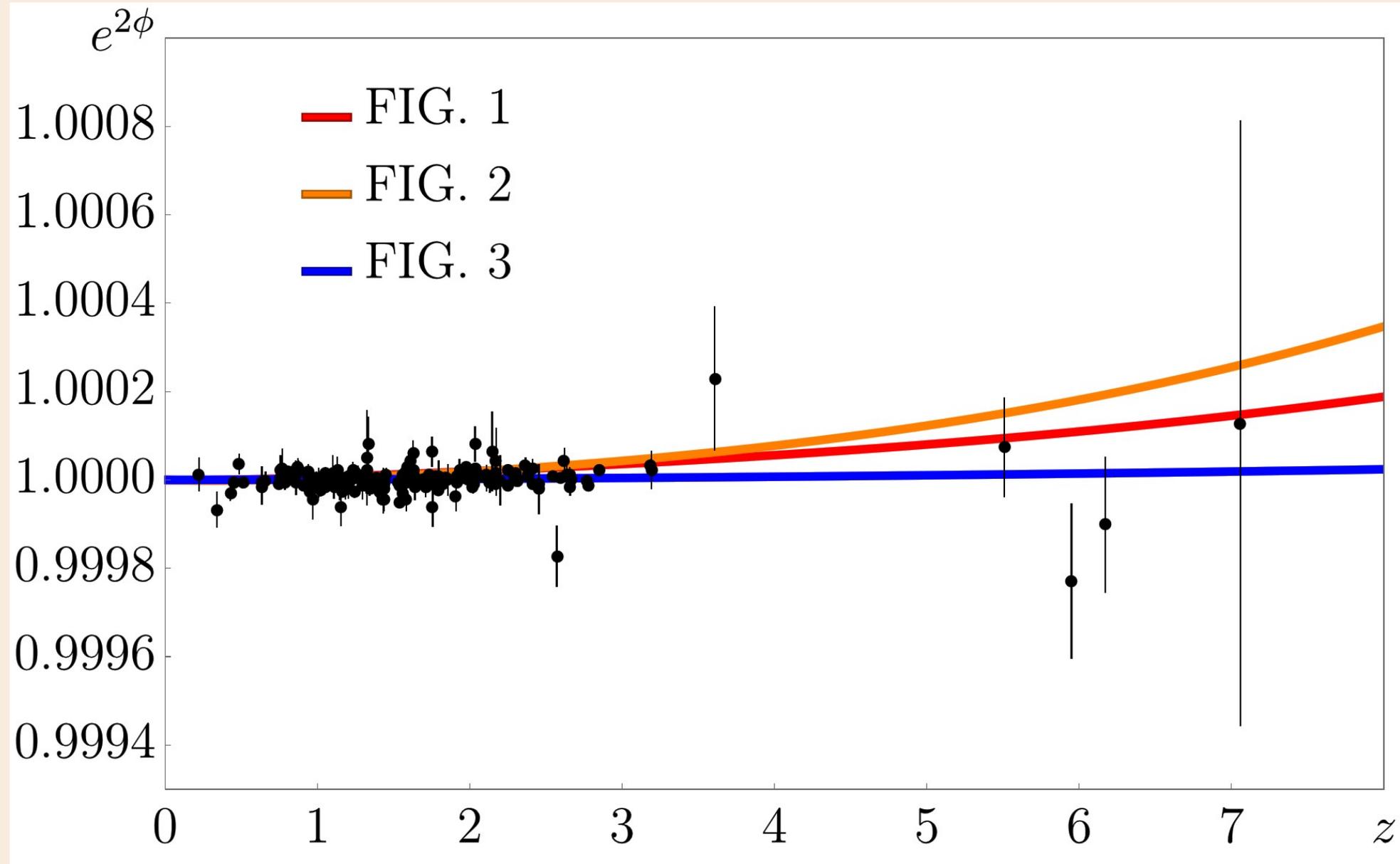
**Fig. 3: 1.02**

**$\chi^2_{\nu} < 1$ : Over-Fitted**

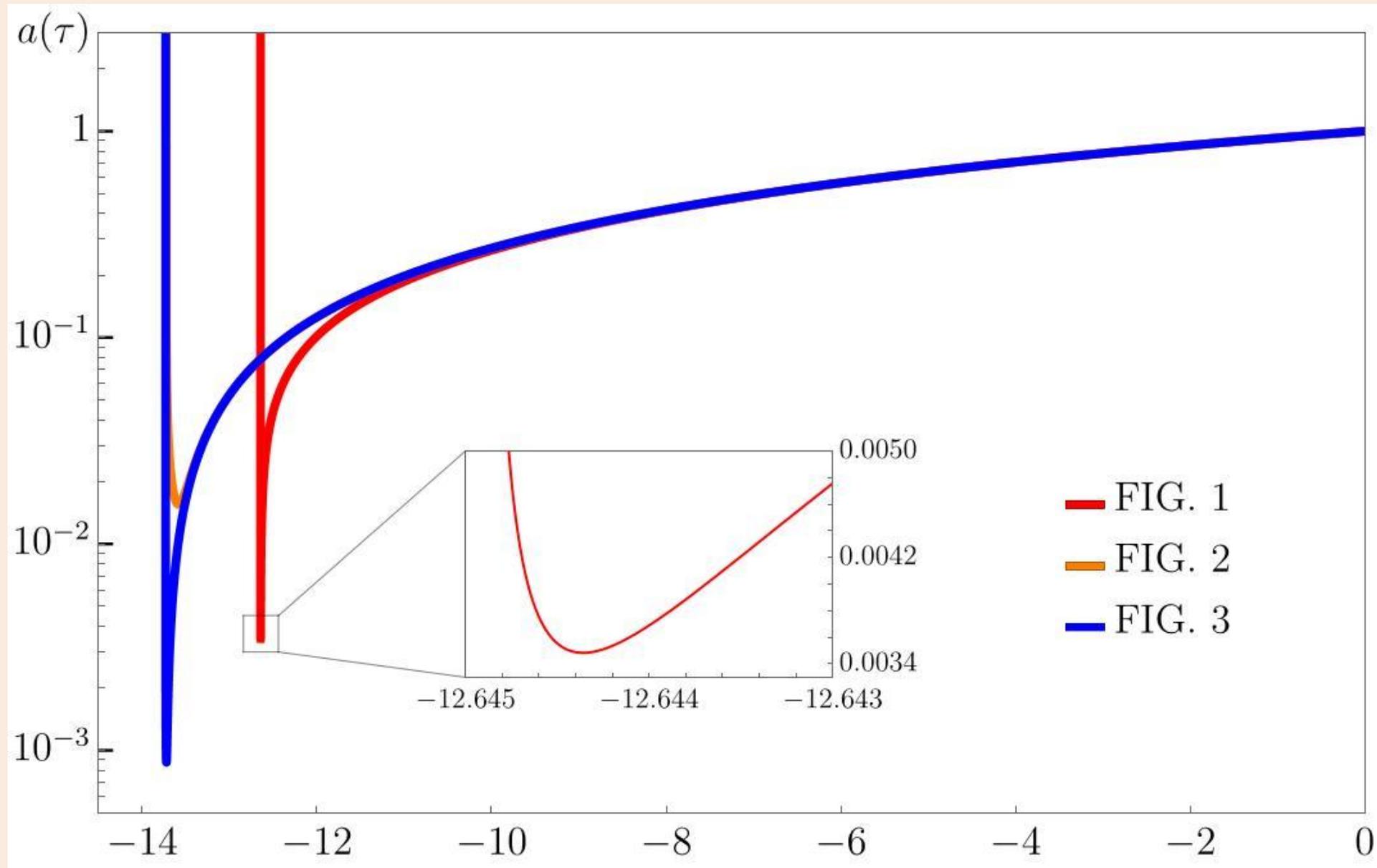
# Results: $z$ vs. $\mu$ (Distance Modulus)



# Results: $z$ vs. $e^{2\phi}$ (Fine-Structure Constant)



# Results: Gyr. vs. Scale Factor (Bouncing Universe)



# Analytic Vacuum Solution

\*E. J. Copeland, A. Lahiri, D. Wands, *Phys. Rev. D* 50 (1994)

\*S. Angus, K. Cho, G. Franzmann, S. Mukohyama, J. H. Park, *Eur. Phys. J. C* 80 (2020)

$$ds^2 = a(\eta)^2 \left[ -d\eta^2 + \frac{1}{\left(1 - \frac{r^2}{4\ell^2}\right)^2} (dr^2 + r^2 d\Omega^2) \right]$$

$$a(\eta)^2 = a_0^2 e^{2(\phi - \phi_0)} \frac{\sinh(2(\eta - \eta_0)/\ell + \zeta)}{\sinh\zeta}$$

$$e^{2\phi(\eta)} = \frac{1}{2} \left( 1 + \sigma \sqrt{1 - \frac{(h\ell \sinh\zeta)^2}{12}} \right) \left( \frac{\tanh\left(\frac{\eta - \eta_0}{\ell} + \frac{\zeta}{2}\right)}{\tanh\frac{\zeta}{2}} \right)^{\sqrt{3}} + \dots$$

# Fate of the Dilaton Field

**Closed Universe:**  $\lim_{\eta \rightarrow \infty} e^{2(\phi - \phi_0)} = \textit{diverge}$

**Flat Universe:**  $\lim_{\eta \rightarrow \infty} e^{2(\phi - \phi_0)} = \textit{linear}$

**Open Universe:**  $\lim_{\eta \rightarrow \infty} e^{2(\phi - \phi_0)} = \textit{converge}$   
➡ ***Stable***

# Summary & Discussion

- The stringy gravity theory of closed string massless sector is consistent with the observation of late-time Universe.
- Our results favor an open Universe without dark sector. Acceleration is natural in string frame.
- $H_0 = 71.22 \text{ km/s/Mpc}$ ,  $\Omega_k = 1 + 6 \times 10^{-7}$
- This is not sufficient to reconstruct the entire history of Universe. The early-time Universe remains an open question for further investigation.