Modern Computational Approaches to Early Universe Modeling

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2025 CAU-IBS Beyond the Standard Model Workshop
Chung-Ang University
20 Feb 2025

CAU ibS Institute for Basic Science

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O. Lebedev, T. Solomko, and **J.-H. Yoon, "Dark matter production via a non-minimal coupling to gravity,"**JCAP, vol. 02, p. 035, 2023.

M.A.G. Garcia, M. Gross, Y. Mambrini, K.A. Olive, M. Pierre and**J.-H. Yoon, Effects of fragmentation on post-inflationary reheating,**JCAP 12 (2023) 028 [2308.16231].

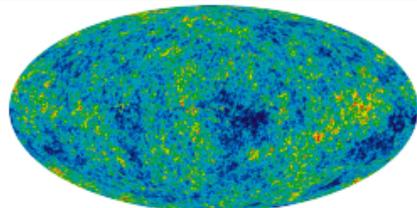**JY**, S.Clery, M.Gross, Y.Mambrini**, "Preheating with deep learning,"** JCAP, vol. 08, p. 031, 2024. [arXiv:hep-ph/2405.08901]
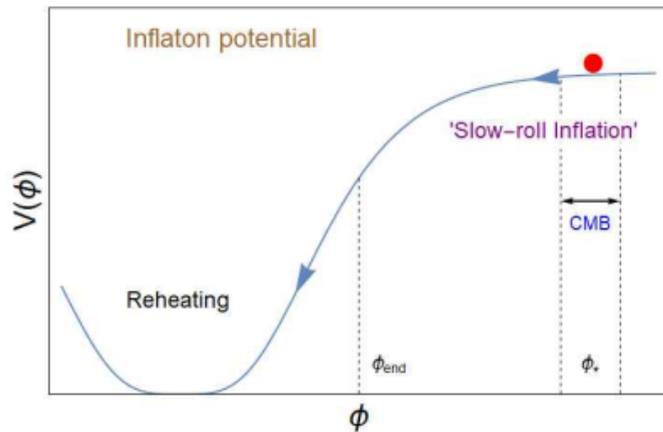
Minimal Cosmological Models in Inflationary Universe

- Inflationary epoch (< 10^-32 s)
- Solution to Horizon & Flatness problem

Nine-year Wilkinson Microwave Anisotropy Probe heat map of temperature fluctuations in the CMB

- Inflaton?
- Real scalar field
- Homogeneity & Inhomogeneity

→ Inflationary Cosmology

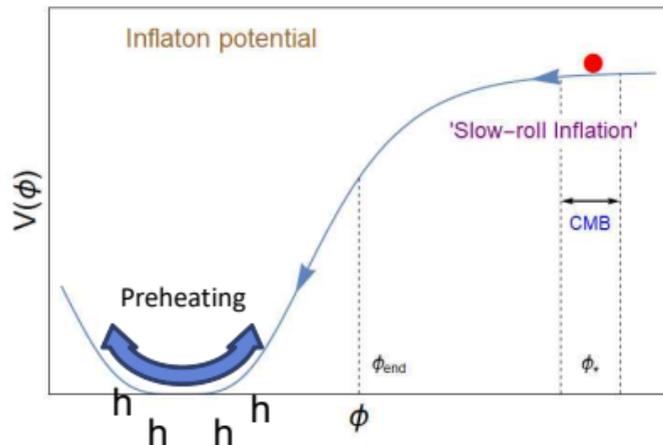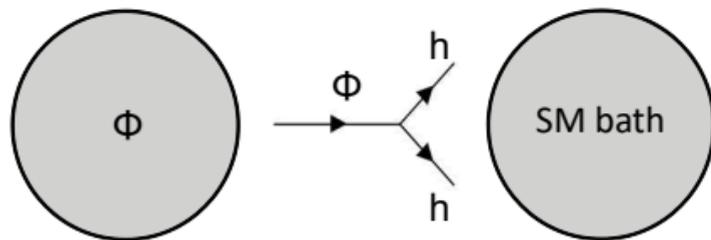(1970~1980s, Alexei Starobinsky, Alan Guth, Paul Steinhardt, and Andrei Linde)

Inflaton potential

'Slow–roll Inflation'

CMB

Reheating

ϕ_{end} ϕ_*

$V(\phi)$

ϕ

Minimal Cosmological Models in Inflationary Universe

- Reheating (Inflaton → SM bath)
(little known: a few MeV < T_R < 10^13 GeV)

- Simplest reheating model
Inflaton quanta → Higgs

- However, the inflaton field oscillates around the minimum of the potential with large field values
→ Turbulent/non-pert. effects
→ Preheating

Minimal Cosmological Models in Inflationary Universe

- While Inflaton → SM (reheating the universe) in the long run,

- DM is produced during preheating:

Inflaton=DM
Inflaton-DM scattering
Inflaton F.O., decay to DM
Inflaton-DM non-renormalizable couplings
Inflaton-DM via gravity

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Minimal Cosmological Models in Inflationary Universe

- Inflaton-DM via gravity

- Non-minimal coupling to gravity

ξ: coefficient
R: Ricci scalar
Φ: Inflaton field
s: scalar DM

$$\mathcal{S} = \int d^4x \sqrt{-g} \left(\frac{1}{2} M_{\mathrm{Pl}}^2 R - \frac{1}{2}\xi R\, s^2 - \frac{1}{2} g^{\mu\nu} \partial_\mu s\, \partial_\nu s - \frac{1}{2} g^{\mu\nu} \partial_\mu \phi\, \partial_\nu \phi - V \right)$$

- R is effectively dominated by Φ, so DM can interact with Φ via

$$R = -\frac{1}{M_{\mathrm{Pl}}^2} T^\mu_\mu$$

O. Lebedev, T. Solomko, and **J.-H. Yoon**, **"Dark matter production via a non-minimal coupling to gravity," JCAP, vol. 02, p. 035, 2023.**
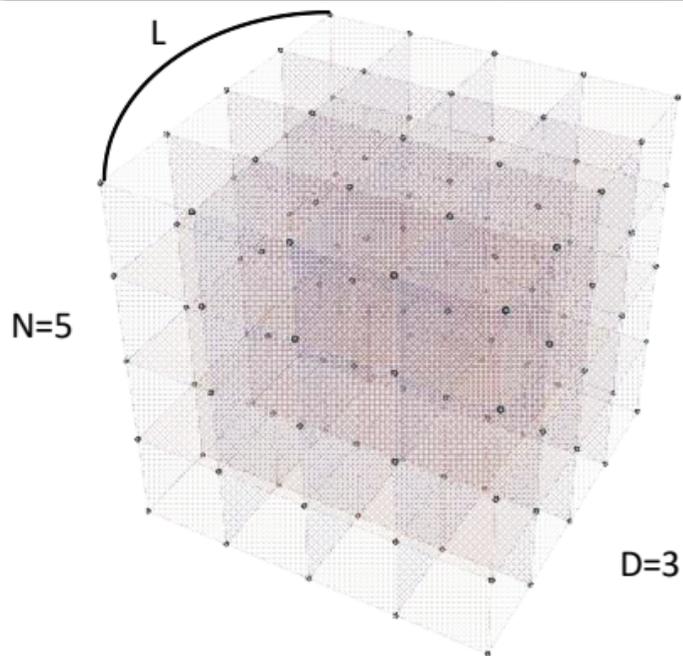
Minimal Cosmological Models in Inflationary Universe

- Equation of motion in momentum space

$$\ddot{Y}_k + \left(k^2 + \xi R a^2 - \frac{\ddot{a}}{a} \right) Y_k = 0$$

$$Y_k \equiv a\, s_k \qquad \text{a: scale factor}$$
$$dt = a\, d\tau \qquad \text{k: comoving momentum}$$

- Analytic Methods
- Boundary Matching, Stokes Phenomenon, etc.
- Resonance Structures (Parametric, Tachyonic, etc.)

- For large ξ, we treat the system semi-classically and solve it numerically to take into account non-perturbative effects

Numerical Approaches with High-Performance Computing

L

N=5

D=3

$$k_{min} = \frac{2\pi}{L} \qquad k_{max} = k_{min} \times \frac{\sqrt{D}}{2}N$$

- Equations of Motion for Particle Production

$$\ddot{f} + 3\frac{\dot{a}}{a}\dot{f} - \frac{1}{a^2}\nabla^2 f + \frac{\partial V}{\partial f} = 0$$

$$\ddot{a} = -\frac{4\pi a}{3}(\rho + 3p)$$

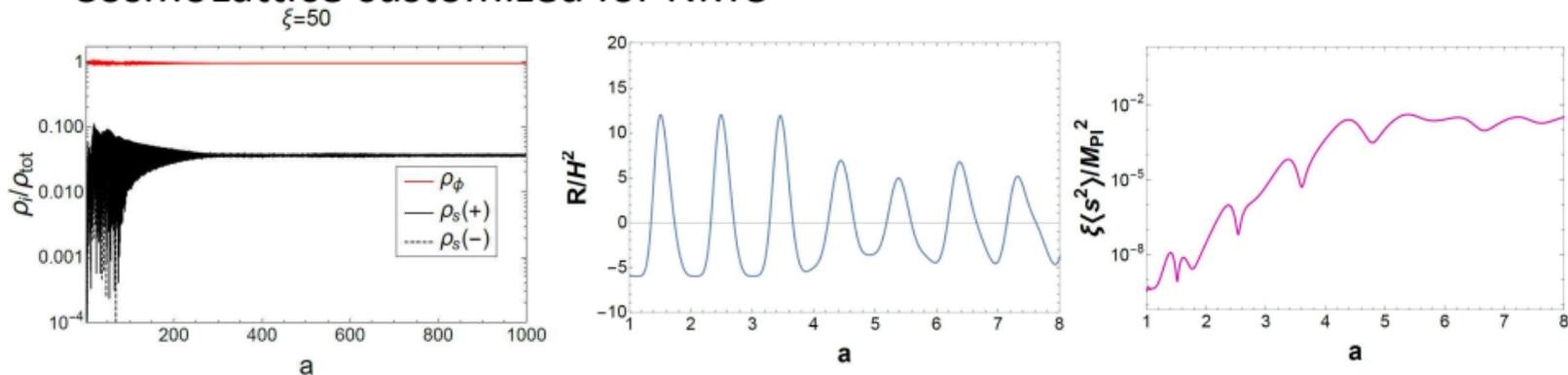$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi}{3}\rho$$

$$\rho = T + G + V \; ; \; p = T - \frac{1}{3}G - V$$

$$T = \frac{1}{2}\dot{f}^2 \; ; \; G = \frac{1}{2a^2}|\nabla f|^2 .$$

Numerical Approaches with High-Performance Computing

Hardware (ruche cluster at Paris-Saclay University)

Software (CosmoLattice)

Numerical Approaches with High-Performance Computing

- CosmoLattice customized for NMC

- Energy distribution, R breakdown, resonant production, etc.

- Simulations provide intuitive insights into events in the early universe

Numerical Approaches with High-Performance Computing

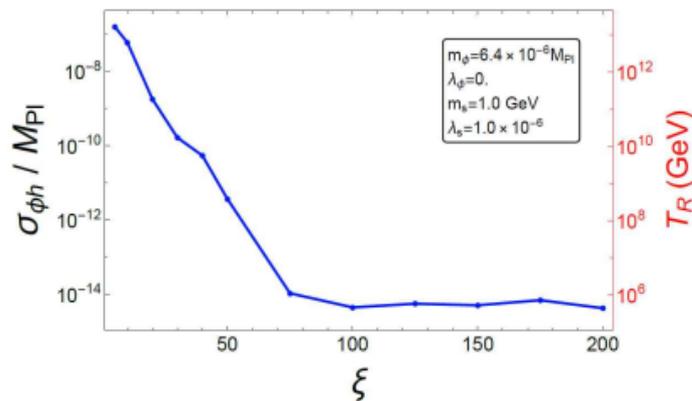- DM relic abundance (conserved since reheating)

$$Y = \frac{n}{s_{\text{SM}}} \quad , \quad s_{\text{SM}} = \frac{2\pi^2}{45} g_{*s} T^3$$

$$Y_\infty = 4.4 \times 10^{-10} \left(\frac{\text{GeV}}{m_s} \right)$$

- Reheating via inflaton decay into Higgs

$$H_R \simeq \Gamma_{\phi \to hh} \quad , \quad \Gamma_{\phi \to hh} = \frac{\sigma_{\phi h}^2}{8\pi m_\phi} \qquad H_R = \sqrt{\frac{\pi^2 g_*}{90}} \frac{T_R^2}{M_{\text{Pl}}}$$

T_R: Reheating temperature

- Early DM production can explain the relic abundance today

Numerical Approaches with High-Performance Computing

- Perturbative Reheating
- Let us consider a simple inflaton potential and reheating channels

$$V(\phi) = \lambda M_P^4 \left(\frac{\phi}{M_P} \right)^k, \qquad \phi \ll M_P$$

$$\mathcal{L} \supset \begin{cases} y\phi\bar{f}f & \phi \to \bar{f}f \\ \mu\phi bb & \phi \to bb \\ \sigma\phi^2 b^2 & \phi\phi \to bb \end{cases}$$

- One may derive the time for reheating when $\rho_\phi(a_{\mathrm{RH}}) = \rho_R(a_{\mathrm{RH}})$

M.A.G. Garcia, M. Gross, Y. Mambrini, K.A. Olive, M. Pierre and **J.-H. Yoon,**
Effects of fragmentation on post-inflationary reheating, JCAP 12 (2023) 028 [2308.16231]

Numerical Approaches with High-Performance Computing

- During inflation, the energy of the universe is primarily stored in the homogeneous part (condensate) of the inflaton, while the inhomogeneous part (fluctuations) is negligible

$$\phi(t, \vec{x}) = \bar{\phi} + \delta\phi(t, \vec{x})$$

$$\rho_{\delta\phi} \ll \rho_{\bar{\phi}}$$

Numerical Approaches with High-Performance Computing

- Under such assumptions, we can work with the oscillations of the inflaton and determine the corresponding decay rate

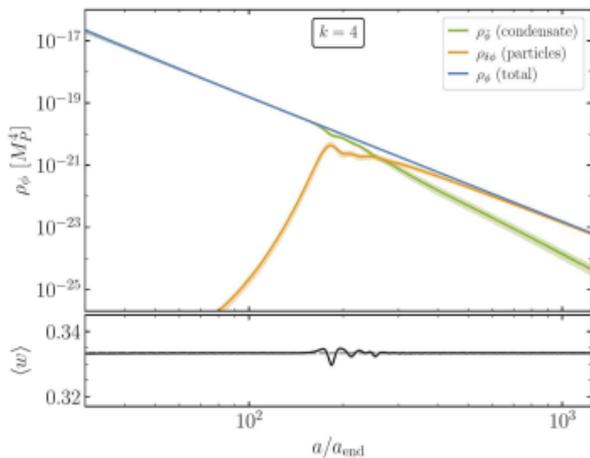- It allows us to proceed with analytical calculations for reheating

$$\Gamma_\phi = \gamma_\phi \left(\frac{\rho_\phi}{M_P^4} \right)^l \quad \longrightarrow \quad \frac{a_{\rm RH}}{a_{\rm end}} = \left[\frac{k+8-6kl}{2k} \frac{M_P^{4l-1} \rho_{\rm end}^{\frac{1}{2}-l}}{\sqrt{3}\gamma_\phi} \right]^{\frac{k+2}{3k-6kl}}$$

$$\rho_\phi(a_{\rm RH}) = \rho_R(a_{\rm RH})$$

$$\gamma_\phi = \begin{cases} \sqrt{k(k-1)}\lambda^{1/k} M_P \frac{y_{\rm eff}^2}{8\pi}, & \phi \to \bar{f}f, \\ \frac{\mu_{\rm eff}^2}{8\pi\sqrt{k(k-1)}\lambda^{1/k} M_P}, & \phi \to bb, \\ \frac{\sigma_{\rm eff}^2 M_P}{8\pi[k(k-1)]^{3/2}\lambda^{3/k}}, & \phi\phi \to bb \end{cases} \qquad l = \begin{cases} \frac{1}{2} - \frac{1}{k}, & \phi \to \bar{f}f, \\ \frac{1}{k} - \frac{1}{2}, & \phi \to bb, \\ \frac{3}{k} - \frac{1}{2}, & \phi\phi \to bb \end{cases}$$

Numerical Approaches with High-Performance Computing

- However, Inflaton breaks on its own
= Inflaton's self-interaction fragments its background
→ Energy transfer from condensate to particles

Numerical Approaches with High-Performance Computing

- Event order matters

- Early perturbative reheating
$\rightarrow$ Conventional analytic estimates hold

- Early fragmentation
$\rightarrow$ Reconsider Reheating via decay of inflaton 'quanta' to SM bath

Numerical Approaches with High-Performance Computing

- Reheating via fermionic decays $y\phi\bar{f}f$

- $y \propto T_{RH}$

- Small y →

Fragmentation precedes perturbative reheating

→ Delayed reheating

→ Excluded by BBN

M.A.G. Garcia, M. Gross, Y. Mambrini, K.A. Olive, M. Pierre and **J.-H. Yoon, Effects of fragmentation on post-inflationary reheating,** JCAP 12 (2023) 028 [2308.16231].

Numerical Approaches with High-Performance Computing

- Intense Particle Production → GWB production

- BSM in the early universe: Sterile neutrino, Axion Inflation, Dark energy, Quantum gravity, etc.

- Thermalization

Beyond Lattice Simulations: Integrating Deep Learning

The Nobel Prize in Physics 2024
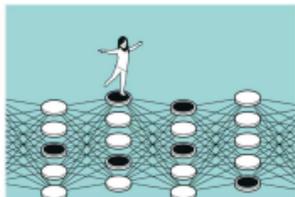
They used physics to find patterns in information

Machine learning has long been important for research, including the sorting and analysis of vast amounts of data. John Hopfield and Geoffrey Hinton used tools from physics to construct methods that helped lay the foundation for today's powerful machine learning. Machine learning based on artificial neural networks is currently revolutionising science, engineering and daily life.

Related articles
Press release
Popular information: They used physics to find patterns in information
Scientific background "for foundational discoveries and inventions that enable machine learning with artificial neural networks"

Nobel Prize in Physics

The 2024 physics laureates

The Nobel Prize in Physics 2024 was awarded to John J. Hopfield and Geoffrey E. Hinton "for foundational discoveries and inventions that enable machine learning with artificial neural networks."

Hopfield created a structure that can store and reconstruct information. Hinton invented a method that can independently discover properties in data and which has become important for the large neural networks now in use.

https://www.nobelprize.org/prizes/physics/

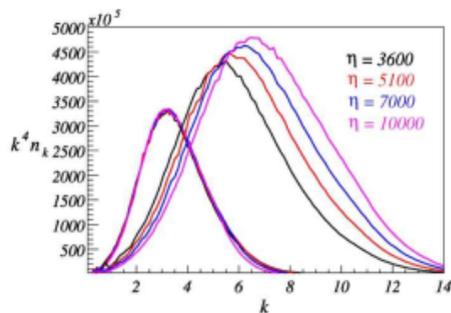- Simulations generate data that can be analyzed by Deep Learning

J.-H. Yoon, S.Clery, M.Gross, Y.Mambrini, **"Preheating with deep learning,"** *JCAP*, vol. 08, p. 031, 2024. [arXiv:hep-ph/2405.08901]

- LatticeQCD, CMB, LHC, DM Exp., etc. wherever we have data

Beyond Lattice Simulations: Integrating Deep Learning

- Late-time preheating dynamics exhibits a universal form:

Self-similar evolution of self- or gauge interacting field

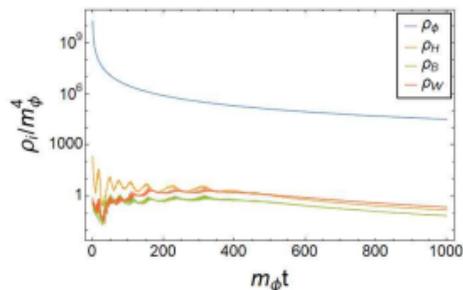→ Implies patterns and trends, which are what DL is all about

Distribution of Φ field in Φ^4 model

R. Micha and I. Tkachev, "Turbulent thermalization," arXiv:hep-ph/0403101

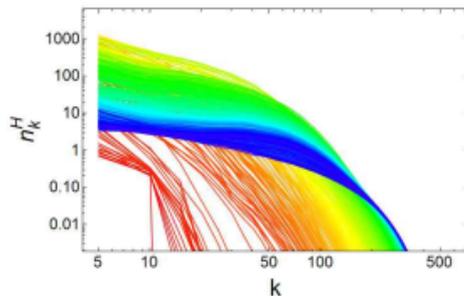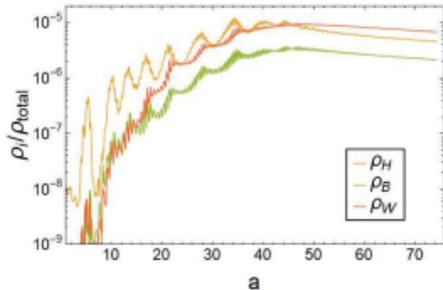# Beyond Lattice Simulations: Integrating Deep Learning

- Implementation
- Preheating model involving Higgs

→ Minimal reheating scenario + self- and gauge interaction

$$\Delta V = \frac{1}{2} m_\phi^2 \phi^2 + \frac{1}{4} \lambda_\phi \phi^4 + \frac{1}{2} \lambda_{\phi h} \phi^2 H^\dagger H + \sigma_{\phi h} \phi H^\dagger H - m_h^2 H^\dagger H + \lambda_h (H^\dagger H)^2$$
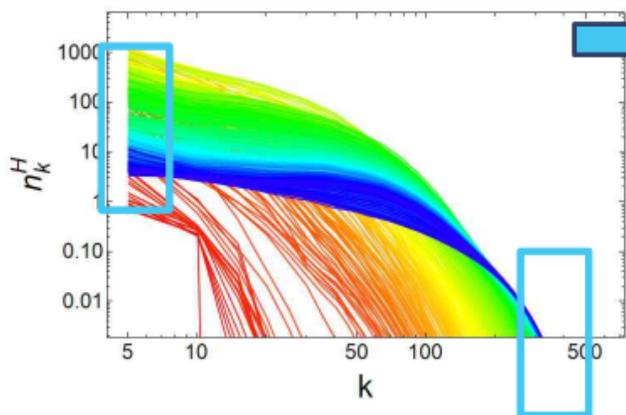
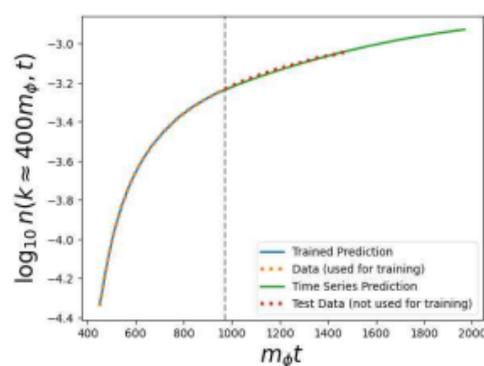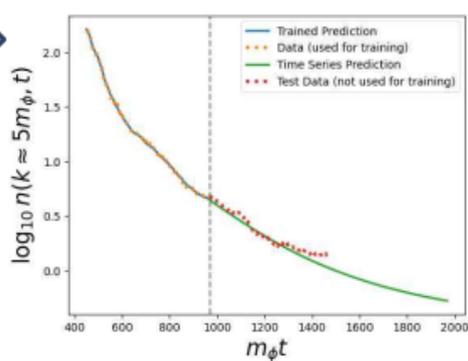

Energy distributions over time/scale factor

Occupation number of Higgs ~ distribution function
(red to blue over time)

Beyond Lattice Simulations: Integrating Deep Learning

- **Training DL model**

15 k-modes used for training the DL model
(only two of them are represented here)

CNN-LSTM time series analysis

Managed to extend simulation outcomes with DL
→ Once trained, the DL model's predictions are almost instantaneous
→ Successfully laid the groundwork for future developments

Conclusions

- We learned about (p)reheating and minimal cosmological models (e.g. DM via gravity)
→ Preheating effects are often unavoidable and require numerical approaches

- Simulating the early universe with HPC is interesting and useful
→ Intuitive insights, GW search, DM production, BSM physics, Reheating, etc.

- Deep learning can be applied to late-time self-similar systems in the early universe
→ It marks the first step toward simulating the entire history of the universe,
from the Big Bang to the present