Spin and symmetry properties of all-charm tetraquarks with CMS

Zhiyuan Huang

Johns Hopkins University

For CMS collaboration

Aug 28, 2025

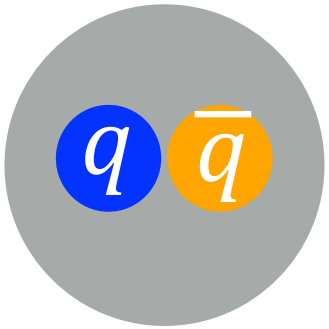
Lepton-Photon2025, Madison

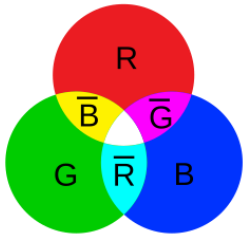
Outline

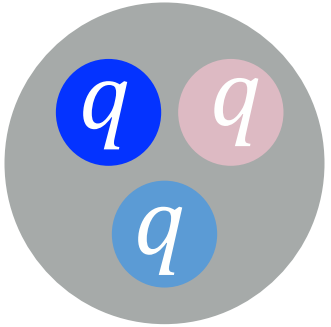
- The Quark Model History
- Motivation
- Polarization in production and decay
- Helicity Amplitudes
- Simulation
- MELA and Combine Tools
- MC simulation and tuning
- Systematics
- Unblind
- Summary

The Quark Model

- > 60 years of classical Quark Model

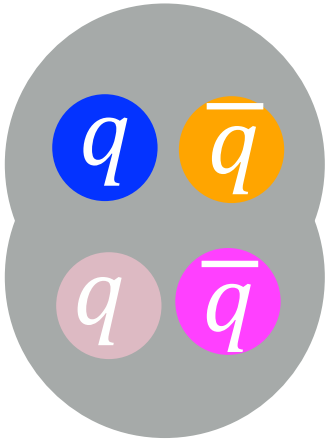
meson 

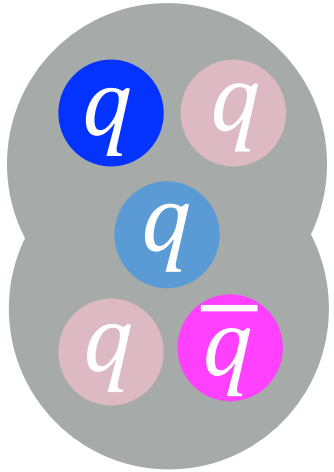
color charge 



baryon

- "exotic" hadrons remained elusive until the XXI century:

tetraquark 

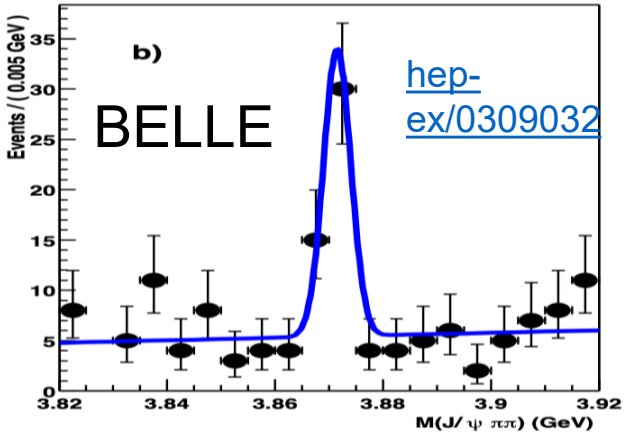


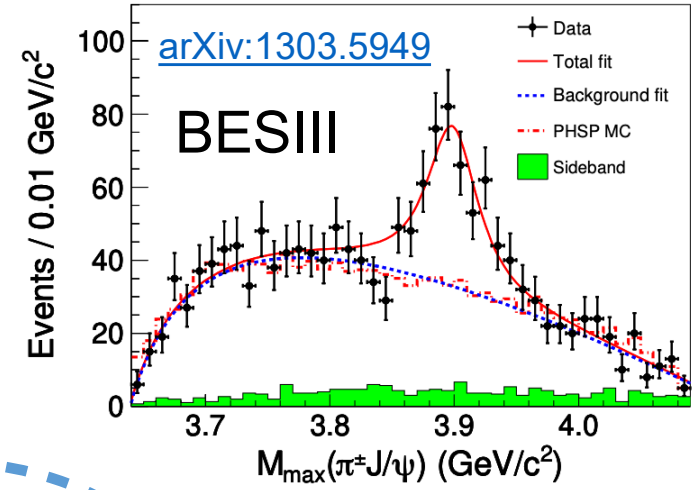
pentaquark

more quarks?

The Tetraquark Candidates

Charm and beauty factories of the XXI century:





$e^+e^- \to \Upsilon(4S) \to B^+B^-$

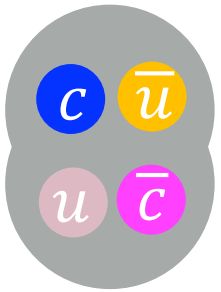
$B^\pm \to X(3872)K^\pm$

$X(3872) \to J/\psi\pi^+\pi^-$

$D\overline{D}^*$ molecule? or compact objects?

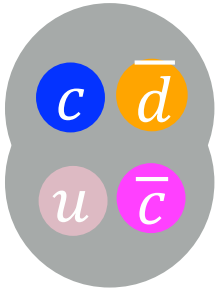
$e^+e^- \to \pi^+\pi^- J/\psi$

$Z(3900)^\pm \to J/\psi\pi^\pm$

$J^{PC} = 1^{++}$ 

+ more candidates

but all heavy + light quarks

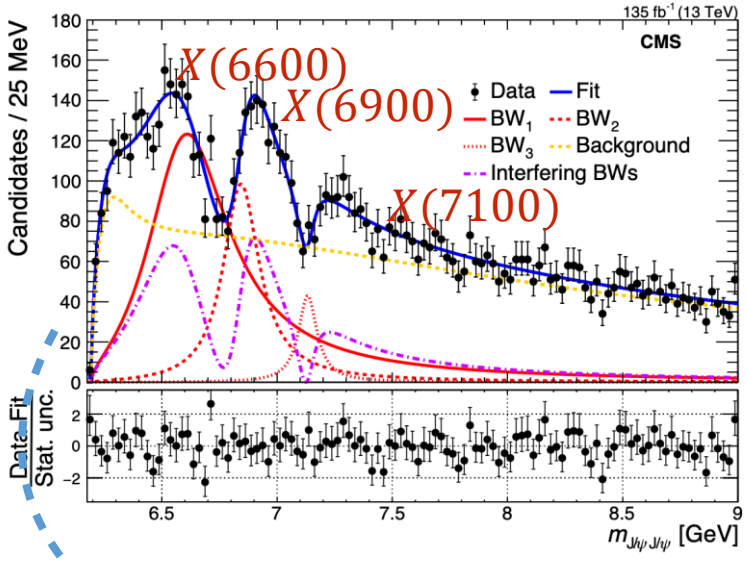


$J^{P(C)} = 1^{+(-)}$

Motivation: CMS Background information

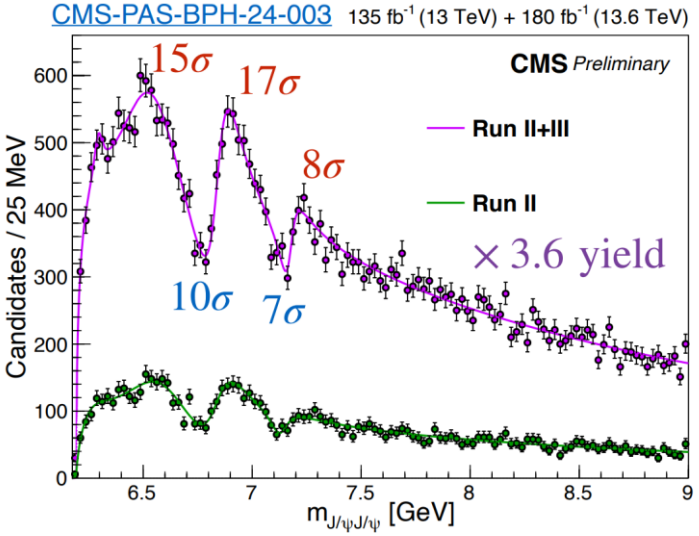
- CMS: found 3 tetraquark candidates in 6.5 - 7.5 GeV range:

published: 2016-18 data BPH-21-003



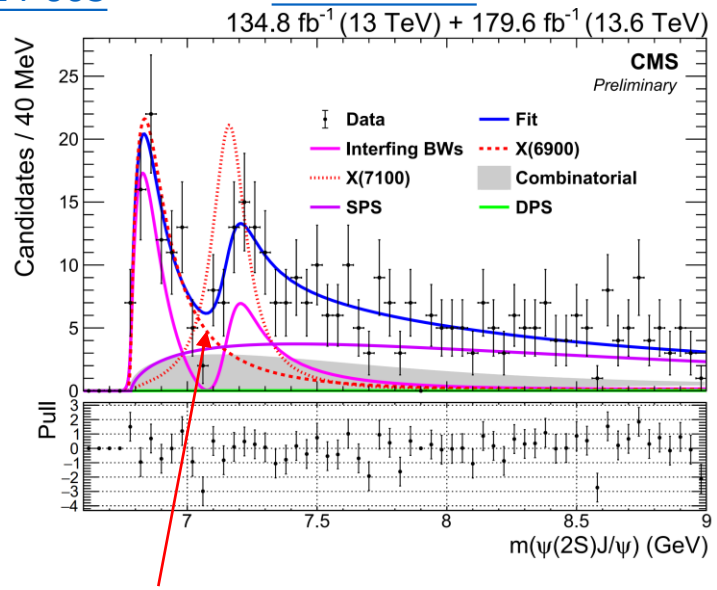
In this talk use only Run2 data

Approved: 2022-24 data BPH-24-003



Dips: > 5σ with Run2+3 data

BPH-22-004

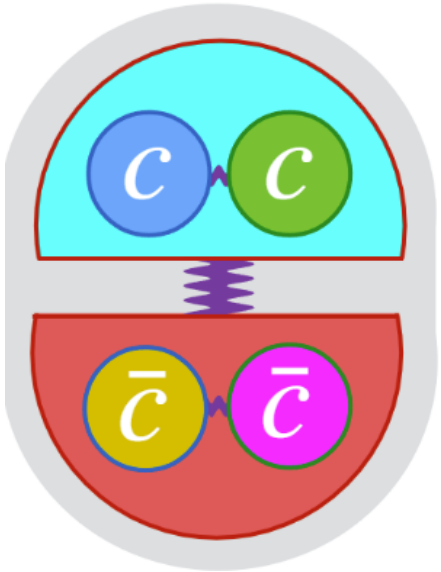


Dip between the peaks and indicates interference

- linear m_n^2 pattern — consistent $n = (1,)2,3,4,..$ radial excitations (Regge trajectory)
- interference pattern — consistent with same J^{PC}
- pattern of Γ_n — also interesting to watch…

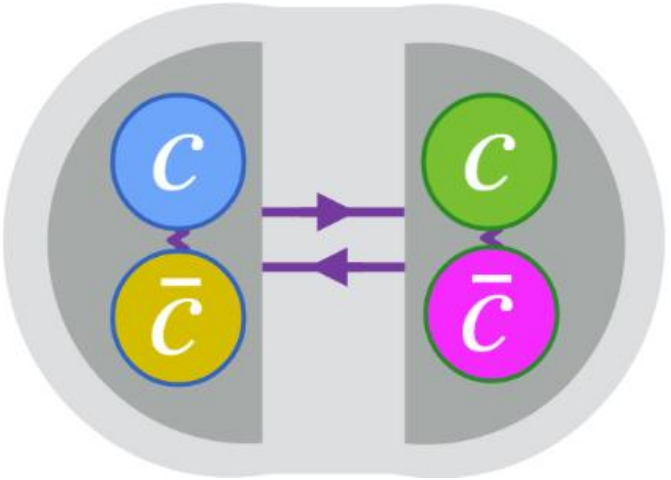
— LHCb and ATLAS: only $X(6900)$, cannot state the above…

Motivation: Our objective



"True" tetra-quark connects two colored objects (cc) and $(\bar{c}\bar{c})$ through direct strong interactions like quarks in a proton or neutron

"Molecule" a bound state of two neutral mesons connected by a strong nuclear force, like proton and neutron in a nucleus



$J/\psi(c\bar{c})$ mass is 22 heavier than π^0

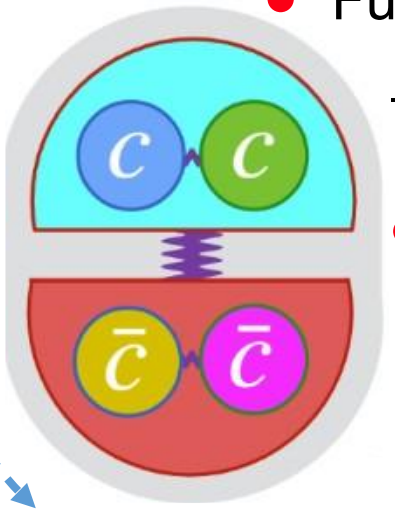
$$V(r) = g \cdot \frac{e^{-\frac{m_{\eta_c} c}{\hbar} r}}{r}$$

binding would be much weaker than in a nucleus or light "tetra-quarks"

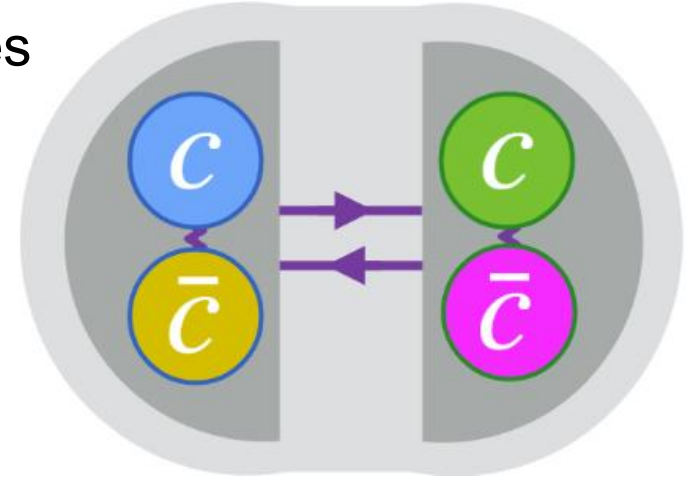
There are other less popular models...

Motivation: Our objective

- Fundamental study of matter at quark level…
  — unique all-heavy tetra-quark states



- Quantum numbers $J^{PC} = ?$
  — we know $C = +1$ from decay
  — closely related to the structure:



(cc) or $(\overline{cc})$ identical ⇒ color anti-triplet (bg-gb,..) with $L = 0$, spin axial vector

$(cc) + (\overline{cc}) \Rightarrow L = 0(\mathrm{n}S): S = 0,2 \Rightarrow J^P = 0^+, 2^+$ — most likely for a tetraquark

$L = 1(\mathrm{n}P): S = 1 \Rightarrow J^P = 0^-, 1^-, 2^-$ — less likely

$n = 1,2,3,4,..$

$L = 2(\mathrm{n}D): S = 0 \Rightarrow J^P = 2^+$

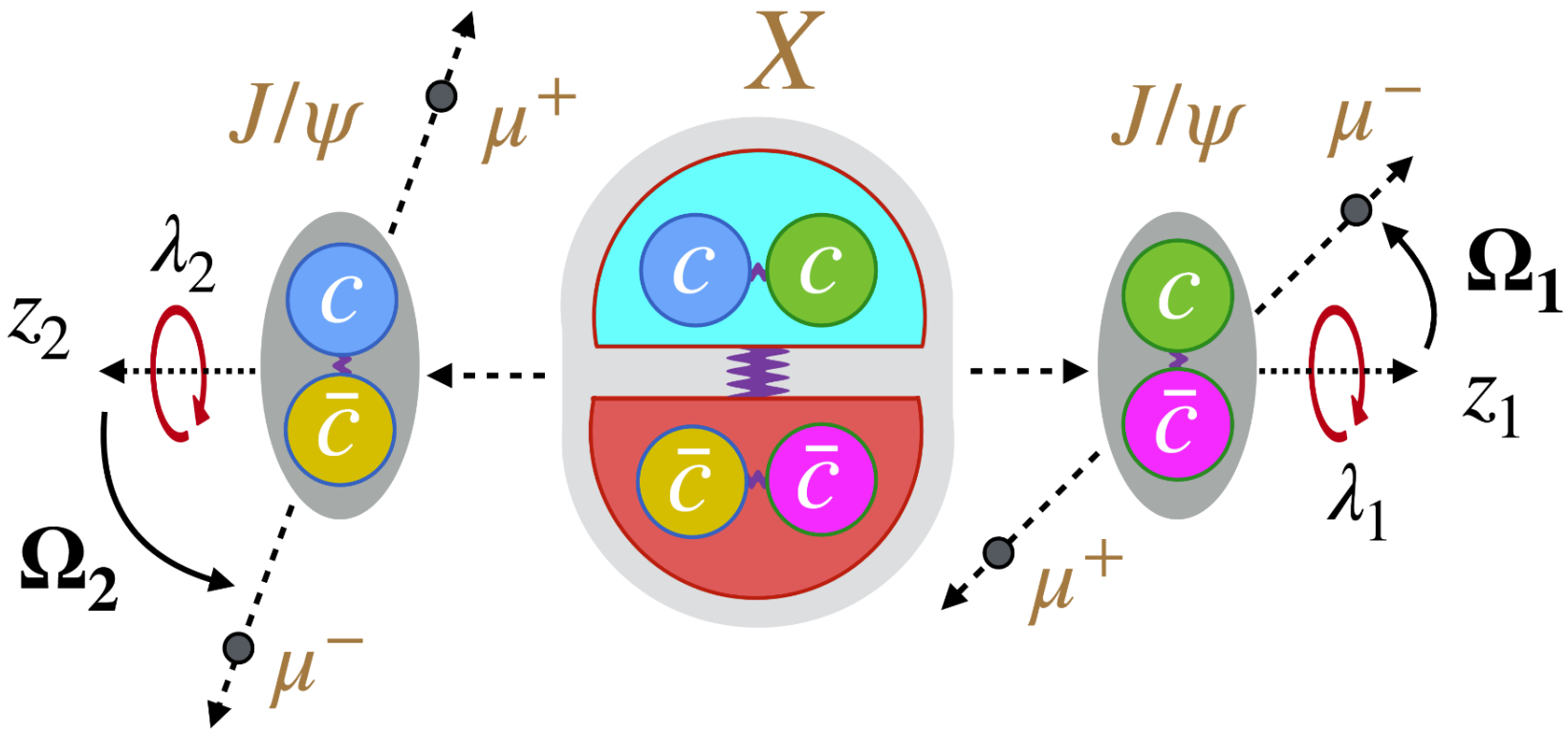
unlikely

$S = 2 \Rightarrow J^P = 0^+, 1^+, 2^+, 3^+, 4^+$ …

- Production considerations:

— likely gluon fragmentation (collision)⇒ $J \neq 1$, but quark fragmentation possible

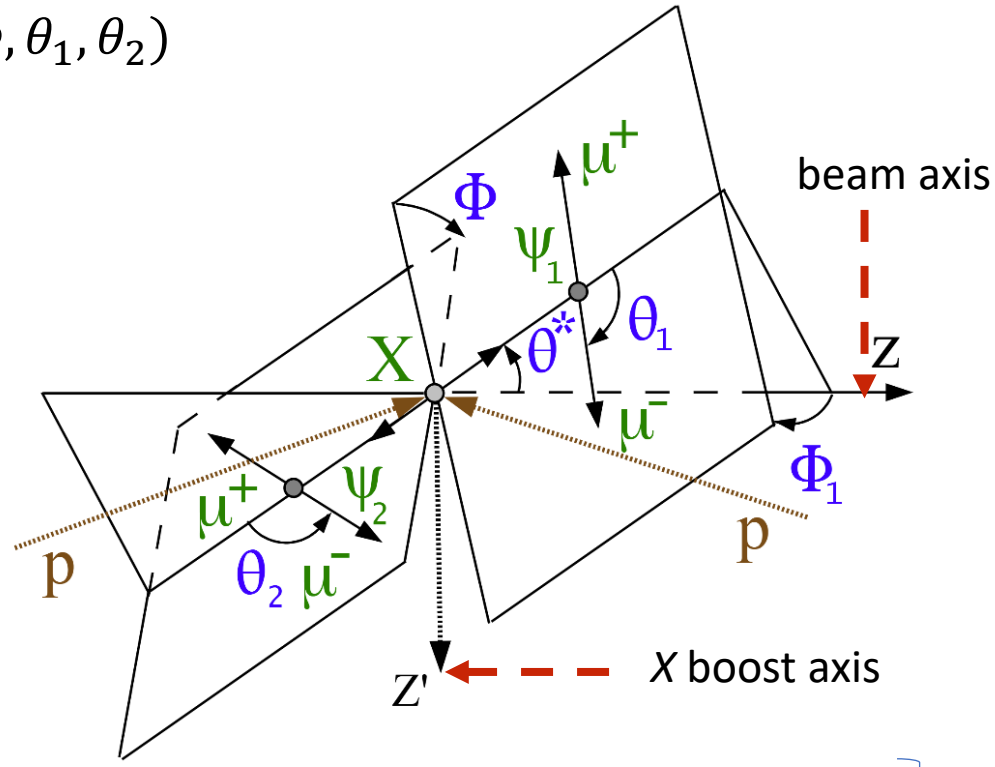
Motivation: Our objective



$$J^P = ?$$

Polarization in production and decay

- Same data sample and event selection criteria in [Phys. Rev. Lett. 132 (2024) 111901]
- Characterized by $\vec{\Omega} = (\theta^*(\theta'^*), \Phi_1(\Phi_1'), \Phi, \theta_1, \theta_2)$



Flat for unpolarized (test in data)

- θ^*: the angle between beam line and J/ψ momentum in the X rest frame
- θ'^*: the angle between boost direction and J/ψ momentum in the X rest frame
- $\Phi_1(\Phi_1')$: the angle between plane $\mathcal{P}$ and $J/\psi \to l^+l^-$ decay plane in the X rest frame

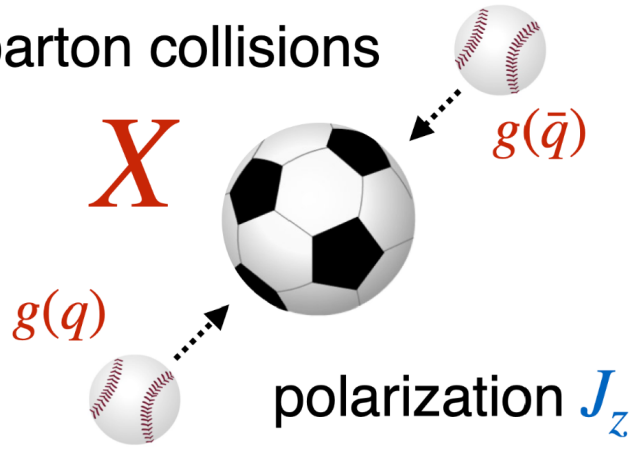
Polarization axis and J/ψ momentum constitute a production plane $\mathcal{P}$

- Φ: the angle between two $J/\psi \to l^+l^-$ decay planes defined in the X rest frame
- θ_1: the helicity angle between J/ψ_1 momentum and l momentum defined in the J/ψ_1 rest frame
- θ_2: the helicity angle between J/ψ_2 momentum and l momentum defined in the J/ψ_2 rest frame

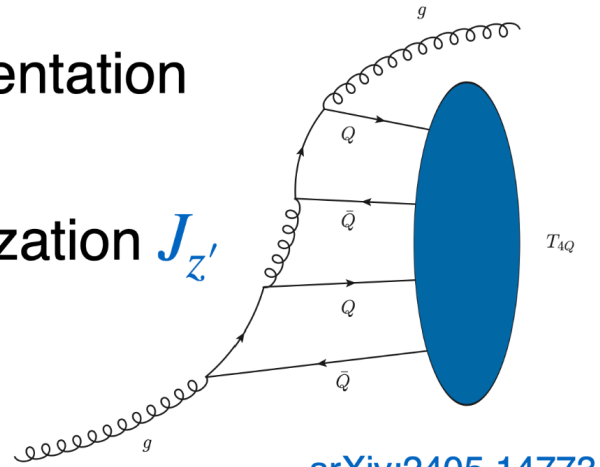
Polarization in Production

- The production mechanism is unknown
- Possible production mechanism for simulating sample

(A) parton collisions



(B) fragmentation



arXiv:2405.14773

- Helicity amplitudes appear in production. For parton collision:
  - spin-0: unpolarized in any case, e.g. $gg \to X$
  - spin-1: $q\bar{q} \to X$ produce $J_z = \pm 1$ (not 0!)
  - spin-2: $gg \to X$ produce $J_z = 0, \pm 2$, minimal coupling: $J_z = \pm 2$
    $q\bar{q} \to X$ produce $J_z = \pm 1$
- Similar ideas in fragmentation of g or Q

Decay: Lorentz-Invariant Amplitude: spin-0,1

Natural to follow Lorentz-invariant amplitude formalism:

$$A(X_{J=0} \to V_1V_2) = \left(a_1(q^2)m_V^2\epsilon_1^*\epsilon_2^* + a_2(q^2)f_{\mu\nu}^{*(1)}f^{*(2),\mu\nu} + a_3(q^2)f_{\mu\nu}^{*(1)}\tilde{f}^{*(2),\mu\nu}\right)$$

0_m^+ 0_h^+ 0^-

$A_{00} = A_{++} = A_{--}$ at $2m_{J/\psi}$ threshold

A_{00} at large m_X $A_{++} = A_{--}$

$A_{++} = -A_{--}$

recall (22 years):

$B \to \varphi K^*$ expect A_{00}

found ~50% A_{++}

Higgs (12 years):

$H \to 4\ell \Rightarrow 0_m^+$

empirical form factors

$$A(X_{J=1} \to V_1V_2) = \left(b_1(q^2)[(\epsilon_1^*q)(\epsilon_2^*\epsilon_X) + (\epsilon_2^*q)(\epsilon_1^*\epsilon_X)] + b_2(q^2)\epsilon_{\alpha\mu\nu\beta}\epsilon_X^\alpha\epsilon_1^{*,\mu}\epsilon_2^{*,\nu}\tilde{q}^\beta\right)$$

1^-

$A_{+0} = -A_{0+} = A_{-0} = -A_{0-}$

1^+

$A_{+0} = -A_{0+} = -A_{-0} = A_{0-}$

more for spin-2

Decay: Lorentz-Invariant Amplitude: spin-2

- Natural to follow Lorentz-invariant amplitude formalism spin-2:

$J_z = 0, \pm 2, \pm 1$

$gg \to X$

$$A(X \to VV) = \Lambda^{-1}\left[2g_1^{(2)} t_{\mu\nu} f^{*1,\mu\alpha} f^{*2,\nu\alpha} + 2g_2^{(2)} t_{\mu\nu} \frac{q_\alpha q_\beta}{\Lambda^2} f^{*1,\mu\alpha} f^{*2,\nu,\beta}\right.$$
$$+ g_3^{(2)} \frac{\tilde{q}^\beta \tilde{q}^\alpha}{\Lambda^2} t_{\beta\nu}(f^{*1,\mu\nu} f^{*2}_{\mu\alpha} + f^{*2,\mu\nu} f^{*1}_{\mu\alpha}) + g_4^{(2)} \frac{\tilde{q}^\nu \tilde{q}^\mu}{\Lambda^2} t_{\mu\nu} f^{*1,\alpha\beta} f^{*(2)}_{\alpha\beta} \qquad J^P = 2^+$$
$$+ m_V^2 \left(2g_5^{(2)} t_{\mu\nu} \epsilon_1^{*\mu} \epsilon_2^{*\nu} + 2g_6^{(2)} \frac{\tilde{q}^\mu q_\alpha}{\Lambda^2} t_{\mu\nu} (\epsilon_1^{*\nu} \epsilon_2^{*\alpha} - \epsilon_1^{*\alpha} \epsilon_2^{*\nu}) + g_7^{(2)} \frac{\tilde{q}^\mu \tilde{q}^\nu}{\Lambda^2} t_{\mu\nu} \epsilon_1^* \epsilon_2^*\right)$$
$$\left. + g_8^{(2)} \frac{\tilde{q}_\mu \tilde{q}_\nu}{\Lambda^2} t_{\mu\nu} f^{*1,\alpha\beta} \tilde{f}^{*(2)}_{\alpha\beta} + g_9^{(2)} t_{\mu\alpha} \tilde{q}^\alpha \epsilon_{\mu\nu\rho\sigma} \epsilon_1^{*\nu} \epsilon_2^{*\rho} q^\sigma + \frac{g_{10}^{(2)} t_{\mu\alpha} \tilde{q}^\alpha}{\Lambda^2} \epsilon_{\mu\nu\rho\sigma} q^\rho \tilde{q}^\sigma \left(\epsilon_1^{*\nu}(q\epsilon_2^*) + \epsilon_2^{*\nu}(q\epsilon_1^*)\right)\right]$$

$J^P = 2^-$

2^-_m — minimal 2^- model, kinematics similar to 0^- $(A_{++} = -A_{--})$

2^-_h — another 2^- model: kinematics similar to 1^- $(A_{+0} = A_{0+} = -A_{-0} = -A_{0-})$

Decay: Lorentz-Invariant Amplitude: spin-2

- Natural to follow Lorentz-invariant amplitude formalism spin-2:

$J_z = 0, \pm 2, \pm 1$

$gg \to X$

$$A(X \to VV) = \Lambda^{-1}\left[2g_1^{(2)} t_{\mu\nu} f^{*1,\mu\alpha} f^{*2,\nu\alpha} + 2g_2^{(2)} t_{\mu\nu} \frac{q_\alpha q_\beta}{\Lambda^2} f^{*1,\mu\alpha} f^{*2,\nu,\beta}\right.$$
$$+ g_3^{(2)} \frac{\tilde{q}^\beta \tilde{q}^\alpha}{\Lambda^2} t_{\beta\nu}(f^{*1,\mu\nu} f^{*2}_{\mu\alpha} + f^{*2,\mu\nu} f^{*1}_{\mu\alpha}) + g_4^{(2)} \frac{\tilde{q}^\nu \tilde{q}^\mu}{\Lambda^2} t_{\mu\nu} f^{*1,\alpha\beta} f^{*(2)}_{\alpha\beta}$$
$$+ m_V^2 \left(2g_5^{(2)} t_{\mu\nu} \epsilon_1^{*\mu} \epsilon_2^{*\nu} + 2g_6^{(2)} \frac{\tilde{q}^\mu q_\alpha}{\Lambda^2} t_{\mu\nu}(\epsilon_1^{*\nu} \epsilon_2^{*\alpha} - \epsilon_1^{*\alpha} \epsilon_2^{*\nu}) + g_7^{(2)} \frac{\tilde{q}^\mu \tilde{q}^\nu}{\Lambda^2} t_{\mu\nu} \epsilon_1^* \epsilon_2^*\right)$$
$$\left. + g_8^{(2)} \frac{\tilde{q}_\mu \tilde{q}_\nu}{\Lambda^2} t_{\mu\nu} f^{*1,\alpha\beta} \tilde{f}^{*(2)}_{\alpha\beta} + g_9^{(2)} t_{\mu\alpha} \tilde{q}^\alpha \epsilon_{\mu\nu\rho\sigma} \epsilon_1^{*\nu} \epsilon_2^{*\rho} q^\sigma + \frac{g_{10}^{(2)} t_{\mu\alpha} \tilde{q}^\alpha}{\Lambda^2} \epsilon_{\mu\nu\rho\sigma} q^\rho \tilde{q}^\sigma (\epsilon_1^{*\nu}(q\epsilon_2^*) + \epsilon_2^{*\nu}(q\epsilon_1^*))\right]$$

$J^P = 2^+$

$J^P = 2^-$

2^+_m — minimal representative model including all amplitudes:

4 d.o.f. $A_{00}, A_{++} = A_{--}, A_{+0} = A_{0+} = A_{-0} = A_{0-}, A_{+-} = A_{-+}$ for 2^+ unique

basis could be equivalent to $2^+_m, 0^+_m, 0^+_h, 1^+$

- Unless we find A_{+-}, A_{-+} or polarization: cannot separate spin-2

Simulation and Fitting Tools

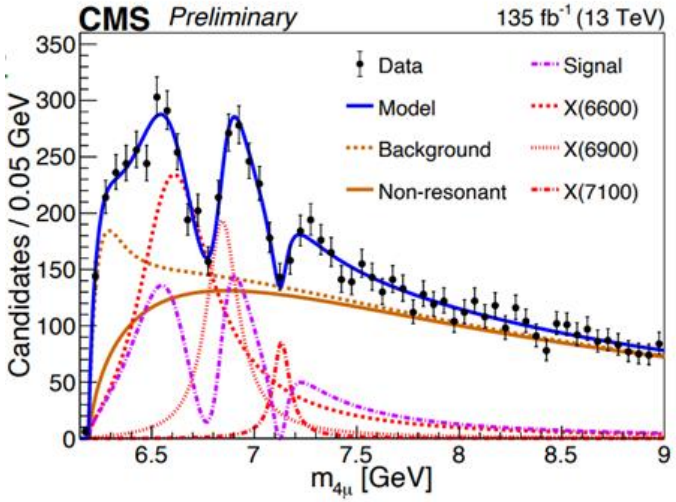
Pythia8: **NRSPS**—Non-Resonant Single Parton Scattering

$$gg(99\%) \rightarrow J/\psi J/\psi \rightarrow \mu^+\mu^-\mu^+\mu^-$$

DPS—Non-Resonant Double Parton Scattering

JHUGen:

- $gg(q\bar{q}) \rightarrow X \rightarrow J/\psi\, J/\psi \rightarrow 4\mu$
  with different J^P : $0^+, 0^-, 1^+, 1^-, 2^+, 2^-$
- Can model interference $\mathcal{P}_k(m_{4\mu})$
- Assume three resonances have same J^P
  - Dips observed in data —— explained by interference which indicates coherent production process and same J^P



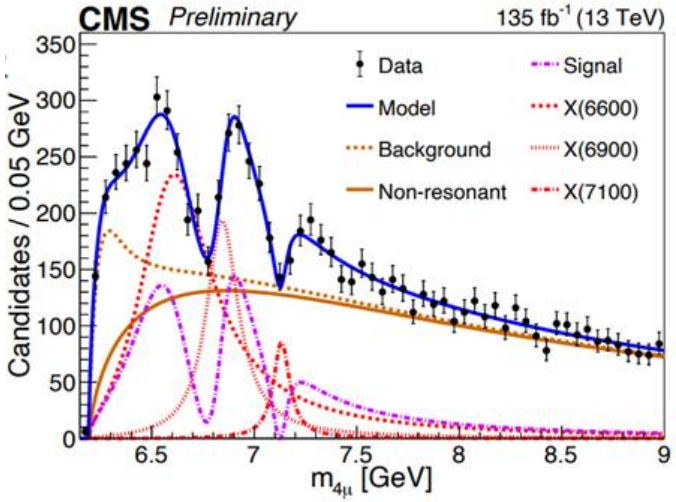
Simulation and Fitting Tools

MELA (Matrix Element Likelihood Approach):

- create $\mathcal{D}_{ij}(\vec{\Omega}|m_{4\mu})$: Probability computed from

$$\mathcal{D}_{ij}(\vec{\Omega}|m_{4\mu}) = \frac{\mathcal{P}_i(\vec{\Omega}|m_{4\mu})}{\mathcal{P}_i(\vec{\Omega}|m_{4\mu}) + \mathcal{P}_j(\vec{\Omega}|m_{4\mu})}$$

- matrix elements($\vec{\Omega} = (cos\theta^*, \Phi_1, \Phi, cos\theta_1, cos\theta_2)$)
- re-weight to any model and polarization (including unpolarized, polarized $\mathcal{Z}$ or $\mathcal{Z}'$)



Combine 2D fit (Follow Higgs analysis):

- $\mathcal{P}_{ijk}(m_{4\mu}, \mathcal{D}_{ij}) = \mathcal{P}_k(m_{4\mu})\mathcal{I}_{ijk}(\mathcal{D}_{ij}|m_{4\mu})$

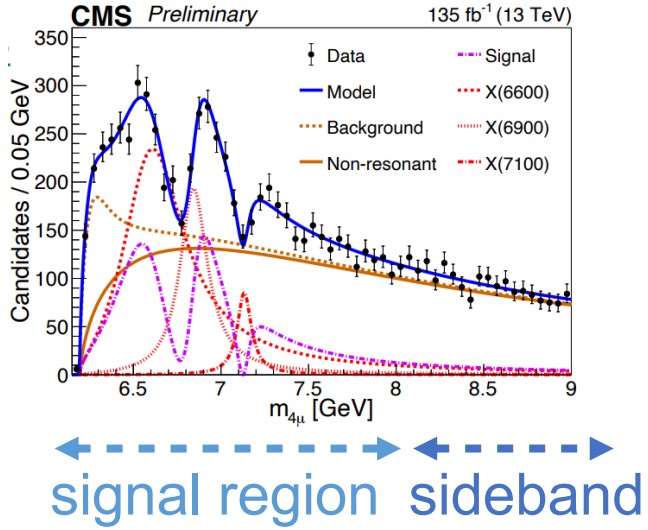
- $pdf_m(m) = N_{NRSPS} \cdot pdf_{NRSPS} + N_{DPS} \cdot pdf_{DPS} + N_{BW_0} \cdot |BW_0|^2 \otimes R(M_0) + |r_1 \cdot \exp(i\phi_1) \cdot BW_1 + BW_2 + r_3 \cdot \exp(i\phi_3) \cdot BW_3|^2$

- i, j represent different hypothesis
- k represents sample including all kinds of background and signal

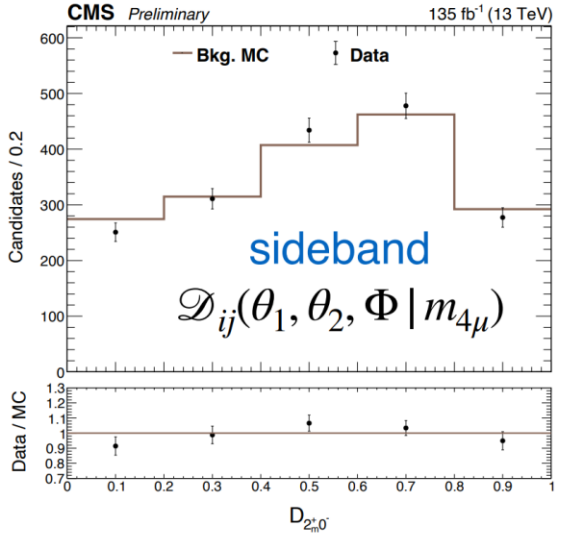
MC simulation and tuning

- Relevant information
  - $m_{4\mu}$ spectrum $X \to 4\mu$ — identical to BPH-21-003
  - Polarization J_z or $J_{z'}$ of X — assume unpolarized
    - For $J = 0$ exactly
    - For $J = 1, 2$ depends on production mechanism, vary J_z or $J_{z'}$ as systematic

(1) $m_{4\mu}$ spectrum



(2) Discriminant distribution comparison (systematics)



Systematics

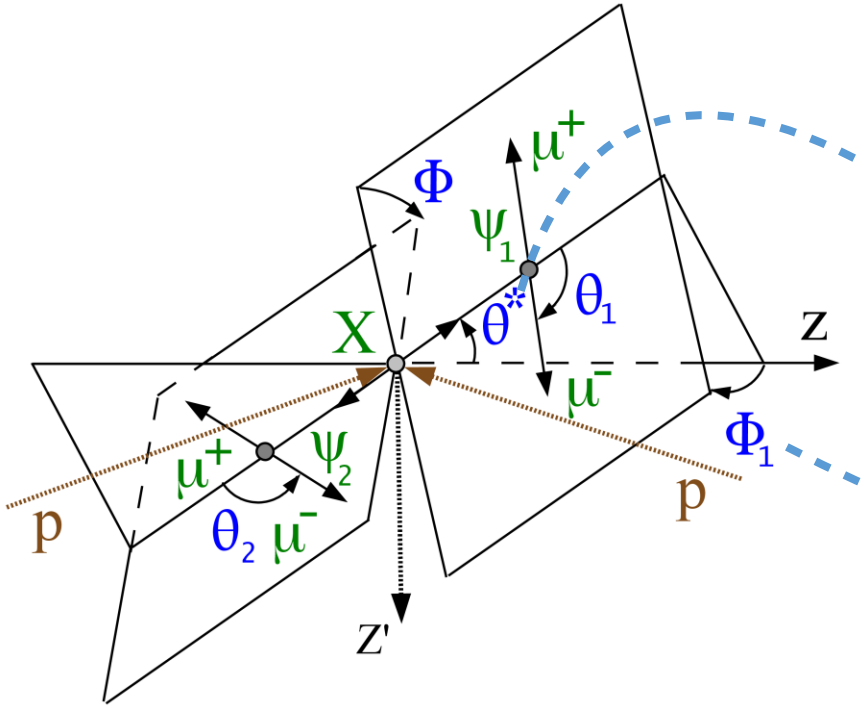
Systematic	
Mass shapes	Signal parameterization
	Statistical uncertainty of nominal shapes
	Mass resolution
	Efficiency correction
	DPS shape
	NRSPS shape
	Feed-down contribution
	Combinatorial background
Discriminant shapes	p_T reweigting
	Discriminant reweighting
	Difference between pre-UL and UL MC
	Polarization

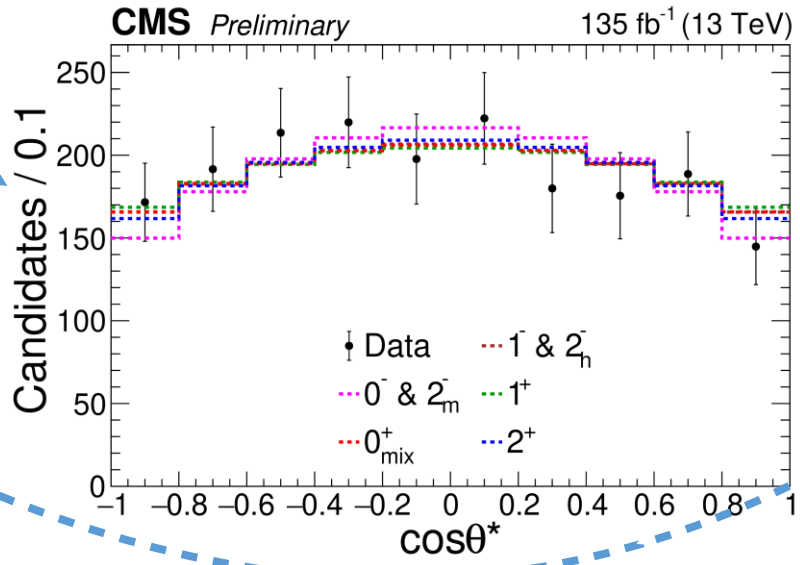
same with BPH-21-003

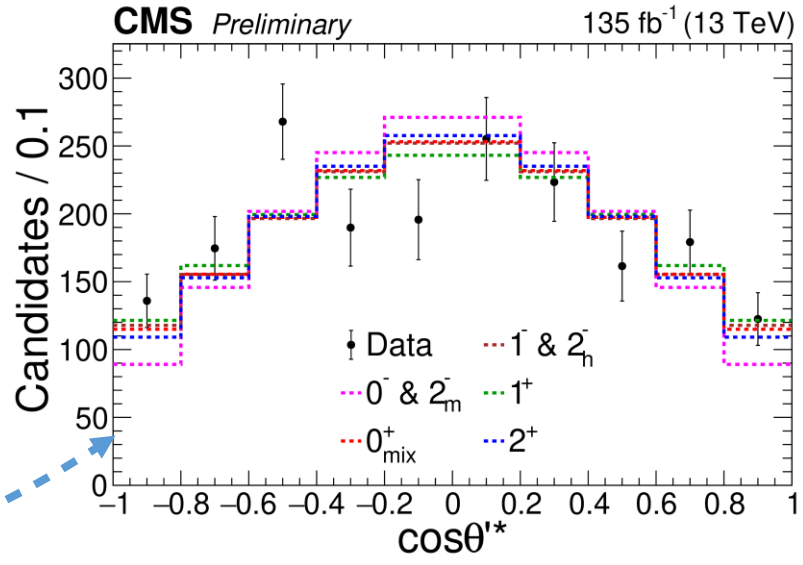
- Get a set of alternative shapes due to a source of systematic
  - Alternative mass shapes
  - Alternative discriminant shapes
- All sets of alternative shapes are input in Combine to estimate systematic

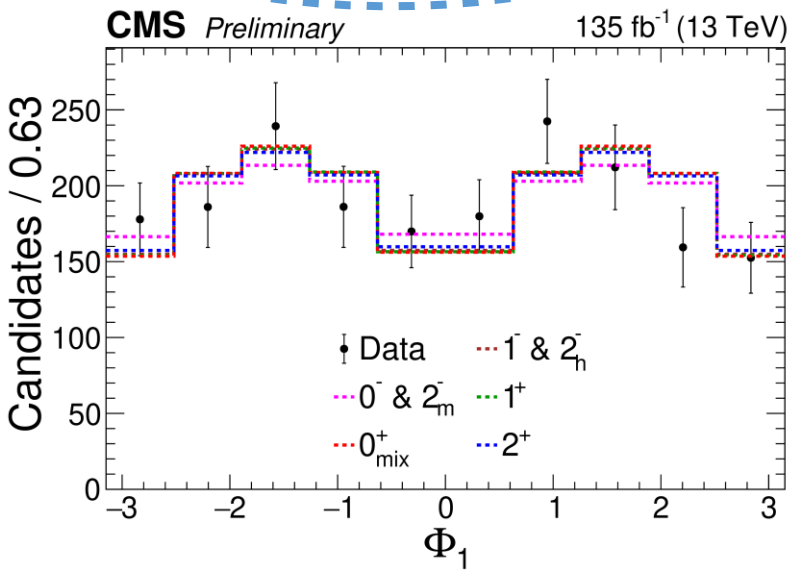
Everything mentioned above are just based on MC Simulation
Now ready to look at the data!

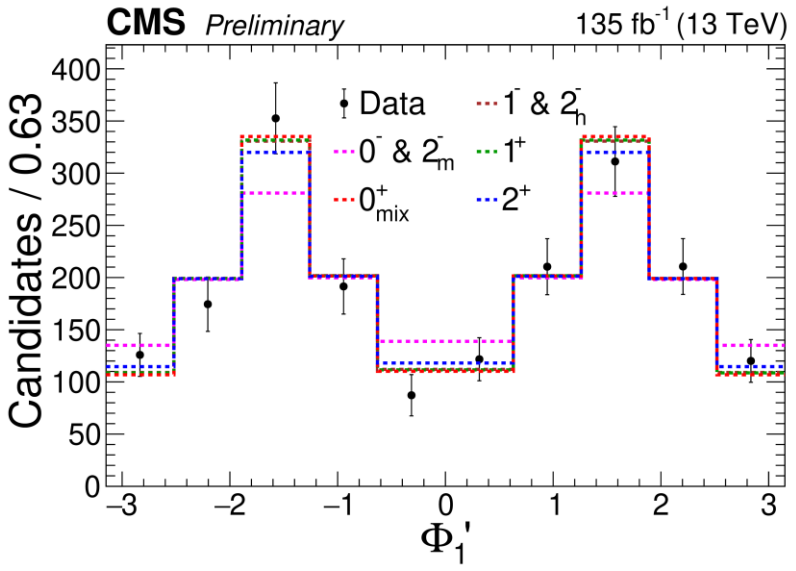
Unblind: production angles





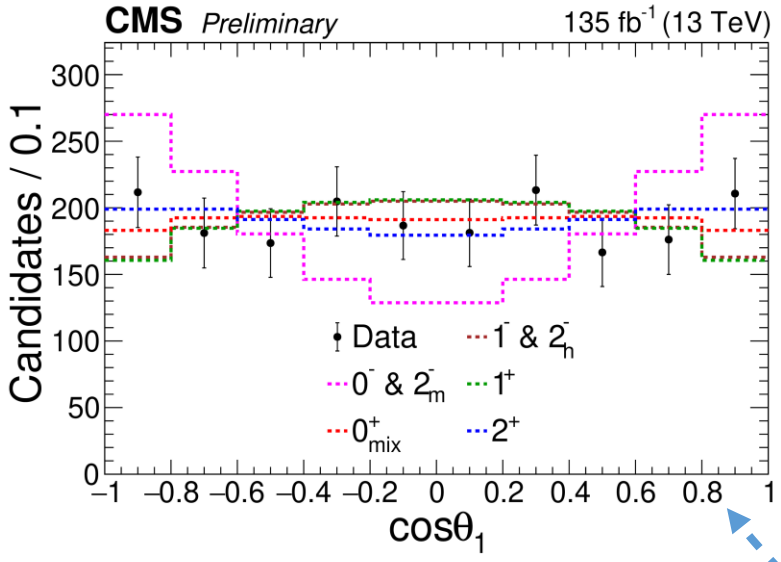


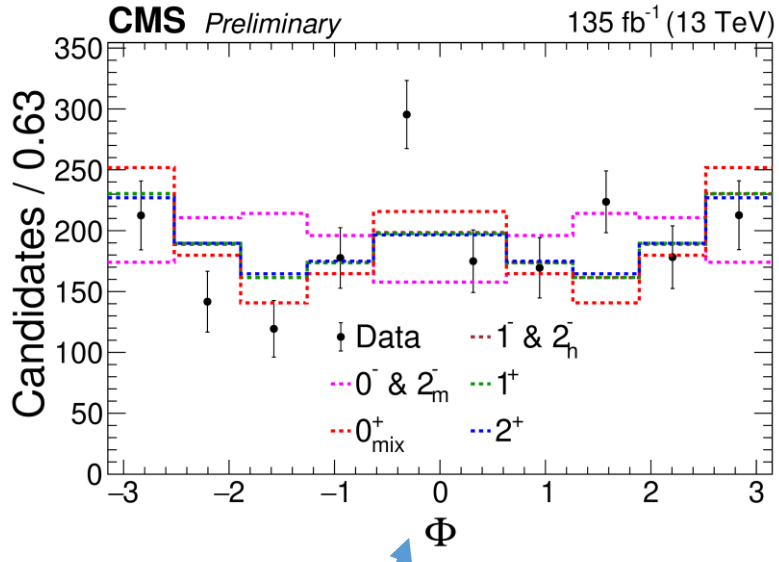


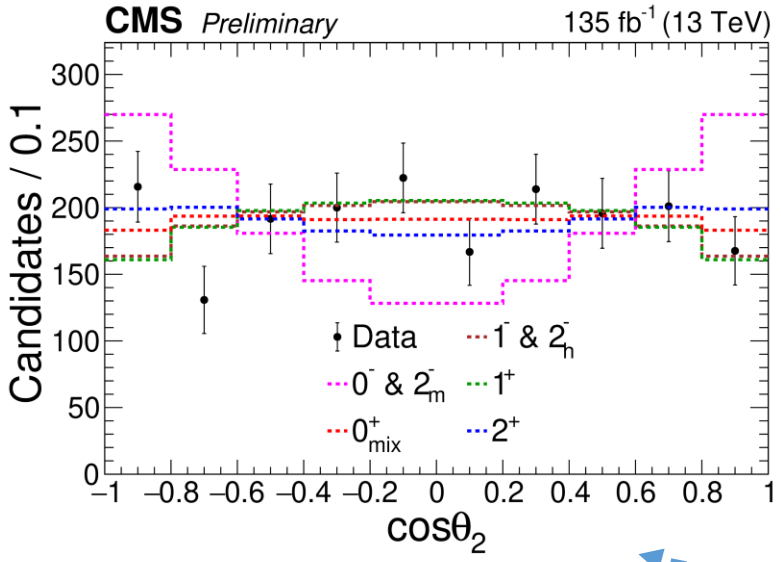


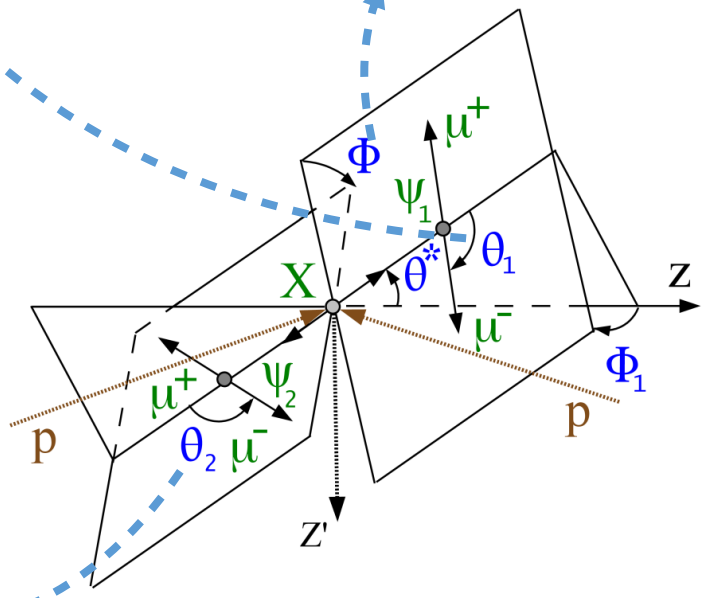
- All plots are background subtracted
- Production angles are consistent with unpolarized case

Unblind: decay angles









- 0^- and 2^- are visually not consistent with data from the angular projections alone.

Hypothesis test result: 0^- vs 2^+

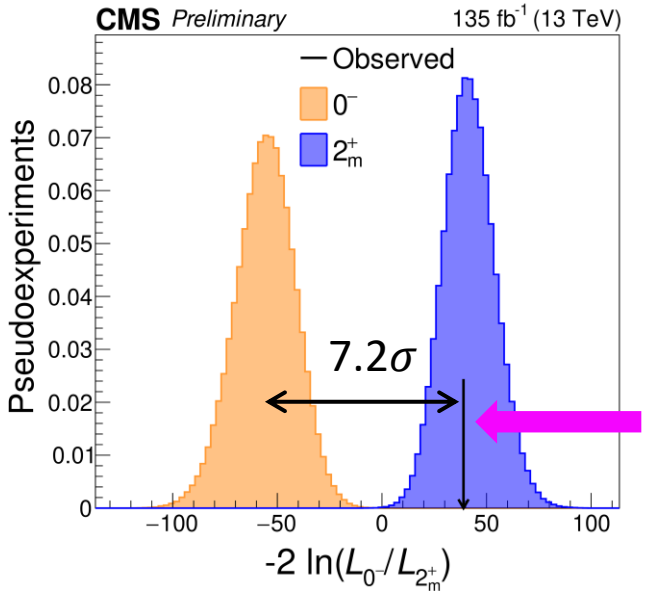
❖ Hypothesis test for $j = 0^-$ vs $i = 2^+_m$

		Observed		Expected	
		p-value	Z-score	p-value	Z-score
0^- vs 2^+_m	0^-	2.7×10^{-13}	7.2	6.5×10^{-14}	7.4
	2^+_m	4.2×10^{-1}	0.2	0.50	0.0

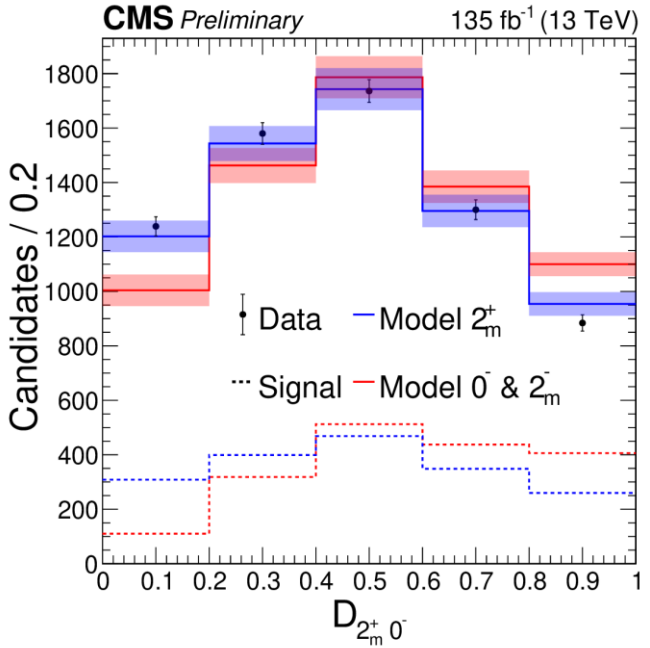
❖ Test statistic:

$$q = -2\ln(\mathcal{L}_{J_j^P}/\mathcal{L}_{J_i^P})$$

❖ $CL_s = \dfrac{P(q \geq q_{obs}|J_j^P + bkg)}{P(q \geq q_{obs}|J_i^P + bkg)}$



observed

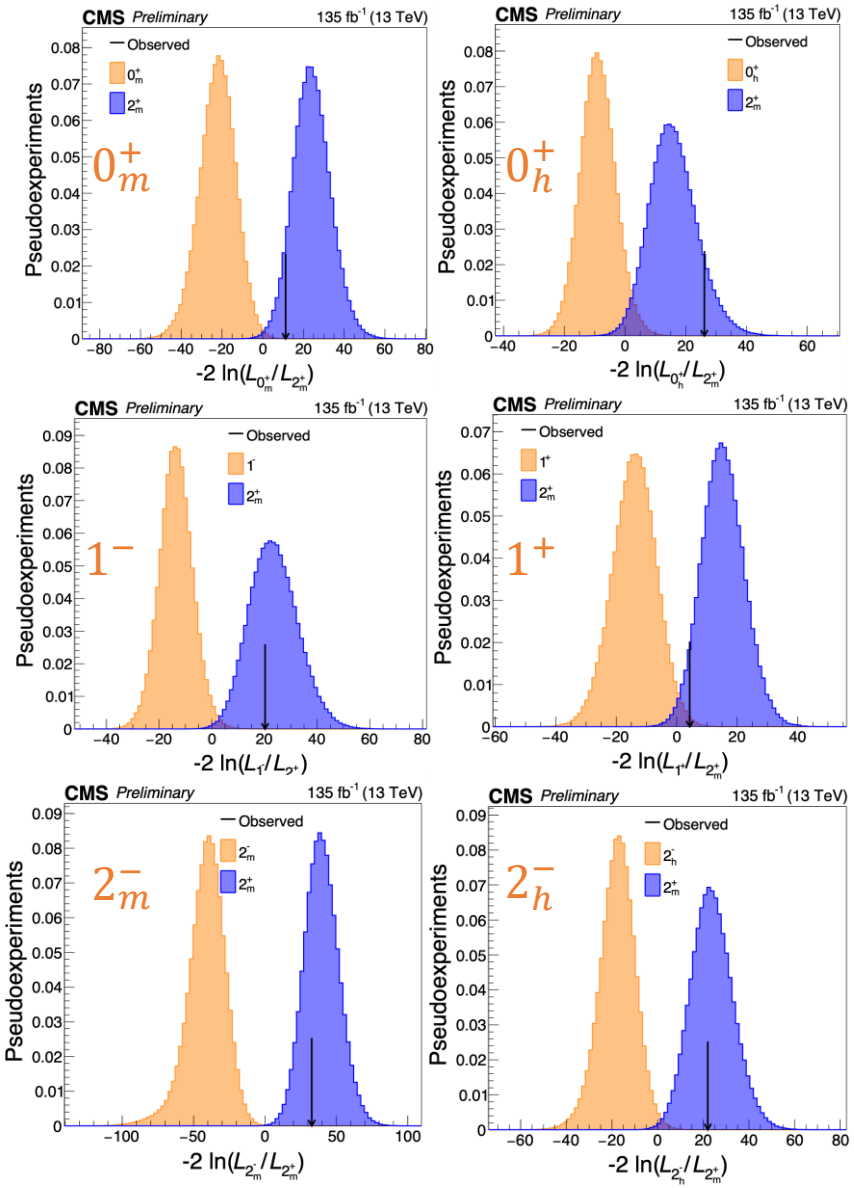


❖ Discriminant distribution in 1D projection for separate illustration

Hypothesis test result: all models

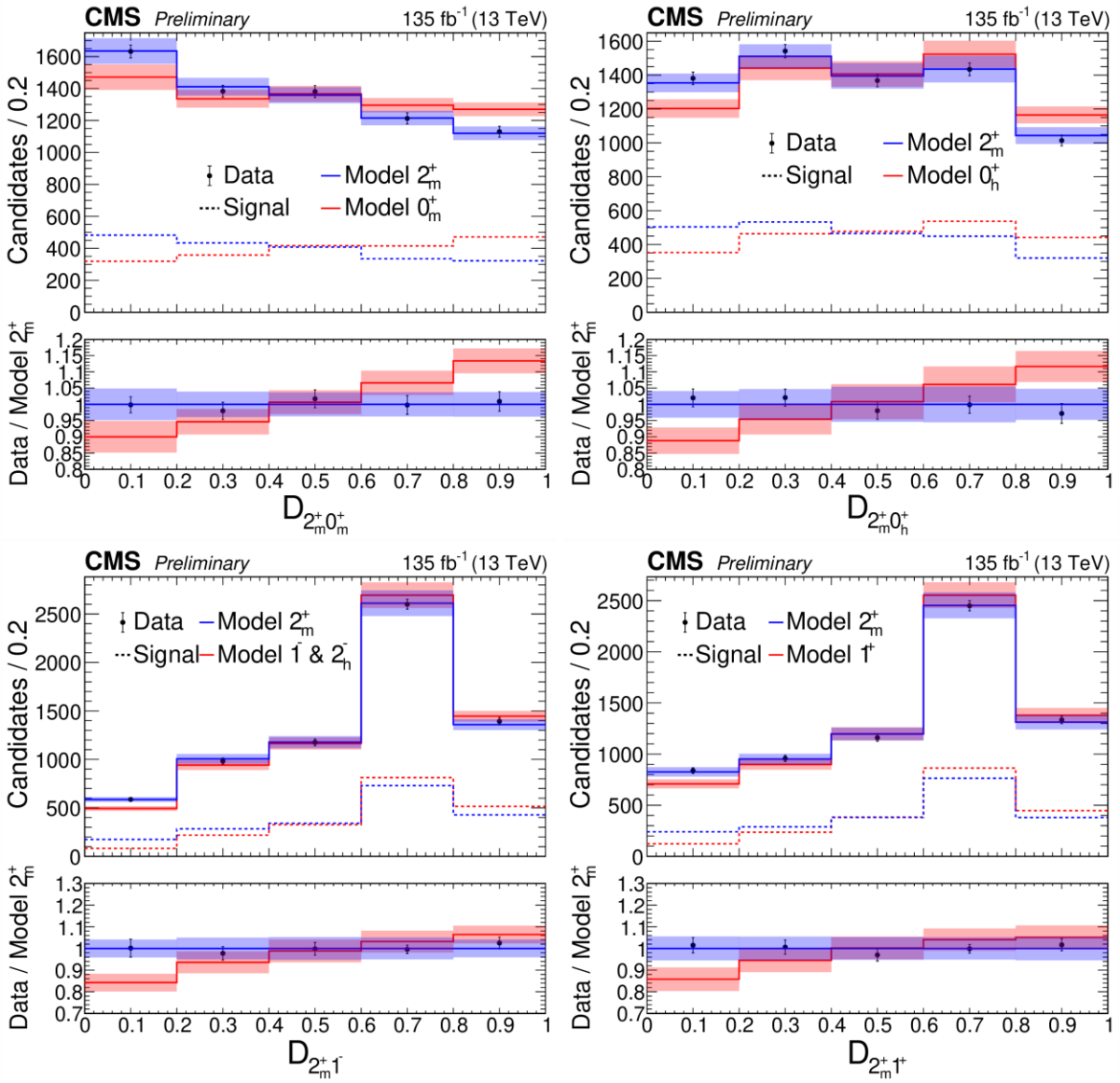
J^P_X	p-value	Z-score reject J^P_X
0^-	2.7×10^{-13}	7.2
0^+_m	4.3×10^{-5}	3.9
0^+_{mix}	1.4×10^{-2}	2.2
0^+_h	3.1×10^{-9}	5.8
1^-	8.0×10^{-8}	5.2
1^+	4.7×10^{-3}	2.6
2^-_m	4.1×10^{-12}	6.8
2^-_{mix}	6.5×10^{-4}	3.2
2^-_h	2.2×10^{-8}	5.5

- Minimum separation > 2σ
- Reject P = -1 > 3σ

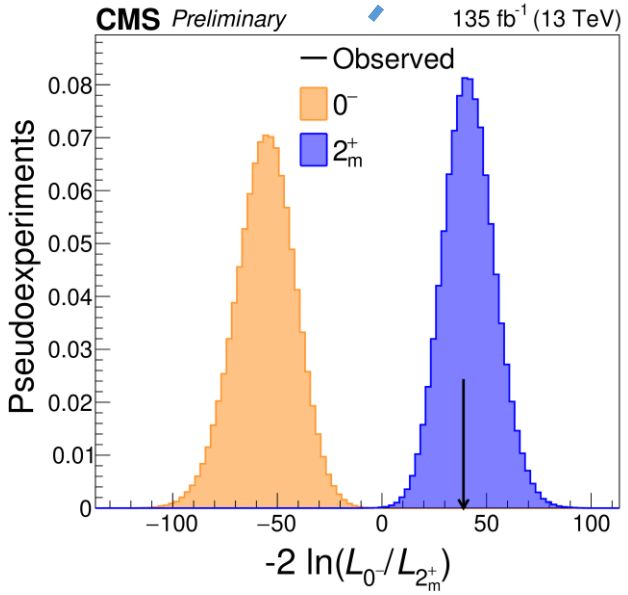


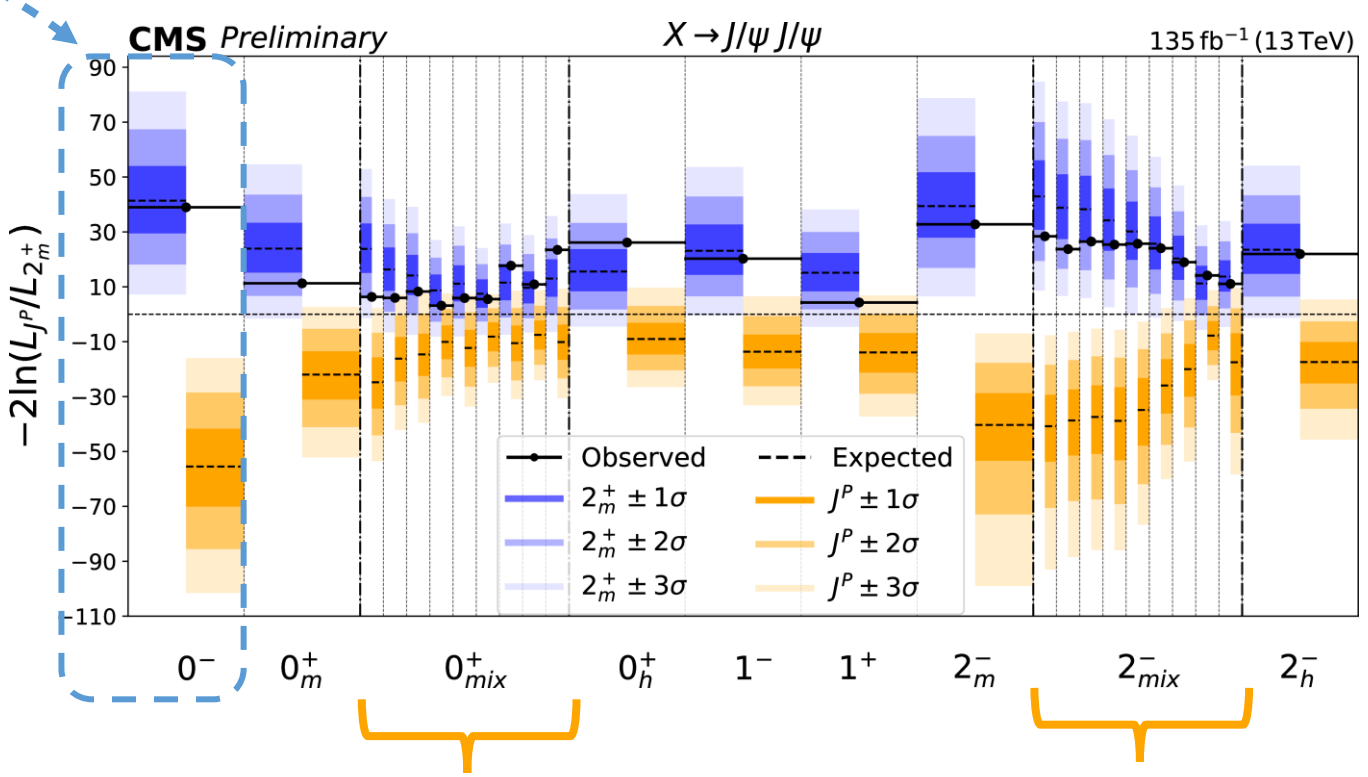
- Model in blue is 2^+_m
- Black arrow is the observed
- 2^+_m is the one survived

Discriminant distribution



Hypothesis test result: summary plot





- Scan mixture of two 0^+_m and 0^+_h model amplitudes (11 steps including phase)
  — constructive interference most conservative
- Scan mixture of two 2^-_m and 2^-_h model amplitudes (11 steps including phase)

Hypothesis test result: summary plot

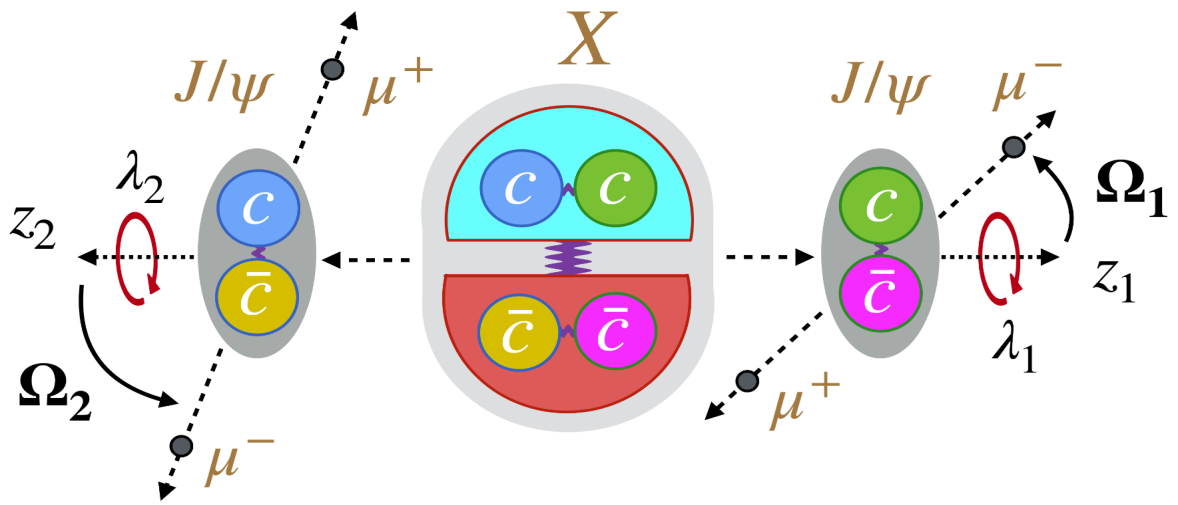
		Observed		Expected	
		p-value	Z-score	p-value	Z-score
0^- vs 2^+_m	0^-	2.7×10^{-13}	7.2	6.5×10^{-14}	7.4
	2^+_m	4.2×10^{-1}	0.2	0.50	0.0
0^+_m vs 2^+_m	0^+_m	4.3×10^{-5}	3.9	5.6×10^{-9}	5.7
	2^+_m	7.2×10^{-2}	1.5	0.50	0.0
0^+_{mix} vs 2^+_m	0^+_{mix}	1.4×10^{-2}	2.2	8.4×10^{-4}	3.1
	2^+_m	1.7×10^{-1}	1.0	0.50	0.0
0^+_h vs 2^+_m	0^+_h	3.1×10^{-9}	5.8	8.5×10^{-5}	3.8
	2^+_m	9.0×10^{-1}	-1.3	0.50	0.0
1^- vs 2^+_m	1^-	8.0×10^{-8}	5.2	6.4×10^{-9}	5.7
	2^+_m	3.8×10^{-1}	0.3	0.50	0.0
1^+ vs 2^+_m	1^+	4.7×10^{-3}	2.6	2.7×10^{-5}	4.0
	2^+_m	5.2×10^{-2}	1.6	0.50	0.0
2^-_m vs 2^+_m	2^-_m	4.1×10^{-12}	6.8	3.9×10^{-14}	7.5
	2^+_m	2.8×10^{-1}	0.6	0.50	0.0
2^-_{mix} vs 2^+_m	2^-_{mix}	6.5×10^{-4}	3.2	1.5×10^{-4}	3.6
	2^+_m	3.1×10^{-1}	0.5	0.50	0.0
2^-_h vs 2^+_m	2^-_h	2.2×10^{-8}	5.5	6.3×10^{-9}	5.7
	2^+_m	4.3×10^{-1}	0.2	0.50	0.0

- $J \neq 0$ at $> 95\%$ CL
- $J \neq 1$ at $> 99\%$ CL
- $P \neq -1$ very certain (exclude J^{-+} including $J \geq 3$)

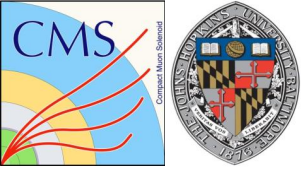
Summary

What we achieve?

- reject 0^- and 1^- at > 5σ
- reject 2^- and J^- with J>2 at > 3σ
- PC=++ very certain
- P-wave (L=1) excluded, S-wave (L=0) most likely
- exclude 0^+ and 1^+ at 95% CL (> 2σ)
- J^{PC} = 2^{++} most likely
- 3^{++} or above possible but unlikely (require L=2)
- novel method for exotic hadron study
- narrow down the internal structure



$$(cc) + (\overline{cc}) \Rightarrow L = 0\ (\mathrm{n}S): S = 2 \Rightarrow J^{PC} = 2^{++}$$

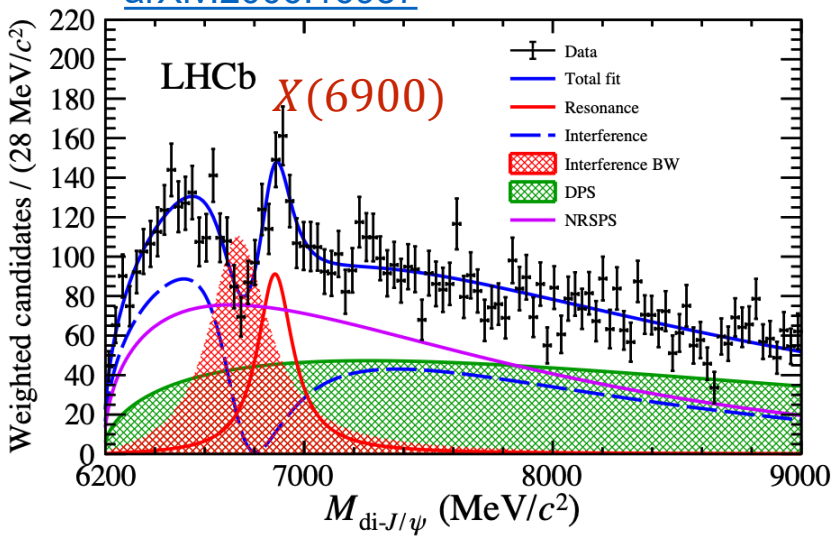


Thank You!

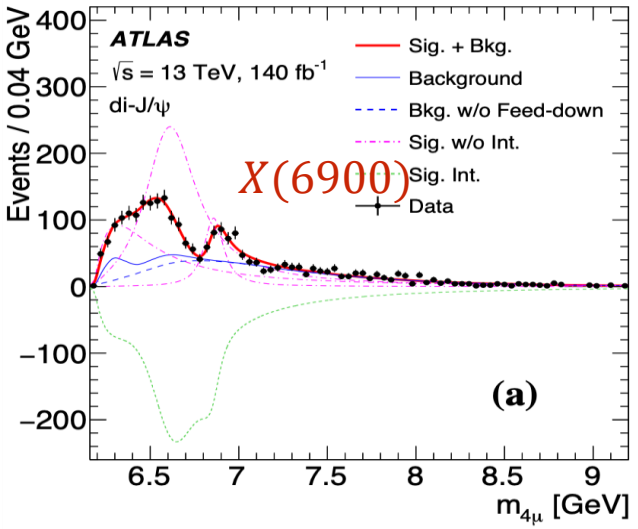
Back up

Motivation: LHC Background information

arXiv:2006.16957



arXiv:2304.08962

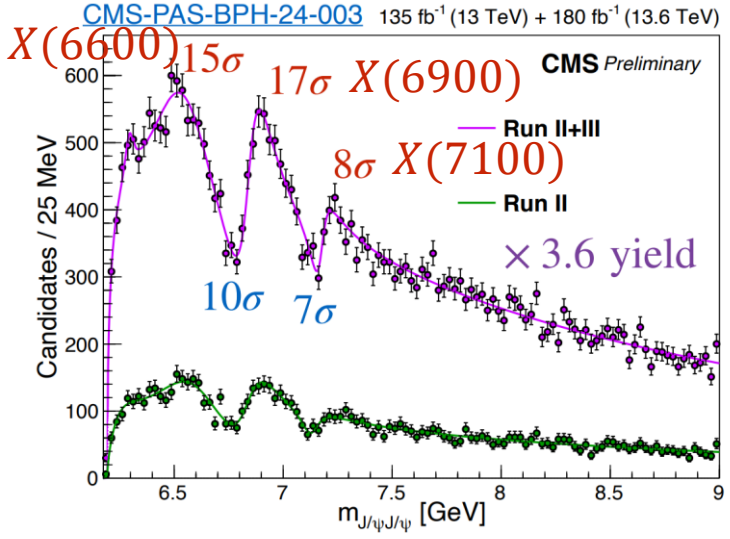


— only $X(6900)$ observed by 3 experiments

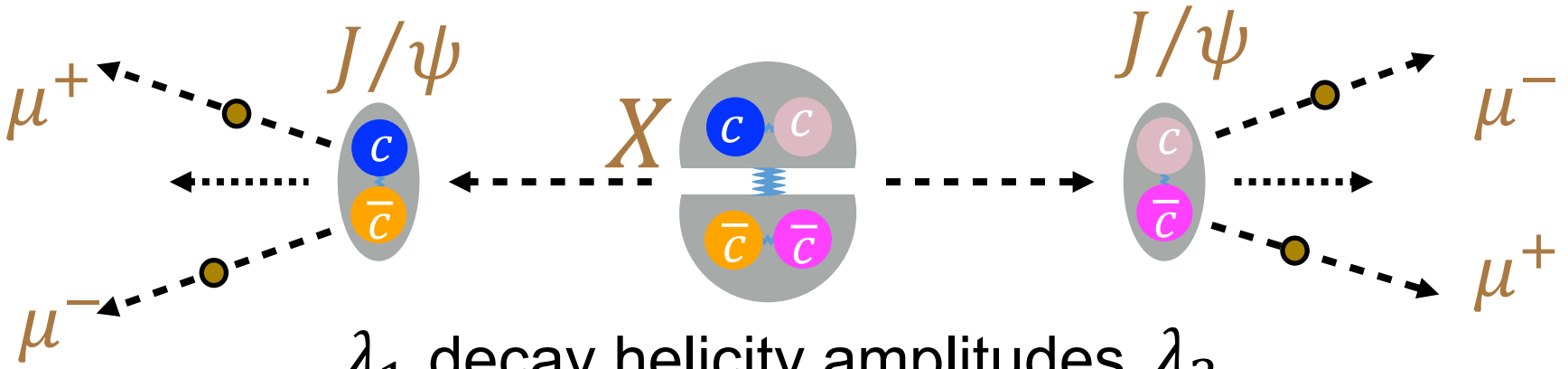
— $X(6600)$ observed by CMS only (7σ)
excess with no claim by LHCb/ATLAS

— $X(7100)$ observed by CMS (> 5σ)
weak hint by LHCb/ATLAS

CMS-PAS-BPH-24-003 135 fb^{-1} (13 TeV) + 180 fb^{-1} (13.6 TeV)



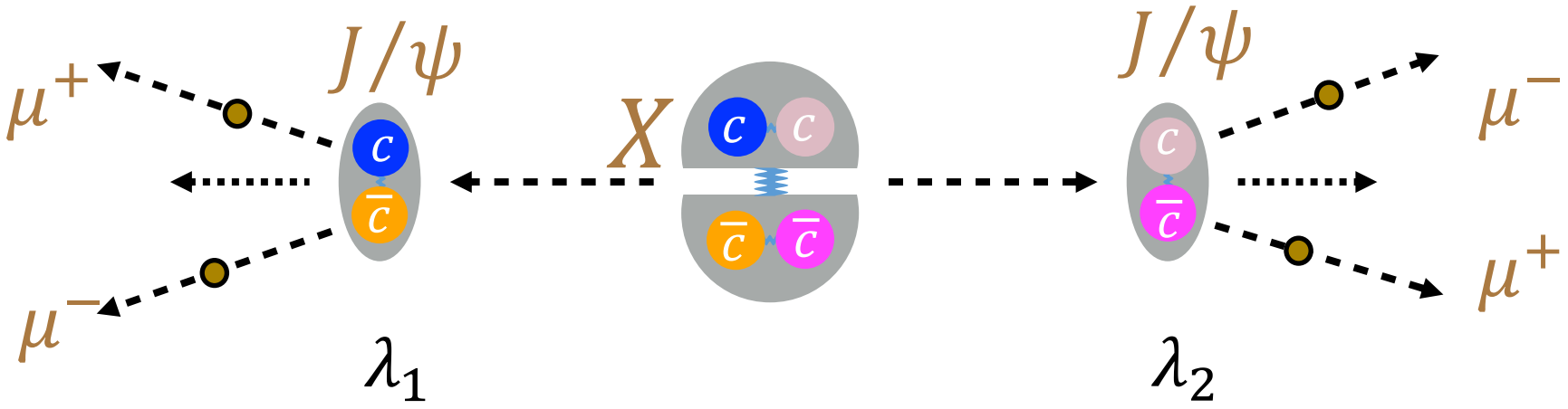
Decay: Polarization of J/ψ



λ_1 decay helicity amplitudes λ_2

$$
\begin{array}{ccc}
\leftarrow & A_{++} & \rightarrow \\
\rightarrow & A_{--} & \leftarrow \\
0 & A_{00} & 0 \\
\leftarrow & A_{+0} & \rightarrow \\
0 & A_{0+} & 0 \\
\rightarrow & A_{-0} & \rightarrow \\
0 & A_{0-} & 0 \\
\leftarrow & A_{+-} & \leftarrow \\
\rightarrow & A_{-+} & \rightarrow
\end{array}
\qquad
\left.\begin{array}{l} J_X = 0 \\ J_X = 1 \end{array}\right\} J_X \geq 2
$$

Decay: Symmetry Constraints



A_{++}
A_{--}
A_{00}
A_{+0}
A_{0+}
A_{-0}
A_{0-}
A_{+-}
A_{-+}

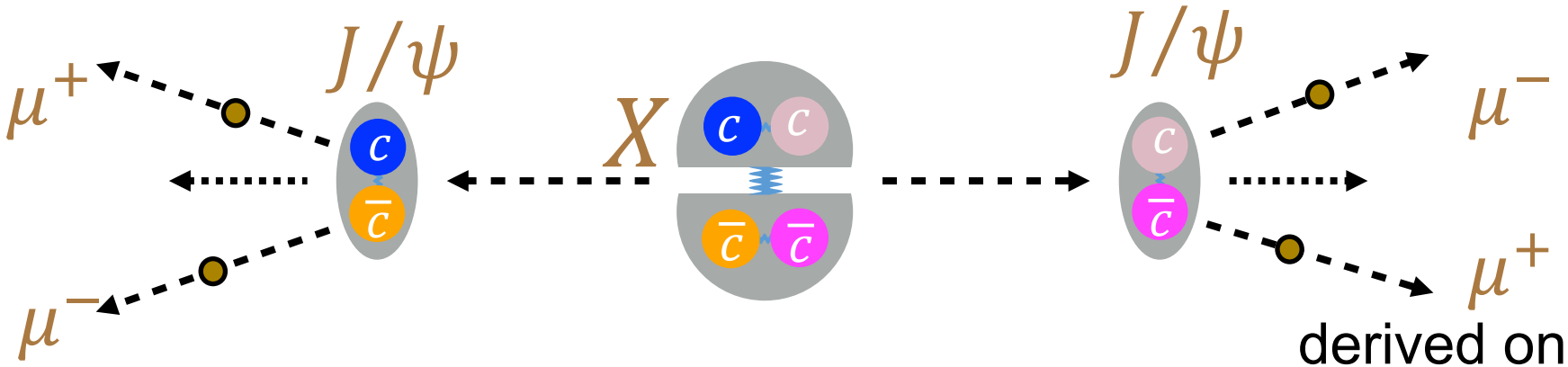
Symmetries:

Test 8+ J_X^P models:

0^{-+}	0^-	$A_{++} = -A_{--}$
0^{++}	0_m^+ and 0_h^+	$A_{++} = A_{--}$ and A_{00} ← note 2 d.o.f.
1^{-+}	1^-	$A_{+0} = -A_{0+} = A_{-0} = -A_{0-}$
1^{++}	1^+	$A_{+0} = -A_{0+} = -A_{-0} = A_{0-}$
2^{-+}	2_m^- and 2_h^-	$A_{++} = -A_{--}$ and $A_{+0} = A_{0+} = -A_{-0} = -A_{0-}$ ← note 2 d.o.f.
2^{++}	2_m^+	$A_{++} = A_{--}, A_{00}, A_{+0} = A_{0+} = A_{-0} = A_{0-}$, and $A_{+-} = A_{-+}$

note 4 d.o.f. for 2^{++}, test one model

Decay: Symmetry Constraints



A_{++}
A_{--}
A_{00}
A_{+0}
A_{0+}
A_{-0}
A_{0-}
A_{+-}
A_{-+}

$$F^J_{0,0}(\theta^*) \times \Big[4|A_{00}|^2 \sin^2\theta_1 \sin^2\theta_2 + 2|A_{++}||A_{--}| \sin^2\theta_1 \sin^2\theta_2 \cos(2\Phi - \phi_{--} + \phi_{++})\Big]$$
$$+ |A_{++}|^2 (1 + 2A_{f_1}\cos\theta_1 + \cos^2\theta_1)(1 + 2A_{f_2}\cos\theta_2 + \cos^2\theta_2)$$
$$+ |A_{--}|^2 (1 - 2A_{f_1}\cos\theta_1 + \cos^2\theta_1)(1 - 2A_{f_2}\cos\theta_2 + \cos^2\theta_2) \qquad \text{spin} = 0\ \&\ \geq 1$$
$$+ 4|A_{00}||A_{++}|(A_{f_1} + \cos\theta_1)\sin\theta_1(A_{f_2} + \cos\theta_2)\sin\theta_2\cos(\Phi + \phi_{++})$$
$$+ 4|A_{00}||A_{--}|(A_{f_1} - \cos\theta_1)\sin\theta_1(A_{f_2} - \cos\theta_2)\sin\theta_2\cos(\Phi - \phi_{--})$$

$$+F^J_{1,1}(\theta^*) \times \Big[2|A_{+0}|^2(1 + 2A_{f_1}\cos\theta_1 + \cos^2\theta_1)\sin^2\theta_2 + 2|A_{0-}|^2\sin^2\theta_1(1 - 2A_{f_2}\cos\theta_2 + \cos^2\theta_2)$$
$$+ 2|A_{-0}|^2(1 - 2A_{f_1}\cos\theta_1 + \cos^2\theta_1)\sin^2\theta_2 + 2|A_{0+}|^2\sin^2\theta_1(1 + 2A_{f_2}\cos\theta_2 + \cos^2\theta_2)$$
$$+ 4|A_{+0}||A_{0-}|(A_{f_1} + \cos\theta_1)\sin\theta_1(A_{f_2} - \cos\theta_2)\sin\theta_2\cos(\Phi + \phi_{+0} - \phi_{0-})$$
$$+ 4|A_{0+}||A_{-0}|(A_{f_1} - \cos\theta_1)\sin\theta_1(A_{f_2} + \cos\theta_2)\sin\theta_2\cos(\Phi + \phi_{0+} - \phi_{-0})\Big] \qquad \text{spin} \geq 1$$
$$+F^J_{1,-1}(\theta^*) \times \Big[4|A_{+0}||A_{0+}|(A_{f_1} + \cos\theta_1)\sin\theta_1(A_{f_2} + \cos\theta_2)\sin\theta_2\cos(2\Psi - \phi_{+0} + \phi_{0+})$$
$$+ 4|A_{0-}||A_{-0}|(A_{f_1} - \cos\theta_1)\sin\theta_1(A_{f_2} - \cos\theta_2)\sin\theta_2\cos(2\Psi - \phi_{0-} + \phi_{-0})$$
$$+4|A_{+0}||A_{-0}|\sin^2\theta_1\sin^2\theta_2\cos(2\Psi - \Phi - \phi_{+0} + \phi_{-0}) + 4|A_{0-}||A_{0+}|\sin^2\theta_1\sin^2\theta_2\cos(2\Psi + \Phi - \phi_{0-} + \phi_{0+})\Big]$$

$$+F^J_{2,2}(\theta^*) \times \Big[|A_{+-}|^2(1 + 2A_{f_1}\cos\theta_1 + \cos^2\theta_1)(1 - 2A_{f_2}\cos\theta_2 + \cos^2\theta_2)$$
$$+ |A_{-+}|^2(1 - 2A_{f_1}\cos\theta_1 + \cos^2\theta_1)(1 + 2A_{f_2}\cos\theta_2 + \cos^2\theta_2)\Big] \qquad \text{spin} \geq 2$$
$$+F^J_{2,-2}(\theta^*) \times \Big[2|A_{+-}||A_{-+}|\sin^2\theta_1\sin^2\theta_2\cos(4\Psi - \phi_{+-} + \phi_{-+})\Big] + \text{other 26 interference terms for spin}$$

where $\Psi = \Phi_1 + \Phi/2$ and $F^J_{ij}(\theta^*) = \sum_{m=0,\pm1,\pm2} f_m\, d^J_{im}(\theta^*) d^J_{jm}(\theta^*)$

derived on CMS
15 years ago
in preparation
for Higgs discovery

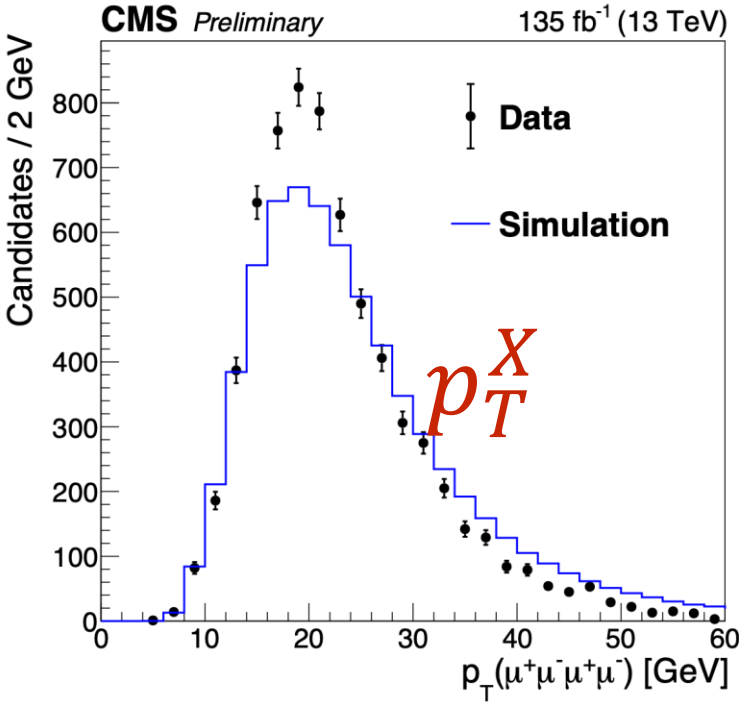
arXiv:1001.3396

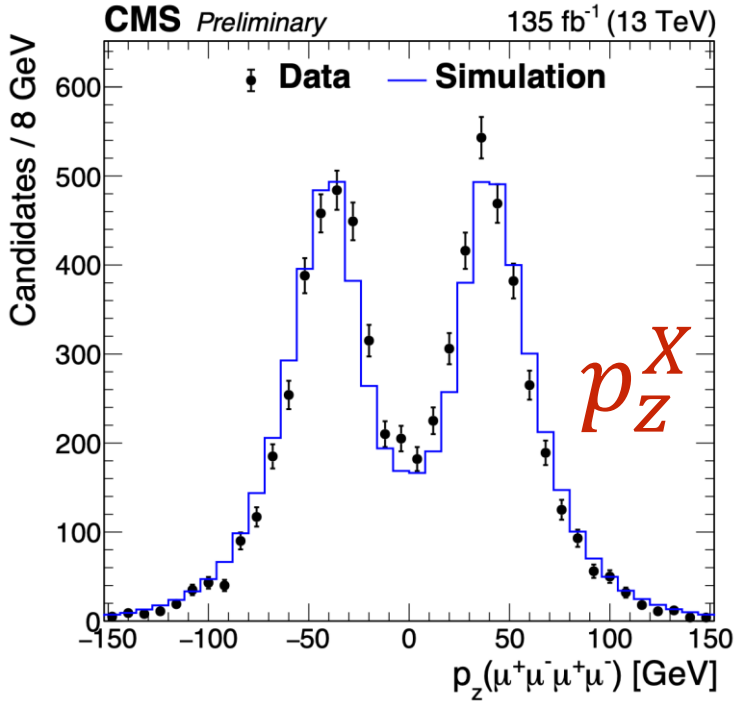
MELA

valid
for any J

MC simulation and tuning

- We do not know the production mechanism
  — empirical model to reproduce p_T^X and p_z^X in data





- tune **Pythia** to match p_T^X in sideband and signal region
- fine-tune re-weighting p_T^X
- residual p_T^X and p_z^X consistency tests coverage in systematics

Simulation and Fitting Tools

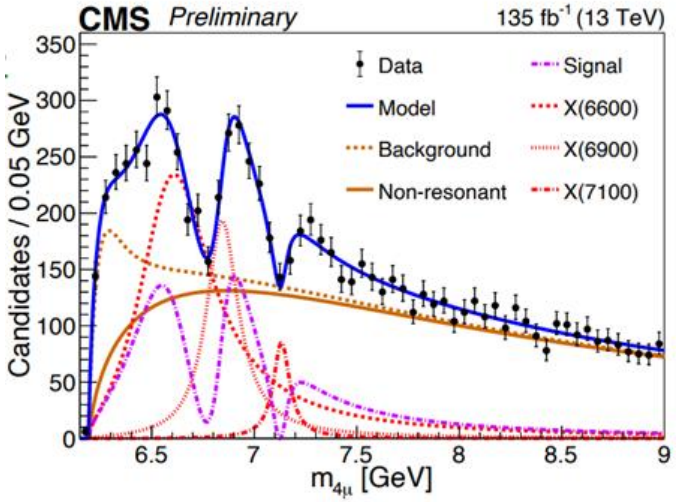
Pythia8: **NRSPS**—Non-Resonant Single Parton Scattering

$$gg(99\%) \rightarrow J/\psi J/\psi \rightarrow \mu^+\mu^-\mu^+\mu^-$$

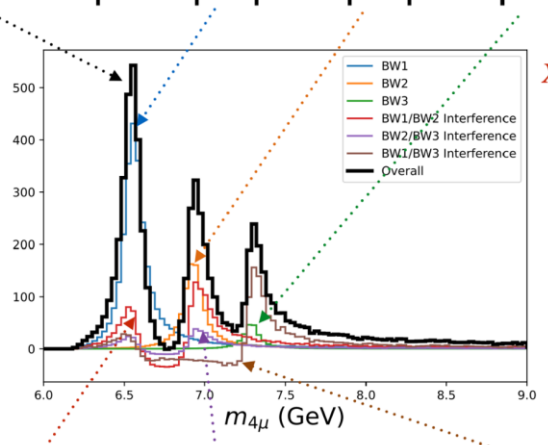
DPS—Non-Resonant Double Parton Scattering

JHUGen:

- $gg(q\bar{q}) \rightarrow X \rightarrow J/\psi\, J/\psi \rightarrow 4\mu$
  with different $J^P : 0^+, 0^-, 1^+, 1^-, 2^+, 2^-$
- Can model interference $\mathcal{P}_k(m_{4\mu})$
- Assume three resonances have same J^P
  - Dips observed in data —— explained by interference which indicates coherent production process and same J^P



$$|A+B+C|^2 = |A|^2 + |B|^2 + |C|^2$$

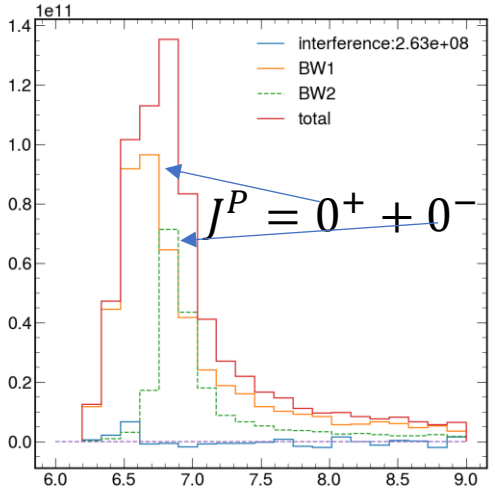


$$+2Re(AB^*) + 2Re(BC^*) + 2Re(CA^*)$$

$X + Y \rightarrow \psi\psi \rightarrow 4f$

$J^P = 0^+$

here $\varphi_i = 0$



- Interference between resonances with different quantum numbers integrates out to zero