



Measurement of the relative phase between strong and EM decays of charmonia

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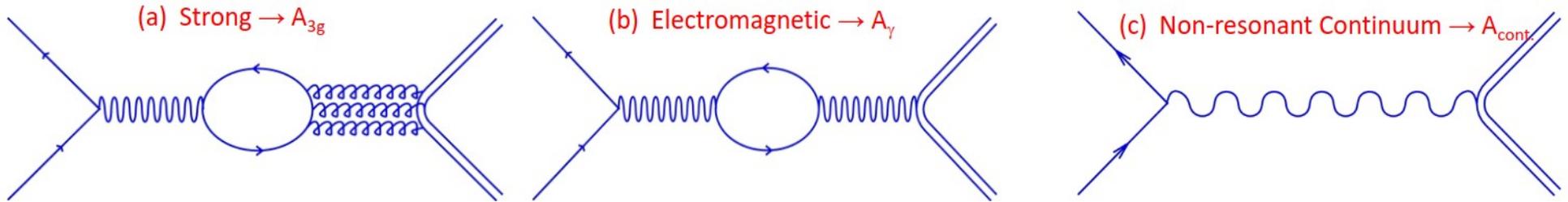
On behalf of the **BESIII** collaboration

Outline

- Introduction
- Some results
 - Around J/ψ resonance peak
 - $e^+e^- \rightarrow \mu^+\mu^-$
 - $e^+e^- \rightarrow 5\pi$
 - $e^+e^- \rightarrow \phi\eta$
 - Around $\psi(2S)$ resonance peak
 - $e^+e^- \rightarrow p\bar{p}\pi^0/p\bar{p}\eta$
 - Around $\psi(3770)$ resonance peak
 - $e^+e^- \rightarrow p\bar{p}$
 - $e^+e^- \rightarrow p\bar{p}\pi^0$
 - $e^+e^- \rightarrow K_S K_L$
- Outlook

A theoretical starting point

When studying the decay of charmonia at e^+e^- colliders, we observe the interplay between three possible diagrams



$$\sigma_{born} = |A_{3g}e^{i\Phi_{g,cont.}} + A_\gamma e^{i\Phi_{\gamma,cont.}} + A_{cont.}|^2$$

Given the quantum nature, they interfere. Two relative phases have to be introduced

The phase and the impact on the branching ratio

With increasing statistics for data sample at quarkonium peak mass, the statistical precision and systematics contribution will reach (or be better than) the percent level for the branching ratio measurements for many final states.

The interference term, and thus the relative phase, enters to modify the branching ratio.

$$\sigma_{\text{tot}}^f(s) = \sigma_c^f(s) + \frac{12\pi\mathcal{B}_{ee}\mathcal{B}_f}{M^2} + 2\frac{\sqrt{\sigma_c^f(s)}\sqrt{12\pi\mathcal{B}_{ee}\mathcal{B}_f}}{M}\sin\varphi,$$

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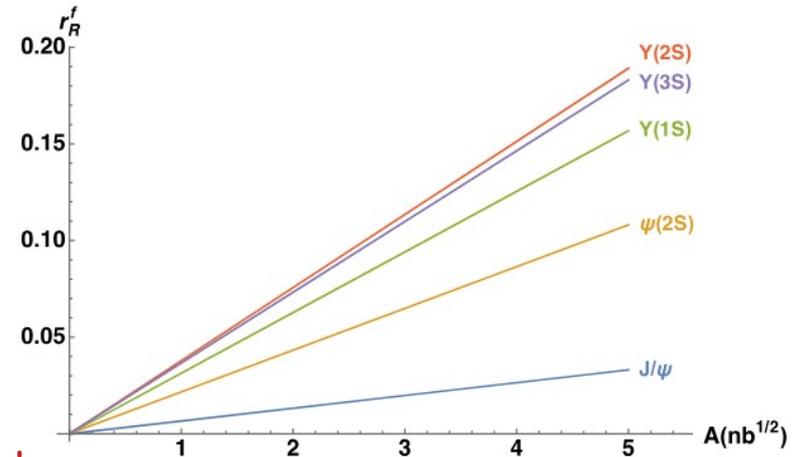
$$\sigma_{\text{tot}}^f(s) = \sigma_c^f(s) + \frac{12\pi\mathcal{B}_{ee}\mathcal{B}_f}{M^2} + 2\frac{\sqrt{\sigma_c^f(s)}\sqrt{12\pi\mathcal{B}_{ee}\mathcal{B}_f}}{M}\sin\varphi,$$

$$r_R^f \equiv \frac{\sigma_{\text{int}}^f(s)}{\sigma_R^f(s)} = \frac{2}{\hbar c} \sqrt{\frac{\sigma_c^f(s)}{\mathcal{B}_f}} \frac{M}{\sqrt{12\pi\mathcal{B}_{ee}}} \sin\varphi \equiv \frac{2}{\hbar c} AB \sin\varphi,$$

$$A = \sqrt{\sigma_c^f(s)/\mathcal{B}_f}$$

$$B = M/\sqrt{12\pi\mathcal{B}_{ee}}$$

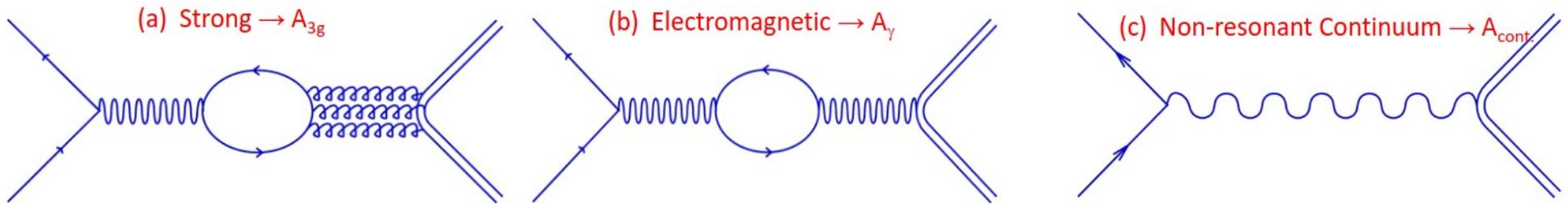
R	$\mathcal{B}[R \rightarrow \Lambda\bar{\Lambda}](10^{-4})$	$\sigma_c^{\Lambda\bar{\Lambda}}$ (nb)	$r_R^f \text{ max } (\%)$
J/ψ	$19.43 \pm 0.03 \pm 0.33$	1.22×10^{-2}	1.7
$\psi(2S)$	$3.97 \pm 0.02 \pm 0.12$	0.57×10^{-3}	2.6



By neglecting it, it may lead to an error comparable to the statistical one!

A theoretical starting point

When studying the decay of charmonia at e^+e^- colliders, we observe the interplay between three possible diagrams

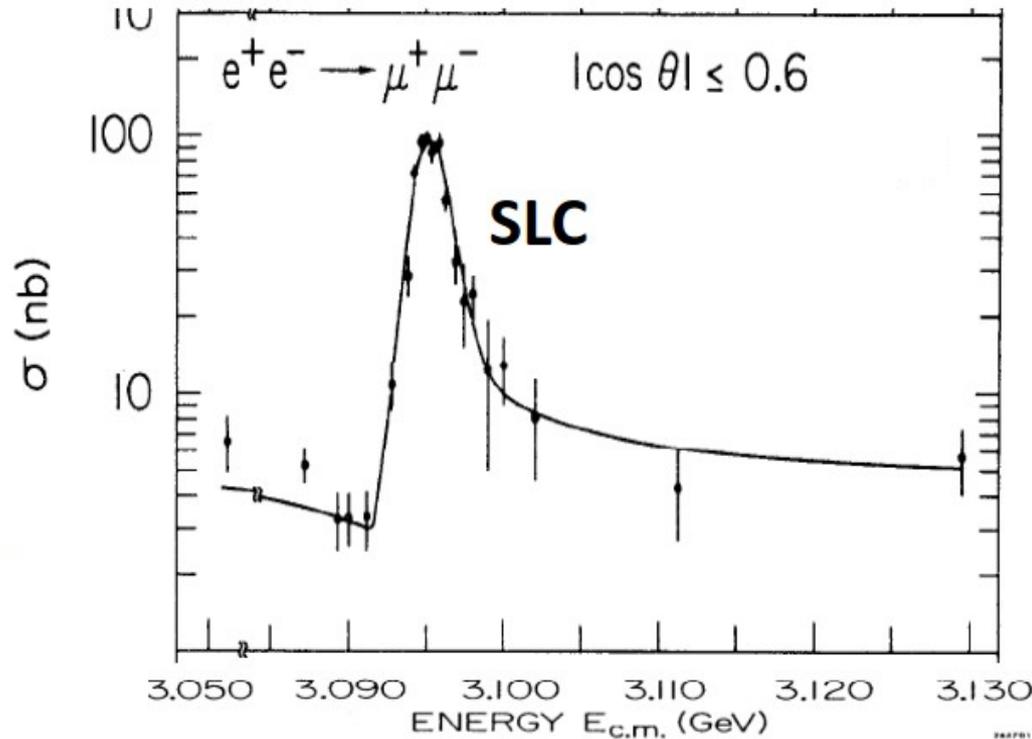


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Given the quantum nature, they interfere. Two relative phases have to be introduced.

How to extract them?

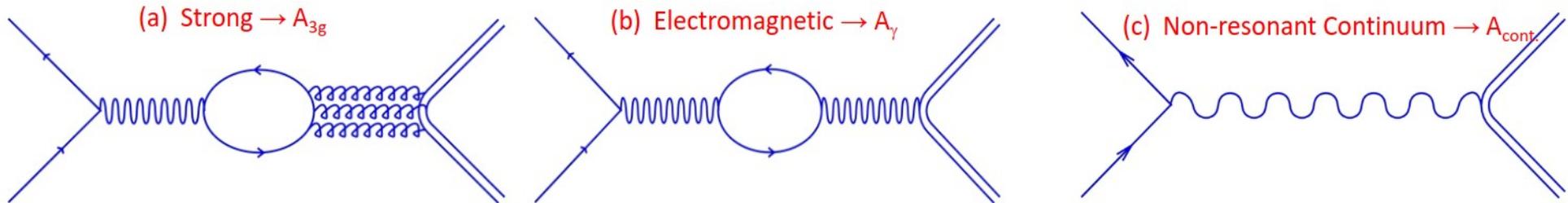
Isolating the first phase $\phi_{\gamma, \text{cont}}$



If we consider a **purely electromagnetic** process, like $e^+e^- \rightarrow \mu^+\mu^-$ we can access the **relative phase** between the **EM resonant** and **continuum** decay modes

The interference pattern has been observed long ago, indicating a relative phase close to 0° (compatible with the perturbative QED predictions)

A theoretical starting point - II



$$\sigma_{born} = |A_{3g} e^{i\Phi_{g,EM}} + A_{\gamma} + A_{cont.}|^2$$

We are then left with the phase between strong and EM resonant amplitudes ($\phi^{g,EM}$)

Following “naive” pQCD predictions: all amplitudes are real, $\phi^{g,EM}$ must be small (0° or 180°)

V.L. Chernyak and I.R. Zhinitsky, Nuclear Physics B 246, 52 (1998)

Description of the inclusive decay to hadrons leads to similar conclusions

J.-M. Gerard, J. Weyers, Phys. Lett. B 462, 324 (1999) ;

P. Wang, C.Z. Yuan, X.H. Mo, Phys. Rev. D 69, 057 502 (2004);

M. Suzuki, Phys. Rev. D 58, 111 504 (1998); etc.

Exclusive experimental measurements

$SU(3)_F$ and $SU(3)_F$ -**breaking** models can be used to study the phase using an “inclusive” list of exclusive decays extracted from **experimental data**

These models, tested for J/ψ to 0^-1^- , 0^-0^- , 1^-1^- , $B\bar{B}$ shows a **large phase**, compatible with **90°**

L. Köpke and N. Wermes, Phys. Rep. 174, 67 (1989)
 G. Lopez Castro, J. L. Lucio M. and J. Pestieau, hep-ph/9 902 300v1 (1999)
 M. Suzuki, PRD 57, 5717 (1998); PRD 58, 111 504 (1998);
 PRD 60, 051 501 (1999); PRD 63, 054 021 (2001);
 J. L. Rosner, Phys. Rev. D 60, 074 029 (1999)
 P. Wang, C.Z. Yuan, X.H. Mo, Phys. Rev. D 69, 057 502 (2004)
 R. Baldini et al, Physics Letters B 444, 111–118 (1998)
 K. Zhu et al., Int. J Mod. Phys. A30, 1 550 148 (2015)

Process	SOZI amplitude	DOZI correction
$J/\psi \rightarrow PV$		
$\rho^+ \pi^-, \rho^0 \pi^0, \rho^- \pi^+$	$g + e$	
$K^{*+} K^-, K^{*0} K^0$	$g(1 - s_g) + e$	
$K^{*0} \bar{K}^0, \bar{K}^{*0} K^0$	$g(1 - s_g) - 2e$	
$\omega \eta$	$(g + e)X_\eta$	$+ \sqrt{2}rg(\sqrt{2}X_\eta + Y_\eta)$
$\omega \eta'$	$(g + e)X_{\eta'}$	$+ \sqrt{2}rg(\sqrt{2}X_{\eta'} + Y_{\eta'})$
$\phi \eta$	$[g(1 - 2s_g) - 2e]Y_\eta$	$+ rg(\sqrt{2}X_\eta + Y_\eta)$
$\phi \eta'$	$[g(1 - 2s_g) - 2e]Y_{\eta'}$	$+ rg(\sqrt{2}X_{\eta'} + Y_{\eta'})$
$\rho \eta$	$3eX_\eta$	
$\rho \eta'$	$3eX_{\eta'}$	
$\omega \pi^0$	$3e$	
$\phi \pi^0$	0	

An example

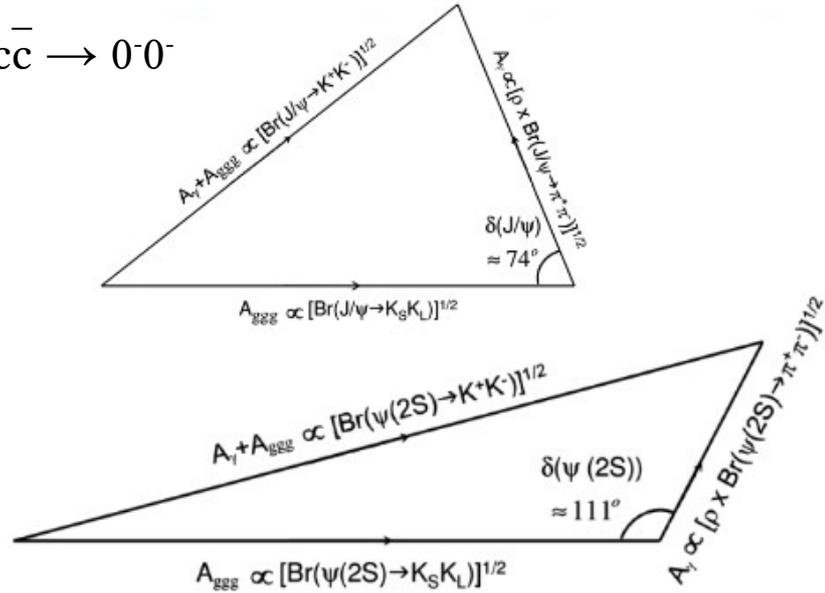
$$g - A_{3g}; e - A_\gamma$$

$X_\eta, Y_\eta, s_g - SU(3)$ breaking items

Other experimental results

Phys.Rev.D 85 (2012) 092 007

$\bar{c}c \rightarrow 0^-0^-$



Large phase for both J/ψ and $\psi(2S)$

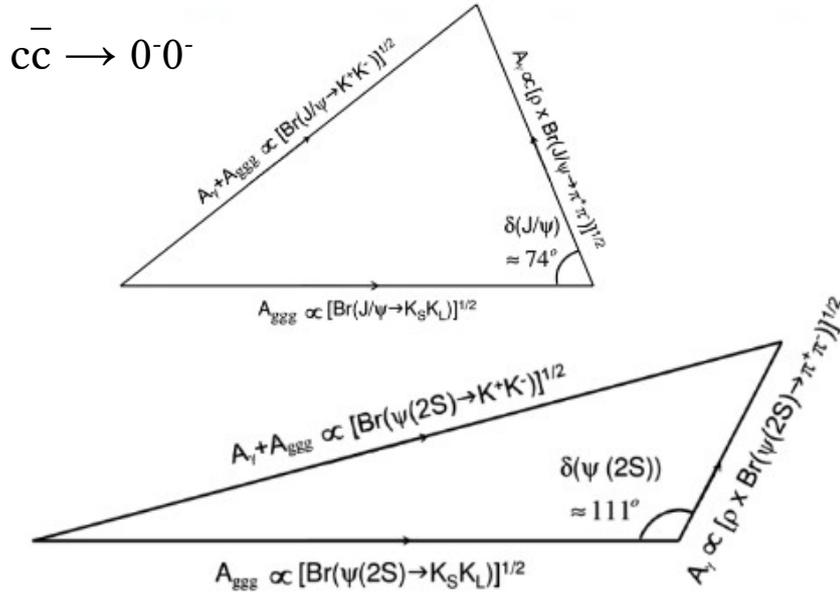
Using PDG 2025:

$\delta(J/\psi) \sim 87^\circ$

$\delta(\psi(2S)) \sim 112^\circ$

Other experimental results

Phys.Rev.D 85 (2012) 092 007



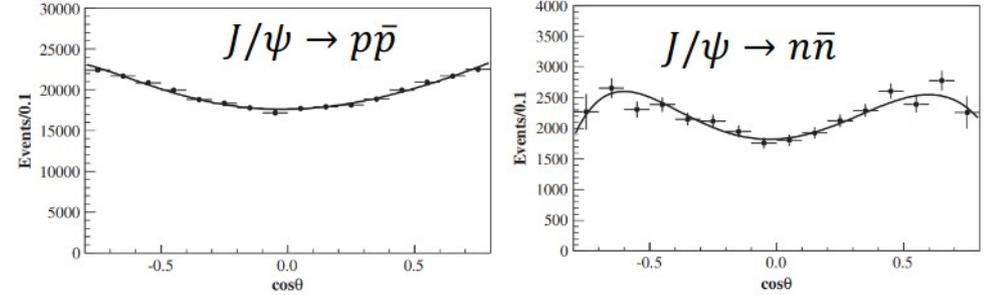
Large phase for both J/ψ and ψ(2S)

Using PDG 2025:

$\delta(J/\psi) \sim 87^\circ$

$\delta(\psi(2S)) \sim 112^\circ$

Phys. Rev. D 86, 032 014 (2012)



$$\phi = \cos^{-1}[(\mathcal{B}(J/\psi \rightarrow p\bar{p}) - S^2 - E_p^2)/(2SE_p)]$$

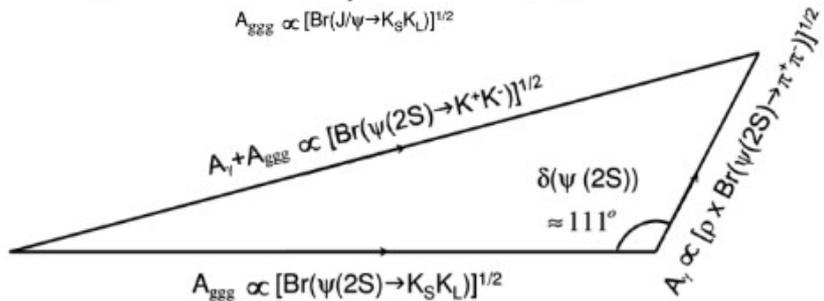
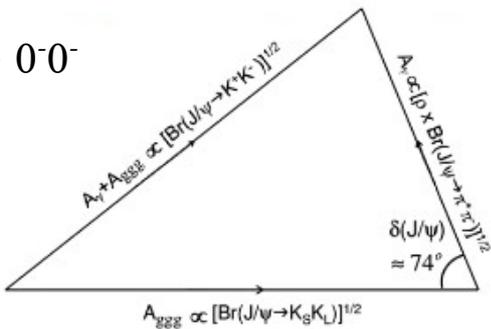
$$= (88.7 \pm 8.1)^\circ.$$

Assuming proton and neutron have equal strong amplitude (S) and opposite electromagnetic one (E)

Other experimental results

Phys.Rev.D 85 (2012) 092 007

$\bar{c}c \rightarrow 0^-0^-$



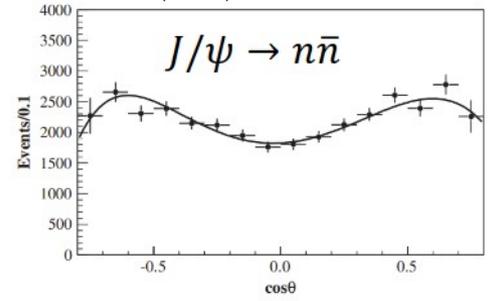
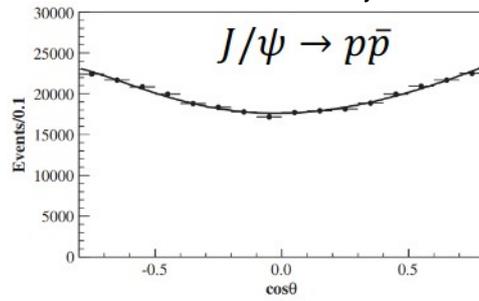
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Phys. Rev. D 86, 032 014 (2012)



$$\phi = \cos^{-1}[(\mathcal{B}(J/\psi \rightarrow p\bar{p}) - S^2 - E_p^2)/(2SE_p)] = (88.7 \pm 8.1)^\circ.$$

Assuming proton and neutron have equal strong amplitude (S) and opposite electromagnetic one (E)

And more for $\psi(2S)$ and $\psi(3770)$:

- small phase for $\psi(2S) \rightarrow 1^+0^-$ (Phys. Rev. D 63 (2000) 054 021)
- large phase for $\psi(2S) \rightarrow K\bar{K}$ (Phys. Rev. Lett. 92, (2004) 052 001)
- large phase in $\psi(2S) \rightarrow B\bar{B}$ (Inter. J. Mod. Phys. A, 30 (2015), 1 550 148)
- -90° phase in $\psi(3770)$ decays (ArXiv: 0 410 028v2 (2004))

Scan of the resonance lineshape

All previous extraction depends on some models about $SU(3)_F$ and $SU(3)_F$ -breaking amplitudes

Ideally, it would be better to determine such phase independently from those assumptions.

BESIII can study the behaviour of the cross section and directly extract the phase, owing to the possibility to scan the resonance lineshape given by BEPCII accelerator,

$$\sigma_{born} = |A_{3g} e^{i\Phi_{g,EM}} + A_{\gamma} + A_{cont.}|^2$$

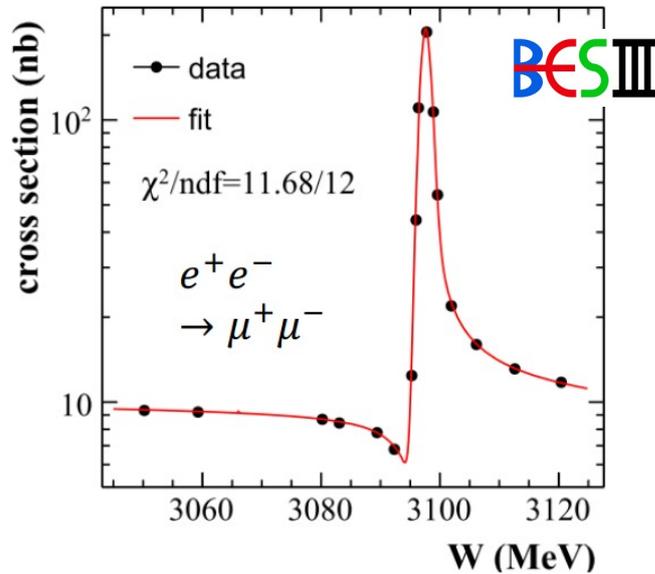
BESIII collected several dedicated data around J/ψ [50/pb], $\psi(2S)$ [500/pb], $\psi(3770)$ [60/pb], and resonance fast scan data used for beam calibration.

Crucial the use of Beam Energy Measurement System for precise energy measurement

BESIII scan measurements

- Around J/ψ resonance peak
 - $e^+e^- \rightarrow \mu^+\mu^-$
 - $e^+e^- \rightarrow 5\pi$
 - $e^+e^- \rightarrow \phi\eta$
- Around $\psi(2S)$ resonance peak
 - $e^+e^- \rightarrow p\bar{p}\pi^0/p\bar{p}\eta$
- Around $\psi(3770)$ resonance peak
 - $e^+e^- \rightarrow p\bar{p}$
 - $e^+e^- \rightarrow p\bar{p}\pi^0$
 - $e^+e^- \rightarrow K_S K_L$

$e^+e^- \rightarrow \mu^+\mu^-$ around J/ψ peak



Re-extracting the phase $\phi^{\gamma, \text{cont}}$ via a well-known $e^+e^- \rightarrow \mu^+\mu^-$ allows:

- to verify the fit model
- check for any additional global energy shift in data

In the fit, fix J/ψ mass and width to PDG

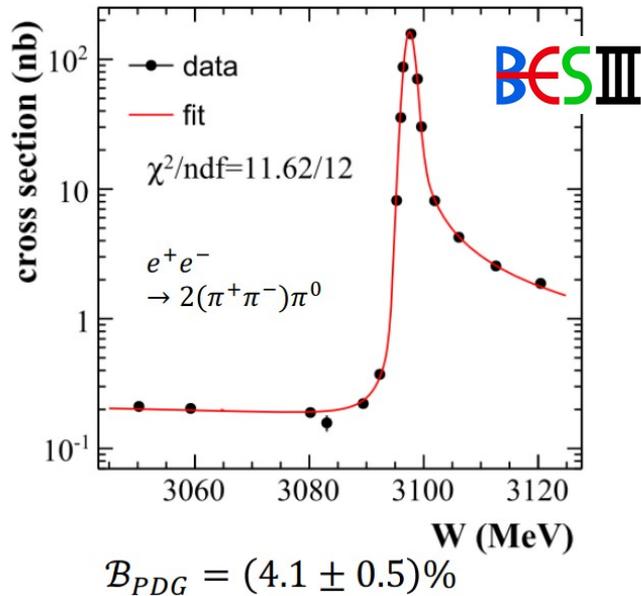
$$\sigma^0(W) = \frac{4\pi\alpha^2}{W^2} \left| 1 + \frac{3W^2 \sqrt{\Gamma_{ee}\Gamma_{\mu\mu}} e^{i\Phi_{\gamma, \text{cont}}}}{\alpha M(W^2 - M^2 + iM\Gamma)} \right|^2$$

Phase is found compatible with 0°

- $\Phi_{\gamma, \text{cont.}} = (3.0 \pm 10.0)^\circ$
- $S_E = (0.90 \pm 0.03) \text{ MeV}$

$e^+e^- \rightarrow 2(\pi^+\pi^-)\pi^0$ around J/ψ peak

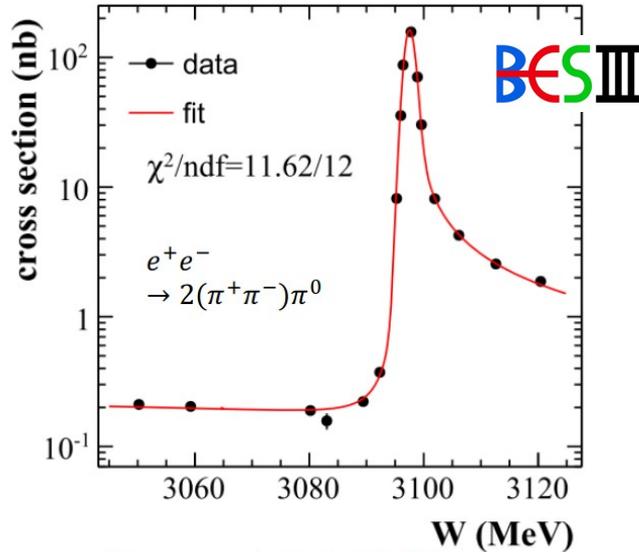
The 5π final state is useful since it has both the case with **strong and EM amplitude**



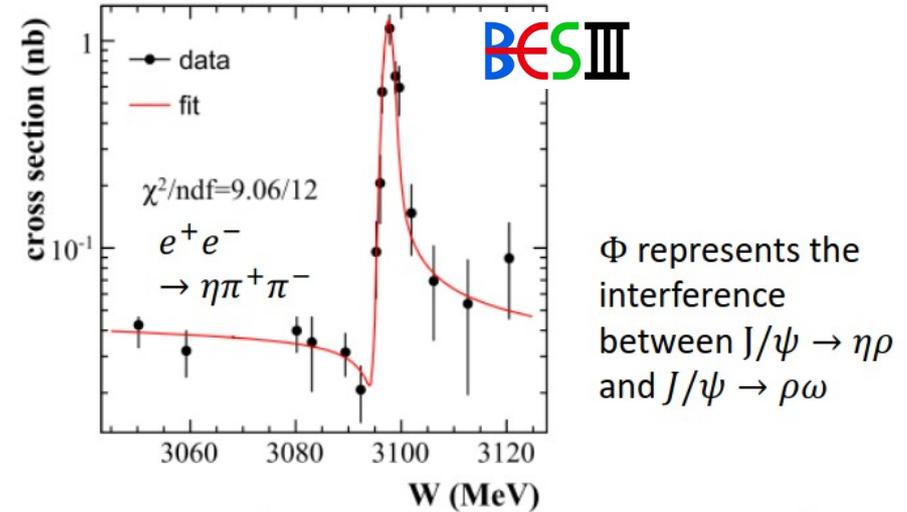
	$\Phi_{g,EM}$	$\mathcal{B}_{5\pi}$ (%)
Solution I	$(84.9 \pm 3.6)^\circ$	4.73 ± 0.44
Solution II	$(-84.7 \pm 3.1)^\circ$	4.85 ± 0.45

$e^+e^- \rightarrow 2(\pi^+\pi^-)\pi^0$ around J/ψ peak

The 5π final state is useful since it has both the case with strong and EM amplitude, but also an **intermediate process purely EM**



$$\mathcal{B}_{PDG} = (4.1 \pm 0.5)\%$$



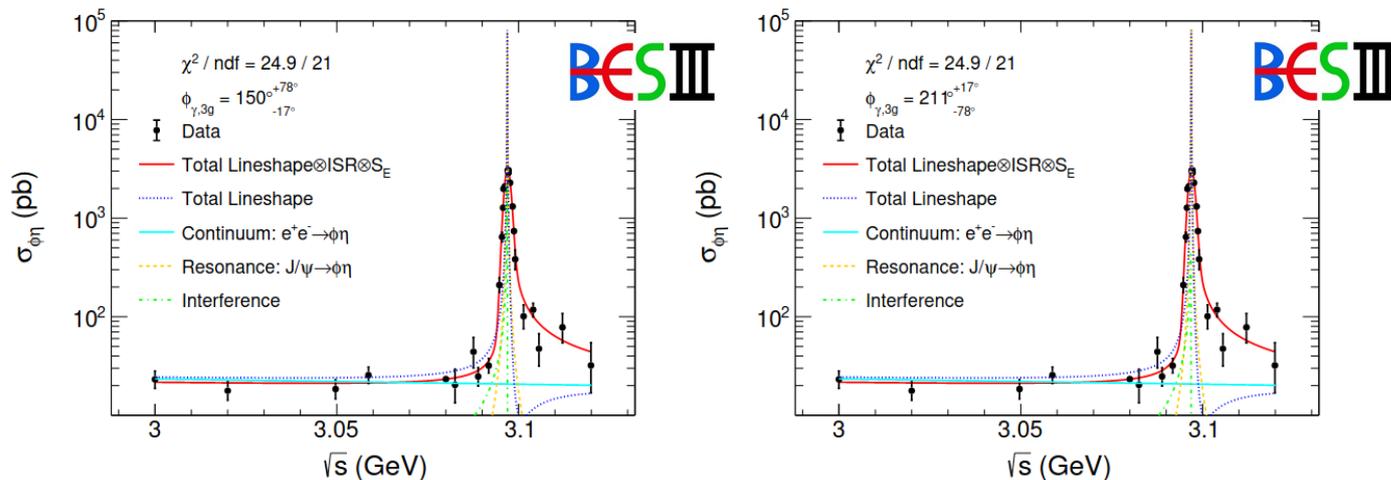
$$\sigma^0(W) = \left(\frac{\mathcal{A}}{W^2} \right)^2 \frac{4\pi\alpha^2}{W^2} \left| 1 + \frac{3W^2 \sqrt{\Gamma_{ee}\Gamma_{\mu\mu}} C_1 e^{i\Phi_{\gamma,cont}} (1 + C_2 e^{i\Phi})}{\alpha M(W^2 - M^2 + iM\Gamma)} \right|^2$$

- $\Phi_{\gamma,cont.} = (-2 \pm 36)^\circ$ or $(-22 \pm 36)^\circ$
- $Br(J/\psi \rightarrow \eta\pi^+\pi^-) = (3.78 \pm 0.66) \times 10^{-4}$
- $Br_{PDG}(J/\psi \rightarrow \eta\pi^+\pi^-) = (4.0 \pm 1.7) \times 10^{-4}$

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$e^+e^- \rightarrow \phi\eta$ around J/ψ peak

First **exclusive** measurement of a $J/\psi \rightarrow 0^-1^-$ final state (relevant for $\rho\pi$ puzzle)

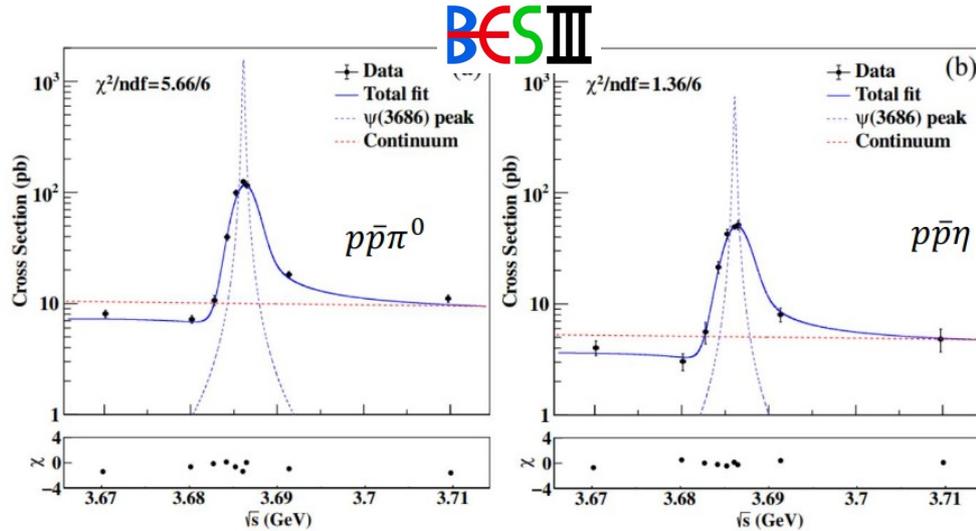


	Positive phase	Negative phase
χ^2/ndf	24.9/21	24.9/21
$\phi_{\gamma,3g}$ ($^\circ$)	150^{+78}_{-17}	211^{+17}_{-78}
\mathcal{F}		0.11 ± 0.01
C		3.3 ± 0.4
S_E (MeV)		0.88 ± 0.03
f		0.99 ± 0.04

Two solutions indistinguishable in terms of fit quality

**Final results for the $\phi^{\text{g,EM}}$ is $[133^\circ, 228^\circ]$
(1σ range)**

$e^+e^- \rightarrow p\bar{p}\pi^0/p\bar{p}\eta$ around $\psi(2S)$ peak



$$\sigma_{\text{Born}} = |A_{\text{con}} + A_{\text{res}} \times e^{i\phi}|^2$$

$$A_{\text{con}}(s) = a/s^n$$

$$A_{\text{res}}(s) = \frac{\sqrt{12\pi\Gamma_{ee}\Gamma_{\text{tot}}\mathcal{B}_f}}{s - M^2 + iM\Gamma_{\text{tot}}}$$

A partial wave is needed to take into account the possible intermediate states.

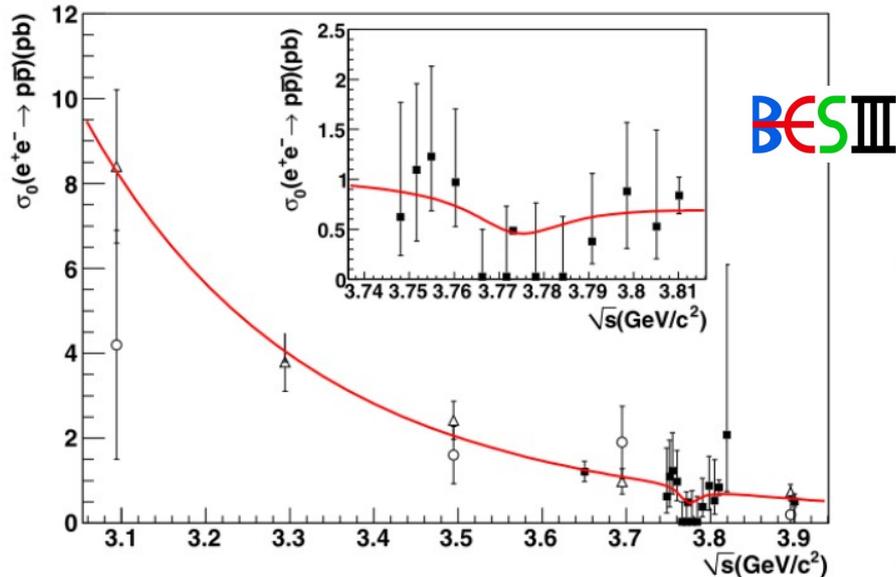
$\psi(3686) \rightarrow p\bar{p}\pi^0$	$\mathcal{B}_f\Gamma_{ee}$ (0.1 eV)	ϕ ($^\circ$)	ΔE (MeV)	\mathcal{B}_f ($\times 10^{-6}$)
Constructive solution	3.12 ± 0.26	65.0 ± 6.7	1.27 ± 0.09	$133.9 \pm 11.2 \pm 2.3$
Destructive solution	4.28 ± 0.32	-68.9 ± 5.7	1.27 ± 0.09	$183.7 \pm 13.7 \pm 3.2$
$\psi(3686) \rightarrow p\bar{p}\eta$	$\mathcal{B}_f\Gamma_{ee}$ (0.1 eV)	ϕ ($^\circ$)	ΔE (MeV)	\mathcal{B}_f ($\times 10^{-6}$)
Constructive solution	1.44 ± 0.15	58.9 ± 14.1	1.39 ± 0.14	$61.5 \pm 6.5 \pm 1.1$
Destructive solution	1.98 ± 0.16	-63.8 ± 12.1	1.39 ± 0.14	$84.4 \pm 6.9 \pm 1.4$

$$\text{BR}_{\text{PDG}} = (153 \pm 7) 10^{-6}$$

$$\text{BR}_{\text{PDG}} = (60 \pm 4) 10^{-6}$$

Large phase. Branching ratio with and without (PDG) interference effect can differ

$e^+e^- \rightarrow p\bar{p}$ around $\psi(3770)$ peak



$$\begin{aligned}\sigma(s) &= |A_{con} + A_{\psi} e^{i\phi}|^2 \\ &= \left| \sqrt{\sigma_{con}(s)} + \sqrt{\sigma_{\psi}} \frac{m_{\psi} \Gamma_{\psi}}{s - m_{\psi}^2 + im_{\psi} \Gamma_{\psi}} e^{i\phi} \right|^2\end{aligned}$$

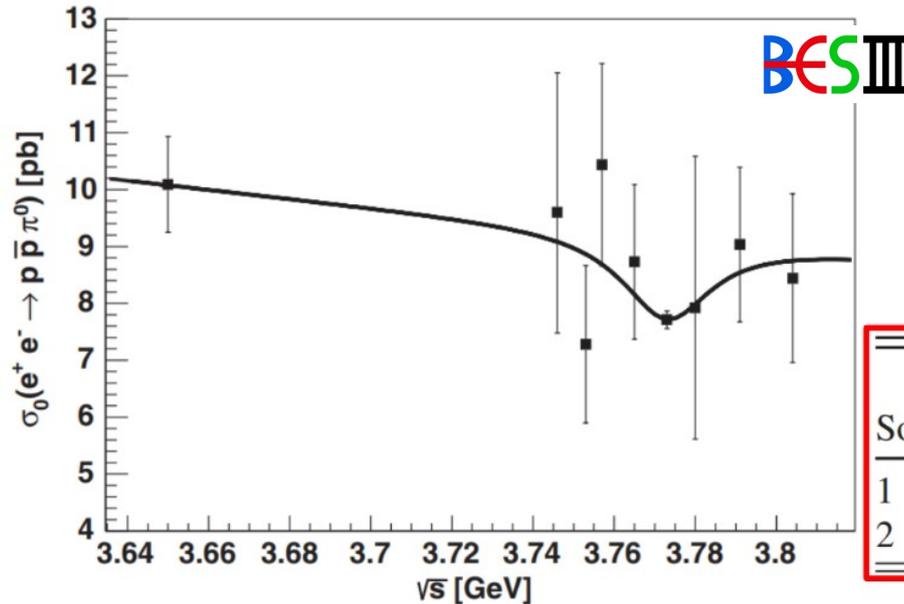
$\sigma_{(\psi(3770) \rightarrow p\bar{p})}^{dressed}$ (pb)	$Br(\times 10^{-4})$	$\phi(^{\circ})$
$0.059^{+0.070}_{-0.020} \pm 0.012$ (< 0.166 at 90% C.L.)	$7.1^{+8.6}_{-2.9}$	$255.8^{+39.0}_{-26.6} \pm 4.8$
$2.57^{+0.12}_{-0.13} \pm 0.12$	3.1 ± 0.3	$266.9^{+6.1}_{-6.3} \pm 0.9$

Old result, to be updated with also data at J/ψ and $\psi(2S)$ peak, in preparation.

Assuming a large A_{3g} wrt A_{γ} , **phase** is compatible with **90°**

Useful input for $p\bar{p}$ collider

$e^+e^- \rightarrow p\bar{p}\pi^0$ around $\psi(3770)$ peak



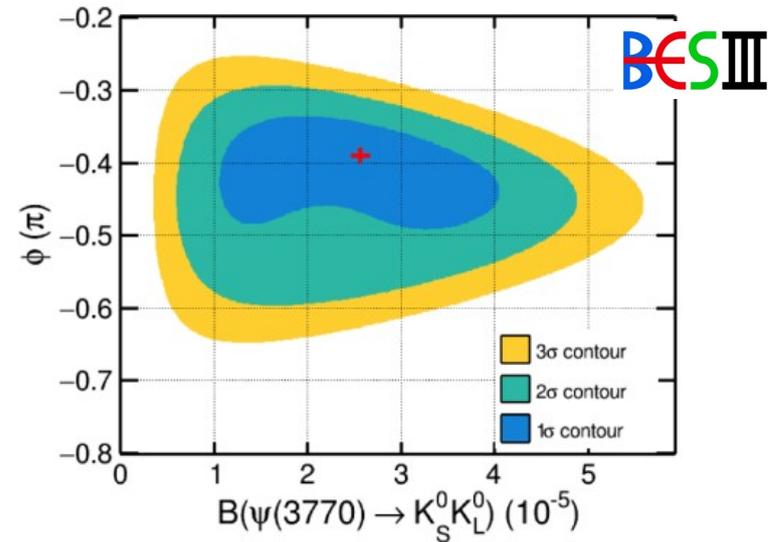
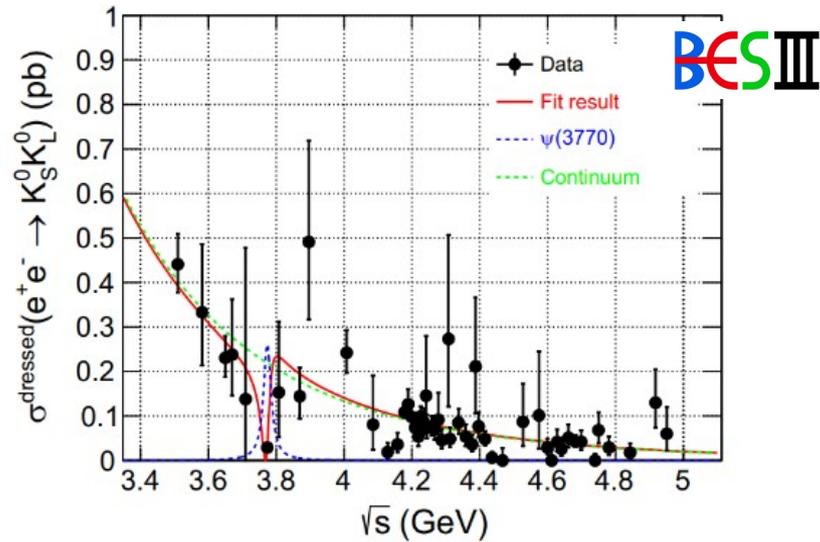
$$\sigma(s) = \left| \sqrt{\sigma_{\text{con}}} + \sqrt{\sigma_{\psi}} \frac{m\Gamma}{s - m^2 + im\Gamma} \exp(i\phi) \right|^2$$

Solution	$\sigma_0^{\psi(3770) \rightarrow p\bar{p}\pi^0}$ [pb]	ϕ_{Fit} [°]	$\sigma_0^{p\bar{p} \rightarrow \psi(3770)\pi^0}$ at 5.26 GeV [nb]
1	< 0.22	$269.8^{+52.4}_{-48.0} \pm 11.0$	< 0.79
2	$33.8 \pm 1.8 \pm 2.1$	$269.7 \pm 2.3 \pm 0.3$	122 ± 10

Old result, the result at $\psi(2S)$ peak just presented

Assuming a large A_{3g} wrt A_{γ} , **phase** is still compatible with **90°**, with large uncertainties

$e^+e^- \rightarrow K_S K_L$ around $\psi(3770)$ peak



Observation (10σ) of the rare decay $\psi(3770) \rightarrow K_S K_L$ via the scan method.

Extracted branching ratio and the phase between the continuum and the resonance.

$$\mathcal{B} = (2.63_{-1.59}^{+1.40}) \times 10^{-5} \text{ and } \phi = (-0.39_{-0.10}^{+0.05})\pi \rightarrow (-22_{-6}^{+3})^\circ$$

Summary and Outlook

- Study of charmonium decays into hadrons allow to provide information on **fundamental quantum nature**
 - pQCD prescriptions and $SU(3)_F$ models provide different results for the phase
 - A **model independent approach can be more universal!**
- **BESIII** can play a unique role owing to dedicated samples obtained by the high luminosity BEPCII
 - Few results presented, more to come with the J/ψ and $\psi(2S)$ samples
 - Dedicated working group inside the collaboration
- **Relative phase between EM and strong components is universal?**
 - Or Is there any dependence on the final state hadrons?
- High precision measurements of charmonium decays will require the **precise knowledge of the interference** to get unbiased **branching ratio results!**